Time-dependent Reliability of Aging Structures: An Overview of Assessment Methods

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ABSTRACT

Reliability assessment of engineered structures is a powerful and useful concept to estimate the structural capacity of withstanding hazardous events during their service lives. Taking into account the time-variation of both structural resistance and the external load processes, the structural safety level is dependent on the duration of service period of interest, due to the accumulation of hazards by exposure in time. This paper presents an overview on the non-empirical assessment methods for time-dependent reliability of deteriorating structures. Generally, these methods can be classified into two types, namely simulation-based and analytical methods. The former is usually brute, and is especially suitable for solving high-dimensional reliability problems. On the other hand, analytical solutions may improve the calculation efficiency significantly, and offer insights into the reliability problem which otherwise could be difficult to achieve through Monte Carlo simulation. Both the simulation-based and analytical methods will be reviewed in this paper. Furthermore, the application of time-dependent reliability methods in practical engineering is discussed. Recommendations for future research efforts are also presented.

Keywords: Structural reliability; Time-dependent reliability; Monte Carlo simulation; Analytical approach; Resistance deterioration; Load stochastic processes; Resilience.

INTRODUCTION

Civil engineering structures (e.g., buildings, bridges, and other facilities) play an essential role in physically supporting the functionalities of a community, and thus are expected to serve with an acceptable safety level during their service lives. However, there are many factors that may threaten the safety of

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in-service structures. For example, the structural resistance (stiffness, strength, etc) may degrade with time due to the impacts of environmental or operational conditions (Ellingwood 2005; van Noortwijk et al. 2007; Ghosh and Padgett 2010; Wang et al. 2017b; Ayyub 2014a). On the other hand, many types of loads have an increasing trend over time in terms of occurrence frequency and/or magnitude, e.g., traffic load (Nowak et al. 2010), wind load (Knutson et al. 2010), among others. These facts indicate that the safety level of structures could decline with time and unavoidably raise concerns from asset owners and managers. This is especially the case for existing structures with a service history, many of which are still in use with a deteriorated performance due to socio-economic constraints. For example, according to the recently released ASCE's 2021 Infrastructure Report Card (https://infrastructurereportcard.org), of the 617,000 bridges across the US, 42% have served for at least 50 years currently, and 7.5% (46,154) are structurally deficient with poor condition. As a result, it is essentially important to estimate the safety level of deteriorating structures with time so as to provide quantitative evidence of structural capability of fulfilling service requirements. Such evaluation will further offer quantitative support for decision-makers regarding the maintenance and repair strategies for the degrading structures (Ellingwood and Mori 1993; Enright and Frangopol 1998b; Jensen et al. 2008).

The impacts of the safety-threatening factors on structural performance are often impossible to predict exactly due to uncertainties. Under this context, probability-based approaches can be employed to describe the randomness associated with these factors and, correspondingly, the structural safety is measured in a probabilistic manner. Conceptually, the structural resistance and load effect, denoted by R and S respectively, depend on many parameters such as geometry, material properties, among others. Letting $\mathbf{X}$ be a random vector of these parameters, R and S would be functions of $\mathbf{X}$, expressed as $R(\mathbf{X})$ and $S(\mathbf{X})$. When both R and S are time-invariant random variables, the structural reliability, $\mathbb{L}$, equals the probability that S does not exceed R, that is,

$$\mathbb{L} = \mathbb{P}(R > S) = \int_{Z(\mathbf{X})>0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{1}$$

where $\mathbb{P}(\,)$ denotes the probability of the event in the brackets, $Z(\mathbf{X}) = R(\mathbf{X}) - S(\mathbf{X})$ is the limit state function, and $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{X}$. Correspondingly, the failure probability, $\mathbb{P}_f$, equals

$$\mathbb{P}_f = 1 - \mathbb{L} = \mathbb{P}(R < S) = \int_{Z(\mathbf{X})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{2}$$

In Eq. (1), if assuming that both R and S are statistically independent, it follows that,

$$\mathbb{L} = \int_0^\infty F_S(r) f_R(r) dr = \int_0^\infty [1 - F_R(s)] f_S(s) ds \tag{3}$$

in which $F_S(s)$ and $F_R(r)$ are the cumulative distribution functions (CDFs) of S and R, $f_S(s)$ and $f_R(r)$ are the PDFs of S and R, respectively. The limit state function in Eq. (1) can be classified into two types: ultimate limit state and serviceability limit state, depending on the problem of interest. The former includes flexural failure, collapse, etc, while the latter focuses on durability, deflection, vibration, and others. More details can be found in many textbooks on structural reliability (Tang and Ang 2007; Ayyub and McCuen 2011; Melchers and Beck 2018; Wang 2021a).

Note that Eqs. (1) and (3) have been based on the assumption that both R and S are time-invariant variables. However, if taking into account the time-variation of both quantities, an additional item of time t should be naturally incorporated. For a reference period of $[0, T]$, the structural reliability, $\mathbb{L}(T)$, is defined as the probability that the load effect does not exceed the resistance at any time $t \in [0, T]$. With this, it follows that,

$$\mathbb{L}(T) = \mathbb{P}\left\{R(t) > S(t), \forall t \in [0, T]\right\} = \int_0^T \int_{Z(t)>0} f_{Z(t)}(z(t)) d(z(t)) dt \tag{4}$$

where $R(t)$, $S(t)$ and $Z(t) = R(t) - S(t)$ are the resistance, load effect and limit state function at time t, respectively, and $f_{Z(t)}(z)$ is the PDF of $Z(t)$. It can be seen that Eq. (4) is more complicated than Eq. (1) due to the introduction of the time scale, reflecting the accumulation of structural failure risks during a service period of interest (e.g., service life). The assessment methods for Eq. (4) and their application, with a focus on the ultimate limit state, fall within the scope of this paper. Notice that in Eq. (4), the probabilistic information on the resistance and load processes plays an essential role in the estimate of structural reliability. Due to the length limit, this paper only discusses the time-dependent reliability problems where the probabilistic characteristics of the processes $R(t)$ and $S(t)$ are fully known. In cases where only imprecise probabilistic information is available, non-probabilistic and hybrid approaches could be employed for reliability assessment (Muscolino et al. 2016; Möller et al. 2006; Beer et al. 2013; Zhang et al. 2010; Wang et al. 2018).

Monte Carlo simulation is a robust and powerful approach to assess the structural reliability in Eq. (4), especially for high-dimensional problems. It uses many sampled trajectories of $Z(t)$ to approximate the true reliability (or failure probability). In each simulation run, a sample of stochastic process $Z(t)$ for $t \in [0, T]$, denoted by $z(t)$, is generated, and is compared against zero. If $z(t) > 0$ holds for $\forall t \in [0, T]$, then the structure is deemed as survival and otherwise failure. Guaranteed by the strong law of large numbers (Ross 2014), the average of these simulation replications converges to the structural reliability (i.e., the solution to Eq. (4)). The main drawback of Monte Carlo simulation is that it imposes a heavy computational burden for low failure probabilities, since the number of required simulation runs is in proportion to $1/\mathbb{P}_f$. Motivated by this, some advanced techniques have been developed, e.g., importance sampling, subset simulation, among others. However, simulation-based methods provide a *black box* for the reliability problems, with input of random parameters/stochastic processes and output of structural reliability. On the other hand, in many occasions the structural resistance and load processes can be well described by mathematical models, based on which analytical solutions can be derived for structural reliability. These analytical approaches will offer insights into the mechanisms of the reliability problems from different angles, improve the calculation efficiency significantly, and may accelerate the development of more complicated simulation-based reliability assessments (Wang et al. 2016a).

In Eq. (4), if $Z(t)$ decreases monotonically with time t, the reliability problem can be simply represented by the case at time T, corresponding to the worst scenario. However, this is often not the case in practice, where the variation of $Z(t)$ is not monotonic (driven by the fluctuation of $S(t)$ over time). Nonetheless, a straightforward idea for solving Eq. (4) is to convert it into a time-invariant reliability problem (taking a similar form of Eq. (1)) by considering the extreme value of the stochastic process $Z(t)$. That is, letting $Z_{\min} = \min_{t \in [0,T]} Z(t)$, then $Z_{\min}$ is a random variable despite the time-variation of $Z(t)$ for $t \in [0, T]$. With this, Eq. (4) becomes

$$\mathbb{L}(T) = \int_{Z_{\min} > 0} f_{Z_{\min}}(z) dz \tag{5}$$

where $f_{Z_{\min}}(z)$ is the PDF of $Z_{\min}$. Similarly, the failure probability equals $\mathbb{P}_f(T) = \int_{Z_{\min} < 0} f_{Z_{\min}}(z) dz$. Eq. (5) is known as the extreme value-based method (Wang and Chen 2016; Li et al. 2007), where the key step (and also the challenging task) is to identify the distribution function of $Z_{\min}$. However, Eq. (5) cannot separately reflect the time-variant characteristics of $R(t)$ and $S(t)$, and it is difficult to tackle the probabilistic behaviour of $Z_{\min}$ in the presence of nonlinear $R(t)$ and $S(t)$.

Another approach to compute the reliability in Eq. (4) analytically is known as the outcrossing-based method. For a reference period of $[0, T]$, if the time-variant resistance is greater than the load effect for all time $t \in [0, T]$, the structure is safe. On the other hand, the failure occurs once the load effect exceeds the resistance at any time, referred to as an outcrossing. By definition, the time-dependent reliability would be equal to the probability of zero outcrossing during the reference period of interest. The computation of the outcrossing rate (probability of outcrossing occurrence during unit time) is the key step in reliability assessment. The resistance process $R(t)$ in Eq. (4) is a continuous one by nature. However, the load process $S(t)$ can be modeled by either a discrete or a continuous one, as illustrated in Fig. 1. In terms of the former, the focus is on the significant events that threaten structural safety directly. For the latter, the load is continuously applied to the structure. It was argued in Wang et al. (2019) that a continuous load process can be converted into a discrete one, and vice versa. When a discrete load process model is used, the outcrossing could potentially occur at the time instant of load occurrence, and it is thus important to describe the time sequence at which load events occur. For a continuous load process, the outcrossing could be at any time within the reference period of interest.

The assessment methods for time-dependent reliability of aging structures have been widely applied in practical engineering. Specifically, the following four aspects are discussed in this paper.

- The structural service life can be predicted by comparing the time-dependent reliability and the target reliability level.
- The reliability assessment can be used to assess the efficiency of maintenance/repair measures to aging structures under the context of life-cycle cost, and to optimize the allocation of limited resources (e.g., funds, workmen) for the enhancement of multiple structures.
- The reliability methods for a single structure can be further extended to those for a system consisting of multiple components (structures).
- The estimate of structural reliability can be used in the resilience analysis of infrastructure systems/the built environment within a community.

This paper presents an overview on the assessment approaches for structural time-dependent reliability. The following four topics will be included: (1) Simulation-based approach for time-dependent reliability assessment; (2) Extreme value-based method for reliability assessment; (3) Outcrossing-based reliability methods in the presence of discrete and continuous load processes respectively; (4) Application of time-dependent reliability methods in practical engineering. For each topic, the state-of-the-art of the researches in the literature will be summarized, and recommendations for future research efforts will be presented. Fig. 2 presents a flowchart for the organization of this paper.

SIMULATION-BASED APPROACHES

Monte Carlo simulation

Monte Carlo simulation methods can be employed to estimate the time-dependent reliability in Eq. (4). Conceptually, with ℓ samples for the stochastic process $Z(t)$, where ℓ is a sufficiently large integer, if there are m scenarios that the structure fails, the failure probability can be approximated by m/ℓ (and the structural reliability equals $1 - m/\ell$). Mathematically, for a reference period of $[0, T]$, $\mathbb{P}_f(T)$ is obtained by

$$\mathbb{P}_f(T) = \mathbb{E}[\mathbb{I}(Z_{\min}(\mathbf{X})) < 0)] \approx \frac{1}{\ell}\sum_{j=1}^{\ell} \mathbb{I}\left\{Z_{\min}(\mathbf{x}_j)) < 0\right\} \tag{6}$$

where $\mathbb{E}[\cdot]$ denotes the mean value of the variable in the brackets, $Z_{\min}(\mathbf{x}_j) = \min Z(\mathbf{x}_j(t)), \forall t \in [0, T]$, ℓ is the number of simulations, $\mathbf{x}_j$ is the jth sample of $\mathbf{X}$, and $\mathbb{I}[\cdot]$ is an indicator function, which returns 1 if the statement in the bracket is true and 0 otherwise. In each simulation run, the sample $\mathbf{x}_j$ can be generated by using the inverse transform method as follows, $\mathbf{x}_j = F_{\mathbf{X}}^{-1}(\mathbf{u}_j)$ for $j = 1, 2, \ldots, \ell$, where $F_{\mathbf{X}}(\,)$ is the joint CDF of $\mathbf{X}$, and $\mathbf{u}_j$ is a sample of standard uniform random vector (Ross 2014; Melchers and Beck 2018). The stabilization of Eq. (6) is guaranteed by the law of large numbers, with which the average of $\mathbb{I}\{Z_{\min}(\mathbf{X})) < 0\}$ associated with the ℓ trials approaches $\mathbb{P}_f(T)$ with probability 1. In fact, with Eq. (6), let $\hat{\mathbb{P}}_f(T)$ be the failure probability obtained from simulation, its coefficient of variation (COV) can be measured by

$$\mathrm{COV}(\hat{\mathbb{P}}_f(T)) = \sqrt{\frac{1 - \hat{\mathbb{P}}_f(T)}{\ell \cdot \hat{\mathbb{P}}_f(T)}} \tag{7}$$

which decreases with $\sqrt{\ell}$ and $\sqrt{\hat{\mathbb{P}}_f(T)}$ approximately linearly for a small failure probability.

It is noticed that some methods for structural reliability assessment may combine both the simulation and analytical approaches. One example is that, in Eq. (4), if the failure probability for a reference period of $[0, T]$ can be expressed by an explicit function of $\mathbf{X}$ and T, denoted by $\mathbb{P}_f(T) = g(\mathbf{X}, T)$, Eq. (6) can be rewritten as follows,

$$\mathbb{P}_f(T) = \mathbb{E}[g(\mathbf{X}, T))] \approx \frac{1}{\ell}\sum_{j=1}^{\ell} g(\mathbf{x}_j, T) \tag{8}$$

where $\mathbf{x}_j$ is the jth sample of $\mathbf{X}$ as before. Employing Eq. (8) is often associated with improved efficiency compared with Eq. (6) since the time discretization is not needed in Eq. (8). Another example is the *extreme value based reliability method*, as will be discussed in the following, where the simulation-based is often incorporated to estimate the probabilistic behaviour of $Z_{\min}$.

The simulation-based approach in Eqs. (6) and (8) provides a robust tool for estimating the time-dependent reliability in Eq. (4), especially when many random variables are involved so that the problem is high-dimensional. However, the simulation method is often critiqued due to its huge computational burdens for relatively low failure probabilities. For example, according to Eq. (7), with a fixed requirement on the COV of the estimated failure probability (say, 0.05), the required number of simulation runs (ℓ) is proportional to $1/\hat{\mathbb{P}}_f(T)$. With this regard, some advanced simulation techniques can be employed to improve the computational efficiency.

One practical approach to improve the simulation efficiency is to employ the importance sampling method, which replaces the joint PDF of the random variables with a new one, known as *importance sampling density function* (ISDF). In such a way, the new joint PDF focuses on the more critical regions (closer to the limit state in Eq. (4)) and thus reduces the sample variance. Note that Eq. (4) can be rewritten as follows (Melchers 1990; Zio 2013),

$$\mathbb{P}_f(T) = \int \cdots \int \frac{\mathbb{I}[Z_{\min}(\mathbf{X}) < 0] f_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{X}}(\mathbf{x})} \cdot h_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{9}$$

where $h_{\mathbf{X}}(\mathbf{x})$ is the ISDF. With this, Eq. (6) becomes

$$\mathbb{P}_f(T) \approx \frac{1}{\ell} \sum_{j=1}^{\ell} \mathbb{I}\left\{Z_{\min}(\mathbf{x}_j^h)) < 0\right\} \tag{10}$$

in which $\mathbf{x}_j^h$ is the jth sample of $\mathbf{X}$ generated according to $h_{\mathbf{X}}(\mathbf{x})$. Similar to Eq. (10), Eq. (8) can be re-evaluated by $\sum_{j=1}^{\ell} g(\mathbf{x}_j^h, T)/\ell$, provided that an explicit solution for the function g is available. Mori and Ellingwood (1993b) employed the importance sampling method to estimate the time-dependent reliability of an aging system. In Singh et al. (2011), the importance sampling was used to compute the cumulative probability of failure for random dynamic systems excited by a stationary random process. Wang et al. (2021) generated instantaneous samples for $Z(t)$ using the importance sampling approach, and then computed the reliability for a service period of interest by incorporating the correlation between the instantaneous samples. However, a challenging step in applying the importance sampling method is the selection of the ISDF, since an improperly selected $h_{\mathbf{X}}(\mathbf{x})$ could result in significant error in the estimate of failure probability by order of magnitude (Melchers 1990; Mori and Ellingwood 1993b). Furthermore, how to choose the ISDF requires *a priori* information regarding the effect of structural parameters on the critical response.

Another remarkable method to improve the simulation efficiency is known as the subset simulation method, which was first developed by Au and Beck (2001). The method is efficient in dealing with small failure probabilities in high dimensions. The basic idea is that, a small failure probability can be expressed as the product of several larger conditional failure probabilities which need much less computational efforts to estimate. Under the context of time-dependent reliability in Eq. (4), a sequence of events E_i is defined as $Z_{\min}(\mathbf{X}) \leq \alpha_i$ for $i = 1, 2, \ldots m$, where m is a properly selected number, and $\alpha_1 > \alpha_2 > \ldots > \alpha_{m-1} > \alpha_m = 0$. It is easy to see that $E_1 \supset E_2 \supset \ldots \supset E_m$. Thus, it follows that (Au and Beck 2003; Wang et al. 2014; Ching et al. 2005; Schneider et al. 2017),

$$\mathbb{P}_f(T) = \mathbb{P}(E_m) = \mathbb{P}\left(\bigcup_{i=1}^{m} E_i\right) = \mathbb{P}(E_1) \cdot \prod_{i=1}^{m-1} \mathbb{P}(E_{i+1}|E_i) \tag{11}$$

As such, the failure probability for reference period $[0, T]$ can be computed by first determining the conditional probabilities $\mathbb{P}(E_{i+1}|E_i)$. Wang et al. (2014) employed a different splitting method to estimate the time-dependent reliability based on subset simulation compared with Eq. (11). The authors first partitioned the time interval $[0, T]$ into several smaller intervals, and only considered the survived sample trajectories from the previous time interval to estimate the failure probability in the current time interval. Li et al. (2019) proposed a sampling-based approach for high-dimensional time-dependent reliability assessment incorporating the subset simulation method, using a similar concept to that in Wang et al. (2014).

Other advanced simulation methods include Latin Hypercube sampling (Olsson et al. 2003), line sampling (de Angelis et al. 2015; Song et al. 2021b), directional sampling (Nie and Ellingwood 2000; Valdebenito et al. 2021), among others (one may refer to Ayyub and McCuen (2011) for a review on simulation methods for reliability assessment). Most of them have been proposed initially for solving classical time-invariant reliability problems. While these techniques can be applied to estimate the reliability problem in Eq. (8) (which takes a form of time-invariant reliability) with improved efficiency compared with brute Monte Carlo simulation, their application in assessing general time-dependent reliability in Eq. (6) has been limited (Wang et al. 2014; Li et al. 2019). The main reason is due to the complexity of the time-dependent reliability problems involving an additional dimension (temporal scale) compared with time-invariant ones.

In summary, simulation-based approaches can be used to handle general reliability problems with significant robustness and accuracy (even for those reliability problems that cannot be solved analytically).

Advanced simulation techniques have also been proposed in the literature to improve the simulation efficiency. However, they provide limited insights into the reliability problems and may be time-consuming compared with analytical solutions for structural reliability. Nonetheless, simulation-based methods are often used to estimate the reliability of benchmark problems when verifying the accuracy of (newly-developed) analytical solutions, where the *true* reliability is approximated by the simulation-based result.

Simulation of stochastic processes

In this section, the simulation techniques for stochastic processes will be discussed. The case of a stationary Gaussian process is relatively simple, since only the first two orders of moments can completely capture its probabilistic behaviour. For a one-dimensional, univariate stationary Gaussian stochastic process $X(t)$ with zero mean, if its two-sided power spectral density function (PSDF) is $\mathbb{S}(\omega)$, the process $X(t)$ can be simulated as follows (Shinozuka and Deodatis 1991),

$$X(t) = 2\sum_{i=0}^{n-1}\sqrt{\mathbb{S}(\omega_i)\cdot\frac{\omega_u}{n}}\cos(\omega_i t + \phi_i) \tag{12}$$

where $n \to \infty$ is the number of harmonics, ϕ_i is a random phase angle (uniformly distributed within $[0, 2\pi]$), $\omega_i = i\omega_u/n$, and ω_u is an upper cut-off frequency with which $\mathbb{S}(\omega)$ may be assumed to be zero for $\omega > \omega_u$. The basis of Eq. (12) is that for $X(t)$, there exist two mutually orthogonal processes $u(\omega)$ and $v(\omega)$ such that the following relationship holds, $X(t) = \int_0^\infty[\cos(\omega t)du(\omega) + \sin(\omega t)dv(\omega)]$. The Gaussianness and ergodicity of the simulated process in Eq. (12) were also shown in Shinozuka and Deodatis (1991). The method in Eq. (12) is in fact built on the spectral representation (SR) method, with discrete spectra approximating the PSDF of the target process $X(t)$. A rigorous derivation of Eq. (12), as well as the form for a non-stationary Gaussian process, can be found in Liang et al. (2007). Grigoriu (1993a) developed two models for the simulation of band-limited Gaussian processes, which were also based on the SR method and the superposition of n harmonics (taking a similar form of Eq. (12)). It was shown that both models in Grigoriu (1993a) are equivalent mutually in the second-moment sense, but differ in the Gaussianness and ergodicity properties, and thus should be selected with caution. Chen et al. (2017) extended the classical SR method so that the frequencies and the associated amplitudes are also random variables.

Another important approach to simulate a Gaussian process is the Karhunen–Loève (K–L) expansion method, which is based on the eigen-decomposition of the covariance function of the stochastic process (Huang et al. 2001). A random process $X(t, \theta)$ with mean $\overline{X}(t)$ and finite variance, which is indexed on a bounded domain $\mathcal{D}$, can be approximated by the following,

$$X(t,\theta) = \overline{X}(t) + \sum_{i=1}^{n} \sqrt{\lambda_i}\xi_i(\theta) f_i(t) \tag{13}$$

where n is the number of K–L terms, λ_i and $f_i(t)$ are the ith eigenvalue and eigenfunction of the covariance function $\mathbb{C}(t_1, t_2)$ respectively, and $\xi_i(\theta)$ is a standard normal random variable, which is independent of other ξ_i's due to the normal distribution of $X(t,\theta)$ (Le Maître and Knio 2010). The eigenfunctions, $f_i(t)$, which are deterministic, can be determined through the Fredholm integral equation of the second kind as follows,

$$\int_{\mathcal{D}} \mathbb{C}(t_1, t_2) f_i(t_1) dt_1 = \lambda_i f_i(t_2) \tag{14}$$

An important feature of the K–L expansion is that it yields the optimal orthonormal expansion in the sense of minimizing the mean square error (Ghanem and Spanos 2003). It was shown in Huang et al. (2001) that the K–L expansion reduces to the SR method when the support $\mathcal{D}$ is large enough. Other methods for simulating a Gaussian process include the wavelet theory (Zeldin and Spanos 1996; Phoon et al. 2004), Polynomial Chaos (PC) expansion (Blatman and Sudret 2010), Auto-Regressive Moving-Average model (Samaras et al. 1985), sampling theorem (Grigoriu 1993b), local average subdivision (Fenton and Vanmarcke 1990), modal decomposition method (Igusa et al. 1984), turning bands method (Mantoglou and Wilson 1982), among others.

The simulation of a non-Gaussian stochastic process, however, is more challenging due to the complexity of the statistical characteristics. Unlike a Gaussian process, the joint multi-dimensional PDFs associated with different time instants are needed to fully capture the probabilistic behaviour of a non-Gaussian process. Furthermore, the marginal distribution (PDF associated with a single time instant) would be time-dependent for a non-stationary stochastic process. Existing methods for simulating non-Gaussian stochastic processes have been established, for the most part, based on the extension of the methods for Gaussian ones. For example, the SR method, which is based on the discretization on the frequency domain, was combined with Grigoriu's translation process approach (Grigoriu 1984) to simulate non-Gaussian processes according to a predefined PSDF and marginal distribution (Stefanou and Papadrakakis 2007; Shields et al. 2011; Shields and Deodatis 2013). The difficulty of this method is the identification of the PSDF for the translated Gaussian process that matches the PSDF of the original non-Gaussian process. The K–L expansion method (c.f.

Eq. (13)) can also be extended to a non-Gaussian stochastic process (Huang et al. 2001; Phoon et al. 2002; Phoon et al. 2005; Liu et al. 2017). In such a case, the item $\xi_i(\theta)$ is no longer a standard normal variable, and can be computed as follows,

$$\xi_i(\theta) = \frac{1}{\sqrt{\lambda_i}} \int_{\mathcal{D}} [X(t,\theta) - \overline{X}(t)] f_i(t) dt \tag{15}$$

It is often a challenging task to find the probabilistic characteristics (PDF, mean value and standard deviation) of $\xi_i(\theta)$ corresponding to an arbitrary distribution type of the original stochastic process. An iteration-based procedure was proposed in Phoon et al. (2002) to determine the variables $\xi_i(\theta)$'s in Eq. (15) to fit the marginal PDF of the original process. In Sakamoto and Ghanem (2002), the K–L expansion is combined with the PC expansion to simulate non-stationary non-Gaussian processes. The authors first used the PC expansion for an arbitrary stochastic process $X(t)$ as follows,

$$X(t) = u_0(t) + u_1(t)\gamma(t) + u_2(t)(\gamma^2(t) - 1) + u_3(t)(\gamma^3(t) - 3\gamma(t)) + u_4(t)(\gamma^4(t) - 6\gamma^2(t) + 3) + \dots \tag{16}$$

where $u_i(t)$ are deterministic functions with respect to t, and $\gamma(t)$ is a standard Gaussian process with zero mean and unit variance. Subsequently, the process $\gamma(t)$ is decomposed using the K–L expansion as in Eq. (13). Dai et al. (2019) proposed a method for simulating non-Gaussian stochastic processes, where the target process is represented by the K–L expansion (c.f. Eq. (13)), and the random coefficients involved in the K–L series ($\xi_i(\theta)$ in Eq. (15)) is decomposed using the PC expansion. The accuracy of using PC expansion to approximate stochastic processes was discussed in Field Jr and Grigoriu (2004). It was shown that the PC approximations could be computationally demanding or even prohibitive for certain processes due to the large amount of coefficients to be calculated.

In terms of application in simulation-based reliability assessment (c.f. Eq. (6)), while the methods for simulating Gaussian/non-Gaussian stochastic processes have been widely discussed in the literature, it is important to extend/modify the methods for some reliability problems by considering: (a) sampling of stochastic processes conditional on time-variant parameters (e.g., generating a sample for $S(\mathbf{X})$, where $\mathbf{X}$ may also vary with time); (b) simulation of stochastic processes considering interaction with accompanying processes (e.g., sampling $S(t)$ in Eq. (4) considering its correlation with $R(t)$). In order to improve the simulation efficiency, it is an interesting topic to apply the advanced simulation methods (as discussed earlier) when generating samples for stochastic processes. Furthermore, the simulation of Gaussian/non-Gaussian stochastic processes plays an important role in the extreme value-based reliability method, in conjugation with advanced techniques such as adaptive Kriging-Monte Carlo simulation, as will be discussed in the next section.

EXTREME VALUE-BASED RELIABILITY METHOD

Recall the time-dependent reliability problem in Eq. (5), which is referred to as the extreme value-based reliability method. It takes advantage of the extreme value $Z_{\min}$ (a random variable) to represent the stochastic process $Z(t)$, with which the time-dependent reliability is converted into a time-invariant one. Under this context, failure is defined as the probability of $Z_{\min}$ being less than 0 (the threshold). Clearly, the probability model of $Z_{\min}$ is the key component in estimating the extreme value-based reliability. Generally, the distribution of extreme value is much more nonlinear compared with the stochastic process $Z(t)$ itself with respect to the input random variables $\mathbf{X}$, e.g., it could be multi-modal with different modes (peaks of PDF) (Bichon et al. 2007).

The efficient global optimization (EGO) studied by Jones et al. (1998) is a useful method to find the global extreme value of a stochastic process. It first fits response surfaces to the data at a few points based on the Kriging surrogate model using the maximum likelihood estimation, and then improves the fitting based on a branch-and-bound algorithm iteratively, where the iteration stops if the expected improvement is less than 1% of the best current function value. However, the EGO method generates samples for the (time-invariant) random variables and time separately, leaving a room for improving the efficiency by considering the interaction between the two types of quantities. Motivated by this, Hu and Du (2015) developed a mixed EGO method by generating the two types of samples (random variables and time sequence) simultaneously, and employed the adaptive Kriging-Monte Carlo simulation to improve the calculation accuracy. However, the mixed EGO method is associated with great computational requirements due to the discretization of the stochastic process. Hu and Du (2013a) proposed a sampling approach to estimate the CDF of $Z_{\min}$, where the expansion optimal linear estimation (EOLE) method (Li and Der Kiureghian 1993) was used to generate samples for $Z_{\min}$, and the saddle-point approximation method (Daniels 1954) was utilized to approximate the CDF of $Z_{\min}$ based on the samples. However, only one stochastic process could be involved in this method. Wang and Wang (2013) proposed a nested extreme response surface (NERS) method to find the distribution of the extreme value, where a Kriging model was employed to establish a NERS corresponding to the extreme value of the limit state function. The authors first used the nested time prediction model (NTPM) to predict the time at which the stochastic process $Z(t)$ reaches its extreme value (that is, to locate the time of worst scenario), and then employed the adaptive response prediction and model maturation algorithm to improve the prediction accuracy and efficiency by enrolling new samples when needed. However, the NERS method is difficult to propagate the random parameters to dynamic responses of engineering systems. This disadvantage was overcome in Wang and Chen (2016), where an equivalent stochastic process transformation method was developed for structural time-dependent reliability assessment. The dimensionality of a random process is reduced by representative standard normal variables based on spectral decomposition, so that the limit state function is reformulated as a function of the random variables and time. Subsequently, the Gaussian process is used to establish a surrogate (Kriging) model for predicting the value of limit state function at the discretized time nodes with an adaptive sampling technique, based on which the extreme response surface can be approximated. However, the efficiency of the method highly depends on the number of discretized time nodes. Ping et al. (2019) proposed a time-dependent extreme value event evolution method for structural reliability assessment, where the random process involved in the limit state function was represented by an improved orthogonal series expansion method. However, this method is only applicable to the case of Gaussian stochastic processes.

Li et al. (2007) proposed a method to obtain the extreme value distribution of the nonlinear response of a structure, where a virtual stochastic process was introduced so that the extreme value of the original stochastic process equals the virtual one evaluated at a certain instant of time. The authors employed the probability density evolution method (PDEM) to estimate the instantaneous PDF of the virtual process, with which the PDF of the extreme value can be obtained accordingly. For a stochastic process $Z(\mathbf{X}, t)$, which is a function of basic random variables $\mathbf{X}$ at time t, its minimum value within time interval $[0, T]$ is a function of both $\mathbf{X}$ and T, denoted by $Z_{\min} = W(\mathbf{X}, T)$. An auxiliary stochastic process $Y(t) = \psi(\mathbf{X}, t)$ is constructed, satisfying $Y(0) = \psi(\mathbf{X}, 0) = 0$ and $Y(t_c) = \psi(\mathbf{X}, t_c) = W(\mathbf{X}, T)$ simultaneously, where t_c is a predefined time. According to the PDEM theory (Li and Chen 2009), it follows that

$$\frac{\partial p_{Y\mathbf{X}}(y, \mathbf{x}, t)}{\partial t} + \dot{\psi}(\mathbf{x}, t) \frac{\partial p_{Y\mathbf{X}}(y, \mathbf{x}, t)}{\partial y} = 0 \tag{17}$$

where $p_{Y\mathbf{X}}(y, \mathbf{x}, t)$ is the joint PDF of $\mathbf{X}$ and $Y(t)$, and $\dot{\psi}$ takes differentiation with respect to t. When

$p_{Y\mathbf{X}}(y, \mathbf{x}, t)$ is obtained from Eq. (17), the PDF of $Z_{\min} = Y(t_c)$ can be computed by

$$f_{Z_{\min}}(y) = p_Y(y, t_c) = \int_{\mathbf{\Omega_X}} p_{Y\mathbf{X}}(y, \mathbf{x}, t_c) d\mathbf{x} \tag{18}$$

where $\mathbf{\Omega_X}$ is the domain for $\mathbf{X}$. However, according to Eqs. (17) and (18), the explicit form for the distribution of the extreme value is unavailable for most engineering cases. In terms of implementation of Eqs. (17) and (18), a numerical algorithm is essentially necessary (Li et al. 2007), which highly depends on the selection of the *seed* points from $\mathbf{\Omega_X}$. Moreover, the PDEM for extreme value distribution is only applicable for problems where the evolutionary mechanism of the stochastic process is explicitly known, so that the value of $W(\mathbf{x}, T)$ can be determined given a $\mathbf{x} \in \mathbf{\Omega_X}$.

In summary, the extreme value-based method can convert the time-dependent reliability problem into a time-invariant one, where some mature techniques for classical reliability problems can be employed. However, the method only considers the combined effect of the resistance and load processes, and thus cannot reflect the probabilistic characteristics of these processes individually. Furthermore, it is typically difficult to derive a closed form solution for the extreme value $Z_{\min}$, and thus is not straightforward to understand the mechanisms of failure due to $Z_{\min} < 0$. Employing simulation-based methods to find/approximate the CDF of $Z_{\min}$ is often associated with significant computational burden, especially when a relatively long reference period is considered and a discretization is needed for reliability assessment. Moreover, the extreme value-based method has been mainly focused on a continuous stochastic process, rather than a discrete one. Future research efforts could be put into (1) improved methods that can adaptively achieve a trade-off between accuracy (error associated with the failure probability) and efficiency (number of discretized time intervals) of reliability assessment; (2) reliability methods that focus on the joint behaviour of the local extreme values in an attempt to improve the computational efficiency, by noting that the global extreme value is among the local ones; (3) extreme value-based method considering a discrete load process, where the discretization of time interval could be replaced by only considering the time sequence at which load events occur.

OUTCROSSING-BASED RELIABILITY ASSESSMENT CONSIDERING A DISCRETE LOAD PROCESS

In this section, the time-dependent reliability assessment methods considering a discrete load process will be discussed. Under this context, the focus will be on the structural performance at the times of load occurrence. Consider the time-dependent reliability within a reference period of $[0, T]$. Significant load events that directly threaten structural safety randomly occur in time with random intensities. The duration of these events is typically short (c.f., τ in Fig. 3), and only occupies a negligible portion of the structural service life. Furthermore, the load intensity can be treated as constant during the interval in which the load event occurs. If not taking into account the impact of dynamic response, the load process can be modeled by a sequence of randomly occurring pulses with random intensities. Assume that there are totally n load events occurring within $[0, T]$ at time instants $t_1, t_2, \dots t_n$, respectively, yielding a sequence of load effects $S_1, S_2, \dots S_n$. With this, the time-dependent reliability in Eq. (4) is rewritten as follows,

$$\mathbb{L}(T) = \mathbb{P}\left\{R(t_1) > S_1 \cap R(t_2) > S_2 \cap \ldots \cap R(t_N) > S_n\right\} \tag{19}$$

The reliability problem in the presence of a discrete load process is illustrated in Fig. 3. At any time $t_i \in \{t_1, t_2, \ldots t_n\}$, if S_i exceeds the corresponding resistance $R(t_i)$, it is deemed that an *outcrossing* occurs, leading to structural failure at time t_i.

A frequently-used model for the load occurrence is the Poisson point process (Mori and Ellingwood 1993a; Li et al. 2015). With this, the probability of n loads occurring within $[0, T]$ is

$$\mathbb{P}(n = k) = \frac{(\lambda T)^k \exp(-\lambda T)}{k!}, \quad k = 0, 1, 2, \dots \tag{20}$$

where λ is the mean occurrence rate of the loads (a time-invariant parameter). When considering the time-variation of the load occurrence rate with time, a non-stationary (non-homogeneous) Poisson process can be employed. With this regard, if the occurrence rate is $\lambda(t)$, which is a function of time t, the probability in Eq. (20) becomes

$$\mathbb{P}(n = k) = \frac{\left(\int_0^T \lambda(t)dt\right)^k \exp\left(-\int_0^T \lambda(t)dt\right)}{k!}, \quad k = 0, 1, 2, \dots \tag{21}$$

If taking into account the temporal correlation in the load occurrence, a modified version of Poisson process can be used, as discussed in Wang et al. (2017a) and Wang and Zhang (2018).

In Eq. (19), the resistance deterioration process (c.f., $R(t), t \in [0, T]$) is an essential component for reliability assessment. Models for resistance deterioration will be reviewed in the next section. Subsequently, the assessment methods for structural reliability will be discussed.

Resistance deterioration models

The description of aging mechanisms of structures is a key step in developing resistance deterioration model of these structures. It was reported in Clifton and Knab (1989) that, for reinforced concrete (RC) structures, the resistance deterioration could be caused by sulfate attack, alkali-silica reaction, corrosion of steel bars, frost attack, chloride (Cl^-) penetration and carbonation. For certain deterioration types (e.g., corrosion of steel bars, concrete carbonation), mechanism-based models have been developed in previous studies. For example, the deterioration process due to corrosion consists of the stages of corrosion initiation, crack initiation and crack propagation (Enright and Frangopol 1998a; Stewart and Rosowsky 1998; Stewart and Mullard 2007; Li and Ye 2018). For the first stage, the time of corrosion initiation is typically determined by comparing the chloride concentration at the surface of steel bars with a predefined threshold, where the ingress process is modeled by the Fick's second law (Collepardi et al. 1972). Let $C(x,t)$ be the chloride concentration at depth x at time t, which is expressed as follows,

$$C(x,t) = C_s\left(1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D(t)t}}\right)\right) \tag{22}$$

in which C_s is the chloride concentration at the structural surface, $D(t)$ is the chloride diffusion coefficient, which is a function of time, and erf is the error function. Based on Eq. (22), the time of corrosion initiation, t_i, is determined by $C(x,t_i) = C_{cr}$, where C_{cr} is the critical value for chloride concentration (a predefined threshold). The second stage (crack initiation) refers to the period from corrosion initiation to the appearance of the first visible crack. The models that focus on the cracking time and the amount of material loss (e.g., loss of cross-section of steel bars) have been mostly built on the accelerated or natural-speed corrosion tests (Alonso et al. 1998; Vidal et al. 2004; El Maaddawy and Soudki 2007), where the observed data are used to calibrate the parameters involved in the crack initiation models. Correspondingly, the structural capacity loss is often measured by the material loss. For example, Frangopol et al. (1997) studied the impact of corrosion on the reliability of reinforced concrete members, and used the following expression to describe the time-variation of the bending reinforced area of reinforce steel bars under uniform corrosion,

$$A_s(t) = \pi[D_b - 2\nu(t - t_i)]^2/4, \quad t \geq t_i \tag{23}$$

where $A_s(t)$ is the cross-section area of the steel bar, having an initial value of $\pi D_b^2/4$, D_b is the diameter of the steel bar, t_i is the time of corrosion initiation, and v is the rate of corrosion. Subsequently, the ultimate bending moment and shear force were expressed as functions of $A_s(t)$. However, the effects of bond deterioration (Mangat and Elgarf 1999) and yield stress deterioration (Du et al. 2005) on the structural resistance were not considered by Frangopol et al. (1997). The models for the third stage, which is typically a complicated process, also rely on experimental results to calibrate the crack propagation progress (Vu and Stewart 2000; Yuan et al. 2010). The development of crack width, which is characteristic of the third stage, will in return accelerate the rate of corrosion by increasing concrete penetrability (Otieno et al. 2010).

The difficulties in using the mechanism-based deterioration models are as follows: (1) The accurate description of the impact of material aging (e.g., material loss, cracking) on the resistance deterioration is not clear, which heavily relies on experimental results to calibrate the involved parameters. (2) The parameters involved in the resistance deterioration process depend on the characteristics of each individual structure (e.g., exposure site, structural geometry, construction practice, materials), and thus it is a significant burden to obtain the probabilistic distributions of the structure-dependent parameters. (3) The resistance deterioration process is often a complex one consisting of multifarious mechanisms, making it difficult to accurately model the deterioration process in the presence of interactions between different deterioration modes.

A resistance deterioration model that can well represent the time-variant characteristics of real-world structures would need extensive in-situ experiments and field measurements. The deterioration conditions of aging structures can be assessed by in-situ inspection and/or load testing (Wang et al. 2011; Lantsoght et al. 2017), and the results are informative of the degraded structural resistance. However, such evaluations are seldom conducted in practice partially due to the relatively high costs. The structural health monitoring (SHM) is a process that monitors a structure over time, collecting the real-time or periodically observed measurements of strain, vibration, and vertical movement from a network of sensors. These collected data can be further used, based on established algorithms, to detect structural damage, evaluate structural resistance deterioration and estimate the remaining service life (Ko and Ni 2005; Seo et al. 2016). However, there is a research gap to combine the SHM data with sophisticated finite element models to dynamically update the estimate of structural resistance (Seo et al. 2016). On the other hand, the current application of SHM in civil engineering structures is still in its infancy (Chen and Ni 2018).

Some studies investigated the deterioration model of structural elements based on experimental data in the laboratory using accelerated corrosion tests (Chung et al. 2008; Ma et al. 2013). While these results can reflect the characteristics of resistance deterioration for the prototypes, it is often questionable to extrapolate the data on real-world structures when the scaling of the prototypes to full-scale structures is nonlinear (Ellingwood 2005). Moreover, the determination of probabilistic models of the parameters involved in the deterioration process needs a large amount of available data, meaning that significant bias could be introduced if the sample size is limited. Furthermore, it was pointed out in a review paper by Kashani et al. (2019) that, in terms of experimental data on corroded RC structures, much fewer have been focusing on the behaviour of corroded RC columns under axial and static lateral loading conditions, compared with those on the flexure and/or shear capacity of corroded RC beams.

A reasonable resistance deterioration model for aging structures is expected to capture both the dominant deterioration mechanism and available observed data. The former determines the overall shape of the deterioration process, while the latter could reduce the epistemic uncertainty associated with the mechanism-based deterioration model. Mathematically, the deterioration process of structural resistance can be expressed as follows,

$$R(t) = R_0 \cdot G(t) \tag{24}$$

where $R(t)$ is the resistance at time t, $R_0 = R(0)$ is the initial resistance, and $G(t)$ is the deterioration function. A simple model for $G(t)$ is as follows for RC structures (Mori and Ellingwood 1993a; Ellingwood 2005),

$$G(t) = 1 - a(t - t_i)^b \epsilon, \quad t > t_i \tag{25}$$

where a and b are two parameters that reflect the rate and shape of the deterioration process respectively, which can be calibrated through observed data, ϵ is a random variable accounting for the randomness of $G(t)$, and t_i is the time of deterioration initiation. It is shown in Eq. (25) that the mean value of $G(t)$ decreases proportional to $(t - t_i)^b$. The exponent b takes a value of 0.5, 1 and 2 respectively corresponding to the dominant mechanisms of diffusion-controlled aging, corrosion and sulfate attack (Mori and Ellingwood 1993a). A similar model to Eq. (25) was used in Ayyub et al. (2015) to describe the impact of corrosion on the ultimate strength of vessels, taking a form of

$$G(t) = 1 - a_1 a_2 (t - t_i)^b \epsilon, \quad t > t_i \tag{26}$$

in which a_1 is the annual thickness reduction factor for general corrosion, a_2 is the strength reduction factor per unit value of a_1, b is a model coefficient to account for the trend nonlinearity, and t_i is the initiation time. The item ϵ in Eq. (26) takes a value of 1 if modeling the deterioration process as deterministic, and is a random variable when considering the uncertainty associated with the deterioration process.

Note that with the models in Eqs. (25) and (26), the deterioration process is either deterministic (with ϵ being equal 1) or fully-correlated. However, this is not consistent with a realistic resistance deterioration process, which is a non-increasing stochastic process by nature with autocorrelation arising from the deterioration functions evaluated at any two time instants. With this regard, the item ϵ was modified to be a stochastic process, denoted by $\epsilon(t)$, which is a zero-mean Gaussian process that reflects the uncertainty associated with the deterioration process (Melchers 2003). However, such a modification could lead to an non-decreasing trajectory of $G(t)$. Bhattacharya et al. (2008) proposed a corrosion rate model $C(t)$ that ensures a non-decreasing trend in time with multiplicative noise term,

$$\frac{dC(t)}{dt} = a(t - t_i)^b \exp(\eta(t)), \quad t > t_i \tag{27}$$

in which a and b are two time-invariant parameters, t_i is the time of deterioration initiation, $\eta(t)$ is a zero-mean stationary process, and is modeled as an Ornstein-Uhlenbeck process as $d\eta(t)/dt = -k\eta(t) + \sqrt{D}\xi(t)$, D and k are two constants, and $\xi(t)$ is a white noise. Subsequently, the deterioration function $G(t)$ was expressed as a function of $C(t)$. The disadvantage Eq. (27) is that many parameters are involved in this deterioration model, which are difficult to calibrate using limited available data.

The Gamma process has been widely used to model the difference between the initial and degraded resistances (c.f., $1 - G(t)$ in Eq. (24)) (Dieulle et al. 2003; Saassouh et al. 2007; Mahmoodian and Alani 2014; Wang et al. 2015). It describes a non-decreasing process with independent and positive increments. Mathematically, for a Gamma random variable X with shape parameter $a > 0$ and scale parameter $b > 0$, its PDF takes a form of

$$f_X(x) = \frac{(x/b)^{a-1}}{b\Gamma(a)} \exp(-x/b), \quad x \geq 0 \tag{28}$$

where $\Gamma(\cdot)$ is the Gamma function. Next, for a Gamma stochastic process $X(t), t \geq 0$ with shape parameter a and scale parameter b, which is a continuous process with independent and Gamma-distributed increments, it has the following properties (Kahle et al. 2016),

- $\mathbb{P}(X(0) = 0) = 1$.
- $\Delta X(t) = X(t + \Delta t) - X(t)$ is also Gamma distributed, with shape parameter $a\Delta t$ and scale parameter b, for any $t \geq 0$ and $\Delta t > 0$.
- For an integer $n \geq 1$ and time instants $0 \leq t_0 < t_1 < \ldots < t_n$, the variables (increments) $X(t_0), X(t_1) - X(t_0), \ldots X(t_n) - X(t_{n-1})$ are mutually statistically independent.

With a Gamma process-based deterioration model, the PDF of $G(t)$ at any time t can be easily determined since $1 - G(t)$ follows a Gamma distribution. However, the critique for a Gamma process-based deterioration model is that, both the autocorrelation and the variance of $G(t)$ are dependent on the mean value of the deterioration process (Wang 2020b).

Some researchers (Klutke and Yang 2002; Sanchez-Silva et al. 2011; Iervolino et al. 2013; Kumar et al. 2015) modeled the resistance deterioration for aging structures as a linear combination of the gradual and shock deteriorations, by considering that the shock events also contribute to the accumulation of structural fragility (Choe et al. 2010; Kumar and Gardoni 2011). However, in these models, both types of deterioration have been assumed to be independent of each other. Wang et al. (2017b) proposed a deterioration model by addressing the mutual correlation between the gradual deterioration, shock deterioration and the load effect. The authors employed the Gaussian Copula function to capture the joint probabilistic behaviour of correlated random variables. However, it is difficult to determine the deterioration-load correlation in the model by Wang et al. (2017b). Table 1 presents a comparison between the mathematical models for resistance deterioration. It is seen that a more advanced model can better describe the characteristics of deterioration process but simultaneously involves more parameters. As a result, it is important to achieve a trade-off between the accuracy and efficiency when choosing a candidate model for the resistance deterioration of aging structures.

Reliability assessment methods

When some simple models for the resistance and load processes are employed, one can derive closed form solutions for structural time-dependent reliability. The explicit solutions are beneficial for their application in practical engineering. With this regard, a useful tool is the hazard function, denoted by $h(t)$. By definition, $h(t)\Delta t$ equals the probability of structural failure within the interval $(t, t+\Delta t]$ conditioned on structure survival within the period of $[0, t]$, where $\Delta t \to 0$. The hazard function can be related to structural time-dependent reliability, $\mathbb{L}(t)$, as follows,

$$h(t) = -\frac{d}{dt}\ln[\mathbb{L}(t)] \tag{29}$$

In the presence of structural deterioration, $h(t)$ characteristically increases with time due to the enhanced risk of failure. Based on Eq. (29), once the hazard function is obtained, the time-dependent reliability is simply as follows,

$$\mathbb{L}(t) = \exp\left(-\int_0^t h(\tau)d\tau\right) \tag{30}$$

If the resistance deterioration process is deterministic (i.e., not considering the uncertainty associated with $G(t)$), and the load process is modeled by a Poisson process with an occurrence rate of λ, the hazard function can be determined by considering the failure probability at time t (due to the occurrence of a load event causing a load effect greater than the resistance at time t) as follows,

$$h(t) = \lambda[1 - F_S(R_0 \cdot G(t))] \tag{31}$$

in which $F_S(\,)$ is the CDF of the load effect. Substituting Eq. (31) into Eq. (30), the time-dependent reliability of the aging structure is,

$$\mathbb{L}(T) = \exp\left\{-\int_0^T \lambda[1 - F_S(R_0 \cdot G(t))]dt\right\} \tag{32}$$

which is further rewritten as follows if taking into account the uncertainty associated with the initial resistance R_0 (Mori and Ellingwood 1993a),

$$\mathbb{L}(T) = \int_0^\infty \exp\left\{-\int_0^T \lambda[1 - F_S(r \cdot G(t))]dt\right\} \cdot f_{R_0}(r)dr \tag{33}$$

where $f_{R_0}(r)$ is the PDF of R_0. Notice that Mori and Ellingwood (1993a) used a different approach to derive Eq. (33), where the authors considered the Poisson distribution of the number of load events within time interval $[0, T]$, and the joint distribution of the occurrence time for load events, and finally obtained the same result as in Eq. (33) by employing the law of total probability. Eq. (33) has been widely used in the literature to perform reliability assessment of aging structures. For example, Ellingwood and Mori (1997) studied the optimal in-service inspection-maintenance strategies for concrete structures in nuclear power plants under the framework of structural time-dependent reliability assessment to meet the performance goals probabilistically. Enright and Frangopol (1998a) investigated the time-dependent reliability of concrete bridge beams considering flexural strength loss due to corrosion of steel reinforcement. Mori and Ellingwood (2006) used Eq. (33) to estimate the time-dependent reliability of a RC structural wall, where the strength degradation was represented by a probabilistic defect growth model based on experimental data. Akiyama et al. (2010) studied the time-dependent reliability of RC structures in a marine environment with Eq. (33), where the impacts of corrosion crack width and the chloride concentration distribution were taken into account. Ayyub et al. (2015) used a similar formula of Eq. (33) to perform structural reliability analysis of stiffened panels and hulls of marine vessels considering the impact of strength deterioration. Remarkably, Eq. (33) has been adopted in the international standard *Bases for design of structures – Assessment of existing structures* (ISO 2010).

The reliability method in Eq. (33) can be simplified when the load effect follows an Extreme Type I (Gumbel) distribution (Wang et al. 2016b). With this regard, the CDF $F_S(s)$ takes a form of

$$F_S(s) = \exp\left(-\exp\left(-\frac{s-u}{\alpha}\right)\right) \tag{34}$$

where u and α are the location and scale parameters respectively. If the deterioration function is linear, $G(t) = 1 - kt$, Eq. (33) becomes,

$$\mathbb{L}(T) = \int_0^\infty \exp(\lambda\xi) \cdot f_{R_0}(r)dr \tag{35}$$

where

$$\xi = \begin{cases} -\exp\left(-\dfrac{r-u}{\alpha}\right) \cdot T, & k = 0 \\ -\exp\left(-\dfrac{r-u}{\alpha}\right) \cdot \dfrac{\alpha}{kr} \cdot \left[\exp\left(\dfrac{krT}{\alpha}\right) - 1\right], & k \neq 0 \end{cases} \tag{36}$$

An improved version of Eq. (33) was proposed by Li et al. (2015), where the non-stationarity of the load process was considered. The authors used a non-homogeneous Poisson process (with a time-variant occurrence rate of $\lambda(t)$) to describe the load occurrence, and a time-varying CDF of load effect (denoted by $F_S(s,t)$) to model the non-stationarity in load magnitude. Under this context, Eq. (32) is generalized as (Li et al. 2015),

$$\mathbb{L}(T) = \exp\left\{-\int_0^T \lambda(t)[1 - F_S(R_0 \cdot G(t), t)]dt\right\} \tag{37}$$

which can be further rewritten as follows if taking into account the uncertainty of the initial resistance,

$$\mathbb{L}(T) = \int_0^\infty \exp\left\{-\int_0^T \lambda(t)[1 - F_S(r \cdot G(t), t)]dt\right\} \cdot f_{R_0}(r)dr \tag{38}$$

Notice that Eq. (37) is a generalized form of Eq. (32), by incorporating the load non-stationarity in terms of both occurrence rate and magnitude. Saini and Tien (2017) employed Eq. (37) to evaluate the impact of climate change on the long-term reliability of infrastructure systems, where the global climate change projections until the end of the 21st century were used, yielding a non-stationary process for the loads.

In Eq. (38), if the mean value of the load effect increases linearly with time with a rate of $\kappa_m > 0$, the load effect follows an Extreme Type I distribution with a constant scale parameter α, and the occurrence rate is time-invariant, denoted by λ, then Eq. (38) can be rewritten as Eq. (35) but the item ξ becomes (Wang et al. 2016b),

$$\xi = -\exp\left(-\frac{r - u(0)}{\alpha}\right) \cdot \frac{\alpha}{kr + \kappa_m} \cdot \left[\exp\left(\frac{T(kr + \kappa_m)}{\alpha}\right) - 1\right] \tag{39}$$

where $u(0)$ is the location parameter of load effect at the initial time. On the other hand, if the load effect is time-invariant in Eq. (38) while the occurrence rate increases linearly with time with a rate of $\kappa_\lambda > 0$, Eq. (38) is rewritten as follows,

$$\mathbb{L}(T) = \int_0^\infty \exp[\lambda(0)\xi + \kappa_\lambda \psi] \cdot f_{R_0}(r)\mathrm{d}r \tag{40}$$

where ξ is as in Eq. (36), $\lambda(0)$ is the occurrence rate of loads at the initial time, and

$$\psi = \begin{cases} -\exp\left(-\frac{r-u}{\alpha}\right) \cdot \frac{1}{2}T^2, & k = 0 \\ -\exp\left(-\frac{r-u}{\alpha}\right) \cdot \left\{\exp\left(\frac{krT}{\alpha}\right) \cdot \left[\frac{\alpha T}{kr} - \left(\frac{\alpha}{kr}\right)^2\right] + \left(\frac{\alpha}{kr}\right)^2\right\}, & k \neq 0 \end{cases} \tag{41}$$

When more complicated models for structural resistance and/or load process are employed, a simulation-aided approach can be employed for structural time-dependent reliability assessment (Kumar et al. 2015; Wang et al. 2017b; Wang and Zhang 2018), since explicit solutions are not necessarily available. With this regard, one may use a semi-analytical approach for reliability assessment by applying the conditional reliability (on a partial set of random variables) and combining the simulation-based method (c.f. Eq. (8)).

In summary, when a discrete load process is considered, the outcrossing-based reliability methods focus on the structural behaviour at the time instants of load occurrence. The modeling of the resistance deterioration and the discrete load process are the key elements in time-dependent reliability assessment. When the Poisson process is employed for load occurrence, and a deterministic deterioration process is used, a closed form solution can be derived for structural reliability assessment (c.f. Eqs. (33) and (38)). In these solutions, the uncertainty associated with the deterioration process can be further incorporated by using the law of total probability. More advanced analysis techniques also include the interaction between the deterioration process and the load effect. Future research efforts could be put into (1) Resistance deterioration models using in-situ observed data of full-scale real-world structures. This will serve as benchmark problems for examining the applicability of mathematical models of resistance deterioration. With this regard, some multidisciplinary techniques such as the machine learning-based predictions (Taffese and Sistonen 2017; Nguyen et al. 2021) can be used to aid the establishment of data-driven deterioration models. (2) Examination of the impact of load process on the resistance deterioration, where a mechanism-based analysis is essentially needed. This is especially the case when the deterioration process consists of multifarious mechanisms. (3) Simplified closed form solutions for structural time-dependent reliability (ideally integral-free), which would be beneficial for inclusion in standards/codes for performance assessment of aging structures. With this regard, some frequently-used distribution types could be used to achieve such a simplified reliability method (c.f. Eqs. (35) and (40)). (4) Time-dependent reliability assessment of durability design-based structures, taking into account the close relationship between the resistance deterioration process and the durability requirements. This will help the designers better understand how the safety levels (associated with ultimate and serviceability limit states respectively) of in-service structures degrade with time simultaneously.

OUTCROSSING-BASED RELIABILITY ASSESSMENT CONSIDERING A CONTINUOUS LOAD PROCESS

In this section, the outcrossing-based reliability methods in the presence of a continuous load process will be discussed. As illustrated in Fig. 4, for a reference period of $[0, T]$, the structural time-dependent reliability equals the probability that the load effect $S(t)$ does not exceed the resistance $R(t)$ for any $t \in [0, T]$, or equivalently, the probability that $S(0)$ is smaller than $R(0)$ and there is no occurrence of outcrossing of $S(t)$ with respect to $R(t)$ for $\forall t \in (0, T]$.

The relationship between the hazard function and the time-dependent reliability (c.f. Eq. (30)) can be used herein to derive the reliability considering a continuous load process with slight modification as follows,

$$\mathbb{L}(T) = \mathbb{L}(0) \cdot \exp\left(-\int_0^T h(t)dt\right) \tag{42}$$

where $\mathbb{L}(0)$ is the instantaneous reliability at the initial time. The difference between Eqs. (30) and (42) is that an emphasis is also put on the structural performance at the initial time in Eq. (42). With this regard, it is important to determine the hazard function for reliability assessment. One practical way is to use the upcrossing (outcrossing with a positive slope) rate of $S(t)$ with respect to $R(t)$ to approximate the hazard function (Rice 1944). Let $\nu^+(t)$ be the upcrossing rate at time t, which is by definition expressed as follows,

$$\nu^+(t) = \lim_{\Delta t \to 0} \frac{\mathbb{P}[Z(t) > 0 \cap Z(t + \Delta t) < 0]}{\Delta t} \tag{43}$$

where $Z(t) = R(t) - S(t)$ as before. Replacing $h(t)$ with $\nu^+(t)$, Eq. (42) becomes

$$\mathbb{L}(T) = \mathbb{L}(0) \cdot \exp\left(-\int_0^T \nu^+(t)dt\right) \tag{44}$$

It is noticed that the definition for upcrossing rate in Eq. (43) has assumed that the occurrence of upcrossings are statistically independent (that is, $\nu^+(t_1)$ is independent of $\nu^+(t_2)$ for two time instants $t_1, t_2 \in [0, T]$). An alternative approach to derive Eq. (44) is to consider the probability mass function (PMF) of $\mathcal{N}^+(0, T)$, which is the number of upcrossings of $S(t)$ with respect to $R(t)$ within the time interval $[0, T]$. Recall Eq. (4), with which

$$\mathbb{L}(T) = \mathbb{P}(Z(0) > 0 \cap \mathcal{N}^+(0, T) = 0) = \mathbb{L}(0) \cdot \mathbb{P}(\mathcal{N}^+(0, T) = 0) \tag{45}$$

where the mean value of $\mathcal{N}^+(0, T)$ is determined by

$$\mathbb{E}(\mathcal{N}^+(0, T)) = \int_0^T \nu^+(t)dt \tag{46}$$

Since the occurrence of upcrossings are statistically independent, $\mathcal{N}^+(0, T)$ follows a Poisson process (c.f. Eq. (20) for its PMF), and the probability of $\mathcal{N}^+(0, T) = 0$ equals $\exp\left(-\int_0^T \nu^+(t)dt\right)$, which is consistent with Eq. (44).

With Eq. (43), it follows that,

$$\begin{aligned}\lim_{\mathrm{d}t\to 0} \nu^+(t)\mathrm{d}t &= \mathbb{P}\left\{R(t) > S(t) \bigcap R(t+\mathrm{d}t) < S(t+\mathrm{d}t)\right\} \\ &= \mathbb{P}\left\{R(t+\mathrm{d}t) - \dot{S}(t)\mathrm{d}t < S(t) < R(t)\right\} \\ &= \int_{\dot{R}(t)}^{\infty} \left[\dot{S}(t) - \dot{R}(t)\right] f_{S\dot{S}} \left[R(t), \dot{S}(t)\right] \mathrm{d}\dot{S}(t)\mathrm{d}t\end{aligned} \tag{47}$$

where $\dot{S}$ and $\dot{R}$ denotes the derivatives of S and R with respect to time t. Rearranging Eq. (47) gives the following Rice's formula,

$$\nu^+(t) = \int_{\dot{R}(t)}^{\infty} \left(\dot{S} - \dot{R}(t)\right) f_{S\dot{S}} \left(R(t), \dot{S}\right) \mathrm{d}\dot{S} \tag{48}$$

in which $f_{S\dot{S}}$ is the joint PDF of $S(t)$ and $\dot{S}(t)$. For an arbitrary distribution type of $S(t)$, it is generally difficult to obtain the closed form solution to $\nu^+(t)$ in Eq. (48) due to the difficulty of expressing $f_{S\dot{S}}$. However, an explicit formula is available for $f_{S\dot{S}}$ when $S(t)$ is a Gaussian process, because the derivative of a Gaussian process is also a Gaussian one. This explains why many researches have focused on the time-dependent reliability problems considering a Gaussian load process (Li and Melchers 1993; Lutes and Sarkani 2009; Wang et al. 2019). For example, if the resistance deterioration process $R(t)$ is deterministic, and the load process is Gaussian with mean $\mu_S(t)$ and standard deviation $\sigma_S(t)$, it follows that (Li and Melchers 1993),

$$\nu^+(t) = \frac{\sigma_{\dot{S}|S}(t)}{\sigma_S(t)} \phi\left(\frac{R(t) - \mu_S(t)}{\sigma_S(t)}\right) \left[\phi\left(\frac{\dot{R}(t) - \mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)}\right) - \frac{\dot{R}(t) - \mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)} \Phi\left(-\frac{\dot{R}(t) - \mu_{\dot{S}|S}(t)}{\sigma_{\dot{S}|S}(t)}\right)\right] \tag{49}$$

where $\Phi(\,)$ and $\phi(\,)$ are the CDF and PDF of a standard normal distribution, $\mu_{\dot{S}|S}(t)$ and $\sigma_{\dot{S}|S}(t)$ are the conditional mean and standard deviation of $\dot{S}(t)$ on $S(t)$. Firouzi et al. (2018) studied the the formulas for $\mu_{\dot{S}|S}(t)$ and $\sigma_{\dot{S}|S}(t)$ in Eq. (49) for two types of autocorrelation function: Gaussian and Sinus cardinal. Li et al. (2016) derived a closed form solution for structural time-dependent reliability considering a lognormal load process. However, the authors assumed that the derivative of a lognormal process follows a lognormal distribution. In Wang et al. (2019), it was argued that for an arbitrary distribution type of load process, it can be converted into a Gaussian one using the Nataf transformation method (Der Kiureghian and Liu 1986). Correspondingly, the upcrossing rate in Eq. (49) can be used to achieve an explicit solution for structural reliability. The method of using equivalent resistance and load in Wang et al. (2019) is a generalized version of the translation process method proposed in Grigoriu (1984), which considers a constant barrier level. However, the use of Nataf transformation method is built on the assignment of Gaussian copula function for the joint distribution of the load process evaluated at different time instants.

In order to compute the upcrossing rate in the presence of non-Gaussian load process, Andrieu-Renaud et al. (2004) proposed a FORM-based method, namely the PHI2 method. The acronym FORM means *first order reliability method*, which is a widely used approach for handling classical time-invariant reliability problems. According to Andrieu-Renaud et al. (2004), the definition of $\nu^+(t)$ in Eq. (43) is modified slightly as follows,

$$\nu^+_{PHI2}(t) = \frac{\mathbb{P}[Z(t) > 0 \cap Z(t+\Delta t) < 0]}{\Delta t} \tag{50}$$

Note that the limit passage in Eq. (43) has been replaced by a finite-differential like operation in (50). Subsequently, the items $Z(t) > 0$ and $Z(t+\Delta t) < 0$ are treated as two correlated events, corresponding to a reliability index of $\beta(t)$ and $\beta(t+\Delta t)$ respectively based on FORM. The correlation between the two events, denoted by $\rho_Z(t, t+\Delta t)$, equals $-\boldsymbol{\alpha}(t) \cdot \boldsymbol{\alpha}(t+\Delta t)$, where $\boldsymbol{\alpha}(t) = \mathbf{u}^*(t)/\beta(t)$ and $\mathbf{u}^*(t)$ is the design point in the standard normal space at time t. Similar to the reliability of a parallel system (Hagen and Tvedt 1991), Eq. (50) becomes

$$\nu^+_{PHI2}(t) = \frac{\Phi_2[\beta(t), -\beta(t+\Delta t); \rho_Z(t, t+\Delta t)]}{\Delta t} \tag{51}$$

where Φ_2 is the bi-normal CDF, which explains the method's name PHI2. However, the PHI2 method may result in significant error in the presence of a nonlinear limit state or improperly-selected time step Δt. Furthermore, the method is often time-consuming with a sufficiently small value of Δt. Sudret (2008) proposed an improved version of PHI2 with greater stability and accuracy. The author first introduced an auxiliary function

$$f(t,h) = \mathbb{P}[Z(t) > 0 \cap Z(t+h) < 0] \tag{52}$$

with which $\nu^+_{PHI2}(t) = \partial f(t,0)/\partial h$. Using a similar approach as in Andrieu-Renaud et al. (2004), it is obtained that

$$\nu^+_{PHI2}(t) = ||\boldsymbol{\alpha}'(t)||\phi(\beta(t))\Psi\left(\frac{\beta'(t)}{||\boldsymbol{\alpha}'(t)||}\right) \tag{53}$$

in which $\phi(x)$ is the PDF of a standard normal distribution, and $\Psi(x) = \phi(x) - x\Phi(-x)$. An alternative approach to derive Eq. (53) can be found in Zhang and Du (2011). It was shown in Sudret (2008) using a numerical example that the new approach (c.f. Eq. (53)) is exact for linear problems if the time increment $\Delta t \to 0$, and is less sensitive to Δt compared with Eq. (51).

Notice that the above mentioned approaches have considered an independent sequence of outcrossings.

This assumption is reasonable if the upcrossings are rare and weakly dependent. However, significant error could be induced in cases where the upcrossings are associated with large occurrence rate and correlation. With this regard, the impact of temporal correlation of the upcrossings has been considered in many studies. Vanmarcke (1975) presented an approximate formula for the upcrossing-based reliability considering a stationary Gaussian process upcrossing a constant barrier (with $R(t) \equiv R$). The author considered two successive time intervals, denoted by T_0 and T_1, corresponding to the time spent in the safe region ($S(t) < R$) and unsafe region ($S(t) > R$) respectively. The sum of T_0 and T_1 is the time between successive upcrossings. The following two assumptions were introduced to derive a closed form solution for the reliability problem: (a) The upcrossings of $S(t)$ with respect to the barrier R are recurrent events with independent and identically distributed recurrence times; (b) The time interval T_0 is exponentially distributed. With this, an approximate solution for structural reliability was obtained, where the upcrossing rate (ν^+ with Poisson assumption) equals $\nu^+/\Phi(R)$, in which $\Phi(\,)$ is the CDF of a standard normal distribution. Preumont (1985) developed an empirical approach for the extreme value of a stationary Gaussian process. The author assumed that the extreme point process is Markovian and fitted an Extreme Type I distribution on the PDF of extreme points. It was shown in Preumont (1985) that the fitting-based distribution converges asymptotically to the Poisson approximation.

Madsen and Krenk (1984) proposed a joint upcrossing rate method for the first-passage reliability problem. The authors considered the PDF of the time to the first upcrossing, $f_1(t)$, conditional on $R(0) > S(0)$, which is related to the upcrossing rate $\nu^+(t)$ as follows,

$$\nu^+(t) = f_1(t) + \int_0^t \nu^+(t|\tau) f_1(\tau) d\tau \tag{54}$$

where $\nu^+(t|\tau)$ is the upcrossing rate at time t conditional on the first upcrossing having occurred at time $\tau < t$. The kernel in Eq. (54), $\nu^+(t|\tau)$, can be expressed as $\nu^{++}(t,\tau)/\nu^+(\tau)$, in which $\nu^{++}(t,\tau)$ is the joint upcrossing rate at time instants t and τ. By noting that the reliability is closely related to $f_1(t)$, the target is to find $f_1(t)$ in Eq. (54) first. A generalized form of the Rice's formula (c.f. Eq. (48)) was used to compute $\nu^{++}(t,\tau)$ as follows,

$$\nu^{++}(t,\tau) = \int_{\dot{R}(t)}^{\infty} \int_{\dot{R}(\tau)}^{\infty} \left(s_1 - \dot{R}(t)\right) \left(s_2 - \dot{R}(\tau)\right) f_{\mathbf{S}\dot{\mathbf{S}}}\left(\mathbf{R}, \mathbf{s}\right) ds_1 ds_2 \tag{55}$$

in which $\mathbf{R} = [R(t), R(\tau)]$, $\mathbf{s} = [s_1, s_2]$, and $f_{\mathbf{S}\dot{\mathbf{S}}}$ is the joint PDF of $\mathbf{S} = [S(t), S(\tau)]$ and $\dot{\mathbf{S}} = [\dot{S}(t), \dot{S}(\tau)]$. For the case of a constant barrier and a stationary narrow-band Gaussian process, an approximate formula is available for Eq. (55), as was presented in Madsen and Krenk (1984). However, for more general cases, it is difficult to estimate Eq. (55) in a closed form.

Engelund et al. (1995) studied the reliability problem by considering the PMF of $\mathcal{N}^+(0,T)$ (c.f. Eq. (45)), denoted by $p_{\mathcal{N}^+}(k)$. The probability of no upcrossing within $[0,T]$ equals

$$\mathbb{P}(\mathcal{N}^+(0,T) = 0) = 1 - \sum_{k=1}^{\infty} p_{\mathcal{N}^+}(k) = 1 - \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} m_k \tag{56}$$

in which m_k is the kth moment of the number of upcrossings. While Eq. (56) presents a theoretically accurate solution for structural reliability (c.f. Eq. (45)), it is difficult to determine m_k, which consists of multiple-fold integral, especially for general distribution types of load process. Moreover, it is implicit to estimate the error of reliability assessment induced by Eq. (56). Nonetheless, it was shown in Engelund et al. (1995) that $\mathcal{N}^+(0,T)$ is asymptotically a Poisson variable when $1 \gg m_1 \gg m_2 \gg \ldots$ (or when $T \to 0$).

Hu and Du (2013b) developed a method for outcrossing-based structural reliability considering the joint behaviour of the upcrossing rates, known as JUR/FORM as it combines the joint upcrossing rates and FORM. The work was developed by extending the joint upcrossing rate method in Madsen and Krenk (1984) so that more general limit state functions involving time, random variables and stochastic processes can be handled. The JUR/FORM transforms the non-Gaussian load processes to Gaussian ones, and discretizes the time interval of interest into many small sections. Corresponding to each small time interval, the upcrossing rate is computed by the PHI2 method and the joint upcrossing rate ($\nu^{++}(t,\tau)$) is calculated via Eq. (55) (note that an approximate, integral-free formula for Eq. (55) is available for a stationary Gaussian process, as proposed by Madsen and Krenk (1984)). Finally, the discretized values of $f_1(t)$ (corresponding to the small time intervals) can be obtained by using Eq. (54). However, the efficiency of the JUR/FORM could be reduced with a large number of discretized time intervals.

Many studied have considered a Markovian process to model the occurrence of upcrossings (Yang and Shinozuka 1971; Ditlevsen 1986; Qian et al. 2020; Wang 2020a). For example, Yang and Shinozuka (1971) employed the concept of Markovian chain to model the extreme point process of a stationary zero-mean narrow-band Gaussian process. Ditlevsen (1986) showed that for a narrow-band stationary Gaussian process, the following relationship approximately holds,

$$1 - F_{T(r)}(t) = \exp\left[\int_r^{r+t} \frac{\nu^+(s)}{\psi(s)} ds\right] \tag{57}$$

in which $T(r)$ is the first outcrossing time after time instant r, $F_{T(r)}(t)$ is the CDF of $T(r)$, and $\psi(s)$ is the instantaneous reliability at time s. Assigning $r = 0$ in Eq. (57) gives

$$\mathbb{L}(T) = \mathbb{L}(0) \cdot \exp\left(-\int_0^T \frac{\nu^+(t)}{\psi(s)} dt\right) \tag{58}$$

Qian et al. (2020) reconsidered the upcrossing rate in Eq. (43) as follows,

$$\nu^+(t) = \mathbb{P}[Z(t) > 0] \cdot \lim_{\Delta t \to 0} \frac{\mathbb{P}[Z(t + \Delta t) < 0 | Z(t) > 0]}{\Delta t} \approx \psi(t) \cdot h(t) \tag{59}$$

in which $\psi(t)$ is the instantaneous reliability at time t, and $h(t)$ is the hazard function (also known as failure rate) at time t (c.f. Eq. (42)). Substituting Eq. (59) into Eq. (42) yields a new approach for structural time-dependent reliability, which takes the same form as in Eq. (58). Furthermore, a similar result to Eq. (58) was presented in Wang (2020a), where a Gaussian load process is considered, and the occurrence of outcrossings of the load process with respect to the resistance is a Markovian process by discretizing the time interval of interest into many small sections. Spanos and Kougioumtzoglou (2014) modeled the response amplitude process of a single-degree-of-freedom oscillator as a Markovian process, based on which a closed form solution was derived for the first-passage reliability problem. However, the Markovian chain can only capture the short-term memory associated with the upcrossing sequence. Nonetheless, the Markovian process-based reliability methods yield improved accuracy compared with Poisson assumption-based ones.

The concept of Markovian process has also been used in the path integration method to solve time-dependent reliability numerically (Iourtchenko et al. 2008; Naess et al. 2011; Kougioumtzoglou and Spanos 2013; Bucher et al. 2016). The method is suitable for handling reliability problems involving such a stochastic process $S(t)$ that can be expressed by the Itô stochastic differential equation as follows ,

$$d\mathbf{W}(t) = f(\mathbf{W}(t))dt + \mathbf{b} \cdot dB(t) \tag{60}$$

in which $\mathbf{W}(t) = [S(t), \dot{S}(t)]^\mathsf{T}$ is the state space vector process, $B(t)$ is a standard Brownian process, $f(\mathbf{W}(t)) =$

$[\dot{S}(t), -q(S(t), \dot{S}(t))]^\mathsf{T}$, $\mathbf{b} = [0, \sqrt{D}]^\mathsf{T}$, D is a positive parameter, and $q(\cdot,\cdot)$ is a function involved in the governing equation for $S(t)$. Eq. (60) indicates that $\mathbf{W}(t)$ is a Markovian vector process and its property of short-term memory will be used in the path integration method. For a time interval of $[0, T]$, it is first subdivided into n small sections, namely $[0, t_1], [t_1, t_2], \ldots [t_{n-1}, t_n]$. With this, the reliability within $[0, T]$ is written as follows,

$$\begin{aligned}\mathbb{L}(T) &= \mathbb{P}[S(t_1) \leq R(t_1) \cap S(t_2) \leq R(t_2) \cap \ldots \cap S(t_n) \leq R(t_n)] \\ &= \int_{-\infty}^{R(t_1)} \int_{-\infty}^{R(t_2)} \cdots \int_{-\infty}^{R(t_n)} f_{S(t_1),S(t_2),\ldots S(t_n)}(s_1, s_2, \ldots s_n) ds_1 ds_2 \ldots ds_n\end{aligned} \tag{61}$$

where $f_{S(t_1),S(t_2),\ldots S(t_n)}(s_1, s_2, \ldots s_n)$ is the joint PDF of $S(t_1), S(t_2), \ldots S(t_n)$. Taking into account the properties associated with a Markovian process, it follows that

$$f_{S(t_1),S(t_2),\ldots S(t_n)}(s_1, s_2, \ldots s_n) = f_{S(t_1)}(s_1) \prod_{i=2}^{n} f_{S(t_i)|S(t_{i-1})}(s_i | s_{i-1}) \tag{62}$$

in which $f_{S(t_i)|S(t_{i-1})}(s_i | s_{i-1})$ is the conditional PDF of $S(t_i)$ conditional on $S(t_{i-1}) = s_{i-1}$. Substituting Eq. (62) into Eq. (61) simplifies the reliability problem, given the explicit expression for $f_{S(t_i)|S(t_{i-1})}(s_i | s_{i-1})$. However, the conditional PDFs are only available in closed form for some specific cases of load process (e.g., Gaussian and external Poisson white noises), making it difficult to implement for general distribution types.

Table 2 presents a summary of the outcrossing-based methods for time-dependent reliability assessment considering a continuous load process. Existing methods that focus on the outcrossing-based time-dependent reliability assessment have been built on the upcrossing rate of the load process with respect to the barrier (the resistance). The pioneering Rice's formula in Eq. (48) has stimulated many improved methods for structural reliability assessment in terms of (a) considering the expression of upcrossing rate for general distribution types of load, and (b) considering the temporal correlation arising from the upcrossing sequence. Future research efforts could be put into (1) Error estimate of time-dependent reliability when transforming the non-Gaussian load processes to Gaussian ones. This will help the users form a straightforward confidence level on the reliability assessment result when transforming the non-Gaussian process. (2) Reliability methods that can better capture the temporal correlation of upcrossings (compared with, say, a Markovian process-based upcrossing sequence) and are simultaneously associated with acceptable computational burden/complexity for practical engineering. (3) The modeling of interaction between the resistance deterioration process and the load process. With this regard, a deterministic resistance process has been considered in most studies, which should be extended to consider the uncertainty associated with $R(t)$ using the law of total probability (Rackwitz 2001). To this end, the first-fold integral is performed for $v^+(t)$ for the considered reference period conditional on the resistance variables, and then the second-fold integral is performed over the domain of the resistance variables. When the resistance uncertainty can be well captured by that of the initial resistance, the ensemble outcrossing rate approach is a possible way to simplify the calculation by exchanging the order of integrals (Wen and Chen 1989; Beck and Melchers 2004). However, this method could overestimate structural failure probability and it application needs further verification.

APPLICATIONS OF STRUCTURAL TIME-DEPENDENT RELIABILITY

In this section, the application of structural time-dependent reliability assessment in practical engineering will be discussed.

Service life prediction

The estimate of structural reliability is immediately informative of structural service life by comparing with the target reliability level (Cheung and Kyle 1996). That is, with a threshold of p for the failure probability, the service life of the structure equals $\mathbb{P}_f^{-1}(p)$, where $\mathbb{P}(t)$ is the structural time-dependent reliability for a reference period of $[0, t]$ (note that $\mathbb{P}(t)$ is a monotonic function of t and thus the inverse of $\mathbb{P}(t)$ exists). This predicted time to failure (service life) is representative of structural performance under no maintenance and can be used by asset owners to rationally plan relevant maintenance strategies for service life extension. As mentioned before, the time-dependent reliability $\mathbb{L}(t)$ may refer to different limit states (i.e., ultimate or serviceability), and so does the prediction of structural service life. For example, Kwon et al. (2009) estimated the service life of wharf structures in a marine environment considering the impact of chloride penetration. The authors first obtained the diffusion coefficients in wharf structures through field inspection, and then employed the Monte Carlo simulation method to estimate the structural service life, considering a limit state that the accumulation of chloride exceeds the predefined critical value. Li and Mahmoodian (2013) analyzed the remaining service life of underground cast iron pipes based on time-dependent reliability assessment, where the limit state is that the time-variant stress intensity factor exceeds the critical value (beyond which the pipe cannot sustain crack growth).

The threshold for failure probability, p, is typically represented by the target reliability index β_T, which satisfies $p = \Phi(-\beta_T)$. The index β_T has been widely developed and accepted for newly-built structures through a calibration procedure (ASCE/SEI 7-10 2013; Melchers and Beck 2018), depending on the failure consequence and importance level of the target structures. However, for an existing structure, the selection of the target reliability index could be different from that for a new structure, and is often difficult to determine in practice (Stewart 2001; Stewart et al. 2001). This is because, the value of β_T for an existing structure also depends on the current state of the structure (including material, geometry and failure mode), load models, and the remaining service life. Determining a reasonable target reliability index for existing structures needs extensive data collection, numerical modeling, data-driven calibration procedure, as well as effective communications with asset owners and managers.

The time-dependent reliability $\mathbb{L}(t)$ can also be used to describe the probabilistic behaviour of the failure time. Let T_f denote the time to failure of a structure, and $F_{T_f}(t)$ the CDF of T_f. By definition, it follows,

$$F_{T_f}(t) = \mathbb{P}(T_f \leq t) = 1 - \mathbb{P}(T_f > t) = 1 - \mathbb{L}(t) \tag{63}$$

Correspondingly, the PDF of T_f can be easily obtained by differentiating $F_{T_f}(t)$. Similar to Eq. (63), the Weibull distribution has been used to model the probabilistic behaviour of the time to failure in some studies (Yang et al. 2004). Mathematically, for a random variable X following a Weibull distribution, its CDF takes a form of

$$F_X(x) = 1 - \exp\left(-(x/u)^\alpha\right), \quad x \geq 0 \tag{64}$$

where $u > 0$ and $\alpha > 0$ are the scale and shape parameters, respectively. Comparing Eqs. (63) and (64) it can be seen that the failure time modeled by Weibull distribution has a hazard function of $h(t) = \alpha(t/u)^{\alpha-1}$, whose shape depends on the value of α. Employing a Weibull distribution for the failure time can establish a snapshot for the dependence of structural reliability on time in a simple form (with only two parameters to be determined), but cannot sufficiently address the temporal characteristics of resistance deterioration and load variation for realistic cases.

From a view of infrastructure system with multiple structures (e.g., a traffic network with multiple bridges), the time-dependent reliability assessment can be used to determine the relative safety level of each structure and prioritize maintenance measures by risk ranking (from highest to lowest probability of failure, c.f. the item $\mathbb{P}_f(t)$). If further taking into account the different consequences of structural failure, the ranking can be performed by considering $C_f \int_0^t (1+r)^{-\tau} \cdot f(\tau) d\tau$ instead of failure probability alone, where C_f is the failure cost, r is the discount rate, and $f(\tau)$ is the PDF of time to failure (Stewart et al. 2001).

However, the failure cost of each structure cannot be determined independently, but should take into account its functionality and importance in the system, where each component (structure) may interact mutually.

Life-cycle cost assessment and optimization

Civil structures and infrastructures are often designed to serve for long service periods, during which the structural serviceability and safety level could degrade with time, indicating the necessity of taking maintenance or repair measures for these structures. The life-cycle cost analysis is a practical method to optimize the initial structural design, to evaluate the benefit of the enhancement strategies for in-service structures, and to determine the efficient allocation of limited resources (funds, workmen, etc) on the degrading structures (Enright and Frangopol 1998b; Val and Stewart 2003; Ellingwood 2005; Frangopol and Liu 2007; Hu and Du 2014; Sajedi and Huang 2019). The life-cycle cost for a reference period of $[0, T]$, $\mathbb{C}(T)$, can be computed by,

$$\mathbb{E}(\mathbb{C}(T)) = C_I + \sum_i \frac{C_{mi}}{(1+r)^{t_i}} + \mathbb{E}\left(\frac{C_f}{(1+r)^{T_f}}\right) \tag{65}$$

where C_I is the initial cost in terms of design and construction, C_{mi} is the maintenance cost in year t_i, C_f is the failure cost (the cost of failing to achieve performance targets), r is the discount rate, and T_f is the time to failure, whose CDF can be obtained by first estimating the time-dependent failure probability $\mathbb{P}_f(T)$ (c.f. Eq. (63)). Based on Eq. (65), the reliability-based life-cycle cost optimization of aging structures can be done by solving the following problem,

$$\min \mathbb{E}(\mathbb{C}(T)), \quad \text{subject to } \mathbb{P}_f(T) \le p \tag{66}$$

in which p is the threshold for failure probability. However, Eq. (66), which is a single-objective problem, should be used with caution in practical engineering as the safety level of the structure (c.f. the threshold p) needs to be dynamically adjusted according to the amount of available budgets. As a result, it is important to achieve a trade-off between the life-cycle cost minimization and the structural performance maximization when making decisions on the performance-based maintenance strategies, by means of considering multi-objective optimization problems (Furuta et al. 2004; Xie et al. 2018). With this regard, one may refer to Frangopol and Liu (2007) for a review on the life-cycle maintenance and management planning for aging structures based on life-cycle cost optimization and reliability. This emphasis herein, however, is that the estimate of structural time-dependent reliability is a critical ingredient in the life-cycle cost assessment and optimization (c.f. Eq. (66)), yet the application of the latest reliability methods has been insufficient in the life-cycle cost analysis. Importantly, in Eq. (66), the PDF of the time to failure is dependent on the maintenance actions at the previous time instants, and this impact should be taken into account when estimating the time-dependent reliability for a subsequent reference period. Ayyub (2014a) provided methods for examining the life-cycle in a broader micro-economic framework in terms of benefits and costs associated with enhancing a system. Economic valuation methods are used for this purposes with the costs and benefits treated as random variables.

System reliability assessment

Most infrastructure systems consist of multiple components (structures) and thus the performance of the system would depend on the joint behaviour of each component. It is naturally interesting to extend the time-dependent reliability method from a single component to that of a system. A simple model is the series system, which, by definition, survives if and only if all the components survive. For a discrete load process, which is represented by a Poisson model, Mori and Ellingwood (1993b) developed an analytical approach for time-dependent reliability of aging series systems, which is an extended form of Eq. (33). For a series system with n components (structures), assume that the initial resistance of component j ($j = 1, 2, \ldots n$) is R_j, and the deterioration function of component j is $g_j(t)$, which is independent of the load process. A load event with magnitude s induces a structural action $c_j \cdot s$ in component j (e.g., moment, shear, etc). With this, the time-dependent series system reliability is given by (Mori and Ellingwood 1993b)

$$\mathbb{L}_{ss}(T) = \int \ldots \int \exp\left[-\int_0^T \lambda \cdot \left\{1 - F_S\left(\min_{j=1}^{n} \frac{r_j g_j(t)}{c_j}\right)\right\} \mathrm{d}t\right] f_{\mathbf{R}}(\mathbf{r})\mathrm{d}\mathbf{r} \tag{67}$$

where $f_{\mathbf{R}}(\mathbf{r})$ is the joint PDF of the initial resistances $\mathbf{R} = \{R_1, R_2, \ldots R_n\}$, $\mathbf{r} = \{r_1, r_2, \ldots r_n\}$, λ is the load occurrence rate, and $F_S(s)$ is the CDF of load effect conditional on occurrence. Enright and Frangopol (1998b) investigated the service life of deteriorating bridges based on Eq. (67). Wang et al. (2017c) further extended Eq. (67) to a non-stationary case with time-variant load occurrence rate and load effect as follows,

$$\mathbb{L}_{ss}(T) = \int \ldots \int \exp\left[-\int_0^T \lambda(t) \cdot \left\{1 - F_S\left(\min_{j=1}^{n} \frac{r_j g_j(t)}{c_j}, t\right)\right\} \mathrm{d}t\right] f_{\mathbf{R}}(\mathbf{r})\mathrm{d}\mathbf{r} \tag{68}$$

in which $\lambda(t)$ is the load occurrence rate at time t, and $F_S(s)$ is the CDF of load effect conditional on occurrence at time t. Clearly, Eq. (67) is a special case of Eq. (68) with $\lambda(t) \equiv \lambda$ and $F_S(s,t) \equiv F_S(s)$.

When the load process is continuous, the outcrossing based reliability methods for a single structure can also be extended to those of a series system as follows,

$$\mathbb{L}_{ss}(T) = \mathbb{P}\left(\cap_{i=1}^{n} Z_i(t) > 0, \forall t \in [0,T]\right) \tag{69}$$

in which n is the number of structures within the system, and $Z_i(t)$ is the time-variant limit state function for the ith structure. Referring to Eq. (43), the upcrossing rate for the series system can be expressed as follows (Jiang et al. 2017),

$$\nu_{ss}^{+}(t) = \lim_{\Delta t \to 0} \frac{\mathbb{P}\left[\cap_{i=1}^{n} Z_i(t) > 0 \cap \cup_{i=1}^{n} Z_i(t+\Delta t) < 0\right]}{\Delta t} \tag{70}$$

which can be solved by using FORM and considering the correlation between different limit state functions of the structures. Similar to Eq. (70), Hu et al. (2018) developed an analytical expression for system reliability with bivariate responses, where the single and joint upcrossing rates were first derived based on FORM. For the more general case of multiple correlated stochastic processes, Hu and Mahadevan (2017) proposed a Vine-autoregressive moving average (Vine-ARMA) model for time-dependent reliability analysis. The Vine copula was used to capture the correlation and tail dependence between different ARMA models.

The system reliabilities as mentioned above can also be used to estimate the reliability of a single structure in the presence of multiple failure modes (caused by, e.g., the joint effect of multiple hazards). In such a case, the occurrence of failure due to any mode would result in structural failure. For example, Liu and Frangopol (2018) studied the time-dependent reliability of ship structures subjected to multi-hazard, where the failure modes of buckling and permanent set were considered, triggered by corrosion and extreme lateral pressure. For a highway bridge, the natural hazards could include earthquakes, hurricane-induced surge and wave, tsunami, riverine flooding, among others (Gidaris et al. 2017). When different failure modes are considered, it is often important to take into account the correlation between these modes due to common causes and ripple effects (Gehl and DAyala 2016).

More generally, the concept of k-out-of-n system has been widely used in engineering, which refers to such a system with n components that it survives if at least k components work. Its special cases include a series system (when $k = n$) and a parallel system (when $k = 1$). The time-invariant reliability of a k-out-of-n

system has been extensively studied in the literature (Sheng and Ke 2020; Zhang et al. 2020). However, the time-dependent reliability of a k-out-of-n system remains limitedly addressed (Coit et al. 2015; Wang 2021b). This is mainly due to the complexity of analyzing the system reliability on the temporal scale, where the state of the system depends on that at the previous time. Illustratively, consider a 2-out-of-3 system, as shown in Fig. 5. The three components are labeled as 1, 2 and 3 respectively. At time t_1, a load event occurs with magnitude S_1 causing the failure of component 3. Subsequently, at time t_2, a load event occurs with magnitude S_2 causing the failure of component 1 and thus the failure of the whole system (although only one component fails at time t_2). As a result, future research efforts are needed on the time-dependent reliability assessment of k-out-of-n systems by extending the reliability methods for a single structure.

More complicated systems are also frequently encountered in practical engineering, e.g., a power grid system consisting of multiple interacting plants, substations and transmission lines (Wang and Zhang 2020). Time-dependent reliability assessment of these systems is a challenging task since the following aspects are to be considered: (1) the interaction/dependency between different components (structures); (2) the correlation between the performance (e.g., resistance deterioration) of different components due to the common design provisions and construction practice (Vitoontus 2012); (3) the spatial correlation of the loads due to the large footprint of the underlying natural hazards such as earthquakes and hurricanes (Goda and Hong 2008; Zeng et al. 2020; Wang and Zhang 2020). One may refer to a recent review paper by Song et al. (2021a) for more details on system reliability assessment.

Application in resilience analysis

The concept of system resilience at different levels (e.g., structure, network and community) has gained much attention from the scientific community and engineers during the past two decades, with an emphasis on the functionality of the target system before, during and after hazardous events (Bruneau et al. 2003). Fig. 6 presents an illustration for the concept of community resilience, which includes the stages of (pre-hazard) preparation, function loss due to hazard occurrence, and post-hazard recovery. Many definitions of resilience and the measuring metrics have been developed in the literature (Miles and Chang 2006; Attoh-Okine et al. 2009; Cutter et al. 2010; Ayyub 2015; Hu and Mahadevan 2016), among which the time to functionality loss (c.f. t_1 in Fig. 6) is a key ingredient in the calculation of resilience merit. For example, Attoh-Okine et al.

(2009) used the following form to compute resilience,

$$\text{Resilience} = \frac{\int_{t_1}^{t_2} Q(t)dt}{t_2 - t_1} \tag{71}$$

where $Q(t)$ is the quality (functionality) of the system at time t. Taking into account the uncertainties associated with t_1, its probabilistic behaviour should be reasonably captured. Subsequently, the estimate of t_2 would be dependent on t_1 and the recovery process. By referring to Eq. (63), the PDF of t_1 can be obtained by first performing time-dependent reliability assessment of the system (Ayyub 2014a; Ayyub 2014b). It was revealed in Eq. (4) and Fig. 6 that, the key factors that influence the distribution of t_1 include the structural aging (performance deterioration) and the variation (including non-stationary and uncertainty) in the load process. These impacts can be quantitatively measured by employing the analytical methods for structural time-dependent reliability assessment, providing an essential ingredient for community resilience analysis.

CONCLUDING REMARKS

This paper has presented a state-of-the-art review on the non-empirical assessment methods for time-dependent reliability of aging civil structures. The necessity of performing time-dependent reliability assessment is due to the time-variation of both the structural resistance and the external load process. Approaches for reliability assessment can be generally classified into two types: simulation-based and analytical methods, while some could combine the two to improve the computational efficiency. The analytical approaches include the extreme value-based and outcrossing-based reliability methods. The key findings from this paper are summarized as follows.

- The Monte Carlo simulation methods provide a robust and powerful tool to estimate the general time-dependent reliability problems, especially for those without an explicit solution for structural reliability. The advantage of the simulation methods is enhanced in the presence of a high-dimensional reliability problem with multiple random variables. However, the drawback is that it is often time-consuming when the failure probability to be evaluated is fairly small, since the number of required simulation replications is proportional to the inverse of the failure probability. Furthermore, the simulation methods provide a black box for the reliability problem, with input of random variables/stochastic processes, and output of structural reliability. Thus, it offers less insights into the reliability problem compared with analytical solutions.

- The extreme value-based reliability methods focus on the worst scenario of the time-varying limit state function (a stochastic process), and thus can convert the time-dependent reliability problem into a time-invariant one. However, with existing methods, it is usually difficult to obtain the closed form solution for the distribution function of the extreme value of the stochastic process, and thus is not straightforward to reflect the mechanisms of failure due to the extreme value falling below zero.
- The outcrossing based reliability methods in the presence of a discrete load process focus on the structural performance at the time instants of load occurrence. The Poisson point process is the most widely used model to describe the load occurrence. Conditional on a deterministic resistance deterioration process, the use of Poisson load process enables an explicit solution to the time-dependent reliability. When a more complicated model for resistance deterioration and/or load process is considered, a semi-analytical approach can be used for reliability assessment by combining the simulation-based methods.
- When a continuous load process is considered, the outcrossing based reliability methods focus on the occurrence of upcrossings of the load process with respect to the resistance (the barrier). The reliability thus equals the probability of no upcrossing occurrence within a service period of interest. Closed form solutions for time-dependent reliability are available for the case of a Gaussian load process, which has also been extended to non-Gaussian cases with modification. The temporal correlation of the upcrossing sequence has also been captured in some studies, and it calls for improved methods that can achieve a trade-off between the accuracy and efficiency of reliability assessment.
- The approaches for structural time-dependent reliability assessment have been widely used in practical engineering, including, (a) the reliability-based service life prediction of aging structures; (b) the life-cycle cost assessment and optimization in terms of structural design and maintenance strategies; (c) the extension of time-dependent reliability methods for a single structure to those for a system consisting of multiple components (structures); (d) the quantitative community resilience analysis with a focus on the infrastructure systems and the built environment. The latest development in the advanced approaches for time-dependent reliability are to be incorporated in the engineering applications.

DATA AVAILABILITY STATEMENT

No data, models, or code were generated or used during the study.

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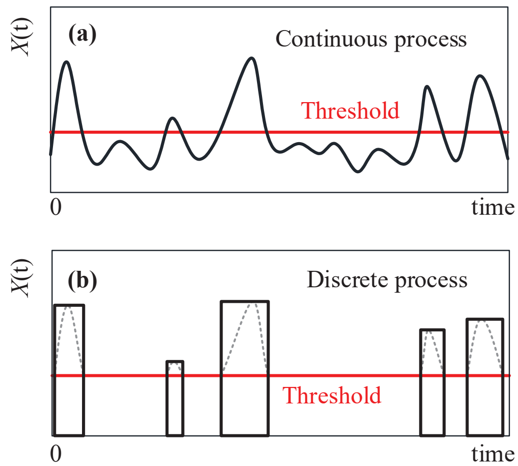

FIG. 1. Comparison of two types of load process (Wang et al. 2019, reproduced with permission from ASCE).

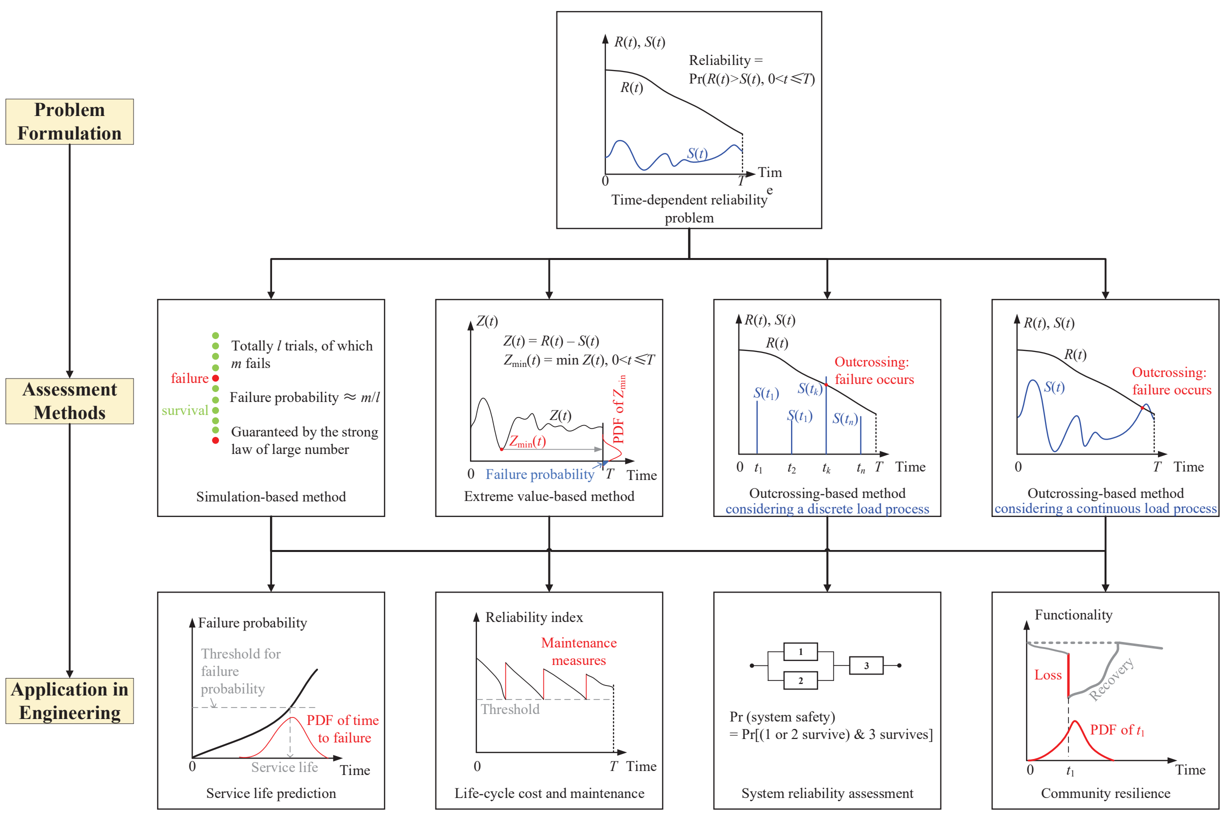

FIG. 2. Organization of this paper: Problem formulation, assessment methods and application in practical engineering.

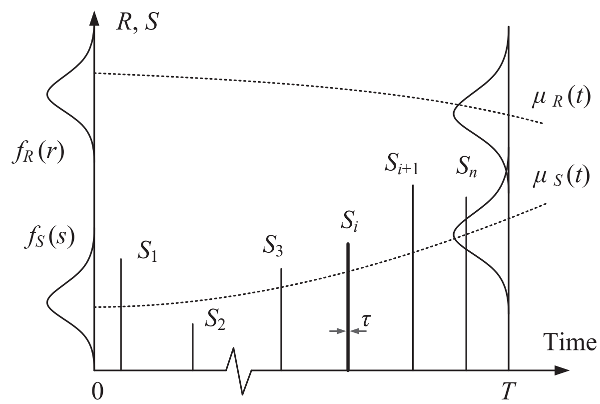

FIG. 3. Illustration of discrete load process and resistance deterioration (Li et al. 2015, reproduced with permission from Elsevier).

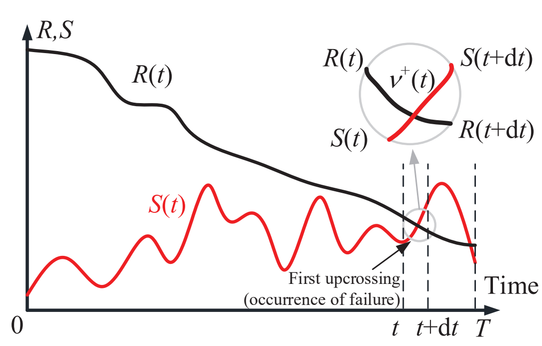

FIG. 4. Outcrossing-based reliability considering a continuous load process.

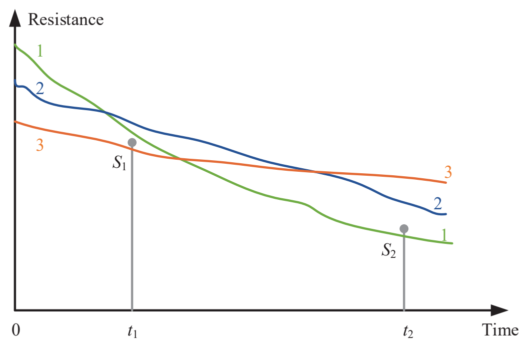

FIG. 5. Time-dependent reliability of a 2-out-of-3 system.

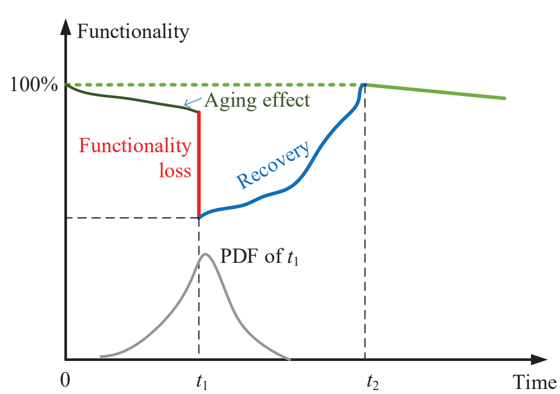

FIG. 6. Illustration of system/community resilience.

TABLE 1. Comparison between mathematical models for resistance deterioration.

References	Description	Advantage	Disadvantage
Mori and Ellingwood (1993a), Ayyub et al. (2015)	The deterioration process is modeled by either a deterministic or fully-correlated stochastic process (c.f. Eqs. (25) and (26)).	It is simple to use, and can reasonably reflect the overall trend of deterioration process.	It cannot capture the autocorrelation between the deterioration function evaluated at different time instants.
Melchers (2003)	The noise item of deterioration process is modeled by a Gaussian stochastic process (c.f. the item ϵ in Eq. (25), which is rewritten as $\epsilon(t)$).	It can capture the uncertainty associated with the deterioration process.	It may result in an increasing trajectory of resistance deterioration, which is inconsistent with the physics of a deterioration process.
Bhattacharya et al. (2008)	The deterioration function is a function of material loss (c.f. $C(t)$ in Eq. (27)), which includes a multiplicative noise term so that $C(t)$ is non-decreasing with time.	It describes a monotonic stochastic process with time, and can well capture the autocorrelation between the values of deterioration function at different time instants.	Too many parameters are involved in this deterioration model, which could be difficult to calibrate using observed data.
Dieulle et al. (2003), Saassouh et al. (2007), Mahmoodian and Alani (2014), Wang et al. (2015)	The resistance deterioration is modeled by a Gamma process. Subdividing the time interval of interest into many small sections, the deterioration increments within each small sections is Gamma distributed with an identical scale parameter.	It describes an non-decreasing stochastic process (which is suitable for modeling the difference between the initial and degraded resistances), and the autocorrelation between different time instants degrades as the time-lag increases.	The autocorrelation and the variance of the deterioration process are dependent on the mean value of the process.
Wang (2020b)	The deterioration process is described by a Gamma process-like model, but the increments within each small interval does not necessarily follow a Gamma distribution.	It releases the dependence of autocorrelation and variance on mean value that arises from a Gamma process-based deterioration model.	It is difficult to explicitly obtain the PDF of the deterioration function evaluated at any time $t > 0$.
Klutke and Yang (2002), Sanchez-Silva et al. (2011), Iervolino et al. (2013), Kumar et al. (2015)	The deterioration process is modeled by the sum (linear combination) of both gradual and shock deteriorations.	It accounts for both the continuous deterioration caused by environmental attacks and the shock deterioration due to the occurrence of shock events.	Both types of deterioration (gradual and shock) are statistically independent of each other.
Wang et al. (2017b)	The deterioration process is modeled by the sum (linear combination) of both gradual and shock deteriorations.	It considers the mutual correlation between the gradual deterioration, shock deterioration and the load process.	It is difficult to determine the correlation between the deterioration process (gradual and shock) and the load effect.

TABLE 2. Comparison between analytical methods for outcrossing-based structural reliability considering a continuous load process.

References	Description	Advantage	Disadvantage
Rice (1944)	A pioneering and inspiring work for outcrossing-based structural reliability assessment.	It provides a framework for reliability assessment considering the upcrossing rate.	The Rice's formula assumes that the occurrence of upcrossings is statistically independent.
Vanmarcke (1975)	An approximate method is proposed for the outcrossing reliability of a stationary Gaussian process with respect to a constant barrier.	It is associated with improve accuracy compared with the Poisson approximation.	Assumptions on the time intervals associated with safe and unsafe states have been introduced. The method is only applicable to a stationary Gaussian load process.
Madsen and Krenk (1984)	A joint upcrossing rate method for the first-passage reliability problem is proposed by considering the PDF of the time to the first upcrossing.	It considers the joint behaviour of the upcrossings at different time instants. A closed form approximate formula is available for a narrow-band stationary Gaussian process.	It is difficult to estimate the joint upcrossing for general distribution types.
Preumont (1985)	An approximate method is proposed for the peak value distribution of a stationary Gaussian process.	It is associated with improve accuracy compared with the Poisson approximation.	The method is based on empirical fitting of an Extreme Type I distribution. The method is only applicable to stationary Gaussian load process.
Engelund et al. (1995)	A moment method is used to estimate the probability of $\mathbb{P}(\mathcal{N}^{+}(0,T) = 0)$ instead of assuming a Poisson distribution for the number of upcrossings.	It provides a theoretically accurate estimate for structural reliability.	It is difficult to compute the moments of number of upcrossings due to the multiple-fold integrals, especially for general distribution types of load process.
Li and Melchers (1993)	A method is developed to estimate the outcrossing rate of non-stationary Gaussian vector process from a convex polyhedral limit-state surface enclosing the origin.	A generalized form of Eq. (48) is presented for multiple limit state functions, which reduces the multidimensional integral associated with multiple limit state functions to a one-dimensional integral.	It is difficult to use when considering nonlinear limit states and non-Gaussian load processes.
Andrieu-Renaud et al. (2004)	The PHI2 method is proposed to compute the upcrossing rate for general distribution types of load process.	It only needs analytical techniques for classical time-invariant reliability problems (e.g., FORM) to compute the upcrossing rate.	Significant error could be introduced for nonlinear limit states or improperly-selected time step. The occurrence of upcrossings is an independent process.
Sudret (2008)	An improved version of the PHI2 method is presented. The calculation of the upcrossing rate does not rely on the selection of the time step.	The method is associated with improved stability and accuracy compared with the PHI2 method.	The occurrence of upcrossings is an independent process.
Hu and Du (2013b)	A JUR/FORM method is proposed for outcrossing-based structural reliability considering the joint behaviour of the upcrossing rates.	It extends the joint upcrossing method in Madsen and Krenk (1984) so that more general limit state functions involving time, random variables and stochastic processes can be handled.	The efficiency could be reduced with a large number of discretized sections of the time period of interest.
Grigoriu (1984), Wang et al. (2019)	The non-Gaussian load process is converted into a Gaussian one so that the closed form solutions for Gaussian process-based reliability can be applied.	It is applicable for non-Gaussian load processes, which are frequently encountered in practical engineering.	The transformation method assumes is built on the assignment of Gaussian copula function for the joint distribution of the load process evaluated at different time instants.
Li et al. (2016)	A closed form solution is presented for structural reliability assessment based on Eq. (48), considering a deterministic resistance deterioration and lognormal load process.	The reliability method is suitable for a lognormal load process.	A lognormal distribution has been assumed for the derivative of a lognormal process. Furthermore, an error could be introduced when the resistance degrades linearly in time.
Yang and Shinozuka (1971), Ditlevsen (1986), Spanos and Kougioumtzoglou (2014), Qian et al. (2020), Wang (2020a)	The upcrossing sequence is modeled by a Markovian process, based on which the Poisson assumption-based reliability method in Eq. (44) can be modified with limited additional computational burden.	The temporal correlation between the upcrossings can be taken into account, yielding improved accuracy compared with Poisson assumption-based methods.	The Markovian chain can only capture the short-term memory associated with the upcrossing sequence.
Iourtchenko et al. (2008), Naess et al. (2011), Kougioumtzoglou and Spanos (2013), Bucher et al. (2016)	The path integration method can be used to compute time-dependent reliability numerically, assuming that the load process is Markovian.	It simplifies the reliability problem for some specific distribution types of load process, where the conditional PDF of load is available in closed form.	It is generally not straightforward to obtain the explicit solution for the conditional PDF of load, making the path integration method difficult to implement.