

Smoothed Normal Distribution Transform for efficient Point Cloud Registration during Space Rendezvous

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Keywords: Point Cloud Registration, Pose Tracking, Normal Distribution Transform, Space Rendezvous

Abstract: Next to the iterative closest point (ICP) algorithm, the normal distribution transform (NDT) algorithm is becoming a second standard for 3D point cloud registration in mobile robotics. Both methods are effective, however they require a sufficiently good initialization to successfully converge. In particular, the discontinuities in the NDT cost function can lead to difficulties when performing the optimization. In addition, when the size of the point clouds increases, performing the registration in real-time becomes challenging. This work introduces a Gaussian smoothing technique of the NDT map, which can be done prior to the registration process. A kd-tree adaptation of the typical octree representation of NDT maps is also proposed. The performance of the modified smoothed NDT (S-NDT) algorithm for pairwise scan registration is assessed on two large-scale outdoor datasets, and compared to the performance of a state-of-the-art ICP implementation. S-NDT is around four times faster and as robust as ICP while reaching similar precision. The algorithm is thereafter applied to the problem of LiDAR tracking of a spacecraft in close-range in the context of space rendezvous, demonstrating the performance and applicability to real-time applications.

1 INTRODUCTION

With more and more satellites orbiting the Earth, some space missions start being oriented towards the maintenance of the satellite infrastructure, whether it is to repair a satellite in orbit (on-orbit servicing) or to bring back on Earth an inactive satellite (debris removal). For such a mission, an active satellite has to autonomously approach an uncontrolled target satellite: This is the rendezvous. A famous example is the “Mission Extension Vehicle” [Pyrak and Anderson, 2022], where a LiDAR was used to track the pose of the target spacecraft. In addition to also providing depth information, LiDARs represent a powerful alternative to visual cameras for space rendezvous because they are less sensitive to the harsh illumination conditions observed in orbit (frequent eclipses and high luminosity contrast).


To perform LiDAR based tracking, a registration algorithm can be used. 3D point cloud registration consists in the alignment of two point clouds repre-


senting the same scene, but taken from different perspectives. By registering point clouds captured by an on-board LiDAR with respect to a 3D model of the uncooperative target, the pose of this satellite can be tracked. Although space rendezvous is the driving use case considered in this paper, these methods find a wide range of applications for navigation problems in autonomous driving and mobile robotics. Registration algorithms are often at the basis of LiDAR based odometry or 3D cartography procedures, or when these two tasks are combined in a simultaneous localization and mapping (SLAM) framework.

The most widespread technique for point cloud registration is the iterative closest point (ICP) algorithm [Besl and McKay, 1992] and its multiple variants such as trimmed ICP [Chetverikov et al., 2002], and generalized ICP [Segal et al., 2009]. However, the normal distribution transform (NDT) algorithm [Biber and Straßer, 2003] is gaining in popularity. It is becoming a second standard for registration problems due to its efficiency. In the context of road vehicle registration [Pang et al., 2018] and mine mapping [Magnusson et al., 2009], it was found to be as precise and faster than the ICP.

Nevertheless, the NDT is sensitive to a good ini-

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tial estimate [Lim et al., 2020]. Solutions to widen the basin of convergence of the algorithm such as trilinear interpolation [Magnusson et al., 2009] lead to an important increase of the computation time, and the loss of the algorithm’s benefit compared to ICP. Thus, performing robust registration in real-time can be challenging, especially for large point clouds, or if the processing hardware has limited capabilities as it is often the case for space applications.

In this work, we introduce a generic smoothed NDT (S-NDT) algorithm aiming at tackling the aforementioned issues, before applying it to spacecraft pose tracking. The main contributions are:

- A Gaussian smoothing technique of the NDT map. This smoothing leads to an increase in the algorithm’s robustness without affecting the processing time of one iteration.
- A kd-tree representation of the NDT mapping. This formulation further widens the algorithm’s convergence basin.
- An evaluation of S-NDT’s precision, efficiency and robustness compared to the ICP, on two datasets from the automotive domain.
- The demonstration of the applicability of S-NDT to real-time applications in the context of pose estimation of an uncooperative satellite for autonomous space rendezvous.

The remainder of this paper is organized as follows: In Section 2, we present the NDT and related work on this algorithm. Section 3 introduces our S-NDT algorithm, and Section 4 its results on outdoor datasets compared to ICP and classical NDT. Section 5 describes the results of the algorithm when applied to the satellite pose tracking problem. Finally, Section 6 summarizes and concludes the paper.

2 NDT REGISTRATION

2.1 Original NDT algorithm

The NDT algorithm was first developed for registration of 2D scan data [Biber and Straßer, 2003]. Later, it was extended to the registration of 3D point clouds [Magnusson, 2009, Takeuchi and Tsubouchi, 2006]. The specificity of the NDT lies in the representation of the target point cloud. The reference scene is divided into several voxels of a certain size. For each voxel, the mean and covariance matrix (μ, C) of the n points with coordinates x_1, \dots, x_n that fall within this voxel is computed as

$$\mu = \frac{1}{n} \sum_{k=1}^n x_k \quad (1)$$

$$C = \frac{1}{n-1} \sum_{k=1}^n (x_k - \mu)(x_k - \mu)^T \quad (2)$$

The probability density function (pdf) of the target point cloud for each voxel with parameters (μ, C) is then estimated by a normal distribution where the normalization constant is left out:

$$p(x) = \exp\left(-\frac{1}{2}(x - \mu)^T C^{-1}(x - \mu)\right) \quad (3)$$

After having built the pdf of the target point cloud, registration of another point cloud (source) consisting of m points z_1, \dots, z_m can be performed. The relative transformation between both point clouds can be parametrized by $\alpha = (R, t)$ where R is a rotation matrix and t a translation vector. Each 3D point z is transformed according to

$$T(\alpha, z) = Rz + t \quad (4)$$

The score of a pose transformation α is

$$s_{NDT}(\alpha) = - \sum_{k=1}^m p(T(\alpha, z_k)) \quad (5)$$

The NDT algorithm iteratively finds the pose transformation α that minimizes s_{NDT} . After each iteration, the points of the source cloud are associated with their new corresponding distributions until convergence. Because it is a local optimization algorithm, it requires a sufficiently good initial estimate to converge.

2.2 NDT variants

The voxel size is an important tuning parameter of the NDT. If it is too small, the algorithm will have difficulties converging and be less robust to initialization errors. If it is too big, the precision of the registration will decrease. Hence, in multi-layered NDT or ML-NDT [Ulaş and Temeltaş, 2013], optimization is first performed on coarse voxel grids before switching to finer resolutions. ML-NDT can effectively increase the robustness of the registration process, but requires more processing time and memory for storing the different submaps.

For speeding up registration of sparse outdoor point clouds, NDT can be combined with ground segmentation and clustering of the remaining features [Das et al., 2013], so that the total number of distributions to evaluate is significantly less than standard NDT. Another approach to reduce the number of voxels (or surfels) is the use of multi-resolution surfel

maps which have a lower resolution when the distance to the sensor increases [Droeschel et al., 2014].

Instead of performing point-to-distribution registration, some authors directly compared two NDT models to perform distribution-to-distribution NDT [Stoyanov et al., 2012, Droeschel et al., 2014], leading to an improvement of the computation time with similar precision results. Likewise, in probabilistic NDT [Hong and Lee, 2017], each point is modelled by its own pdf. This avoids degenerate distributions and can also be viewed as a distribution-to-distribution NDT. A similar representation of the points is used in voxelized generalized ICP [Koide et al., 2021], where each point of the source cloud is matched with the multiple distributions originating from the different points contained in a voxel of the target cloud.

In most approaches, the negative sum of probabilities (5) is minimized. This non-linear optimization can be achieved by a Gauss-Newton method [Magnusson, 2009]. However, minimizing the sum of squared residuals in order to have a standard non-linear least squares formulation can lead to faster convergence [Schulz et al., 2018]. If the squared Mahalanobis distances are minimized [Ulaş and Temeltaş, 2013], the problem takes the form of a simple weighted least squares problem. It can be solved very efficiently, but for good convergence then the method strongly relies on the distributions being non-degenerate and not too discontinuous from one voxel to the other.

2.3 Discontinuity problem

One problem of the NDT can be the discontinuity of the cost function when a point is moved from one voxel to another during the registration process. Too big discontinuities are problematic for the optimization routine. To overcome this problem for 2D scan registration, the original NDT implementation [Biber and Straßer, 2003] makes use of 4 overlapping voxel grids. The contribution of each point is not evaluated taking into account the distribution within one voxel only, but of 4 partially overlapping voxels.

In 3D [Takeuchi and Tsubouchi, 2006, Schulz et al., 2018], this implies evaluating 8 overlapping voxels. If it enables to increase the convergence basin and smoothness of the optimization, this method implies to create and evaluate 8 additional submaps, leading to an increase in computation time about around 8 times. Magnusson uses an analogous method: No additional submaps are created, but each point is evaluated by dynamically weighting the distributions of the 8 nearest voxels using trilinear interpolation [Magnusson et al., 2009]. This also leads

to an increase in computation time between 4 and 8 times. The next section introduces an alternative solution, which consists in applying a Gaussian blur to the NDT map in order to smooth out big discontinuities between adjacent distributions.

3 SMOOTHED NDT

3.1 Smoothing of the NDT map

3.1.1 Gaussian kernel smoothing

We propose a solution to tackle the discontinuity problem without increasing the computation time of one iteration. The idea is to apply a smoothing kernel to the NDT voxel grid, as illustrated in Figure 1. In consequence, the distribution held by one voxel does not only represent the distribution of the points within this voxel any more, but the distribution of the points in the surroundings of this voxel too.

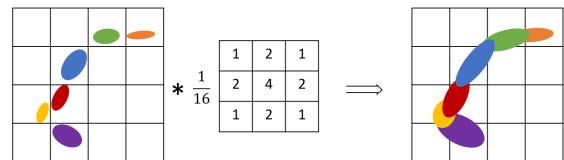


Figure 1: Schematic representation of the smoothing step for a 2D voxel grid to which a 3×3 sized Gaussian kernel is applied. The distributions are represented by their confidence ellipses. The initial distributions (left) are blurred to obtain the smoothed distributions (right).

When applying a smoothing kernel, the new random variable X represented by aggregating the random variables X_1, \dots, X_l with weighting coefficients w_1, \dots, w_l is characterized by the mean and covariance

$$\mu_X = \sum_{i=1}^l w_i \mu_{X_i} \quad (6)$$

$$C_X = \sum_{i=1}^l w_i (C_{X_i} + \mu_{X_i} \mu_{X_i}^T) - \mu_X \mu_X^T \quad (7)$$

Each weight w_i is also multiplied by the number of points n_i that fell into this voxel, before the sum of weights is normalized to 1. This smoothing method can be applied to the NDT map as a pre-processing step. No additional storage costs are required, as after construction only the smoothed distributions need to be stored by each cell for the registration.

It might be noticed that the distributions are obtained as a weighted sum of other distributions (“mean of means”). Our experiments showed that it produces better precision results than when taking directly a weighted sum of the points. Indeed, in the

latter case, more weight is artificially given to points which happen to be close to the middle of a cell.

3.1.2 Covariance matrix regularization

Despite the smoothing, the covariance matrix of a distribution can still be singular, for instance for a set of coplanar points. This happens quite often in practice, as most scenes contain planar surfaces. To avoid such cases, in probabilistic NDT each point is represented by its own uncertainty distribution [Hong and Lee, 2017]. Another possibility is to inflate the smallest eigenvalues of singular or nearly singular covariance matrices [Magnusson, 2009].

The proposed approach here is similar. For a positive semi-definite matrix C with maximal and minimal eigenvalues $\lambda_{\max}(C) \geq \lambda_{\min}(C) \geq 0$, the condition number is defined as:

$$\text{cond}(C) = \frac{\lambda_{\max}(C)}{\lambda_{\min}(C)} \in [1, +\infty] \quad (8)$$

Condition number regularization consists in modifying each covariance matrix C to satisfy the condition $\text{cond}(C) \leq \kappa$, for $\kappa > 1$ (typically $\kappa = 50$). This can be achieved by replacing C by $C + \delta I$, with δ given by [Olive, 2022]:

$$\delta = \max\left(0, \frac{\lambda_{\max}(C) - \kappa \lambda_{\min}(C)}{\kappa - 1}\right) \quad (9)$$

3.2 Optimization problem formulation

3.2.1 Optimization on the $SO(3)$ manifold

Euler angles are commonly used for representing the rotation when using the NDT [Hong and Lee, 2017, Lim et al., 2020]. However, they come with the gimbal lock problem and necessary convention setting. We make use of an alternative and convenient representation. The rotation group $SO(3)$ is a matrix lie group, so that each element can be parametrized by a 3-dimensional vector ω of the manifold. The corresponding rotation matrix is obtained by taking the exponential of the cross-product matrix of ω

$$\exp(\omega^\times) = \exp\left(\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\right) \quad (10)$$

In this way, the optimization problem in $S = SO(3) \times \mathbb{R}^3$ can be reduced to a problem in $\mathbb{R}^3 \times \mathbb{R}^3 = \mathbb{R}^6$. The ‘‘boxplus’’ operator [Hertzberg et al., 2013] is defined to compute the new rotation and translation obtained after applying an increment from the

6D-manifold:

$$\boxplus = \begin{cases} S \times \mathbb{R}^6 & \rightarrow S \\ (R, t) \boxplus (\omega, \tau) & = (\exp(\omega^\times)R, t + \tau) \end{cases} \quad (11)$$

The Jacobian matrix of a transformation with parameters $\alpha = (R, t)$ evaluated at a 3D point z respectively to the elements of the manifold $\varepsilon = (\omega, \tau)$ yields

$$\left. \frac{\partial T(\alpha \boxplus \varepsilon, z)}{\partial \varepsilon} \right|_{\varepsilon=0} = \begin{bmatrix} 0 & v_3 & -v_2 & 1 & 0 & 0 \\ -v_3 & 0 & v_1 & 0 & 1 & 0 \\ v_2 & -v_1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

where $v = Rz$. This Jacobian will be needed in the next section.

3.2.2 Weighted least squares optimization

For a simpler formulation of the optimization problem, we consider the relaxed problem where the non-linear normal distribution representation has been omitted. The residuals are

$$r_i = T(\alpha, z_i) - \mu_i \quad (13)$$

and the cost function is the sum of squared Mahalanobis distances:

$$s(\alpha) = \frac{1}{m_a} \sum_{i=1}^{m_a} r_i^T C_i^{-1} r_i \quad (14)$$

where μ_i , C_i are the mean and covariance of the distribution associated with point $T(\alpha, z_i)$, and m_a is the number of points from the source point cloud which could be associated with a distribution. The Jacobian of each residual is

$$J_i = \left. \frac{\partial T(\alpha \boxplus \varepsilon, z_i)}{\partial \varepsilon} \right|_{\varepsilon=0} \quad (15)$$

Stacking the Jacobians into the matrix $J \in \mathbb{R}^{3m \times 6}$, the inverse covariances into the block-diagonal $W \in \mathbb{R}^{3m \times 3m}$, and the residuals into $r \in \mathbb{R}^{3m}$, the increment to perform according to the Gauss-Newton algorithm is given by

$$J^T W J \varepsilon = -J^T W r \quad (16)$$

After each iteration, the current estimate of the transformation is updated according to $\alpha \leftarrow \alpha \boxplus \varepsilon$. The optimization stops whenever one of the following three conditions is reached:

- The maximum number of iterations is reached
- The norm of the increment $\|\varepsilon\|$ is below a certain threshold ε_{\min}
- The number of points m_a matched with the NDT map does not increase, and the cost function (14) increases.

3.3 Kd-tree adaptation

3.3.1 Kd-NDT map

An NDT map is usually constructed using a regular grid structure, typically an octree [Ulaş and Temeltaş, 2013, Hong and Lee, 2017]. Nevertheless, octrees are not optimal because they map a lot of free space. A strategy can be to store the cubical cells from the regular grid subdivision into a kd-tree [Schulz et al., 2018]. Yet to the best of our knowledge, directly using a kd-tree with non-cubical cells for constructing the NDT map has never been done. This is mainly due to the fact that the distributions can be distorted by the shape of kd-tree cells which can have a high aspect ratio. However, this problem vanishes with the smoothing method presented in the following Section 3.3.2.

The kd-tree is constructed as follows: At each subdivision step, the new bounding box of all points is computed. This is important for the efficiency of the tree search at later steps. Then, the cell is split in two children along its longest edge and in the middle. This ensures that the cells keep a good aspect ratio. The subdivision stops once the kd-tree cells have reached a certain size. The size of a kd-tree cell being its longest edge l , if we want all cells to have an approximate cell size of r , the subdivision stops whenever the criteria $l < \frac{4}{3}r$ is met. Consequently, all cells have a size $l \in [\frac{2}{3}r, \frac{4}{3}r]$.

3.3.2 Smoothing of a kd-NDT map

In the case of a kd-tree, the neighbouring cells to be selected for performing the Gaussian smoothing step are the ones which lie within a certain radius d_r of the center of the considered cell. Figure 2 illustrates this idea.

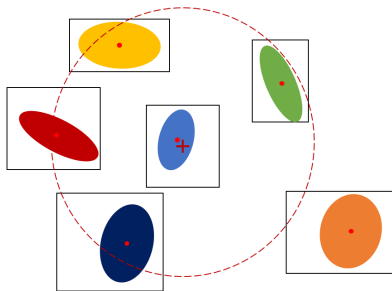


Figure 2: Schematic representation of the smoothing step in the case of a 2D kd-grid with non square cells. All the cells within the radius are taken into account for computing the smoothed distribution of the central cell. Here, only the bottom right-cell is not considered because it lies too far away

Instead of using a discrete Gaussian kernel, a continuous Gaussian blur can be used. The contribution of a distribution X_i with n_i points and mean μ_{X_i} to the smoothed distribution of a cell with center c is

$$w_i \propto n_i \exp\left(-\frac{\|\mu_{X_i} - c\|^2}{2\sigma^2}\right) \quad (17)$$

where σ is the standard deviation of the Gaussian function. If r is the approximate resolution of the grid map, in order for distributions located at a distance r from the center to only have half the weight of distributions located at the center, σ can be chosen as:

$$\exp\left(-\frac{r^2}{2\sigma^2}\right) = \frac{1}{2} \iff \sigma = \frac{r}{\sqrt{2\ln(2)}}. \quad (18)$$

In this way, a kd-tree cell of the smoothed NDT tree holds the weighted distribution of all points located in the influence sphere. Therefore, the actual non-cubic but rectangular shape of the kd-tree cell does not matter any more, and does not negatively influence the shape of the distribution, what makes the use of this kd-NDT map possible. The radius is typically set to $d_r = 3\sigma$ to capture all significant contributions.

3.3.3 NDT algorithm adaptation

The algorithm described in Section 3.2.2 can be applied in the same way to kd-tree NDT maps. Each point of the source cloud is associated to a distribution by performing a depth first search. In our experiments, this results in similar precision than exact nearest cell search, and a speed-up by a factor approximately 1.5. Additionally, we define a maximum point-to-cell distance d_{p2c} . A point z is only associated with a distribution held by a cell with center c if $\|z - c\| < d_{p2c}$.

4 RESULTS ON OUTDOOR DATASETS

4.1 Datasets and methodology

We compared the results of the algorithm on two datasets of the Robotic 3D Scan Repository [Nüchter and Lingemann, 2016]. The first dataset, *Hanover 2*, was recorded with a Velodyne sensor at the campus of the Leibniz University in Hanover. It contains 924 scans of each approximately 13000 points (Figure 3). Initial pose estimates are provided and are precise at the order of a few degrees ($< 5^\circ$) and a

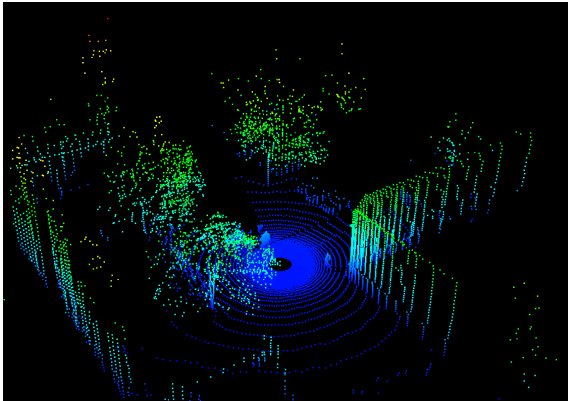


Figure 3: Scan 22 of the *Hanover 2* dataset

few cm ($\sim 10cm$). Ground truth data is also available [Nüchter et al., 2010] starting from scan 22.

The second dataset contains 333 scans recorded by a Velodyne scanner mounted on a car at the University of Koblenz-Landau. Each cloud contains about 100000 points (Figure 4). The initial pose estimates are less precise than the previous ones ($< 7^\circ$ in rotation, and $\sim 1m$ in translation). Ground truth is not available, however using the SLAM-6D algorithm from the 3D toolkit [3DTK, 2022], we can generate highly precise pose estimations which can be considered as ground truth for our problem. Simultaneous localization and mapping (SLAM) is a global mapping method not suited for pairwise scan-registration. Still, a core component of SLAM-6D is an ICP implementation which we will use for comparison.

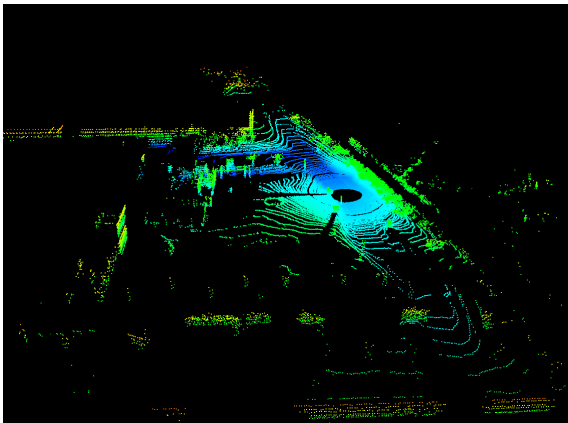


Figure 4: Scan 98 of the *Koblenz University* dataset

We compared four algorithms for pairwise scan registration, i.e. LiDAR odometry: The efficient state-of-the-art ICP implementation of the 3D toolkit [3DTK, 2022] as reference, against our implementations of a classical NDT, the octree-based S-NDT and the kd-tree based S-NDT. The implementations of the “classical” NDT and the octree S-NDT only differ in that the NDT map is being smoothed for the latter.

The algorithms were run on one core of an Intel Core i7 CPU. It has to be noted that the ICP version of 3DTK is also optimized for multi threading and could run on more cores.

4.2 Pairwise scan matching

The performance was evaluated by comparing the result of the registration of each pair of consecutive scans with the expected result according to ground truth. For all four algorithms, the scans were down-sampled using a voxel grid filter in a pre-processing step. The parameters used for the experiment are summarized in Table 1.

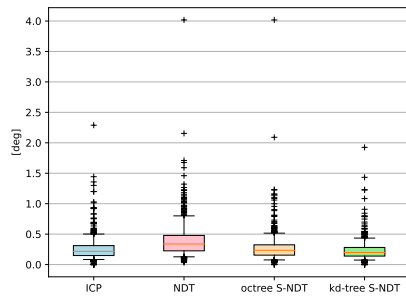
Table 1: Parameters settings for the outdoor datasets.

	Hanover 2	Koblenz University
max iterations n_{it}^{max}	100	100
min increment ϵ_{min}	10^{-5}	10^{-5}
voxel grid filter resolution d_{filter}	10cm	20cm
(S-)NDT cell size r	50cm	150cm
ICP max point to point distance d_{p2p}	75cm	150cm
S-NDT max point to cell distance d_{p2c}	75cm	150cm

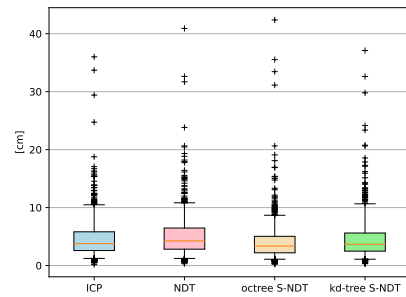
The results are presented on Figure 5 for the *Hanover 2* dataset and Figure 6 for *Koblenz University*. The ICP and S-NDT algorithms exhibit very similar results for the translational and angular error (Figures 5a, 5b, 6a, 6b). The precision of the classical NDT is close as well, but the angular errors are slightly higher on the *Hanover 2* dataset, and present more outliers on the *Koblenz University* dataset. The NDT and S-NDT implementations are significantly faster than the ICP, approximately by a factor 4 (Figures 5c, 6c). They obtain a result in typically around 10 iterations, while the ICP converges within around 30 to 50 iterations (Figures 5d, 6d). For small point clouds, the classical NDT is slightly faster than the S-NDT, because then the smoothing step represents a non-negligible part of the total processing time.

4.3 Robustness evaluation

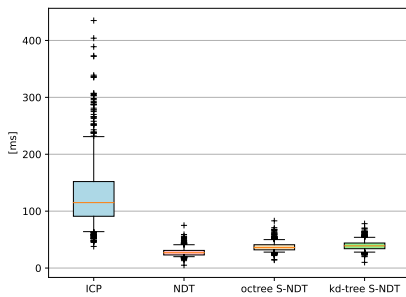
The robustness of an algorithm can be defined in various ways: Sensitivity to noise, to initial conditions, to different scenarios etc. In this section, we analyse the sensitivity of the different algorithms to the precision of the initial estimate. Each algorithm being a local optimization method, its convergence basin can be seen as the set of initial translational and angular



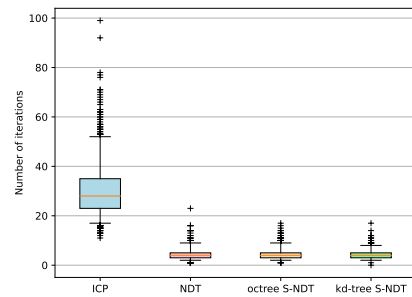
(a) Angular errors



(b) Translational errors

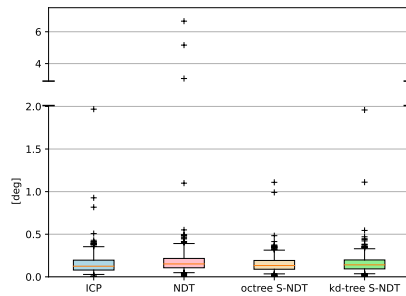


(c) Execution times

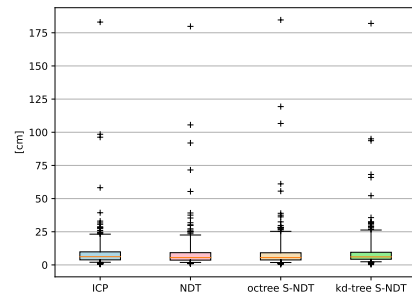


(d) Number of iterations

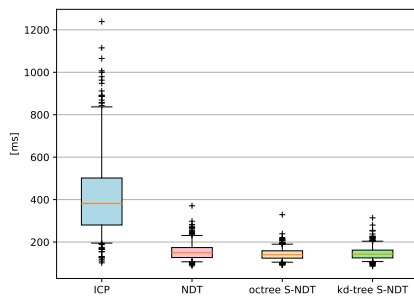
Figure 5: Distribution of the errors and key characteristics of each algorithm, when performing pairwise scan registration from scan 22 to 923 of the *Hanover 2* dataset.



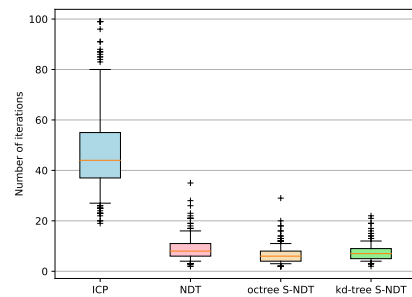
(a) Angular errors



(b) Translational errors



(c) Execution times



(d) Number of iterations

Figure 6: Distribution of the errors and key characteristics of each algorithm, when performing pairwise scan registration from scan 1 to 328 of the *Koblenz university* dataset.

errors for which the algorithm converges.

The convergence basin is estimated using a Monte-Carlo method. The 2D space of initial translation and angular errors is divided into discrete data points. For each of these points, the probability that each algorithm converges is estimated. The estimation is the average of successes observed when repeating 50 times the following random experiment: Select two consecutive scans at random, and starting from the perfect alignment, rotate and translate one scan along a random axis and a random direction in order to reach the desired angular and translational errors. Afterwards start the algorithm from this initial pose, and evaluate the result.

The criteria for defining if a method was successful is based on the results obtained on the two datasets (Figures 5 and 6): For *Hanover 2*, a registration is considered to be successful if the error is below 1.5deg and 30cm. For *Koblenz University*, the limits were set to 1.2deg and 75cm. The heat maps representing the estimated convergence basins of each algorithm for both datasets following this method are presented on Figures 7 and 8. On the *Hanover 2* dataset, the ICP algorithm shows to be the most robust, followed closely by the kd-tree S-NDT. The octree S-NDT turns out to have poorer convergence results, but they are still better than the classical NDT. On the *Koblenz university* dataset however, both S-NDT implementations demonstrate better robustness than the ICP and NDT.

4.4 Discussion

If all four algorithms are suited for precise scan matching, the speed advantage of the NDT and S-NDT versions makes them particularly interesting for real-time applications. Nevertheless, if the ICP is such a popular algorithm, it is also because of its robustness. The NDT shows to have a smaller convergence basin compared to the ICP. The challenge was to develop an algorithm profiting from the speed-up potential of the NDT formulation, while achieving similar robustness as the ICP. The experiments demonstrate that the smoothing step of the S-NDT increases the convergence basin of the algorithm. If the octree S-NDT formulation still has poorer convergence results then the ICP on the *Hanover 2* dataset, the kd-tree version shows consistently good results on both datasets.

The performance of the ICP in terms of robustness differs depending on the considered dataset (Figures 7a and 8a). A possible explanation resides in the fact that the *Hanover 2* dataset represents rather small outdoor scenes, while *Koblenz University* contains large

scale scans with considerably more points (see Figures 3 and 4 for comparison). The ICP being an algorithm that considers every point to point association, a hypothesis is that it fails to get the bigger picture on such scans, resulting in it being stuck in a local optimum. In contrast, the NDT algorithm computes voxel-wise distributions for a more coarse representation of the point cloud.

The main advantage of the kd-tree formulation is the flexibility in the implementation. Specifically, being able to set a maximum point-to-cell distance for considering or rejecting associations enables to widen the attraction domain of a distribution, and thus the overall algorithm's convergence basin. Of the four methods, the kd-tree S-NDT appears to be the most suited method for precise, reliable and efficient point cloud registration.

5 APPLICATION TO SPACECRAFT POSE TRACKING

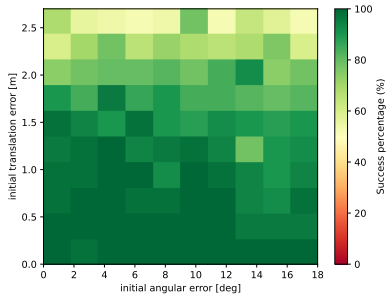
5.1 Testing environment

5.1.1 European Proximity Operations Simulator

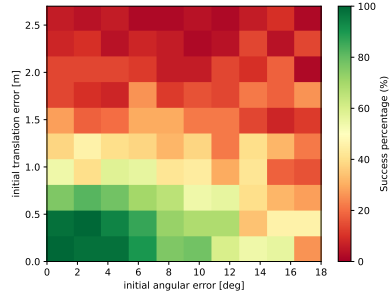
The European Proximity Operations Simulator or EPOS [Benninghoff et al., 2017], is a high-fidelity hardware in the loop simulator for spacecraft proximity operations, maintained by the German Space Operations Center. This simulator can be used to generate test data for the context of LiDAR pose tracking of an uncooperative satellite during autonomous space rendezvous.

The facility consists of two robotic arms carrying a load (Figure 9), which can move with 6 degrees of freedom. The robot on the left on the picture represents the active and controlled satellite, also called servicer satellite. It has a Livox-Mid40 LiDAR installed on its plate. Additionally, this robot is mounted on a 25m long linear rail to simulate translational movement. The second robot holds a mock-up of the target satellite at scale 1:1, which is made of realistic materials (solar panels, golden MLI sheets). To reproduce space dynamics, the facility is coupled to a dynamics simulator. A guidance and control system is used to follow a desired rendezvous trajectory.

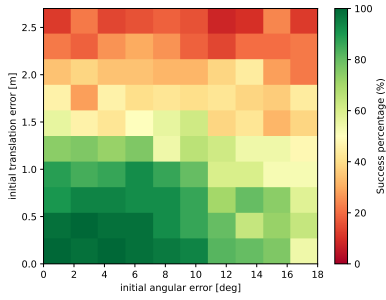
For estimating the relative pose of the target satellite, a navigation filter uses as input the result of the tracking of multiple sensors such as an RGB camera, PMD camera and a LiDAR [Frei et al., 2022]. Ultimately, the goal of the LiDAR pose estimation is to



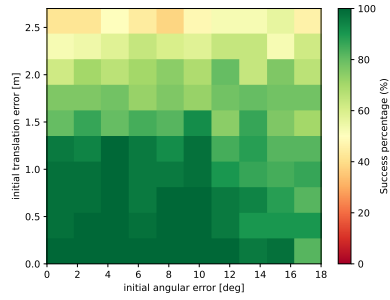
(a) Convergence basin for the ICP



(b) Convergence basin for the classical NDT

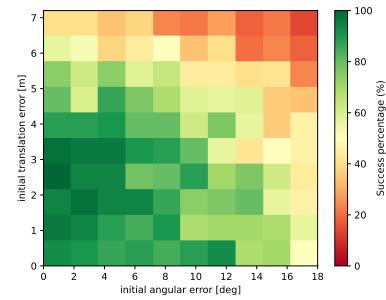


(c) Convergence basin for the octree based S-NDT

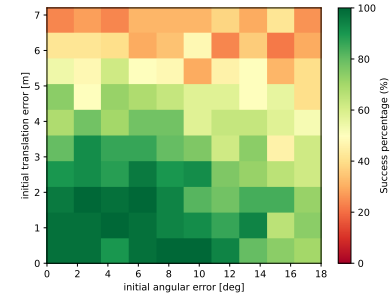


(d) Convergence basin for the kd-tree based S-NDT

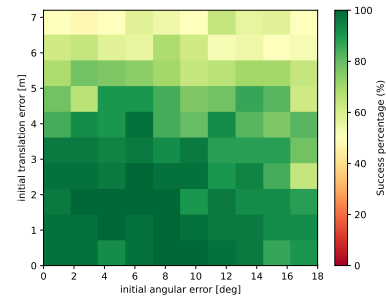
Figure 7: Heat maps representing the convergence basin of each algorithm on the *Hanover 2* dataset. Scans were selected at random following a Monte-Carlo method. The maximum registration error for defining success was set to 1.5deg and 30cm.



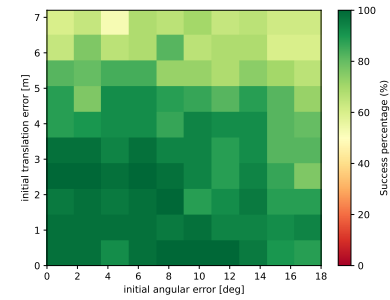
(a) Convergence basin for the ICP



(b) Convergence basin for the classical NDT



(c) Convergence basin for the octree based S-NDT



(d) Convergence basin for the kd-tree based S-NDT

Figure 8: Heat maps representing the convergence basin of each algorithm on the *Koblenz university* dataset. Scans were selected at random following a Monte-Carlo method. The maximum registration error for defining success was set to 1.2deg and 75cm.

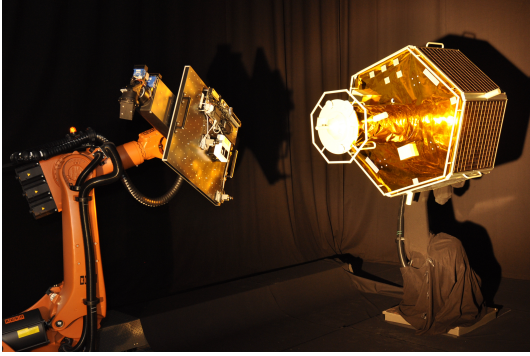


Figure 9: Picture of the EPOS facility robots

provide input for this navigation filter on-line. However, for this experiment, the data was first collected separately and the tracking was done off-line.

In order to obtain realistic point clouds representative of space conditions, the points which correspond to the robotic arm or the background of the testing facility are cut out automatically. Because of the quality of the sensor and the reflecting materials on the satellite such as the golden MLI sheets, the point clouds are noisy and distorted, as can be observed on Figure 11.

5.1.2 Rendezvous scenario

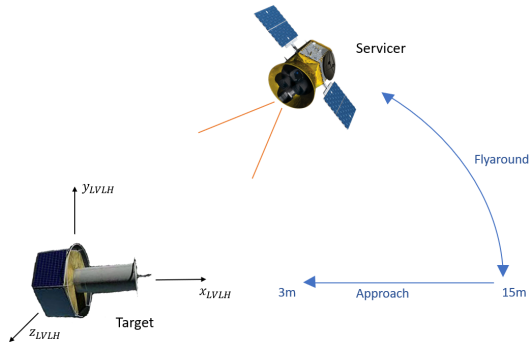


Figure 10: Schematic representation of the rendezvous trajectory

The LiDAR data was collected during a rendezvous simulation at EPOS with following scenario, schematically represented on Figure 10: First, the servicer satellite performs a fly-around of the tumbling target satellite at a steady distance of 15m in order to inspect it from different angles. Then, it flies back to its initial point and the approach is started. The first approach part, until 8m, is performed with a velocity of 2cm/s, and the final approach to 3m with a velocity of 1cm/s. Throughout the whole experiment of around 20 minutes, the target satellite spins around its main axis ($x_{LV LH}$) at a rate of 1 deg/s.

During the rendezvous, the LiDAR captures point clouds with an adaptive frequency, such that every

point cloud contains around 10000 points. Therefore, at 15m, the frequency of the LiDAR is of around 1Hz, while it can go up to 4Hz for the shortest ranges in this experiment (3m). In total, the dataset for this trajectory contains 1644 point clouds.

5.2 Pose tracking

5.2.1 Strategy

In this scenario, a geometrical model of the target satellite is known in the beforehand. Thus we do not register each acquired point cloud with the previous one, but perform registration of each scan respectively to a model point cloud of the target as shown in Figure 11. The tracking is performed with the kd-tree based S-NDT algorithm, and the smoothed NDT map of the model point cloud only has to be computed once for the whole experiment.

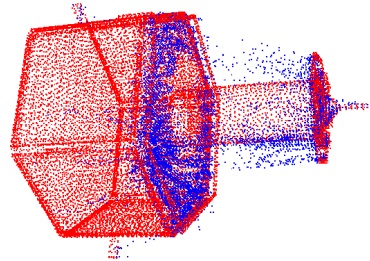


Figure 11: Point cloud captured by the LiDAR at EPOS (blue) superimposed with a model point cloud of the target (red). The captured point cloud is noisy and distorted due to reflections on the golden MLI sheets.

For every new incoming point cloud, the result of the previous registration step is used as initial estimation. Still, for the very first point cloud, an initial guess has to be formulated. This would be the task of a dedicated pose initialization algorithm. In the context of this work, such an algorithm was not developed. The pose initialization is assumed to be perfect and is obtained using the ground truth available at the EPOS facility.

5.3 Results and discussion

The experiment was run on one core of an Intel Core i7 CPU, with following parameters for the S-NDT:

$$\begin{aligned} n_{it}^{max} &= 50, & \epsilon_{min} &= 10^{-3}, & d_{filter} &= 2\text{cm} \\ r &= 7.5\text{cm}, & d_{p2c} &= 7.5\text{cm} \end{aligned} \quad (19)$$

The results are presented on Figure 12. The first phase of the flight is the fly-around, and the approach from 15m to 3m starts at $t = 670\text{s}$. The position of

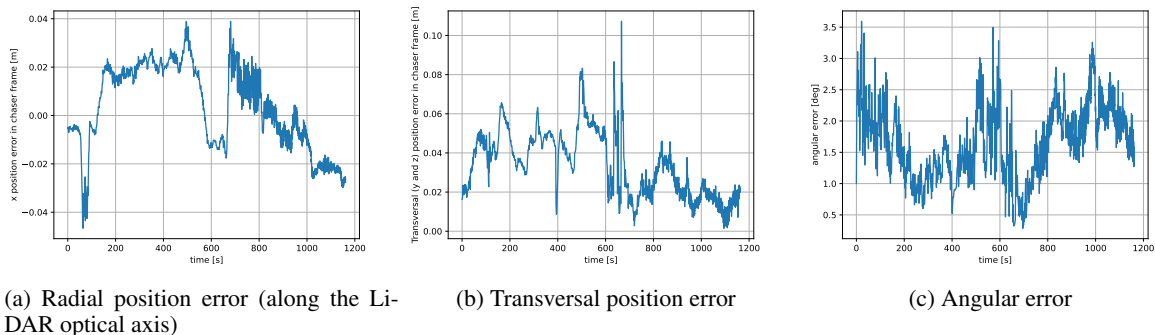


Figure 12: Translational and angular errors of the tracking of the spacecraft’s center of mass and orientation when compared to the ground truth at EPOS. The oscillation of the chaser when initiating the approach at $t = 670$ s leads to the biggest tracking error.

the target satellite’s center of mass is tracked. Both the frontal (along the x axis) and the transversal (y and z axes) errors are below 11cm throughout the experiment. When getting closer to the target satellite, this error further decreases to a few cm. The angular error of the spacecraft’s estimated orientation is below 3.6deg.

The translational and angular errors of the tracking are sufficient for performing an approach up to a distance of a few meters of the target. Nevertheless, some improvement could probably be obtained by using a more accurate 3D model of the target satellite than the current one. The LiDAR’s frame rate is also an important parameter. If it is too high, the captured point cloud is not dense enough, but if it is too low, the measurement is delayed and might be distorted (motion blur). This motivated the current choice of an adaptive frame rate ensuring that all point clouds are equally dense. Nevertheless, when manoeuvres are performed, the delay and motion blur effects can already be observed and lead to loss of precision in the tracking, for instance at $t = 670$ s.

The average processing time for down-sampling a point cloud and performing the S-NDT registration was of 18 milliseconds. This demonstrates that the algorithm is suited for real-time tracking requirements. However, currently the processing was done on a standard computer, and further work might include testing the algorithm on space-representative hardware.

6 CONCLUSIONS

In this paper, we proposed a modified S-NDT algorithm in order to reach better efficiency for real-time tracking applications. We introduced a novel smoothing method of the NDT map based on Gaussian blurring. Combined with a kd-tree representation of the map, it leads to a better robustness while maintaining the algorithm’s speed. The S-NDT was tested in different contexts. When compared to ICP on outdoor datasets from the automotive domain, it is significantly faster while achieving the same accuracy. The experiments show that it is also a fast, precise and reliable registration method for satellite pose tracking during autonomous space rendezvous.

Nevertheless, the method remains a local optimization method, which has to be coupled with a global optimization procedure in order for a system to navigate autonomously. In the context of autonomous driving, a loop detection procedure such as SLAM would be required to detect already scanned environments and correct the accumulated drift. For spacecraft pose tracking, a global pose initialization method is needed to initiate the tracking, and will be the subject of future work. Other potential limitations could arise when tracking faster objects. If the relative movements in the context of satellite rendezvous are rather slow, with faster dynamics, effects such as motion blur could arise and lead to a distortion of the point clouds which would have to be taken into account.

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