

# Supplementary information “Model-based analysis of the dynamic capacity ramp-up of closed-loop supply chains for lithium-ion batteries in Japan and Germany”

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## Mathematical formulation a capacity planning model

### Sets and indices

|       |   |
|-------|---|
| $t$   | Periods ( $t \in T$ )                                 |
| $t_s$ | Start of the investment period ( $t_s \in T$ )        |
| $t_e$ | End of the investment period ( $t_e \in T$ )          |
| $m$   | Modules of the closed-loop supply chain ( $m \in M$ ) |
| $s$   | Size of the capacity expansion ( $s \in S$ )          |

### Parameters

|                         |   |
|-------------------------|---|
| $k_{m,s,t_s,t_e}^{inv}$ | Net present value. Discounted cash flows of initial expenses for installation and start-up of a module $m$ in capacity size $s$ from the beginning of period $t_s$ to the end of period $t_e$ |
| $k_{m,s,t}^{fix}$       | Fixed expenses of a module $m$ of the size $s$ in period $t$  |
| $k_{m,s,t}^{var}$       | Variable expenses arising in a module $m$ of the size $s$ in period $t$   |
| $p_{m,t}^{Process}$     | Amount of products processed in the module $m$ in period $t$  |
| $c_{m,s}$               | Capacity of module sizes. Maximum technical capacity of a module $m$ in the size $s$  |
| $t^{min}$               | Pre-set minimum operating time in periods ( $t^{min} \in T$ )   |
| $t^{max}$               | Physical maximum operating time in periods ( $t^{max} \in T$ )  |

### Variables

|  |   |
|--|---|
| $x_{m,s,t_s,t_e}^{inv} \subseteq \mathbb{N}$ | Investment program variable: Number of modules $m$ of the size $s$ operated from the beginning of period $t_s$ to the end of period $t_e$ (with $t_s < t_e$ ) |
| $x_{m,s,t} \subseteq \mathbb{N}$             | Number of modules $m$ in size $s$ operated in period  |
| $p_{m,s,t}$                                  | Number of products processed in the module $m$ with the size $s$ in period $t$  |

## Objective function

$$\text{Min } Z = \sum_m \sum_s \sum_{t_s} \sum_{t_e} k_{m,s,t_s,t_e}^{inv} * x_{m,s,t_s,t_e}^{inv} + \sum_m \sum_s \sum_t k_{m,s,t}^{fix} * x_{m,s,t} + \sum_m \sum_s \sum_t k_{m,s,t}^{var} * p_{m,s,t}$$

The objective is to minimize the net present value ( $Z$ ) of the outflows in the planning period. For this purpose, objective function consists of three terms:

The first term describes the discounted investment-related cash flows for all modules  $m$  of capacity  $s$  installed in the planning horizon. To ensure that the sum can specifically represent each module  $m$  in each size  $s$  at each planning period from  $t_s$  to  $t_e$  in the planning horizon,  $k_{m,s,t_s,t_e}^{inv}$  must be calculated in advance. This procedure reduces the complexity of the model and allows higher flexibility for the model, as it is easier to change specific scenarios and observation periods.

The second term represents the specific fixed expenses that are incurred by each installed module for each period. In this case, the fixed cost is incurred specifically for each module  $m$  in the respective period  $t$ . Again, the period-related fixed costs are calculated and discounted in advance to reduce complexity and facilitate the adaptation of changing scenarios.

The third term represents the period-related variable payments. The payments result from the quantity of products  $p$  that runs in period  $t$  through module  $m$  with size  $s$  and the variable operating payments  $k_{m,s,t}^{var}$ .

## Restrictions

$$p_{m,s,t} \leq x_{m,s,t} * c_{m,s} \quad \forall m \in M, s \in S, t \in T \quad (1)$$

In each period  $t$ , the capacity requirement in the form of variable  $p_{m,s,t}$ , the number of products per module  $m$  and module size  $s$  must always be covered by the capacity size built up in the period. For this purpose, the number of plants is multiplied by the specific maximum capacity  $c_{m,s}$ .

$$p_{m,t}^{Process} = \sum_s p_{m,s,t} \quad \forall m \in M, s \in S, t \in T \quad (2)$$

This restriction ensures that each module's products processed per period  $p_{m,t}^{Process}$  are divided among the different capacity-size plants. Thus, the sum of the products processed in the modules of the individual capacity sizes  $s$  equals the sum of the given products for the entire module  $m$ .

$$x_{m,s,t_s,t_e}^{inv} = 0 \quad \forall m \in M, s \in S, t_s \in T, t_s + t^{min} - 1 \geq t_e \geq t_s + t^{max} + 1 \quad (3)$$

The installed plants' minimum and maximum operating time must not be exceeded or underrun. Therefore, operating periods are excluded in which the end time  $t_e$  is smaller than the start time  $t_s$  and the minimum  $t^{min}$  duration or the end time is larger than the start time plus the maximum operating time  $t^{max}$ .

$$\sum_{t_s=1}^t \sum_{t_e=t}^T x_{m,s,t_s,t_e}^{inv} = x_{m,s,t} \quad \forall m \in M, s \in S, t \in T_s, t_s \in T, t_e \in T_e \quad (4)$$

Restriction four defines that the number of plants of module  $m$  and size  $s$  in a period must equal the built modules  $m$  of size  $s$  in the period from  $t_s$  to  $t_e$ .

$$x_{m,s,t_s,t_e}^{inv}, x_{m,s,t}, p_{m,s,t} \geq 0 \quad \forall m \in M, s \in S, t \in T_s, t_s \in T, t_e \in T_e \quad (5)$$

$$x_{m,s,t_s,t_e}^{inv}, x_{m,s,t} \in \mathbb{R}^+ \quad \forall m \in M, s \in S, t \in T_s, t_s \in T, t_e \in T_e$$

The variables  $x_{m,s,t_s,t_e}^{inv}$  and  $x_{m,s,t}$  must be integer and  $p_{m,s,t}$  must be greater than or equal to zero.