SSMS: A Split Step MultiBand Simulation Software

P. Khare[1*], N. Costa[2], J. Pedro[2,3], A. Napoli[4], F. Arpanaei[5], J. Comellas[1], M. Ruiz[1], and L. Velasco[1]

[1] *Optical Communications Group (GCO), Universitat Politècnica de Catalunya (UPC), Barcelona, Spain*

[2] *Infinera Unipessoal Lda., Carnaxide, Portugal,* [3]*Instituto de Telecomunicações, Instituto Superior Técnico, Lisboa, Portugal,* [4] *Infinera, Munich, Germany,* [5]*Universidad Carlos III de Madrid, Spain*

**e-mail: prasunika.khare@upc.edu*

ABSTRACT

We introduce SSMS, a multiband optical fiber simulator entirely developed in MATLAB. SSMS solves the generalized nonlinear Schrödinger equation relying on the 4th order Runge-Kutta method in Interaction Picture (RK4IP) with adaptive step size approach and compare it with the widely used split-step Fourier method (SSFM). The simulator is validated considering S+C+L multiband transmission. Results show that the RK4IP method is approximately 10× faster than the traditional SSFM model for a similar level of accuracy.

Keywords: multiband optical transmission, multiband simulation.

1. INTRODUCTION

5G and beyond networks are drastically increasing the capacity requirement of the optical transport network. Several solutions have been proposed to cope with this requirement, such as: *i*) spatial division multiplexing (SDM) with Multi-core/multi-fiber systems; and *ii*) multiband (MB) fiber transmission [1]-[2]. The SDM technique consists in increasing the number of used fibers/cores accordingly to the capacity requirement, whereas MB transmission uses additional optical transmission bands of existing optical fibers. Since MB leverages the already existing optical fiber infrastructure, it can be considered as an interesting solution, at least in the short term, if the key enabling optical components become cost-effective and mature in a timely manner.

In online network provisioning, accurate estimation of quality of transmission is a crucial task for the control plane. In the frequency domain, there have been several proposed closed-form tools, such as those studied in specific scenarios [3], but these tools may not be applicable for all network planning scenarios. Alternatively, the GNPy tool utilizes the Generalized Gaussian Noise (GGN) model [4], which takes an integral-based approach and provides higher accuracy than the closed-form formulas, at the expenses of being more complex than the close-form of [3]. On the other hand, the original generalized nonlinear Schrödinger equation (GNLSE) solved using the split-step Fourier method (SSFM) has higher computational complexity than GGN, but with the benefit of achieving even higher accuracy compared to other methods, and provides results in the frequency-time domain. Therefore, various faster SSFM-based approaches were studied; one of them is the 4th order Runge-Kutta in Interaction Picture (RK4IP) method with adaptive step-size. The RK4IP method was originally created for the study of Bose-Einstein condensates and it gained popularity after being used in supercontinuum simulation [5]. In this work, we implement the RK4IP method with adaptive step-size to solve the GNLSE considering a MB system and propose a split-step multiband simulator (SSMS) software, which has been developed in MATLAB.

	L Band	C Band	S Band
Freq. range (THz)	185.4 – 191.5	191.5 – 195.9	195.9 – 202.2
Num of Channels	101	75	105

Figure 1. Illustration of ITU-T band definition for SSMF [1].

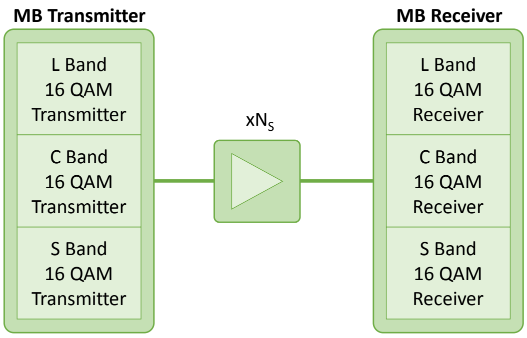

Figure 2. Illustration of the considered MB system.

Table 1. Summary of parameters.

Parameters	Values
Span Length	80 km
Input Power	-1 dBm
No of bits	2^{10}
Non-Linear Coefficient γ	1.3 1/W/km
Symbol Rate	32 Gbaud

2. C+L+S BAND SIMULATION SETUP

In this work, we consider a C+L+S MB transmission system. The frequency range and number of channels considered are presented in Fig. 1. The optical line system is loaded with a total of 281 channels with 60 GHz channel spacing. An illustration of the multiband setup is shown in Fig. 2. Three 32-Gbaud 16QAM transmitters are used to generate a wavelength-division multiplexing (WDM) comb, which includes L, C and S frequency bands. The raised cosine filter has a roll off factor of 0.06. The remaining parameters used in the simulation setup are shown in Table 1. The propagation of the resulting WDM optical signal travelling through optical fiber is governed by the GNLSE equation.

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2.1 Generalized Non-Linear Schrödinger equation (GNLSE)

The accurate modeling of optical signals propagation in optical fiber should include linear, as well as non-linear transmission effects. The linear effects considered are chromatic dispersion and fiber attenuation. An optical fiber chromatic dispersion parameter of 16.7 dB/km and attenuation of 0.19 dB/km are considered at 1550 nm. The non-linear Kerr-effects included in this work are self-phase modulation (SPM) and cross-phase modulation (XPM).

We also consider energy transferred from incident photons to the vibrational modes, which results in the generation of new photons with different frequencies that originates the stimulated Raman scattering (SRS), and self-steepening effect. The Non-Linear Schrödinger equation can be written in time-domain as [5], [6]:

$$\frac{dA(z,t)}{dz}+\beta_1\frac{dA(z,t)}{dt}+i\beta_2\frac{d^2A(z,t)}{dt^2}-\beta_3\frac{d^3A(z,t)}{dt^3}=i\gamma\left(1+\frac{i}{\omega_0}\frac{d}{dt}\right)\left(A(z,t)\int R(t')\mathrm{A}(\mathrm{z},(\mathrm{t}-t'))^2dt'\right) \tag{1}$$

where ω_0 is the carrier frequency, β_k are the coefficient representing the different order of dispersion, *A(z,t)* is the field of the optical signal at propagation distance z, and $R(t)$ is the non-linear response function, which is given by $R(t)=(1-f_R)\delta(t)+f_R h_R(t)$ with $h_R(t)=(f_a+f_c)h_a(t)+f_b h_b(t)$, where $h_a(t)=\tau_1(\tau_1^{-2}+\tau_2^{-2})e^{-(t/\tau_2)}\sin(t/\tau_1)$, $h_b(t)=[(2\tau_b-t)/\tau_b^2]e^{(t/\tau_b)}$, h_R is the Raman response function, f_R is the peak Raman gain, and $\tau_1$and τ_2 are adjusting parameters for the Raman response function.

In line with [5], in this work, we consider $\tau_1=12.2\,f_s\,[ps]$, $\tau_2=32\,f_s\,[ps], \tau_b=96\,f_s\,[ps], where\ f_s=10^{-3}, f_a=0.75, f_b=\ 0.21, f_c\ =0.04$, and $f_R=0.18$.

In equation (1), we can observe that the optical signal amplitude depends on time, as well as with transmission distance. Thus, besides the numerical error resulting from the selected step size, additional numerical error origins from this time derivative, which also depends on the step size. This issue can be circumvented by writing eq. (1) in the frequency domain as:

$$\frac{d\tilde{A}}{dz}+i\tilde{A}(\beta(\Omega)-\beta_0-\beta_1\Omega)=-i\gamma\left(1+\frac{\Omega}{\omega_0}\right)F\left[(1-f_R)A|A|^2+f_R A F^{-1}\left[\widetilde{h_R}\,F[|A|^2]\right]\right] \tag{2}$$

where $\tilde{A}(z,\Omega)=FT(A(z,t))$ and $\widetilde{h_R}=FT(h_R(t)$, where FT denotes the Fourier transform operator. Instead of using the traditional SSFM approach to solve equations (1) and (2), they can be solved much faster by using the RK4IP integration method [4].

2.2 Fourth Order Runge-Kutta in Interaction Picture (RK4IP) Method

The Interaction Picture term originated from quantum mechanics [5]. When combined with the RK4 method, it results in the RK4IP method, which has been recently used in supercontinuum generation in optics [7]. The RK4IP method can be expressed in time domain as:

$$A(z+h,t)=F^{-1}\left[exp(\frac{h}{2}\widetilde{D})F[A_1+K_1/6+K_2/3+K_3/3]\right]+K_4/6$$
$$A_1=F^{-1}\left[exp(\frac{h}{2}\widetilde{D})\tilde{A}(z,\Omega)\right] \qquad K_1=F^{-1}\left[exp(\frac{h}{2}\widetilde{D})F[hN(A(z,t))]\right] \tag{3}$$
$$K_2=\ hN\left(A_1+\frac{K_1}{2}\right), K_3=hN(A_1+K_2/2) \qquad K_4=F^{-1}\left[hN(exp\left(\frac{h}{2}\widetilde{D}\right)F[A_1+K_3])\right]$$

where h is the step size, $\widetilde{D}$ is the dispersion operator, K is the RK method coefficient, and $N(A(z,t))$ is the nonlinear operator. The RK4IP method can also be applied in frequency domain. By defining the optical signal amplitude as $\tilde{A}$ and nonlinear operator as $\widetilde{N}\left(\tilde{A}(z,t)\right)$ the frequency domain expression is:

$$\tilde{A}(z+h,t)=exp(\frac{h}{2}\widetilde{D})(\tilde{A}_1+K_1/6+K_2/3+K_3/3)+K_4/6$$
$$\tilde{A}_1=F^{-1}\left[exp(\frac{h}{2}\widetilde{D})\tilde{A}(z,\Omega)\right], K_1=exp\left(\frac{h}{2}\widetilde{D}\right)h\widetilde{N}\left(\tilde{A}(z,t)\right) \tag{4}$$
$$K_2=h\widetilde{N}\left(\tilde{A}_1+\frac{K_1}{2}\right), K_3=h\widetilde{N}\left(\tilde{A}+\frac{K_2}{2}\right),\ K_4=h\widetilde{N}(exp\left(\frac{h}{2}\widetilde{D}\right)(\tilde{A}_1+K_3))$$

The implementation of RK4IP method in frequency and time domain will be discussed in the results section. An adaptive step size algorithm is also included to further reduce the computation time required by the RK4IP method. A brief description of the implemented adaptive step size algorithm is given in the next section.

2.3 Adaptive Step Size Algorithm for MB WDM signal propagation

The adaptive step size algorithm for MB WDM signal propagation proposed in this work is described in this section. The first step of Algorithm 1 computes of the signal waveform error. We have considered two different approaches to estimate this error at each step and, consequently, update the step size, which is described in step 2 of Algorithm 1. The global error/ tolerance limit (δ_G) considered in this work is 10^{-4} and η is set to 5.

The two considered error estimation approaches are the Local Error Method (LEM) and the Conservative Quality Error Method (CQEM).

Algorithm 1. Adaptive Step Size for MB WDM signal propagation.

Input: *Step size (dz), global error/tolerance (δ_G), MB WDM signal coarse solution u_c, MB WDM signal fine solution (u_f),*
Output: *Step size$_{new}$*

Step1: *Error Calculation Method select*
Case1: *Local Error Method: Error* $(\boldsymbol{\delta}) = \frac{\|u_f - u_c\|}{\|u_f\|}$
Case2: *Conservative Quality Error Method*
Error $(\boldsymbol{\delta}) = \left| \int \left(\left| \tilde{A}_{cal}(z+h,\omega) \right|^2 - \left| \tilde{A}_{cal}(z,\omega) \right|^2 \right) \frac{S(\omega)}{\omega} d\omega \right|,$
where $\frac{S(\omega)}{\omega}$ *is a scaling factor*
Step2: *Step update*
if *(δ < (δ_G/2))* ***then return*** *step size*(2^η);*
if *(δ_G <= δ <= (2*δ_G))* ***then return*** *step size/(2^η);*
if *δ > (2*δ_G)* ***then return*** *step size/2;*
return *step size;*

Local Error Method (LEM):

The LEM approach estimates the step size at each iteration by using a coarse solution (u_c) and a fine solution (u_f) [7]. To calculate the local error (δ) at each step, two independent half steps are used to obtain the finer solution and one full step is used to obtain the coarse solution.

Conservative Quality Error Method (CQEM)

When using the CQEM approach, the error is calculated by using the photon number before and after each iteration [8]. In this method the error is calculated by using the current iteration solution $\tilde{A}_{cal}(z+h,\omega)$ and the previous iteration solution $\tilde{A}_{cal}(z,\omega)$.

3. RESULTS AND DISCUSSION

The SSMS software proposed in this work is used in this section to solve the GNLSE. The presented simulations are performed solely using the CPU of a desktop PC with an Intel Core i7-6700 processor. We compared the adaptive step size based RK4IP method approach with the traditional SSFM approach. Four RK4IP methods were tested: 1) RK4IP LEM in the frequency domain; 2) RK4IP LEM in the time domain; 3) RK4IP CQEM in the frequency domain; and 4) RK4IP CQEM in the time domain.

Figure 3 shows the received power in a C+L+S MB transmission system after nonlinear fiber transmission along 80 km of optical fiber. The uniform step size considered in the SSFM approach is increased from 0.01 to 10 meter in Fig. 3(a) to (d). We observe that the accurate modeling of the SRS effect requires a uniform step size as short as 0.01 meter when using the SSFM. Although a uniform launch power of -1 dBm only is considered, the SRS effect already leads to a power tilt of about 9 dB when considering S+C+L MBT, which highlights the high

(a) Step Size: 0.01 m
(b) Step Size: 0.1 m
(c) Step Size: 1 m
(d) Step Size: 10 m

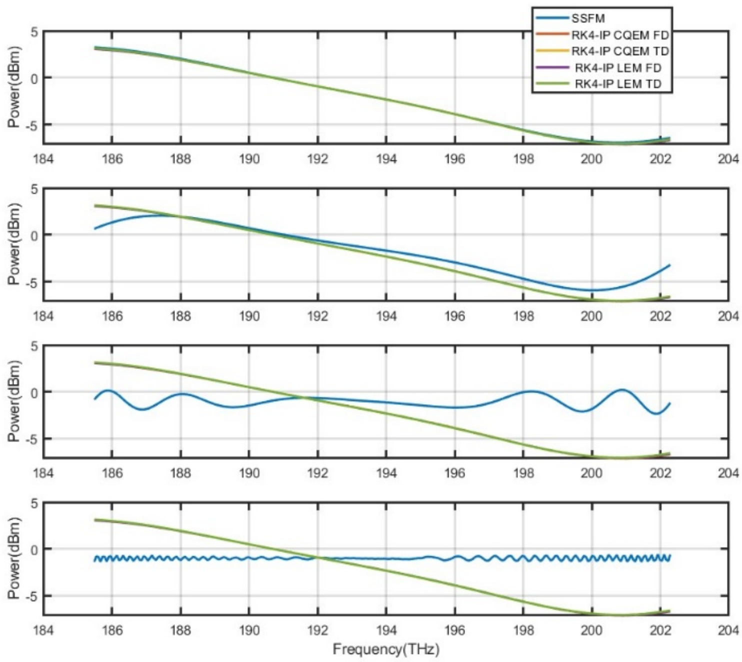

Figure 3. Received power [dBm] for each channel.

impact of SRS in MBT systems. Due to the required small step size to accurately model the impact of SRS, the traditional SSFM model took about 95 hours to simulate nonlinear fiber transmission along the 80 km fiber.

The computation time considering different step sizes is reported in Fig. 4. The analysis of Fig. 4 shows that the time required by the RK method in the time domain (TD) is higher than the one required by the same method in the frequency domain (FD). Moreover, the RK4IP CQEM in the FD has proven to be the method requiring the lower computation time from all considered approaches. Although the computation time is a critical metric, the resulting accuracy/error of the simulator must also be assessed simultaneously. In this work, the error is computed with respect to the results obtained with the SSFM using a 0.01-meter step size. In line with [4], the error is defined in eq. (5), where P_{method} denotes power of method like RK4IP CQEM TD, RK4IP CQEM FD, RK4IP LEM TD, RK4IP LEM FD.

$$error = \sum_{k=1}^{n} \frac{||P_{method}|^2 - |P_{SSFM}|^2|}{\max(|P_{SSFM}|^2)} \tag{5}$$

Figure 5 shows the error as defined in eq. (5) for all the considered approaches. The analysis of this figure shows that the RK4 IP methods with adaptive step size usually led to a similar level of accuracy independently of the considered initial step size. Moreover, it highlights that although considering a longer step size, when using the SSFM, may considerably reduce the computation time, the resulting error may be too high.

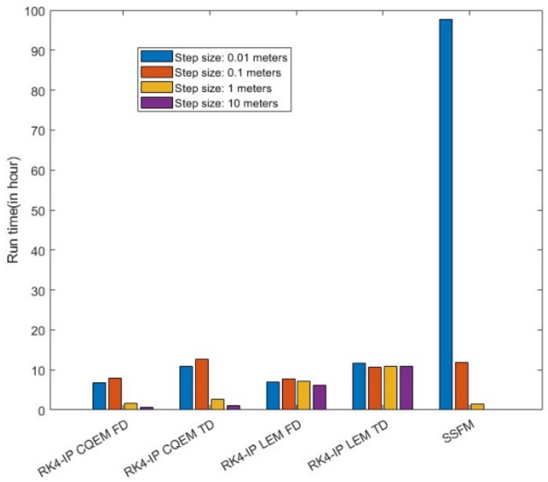

Figure 4. Computation time of RK and SSFM methods.

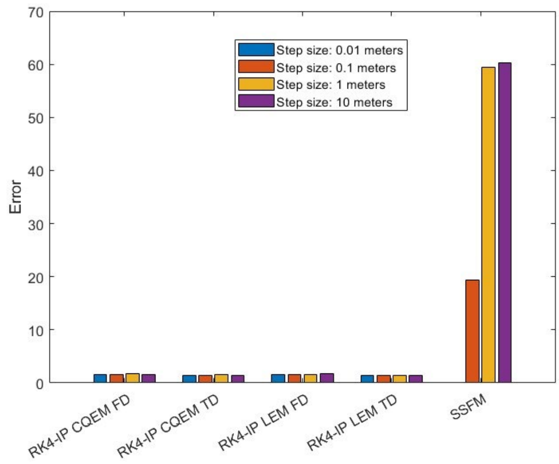

Figure 5. Simulation accuracy for RK and SSFM methods.

4. CONCLUSION

A Split Step Multiband Simulation software tool has been proposed in this work. The SSMS tool solves the nonlinear Schrödinger equation using the fourth order RK4IP method. An adaptive step size algorithm has also been proposed to further reduce the computation time when using the RK4IP methods. The performance of the tool was benchmarked against the traditional SSFM approach in a S+C+L MBT system, i.e., considering a total transmission bandwidth of 16.8 THz. The obtained results have shown that the RK4IP methods are much faster than the SSFM, while still achieving a similar level of accuracy.

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