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TYPES OF QUESTIONS TEACHERS ASK TO ENGAGE STUDENTS IN MAKING SENSE OF A STUDENT CONTRIBUTION

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Table of Contents

List of Figures

List of Tables

To my parents,
for always believing in me

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Abstract

In the student-centered classroom, a teacher's interpretation and response to student mathematical contributions plays an important role to shape and direct students' opportunities for sense-making. This research used a scenario-based survey questionnaire to examine what types of questions middle and high school mathematics teachers indicate they would ask to engage students in making sense of a high-leverage student mathematical contribution and their reasoning about why particular questions are or are not productive. From the results, it could be concluded that teachers asked more productive questions after seeing a set of possible questions. Their beliefs about the productivity of the questions related to a variety of factors, including the specificity of the question, student participation, student ability and whether incorrect solutions should be discussed. The results could inform future work with teachers to productively use student thinking in their teaching.

1 Introduction

The National Council of Teachers of Mathematics (NCTM, 2000; 2009; 2014) has emphasized reasoning and sense-making as necessary elements in the secondary school curriculum. Their principles have led to the reform of the traditional teacher-centered classroom to a student-centered classroom. A traditional teacher-centered instruction implies a “high degree of teacher direction and a focus of students on academic tasks and it vividly contrasts with student-centered or constructivist approaches” (Schug, 2003, p. 94). On the contrary, student-centered instruction includes teachers making students’ mathematical ideas the center of whole class discussion (e.g., NCTM, 2014). By making the students’ ideas the center of discussion, teachers create an environment where students feel welcome to share and discuss their mathematical ideas with the whole class (Bansal, 2018).

The role of teachers in student-centered classrooms is important in fostering an active and engaging learning environment. As facilitators, teachers must continuously make decisions to create an environment where students are encouraged to participate, share their ideas and expand on their reflective and critical thinking (Serin, 2018). However, it is also crucial that the teacher provides feedback in a way that maintains students’ focus and direction in the discussion (NCTM, 2014). If teachers do not engage with the important ideas shared by their students, it may send a negative signal, leading students to disengage and shift their direction of thought. This, in turn, could result in students coming up with entirely different ideas that may not be in line with the goals of the lesson.

One particularly important decision a teacher has to make in a student-centered classroom is how to engage the entire class in a sense-making discussion about important student contributions (Kooloos et al., 2022a). They must decide what type of question to ask to prompt students to make sense of the student contribution. These decisions are crucial to promoting a collaborative student-centered learning environment and depend on the teacher's interpretation of students' thinking (Jacobs et al., 2010). Their interpretation of students' mathematical contributions and how they respond to them can shape and direct students' mathematical thinking significantly by enhancing cognitive opportunities for sense-making (Boaler & Brodie, 2004; Van Zoest et al., 2022). Given the importance of this decision, teachers must become adept at developing effective questioning strategies that help to engage students, deepen understanding, and encourage productive discussion in the classroom.

An important way that teachers foster productive discussion is through their questioning. Asking different types of questions leads students to different sense-making opportunities (Van Zoest et al., 2022). Asking productive questions could foster a more engaging class discussion to facilitate students in making sense of important ideas. For example, when asking students for an explanation of their contribution, targeted, specific questions can encourage a deeper analysis of the topic (Bansal, 2018; Franke et al., 2009), which leads to a better understanding of the information. On the other hand, vague or poorly worded questions that are only asked to gather more information (Bishop, 2008) can cause confusion and lack of focus, thus hindering the learning process. This highlights the importance of teachers' questioning and its impact on student learning and understanding.

A second important decision a teacher has to make is to create a safe and effective learning environment where all students can share their mathematical ideas. Not only does a productive discussion depend on what question is asked, but also who gets the opportunity to share (Van Zoest et al., 2022). In other words, is only the teacher engaging with the student contribution or is the teacher asking other students to engage with the contribution? These subtle differences in engagement prompted by the teacher's questioning can influence students' engagement in conceptual thinking (Kazemi & Stipek, 2009). Encouraging students to participate in whole-class discussions is consistent with the NCTM's (2014) goal of having teachers support their students' mathematical thinking. For example, Van Zoest et al. (2022) observed in their study that asking a student to evaluate another student's contribution creates a different learning opportunity from having a teacher evaluate it. It gives students the opportunity to share their thinking as well as help other students to have a better understanding of the contribution.

Schoenfeld (2011) found that teachers make decisions about an unpredictable topic or question by using their resources, goals, and orientations. Teachers also often develop a classroom routine that helps them to communicate with their students (Schoenfeld, 2008) and determine how to approach students' thinking. Developing these routines can boost teachers in the moment decision making by reducing the time and increasing the effectiveness of their decision. Teaching becomes automated among expert teachers through the repeated usage of a set of routines in the classroom and supports teachers in an unexpected event (Leinhardt et al., 1986). A teacher could have a specific classroom routine that might not work for all the students, however, as every student is different and has different thinking than others (Tomlinson, 2001). Thus, a classroom routine could be supportive of students' learning or else these routines can also get in the way. Thus, it is crucial to understand the factors that influence teachers' decisions, including how these decisions are influenced by their knowledge, goals and beliefs (Schoenfeld, 2008). By identifying the criteria and rationale for teacher decisions—including the questions they ask—we can not only better support teachers in their role as facilitators of learning, but also develop a framework or training program to train teachers to transfer the automated knowledge.

Although research has identified a broad understanding of teachers' decision making and the type of questions teachers ask, there is not any work specifically focused on how teachers would turn a student idea over to the class to make sense of it. In this research, I am focusing on a small part of teacher's decision-making in student-centered classrooms. In particular, this research aims to investigate the types of questions teachers ask the student(s) after a teachable moment occurs as well as the reasons behind their choices of asking that question. Understanding teachers' reasoning in their pedagogical decision-making process is vital in developing effective classroom practices that promote student learning consistent with student-centered instruction. The results of this study may inform the researchers how they could develop evidence-based training programs that support both novice and experienced teachers in improving their teaching practice and ultimately enhance the quality of education provided to students.

2 Literature Review

2.1 Responsive Teaching Practice

Hammer et al. (2012) said a responsive approach to teaching "is to adapt and discover instructional objectives responsively to student thinking" (p. 55). In responsive or student-centered teaching, first students engage in a provocative task or situation where they actively capture their interest and encourage them to become invested in the lesson. Then teachers support their engagement by observing and listening to the students' thinking from what they are doing. In this way, teachers may be able to select and pursue students' thinking in a strategic way by recognizing and building on a more specific target that students are already working on. Dyer & Sherin (2016) found that teachers' responsive practice over time becomes stable and teachers use three common types of instructional reasoning to interpret student thinking such as (a) making connections between more than one specific moment of student thinking, (b) considering the relation between the mathematics of student thinking and the structure of a mathematical task, and (c) developing tests of student thinking. By engaging in responsive teaching, teachers can bolster students' reasoning and argumentation by providing opportunities for students to engage in collaborative discussion (Conner et al., 2014). The researchers found that in the study, the teachers employed three forms of support for collective argumentation: "directly contributing argument components, asking questions that elicited parts of arguments, and using other supportive actions" (p. 417). Teachers can support students by building on their reasoning as well (Boaler & Brodie, 2004; Conner et al., 2014; Leatham et al., 2015).

2.2 Teachers' Noticing and Decision-making

Kooloos et al. (2022b) found that five factors affect how teachers make sense of the students' thinking process: preoccupation, flexibility, exemplification, projection, and incomprehension. These categories showed how teachers' own thinking can support or impede their ability to make sense of students' ideas. They also found that sense-making of students' thinking requires teachers' reflective thinking of engaging or reengaging in the mathematical content and their own thinking. Teachers often engage in patterns of responses to student thinking (Serin, 2018). These responses require decision-making depending on many key factors. In a study of decision-making, Kahneman (2003) found that two distinct processes manipulate human decision-making. System 1 is "fast, automatic, effortless, associative, implicit (not available to introspection) and often emotionally charged" (p. 698), while system 2 is "slower, serial, effortful, more likely to be consciously monitored and deliberately controlled" (p. 698). They concluded that most judgments are made based on "intuitive, skilled, unproblematic, and successful" (p. 717) processing of the information. By closely investigating teachers' classroom routines, it could be observed that the same phenomena of fast and intuitive judgment are going on in their decision-making process, such as teaching becoming automated among expert teachers as they are accustomed to relying on the established classroom routines (Leinhardt et al., 1986). For example, Stockero et al. (2017a; 2022) found that teachers tend to direct their responses to the student who asked a question and try to include students' thinking in their responses. Thus, we can conclude that practice can reduce teachers' cognitive load and become more adept at implementing classroom routines as they develop a greater degree of flexibility (Schoenfeld, 2011).

2.3 Teacher’s Questioning

Teachers’ questioning strategies are a crucial aspect of student-centered learning environments by creating and maintaining the quality and the nature of the classroom discourse. Boaler & Brodie (2004) observed that even teachers who were using the same curriculum generated very different classroom discourses from each other. For example, they observed that some teachers used "closed" questions that lead to a single correct answer and were used to check for understanding, while others used "open" questions that allowed for multiple possible answers and encouraged student thinking and exploration. They also found that some teachers used "recitation" style questions that focused on memorization and repetition, while others used "exploratory" questions that promoted reasoning and reflection. They found that if teachers were unable to get many students to engage in the whole class discussion, then they tend to revise their initial question by again asking the contributor about the contribution to ensure that every student has a clear idea about the ongoing discussion. In this way, teachers were looking to engage more students. Bansal (2018) found that teachers create specific opportunities such as open conversational space for students to participate in classroom discourse. They also found that what type of questions teachers ask can push the student to make their thinking explicit. Analyzing different types of teachers' questioning can help us understand how pre-service and in-service teachers engage with their students (Ellis et al., 2019).

Boaler & Brodie (2004) also found that what questions teachers ask shape students' mathematical thinking significantly and direct the class discussion by enhancing cognitive opportunities. They found that some questions influence students to participate in the discussion. However, questions can also deteriorate opportunities for sense-making if teachers fail to ask questions that engage students in such activity (Bishop, 2008; Franke et al., 2009). Thus, asking productive questions that engage students in sense-making is an important part of classroom instruction.

Productive teacher questioning is important to help both elicit a student's thinking completely and to build on that thinking by engaging the whole class in a discussion. For example, sometimes teachers' sequential questioning (back-and-forth specific questioning) can help students to explain their ideas more completely by joining the missing parts together or by elaborating more explicitly (Bansal, 2018; Franke et al., 2009). Franke et al. (2009) found that students respond accordingly to the specific questions that a teacher asks. For example, when a teacher tries to make explicit a student's idea by continuously asking specific questions like a revoicing or repeating question, the student typically elaborated on their contribution by completing or adding words to the earlier comment to make it clear and complete. Similarly, when asked to describe strategies, the student explained the steps that they have taken to solve the problem to the class. When the teacher asked questions to the whole class about a strategy, and whether it was the right way to solve the problem, students gave their opinion and engaged in the discussion by agreeing or disagreeing with the strategy. Teachers' in-the-moment decision-making can support building on student thinking when they use questions that, for instance, seek more information from the class about the students' thinking (Kooloos et al., 2022a) or invite students to evaluate a claim, reflect on their shared ideas, give their reasoning behind the contribution, solve a new problem using earlier ideas or provide counterexamples for their claims (Cengiz et al., 2011).

Students also build knowledge about how to question their own work from teachers' questioning and try to analyze their own work (Boaler & Brodie, 2004). In general, productive questioning is essential in helping teachers to build on students' thinking.

However, not every question is helpful for developing students' thinking. Bishop (2008) found that experienced teachers, when faced with challenges, sometimes try to overcome them by buying some time by asking questions to other students or asking for an explanation from the contributor who asked the question to gather more information or find counterexamples. Additionally, sometimes experienced teachers asked questions when they find themselves having continuous classroom discourse, trying to have a gap between the continual dialog and "stand back from the action" (Bishop, 2008, p. 34). These questions to buy time or have a gap in the dialog may not be helpful if they do not engage students in mathematical thinking. Often teachers asking specific questions to students, like clarifying or elaborating on their ideas when their ideas are incomplete and ambiguous, is not enough to get a correct and complete explanation of the student's thinking (Franke et al., 2009).

2.4 Engaging Students in Sense-making

Engaging students in the sense-making process through whole-class discussion follows NCTM's (2014) vision of teachers supporting students' mathematical thinking. Researchers (Stockero et al., 2017a; 2022; Van Zoest et al., 2022) have uncovered subtle but significant variations in how teachers act as a facilitator and engage students with student contributions.

In a study of how teachers respond to a common set of student contributions, Stockero et al. (2022) examined whether teachers were differentiating their responses based on the mathematical potential of the contribution, as well as whether they were directing their responses based on that same mathematical potential. They found that regardless of the mathematical potential of a student contribution, teachers tended to direct their responses to the student who contributed the idea and occasionally directed their responses to the whole class (Stockero et al., 2017a; 2022). While directing responses to the same student can be productive in cases where clarification is needed, it may hinder teachers from engaging the class in a discussion of a high-potential contribution, not allowing all students to collectively make sense of the mathematical idea behind the student contribution by sharing their thinking (Stockero et al., 2017a). However, if they did direct their response to the whole class then it was mostly in response to high-potential contributions (Stockero et al., 2022). On the other hand, in response to lower potential contributions, teachers themselves often became the actors in response to the student thinking. This led to the conclusion that teachers were able to differentiate who they directed their responses to based on the mathematical potential of the instances (Stockero et al., 2017a; 2022).

Van Zoest et al. (2022) found that teachers were trying to take actions that could facilitate productive discussions in the classroom and prioritize conceptual understanding among the students rather than simply seeking correct answers. For example, asking for clarifying and connecting questions to clarify and connect student ideas, supports the development of students' mathematical concepts. The researchers anticipated that connecting and clarifying the students' ideas first and then asking them to justify their answers as a routine could serve as a template for novice teachers in facilitating classroom discourse. The researchers found that most often teachers ask justifying and developing questions which could lead to the potential of productive discussion. Depending on the students' thinking, the researchers established that teachers ask repeat and clarify questions for seeking more clarification to make the content of the idea understandable to the students before asking them to make sense of it. Studying teacher responses provides insight into the subtle differences resulting from variations in teacher responses to the mathematics of the student contributions (Van Zoest et al., 2022).

Stockero et al. (2022) also examined teacher actions in response to student contributions. Their findings revealed that teachers employed developing questions in approximately one-third of MOST related instances to actively engage students in the discourse. Teachers also asked adjourn, clarify, and justify questions for MOST related instances. Interestingly, the researchers observed a higher percentage of teachers were asking developing and justifying questions in response to MOSTs than non-MOSTs. However, as noted above, these questions were predominantly directed towards the same student (Stockero et al., 2017a; 2022). Conversely, adjourn and clarify questions were addressed to the same student for non-MOSTs, indicating that teachers adapt their responses based on differences in the student thinking. Stockero et al. (2017a) argued that teachers who prompted the whole class to justify the responses about the MOST seemed to be focused on the broader context, whereas those who directed justifying questions to the contributing student (same student) aimed to gather more information. This argument highlights the importance of both the actor and action with the student contribution in a classroom discourse.

Van Zoest et al. (2022) found that how teachers include students' ideas and actions in their comments expresses if teachers are responsive as well as help to identify the nuanced ways in which responsiveness occurs. For example, if we looked into Stockero et al. (2022) findings, it is noticeable that a substantial number of teachers more often either explicitly or implicitly used the student's own words when responding to MOSTs compared to non-MOSTs. The findings indicated that the ideas expressed by students played a fundamental role in the responses provided by the teachers, particularly in response to the MOSTs. Additionally, Stockero et al. (2017a; 2022) found that a significant proportion of teachers incorporated student ideas as a core part of their responses, thus highlighting their consideration of student contributions, particularly with respect to MOSTs. This evidence supports the notion that teachers actively take into account the ideas and actions of their students by differentiating their contribution based on their level of productivity.

2.5 Reasoning Behind Teacher's Decision-making

The University of California at Berkeley's Teacher Model Group (TMG) established a general cognitive theory that hypothesized that one can understand why a teacher does what they do and how, by exploring teachers' knowledge, goals, and beliefs (Schoenfeld, 2008). How teachers interpret and perceive student contributions influences their decision-making process.

Teacher beliefs about their role and students' ability to understand mathematical thinking is the crucial element of a student center classroom (Beswick, 2007). In student-centered classrooms, students are active learners and teachers facilitate and guide students to explore, experiment, and reflect on their own thinking, leading to a deeper and more meaningful understanding (Seng, 2014). Beswick (2007) found that beliefs about the discipline of mathematics, the teacher's role, and students' capabilities were central to creating student centered classroom. These beliefs emerged in diverse classroom contexts and may interact with other beliefs to influence teacher behavior (Beswick, 2007).

Research has found that teachers often do not ask questions that will engage students in the sense-making process (Stockero et al., 2022). One reason that teachers do not capitalize on teachable moments may be that teachers often develop a classroom routine –either productive or unproductive–which is consistent with their knowledge, goals, and beliefs that helps them to communicate with students and sort out how to approach students' thinking (Schoenfeld, 2008). Another reason is that teachers make decisions about any unpredictable event based on their beliefs and the degree of importance to their specific goals (Schoenfeld, 2011). However, this is problematic if their decisions are influenced by biases or norms that lead students' mathematical thinking in a direction where students are not positioned to make sense of the teachable moment. Furthermore, their high and low-priority goals also influence their decision on how to respond to student's questions and they analyze it based on the cost-benefit relationship among other factors and students' thinking (Schoenfeld, 2008).

Not all teacher beliefs would support building on the students' thinking with whole-class interaction. Stockero et al. (2020) found that teachers' beliefs can either "support or hinder the development of the practice of building on student mathematical thinking" (p. 256). These researchers added that teachers' beliefs that it is productive when students interact with other students' ideas and critique, discuss, and compare them could be productive for sense-making as well as support the building practice of student thinking.

However, when teachers believe that a student's contribution is required to be corrected by a teacher or needs evaluation before the other student could understand it, could be counted as unproductive and led to hindering the building practice of student thinking.

Bray (2008; 2011) found that certain aspects of instruction were more influenced by teacher beliefs, while others were more influenced by teacher knowledge. For example, teachers' beliefs affect the ways in which they structure class discussions when errors arise, including the roles they take on for themselves and assign to students, while teacher knowledge is the primary determinant of the mathematical and pedagogical quality of teachers' responses (Bray, 2011). Teachers’ responses to students’ questions depend on both their general and specific knowledge about the subject (Schoenfeld, 2008, 2011). The chances of teachers responding to students in ways that promote conceptual understanding is related to the teacher's knowledge of relevant mathematical concepts, student strategies and misconceptions, teaching strategies, and the ability to interpret student work in the moment (Bray, 2011).

A visible discrepancy is sometimes noticeable between how teachers structure their lessons and how they teach their classes. Cross (2009) found that teachers' beliefs about mathematics instruction were generally consistent with broader educational goals, but there were some discrepancies between their beliefs and their actual teaching practices. The researcher also found that teachers' beliefs were cohesive and interconnected. Teachers with strong beliefs about specific instructional practices were less likely to change their practices in response to new information or experiences. However, some teachers were able to revise their beliefs and practices in response to professional development and collaboration with colleagues. Therefore, efforts to improve mathematics education should focus on supporting teachers in developing more cohesive and consistent beliefs about student-centered instruction by promoting flexibility and openness to change. More work needs to be done to understand the professional learning experiences that can support changing beliefs (Beswick, 2007).

3 Theoretical Framing of the Study

Researchers (e.g., Fennema et al.,1996) have discussed teachers building on students' mathematical thinking. However, practices such as eliciting and extending student thinking (Cengiz et al., 2011) and simultaneously ensuring students understand what is being taught and engaging them in a productive discussion about other students' ideas is challenging for teachers (Sherin, 2002; Silver et al., 2005). Teachers' approaches often don't limit to only building on student's thinking but tend to go beyond their interpretation of students' mathematics to solve the problem (Milewski et al., 2021).

There are several key ideas from prior research that were used to frame my research, such as the MOST analytic framework, the practice of building on student thinking, the Grapple Toss element of building, and the Teacher Response Coding framework. These ideas are important to understand the theoretical foundation of this research and are discussed in the following sections.

3.1 Background of MOSTs

Building on the students' ideas depends on recognizing and interpreting the students' contributions (Leatham et al., 2015). Sometimes, teachers are unable to notice important student mathematical thinking or are not able to understand student ideas (Peterson & Leatham, 2009; Stockero & Van Zoest, 2013). Teachers can sometimes decide not to build on some student ideas, but they also miss opportunities to recognize an important student idea which could be helpful to build on during a whole class discussion. This is especially true for novices, as they lack the knowledge of representations or are unable to make a connection of the student contribution to the context (Peterson & Leatham, 2009; Stockero & Van Zoest, 2013). Leatham et al. (2015)

asserted that teachers need to focus on *which* student contributions are productive to engage students with during instruction, not just on *how* they use student thinking. These limitations identified in earlier research about teachers' use of student thinking inspired Leatham et al. (2015) to create the MOST Analytic Framework to help teachers to notice high-leverage student instances.

A Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) (Leatham et al., 2015) is a high-leverage student contribution that, if turned over to the whole class by the teacher, would engage students with each other in a whole-class conversation and help students make sense of the student's contribution. A MOST is a teachable moment and could foster students' sensemaking if questions are asked that center the MOST in the whole class discussion. Not every student contribution is important enough that could facilitate students in making sense of the mathematics, so not all contributions are MOSTs (Stockero et al., 2017b).

MOSTs are instances that occur in the classroom and have three characteristics: (a) student mathematical thinking, (b) significant mathematics, and (c) present a pedagogical opportunity for the teacher. In simple terms, an important student contribution, a teachable moment that the teacher could turn over to the class to make sense of is a MOST. Leatham et al. (2015) initially thought in their theory that it is normal for teachers to productively use students' thinking but in reality, studying the MOSTs suggested that this type of practice rarely happens in the classroom. These instances thus create important decision points where teachers have to decide how to use the student contribution to support their instruction.

3.2 Building

Once a MOST is recognized by the teacher, they have to introduce the MOST to the whole class by asking a question to engage students in the sense-making process (Leatham et al., 2021). Instead of engaging students in making sense of a contribution, if a teacher used other moves, such as correcting the student's contribution, then it could hinder students' opportunities to make sense of the mathematics of the MOST (Van Zoest et al., 2022). Van Zoest et al. (2016) have theorized that the most productive way to use a MOST during instruction is to *build* on it. They conceptualized the teaching practice of building as "several teacher moves woven together to engage students in the intellectual work of making connections between ideas and abstracting mathematical concepts from consideration of their peers' mathematical thinking" (p. 1284).

In other words, the teaching practice of building on the MOSTs includes the teacher turning a MOST over to the class for students to make sense of it. An engaging whole classroom discussion is important to build on the MOSTs. The MOST project team's conceptualization of building consists of four elements of the practice (Leatham et al., 2021, p. 1393):

- *Establish* the student mathematics of the MOST as the object to be discussed.
- *Grapple Toss* that object in a way that positions the class to make sense of it.
- *Conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST.
- *Make Explicit* the important mathematical idea from the discussion.

3.3 Grapple Toss

I am focusing my research on the second element of the practice of building on a MOST, which is Grapple Toss. After the teacher establishes an important student contribution– a MOST–the teacher needs to ask a question to the whole class to prompt them to make sense of the MOST. This question is known as the Grapple Toss.

Building on students' thinking depends on how a teacher turns the MOST over to the class and what question a teacher asks the class. In other words, how teachers decide to Grapple Toss a MOST to the class will directly affect the engagement of the students with the MOST by making sense of it and not going away from the MOST. So, teachers have to decide carefully what they want students to do to understand the MOST. Grapple tossing a clear, but open-ended question will help students to think about the MOST by making sense of it and directing them toward a high-leverage conversation. A good Grapple Toss question will position students to grapple with the MOST and support students' mathematical thinking by making sense of the MOST. Types of grapple toss questions and examples of each that have been identified in prior research (Leatham at al., in press) are shown in Table 3.1.

Table 3.1 Grapple Toss questions determined by the nature of the MOST (Leatham et al., in press).

Generalized Grapple Toss Questions	**Example**
What do you think, [established object]?	What do you think, can a linear equation have two y-intercepts?
What about [established object] holds up mathematically?	What about this reasoning [references argument written on the board: Point A is (3,0), so x+y=6 would be 3+y=6, which means y=3. So, point B would be (0,3).] holds up mathematically?
What is going on here, [action on established object]?	What is going on here? Why is 9 wrong?
How do you decide which of [established objects] are correct?	How do you decide which of the two arguments that Susan and James have provided is correct?

3.4 Teacher Response Framework

To promote and support students' thinking, teachers employ various strategies including asking different types of questions, such as probing questions, encouraging student explanations, and providing opportunities for students to articulate their reasoning (Stockero et al., 2017a). Not only asking the questions is important, but also who gets a chance to engage with it, whether students could recognize their word and ideas in the teacher response, and if teachers' responses are aligned with the mathematical points to the student contribution are as important (Van Zoest et al., 2022). Therefore, to make the difference between engagement obvious the researchers developed the Teacher Response Coding (TRC) framework to understand how teachers respond to student thinking in the classroom. Peterson et al. (2017) defined the purpose of the TRC as disentangling the teacher's move from other aspects of the teacher response.

The TRC considers four aspects of teacher responses. The first TRC category is actor; this category captures who is given the opportunity to engage with a student's mathematical contribution. For example, if a teachers' response was adding additional information by not considering the student idea or only extending the student's reasoning by themselves as a part of their response then the Teacher is the actor. However, if a teacher response invites the same student who contributed an idea to add information to the context, then the actor is the same student. In a grapple toss question during building, the teacher response to the MOST should be positioning the whole class to make sense of the MOST and the whole class would be the actor.

The second TRC category is action, which the researchers defined as how students might interpret the teacher's response rather than inferring the teacher's intent or what the actor will do in response. For example, if a teacher asks a question inviting students or pausing for a few seconds for students to share their ideas, this is Allow for the action category. However, if the teacher was asking a question indicating students to share additional examples, this is Collect. If a teacher was asking the class to provide information to prove the correctness, expand the student contribution more than a simple clarification, or justify or give their reasoning why they agree or disagree with the MOST, this is known as Evaluate, Develop, and Justify respectively. These actions known as Evaluate, Develop, and Justify would be desirable for building as this gives the student an opportunity to share their reasoning about the contribution.

The researchers used the definition of responsive teaching to draw attention to how teacher responses connect with a student contribution. The authors divided the third TRC category, "how" into two aspects: Student Recognition and Mathematical Alignment. To what extent the student would recognize their contribution in the teacher's response is what Student Recognition captures. The researchers noticed that the student who contributed an idea might recognize their contribution in the teacher’s response if the teacher provided attention to Student Actions and attention to Student Ideas. A good grapple toss question would be if the student’s contribution was the core or peripheral idea in the teacher’s response as well as at the same time the teacher’s actions included the student words explicitly or implicitly enough for a student to recognize their idea in the teacher response. Mathematical Alignment is the degree to which “the mathematics in the teacher response – the mathematics the teacher seems to be moving toward – aligns with the mathematical idea most closely related to the [student contribution]” (p. 2594). In a grapple toss question, the teacher’s mathematical focus would be core or peripheral to the mathematical point of the MOST. In this study, I used the TRC framework to code the teacher’s responses to understand to whom, what, and how the questions are presented.

3.5 Research Questions

What types of questions teachers ask to engage students in making sense of MOSTs is an important part of their decision-making during classroom instruction. Teachers often do not ask questions that would engage students in a whole-class sense-making discussion, which is an important component of student-centered instruction. In Stockero et al.'s (2017) scenario interview, the researchers asked an open ended question to the teachers about how they would respond to a student contribution (i.e., What would you do next?), whereas in this research, teachers were specifically asked to engage the whole class with a student contribution (i.e., what would be your next move or question to engage the class in making sense of the mathematically important idea raised by a student contribution?). This question has the ability to give teachers a little bit of direction and let the teachers know what they should be doing with a high potential student contribution. In this research, teachers' belief and their thoughts behind the productivity of a set of questions that teachers were found to ask in prior work (e.g., Leatham et al., in press; Peterson et al., 2022) were also captured, which could be supportive to fill the gap between the theories and the teaching practices.

To better understand teachers' questions which they ask to engage the class in sense-making discussion, this research will be taking into consideration the questions teachers ask and their reasoning behind asking these questions by answering the following research questions.

- What types of questions do teachers indicate that they would ask to engage students in making sense of a given MOST? (Research Question 1)
- Given a set of questions that teachers have been found to ask to engage students with MOSTs in prior data, to what extent do teachers think asking each question would be a productive next move to engage students in a whole-class discussion around a given MOST? (Research Question 2)
- What influences teachers' decision to select or not select specific types of questions to engage the class with a MOST? (Research Question 3)

4 Methodology

4.1 Participants

In this study, an invitation to participate in a scenario-based survey questionnaire (see Appendix) was emailed to 12 middle and high school mathematics teachers who had participated in a prior research study requesting them to forward the email to their colleagues. The target sample size was at least 50 teachers. I contacted the teachers again when the achieved sample size was less than 50 to get my target sample size. A total of 65 middle and high school mathematics teachers from different areas in the US participated in the scenario-based questionnaire. The participants were all in-service teachers, and not filtered out based on their level of experience or any other criteria.

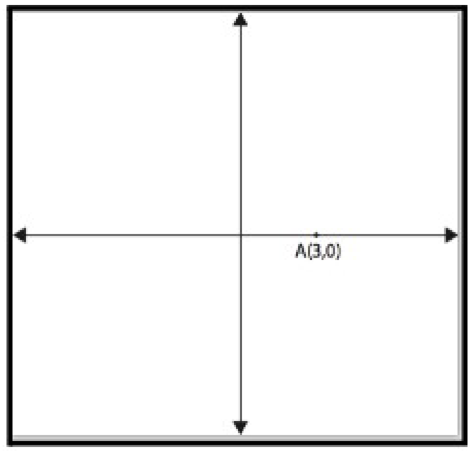
4.2 Data Collection

A survey was distributed using Qualtrics. Participants were first asked to give their consent to participate in the study. If they agreed, they could proceed to the next part of the study. If they disagreed, they would not be able to continue. After giving their consent, they were presented with a scenario in which a mathematically important student contribution, a MOST, had surfaced, and they were asked three questions about that scenario (see Appendix).

In the scenario, the teachers were told that students in Ms. Kamara's mathematics class had worked on a task and had been invited to share their ideas. Alexis shared an idea that Ms. Kamara recognized as a MOST (see Figure 4.1).

Ms. Kamara's mathematics class was working on this task:

Points on a Line Mini-Task
Is it possible to select a point B on the y-axis so that the line $x + y = 6$ goes through both points A and B? Explain why or why not.



After several minutes of individual work and an invitation to share ideas, one student, Alexis, made the following contribution:

Alexis: "If you plug $x = 3$ into the equation, you get $y = 3$. So, B is (0, 3) because 3 + 3 = 6."

Ms. Kamara recognized that Alexis's comment provided an opportunity for the class to make sense of a mathematically important idea in the lesson and established the contribution by recording it on the board.

Figure 4.1. Grapple Toss Scenario-Based Questionnaire (adapted from a survey developed by Horizon Research, Inc.)

After reading the scenario, the teachers were asked an open-ended question in part 1: "What would be your next move or question to engage the class in making sense of the mathematically important idea in the student contribution? Please explain your reasoning."

In part 2, teachers were presented with five potential questions drawn from prior classroom observations by the MOST project research team (see Building on

Mathematical Opportunities in Student Thinking, 2023; Leatham et al., in press; Peterson et al., 2022) (see Table 4.1). Each question was found in prior work to vary in productivity. Participants were asked to indicate how productive they believed each prompt was as the next move to engage the class in making sense of the mathematically important idea raised by MOST. They were also asked to describe why the prompt was or was not a productive next move. Table 4.1 lists the Part 2 survey question prompts and an explanation of their productivity based on prior data.

In part 3 of the questionnaire, participants were asked if their initial thinking had changed or if they would ask a different question after seeing the list of potential questions from the second prompt. They had the opportunity to change their grapple toss question (or not) and provide the reasoning behind their decision.

Table 4.1 Part 2 Survey questions with notes on the productivity of the questions.

Number	Part-2 Survey Question	Notes about Productivity
1	Ask the whole class, "Can I have a volunteer rephrase the idea Alexis has shared?"	This question is often not productive. When students rephrase another student's contribution, they often change the meaning or substance of the contribution.
2	Ask a student who solved the task differently "I saw that some of you had different ways to solve the task. Brian, can you share a way that is different from Alexis's idea?"	This question is not productive as sharing a different way of solving the problem won't help students to make sense of the MOST.
3	Tell the class, "Turn to your table group and discuss whether you agree or disagree with Alexis's idea, and why."	This question may be productive for the small group but less productive for the whole class discussion as students often just report back to the whole class about their conclusion, which could be counted as a takeaway from the discussion.
4	Ask the whole class, "How might you argue whether Alexis's claim and reasoning hold up mathematically?"	This question is often productive. This question encourages students to share their reasoning about the claim.
5	Ask the whole class, "What do others think?"	This question is often not productive. When asked a vague question like this, students often share anything they have in their mind which could distract the class from engaging with the MOST.

4.3 Data Analysis

4.3.1 Research Question 1

To answer Research Question 1, I looked at what types of questions teachers say they would ask to engage the whole class in a sense-making discussion about the MOST. For the open-ended responses from Parts 1 & 3 of the survey, I coded the collected teacher responses with the TRC framework (Van Zoest et al., 2022). The TRC framework was used to analyze teachers' in-the-moment responses to observe how teachers facilitate student thinking, including the types of questions they asked, to whom they directed those questions to, as well as how they use the student contribution in their response and how close the mathematics they are focusing on in their response is to the MOST mathematical point. For ensuring reliability in the coding, two researchers individually coded the data. Each coder independently coded the data based on the predetermined TRC framework. The coders engaged in a collaborative process through discussions and meetings to reconcile and reach a consensus on the coding decisions (Saldana, 2013). If there was disagreement between coders during the reconciling process, we used the common approach to involve a third coder for resolution (Stockero et al., 2022). The involvement of a third coder was to mitigate any potential bias in the coding process and aims to reach a consensus on the coding. In this coding, the third coder reviewed the coding of the two coders and any comments they captured about their coding and made their own independent judgments. By involving a third coder for resolution, the aim is to enhance the reliability and validity of the coding process.

In research question 1, the teachers were asked, what would be their next move to engage the class in making sense of the mathematically important idea raised by Alexis' comment. Therefore, to code their responses the Teacher Response Coding Scheme (TRC) (Van Zoest, et al., 2022) was used to code teachers' responses to part 1 & 3 of the survey. Two coders have coded each teacher's responses based on four TRC categories:

Actor: Who is engaging with a student's contribution

Action: What the actor will do in response

Student Recognition: The extent to which students would recognize their contribution in teacher's responses.

Mathematical Alignment: The degree to which the mathematics in the teacher response aligns with the mathematics in the student contribution.

The two coders met weekly to reconcile their individual coding and came to an agreement about the coding. In Figure 4.2, the TRC framework codes are provided.

TRC Category	**Codes**	
Actor	Teacher Same Student(s) Other student(s)	Whole class Whole class (small group) Indeterminate
Action	Adjourn Allow Develop Repeat Dismiss Evaluate Justify Literal	Check-in Clarify Collect Connect Correct Validate Indeterminate
Student Recognition	Student(s) Actions	Explicit Implicit Not Indeterminate
	Student(s) Ideas	Core Peripheral Other Indeterminate
Mathematical Alignment	Core Peripheral	Other Indeterminate

Figure 4.2 The Teacher Response Coding Framework (TRC) (Van Zoest, et al., 2022)

To provide a sense of the TRC coding, I will discuss here examples (see Table 4.2) of responses coded with the different categories (actor, action, student recognition and mathematical alignment).

In the first response in Table 4.2, the teacher asked the question, “Why did you choose x=3?” to the contributing student. Therefore, the Same Student is coded as the actor. After analyzing the teacher’s response, it was visible that the teacher was asking the student to justify for employing the specific value of x. This leads to code the teacher action as Justify. From the teacher’s action it would be apparent to the student that the teacher is using their words and idea as the core of the question, therefore the student recognition action was coded as Explicit and the student recognition idea was coded as Core, respectively. However, the teacher’s responses did not directly focus on the mathematical point of the contribution, as the students would need to take a leap of logic, so mathematical alignment was coded as Peripheral. In the second response, the teacher asked, “I would ask the class who agrees with Alexis' comment and who disagrees with her comment” to the Whole Class. By looking into the teacher’s response, one can easily observe that the teacher was asking students to determine the correctness of the student contribution. This action is defined as Evaluate. From the teacher’s response, the students would be able to understand that the teacher is talking about Alexis’s contribution as they use their name but stop short of using their words because of conversational conventions.

Table 4.2 Survey Part 1: Examples of teachers' responses with TRC coding

Teacher's Response	**Actor**	**Action**	**Student Recognition - Action**	**Student Recognition - Idea**	**Mathematical Alignment**
Why did you choose x=3?	Same Student	Justify	Explicit	Core	Peripheral
I would ask the class who agrees with Alexis' comment and who disagrees with her comment.	Whole Class	Evaluate	Implicit	Core	Core
I would write the equation with the substitution of 3 and 3 for x and y and add the x and y labels for both numbers. Then I would graph (0,3).	Teacher	Clarify	Explicit	Core	Indeterminate

Therefore, student recognition action is coded as Implicit, and idea is coded as Core as the student could recognize their idea from the response. Additionally, the teacher's response was closely aligned with the mathematical point of the students' contribution because it asked students to evaluate it, which led to code mathematical alignment as Core. In the third response, a teacher said, "I would write the equation with the substitution of 3 and 3 for x and y and add the x and y labels for both numbers. Then I would graph (0,3)", where Teacher is the actor as only the teacher will be interacting with the student contribution. It's evident that the teacher was providing information to Clarify the contribution. From the teacher's action it would be apparent to the student that their words were Explicitly used by the teacher in their response. Additionally, the teacher was using the students' idea as the core of the response, so the student would recognize their idea as the Core of the response. However, it is unclear whether the teacher's response is going toward the mathematical point of the contribution, so mathematical alignment was coded as Indeterminate.

4.3.2 Research Question 2

To answer Research Question 2, I summarized the data from part 2 of the survey to notice to what extent in their response do teachers think asking each question would be a productive next move to engage students in a whole-class discussion around a given MOST. I used a 5-point Likert scale to measure which questions teachers found to be productive, ranging from "Very Unproductive" to "Very Productive". This scale allowed me to capture varying levels of productivity and obtain quantitative data on teachers' opinions. I calculated the frequency of each response for the five questions. This frequency provided an insight into teachers' perceptions and preferences regarding the productivity of different types of questions asked in this part of the survey. This helped me to analyze which questions teachers think are most productive, neutral, and most unproductive.

4.3.3 Research Question 3

To answer Research Question 3, I analyzed teachers' justifications provided in part 2 of the survey to see what beliefs influence teachers' decision-making for selecting or not selecting specific types of questions. To code the reasoning for teachers' questioning, I drew on Schoenfeld's (2008) work related to knowledge, goals, and beliefs. Without a predefined framework for coding teacher's beliefs, an inductive coding approach (Thomas, 2006) was employed to analyze and categorize their reasoning.

In the initial phase of coding, the teachers' reasonings were categorized into broad categories to provide an initial framework for analysis. In the second phase, belief statements were written based on the broad categories for each response as well as the teacher's words. Teachers' responses from the same broad categories were coded similarly when appropriate to identify commonalities among the beliefs. In Table 4.3 below, a sample of teacher beliefs coding is provided. For example, a teacher responded to prompt 5 (What do others think?) as, "It is too open ended and while it may get some people thinking it won't grab quite as many on a deep level." This teacher's belief was coded as "Asking what others think is too vague for students to be able to think in a deeper level." In the third phase, similar teacher's beliefs were grouped together to form themes, aiming to identify common beliefs among the teachers. For example, in the above example the teacher's belief was themed as Nature of Question as the teacher is sharing their concern about the productivity of the wording of the question.

It is important to note that the focus of this analysis became solely focused on teachers' beliefs, as the responses provided by the teachers only gave insights into their beliefs rather than their level of knowledge regarding the mathematical concepts or their objectives for the classroom or curriculum. This coding process helped to establish a cohesive and overarching perspective on the teachers' reasoning. To ensure the reliability and validity of the coding process, a second coder independently reviewed the codes and marked their agreement or disagreement with the initial coding. Any disagreements in coding were resolved through discussions between the coders. These discussions aimed to reach a consensus and ensure consistency in the interpretation and application of codes. By employing this collaborative approach and involving a second coder, the study sought to enhance the rigor and reliability of the coding process, promoting a more robust analysis of the teachers' beliefs.

Table 4.3 Survey Part 2: Examples of teachers' responses with Belief coding.

Ask the whole class, "What do others think?"	Teacher's Response	Teacher Belief	Theme
Somewhat Productive	It is too open ended and while it may get some people thinking it won't grab quite as many on a deep level.	Asking what others think is too vague for students to be able to think in a deeper level.	Nature of Question
Very Productive	This allows others to state what they think about Alexis's ideas. They have to make sense of her ideas in order to state what they think about it.	Asking what others think is a productive way to open up a discussion for students to provide their mathematical reasoning/ justifications for the claim.	Justify
Very Unproductive	Not likely to gain many responses from class- other than the couple of students who are always, or almost always, actively engaged.	Asking what others think may only engage a few students with the idea.	Participation

5 Results

5.1 Research Question 1

Research Question 1 focused on understanding the types of questions teachers indicate they would ask to engage the whole class in a sense-making discussion about the MOST. Part 1 of the survey provides information about the questions they would spontaneously ask. From the result of part 1 of the survey, about half of the teachers (34 of 64; see Table 5.1) provided responses that would engage the whole class with the MOST, including two who said they would engage the whole class in small groups. Engaging the whole class with the MOST would be productive for building on the student thinking (Table 5.1). This suggested that half of the teachers were explicitly following the prompt to engage the class in a whole class discussion. The most common actions provided by teachers to engage the class with the student contribution were to evaluate (10 of 64 teachers), dismiss (8 of 64 teachers), correct (8 of 64 teachers), collect (7 of 64 teachers), and develop (6 of 64 teachers). However, prior research suggests that the actions of dismissing, correcting, and collecting may not be effective in engaging students in sense-making discussion as these actions could hinder student thinking as by taking these actions teachers were restricting the class from the opportunity to make sense of the MOST. On the other hand, by asking questions that require evaluation, development, and justification, teachers provide students with more opportunities to articulate and elaborate on their reasoning about the MOST which is productive for building. Therefore, it was evident that 30% of the teachers (19 of 64) were asking productive questions to engage the class in a sense-making discussion. This indicates that only about one third of the

Table 5.1 Summary of teacher response coding of moves to engage students from part 1 of the survey (n* = 64 responses).

Actor		**Action**		**Student Recognition - Action**		**Student Recognition - Idea**		**Mathematical Alignment**	
Whole Class	32	Evaluate	10	Explicit	21	Core	33	Core	24
Whole Class (small group)	2	Dismiss	8	Implicit	23	Peripheral	21	Peripheral	16
Same Student	12	Correct	8	Not	19	Other	7	Indeterminate	24
Teacher	14	Collect	7	Indeterminate	1	Indeterminate	3		
Indeterminate	4	Develop	6						
		Connect	5						
		Literal	4						
		Repeat	4						
		Clarify	3						
		Justify	3						
		Validate	3						
		Allow	2						
		Indeterminate	1						

*Note: One teachers' response from part 1 of the survey was not coded as it was an incomplete response.

teachers naturally asked productive questions in promoting a deeper understanding and critical thinking among students to facilitate an effective learning environment. About 69% of the teachers primarily used students' words, either explicitly or implicitly (44 of 64), and also 84% of teachers mostly centered their responses around student ideas (54 of

64). This approach is likely to help position the students' ideas as valuable. Additionally, 63% of teachers' responses were mostly aligned to the mathematical points of the instance of student thinking (40 of 64) which showed that teachers were trying to ask questions that focused on the mathematics in students' contributions. These findings indicate that the majority of the teachers were spontaneously using students' words explicitly or implicitly and mostly centered students' ideas in their responses and mostly aligned their responses with the mathematical points of the student contribution, all of which are productive for building on student thinking.

From the result of part 3 of the survey, after the teachers had seen and responded to the part 2 questions that were drawn from prior research, it was evident that the majority of teachers in the survey (35 of 65) said that they would choose to actively engage the whole class in a small group to discussion about the MOST (Table 5.2). They preferred having the students work in small groups rather than as a whole class, which might be productive for building on the student's thinking if the small group later shared their reasoning with the whole class, not only the answers. Notably, this finding differed from the responses in part 1 of the survey, suggesting a shift in teachers' perspectives over the course of the survey.

Teachers' common actions when grappling the student contributions included evaluating, collecting, justifying, and repeating. However, prior research (Van Zoest et al., 2022) has indicated that actions such as collecting and repeating may not be as effective in engaging students in sense-making discussions. These actions might hinder student thinking and the opportunity to build upon students' ideas. On the other hand, as

Table 5.2 Summary of teacher response coding of moves to engage students from part 3 of the survey (n = 65 responses).

Actor*		**Action***		**Student Recognition - Action**		**Student Recognition - Idea**		**Mathematical Alignment**	
Whole Class	17	Evaluate	33	Implicit	63	Core	50	Core	39
Whole Class (small group)	35	Collect	12	Explicit	1	Peripheral	2	Peripheral	1
Same Student	2	Justify	8	Not	1	Other	13	Indeterminate	25
Other Student	13	Repeat	8						
Teacher	1	Connect	4						
		Develop	1						
		Literal	1						
		Dismiss	1						

*Note: The total actor and action codes are n = 68 respectively because some teachers responded to the part 3 question by selecting two responses from the part 2 of the survey, as they thought both of the questions could be productive to engage students in the discussion. This led to some double coding of the actor and action.

noted above, actions like evaluating and justifying (proposed by 39 of 65 teachers, including two teachers who were double coded Evaluate and Justify in their response) were found to be more productive in fostering a deeper understanding of student thinking. This finding suggests that after seeing the different questions, teachers were leaning toward asking more productive questions that aim to uncover and understand students' thinking.

Furthermore, it was observed that teachers implicitly used students' own words (63 responses) and centered their responses around student ideas (50 responses). These findings suggested that all the teachers tried to include students’ words, even though implicitly, except one teacher, in their response more often than from part 1 of the survey. In part 1, teachers used students’ words explicitly (21 responses) and implicitly (23 responses) which suggested that teachers have included students’ words both ways. However, in part 1, 19 teachers’ responses did not include any students’ words explicitly or implicitly and one response was not possible to infer what the teacher’s action was toward students’ words. This suggested that nearly one third of the teachers were not incorporating students’ words either explicitly or implicitly in their first response. However, the findings of part 3 showed that all but one teacher included students’ words implicitly in their response, suggesting that teachers’ responses shifted toward honoring student thinking more in part 3. Additionally, the teachers' responses were more aligned with the core mathematical points of the student thinking in part 3 than in part 1. This finding indicates that after teachers saw the questions in part 2, they were making efforts to ask questions that were closely related to the mathematics in students' contributions. It reflects a deliberate attempt to address the specific mathematical aspects of the students' thinking.

The results from both parts of the survey provide valuable insights into teachers' proposed actions when turning the MOST over to the class. The findings highlight a shift in teachers’ responses from part 1 to part 3. It could be noticed that this shift was evident after teachers were presented with the part 2 questions, as 57 of the 65 teachers selected questions from part 2 (especially Question 3 alone or some variation of Question 3) in their response to part 3 of the survey:

> W*hat do you now think would be the most productive next move to engage the class in a discussion about Alexis's comment? You can choose any of your answers from parts 1 and 2, or you can write a new next move that you think would be productive.*

An additional two teachers used their part 1 question along with a question from part 2 to engage the class in a sense-making discussion in part 3.

It is visible from the part 1 and part 3 results that teachers came to prefer engaging the whole class with the MOST by using small group work to build on student thinking. Teachers selected more productive questions that promote sense-making and understanding of student reasoning among the students after they were exposed to the alternative questions that were drawn from the prior research (Leatham et al., in press; Peterson et al., 2022). They also centered their responses around student ideas. In part 1, 69% of the teachers (44 of 64) used students' words explicitly or implicitly but in part 3 almost all the teachers (63 of 65) used students' words implicitly in their responses after seeing the alternative questions, and better aligned their responses with the mathematical points raised in student thinking than the earlier part of the survey. These findings point out that after seeing a variety of possible questions, teachers selected some more productive questions which prioritize student engagement, understanding, and meaningful mathematical discourse.

5.2 Research Question 2

In general, the teachers thought that the majority of the questions provided in part 2 of the survey were productive. However, from the teacher responses to part 2 of the survey (Table 5.3), it was visible that overall teachers rated Question 3 as the most productive. For this question, 57 teachers rated the response as very productive (42) or somewhat productive (15). Only 6 teachers rated it as unproductive. The majority of teachers also rated three other questions as productive: Question 1- asking to rephrase, Question 2 - asking for a different solution method, and Question 4 - asking how the claim holds up mathematically.

For Question 3, 42 teachers rated the response as very productive (15) or somewhat productive (27) and 17 teachers rated it as unproductive. Additionally, for Question 2, 49 teachers rated the response as very productive (28) or somewhat productive (21) and 10 as unproductive. Furthermore, for Question 4, 35 teachers rated it as productive and 9 as unproductive. However, interestingly 21 teachers thought that this question was neutral. The question that the teachers rated as least productive overall was question 5. For this question, teachers had a mixed response in the productivity of this question; 20 teachers rated it as productive, 23 teachers rated it as unproductive, and 22 teachers rated it as neutral. Therefore, it was evident that the teachers thought that this question could be both productive and unproductive to engage students in whole class discussions.

Among the productive moves, the teachers rated asking students to discuss their contributions in small groups as the most productive overall. This strategy would likely allow students to engage in a collaborative sense-making process by actively

Table 5.3 Summary of teachers' perceptions about the productivity of given responses from part 2 of the survey.

Proposed Teacher Response	**Very Unproductive**	**Somewhat Unproductive**	**Neutral**	**Somewhat Productive**	**Very Productive**
1. Ask the whole class, "Can I have a volunteer rephrase the idea Alexis has shared?"	4	13	6	27	15
2. Ask a student who solved the task differently "I saw that some of you had different ways to solve the task. Brian, can you share a way that is different from Alexis's idea?"	2	8	6	21	28
3. Tell the class, "Turn to your table group and discuss whether you agree or disagree with Alexis's idea, and why."	0	6	2	15	42
4. Ask the whole class, "How might you argue whether Alexis's claim and reasoning hold up mathematically?"	0	9	21	19	16
5. Ask the whole class, "What do others think?"	10	13	22	17	3

participating in the discussion. However, it is noted that this approach may not be as productive for a whole class discussion as for a small group discussion, as it can lead the task to be resolved within the small group, limiting the opportunity for the whole class engagement as well as conceal students' thought processes. This could hinder building on the student thinking in general. On the other hand, asking students if they have other thoughts was rated as the most unproductive strategy. This type of question can potentially divert the discussion away from the student contribution. It may result in a scattered discussion that lacks focus and does not provide opportunities to consider the MOST and share their thinking to the class. Interestingly, the question that in the prior research (Leatham et al., in press; Peterson et al., 2022) have found to be most productive for building on the students' thinking, which involves asking students if a claim holds up mathematically, was overall rated as productive for engaging students, although about one-third of the teachers rated is as neutral.

5.3 Research Question 3

From the survey results, various themes emerged regarding teachers' beliefs about the productivity of asking different types of questions to engage students with an idea. In the following paragraphs, I will be discussing the teachers' common beliefs that underlie their responses in the survey in comparison to each question.

Rephrasing the Student Idea. Among the 42 teachers who thought that asking a student to rephrase Alexis' idea (Question 1) was productive, 9 (21%) of them believed that rephrasing increases students' participation or engagement with the MOST. For example, one teacher said, "Asking this question can produce a large amount of response from others." Some teachers, however, thought the question was only somewhat productive because they believed that rephrasing engaged some, but not all students: "General questions to the class often end up with the same students engaging or being disengaged."

Another common belief, shared by 8 (19%) of the teachers who thought the rephrasing question was productive, is that asking students to rephrase helps the teacher gather feedback on how students interpreted the MOST. For example, one teacher stated that, "Rephrasing her idea allows teachers insight into what students understood and solidifies understanding." Additionally, 8 (19%) of the teachers believed that the rephrasing question was productive, as asking this question provides students the opportunity to make sense of the MOST. For example, one teacher said, "Having someone else rephrase what Alexis said will help other students understand what she said." One teacher thought the question was very productive but concluded that rephrasing alone was not enough for students to make sense of the idea: "Making sure students understand what Alexis's ideas is is key to helping students think deeply about the concept. Rephrasing can help accomplish this. With that being said, rephrasing alone won't help much at all."

On the other hand, the most common belief shared by teachers who thought that the rephrasing question was unproductive (9 teachers; 21%) is that students should not be asked to rephrase an incorrect idea and overall engage in a discussion about it. For example, one teacher commented that, "I find that it would be unproductive to rephrase a student's thinking if it is mathematically incorrect because it confuses other students." Another teacher stated, "Alexis' idea is incorrect, but we haven't yet established that as a class, so by asking a student to rephrase the idea you run the risk of marking the idea as correct. I would rather the students consider whether they agree with her thinking or not rather than trying to rephrase her incorrect thinking."

Asking for a Different Way of Thinking. Among the 49 teachers who thought that asking a student for a different way of thinking than Alexis' idea (Question 2) was productive, 13 (26.5%) of them believed that asking for a different way of thinking helps students to see that the task could be solved in different methods. For example, one teacher said, "Lets students know that there is more than one way to solve problems and inspires them to find another way to look at it and solve it." Another common and related belief, shared by 11 (22.4%) of the teachers who thought that asking for a different way of thinking was productive, is that this generates students sharing different ways of thinking for solving the problem. For example, one teacher stated that, "Seeing multiple ways of thinking and approaching a problem allows students to make different connections with their own work and can lead to students being able to clarify their own ideas."

On the contrary, teachers had mixed beliefs about discussing Alexis's idea before eliciting different ways of thinking. From Table 5.3, it is noticeable that among the 10 teachers who thought that asking different ways of thinking was unproductive, 5 believed that this was unproductive before discussing Alexis's idea as they thought that moving into a new idea would not help students to make sense about Alexis's idea. For instance, a teacher included in their comment that,

> By asking another student for a different way of solving the problem you miss out on the rich mathematical discussion you could have about why Alexis' idea

doesn't quite work. Students learn more from trying to make sense of a mistake or misconception than they do from seeing correct work. So, I wouldn't want to move on from Alexis' thinking until we have explored whether or not it works and why as a class.

However, 4 (8%) teachers with similar beliefs thought the question was somewhat productive because they believed that asking different ways of thinking might be productive as getting other ideas will support teachers to connect different ideas with Alexis's; but first students need to discuss Alexis's idea: "I don't think we want to move away from Alexis's idea completely yet, there are still things that need to be resolved. If Ms. Kamara plans to come back and connect the different ways of thinking later, it could still be somewhat productive." Teachers who thought that discussing Alexis's idea before asking for different ways of thinking was unproductive have similar beliefs with the teacher who found it productive. Additionally, 4 teachers among 6 teachers who thought that the question was neutral also believed that including different ways of thinking is helpful for students to notice and compare with other ideas but does not provide the opportunity to make sense of Alexis's idea: "While it is good to see what other students have done, it does not give students the opportunity to think about the mathematics in Alexis' response."

Discuss in a small group. Among the 57 teachers who thought that asking students to discuss Alexis's idea in a small group (Question 3) was productive, 20 (35%) of them believed that asking students to talk in a small group engages more students with the MOST. For example, one teacher said, "This is allowing students to analyze with the support of others and for all to engage in the process of deciding what they believe."

However, some teachers commented that this question was only somewhat productive as this question might not engage all the students with Alexis's contribution by stating this: "I don't feel like this is always a good idea as usually table groups have a main talker who always shares his/her thoughts but never allows others to share their ideas."

Another common belief, expressed by 13 (23%) teachers, was that asking to discuss Alexis's idea in a small group is productive as this question creates a safe space where students feel comfortable and get more opportunities to share their ideas with each other with less pressure. For example, one teacher stated that, "This allows students to discuss ideas in a small setting with less pressure. Students can have more opportunity to engage with each other and consider alternative points of view." Another common belief among 9 (16%) teachers was that asking to discuss Alexis's idea in a small group is very productive as this question makes students provide evidence to justify their reasoning. For example, one teacher added, "This option makes all students provide evidence or justification for their position which will encourage them to think through the whole process."

Holds up mathematically. Among the 35 teachers who thought the "How does the claim hold up mathematically?" question (Question 4) was productive, 9 (26%) teachers believed that asking this question to the students put them in a position to justify their thinking by reasoning about the MOST. For instance, a teacher commented that, "This question provides a way for students to prove whether or not Alexis is correct and therefore provide more discussion about the mathematics."

Another common belief about this question was related to the nature of the question, specifically how it's worded. Among 9 teachers who rated this response as unproductive (Table 5.3), 5 teachers did so because of how this question was worded. For example, one of the teachers said that "I think this question is too broad or weird for students to understand and will solicit responses that only a few students will answer." Additionally, 7 (20%) of the 35 teachers who thought that this question is productive as asking this question generates students reasoning why they agree or disagree with Alexis's claim still believed the wording of the question could be confusing: "I like the idea of this, but not necessarily how it's worded. I don't think the class would be as clear on what to do or talk about." 9 teachers rated this question as neutral for similar reasons. They thought without developing a classroom culture or training, this question could sound intimidating:

> I'd like to rephrase the question. Arguing for or against a claim mathematically may sound intimidating, but I'm really just asking them to explain their reasoning for or against. They'll need training to know what I mean when I say "argue mathematically".

What do others think. Among the 65 teachers, 34 teachers had concerns about the nature of the question, "What do others think?". This response was rated as unproductive by the most teachers in the survey. Among the 23 teachers who rated it unproductive, 15 (65%) teachers believed that asking "what do others think" (Question 5) was unproductive as the nature of this question is a broad and open-ended question which puts students in a situation where they can share anything, but not particularly about the contribution. For example, one teacher emphasized, "This response is broad so it doesn't give enough directions for students. If we want to stay on topic about what Alexis said then we shouldn't open it up to general discussion.” Five teachers who thought that this question was somewhat productive agreed by pointing out that “You might get some good responses from this question, but it is also possible that with little direction or focus that this question goes nowhere.” Additionally, 14 teachers from 22 teachers believed that this question is neutral by emphasizing the fact that this question is not as direct of a question as asking for different methods. For instance, a teacher added that, “In my opinion this question is based too much on student’s opinions rather than the math. Maybe I’m rewarding it with the wrong tone implied, and maybe with a good follow up question it would be fine; but this question alone isn’t going to advance any thinking.”

6 Discussion

This research focused on the questions that teachers think are productive to engage students in a sense-making whole class discussion and their reasoning behind their thinking. To understand teachers' belief further, in this study, teachers' responses and beliefs about different questions' productivity to engage students in a sense-making discussion were analyzed. The study built upon previous studies that emphasized the significance of aligning responses with student contributions and fostering meaningful mathematical discourse. In the following paragraphs, I will be discussing the major findings of the studies about teachers' beliefs about productive questions and connecting those findings to prior research.

In this study, initially teachers proposed questions to engage the whole class with the MOST, but after seeing several options for the potential questions, they seemed to prefer engaging the whole class in a small group discussion first. The comparison between the current research and the study by Stockero et al. (2017a) reveals differences in their approaches and findings. The findings of Stockero et al. (2017a) indicated that most teachers directed their responses to the same student who made the original contribution, possibly due to an open-ended question about how they would respond to a student contribution, with less guidance in the questions asked by the researchers. This may have resulted in teachers asking clarifying questions or seeking more information from the contributing student. However, in the current research, teachers were given a structured open-ended question with clear guidance on what to do next after the student contribution was presented—engage the class in making sense of it. Therefore, it might be concluded that the prompt seemed to support teachers in engaging the whole class with the MOST.

As from prior research (Van Zoest et al., 2022), engaging students in a small group could be productive for building if the students later share their thought process and justify their reasoning to the whole class about their conclusion about the MOST. Without sharing their reasoning or thought process behind the conclusion to the whole class, however, it may not support the whole class to make sense of the MOST. Therefore, even though asking to discuss in a small group may be able to engage all the students, without sharing student thinking with the class, building on student thinking will not take place since building on a MOST is making a MOST "the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36).

In the beginning of the survey, less than 30% of the teachers (19/64) proposed questions that would support sensemaking (e.g., evaluate, develop, justify) to engage the class with MOST. After they were presented with a collection of possible questions, 62% of the teachers (40/65, including two teachers who were double coded Evaluate and Justify in their response) were asking sense-making questions. Regarding the types of moves used by teachers, Stockero et al. (2017a) found that teachers most frequently used develop and justify moves in their responses toward MOSTs. Recall that prior research (Van Zoest et al., 2022) has shown that actions like developing, justifying, and evaluating are effective in fostering a deeper understanding of student thinking. This suggests that teachers in the current research were asking productive questions, but only after they had questions to choose from, that aimed to uncover and understand students' thinking as well as building on the student thinking by making students justify and share their reasoning with each other.

Stockero et al. (2017a) argued that if a teacher is asking a question to the same student to justify the original thinking focuses on the details of the thinking whereas if teachers ask the whole class to justify focuses more on the general idea behind the contribution. From this finding of Stockero et al. (2017a), we may conclude that depending on whom a question is asked to, the effectiveness of sense-making question could differ. In this research, teachers most often have proposed to engage the whole class in a small group discussion by intending to ask students to evaluate the original contribution. This type of question will support building on student ideas because students would be providing information about the correctness of the MOST, not only just agree or disagree.

Initially, teachers were using students' verbal or non-verbal actions explicitly or implicitly in their responses. In the later part of the survey, it is noticeable that teachers had a shift in including students' words implicitly in their responses. Stockero et al. (2017a) found that teachers used students' words explicitly or implicitly and the majority of the student ideas were the core in their responses. In the current research, almost all of the teachers implicitly used students' words in their responses and centered their responses around student ideas, indicating that almost all the teachers were considering students' words in their responses to utilize students' thinking. The finding of almost all the teachers using students' words implicitly in this research contradict with the findings of Stockero et al. (2017a); however, in the beginning of the survey they have included students' words both explicitly and implicitly. Thus, this shift might happen as the result of teachers' selecting questions from the questions given to them in the second part of the survey. Using students' ideas as the core of the response aligns with Stockero et al.'s

(2017a) findings. Teachers were focusing on integrating student's words and ideas in their responses which would be helpful for building.

Additionally, in the current research, teachers' responses at the end of the survey were more closely related to the mathematical points of the student contribution than before having different options for selecting questions from. In other words, more of the teachers' responses were core to the mathematical point of the contribution at the end than in the beginning. This finding suggests that after teachers were presented with a set of questions, they did better in asking questions core to the mathematical points of the students' contributions and addressed specific mathematical aspects. This alignment between teachers' responses and the mathematical points raised by students could foster meaningful sense-making mathematical discourse.

The comparison with Stockero et al. (2017a) highlights the potential impact of the structure of the survey prompt on teachers' responses, particularly related to the engagement of the whole class. The current research provided teachers with more guidance and direction, which may have supported teachers' practice towards engaging the entire class in a whole class small group discussion. The findings also indicate that at the end of the survey teachers were selecting more productive questions, closely aligning their responses with mathematical points, and utilizing students' thinking more effectively. In conclusion, it is viable to imply that by providing a productive set of questions to teachers to select from, we could support them to productively engage the class in a sense-making discussion. This may be a way researchers and teacher educators can facilitate teachers by developing a professional training program with a focus on productive student-centered classroom discourse.

In part 2 of the survey, teachers considered most of the questions provided to them to be productive for engaging students in sense-making discussions about the MOST except Question 5 (e.g., "What do others think?"). Understanding the factors that influence teachers' decisions, including the role of their beliefs, is crucial (Schoenfeld, 2008). Teachers' beliefs about the productivity of a question will help future researchers and teacher educators to develop a training program for their professional development.

Teachers rated asking students to discuss in a small group as the most productive, as this question could help students to overcome their uneasiness to participate in a discussion when they're in a group. Teachers also included that working in a small group could make students feel less pressure than sharing in front of the whole class and creating an open space for collaborative sense-making discussion. This could be productive for building, but unless they later share their idea in front of the whole class and other students make sense of the mathematics behind the shared idea, the class won't be building on the shared idea (Van Zoest et al., 2017). These findings align with Bansal's (2018) research, which emphasized the creation of open conversational spaces to foster student participation. Some teachers believed that asking students to discuss contributions in small groups would engage more students, facilitate analysis and support each other's thinking. This aligns with Stockero et al. (2020) findings that teachers believed that students critiquing, discussing, and comparing each other's ideas could be productive for sensemaking and could support the building practice of student thinking.

Asking if the student's claim holds up mathematically was also rated as productive by teachers in this study, which was theorized to be a productive next move in prior work (Leatham et al., in press; Peterson et al., 2022). While teachers generally believed this question to be productive for building on student understanding, the data suggests that its effectiveness to the teachers may vary depending on the context or the students' mathematical abilities. Teachers shared that this question could be productive if students have a training of asking and answering this type of question. Therefore, students will be able to understand the questions and won't spend time making sense of the question. If the students need to try to make sense of the terms the question is using, not the student contribution, then this question may not be productive to ask. Some teachers found that this question is unproductive as the question is worded differently. They think that a simply worded, straightforward question would be more productive to engage students with the contribution and won't be confusing. In line with Van Zoest et al.'s (2022) observations, teachers believed that asking students to evaluate another student's contribution allowed for sharing of thinking and better understanding among students. This question supports building on students' thinking by engaging the whole class with the discussion when the students share their ideas to justify their reasoning of the current student contribution.

Question 5 was the only question rated as generally unproductive among the other questions which is consistent with Peterson et al.'s (2022) findings that teachers asking if students have other thoughts could potentially divert the discussion away from the student contribution. Teachers in this study thought that this question could lead to students sharing their own methods or any other irrelevant thinking in the class. Bishop (2008) found that vague or poorly worded questions aimed solely at gathering information, "buying time" or creating a gap in the continuous discourse could hinder the learning process by causing confusion and lack of focus. In this study, teachers expressed similar thoughts about the question "What do others think?", perceiving it as broad and open-ended, lacking clear direction for students. Boaler and Brodie (2004) observed that some teachers used open questions that allowed for multiple possible answers and encouraged student thinking and exploration. However, teachers in this study noted that such questions could be unproductive as this question won't support students making sense of the student contribution and would lead to any direction depending on the students' understanding or scope of thinking.

Bansal (2018) and Franke et al. (2009) found that targeted and specific questions when asking students for an explanation of their contribution can encourage a deeper analysis of the topic and could lead to a better understanding of the contribution. The result of this study showed that most of the teachers also found targeted and specific questions (i.e., "Turn to your table group and discuss whether you agree or disagree with Alexis's idea, and why.") more productive than vague ones (i.e., "What do others think?") as well as confusingly worded questions (i.e., "How might you argue whether Alexis's claim and reasoning hold up mathematically?"). Teachers' recognition that some questions are not as productive as others is positively impactful for generating student-centered instruction.

The research also illuminated other beliefs that would support or hinder building on student thinking. For example, the belief among teachers that rephrasing questions is unproductive was often based on the concern that asking students to rephrase an incorrect idea might confuse them or lead them to believe that the contribution is correct when it hasn't been discussed yet in the class. This belief aligns with the findings of Bray (2011), who discovered that teachers' beliefs influence how they handle class discussions when errors occur. This includes the roles they assume for themselves and assign to their students. However, by not considering the current student thinking, teachers are missing the point of building and not giving students the opportunity to make sense of why the student contribution is incorrect. Discussing an incorrect idea is as important to build on as a correct idea for a sense-making class discussion.

Regarding asking for different ways of thinking, most teachers thought that this question would generate students sharing different ways of thinking and multiple approaches that students had taken to solve the problem. Teachers think that this way students could see there are more ways to solve this problem and they will try to make connections with their own work as well as clarify their understanding about their own ideas which could help the students to clarify their ideas but does not support building on the MOST. Others believed that this question was unproductive, as it may divert attention from the current contribution and hinder sense-making. In other words, teachers believed that students should not move away from the current idea, which is productive since moving away from the idea would not support building and could result in a different learning environment.

Finally, teachers' beliefs related to student engagement and participation surfaced in response to several questions. Teachers thought that question 3 (small groups) is most productive as it creates a safe space for students to engage with the MOST, but teachers were not focusing on whether students are making sense of the mathematics behind the MOST. As from the conceptualization of building, we know that students have to engage in the sense-making process and make connections between the student contribution and the mathematical concepts about the other students mathematical thinking. Other teachers also measured a question's productivity based on students' engagement. They believed rephrasing might only engage or disengage the same students with the contribution, consistent with the findings of Boaler and Brodie (2004) that certain questions influence student participation. However, as with the small group response, they didn't consider if this question would support students to understand and make sense of the student thinking and thus build on it.

7 Limitations

One limitation of this study is that the responses provided by teachers were hypothetical rather than based on actual classroom interactions with student contributions. The study relied on teachers' anticipated responses to student thinking, which may not fully reflect their actual responses in a real classroom setting, as Stockero et al. (2022) have pointed out in their study. It is important to consider that teachers' responses in the classroom can be influenced by various factors such as their beliefs, the available time, the specific context, and the students' ability to engage in the class activities.

Furthermore, another limitation of the study is that the coding and analysis focused specifically on teachers' beliefs about the alternative options provided in Part 2 of the survey. Other parts of the survey were not included in this analysis, and it is possible that examining and analyzing those parts could reveal additional insights into teachers' beliefs that were not evident in the current analysis. These limitations highlight the need for future research that aims to incorporate observations of real classroom interactions and explore teachers' beliefs across various aspects of questioning and student contributions to provide a more comprehensive understanding of their instructional practices.

The researcher acknowledges that understanding the intricacies of effective teaching and student engagement requires a holistic perspective. While asking teachers about their productive next moves to foster whole class sense-making discussions based on an individual student contribution can be insightful, it may not provide a complete picture about how they would engage students in discussing the given student contribution.

Teaching is a dynamic and evolving process that extends beyond isolated moments of interaction.

Finally, teachers' strategies and practices may change over time as they gain experience and engage in professional growth (Leinhardt et al., 1986). Thus, not considering the teachers' experience levels in this research may have impacted the findings. Experienced teachers may possess a broader repertoire of instructional techniques to make in-the-moment decisions to engage students in a whole class sense-making discussion. Thus, to gain a more comprehensive understanding, it is essential to recognize that the results of this study might differ based on teachers' experience level.

8 Conclusion

Teachers' reasoning about selecting questions can provide valuable insights into their beliefs about questions that effectively lead students toward sense-making discussions. By understanding teachers' decisions and beliefs about productive questions, researchers and teacher educators can gain valuable information that can inform the development of effective strategies for teachers' professional development. Teachers' beliefs about productive questions revealed their understanding of how questioning can support student engagement, critical thinking, and sense-making. The contrast in teachers' questioning strategies aimed at stimulating students' sense-making discussions became apparent when comparing their initial approach at the beginning of the survey to their subsequent responses at the survey's conclusion.

In the beginning of the survey, only one third of the teachers were naturally asking productive questions that would engage the whole class in sensemaking and only half of the teachers directed their questions toward the whole class, even though in the prompt teachers were explicitly asked to engage the whole class with the student contribution. Whereas in the end of the survey, after teachers have rated the productivity of the given questions, the majority of the teachers changed their answer and selected productive questions from the given options and indicated they would engage more students in the discussion, either engaging the whole class in small groups or in a whole class discussion. This suggested that teachers recognized that the given questions were more productive to engage the students in a sense making discussion than the earlier questions that they stated. These findings suggest that researchers and teacher educators could support teachers by providing and discussing with them a given set of productive questions to engage the whole class in a sense making discussion.

Examining teachers' reasoning behind question selection has shed light on that teachers think most of the questions provided were productive. However, they recognized that vague and poorly worded questions are mostly unproductive in promoting productive classroom discussions. This information can be valuable for researchers and educators involved in teacher professional development. By identifying the questions that teachers consider productive and their reasoning behind it, professional development programs can incorporate targeted training and support to enhance teachers' questioning strategies. This may involve providing teachers with examples of effective questions, demonstrating how these questions can guide students toward sense-making, and offering opportunities for teachers to practice and reflect on their own questioning techniques.

Researchers and teacher educators could develop a comprehensive training program to demonstrate to teachers the characteristics and benefits of productive questions by providing teachers with insights into the types of questions that promote sensemaking discussions. This training might empower teachers to move beyond thinking that specific, structured questions might be productive and delve into more thought-provoking inquiries, encouraging critical thinking and active whole class student engagement. The findings of this research also help to understand teachers' decisions and beliefs about productive questions that can help researchers and professional development providers identify any misconceptions or gaps in teachers' understanding and beliefs related to effective questioning practices. This knowledge can inform the design of interventions that address these gaps and support teachers in refining their questioning skills.

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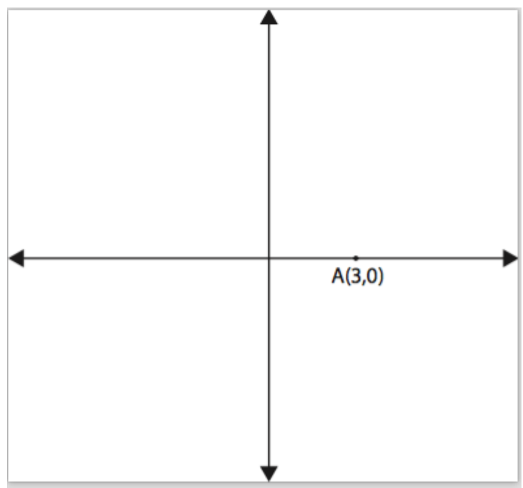
Appendix: Grapple Toss Scenario-Based Questionnaire
(adapted from a survey developed by Horizon Research, Inc.)

Ms. Kamara's mathematics class was working on this task:

Points on a Line Mini-Task

Is it possible to select a point *B* on the y-axis so that the line $x + y = 6$ goes through both points *A* and *B*? Explain why or why not.



After several minutes of individual work and an invitation to share ideas, one student, Alexis, made the following contribution:

Alexis: "If you plug $x = 3$ into the equation, you get $y = 3$. So, *B* is (0, 3) because 3 + 3 = 6."

Ms. Kamara recognized that Alexis's comment provided an opportunity for the class to make sense of a mathematically important idea in the lesson and established the contribution by recording it on the board.

Part 1

After reading Alexis's comment, what would be your next move or question to engage the class in making sense of the mathematically important idea raised by that comment? Please explain your reasoning.

Part 2

For her next move, Ms. Kamara might consider each of the following prompts.

For each, please use the Likert scale to indicate how productive you believe the prompt is as a next move to engage the class in making sense of the mathematically important idea raised by Alexis's comment and explain your reasoning for your selection.

Ask the whole class, "Can I have a volunteer rephrase the idea Alexis has shared?"

Very Unproductive Somewhat Unproductive Neutral Somewhat Productive Very Productive

Ask a student who solved the task differently "I saw that some of you had different ways to solve the task. Brian, can you share a way that is different from Alexis's idea?"

Very Unproductive Somewhat Unproductive Neutral Somewhat Productive Very Productive

Tell the class, "Turn to your table group and discuss whether you agree or disagree with Alexis's idea, and why."

Very Unproductive Somewhat Unproductive Neutral Somewhat Productive Very Productive

Ask the whole class, "How might you argue whether Alexis's claim and reasoning hold up mathematically?"

Very Unproductive Somewhat Unproductive Neutral Somewhat Productive Very Productive

Ask the whole class, "What do others think?"

Very Unproductive Somewhat Unproductive Neutral Somewhat Productive Very Productive

Part 3

No matter what you have answered in parts 1 and 2, what do you now think would be the most productive next move to engage the class in a discussion about Alexis's comment? You can choose any of your answers from parts 1 and 2, or you can write a new next move that you think would be productive. Please explain your reasoning for selecting that move.