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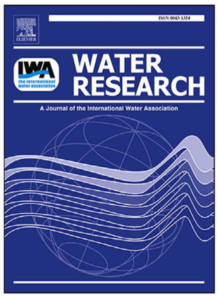

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Highlights

- Formulation of a two-player optimisation for RWH and flood mitigation system design.
- Utilises generalised rainfall parameters, reducing need for high resolution data.
- System design, catchment type and climate independent system design framework.
- Outperforms traditional design methods, eliminating overflow events in simulation.
- 32% improvement in harvested water yields in comparison to traditional systems.

Robust optimisation of combined rainwater harvesting and flood mitigation systems

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Abstract

Combined large-scale rainwater harvesting (RWH) and flood-mitigation systems are promising as a sustainable water management strategy in urban areas. These are multi-purpose infrastructure that not only provide a secondary, localised water resource, but can also reduce discharge and hence loads on any downstream wastewater networks if these are integrated into the wider water network. However, the performance of these systems is dependent on the specific design used for its local catchment which can vary significantly between different implementations. A multitude of design strategies exist, however, there is no universally accepted standard framework. To tackle these issues, this paper presents a two-player optimisation framework which utilises a stochastic design optimisation model and a competing, high intensity rainfall design model to optimise passively operated RWH systems. A customisable tool set is provided, under which optimisation models specific to a given catchment can be built quickly. This reduces the barriers to implementing computationally complex sizing strategies and encouraging more resource-efficient systems to be built. The framework was applied to a densely populated high-rise residential estate, eliminating overflow events from historical rainfall. The optimised configuration resulted in a 32% increase in harvested water yield, but its ability to meet irrigation demands was limited by the operational levels of the treatment pump. Hence, with the inclusion of operational levels in the optimisation model, the framework can provide an efficient large-scale RWH system that is capable of simultaneously meeting water demands and reducing stresses within and beyond its local catchment.

Keywords: Rainwater harvesting, Flood mitigation, Robust stochastic optimisation, Sustainable environmental engineering, Decision tool, Urban residential estates

1. Introduction

Rainwater harvesting (RWH) systems are a strong candidate for sustainable urban water management infrastructure as they allow both the provision of a secondary water resource to tackle issues of increasing water demands, and protection against localised flooding when rainfall intensities and volumes increase

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with climate change[1, 2]. This is increasingly important with climate change heightening water stresses around the globe, with both the number and severity of extreme weather events rising steadily. The UN Office for Disaster Risk Reduction found that, between 2011-21, 75% of people who were affected by natural disasters were impacted by droughts or flooding[3]. Water conservation through efficient use of existing water resources, along with effective flood management strategies, are key towards minimising losses that may occur following these extreme weather events[4].

The system of interest in this study is a RWH and flood mitigation system that is also connected to the wastewater treatment network, hereon referred to as a large-scale RWH system. This is differentiated from smaller, harvesting-focused systems which are typically implemented on the domestic household scale. However, the implementation of these large-scale RWH systems, especially in densely populated and highly developed urban areas, has been slow for its potential[5]. This can be attributed to difficulties in the standardization and hence sharing of knowledge and expertise for these systems[6]. Performance guarantees necessary for investments to be made can be difficult to achieve without expert knowledge or detailed simulations for each RWH system implementation and can involve lengthy processes that only add to the barriers for implementing these systems[7, 8].

The performance of integrated RWH-flood mitigation systems is highly sensitive to its local environment and this is reflective in the range of possible system designs seen around the world[9, 10]. This is a key contributor towards the difficulties in sharing expertise for these systems[11, 12]. Hence, appropriately sizing these systems for their catchment of service remains a significant challenge for urban planners and water managers around the globe. The design of RWH systems are key towards their effectiveness as a sustainable urban water management strategy, especially when they are expected to be operated passively[13]. A one-size-fits-all solution therefore does not exist and each implementation of a large-scale RWH system would require significant effort to guarantee that the configuration of the system infrastructure would not only be cost-effective, but also be able to adequately satisfy both the local water reuse and flood prevention objectives. Whilst a multitude of simulation and optimisation models have been developed to evaluate system performance and design high-performance systems, these are not easily transferable or applicable between catchments and specific RWH system implementations. Thus a gap remains for a standardised strategy for designing RWH systems[14].

Cities, industry partners, and academics alike have sought to address the challenge of adequately sizing RWH systems within the realms of their abilities and available resources. As such, there exists a wide range of sizing strategies which vary widely in their complexity. On one hand, design guidelines laid out by cities and other urban centres typically present highly simplified methods with the aim of decreasing the barriers towards implementing these systems. This is seen in the system sizing guidelines used in Germany and Portugal[15] where RWH systems are sized using generalised parameters such as annual non-potable water demand and/or annual rainwater yield. These design strategies can lead to unforeseen failures or stresses in the resulting system design since rainfall seasonality and short-term tail-events are not at all considered[1].

Conversely, strategies developed by industry partners and academic studies would commonly look to develop more complex algorithms and models since these institutions are normally better equipped, with greater access to computational resources and expertise. These methods can be divided generally into simulation-model-focused approaches, and optimisation-model-focused approaches. Simulation-based approaches[16, 17] use models to derive values for specific desired key-performance indicators for an indicative rainfall profile, recording the changes over a range of possible tank sizes and graphically identifying an optimal selection[18]. Common indicators used include measures of reliability[19], supply efficiency[1], and water savings[20, 21]. More sophisticated models that use probabilistic and/or optimisation methods have since also been shown to be capable of determining an optimal RWH tank system, with the objectives typically minimising total costs [22, 23], or in deriving an estimated probability of satisfying local water demands [24] or overflow volumes[25, 26].

However, existing models tend to have low time resolution, which could be sufficient if the objective was solely to optimise for a system size suitable for satisfying local water demands. Within that scope, the simulation time step was found to be insignificant [27] and a coarsely discretized mass balance such as the Yield-After-Spill or Yield-Before-Spill algorithms using daily or monthly rainfall would be adequate for estimating an optimal system size[28]. Under these models, the possibility of using RWH systems as a flood mitigation strategy through the use of coarse time resolution models is hence at best a secondary objective, rather than an objective of equivalent priority. In order to capture the occurrence of flash floods, which occur within the time frame of hours instead of days[29], models that have higher temporal resolution are needed.

More recently, there has been increased interest in game-theoretic methods in optimisation, especially for problems with multiple stakeholders with individual objectives [30]. Game theory is commonly understood as the mathematical modelling of interactions and strategies amongst rational agents, allowing the characterisation of how choices of one agent can impact that of another. This is an attractive feature as most global optimisation strategies have the implicit assumption that there is complete information, with clear strategies and mechanisms for finding a consensus between the multiple objectives [31]. More specifically to water management systems, game theoretic optimisation approaches have been studied as a water resource allocation strategy and have been applied to the control and operation of a reservoir[32], a river basin[33], and drinking water transport networks[34]. Results from these studies have consistently shown that game theoretic optimisation can improve the both the individual and overall benefits received by their stakeholders and reduce the overall computational time to solution.

This paper presents a two-player game-theoretic optimisation framework for sizing passively-operated RWH systems, which implements two competing optimisation models to ensure that a robust system configuration is derived. Alternative strategies towards generating synthetic rainfall data would require detailed, statistical simulations of the rainfall process which can be time-consuming, with the quality dependent on the available rainfall data[35]. The competitive game allows an efficient derivation of rainfall patterns which removes the need for high resolution historical rainfall data for the optimisation-based sizing strategy to work effectively. This is aimed at improving the RWH system design process, allowing system designers to quickly derive an optimal design configuration using only a set of generalised rainfall parameters from the local catchment and a range of candidate system parameters. This work builds on the literature by providing novel presentations of:

- An intuitive tool which allows optimisation models to be built and customised for individual large-scale RWH systems.
- An optimisation-based system design and sizing strategy that minimises the prerequisites of familiarity and expertise in optimisation and computational methods.
- Reducing reliance of optimal system sizing on the accessibility and availability of high-resolution rainfall data.

These highlighted areas would contribute towards reducing the barriers to implementation and encourage improved designs of sustainable and efficient large-scale RWH systems in urban settings.

The rest of the paper is organised as follows. Section 2 describes the methodology and components of the two-player optimisation framework and outlines a case study to which the framework has been applied. Section 3 demonstrates the key results in characterising the behaviours of the framework and the performance of the derived systems in comparison to alternative system sizing methodologies. Finally, Section 4 summarises the main conclusions, areas for improvements to the proposed framework, and its associated future developments.

2. Methods

A two-player approach is presented in this paper for optimally sizing and designing a multi-tank rainwater harvesting system, following a competitive game framework with two opposing players. The framework is implemented in Python using the Pyomo library[36] and the code is available upon request. Player One is a stochastic design optimisation model where its output strategy set is in the form of optimal system configurations, seeking an optimal tank design for a given set of rainfall patterns. Player Two is a deterministic optimisation model that derives a high intensity, high volume rainfall signal that can overwhelm a given tank design. Therefore, Player One's objectives are to minimise overflow volumes and maximise rainwater yield over all rainfall input scenarios designed by Player Two, whilst Player Two aims to design a rainfall signal that would result in the maximum volume of overflow in the system configuration played by Player One.

The overall workflow of the two-player framework is shown in Figure 1. The algorithm begins with Player Two designing a rainfall signal which results in the highest overflow volumes for the smallest system capacity from Player One's set of possible strategies. Player One then uses that rainfall signal to design a system configuration which minimises overflow volumes. With each rainfall profile designed in each iteration by Player Two, it is added to the set Player One considers in its optimisation process. This continues until one of the two possible convergence criteria is reached:

(a) When Player Two is unable to find a rainfall pattern within the stipulated bounds that can generate overflow above a given acceptable threshold in the system configuration derived by Player One.

(b) When Player One produces the same tank configuration as in the previous iteration, signifying that it is not possible to achieve any further improvements to the overflow reduction performance through the system design.

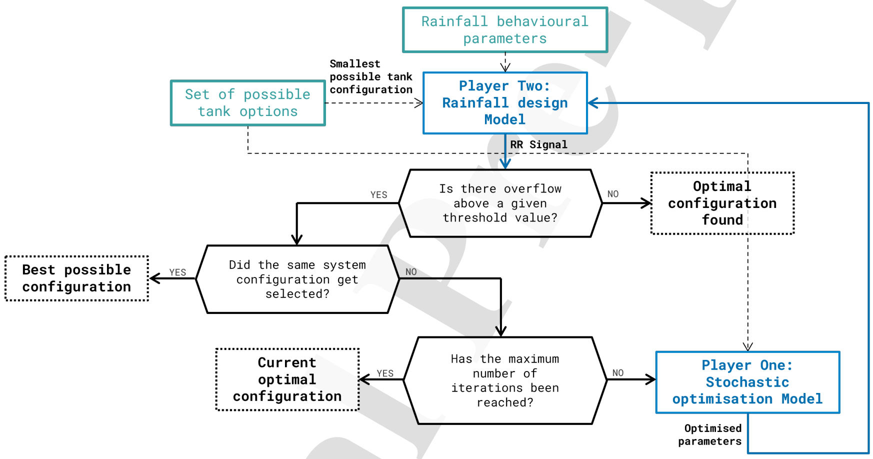

Figure 1: Flowchart summarising the proposed Two-player workflow.

A case study system based on an existing large-scale RWH system is used to demonstrate the behaviours and performance of the proposed two-player design framework. The behaviours of the algorithm under various possible sets of user inputs are then characterised to ensure the framework behaves as intended. A selection of output system configurations are then evaluated using a simulation model to determine the effectiveness of the algorithm in producing well-performing RWH system configurations.

The following subsections present the components of the two-player framework in further detail. Section 2.1 demonstrates the system configuration used to illustrate the working principles of the proposed two-player framework. Detailed formulations of the constraints used in describing the system dynamics implemented in the optimisation models of both Player One and Two are then presented in Section 2.2 and further behavioural constraints for each player are described in Section 2.3 and Section 2.4. Finally, optimisation failure prevention and defaulting behaviours are outlined in Section 2.5 which helps to ensure the framework is not inhibited by a lack of output solutions from the optimisation models, for example as a result of a lack of solution time.

2.1. Case study configuration

The system configuration used in this study is a passively operated, underground large-scale RWH system which comprises a total of four distinct storage tanks, each dedicated to a specific performance goal as illustrated in Figure 2. The system serves a densely populated, high-rise residential estate in a tropical climate and is connected to the wastewater network to prevent long-term water retention to minimise pest growth. The 'Separation' tank serves as the initial receiver and filters the water into the 'Harvesting' and 'Detention' tanks based on the size and height of the openings between these tanks. The 'Detention' tank is the main container used to temporarily hold excess water to be discarded from the catchment before it is released into the downstream public wastewater network. The 'Harvesting' tank stores captured water for local reuse, whilst the 'Treatment' tank stores water that has been processed and is directly ready for use. The orifice which allows flows into the 'Harvesting' tank in this set up is designed to be above the bottom of the tank, such that any initial surface runoff containing sediments and dirt can be discarded, improving the cleanliness and quality of the water being harvested.

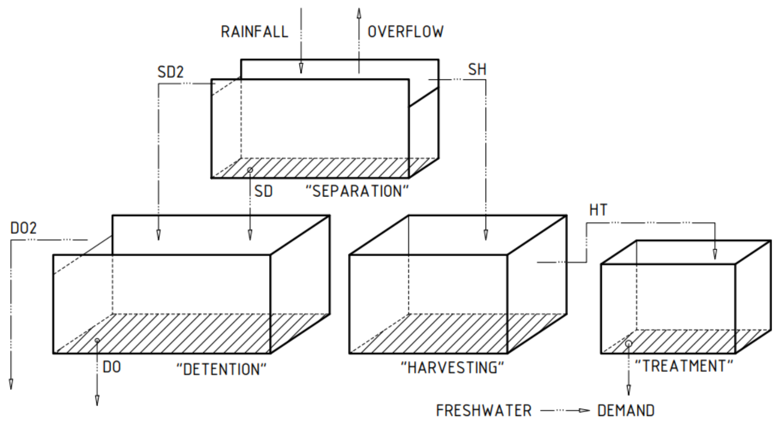

Figure 2: Multi-tank RWH system configuration.

Under this configuration, freshwater is used to supplement when there is insufficient volumes of water captured in the system to meet local non-potable water demand. In this study, the daily irrigation of green spaces within the residential estate is used as the only source of demand. This is assumed to be achieved through a drip irrigation system activated in the evenings when insolation levels are low. There are three main objectives for this system, which are:

1. Minimise risk of surface overflows;

2. Minimise tank capacity required;
3. Maximise water availability for local reuse.

The design variables for the optimisation models in Player One are therefore the tank areas and tank heights of all four tanks in the system, and the heights of the orifices that have an associated height parameter. More specifically, these are the flows that are labelled 'SD2', 'SH' and 'DO2' in Figure 2. Orifice areas are not implemented as a decision variable as these come in standard sizes and provide a smaller degree of freedom than the orifice heights. The inclusion of both the orifice heights and areas as decision variables would increase the size of the optimisation problem by introducing additional variables and a large set of constraints in accompaniment as more linearisations will be required. In this current implementation, the operational levels of the pump located at 'HT' are also not implemented as a decision variable, since this is viewed as an operational variable rather than a system design variable.

2.2. Formulating optimisation model constraints for tank dynamics

Both players in the algorithm abide by the same constraints surrounding the system dynamics. These include a mass balance for the tank, linear approximations of orifice flow equations for each opening in the tank, the definition of when overflows occur in the tank, as well as a total discharge variable that adjusts the final outflow volumes to be consistent based on the available volume of water available within the tank when there are multiple flow openings from the tank. The model is formulated as a Mixed Integer Linear Programme (MILP) to reduce the required computational time and increase the feasibility of the derived models. As the model is also stochastic, the problem size would increase exponentially as the number of scenarios increase, thus establishing that the derived model is linear is the most effective in ensuring that it remains tractable, especially in comparison to other optimisation model types such as the Mixed Integer Non-Linear Programme (MINLP), even if these may provide a more accurate representation of the system.

The code structure for the two-player algorithm is built around a core file describing the system dynamics, which is used to build a few generic tank system blocks. These blocks allow for optimisation models for different tank configurations to be built quickly and can be quickly populated with data specific to the configuration to form a viable optimisation model. These basic blocks are implemented by the model files for both optimisation models used in this two-player algorithm, which ensures both players are constrained by the same system dynamics.

The models are implemented in discrete time steps since rainfall data is typically collected using a relatively coarse temporal resolution. This however produces an inventory modelling problem where flows from the openings, which are dependent on the volume of water within the tank, V^t, can change within the large time delta between time steps, resulting in an inaccurate representation of the mass flow dynamics. Therefore, an optimistic-pessimistic index α and β for the inflows I^t and outflows O^t from the tank respectively is introduced in the mass balance in Equation (1) as an offset of the inflows and outflows to better represent this phenomenon. The values of α and β are derived empirically to improve the model performance. Whilst the overflow volumes W^t are defined in this equation, this is insufficient to ensure that the model remains consistent, especially with the objective of maximising overflow volumes in the rainfall design model. Hence, a further definition of this behaviour is required and shown in Table 1.

$$\begin{aligned} V^{t+1} + W^{t+1} =& \alpha(V^t + I^t + W^t) + (1-\alpha)(V^{t+1} + I^{t+1} + W^{t+1}) \\ & - \beta O^t - (1-\beta)O^{t+1} \end{aligned} \tag{1}$$

The discharge volumes from each orifice are characterised through non-linear orifice flow equations, which are functions of the coefficient of discharge C_d, orifice area θ_o, the gravitational constant g and the water level above the opening L^t in the form represented in Equation (2). However, the implementation of these equations would require the use of a Mixed Integer Non-Linear Programme (MINLP), which can be intractable and take a much longer solution time. To reduce the computational time and requirements of the derived optimisation models, a linear approximation is used in the form of Equation (3) such that a MILP can be implemented. The error resulting from such an approximation is low as the range of values used within these systems are typically limited by the depth in which these underground systems are allowed to reach. Since this is an approximation, the actual discharge from the tank is calculated later using a total discharge variable δ which ensures that the sum of all discharges calculated individually using Equation (3) is below the available volume of water in the tank at each time step.

$$D_o = C_d \theta_o \Delta T \sqrt{2gL^t} \tag{2}$$

$$D_o^t = C_d \theta_o L^t \Delta T \sqrt{2g} \tag{3}$$

These individual orifice discharge rates also need to be bound by the maximum possible discharge rate of the orifice. This is dependent on the tank height, a decision variable, therefore these discharge bounds need to be implemented as constraints for the values to be calculated within the model.

The remaining tank dynamics, such as the flow through orifices that sit above the bottom of the tank, pump operations, and overflow volume definitions are conditional events and are described by discontinuous dynamic equations. A summary of the discontinuous dynamics that can occur in a RWH system is described in Table 1, which follows either the $f = \max(x, y)$ selection logic, or a cases formulation type. For example, overflow events are discontinuous since these can only occur when the sum of the net inflow and existing volumes of water exceeds the capacity of the tank and should have a value of zero otherwise. The discontinuous expressions need to be formulated into algebraic expressions since conditional expressions are not valid as optimisation constraints, which are discussed further below.

Equation (4) shows the formulation used to represent a max-value selection behaviour in the optimisation models, with minimisation behaviour following

Table 1: Dynamics and formulation types of discontinuous dynamics in RWH system.

Dynamics	Mathematical Representation	Formulation type
Conditional flows	$D_o^t = \begin{cases} C_d\theta_o\sqrt{2g}(L^t - \eta_o)\Delta T & L^t > \eta_o \\ 0 & L^t < \eta_o \end{cases}$	$\max(D_o^t, 0)$
Overflow	$W^t = \begin{cases} I^t - C & I^t - V > C \\ 0 & \text{otherwise} \end{cases}$	$\max(W^t, 0)$
Total Discharge	$\delta^t = \begin{cases} \delta_1 = V^t + I^t & \sum_o D_o^t > V^t + I^t \\ \delta_2 = \sum_o D_o^t & \text{otherwise} \end{cases}$	$\min(\delta_1, \delta_2)$
Pump (On/Off)	$D^t = \begin{cases} 0 & 0 \leq L^t \leq r_{off} \\ D^{t-1} & r_{off} \leq L^t \leq r_{on} \\ R & r_{on} \leq L^t \leq \overline{L^t} \end{cases}$	Cases

the opposite logic set. In this case, the selection logic assigns the variable f the value of x_1 or x_2 depending on which is larger, using the Big-M method where M is a large scalar value used to define a boundary that contains the feasible region of f, and ϕ is a binary variable that serves as the switch between the two possible values.

$$f = \max(x_1, x_2) \qquad \text{generalised max equation} \tag{4}$$
$$f \geq x_1 \tag{4a}$$
$$f \geq x_2 \tag{4b}$$
$$f \leq x_1 + (1 - \phi)M \tag{4c}$$
$$f \leq x_2 + M\phi \tag{4d}$$

The second discontinuous event type of operational cases can be described through the behaviours of the pump, which moves water through the treatment system, operated using an on-off controller. The pump activates when the water level in the tank exceeds a high 'on-point' level and turns off when it falls below the 'off-point' threshold. If the water level lies between these two points, it maintains its previous state. This can be represented mathematically by Equation (5), which demonstrates three possible dynamic equations $f_i(x)$ for each operational region in $i = \{1, 2, 3\}$.

$$D^t = \begin{cases} f_1(x^t) & a \le x^t \le b \\ f_2(x^t) & b \le x^t \le c \\ f_3(x^t) & c \le x^t \le d \end{cases} \qquad \text{generalised case equation} \tag{5}$$

$$\rho_1^t + \rho_2^t + \rho_3^t = 1 \tag{5a}$$

$$f = f_1(x_1^t)\rho_1^t + f_2(x_2^t)\rho_2^t + f_3(x_3^t)\rho_3^t \tag{5b}$$

$$x^t = x_1^t + x_2^t + x_3^t \tag{5c}$$

$$a\rho_1^t < x^t < b\rho_1^t \tag{5d}$$

$$b\rho_2^t < x^t < c\rho_2^t \tag{5e}$$

$$c\rho_3^t < x^t < d\rho_3^t \tag{5f}$$

The operational region that the monitored value x^t lies in at time t is given by a binary variable ρ_i^t, which is determined through the set of Equations (5d) to (5f) which establishes the bounds that define each region. Equation (5a) stipulates that only one region can be selected at any one time, and the discharge function is finally calculated as a linear combination of the three case functions as in Equation (5b), where only one function should be activated at any one time, corresponding to the values of their binary variables.

Finally, to ensure that the model is physical, a volume conservation constraint is implemented for each tank model. This follows the logic presented in Equation (6) and prevents the system from generating nonphysical sources of water within the model.

$$\begin{aligned} \alpha I_s^0 + (1-\alpha)I_s^\tau - \beta\delta_s^0 - (1-\beta)\delta_s^\tau + \sum_{1}^{\tau-1} I_s^t \\ = V_s^\tau - V_s^0 + (1-\alpha)W_s^0 + \alpha W_s^\tau \end{aligned} \tag{6}$$

For both the stochastic design optimisation and the rainfall design models, additional model constraints are implemented to define their individual design spaces. The stochastic design model requires additional constraint definitions to aid the system parameter selection, whilst the rainfall design model requires generalised rainfall parameters that define the feasible space for generating reasonable rainfall patterns.

2.3. Player One: Stochastic design optimisation model

The stochastic optimisation model is given a set of possible values for each of the system design parameters, which are the tank areas, tank heights, and orifice heights. The model then selects an optimal system parameter set from this collection that would be best to handle the set of rainfall profiles considered in each iteration, further constrained through a given upper bound on the total system capacity.

The design optimisation model in Player One has been decomposed into individual tank blocks, each corresponding to a single water tank in the multi-tank

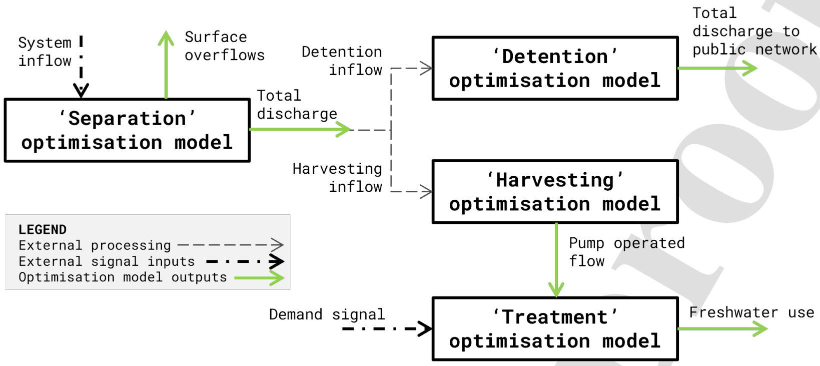

Figure 3: Flow of information between modularised optimisation models.

system presented in Section 2.1. This allows non-linear flows between tanks to be excluded from the optimisation models and processed using a Python function outside of the optimisation models, such that each individual tank optimisation model is implementable as a mixed integer linear programme (MILP)[37]. The interactions and flow of information between each individual optimisation model are demonstrated in Figure 3.

The modularised optimisation framework further allows the dedicated tanks to be designed accordingly with each of their corresponding individual purposes, on top of the overall system objectives outlined in Section 2.1. The specific tank purposes and their corresponding implemented model objectives of each of the component tanks for the multi-tank RWH system are shown in Table 2.

However, in the practical implementation of the schematic shown in Figure 3, the optimisation models solved first would see a larger allowable capacity, and hence be given a larger possible design space. Meanwhile, downstream tanks can only use any remaining allowable capacities to determine an optimal configuration for their respective objectives. Hence, there is inherently a prioritisation and bias towards designing the tanks which are optimised first. This means that the 'Separation' tank, and its objective would inevitably be given the highest priority. As such, the model was designed to minimise surface overflows as this is a critical system objective. In the implemented case study, the next priority was to provide higher harvested water yields and satisfy demands. Therefore, the Harvesting and Treatment tank branches have been set to be solved next, with the Detention tank being implemented as the last step in the optimisation chain.

Each tank model is given a set of tank parameter options that it is allowed to select from to form an optimal tank configuration. This is implemented using a binary selection logic shown in Equation (7) for each system design parameter P^k, each with a possible set of n values $p_i^k = \{p_1^k, p_2^k, \ldots, p_n^k\}$, and a corresponding set of binary variables $\psi_i^k = \{\psi_1^k, \psi_2^k, \ldots, \psi_n^k\}$ to select one

Table 2: Summary of design objectives for individual tank components in the multi-tank RWH system.

Tank	Design Specifications	Model Objectives
Separation	• Prevent flooding on catchment surface. • Reduce required tank sizes.	Minimise total overflow over all input scenarios, with a wasted capacity penalty. $$\min\left(\sum_{t,S} W_S^t + P_C\right)$$
Detention	• Ensure discharge from system is below allowable rates.	Minimise tank capacity $$\min\left(C + P_W\right)$$
Harvesting	• Increase water availability • Increase volume of usable water	Maximise pump output over all scenarios $$\max\left(\sum_{t,S} D_s^t\right)$$
Treatment	• Increase water availability • Satisfy local water demands	Minimise freshwater use over all scenarios $$\min\left(\sum_{t,S} F_S^t\right)$$

optimal parameter value, as defined in Equation (8).

$$P^k = \sum_i p_i^k \psi_i^k \tag{7}$$

$$\sum_i \psi_i^k = 1 \tag{8}$$

The capacity C and volume V^t variables need to be defined such that the mass balance equations in Equation (1) and overflow equations in Table 1 can be represented linearly. As the tank area and tank heights of the system are decision variables and are defined through a binary selection of an optimal value from a given parameter set, the multiplication of these design variables needs to be defined through the following sets of constraints.

The relationship between the volume V^t and water levels L^t in the tank is a function of the decision variable set $A = \sum a_i \lambda_i^a$, the tank area. Whilst this multiplication of variables results in a non-linear constraint, it can be linearised with the introduction of a dummy variable z_i^t and a large upper bound value M. The value of the upper bound value M needs to be sufficiently large to contain all possible values of L^t. The relationship is defined in Equation (9) and follows the logic set in Equations (10a) to (10d).

$$\textit{function definition} \qquad V^t = L^t A = L^t \sum_m a_m \lambda_m \tag{9}$$

$$\textit{function implementation} \qquad V^t = \sum_i a_m z_m^t \tag{10a}$$

$$z_m^t \leq \lambda_m M \tag{10b}$$

$$\textit{dummy variable definitions} \qquad z_m^t \leq L^t \tag{10c}$$

$$z_m^t \geq L^t - (1 - \lambda_m) M \tag{10d}$$

The volume variable is bound by the tank capacity and is a function of the discretised tank height $H = \sum h_i \phi_i^h$ and tank area A, as demonstrated in Equation (11). With the binary selection formulation of both decision variables, this can be similarly linearised into the form shown in Equation (12a) with the aid of a dummy variable $y_{m,j} = \lambda_m \phi_j$ that behaves according to the logic of multiplying two binary variables, achieved through the set shown in Equations (12b) to (12d).

$$\textit{function definition} \qquad C = HA = \sum_i h_i \phi_i \sum_m a_m \lambda_m \tag{11}$$

$$\textit{linear formulation} \qquad C = \sum_{i,j} h_i a_j y_{im} \tag{12a}$$

$$y_{mj} \leq \lambda_m \tag{12b}$$

$$\textit{dummy variable definitions} \qquad y_{mj} \leq \psi_j \tag{12c}$$

$$y_{mj} \geq \lambda_m + \psi_j - 1 \tag{12d}$$

The height of the orifices η_o must be within the height of the tank. Since this is a design variable, the optimal orifice height needs to be determined by multiplying a scalar value ω in the range of $[0,\ 1]$ with the final selected orifice height, as defined in Equation (13). The algebraic formulation of this equation would follow the same logic as in the calculation of the capacity variable.

$$\eta_o = \sum_i \omega_i \psi_i H = \sum_i \omega_i \psi_i \sum_j h_j \phi_j \tag{13}$$

With the competing objectives of minimising surface overflows and minimising tank capacities, penalties are designed for each of the individual tank systems as a mechanism to adjust the prioritisation of the objectives within the models. For the 'Separation' tank, the objective is to minimise the total overflows with the lowest possible tank capacity and a function was implemented to penalize any completely unused, and hence wasted tank capacities. This is realised by defining the maximum wasted tank capacity Ω_S as the difference between the tank capacity and the maximum volume of water held in the tank for each provided scenario, as in Equation (14) and assigning a penalty cost based on a tiered cost function, following Equation (15).

$$\Omega_S = C - \max V_S^t \tag{14}$$

$$P_C = \begin{cases} \rho_1 & 0 \leq \Omega_S \leq 0.1C \\ \rho_2 & 0.1C \leq \Omega_S \leq 0.5C \\ \rho_3 & 0.5C \leq \Omega_S \leq C \end{cases} \tag{15}$$

Finally, whilst the main objectives of the remaining tanks are not related to the surface overflows, overflows from these tanks would back-flow into their source tanks and can result in surface overflow. A threshold-based overflow penalty function is therefore implemented following Equation (17) which penalises excess overflow volumes χ^t, defined as overflow volumes above a given threshold value ζ with a scalar cost C_W as shown in Equation (16).

$$\chi^t = \max(W^t - \zeta, 0) \tag{16}$$

$$P_W = \sum_t \chi^t C_W \tag{17}$$

These penalty functions are implemented to achieve a balance between the competing overall system objectives and individual component objectives and can be adjusted on an individual implementation basis to reflect the desired prioritisation and biases required in a given tank system design.

2.4. Player Two: Rainfall design model

Acting as Player Two in the framework, the rainfall design model is a second optimisation model which looks to maximise an overflow response for a given input system configuration. With the system presented in Section 2.1, the tank component responsible for directly demonstrating surface overflows is the 'Separation' tank. Hence, only this tank is implemented in the rainfall design optimisation model to reduce the complexity of the optimisation problem and in turn to reduce solution times.

By only implementing the 'Separation' tank in the rainfall design model in Player Two, the possibility of water not being able to flow into the downstream tanks when these are full is ignored. As such, the partial system model would provide a conservative calculation of overflow in comparison to a full system implementation. This means that all overflow calculated from this partial system model will result in overflows from a full model, but overflows could still occur for profiles which this model calculates no overflows for.

In ensuring the rainfall designed by the model are feasible, a set of generalised rainfall description parameters need to be implemented on top of the tank dynamics described in Section 2.2. Values of these parameters in this study are derived using historical data collected for the given service catchment area, and are:

- maximum simulation horizon volume;
- maximum 2-hour window volume;
- maximum increment and decrement rates between time steps;
- maximum volume in a single time step.

Other generalised parameters can also be utilised in the rainfall design model, such as the average rainfall volumes, or time-based constraints to dictate the shape of the derived rainfall, for example when rainfall would only occur during certain periods in the day, such as to represent different storm types. The rainfall design parameters used can be altered based on what data and information are available for the region, as long as the set is sufficient in characterising realistic rainfall patterns.

2.5. Failure prevention and substitution function

With the modularised design optimisation model, failures in the optimisation of downstream tanks can occur if there is no feasible combination of parameters from the parameter options after solutions have been found for the upstream tanks. The risk of such occurrences is reduced by reserving sufficient capacity for at least the smallest total capacity for all downstream tanks.

In the event of a failure, however, a set of parameters would still be required to allow the two-player algorithm to continue. As such, an output substitution function was implemented to return a set of tank parameters whenever an optimisation model fails to output a feasible solution. After every tank optimisation attempt, the algorithm determines if a valid output has been produced. If the tank has downstream tanks, the optimisation model for these is skipped, and the source of the failure is recorded. This allows optimisation models for parallel flow streams to continue running.

The defaulting algorithm is called after all optimisation models have been attempted. This is implemented such that the derived substitute configuration would maximise the total allowable capacity of the system and distributes this capacity between the tanks within the set that needs substitute parameters. This procedure is outlined below.

1. Calculate the remaining capacity that the algorithm is allowed to assign, T_A, to the defaulting set S.
2. Determine a matrix $\boldsymbol{C}_j$ of possible capacities for each tank j in the set using their input parameters, and all possible total capacities $\boldsymbol{T}$.

$$\boldsymbol{T}_S = \sum_j \boldsymbol{C}_j = \sum_j A_j H_j \tag{18}$$

3. Identify the selected capacity C_S, defined as the maximum possible capacity from the parameter combinations, that is under the total capacity

threshold. This is achieved by finding the maximum element value in $\boldsymbol{T}_s$ after setting all ineligible options to zero.

$$C_s = \max(\boldsymbol{T}_s > T_A) \tag{19}$$

4. If the size of set $S \geq 1$, find the set of individual tank capacities that has the lowest set variance to distribute the total capacity between all the tanks that are in the defaulting set S. This minimises presumptions made about the system behaviour and dynamics, providing a default configuration unbiased to any of its objectives.
5. For each tank, identify the row and column indices for elements in the matrix that corresponds to the capacity selected based on Equation (19). If there are multiple possible combinations of tank and area parameters which correspond to the same tank capacity, select the combination with the largest area parameter which should provide a lower outflow rate from the tank. This reduces the risk of overflows and stresses to any downstream tanks and systems.

In summary, the behaviour of the substitution function implements an intuitive design principle for the RWH systems. This sought to maximise the allowable capacity to reduce overflow risks, but also opting for a tank system which maximises the area parameter over the height parameter, which serves to reduce discharge rates that can result in downstream stresses.

3. Results and Discussion

In order to determine the performance and behaviours of the proposed algorithm under different possible input sets that a user might provide, the impact of two framework parameters were characterised using the case study described in Section 2.1. These framework parameters are:

- **Solution times:** The time limit provided for both optimisation models to find an optimal solution. Sufficient time should be given to ensure that the optimisation algorithm can search the given design spaces thoroughly, whereas a longer search time can help with identifying a higher-quality solution.
- **Convergence threshold:** The overflow volume at which the system is defined to have converged. For a system to be robust under the time step sizes of both the optimisation model and a high-resolution simulation model, this value would be zero.

The behaviour of each optimisation model parameter is characterised individually. An initial input set of parameters, denominated here as a 'Control' experiment, is used as the basis from which parameter values are adjusted for each of the parameters characterised. This is shown in Table 4 and provides a platform for comparing behaviours between all characterisation experiments.

Each of the optimisation framework parameters is evaluated and discussed in the following subsections to characterise the behaviour of the two-player algorithm. This is measured using the following model behavioural indicators:

- **Time taken:** The total time required for the algorithm to complete, whether this is through achieving convergence or in reaching the maximum possible number of iterations. This is ideally minimised to reduce computational requirements.
- **Number of iterations:** The number of global iterations required to find convergence. A smaller number of iterations would lead to faster solution times. However, a larger iteration number does imply multiple rainfall profiles have been used to determine the optimal system configuration, and hence could be preferable as a measure of the reliability of the system output.
- **Convergence and type:** Indicates whether convergence has been found through the system design or rainfall overflow maximisation models. Convergences from the rainfall design phase would be preferable for a robust solution, whilst convergences in the design would provide the best possible overflow reduction performance under the given design constraints.
- **Failures in stochastic design phase:** A list of tank nodes where no solutions were found within the given time limits. Ideally, there would be no failures in the optimisation models as that would allow the algorithm to best be able to calculate an optimal balance for all the tank modules.

Finally, a selection of the derived system capacities from the characterisation experiments is evaluated for their performance under 700 different historical rainfall profiles, which represents the upper-ranges of rainfall volumes that the system can expect to service. This was executed using a high time-resolution simulation model which represents the detailed system dynamics through implementing mass balance and orifice flow equations. The rainfall profiles were extracted, based on their date-stamp, from a historical time series to generate 24-hour long rainfall profile segments with timestep sizes of 5 minutes. Each of these segments is utilised as a possible rainfall profile and simulated to determine the performance of the system under a wide range of rainfall types it can expect to manage. The performance metrics of interest for the case study system, as outlined in Section 2.1 pertains to the system's ability to reduce overflow risks, provide water availability, and satisfy a given irrigation demand. These metrics and the system performances are discussed further in Section 3.4.

3.1. Impact of time limits

Both optimisation models will require sufficient time to find a solution, and a lower time limit can reduce the solution quality as the optimiser may be unable to search the design space thoroughly. To characterise the impact and determine the minimum solution time, experiments running both higher and lower solution time limits to the values used in the 'Control' experiment are discussed in this section. The time limit shown for the stochastic design models is for each individual tank optimisation module.

The results are summarised in Table 3, which demonstrates that the stochastic optimisation model generally requires more than 600s for each tank optimisation, to ensure that there is sufficient time for a solution to be found. This is especially important in reducing the reliance on the defaulting algorithm, for when the number of iterations increase and the problem grows exponentially. Solutions containing a defaulted parameter would not be a guaranteed optimal solution and hence should be avoided as much as possible.

Table 3: Summary of behaviours for optimisation model time limits. Altered parameter values highlighted in bold.

	Experiment				
	RR1	RR2	C	SO1	SO2
Parameters					
RR Time limit (s)	**30**	**180**	60	60	60
SO Time limit (s)	600	600	600	**300**	**900**
Indicators					
Time taken	2:43:01	1:56:23	2:47:41	1:40:34	2:07:12
Number of iterations	8	6	8	9	6
Convergence type	RR	SO	RR	SO	SO
Failures	Det	Det	Det	All	-
Total capacity (m^3)	900	890	900	900	840

When examining the total algorithm run time and number of iterations used to determine a solution in these experiments, it can be observed that an increase in model allowable time limits can help to improve the derived solution quality, and in turn, reduce the number of iterations required to find an optimal solution. This reduction in the number of required iterations can help to reduce the total algorithm run time, as observed by comparing these behaviours from the RR1 experiment with that of RR2, and separately when comparing the Control and SO2 experiments. Lastly, the total capacities found by each of the experiments have shown that it is highly impacted by the defaulting algorithm, with all experiment runs that showed failure demonstrating capacities much either exactly or very close to the total allowable capacity given for the system, whilst the only experiment with no failures from the stochastic design optimisation model providing a smaller tank option. This shows that in preventing failures from the optimisation models, a better balance between the multi-objectives of the system will be derived, as expected from the design of the defaulting algorithm behaviours.

On the other hand, changes to the time limits allowed for the rainfall design model have shown that it has minimal impact on the output rainfall patterns. An inspection of these outputs demonstrated that the rainfall patterns generated during the RR time limit experiments are identical to those found in the control experiment, with the only difference with the design model producing more iterations when the algorithm has deemed it necessary. This shows that under the given set of rainfall parameters, there is a set of rainfall patterns that is globally optimal in generating overflow events.

3.2. Impact of overflow convergence threshold

The algorithm stops when the rainfall design model is unable to find a rainfall pattern that can create overflows above a given convergence threshold value. This parameter is key towards how quickly the algorithm can converge and produce a solution and is significant in ensuring that the output system configuration adequately satisfies the overflow prevention objectives.

Figure 4 demonstrates the maximum overflow volumes calculated in each iteration of the rainfall design model in the ‘Control’ experiment and shows that there is a rapid decay in the overflow generated within the first optimisation iteration. Increasing the threshold levels at which convergence is defined was demonstrated to reduce the number of iterations required and hence the time and computational resources required to produce a solution. For the same set of input parameters, the convergence behaviour follows the overflow graph exactly, where termination was called after iterations 1, 4, and 5 for threshold definitions of 100 m^3, 50 m^3 and 10 m^3 respectively. The outputs from each

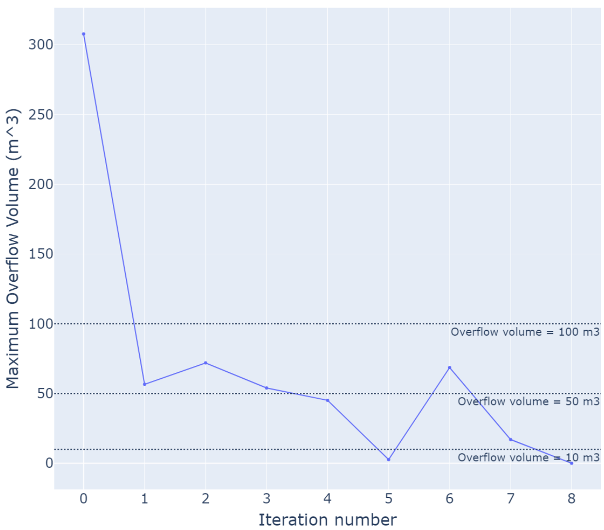

Figure 4: Calculated overflow levels from RR model.

of the threshold definitions also provided an insight into the behaviours of the algorithm when there is a softer threshold on the overflow constraint. As the threshold is increased and more overflow is allowed in the optimisation model, the ‘Separation’ tank size is reduced from 450 m^3 to 360 m^3, freeing up allowable capacities to be used in the other tanks in the system. In addition, with lower threshold overflow volumes, both the height of the secondary outlets of the ‘Separation’ and ‘Detention’ tanks, ‘SD2’ and ‘DO2’ are reduced significantly from 1.2 m and 0.6 m, to 0.9 m and 0.1 m respectively to reduce the risk of overflow events in both tanks.

It is also demonstrable that overflows in the optimisation model do not necessarily correspond to any overflow event in the simulation model. Whilst the same system dynamics are implemented in both models, the optimisation model utilises a much coarser 5-minute step size in comparison to the one-second step size used in the simulation model. This results in a disparity of overflow behaviours, where a higher rainfall inflow volume is required to generate an overflow event in the simulation model than in the optimisation model. Since the simulation model is of much higher time resolution and hence accuracy, this means that the convergence threshold used in the two-player algorithm can be much higher than the actual desired threshold volume. Through adjusting this optimisation model parameter, the algorithm run time can be improved and computational requirements reduced by requiring a smaller number of iterations without significantly impacting the overflow risk reduction performance of the derived optimal configuration.

3.3. Algorithm behavioural dependencies and tuned input parameter set

With each set of characterisation experiments, the main dependencies for each behaviour can be summarised through the following:

- The number of iterations required to find convergence is a major factor in determining the probability of receiving an optimisation error and the time taken to derive a solution, but this is also dependent on the convergence threshold of the two-player optimisation framework.
- The number of failures encountered in the stochastic optimisation models increases with the number of iterations required as the time required to derive a solution increases exponentially between each iteration. Thus, there is a need to ensure there is sufficient time for the stochastic optimisation model to find a solution and maximise the capabilities of the stochastic optimisation model since the defaulting algorithm would always aim to maximise the allowed system capacities. This suggests that there should be an implemented lower bound for the time limit allowable for this parameter.
- The maximum number of iterations allowed for the algorithm is a key factor in ensuring that the algorithm is not allowed to run forever and should be a sufficiently large number to best allow the algorithm to converge. Through the characterisation experiments, it was found that 6-8 iterations were sufficient in providing convergence in most cases.

With the behaviours of the algorithm characterised, a 'tuned' input set as shown in Table 4 was used to derive an optimal system design for the case study to demonstrate the potential of utilising the two-player algorithm, in comparison to an alternative system design strategy. The objectives were to minimise the total system capacity as much as possible on top of the overflow and harvesting objectives. A non-zero overflow threshold value was implemented to reduce the computational requirements, and a value lower than the time step size difference was selected to decrease the likelihood of observing overflows in the more accurate simulation model. As the characterisation experiments also showed that the stochastic design model has lower rates of failure when given more time, this was increased to 900 s for each tank optimisation. As the rainfall design model was minimally impacted by the solution time, hence this was maintained at 30 s. Under this set of 'tuned' parameters, the stochastic optimisation model had succeeded in all iterations and converged following the non-zero threshold level. This gave a total system capacity of 590 m^3.

Table 4: Input parameter values and sets used in the 'Control' and 'Tuned' experiments.

Input Parameters	'Control'	'Tuned'
Total Allowable Capacity	900 m^3	600 m^3
SO Time Limit	600 s	900 s
RR Time Limit	60 s	60 s
Parameter Set Space	'Large'	'Small'
Penalty Weights	Unscaled	High
Convergence Threshold	0 m^3	180 m^3

3.4. Performance of output configurations

A selected set of output configurations, generated using a range of different input parameters, was evaluated using the simulation model for their performance in meeting the objectives of overflow reduction and demand satisfaction. These are derived from the behavioural characterisation experiments, where experiments 1 and 2 tested the size of the set of input design space, experiments 3 to 5 adjusted the weight given to the penalty function of the 'Separation' tank, while experiments 6 to 8 were derived under increased overflow convergence threshold levels. The configurations were selected to determine (a) the overflow mismatch levels between the optimisation and simulation models, and (b) the performance of a wide range of possible system configurations that is found through the two-player algorithm. Four statistical indices were collected, each defined as:

- **Overflow risk:** The percentage of simulated scenarios that demonstrated any occurrences of overflow.
- **Harvesting potential:** The percentage of simulated scenarios that harvested any amount of rainwater.
- **Demand satisfaction:** The percentage of simulated scenarios where the irrigation demand is fully met.
- **Demand non-fulfilment:** The percentage of simulated scenarios where all 47 m^3 of the irrigation demand is met completely using the freshwater supply.

Figure 5 plots the four statistical indices found for each of the simulated configurations derived from the selected experiments. It demonstrates that the configuration 2, derived under the 'Small Parameter Set' inputs has the best performance for meeting demands and ensuring water availability, but demonstrates overflow, albeit only in 1% of the simulated cases and with a maximum volume of 49.4 m^3.

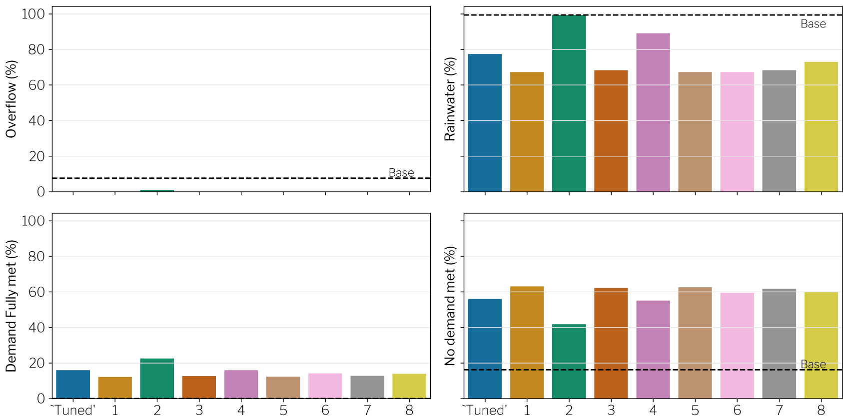

Figure 5: Statistical indicator values for each experiment.

All other experiments eliminated overflow completely from the simulated historical rainfall days. This includes configurations 6 to 8 which are derived under non-zero overflow thresholds, each corresponding to an optimisation instance that demonstrated overflow. This confirms that a higher overflow threshold value can be used with the optimisation models without impacting the robustness of the system whilst improving the harvesting potential of the output system configuration. The threshold overflow level at which this is true is a function of the time step size difference between the simulation model and the optimisation models.

Finally, the performance of the system derived using the 'tuned' input parameter set was compared to that of an existing system. The exact method for deriving the existing system is not known and is assumed to be a more traditional sizing strategy, without the use of optimisation-based methods. The statistical indicator values are shown in Figure 6, demonstrating that the 'tuned' system configuration can completely eliminate overflow for historical rainfall days, with more days in which it is capable of completely meeting irrigation demands with the harvested rainwater.

However, the existing configuration can harvest water from a wider range of rainfall types, having collected any amount of water from 99.3% of the simulated scenarios in comparison to the 77.4% under the 'tuned' input configuration. Additionally, the 'tuned' input configuration had demonstrated significantly more days in which it required freshwater from the mains supply to completely satisfy the daily irrigation demand, with 56% of the simulated scenarios showing no demand being met by harvested rainwater in comparison to 16% from the existing configuration. The poorer performance in satisfying demand in more

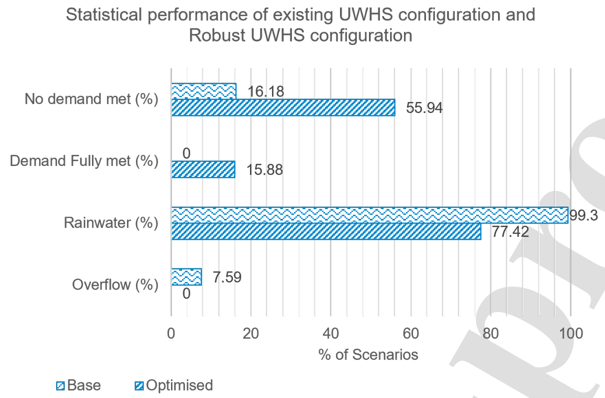

Figure 6: Statistical performance indicator values for two-player optimised system configuration and an existing configuration.

scenario types for the 'tuned' input parameter set was found to be a result of the on/off operating levels of the pump that connects the 'Harvesting' and 'Treatment' tanks. These levels were not included in the optimisation models as these were viewed as part of the operational policy of the system, rather than the design of the system configuration. With the operational levels calibrated for a smaller 'Harvesting' tank with a smaller tank area, the implementation of these levels in a tank with a larger area implies that a much higher rainfall volume, which is not attainable beyond a given total rainfall volume, would be required to activate the pumps.

4. Conclusions

This paper presented a two-player algorithm for the robust sizing and design of a multi-tank RWH and flood-mitigation system, incorporating a stochastic design optimisation model and a high intensity rainfall design model. This was developed to reduce reliance on the availability of high-resolution rainfall data in computationally-intensive optimisation-based system design and sizing strategies and allows efficient systems to be designed even for catchments with minimal rainfall data. The optimisation tools presented in this study were structured to improve the system design process, in turn reducing the barriers to implementation of more efficient and sustainable large-scale RWH systems. This sought to build on existing literature and work by improving the accessibility and availability of more advanced computational methods to system designers and experts that may not be familiar with such strategies, allowing the benefits of implementing these methods to be reaped without a significant learning curve. This is crucial towards building a more sustainable and resilient urban living environment that is efficient at both utilising existing water resources and minimising possible flood risks.

The framework was applied to a large-scale RWH system that would service a densely populated high-rise residential estate which uses the harvested water yields for irrigating its green spaces. Using this case study, the behaviours and interactions of the algorithm were characterised for a range of possible model inputs which limits search times and convergence thresholds. From the characterisation experiments, it was found that:

- If failures are encountered in the stochastic optimisation model, the defaulting algorithm will always seek to maximise the allowable design space.
- Failures were found to be mostly a function of the time limit provided to the stochastic optimisation models, hence the stochastic optimisation model needs to be given sufficient time to search for a solution. This was found to be approximately 900 s for each individual tank optimisation module and would help in producing higher quality solutions, which in turn can reduce the number of iterations required for finding a converged optimal solution.
- The overflow convergence threshold can be larger than zero and still provide a robust performance under the simulation model due to the large time-step size used in the optimisation models. This is a function of the time step size difference between the simulation and optimisation models.

The performance of the output configurations were evaluated, which showed that a stochastically optimal configuration can significantly reduce overflow risks, with no overflow events in the simulated historical rainfall scenarios. In comparison, the existing system configuration, derived using more traditional sizing strategies demonstrated overflow events in 7.6% of the simulated scenarios. Whilst the rainwater harvesting potential of the configuration derived under the proposed can be improved to provide water over a wider range of possible rainfall types, it was found that this had been limited by the operational levels of the pump that serves between the ‘Harvesting’ and ‘Treatment’ tanks and is a parameter that can be easily transformed into a decision variable in the optimisation modules.

Therefore, further improvements to the algorithm in achieving the technical objectives of controlling water volumes could be implemented and are summarised through the following:

- The inclusion of more system parameters and hence the degrees of freedom, such as the operational levels of pumps so that the solutions are not constrained by assumptions and designs made with previous systems.
- A reward function can be implemented to provide more information to the optimisation model about the multi-objectives.

Further work looks at integrating the algorithm and models demonstrated in this paper with the derivation of optimal control policies, such that a holistic tool can be developed for the simultaneous optimisation of the design and control of RWH systems. The implementation of optimal control policies would require active operation and can be realised for example with the use of sensors and actuators. With the integrated design and control optimisation providing a high degree of freedom, improvements to RWH system performances can be maximised by providing the system with efficient infrastructure design with added flexibility to contain and provide water availability over a large range of possible rainfalls.

List of Symbols

Sets

j	Tank index
o	Orifice index
S	Scenario index
t	Time step index
τ	Time set index subset without initial timestep

Parameters

a_m	Tank area option m (m^2)
C_d	Coefficient of discharge
C_W	Cost of excess overflow (\$)
g	Gravitational constant
h_i	Tank height option i (m)
M	Scalar value of big-M method
R	Pump Rate (m^3/s)
r_{on}	Pump operation on-level (m)
r_{off}	Pump operation off-level (m)
ω_k	Orifice height option k (m)
α	Inflow weighting factor
β	Outflow weighting factor
ΔT	Time step size (s)
ζ	Threshold overflow volume (m^3)

Continuous Variables

θ_o	Area of orifice o (m^2)
A	Tank Area (m^2)
C	Tank Capacity (m^3)
D_o^t	Discharge from orifice o during time step t (m^3)
F^t	Freshwater used in time step t (m^3)
H	Tank height (m)
I^t	Inflow to tank during time step t (m^3)
L^t	Level of water in tank during time step t (m)
O^t	Outflow to tank during time step t (m^3)
P_C	Penalty cost for wasted capacities
P_W	Penalty cost for excess overflow volumes
V^t	Volume of water in tank during time step t (m^3)
W^t	Overflow volume from tank during time step t (m^3)
δ^t	Total discharge from tank in time step t (m^3)

η_o Height of orifice o (m)
χ^t Excess overflow in time step t (m^3)

Binary Variables

ϕ Binary variable for tank height selection
ψ Binary variable for orifice height selection
λ Binary variable for tank area selection

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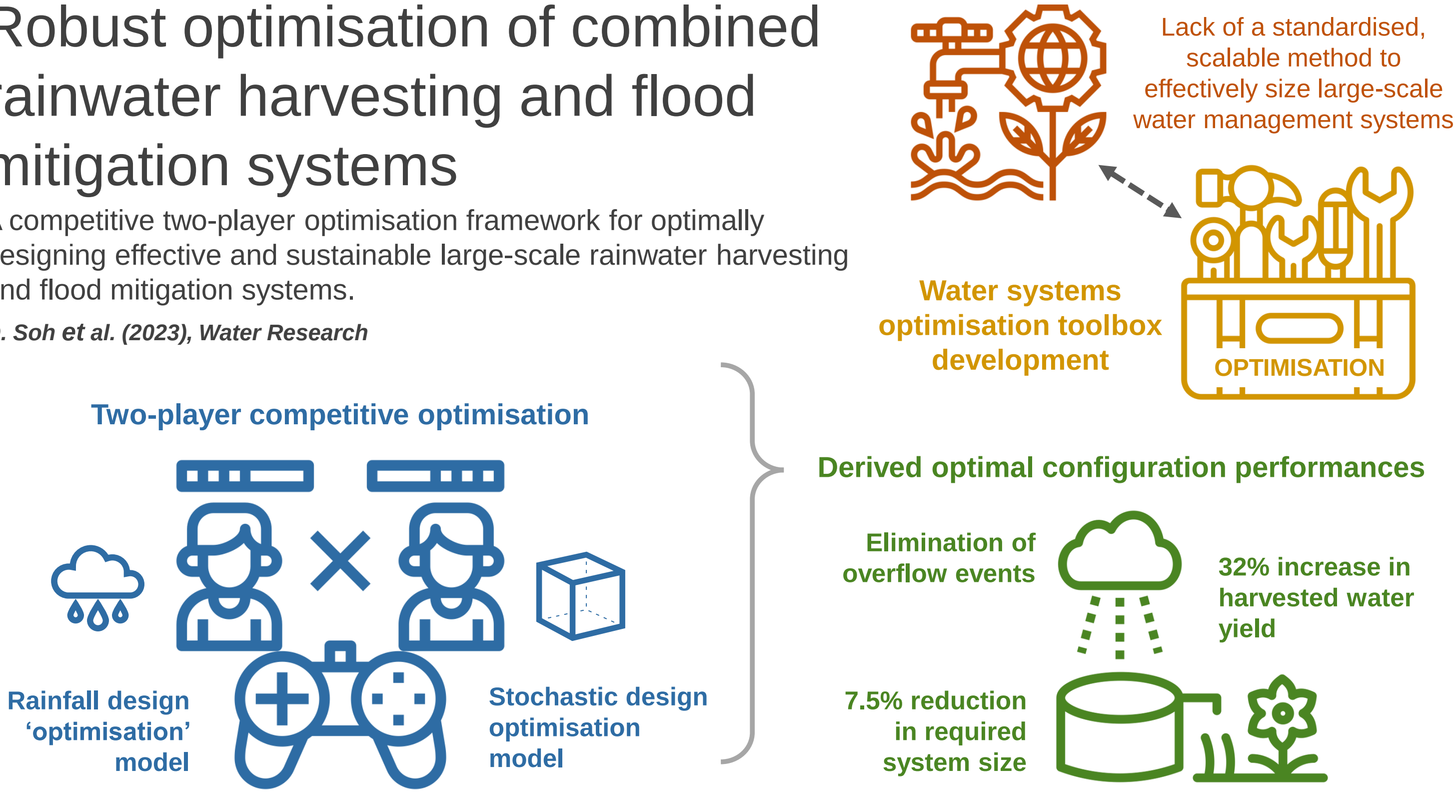
Graphical Abstract

Robust optimisation of combined rainwater harvesting and flood mitigation systems

A competitive two-player optimisation framework for optimally designing effective and sustainable large-scale rainwater harvesting and flood mitigation systems.

Q. Soh et al. (2023), Water Research



Images taken from Noun Project: Toolbox by HNTRY; PVP by Template; Rainwater harvesting by SBTS; Water Management by EUCALYP;

Declaration of interests

☐ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☒ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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