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An Investigation of Control Allocation Methods for the ADMIRE Simulation Model

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This paper presents a comparative study of various control allocation methods, using ADMIRE as a benchmark simulation model. The Ganged Pseudo-Inverse, Weighted Pseudo-Inverse, Cascaded Generalized-Inverse, Daisy Chain, and Linear Programming approaches are evaluated and compared against each other using open loop and closed loop analysis with Euclidean-Norm. In open-loop analysis, control allocation methods are analyzed for each approach that can produce an admissible solution and be able to attain commanded moments. Then, in closed-loop analysis, control allocation methods are compared using ADMIRE nonlinear simulation model for predefined maneuvers which are defined by multiple points in the flight envelope.

Nomenclature

ADMIRE = The Aero-Data Model in Research Environment
FFA = Aeronautical Research Institute of Sweden
FOI = Swedish Defense Research Agency
GAM = The Generic Aero-Data Model
FOSIM = Forskningsimulator (Research Simulator) at FFA
GPI = Ganged Pseudo-Inverse
WPI = Weighted Pseudo-Inverse
CGI = Cascaded Generalized-Inverse
DC = Daisy Chain
LP = Linear Programming

I. Introduction

In recent years, control allocation approaches have attracted much attention because modern fighter aircraft have large numbers of control effectors. The control allocation problem can be defined as the determination of the control effector position to obtain desired or commanded moments by aircraft control law [1]. Traditional fighter aircraft are designed to use only one control effector for each rotational degree of freedom, with three independent control surfaces used to control three desired or commanded moments. Specifically, the elevator generates a pitching moment, the ailerons produce a rolling moment differentially, and the rudder controls the yawing moment of the aircraft. However, modern fighters have additional control effectors, including canards, thrust vectoring, differential flap, etc., which are combined to generate rotational moments in three axes. Combinations of these control effectors are used to obtain rotational moments in three axes. Since there are many solutions to obtain desired moment, the control allocation

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problem is complex. An optimal solution can be obtained by considering a secondary objective, such as minimizing control surface deflection to decrease drag on the aircraft.

In this study, ganged pseudo inverse, weighted pseudo inverse, cascaded generalized inverse, Daisy Chain, and Linear Programming [2] approaches are compared with each other. In a comparison study, ADMIRE (The Aero-Data Model in Research Environment) [3] nonlinear aircraft model is used as a benchmark for comparison. ADMIRE is the main aircraft model which includes the flight control system, actuators, sensor models, and uncertainty parameters with respective limits. The performance of control allocation approaches is measured using open-loop and closed-loop analysis [4]. In open-loop analysis, the performance of approaches is measured by the ability to attain commanded moments. In this open-loop comparison, a predefined time history of moment demands is passed through candidate control allocation algorithms. Performance is then measured by comparing the results using Euclidean-Norm criteria [1]. On the other side, in the closed-loop comparison, the aircraft system and control law dynamics are included.

The rest of the paper is organized as follows: brief information is given about ADMIRE simulation model in section II, the methodology of the different control allocation approaches and used restoring method is defined and explained in section III, open-loop and closed-loop analysis is explained in section IV, and results are presented in section V. Finally, in section VI, the conclusion is given.

II.Admire Simulation Model

To analyze the control allocation approaches more effectively, ADMIRE benchmark model should be recognized. The ADMIRE model was developed by the Aeronautical Research Institute of Sweden (FFA) in 1997, to construct a complete aircraft model for use in the research simulator FOSIM, based on the generic aero data model (GAM) developed by Saab AB. The ADMIRE model is a generic model of a small single-seat fighter aircraft with a delta-canard configuration, consisting of various control effectors such as left and right canards, leading edge flaps, left and right inboard elevons, left and right outboard elevons, and rudder, as illustrated in Figure 1.

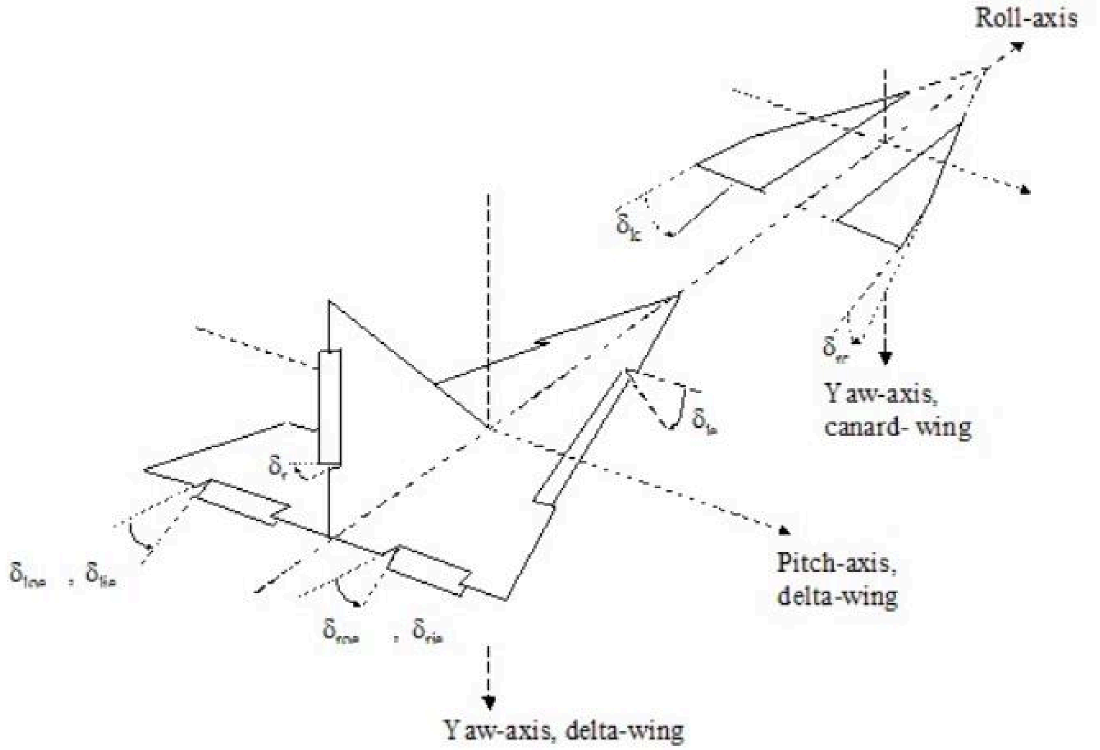

Figure 1 Definition of Control Surfaces Deflections.

The ADMIRE model is augmented with a flight control system (FCS) to provide stability and sufficient handling qualities within the operational envelope. FCS has a longitudinal and lateral-directional controller. The pitch rate controller is used in a longitudinal controller for below Mach number 0.58. The load factor controller is used for a

Mach number greater than or equal to 0.62. The corner speed is close to Mach number 0.60. In the Mach number region between 0.58 and 0.62, a blending is performed. There is an α-limiter functionality active during pitch rate mode. The lateral controller enables the pilot to perform initial roll control around the velocity vector of the aircraft and the angle of sideslip control. FCS is designed for nominal aircraft configuration which is given in Table 1.

Table 1 Aircraft Nominal Configuration.

Property	Symbol	Value	Unit
Mass	m	9100	kg
Wing area	b	45.00	m^2
Wingspan	S	10.00	m
Mean aerodynamic chord	$\bar{c}$	5.20	m
Aircraft inertia	I_x	21000	kg-m^2
Aircraft inertia	I_y	81000	kg-m^2
Aircraft inertia	I_z	101000	kg-m^2
Aircraft inertia	I_{xz}	2500	kg-m^2

The ADMIRE flight envelope is defined within less than 1.2 Mach and below 6 km altitudes. Within the flight envelope, there are other limitations depending on the aero database. Aero database limitations originate from the angle of attack, angle of sideslip, and control surface deflections. Because of structured limitations and pilot health concerns, the normal load factor is constrained to $-3g \leq n_z \leq +9g$.

The maximum allowed deflections and recommended angular rates for the control surfaces of the ADMIRE model are presented in Table 2. It should be noted that the allowable deflections of the control surfaces are a function of the Mach number and vary accordingly. In Table 2, the maximum allowable deflection limit is given for the subsonic Mach region. The GAM is described in [5]. In Ref [6], more detailed information can be found about the atmospheric model, actuator model, sensor model, and engine model. Please find a related reference for detailed information.

Table 2 Control Surface Deflection Limits.

Control Surface	Min [°]	Max [°]	Angular Rate [$°/s$]
Canard	-55	25	±50
Rudder	-30	30	±50
Elevons	-25	25	±50
Leading Edge Flap (LEF)	-10	30	±50

III.Restoring Method and Control Allocation Approaches

In this section, restoring method and control allocation approaches are mentioned briefly.

A. Restoring Method

In modern flight control systems, control effector deflections are determined multiple times per second (around 100 times in modern fighter aircraft) in a frame-wise manner. This means that control allocation depends on the sequence of moments generated between the start and finish of maneuvers, resulting in path dependency. Restoring methods can be used to return control effector positions to desired configurations when they may be undesirable for some reason, despite providing the desired moment. While the original approach was to minimize deflection, it has been found that restoring toward effectors configurations that minimize aerodynamic drag can be more effective. This approach may also be used to minimize cross-section and hydraulic power requirements, and more generally, preferred solutions can be restored based on specific performance criteria [2].

Figures 2 and 3 illustrate the impact of the restoring method. The Ganged Pseudo-Inverse method was employed in the analysis, which was performed for doublet lateral stick input. Figure 3 displays the obtained moments, alongside the desired moments, for the Ganged Pseudo-Inverse algorithm with and without the restoring method. The results indicate that the desired moment can be achieved using the Ganged Pseudo-Inverse algorithm, with or without the restoring method. Therefore, we conclude that the restoring method has no significant effect on the obtained moments.

The restoring method aims to minimize the deflection of control surfaces while ensuring that the obtained and desired moments remain consistent. Figure 2 provides a visual representation of control surface deflection with and without the restoring method. With the restoring method applied, each control surface deflection is minimized while maintaining the moments at their desired levels.

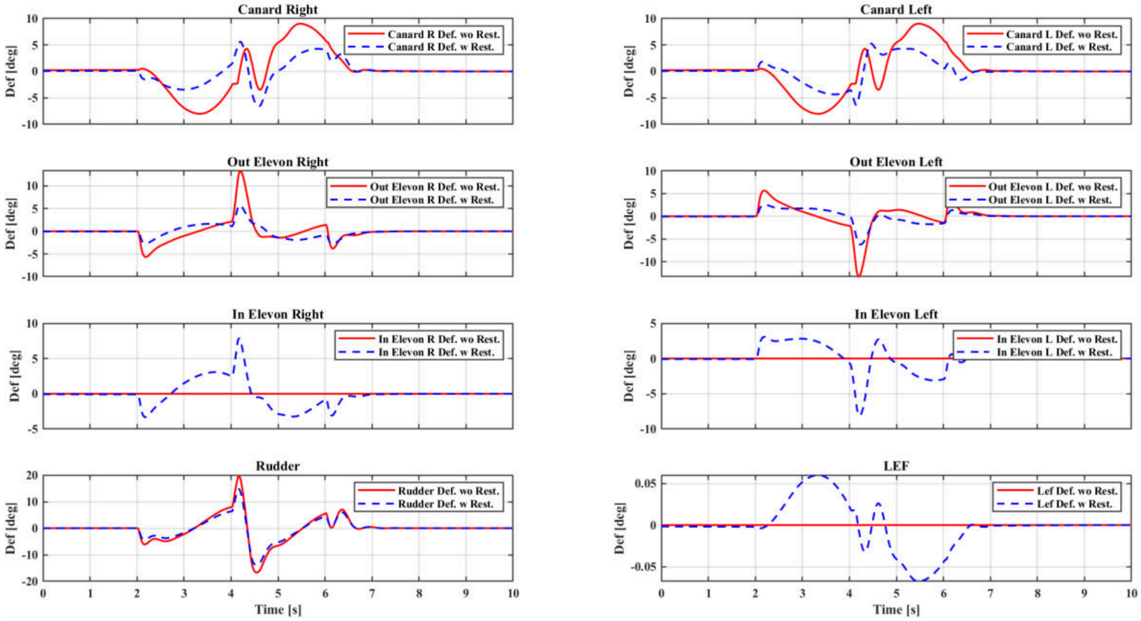

Figure 2 Control Surface Deflections with and without the Restoring Method for Obtaining Desired Moments using the Ganged Pseudo-Inverse Approach.

Figure 2 displays the results obtained using the Ganged Pseudo-Inverse method, which exclusively utilizes primary control effectors to provide the desired moments. Each aircraft's main axis is controlled using a single control effector; the canard is used for the pitch axis, the outboard elevon for the roll axis, and the rudder for the yaw axis. As shown in Figure 2, the inboard elevon and leading edge flap are not employed for allocation purposes. Therefore, the aircraft controller tries to provide a desired moment in each axis with a single control effector, resulting in more control effectors being deflected. However, when the Ganged Pseudo-Inverse approach is implemented with the restoring method, the deflection of each control effector decreases. This phenomenon is particularly evident in the rudder deflection. The restoring method aims to minimize the deflection of control surfaces which provides the same moments. As a result, the deflection of the canard and outboard elevon is also decreased. The restoring method deflects unused control effectors to provide necessary moments for each aircraft’s axis.

As shown in Figure 2, the methodologies used in this study do not reflect the Ganged Pseudo-Inverse approach. Because all control surface is deflected to find the minimum deflection. The study aims to compare control allocation methodologies, and therefore, the restoring method is not used in the rest of the study.

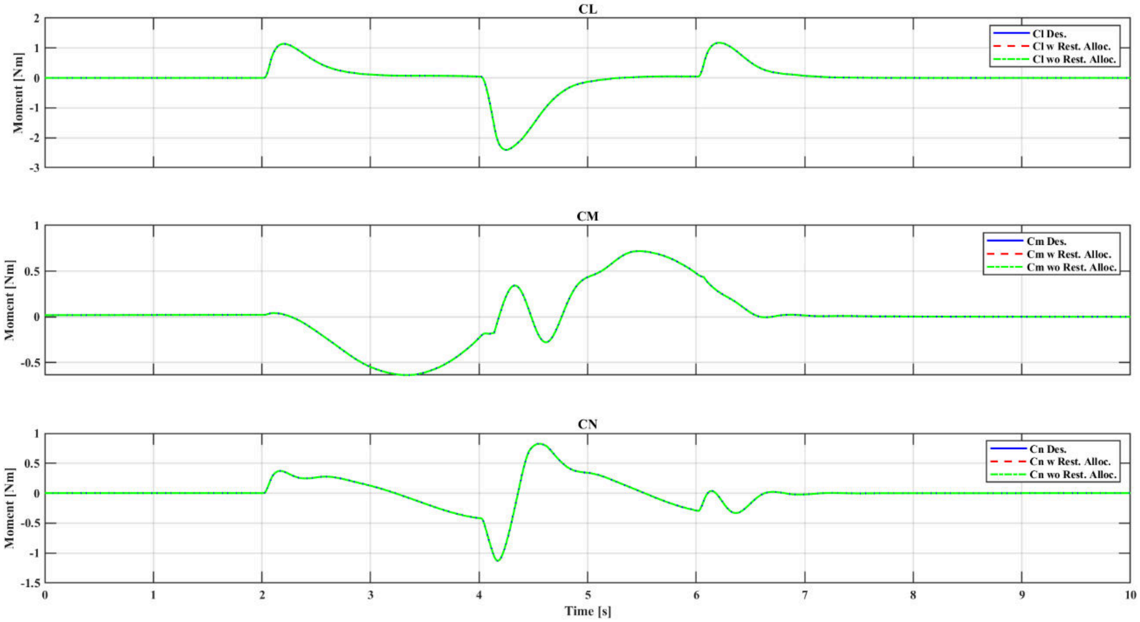

Figure 3 Obtained Moments by Ganged Pseudo-Inverse Approach with and without Restoring.

In this study, the minimum-norm restoring method is implemented. Formulation of the minimum-norm restoring method is out of the scope of this study. Therefore, a curious reader can find more information about the minimum-norm restoring method in [2].

B. Control Allocation Approaches

Control allocation is the process of deciding how much control effort should be allocated among each actuator in a control system with many actuators, such as an aircraft with several control surfaces. As control allocation approaches for this study, the Ganged Pseudo-Inverse, the Weighted Pseudo-Inverse, the Cascaded Generalized-Inverse, the Daisy Chain, and the Linear Programming [2] are selected. Detailed information on these control allocation algorithms is provided in the following subsections.

1. *The Ganged Pseudo-Inverse (GPI)*

The ganged pseudo inverse approach uses only the primary control surfaces to allocate the given desired moments. This method deflects the primary control surfaces such that each control surface generates moment only in one axis. Canards are deflected symmetrically to generate pitch moment; outboard ailerons are deflected symmetrically to generate roll moment and rudder deflected to generate yaw moment. Secondary control surfaces are not considered in this method.

2. *Weighted Pseudo-Inverse (WPI)*

The weighted pseudo-inverse approach is a method that considers a quadratic cost function to minimize the control effort needed to achieve the desired moments. This method involves finding a unique combination of control inputs that satisfies the desired moments while minimizing a weighted quadratic cost function. The optimization problem is solved using a pseudo-inverse matrix of the non-square matrix B that relates the control inputs to the moments. It is a weighted 2-norm solution to the control allocation problem, and the challenge is to reduce $u^T W^T W u$, where W is a positive diagonal matrix. The diagonal terms typically are the form of [2]

$$W_{ii} = \frac{1}{\left(u_{i_{max}} u_{i_{min}}\right)} \tag{1}$$

Alternatively,

$$W_{ii} = \frac{1}{\left(u_{i_{max}} u_{i_{min}}\right)^2} \tag{2}$$

3. *Cascaded Generalized-Inverse (CGI)*

Among the various control allocation techniques, the cascaded generalized inverse (CGI) stands out as a superior method that is also relatively quick and simple to use. This method was used to create the control rules for the X-35 aircraft, [2]The weighted pseudo-inverse allocation technique is a key component of the Cascaded Generalized-Inverse approach, which eliminates all saturated controls at each step and subtracts their effect from the desired moment. This process is repeated until either no new controls saturate (meaning the moment is attainable), all remaining controls saturate (meaning the moment is unattainable), or there are fewer remaining controls than desired moments. [4]

4. Daisy Chain (DC)

At the NASA Dryden Flight Research Facility's High-Angle-of-Attack Projects and Technology Conference in April 1992, daisy chaining was first discussed. The primary concept was to start with traditional controls and use them until one or more of them are ordered past their limits before adding other controls to the solution. The controls are split up into two or more divisions to carry out the procedure. The procedure can be seen in Figure 4 [2].

$$u = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{3}$$

Control effectors are included in the first set, known as u_1, and they must be used continuously. Typically, they are the rudders and ailerons, which are more traditional control effectors. If the first grouping is unable to meet the demand at hand, the second grouping, u_2, should only be used. The two control categories are represented by B_1 and B_2, respectively, in the control effectiveness matrix, B. [7]

$$B = \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix}^T \tag{4}$$

Each control group constitutes its control allocation problem

$$\begin{aligned} m_1 &= B_1 u_1 \\ m_2 &= B_2 u_2 \end{aligned} \tag{5}$$

And, the control solution should satisfy the following equations

$$\begin{aligned} u_1 &= P_1 m_1 \\ u_2 &= P_2 m_2 \end{aligned} \tag{6}$$

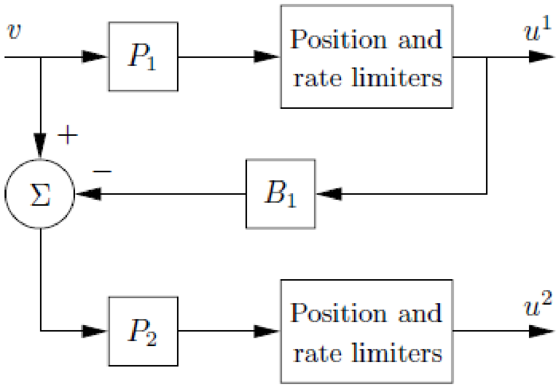

Figure 4 Daisy Chain Control Allocation Approach.

5. Linear Programming (LP)

Linear programming is used as a technique to obtain the best solution for linear systems of equations with restrictions on some or all of the variables. This can be done by allocating limited resources to minimize or maximize the outcome of a process. There are various approaches to linear programming. Some are referred to as traditional or normative. A standard linear programming (LP) problem consists in finding a control deflection vector, x such that [8]

$$\begin{gathered}\text{minimize } J = c^T x \\ \text{subject to } 0 \leq x \leq h \text{ and } Ax = b\end{gathered} \tag{7}$$

$$h = x_{max} - x_{min} \tag{8}$$

where,

- x is control surface deflection
- J is cost fuction
- b is desired moment
- A is control effectiveness matrix
- h is difference of the maximum and minimum deflection

IV.Methodology

A. Representative Maneuvers

Predetermined inputs are given to control inceptors of aircraft. Therefore various control allocation methodologies can be compared to each other. Aircraft responses are examined after the same predefined stick inputs are given to the nonlinear simulation model for each control allocation methodology.

1. Maneuver with Lateral Doublet Stick Input

A lateral stick doublet input is used as one of the predetermined stick inputs for the aircraft control system. The stick doublet input is applied at 0.6 Mach and 3000-meter flight conditions. The doublet input is initiated at a time of two seconds, and each portion of the doublet is held for two seconds. The magnitude of the stick force input is set to half of the maximum stick force, which is 80 newtons. This input is selected as it exercises all control effectors in the aircraft's flight envelope. The lateral stick doublet input is presented in Figure 5.

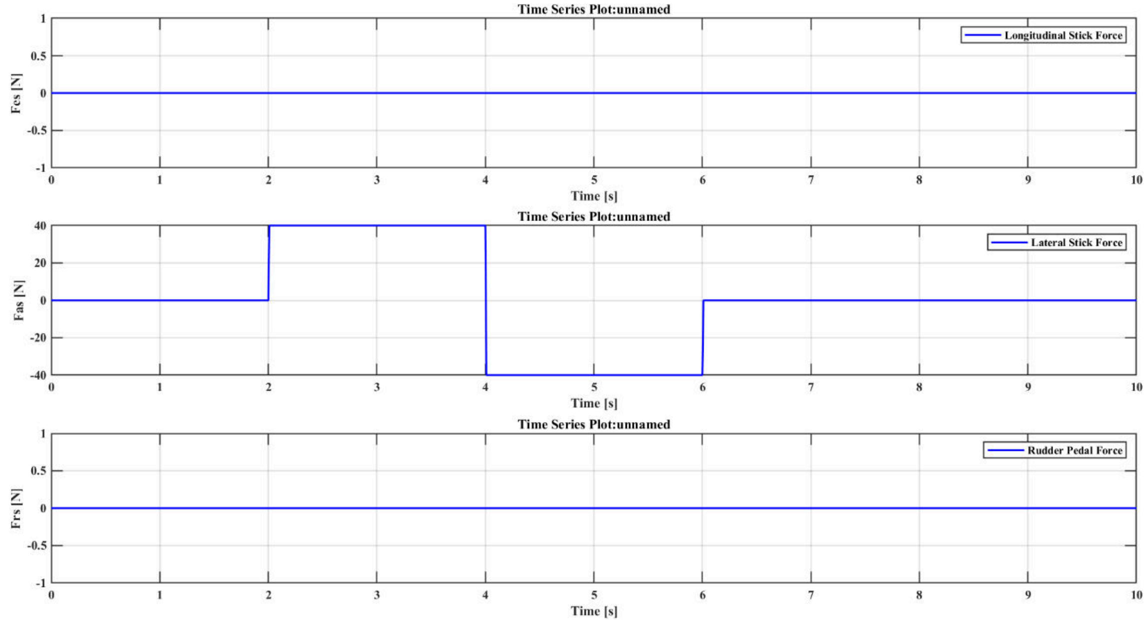

Figure 5: Nonlinear Aircraft Simulation Lateral Stick Doublet Input.

2. *The half-Cuban Eight Maneuver*

The Cuban eight is a well-known aerobatic move that involves making loops in both the lateral and longitudinal directions, then rotating 180 degrees to create the shape of an "eight" in the 2D plane. For this maneuver, the aircraft starts and finishes at the same height. The aircraft performs a circle at the start of the move, but instead of finishing the loop when it is 45 degrees nose down, it rolls to wing level. The 45-degree nose-down descent to the initial height is continued but in the opposite direction. [9] [10]. The half-Cuban eight is decomposed into a half-loop. Figure 6 shows the side view of a half-Cuban eight maneuver. In this study, half Cuban eight maneuver is performed as a representative maneuver and, stick force inputs are given in Figure 7.

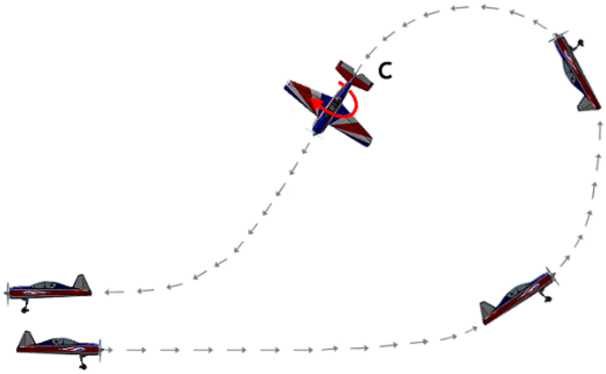

Figure 6: The half-Cuban Eight Maneuver.

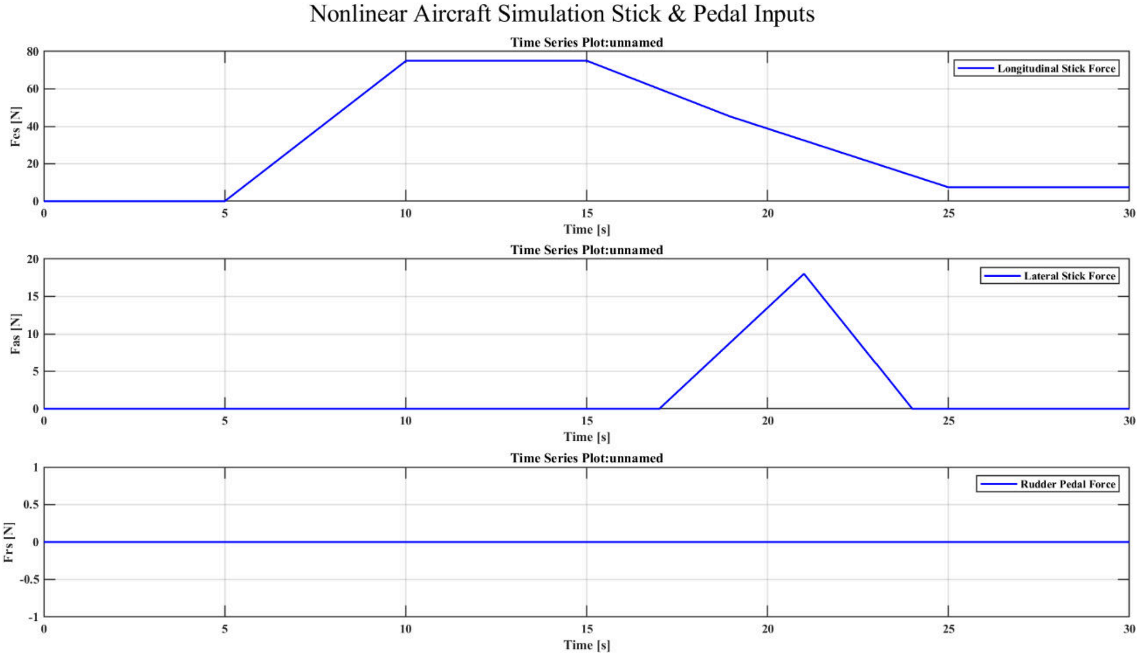

Figure 7: Cuban Eight Maneuver Inputs.

B. Performance Criteria

The paper discusses how control allocation algorithms are used to determine which effector or combination of effectors should be used to generate a given set of desired or commanded moments. However, comparing the performance of these algorithms is difficult, so the paper suggests specific steps for comparing them. The steps include defining performance metrics, implementing the algorithms, collecting, and analysing data, comparing results, and refining algorithms based on the results. The comparison may involve statistical analysis to determine the significance of differences between the algorithms. These steps can help evaluate which control allocation algorithm performs the best based on specific aircraft and mission requirements.

The performance metrics or indicators used to compare the performance of multiple control allocation algorithms for aircraft depend on the specific mission requirements and aircraft configuration. However, common metrics include control effort, control surface deflection, control surface rates, aircraft stability, aircraft performance, pilot workload, and safety. [11]

- Control Effort: Control effort refers to the magnitude of the control inputs required to achieve a desired aircraft response. Lower control effort indicates that the control allocation algorithm is efficient and uses the control surfaces effectively.
- Control Surface Deflection: Control surface deflection refers to the amount of movement of the control surfaces required to achieve a desired aircraft response. Lower control surface deflection indicates that the control allocation algorithm is efficient and minimizes the control surface movement.
- Control Surface Rates: Control surface rates refer to the speed at which the control surfaces move to achieve a desired aircraft response. Higher control surface rates indicate that the control allocation algorithm is responsive and can quickly adjust to changes in the aircraft's flight conditions.
- Aircraft Stability: Aircraft stability refers to the aircraft's ability to maintain its desired flight path. The control allocation algorithm should be able to maintain the aircraft's stability under various flight conditions and disturbances.
- Aircraft Performance: Aircraft performance refers to the aircraft's ability to meet the mission requirements, such as speed, altitude, and range. The control allocation algorithm should be able to optimize the aircraft's performance under various flight conditions.
- Pilot Workload: Pilot workload refers to the amount of mental and physical effort required by the pilot to fly the aircraft. The control allocation algorithm should be designed to minimize the pilot workload and allow for efficient control of the aircraft.
- Safety: Safety is a critical performance metric that measures the ability of the control allocation algorithm to prevent accidents and ensure the safety of the aircraft and its crew.

To quantify the performance of the control allocators, a set of indicators is introduced. It includes some of the traditional indicators such as convergence time and iteration count. [12]. To be considered flight worthy, the control allocator must converge in a minimum amount of time (less than the sample time) and iterations. Also, all the virtual controls should be attained within the capacity of the actuators. Finally, actuator usage should be minimized. As a result, the following indicators are used to evaluate the performance of control allocation techniques [13]:

- Mean and maximum number of iterations,
- Mean and average of 10% maximum convergence time,
- The integral of squared error
- Normalized consumption

The possible performance metrics are already mentioned above. What will be used in this study are as follows:

1. *Euclidean-Norm Comparison*

The purpose of this comparison is to evaluate how far from the neutral position a particular allocator keeps its control surfaces during manoeuvres, providing an analytical explanation of the allocator's performance in terms of control power [1]. The presented data provide insights into the performance of different control allocation methods, allowing for a more thorough evaluation of their effectiveness.

The Euclidean-Norm of the control effector is calculated by taking the norm of all control surface deflection during each timestep. In Eq. 1, the capital X represents a vector of all control effectors' deflection.

$$\|X\|_2 = [{x_1}^2 + {x_2}^2 + {x_3}^2 + {x_4}^2 + \cdots + {x_n}^2]^{\frac{1}{2}} \tag{9}$$

2. *Execution Time Comparison*

A control allocator needs to execute its actions quickly because it affects the control loop budget, which can impact controller performance. If the execution time is slow, it can decrease the controller's effectiveness. Hence, to choose the best control allocator, it's necessary to compare their execution times.

3. *Moment Error Comparison*

The primary goal of every control allocation method is to generate the required moment using control effectors. To evaluate this, the difference between the desired and obtained moments is calculated for each allocator. Then, these calculated errors are compared with each other to determine the effectiveness of each control allocation method.

4. *Aircraft State Comparison*

To ensure aircraft safety and maintain minimum performance loss with each control allocator, the aircraft states are analysed in a closed-loop analysis. This involves comparing the aircraft's performance under different control allocation methods to identify any potential unsafe situations or loss of performance. The goal is to select an allocator that can guarantee safe flight operations with minimal performance degradation.

C. Open- and Closed-Loop Analysis

The main purpose of control allocation approaches is to provide an instantaneous desired moment. Control allocation approaches do not try to find the optimum solution over time. As a result, there may be significant differences in results, when the controller closes the loop. So, control allocation approaches should be examined in both open-loop and closed-loops. In this section, control allocation approaches are analyzed for both of them.

1. *Open-Loop Analysis*

The open-loop analysis method is utilized to evaluate the control allocation algorithm's ability to achieve the desired commanded moments. In this method, the control allocation algorithm is passed a predefined time history of commanded/desired moments. The algorithm decides the control effector's deflection by multiplying it with control effectiveness matrices to obtain the required moment. This provided moment is then compared with the commanded/desired moment to measure the control allocation algorithm's success. Figure 8 illustrates the complete procedures involved in this analysis.

To carry out the analysis, the time history of desired commanded/desired moments is defined using ADMIRER's nonlinear model. The desired moments are recorded for representative maneuvers that were previously defined. In this analysis, the candidate control allocation algorithms are treated as a unit test. The recorded desired moments are fed into the algorithm, and the controller surface deflections are observed. The resulting control surface deflections are then multiplied with the control effectiveness matrix to obtain the provided moments. Finally, the desired and provided moments are compared using performance criteria defined in the performance criteria section. This process allows for a thorough evaluation of the control allocation algorithms.

The main purpose of open loop analysis is to examine the ability to attain desired moments for each candidate control allocation algorithm in the same unit test environment. Therefore, the unit test environment is defined as the same for all of them. Predefined moments are recorded from a nonlinear model of Admirer for the same representative maneuvers at the same trim condition. Also, control effectiveness matrices are obtained for the same trim condition. Control surface initial condition is taken zero instead of trim surface deflection because of preventing saturation.

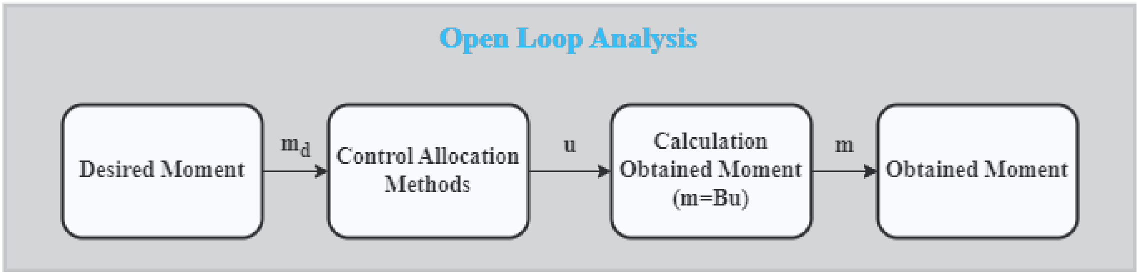

Figure 8: Open-Loop Analysis.

2. *Close-Loop Analysis*

In theory, all control allocation algorithms work similarly when given unlimited control power and perfect knowledge of the control effectiveness matrix with no secondary objective. However, this is not always the case due to aircraft control effectors having position and rate limits. When the desired moment cannot be achieved, each control allocation algorithm can handle the situation differently. In the closed-loop analysis, desired moment's time history is changed because of the closed-loop interaction of control law and control allocation algorithm.

In the control law design procedure, control law and control allocation should be considered together. Because of that, they affect each other. This is why controller performance is affected when the control actuator method is changed. In our study, our main focus is a comparison of the various control allocation methodologies. The study focuses on comparing various control allocation methodologies, and the controller design for each method is outside the scope of this study. Benchmark controller design of ADMIRE is used for the comparative study of closed-loop analysis.

The analysis setups are illustrated in Figure 9, where representative stick inputs are given to a nonlinear simulation model, the control law generates the desired effect, which is then converted to control surface deflection using various control allocation methodologies. Nonlinearities in actuator positions and rate limits also affect the performance of control law and allocation methods.

To evaluate the performance of control allocation methods, the desired effects can be considered as desired control inputs, such as roll rate, pitch rate, and angle of sideslip in the case of ADMIRE. The desired effects obtained by the control allocation methods need to be calculated to assess their performance. To obtain the desired effects, the control surface deflection (output of the control allocation method) is multiplied by the control effectiveness matrix, resulting in the desired effects obtained on the aircraft. By comparing the desired and obtained effects, the performance of the control allocation methods can be evaluated.

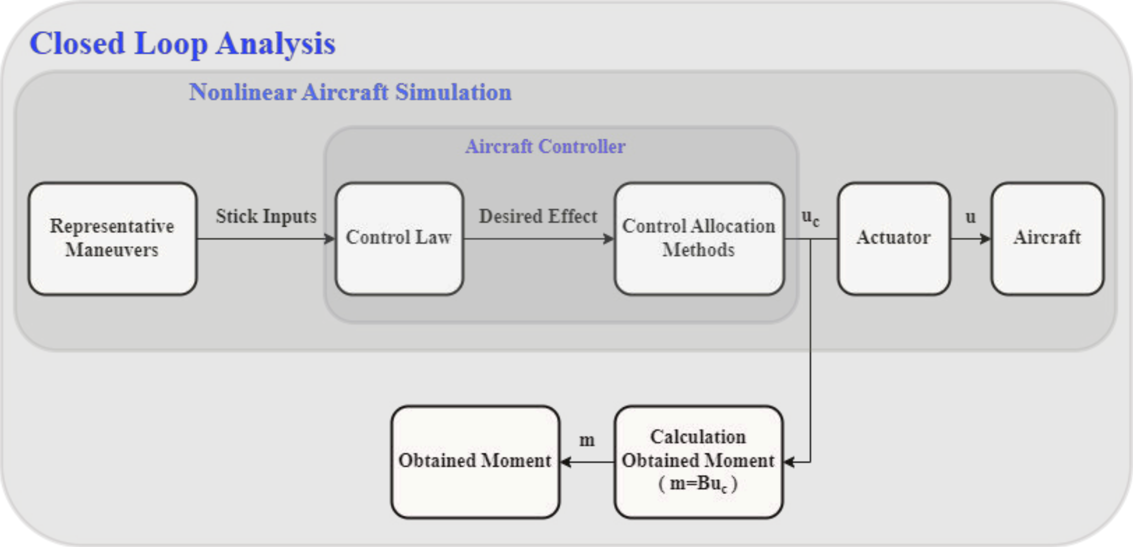

Figure 9: Closed-Loop Analysis.

V.Results and Discussion

There are two specific maneuver scenarios in this study which are namely 1) Lateral Doublet Stick Input and 2) half-Cuban Eight. In this section, control allocation methodologies are compared using the open- and closed-loop analysis for each scenario. Firstly, open-loop analysis results are given, and then closed-loop results are presented and discussed.

A. Scenario 1: Lateral Doublet Stick Input

1. Scenario1 - Open Loop Analysis Result

In an open-loop analysis, each controller allocation approach is evaluated using identical moment inputs. These moment inputs are measured for specific maneuver with lateral doublet stick input. For each allocation approach, the control effector's position and the moments obtained compared to the desired moments are graphed. The plots for each allocation approach are then combined into a single figure to allow for easy comparison between them.

The right/left canard has a positive deflection (control surface edge down) to generate a positive pitch moment and a negative deflection (control surface edge up) to generate a negative pitch moment. The Ganged Pseudo-Inverse method deflects the symmetrically to generate the pitch up/down moment. The left outboard elevon has a positive deflection (control surface edge down), and the right elevon has a negative deflection (control surface edge up) to generate a positive roll moment. To generate a negative roll moment, control surfaces are deflected opposite direction of the previous one. The Ganged Pseudo-Inverse method deflects the asymmetrically to generate the roll moment. The rudder has a positive deflection (control surface edge left) to generate a negative yaw moment and a negative deflection (control surface edge right) to generate a positive yaw moment.

The Ganged Pseudo-Inverse (GPI) method uses primary control effectors. For ADMIRE aircraft model, the canard, outboard elevon, and rudder are selected as the primary control surface. As seen in Figure 10, the Ganged Pseudo-Inverse method does not use any inboard-elevon or leading-edge flap surface to provide a moment. The Ganged Pseudo-Inverse uses directly the canard for the pitch axis, outboard-elevon for the roll axis and the rudder for the yaw axis.

There are some drawbacks to the Ganged Pseudo-Inverse method. Control surfaces can stick in position or rate limit in nonlinear simulation due to using one control surface for one axis moment. Because the Ganged Pseudo-Inverse method probably deflects each control surface so much. In this methodology, position and rate limits are not considered, when calculating the control surfaces. So, the found solution may not be admissible.

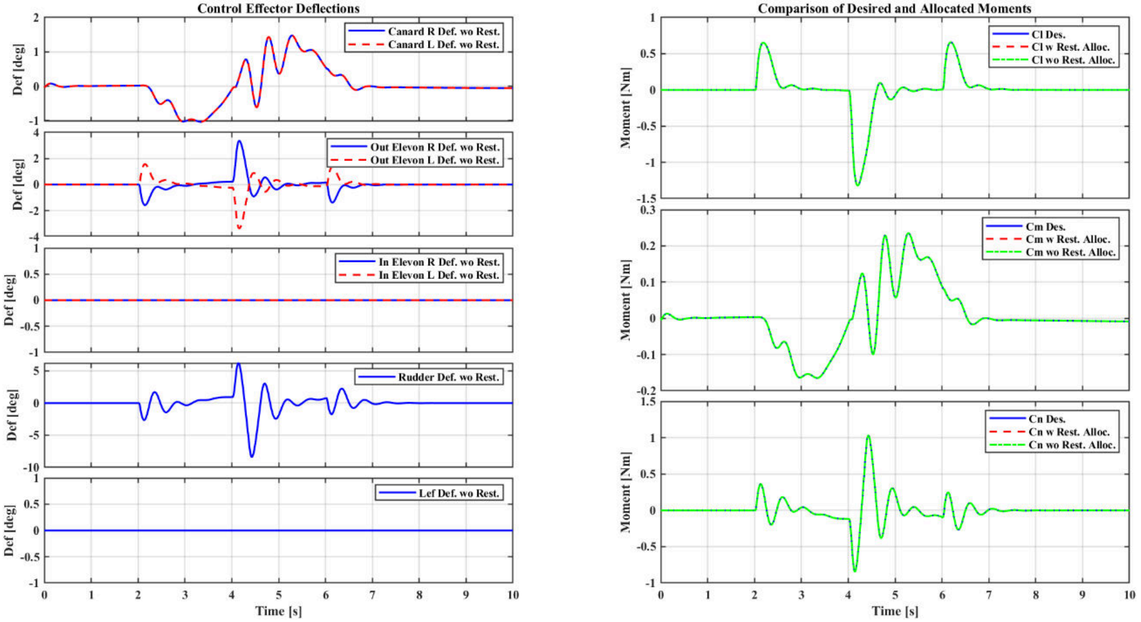

Figure 10: Scenario 1: Open-Loop Results for Ganged Pseudo-Inverse Approach.

The clipped and scaled Weighted Pseudo-Inverse methods are two different versions of the Weighted Pseudo-Inverse (WPI) approach. These methods are used when the commanded values exceed their limits. In the clipped variant, if the commanded values go beyond their limits, the values are clipped or truncated at the limit. This ensures that the commanded values stay within their limits. In contrast, in the scaled variant, the commanded values are uniformly scaled down so that they are all within their limits. This scaling is done in a way that preserves the direction of the commanded moments. This ensures that the system behaves consistently and does not produce undesired behavior due to exceeding the limits.

In the case of the clipped variant, if the commands are clipped, the direction of desired moment combination is changed. Therefore, clipped commands do not generate desired moments in at least one-moment direction anymore. On the other hand, scaled commands generate desired moment direction, but not the desired moment magnitude. In our case, commanded values do not exceed their limits, therefore both methods behave as Weighted Pseudo-Inverse and give the same control surface deflection.

Figures 11 and 12 demonstrate that during the maneuver, the canard is deflected unevenly. By deflecting the left and right canard asymmetrically, a rolling moment is generated by the canard, which reduces the necessary deflection of the elevons. The leading-edge flap provides the remainder of the required pitch moment in both figures. The inboard and outboard elevons provide rolling moments when they deflect unevenly. Furthermore, the elevons assist the canard at two distinct time intervals: 3-4 seconds and 5-6 seconds. During this time, their uneven deflection generates both rolling and pitching moments. Finally, the rudder is the only control surface that generates yaw moments.

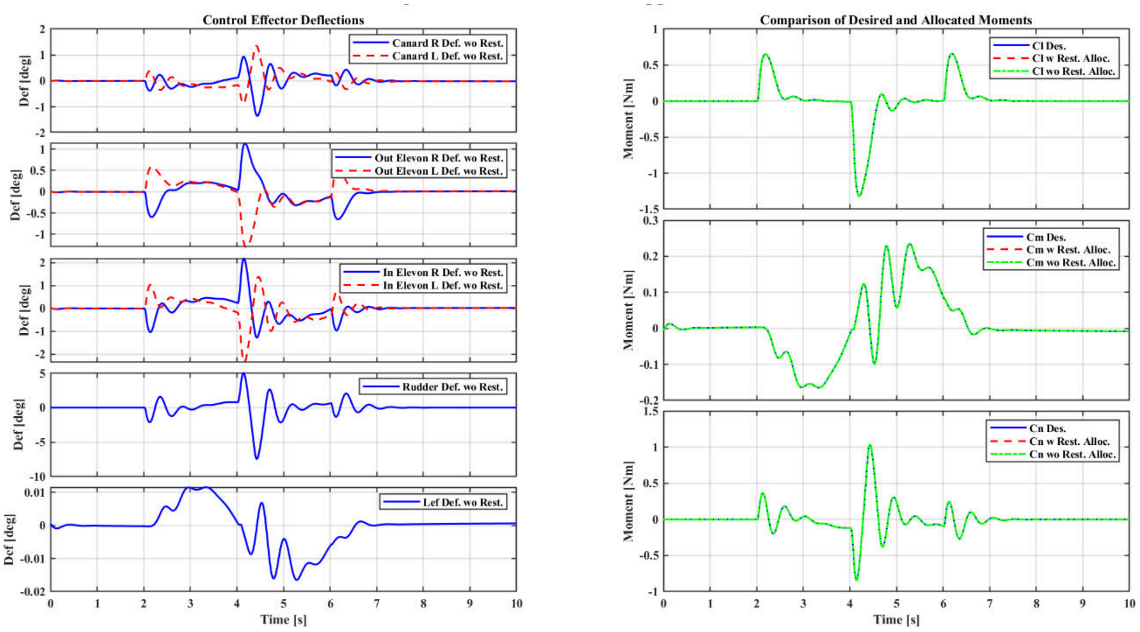

Figure 11: Scenario 1: Open-Loop Results for Weighted Pseudo-Inverse (Scaled) Approach.

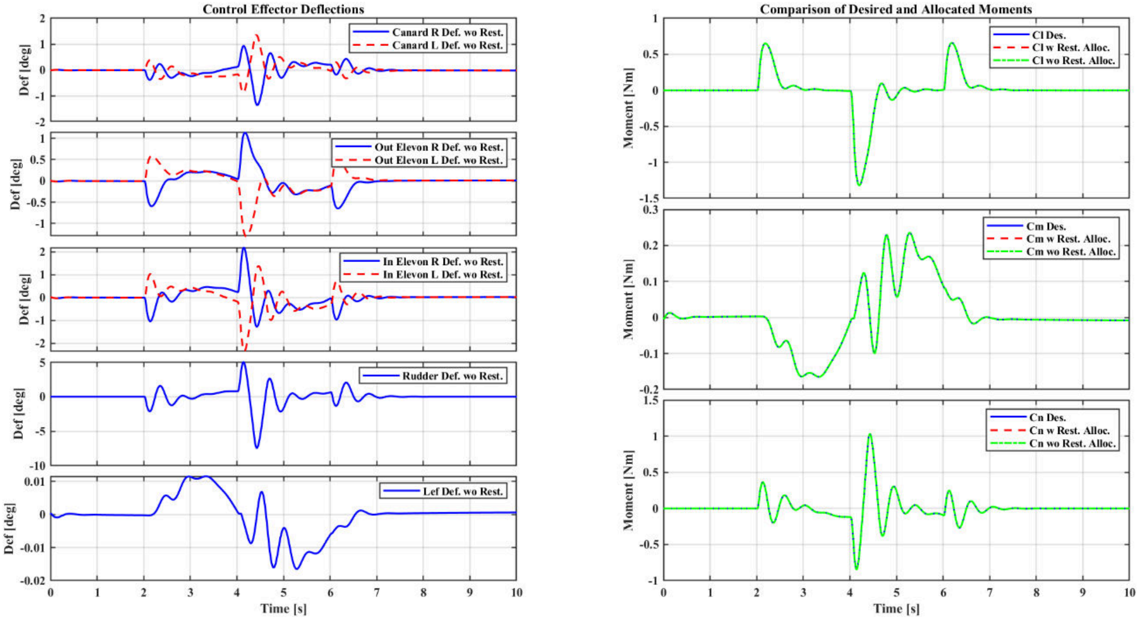

Figure 12: Scenario 1: Open-Loop Results for Weighted Pseudo-Inverse (Clipped) Approach.

The Cascaded Generalized-Inverse (CGI) method uses the Moore-Penrose pseudo-inverse to calculate moments until one or more control effectors are commanded beyond their saturation limit. At this point, the saturated control effectors are removed from the control effectiveness matrix, and the remaining moments are calculated. The unsaturated control surfaces then try to provide the remaining moments. As shown in Figure 13, the cascaded generalized inverse method deflects control surfaces similarly to the Weighted Pseudo-Inverse method, as both methods use the pseudo-inverse approach to calculate control deflection. In the saturated case, if any commanded value exceeds its limit, the Cascaded Generalized-Inverse method attempts to compensate for the remaining necessary moment using the unsaturated control surfaces until all control surfaces exceed their limits. However, the clipped and scaled Weighted Pseudo-Inverse methods may provide a moment with the wrong direction or with less magnitude but the correct direction.

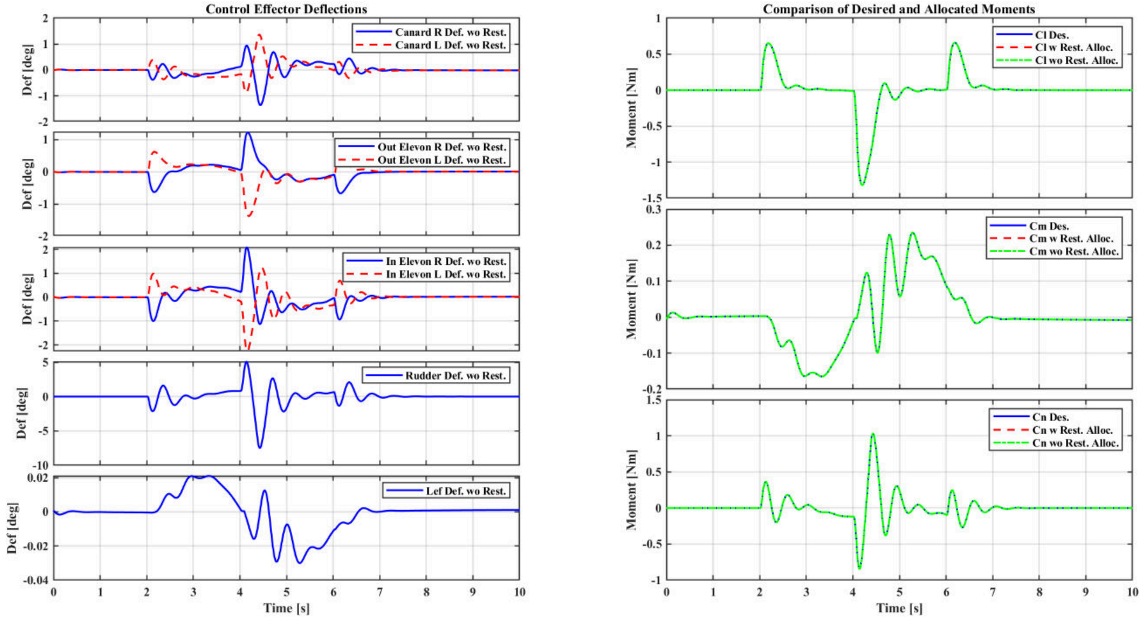

Figure 13: Scenario 1: Open-Loop Results for Cascading General-Inverse Approach.

The Daisy Chain (DC) method is a sequential control allocation method that prioritizes the use of primary control surfaces until they reach their limits, after which secondary control surfaces are brought into the solution. In the case where primary control surfaces do not exceed their limits, only they are used. The Daisy Chain method behaves similarly to the Ganged Pseudo-Inverse method in the cases where it is used. One important point to note is that while Figure 14 only uses symmetric canard and asymmetric elevon deflection, the Daisy Chain method uses symmetric canard deflection with a small shift, allowing the canards to generate not only pitch moment but also roll moment. In addition, elevons are used to generate both pitch and roll moments. This can be seen in the elevon deflection at 3-4 seconds and 5-6 seconds in the results. The rudder is only used to generate yaw moments.

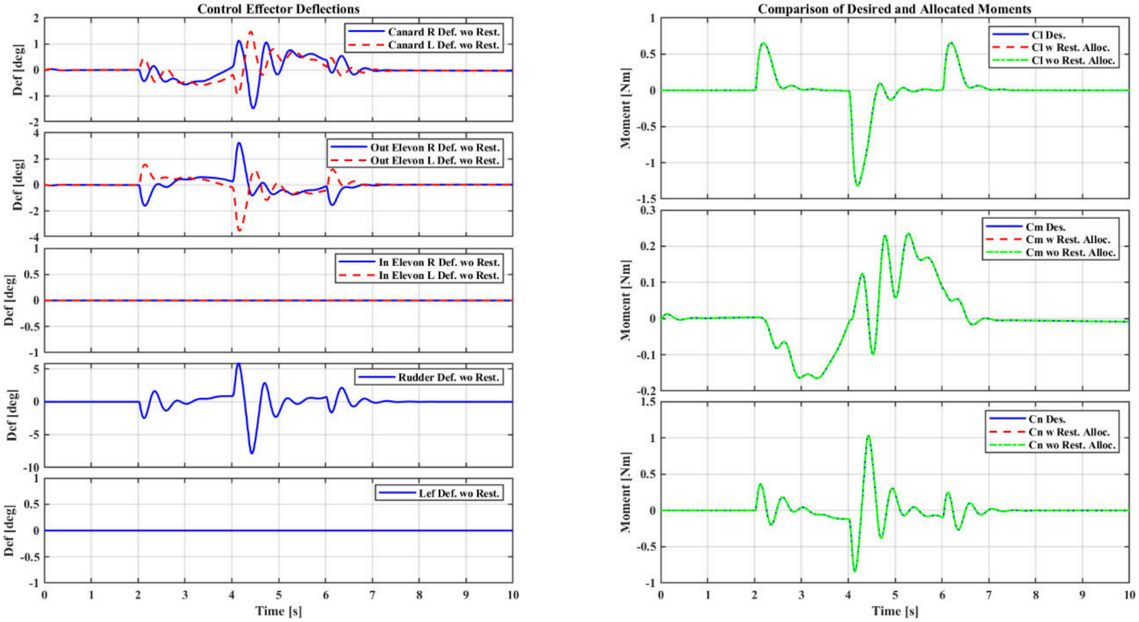

Figure 14: Scenario 1: Open-Loop Results for Daisy Chain Approach.

Figure 15 displays the results of the Linear Programming (LP) control allocation method, which is characterized by its direct utilization of the inboard-elevon for generating both pitch and roll moment. Unlike other methods, it does not employ the canard for generating any moment. The outboard elevon is also utilized for supporting the roll moment, while the rudder generates the yaw moment. Notably, Linear Programming seeks to provide the necessary moments with minimum control deflection, as opposed to separating the necessary moment from all control deflection.

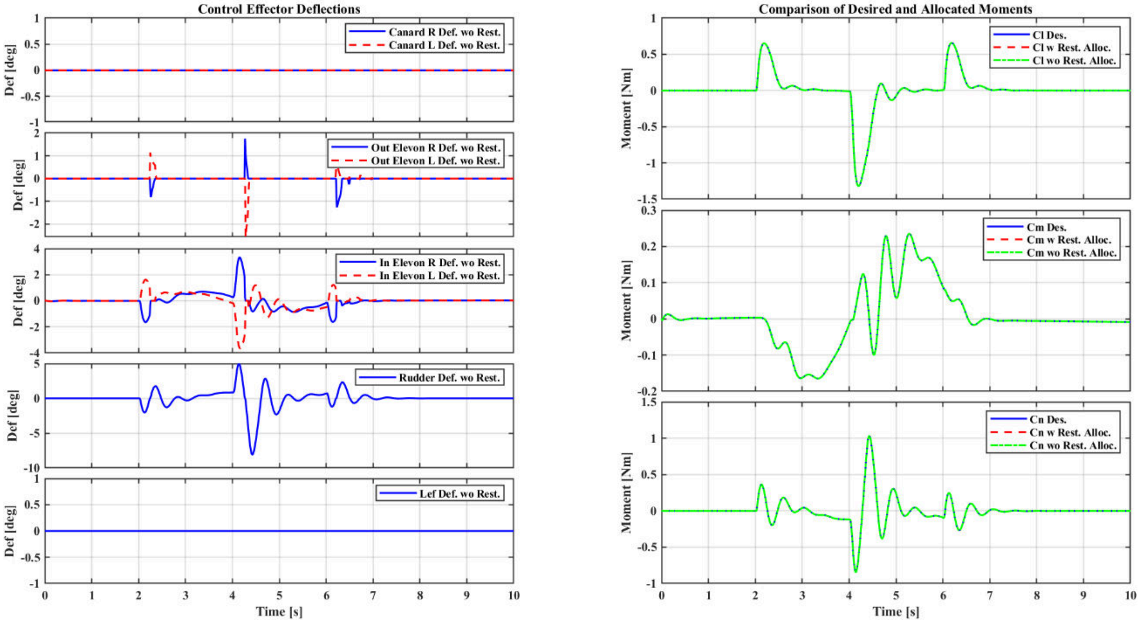

Figure 15: Scenario 1: Open-Loop Results for Linear Programming Approach.

In Figure 16, a comparison of the desired and obtained moments for each control allocation method is presented. It can be observed that all the considered methods perform similarly in terms of obtaining the desired moment. The plot shows that the obtained moments closely follow the desired moments for all the methods. However, as discussed earlier, the methods differ in terms of their computational complexity, execution time, and control deflection.

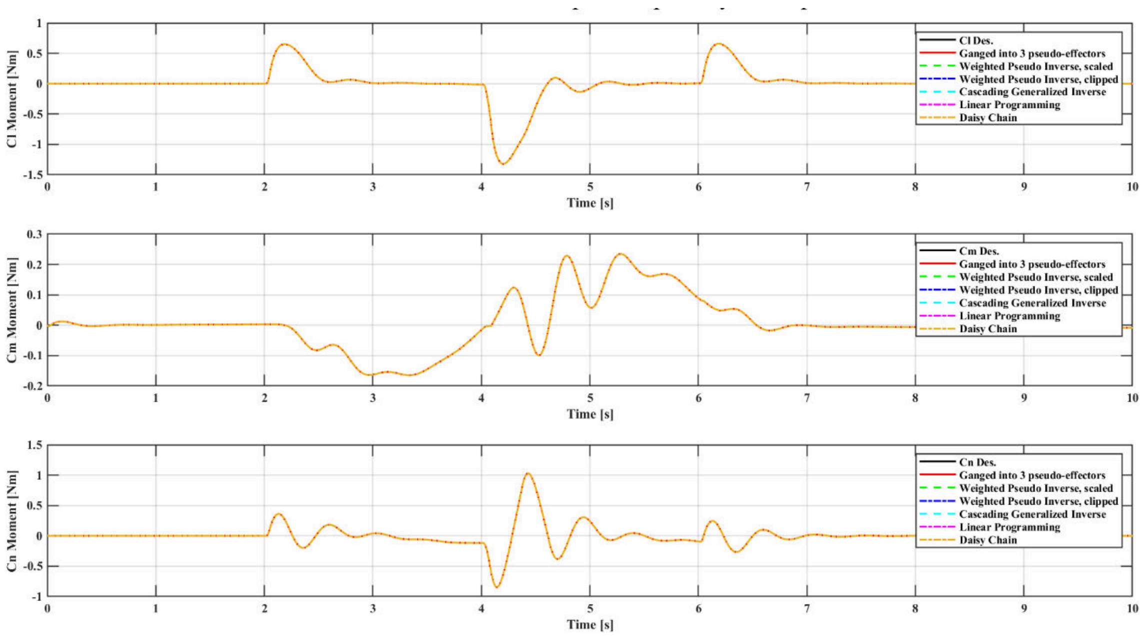

Figure 16: Scenario 1: Comparison of Open-Loop Desired/Obtained Moments for Each Allocation Approach.

In Figure 17, the control surface deflections for each control allocation method are shown. The Ganged Pseudo-Inverse method shows the highest maximum deflection for canard and outboard-elevon deflection. For inboard-elevon deflection, the maximum deflection belongs to the Linear Programming method. This is because the Linear Programming method aims to provide all pitch and roll moments using only inboard-elevon deflection. Only the scaled/clipped Weighted Pseudo-Inverse and cascaded generalized inverse methods use the leading edge flap to support the canard to generate the pitch moment. The Daisy Chain method has the second-highest deflection after the Ganged Pseudo-Inverse method. For rudder deflection, all methods show almost the same deflection because all of them use only the rudder to generate the yaw moment.

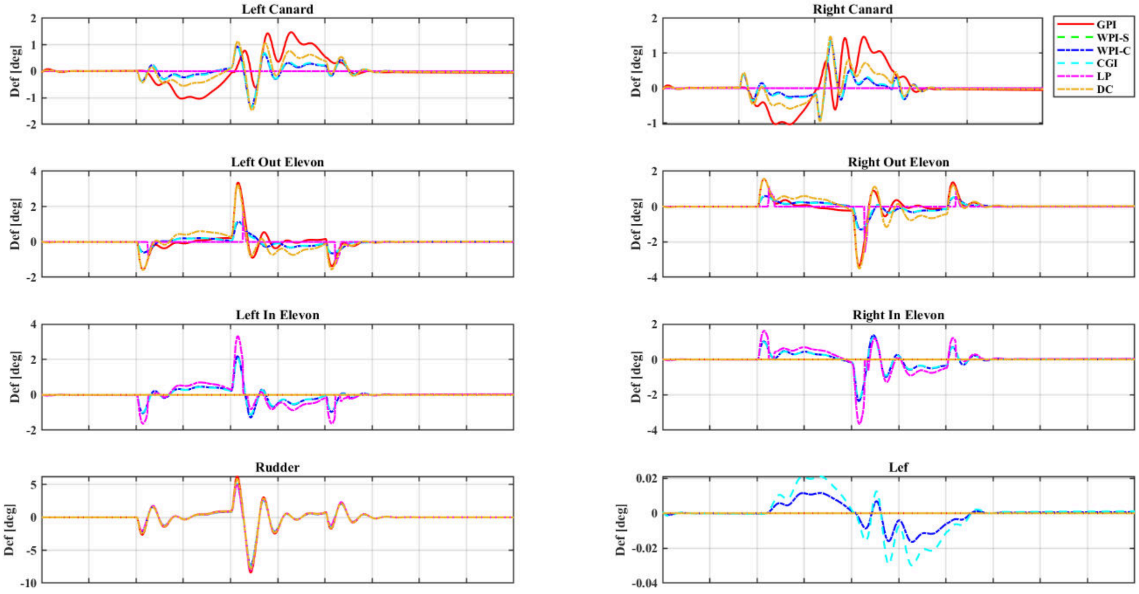

Figure 17: Scenario 1: Comparison of Open-Loop Control Surface Deflections for Each Allocation Approach.

Figure 18 presents the total execution time for ten seconds of simulation time for each control allocation method. The scaled-Weighted Pseudo-Inverse approach performs the best in terms of execution time, while Linear Programming exhibits the worst performance, taking almost three times longer than the other methods. The clipped Weighted Pseudo-Inverse and Ganged Pseudo-Inverse approaches have similar execution times to the scaled Weighted Pseudo-Inverse method. On the other hand, the cascaded generalized pseudo-inverse and Daisy Chain methods have average execution times compared to the other approaches.

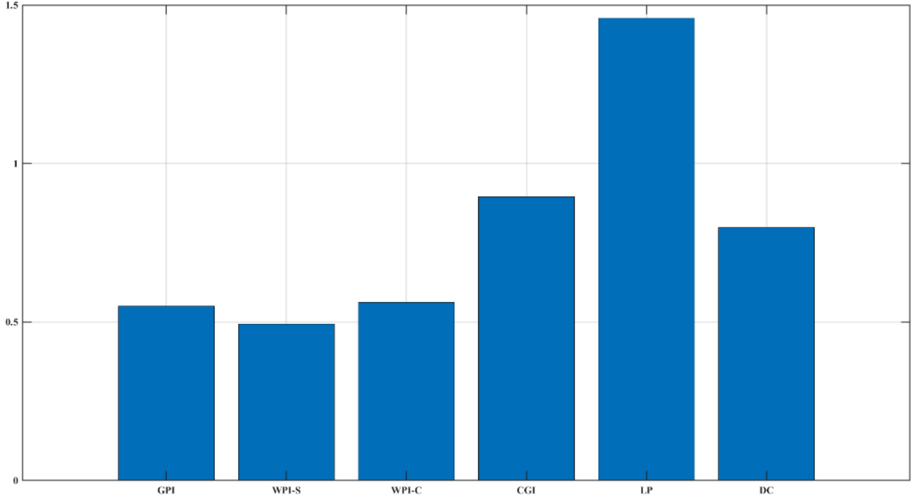

Figure 18: Scenario 1: Comparison of Open-Loop Execution Time for Each Allocation Approach.

2. *Scenario1 - Closed-Loop Analysis Result*

In the closed-loop analysis, each control allocation approach is analyzed for the same lateral doublet stick input. The input is provided to a nonlinear simulation model. A controller generates desired effects, and the control allocation methods generate control surface deflections for a given control signal. This section analyzes the simultaneous effect of controller law and control allocation method.

It is important to note that the ADMIRE benchmark controller sends pitch rate, roll rate, and angle of sideslip as an actuator command to the control allocation methods as a desired effect. However, these pitch rates, roll rates, and angle of sideslip are not the aircraft's final states. Rather, they represent pitch, roll, and yaw moment as in open-loop analysis. Also, because of simplicity, control surface deflection is given in delta effect to trim deflection for the closed loop analysis plots in y axis.

The Ganged Pseudo-Inverse approach is effective in providing the desired moments in open-loop analysis. However, when used in closed-loop analysis with the ADMIRE benchmark controller, the performance of the controller deteriorates due to the saturation of the canard rate limit. This is because the Ganged Pseudo-Inverse approach uses primary control surfaces for each axis, which causes rapid deflections that exceed their rate limits. As a result, the controller fails to properly control the pitch axis and the canards oscillate.

It is essential to consider both the controller and control allocation methodologies together since Figure 19 shows that changing the control allocation method can significantly affect the controller's performance due to nonlinearities. Each control allocation method can use different control surfaces to provide the same desired effect, and each actuator has its rate and position limits. Some allocation methods consider these limits, while others do not. Consequently, some methods can handle these nonlinearities, while others cannot. These effects are not seen in open-loop analysis, but in closed-loop analysis, they appear because the controller closes the loop.

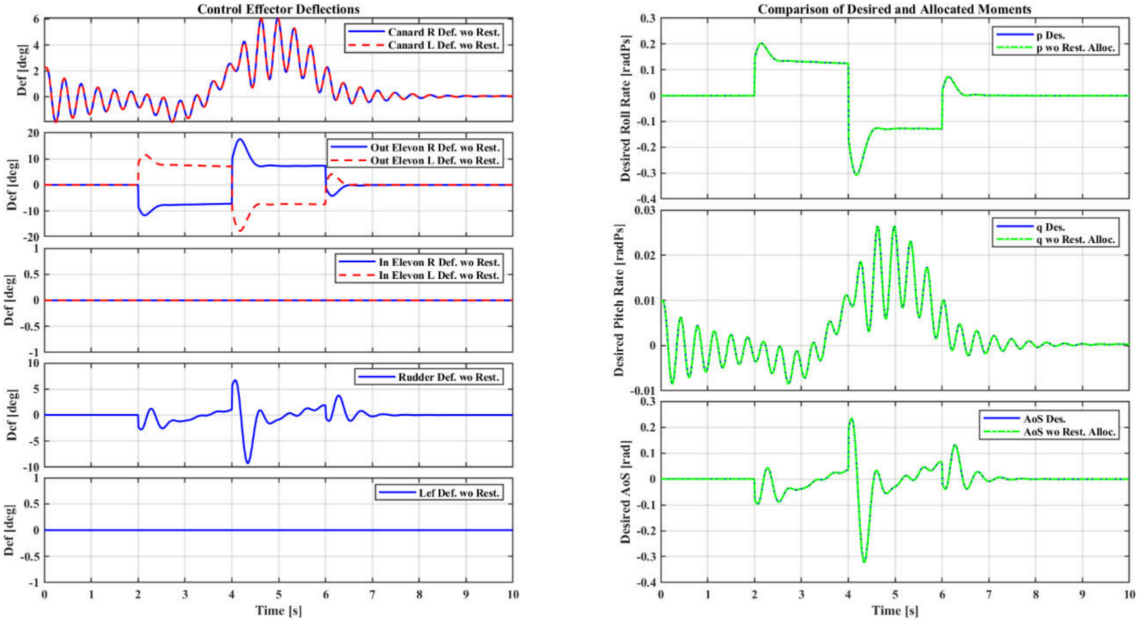

Figure 19: Scenario 1: Closed-Loop Results for Ganged Pseudo-Inverse Approach.

As depicted in Figures 20 and 21, the clipped/scaled Weighted Pseudo-Inverse approaches yield identical control deflections. In open-loop analysis, canards exhibit asymmetric deflection to aid elevons. In contrast, in closed-loop analysis, canards deflect symmetrically, thereby generating only a pitch moment. Consequently, in open-loop analysis, the leading edge flap assists in producing the pitch moment. However, in closed-loop analysis, the leading edge flap remains stationary since the symmetric canard deflection generates all pitch moments. The outboard- and inboard elevons experience asymmetric deflection to generate a roll moment without any lateral shift. Regarding rudder deflection, it experiences nearly the same degree and direction of deflection. However, positive rudder deflection leads to a positive angle of sideslip but a negative yaw moment in open-loop analysis.

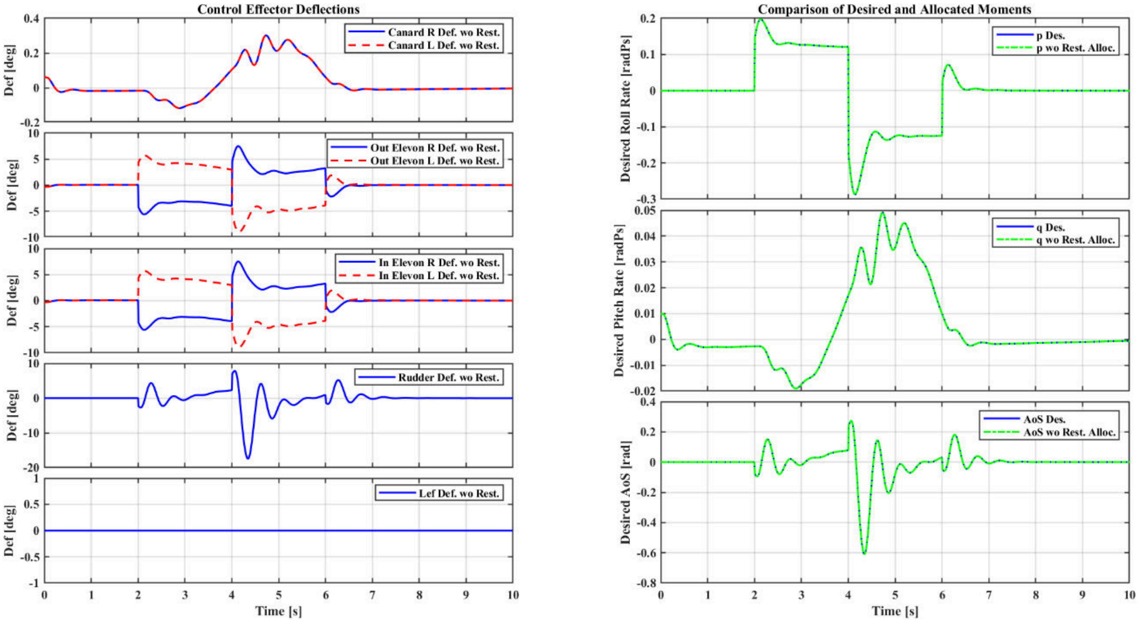

Figure 20: Scenario 1: Closed-Loop Results for Weighted Pseudo-Inverse (Scaled) Approach.

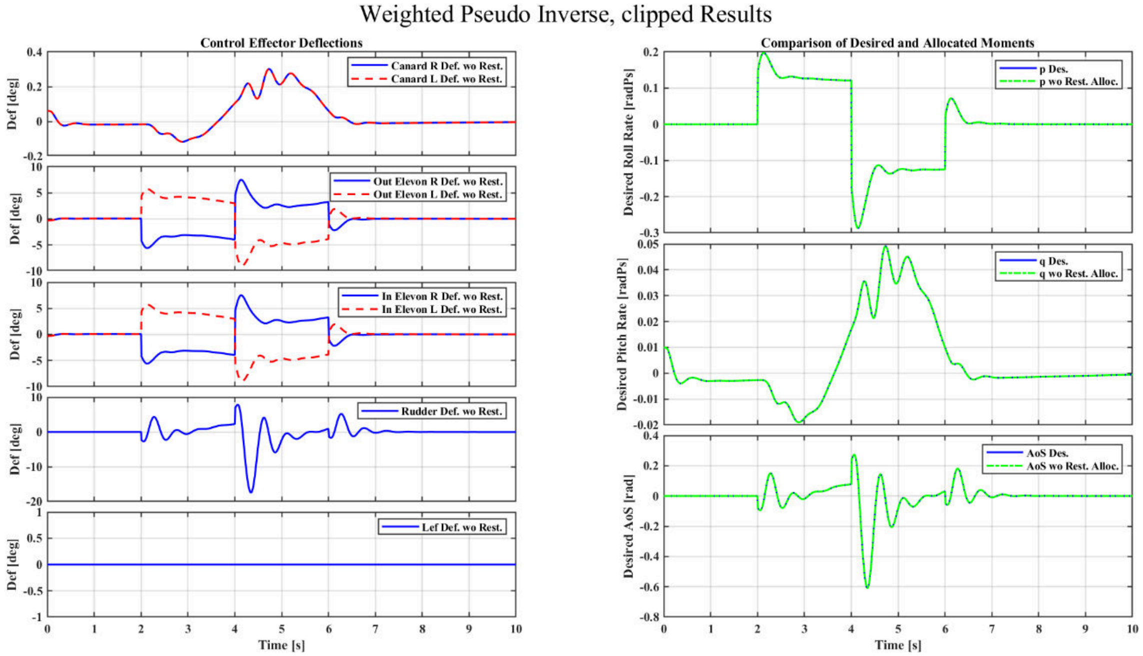

Figure 21: Scenario 1: Closed-Loop Results for Weighted Pseudo-Inverse (Clipped) Approach.

The findings illustrated in Figure 22 demonstrate that the Cascaded Generalized-Inverse approach utilizes the canard to generate the pitch moment, while the two elevons are employed solely for generating the roll moment, and the rudder for generating the yaw moment. In this approach, the deflection angles of the controller surfaces are similar to those obtained with the Weighted Pseudo-Inverse approaches.

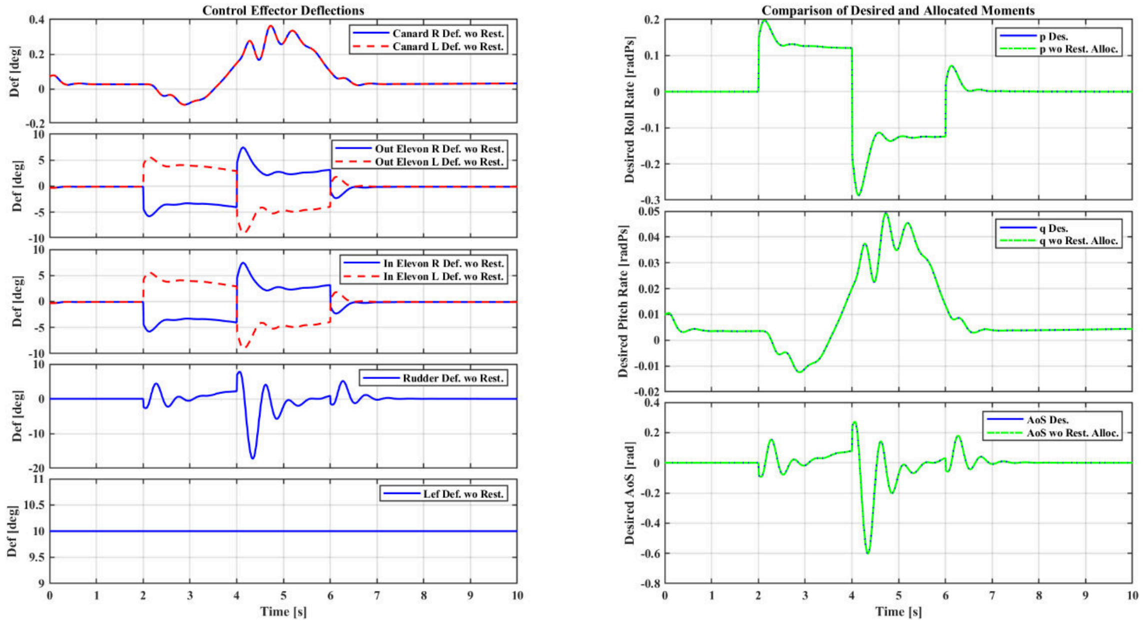

Figure 22: Scenario 1: Closed-Loop Results for Cascaded Pseudo-Inverse Approach.

The Daisy Chain method, shown in Figure 23, deflects primary control surfaces in closed-loop analysis. Unlike the Ganged Pseudo-Inverse method, which uses primary control deflection without any shifting, the Daisy Chain method deflects outboard elevon symmetrically to support the canard at the beginning of the maneuver. Additionally, the Daisy Chain method deflects the outboard elevon with different deflection degrees, as can be observed between 4-5 seconds into the maneuver.

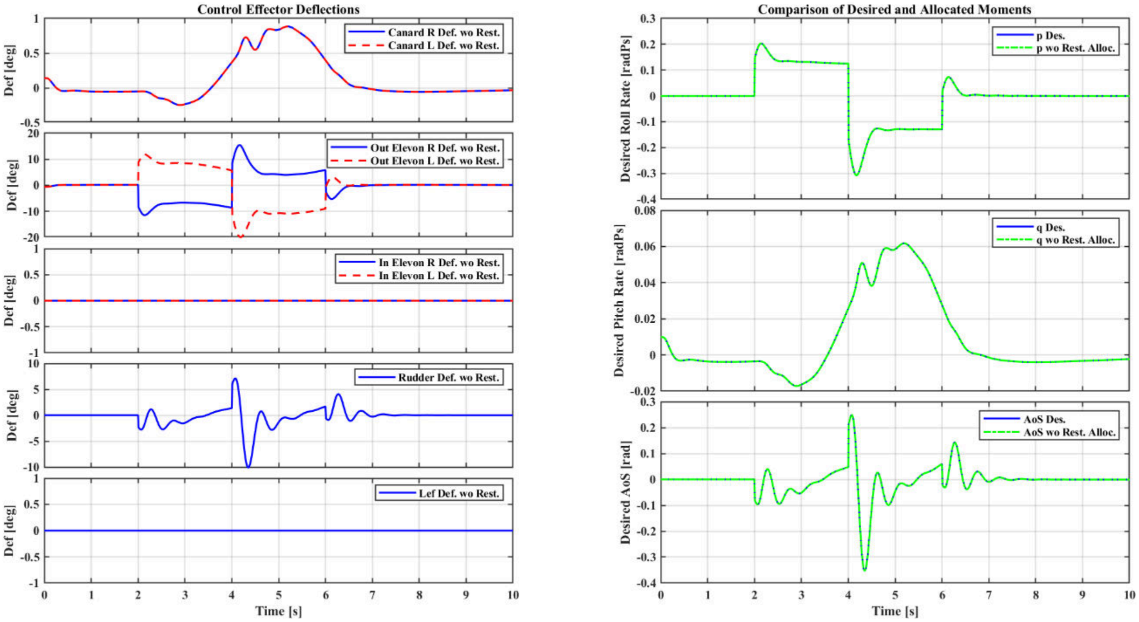

Figure 23: Scenario 1: Closed-Loop Results for Daisy Chain Approach.

Figure 24 demonstrates that in the Linear Programming approach, the rudder is deflected to achieve the desired angle of the sideslip effect. The desired pitch and roll rate are achieved solely by deflecting the right outboard elevon and left inboard elevon, respectively.

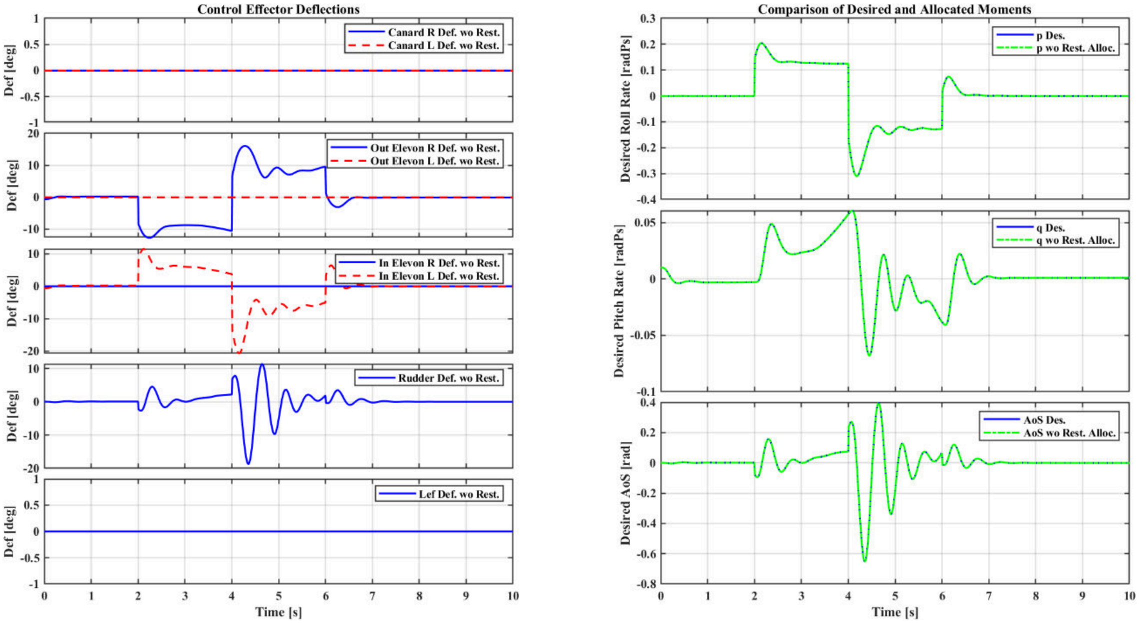

Figure 24: Scenario 1: Closed-Loop Results for Linear Programming Approach.

Figure 25 compares the effectiveness of various control allocation methods. The figure shows that the roll rate commanded by the controller is almost identical for all methods. However, the performance of the controller in other axes is not the same for all methods. In the pitch axis, the Ganged Pseudo-Inverse method results in oscillatory behavior, as seen in Figure 25, which can lead to undesirable oscillations in aircraft states. While the Ganged Pseudo-

Inverse method provides the desired effect on both the pitch rate and angle of the sideslip, it causes oscillation in aircraft states. The Weighted Pseudo-Inverse and Cascaded Generalized-Inverse approaches have almost identical controller commands for the pitch rate. The Daisy Chain method has a higher controller command for the pitch rate than the Weighted Pseudo-Inverse and Cascaded Generalized-Inverse approaches. The Linear Programming approach provides a different pitch rate behavior because it tries to generate the necessary pitch rate using the outboard and inboard elevons.

The Linear Programming approach provides the highest maximum commanded angle of sideslip among all the methods. The Weighted Pseudo-Inverse and Cascaded Generalized-Inverse approaches have almost the same controller commands for the angle of the sideslip. The Daisy Chain and Ganged Pseudo-Inverse methods have lower commanded control signals than the Weighted Pseudo-Inverse and Cascaded Generalized-Inverse methods.

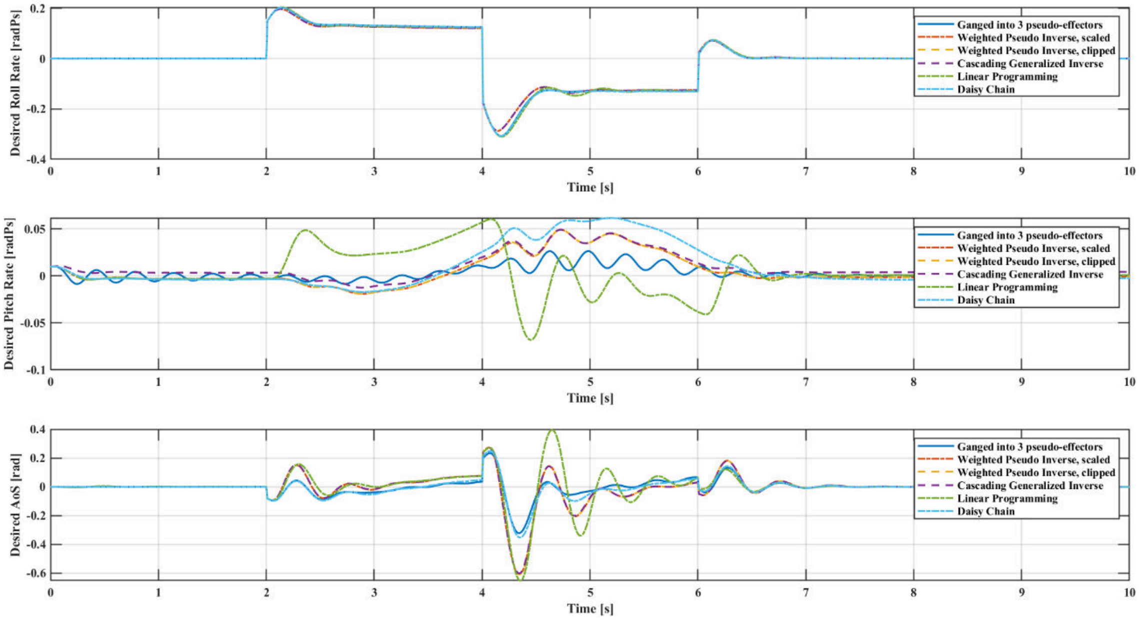

Figure 25: Scenario 1: Comparison of Closed-Loop Desired/Obtained Rates for Each Allocation Approach.

In Figure 26, it is observed that the canards exhibit oscillatory behavior for the Ganged Pseudo-Inverse method, indicating that this method is attempting to achieve the desired pitch rate effect using only primary control surfaces. The deflection of outboard elevons is smaller for the Weighted Pseudo-Inverse and Cascaded Generalized-Inverse approaches, as these methods use inboard elevons to support the outboard elevons for the desired roll rate.

Linear Programming uses only one-sided inboard and outboard elevons, and more rudder deflection is used for controlling the yaw moment. The usage of only one-sided inboard and outboard elevons is preferred over both-sided usage because both-sided usage eliminates the yaw moments generated by the elevons.

It is important to note that the Daisy Chain approach uses outboard elevons with different deflection magnitudes for the desired roll rate, which in turn creates a pitch rate to support the canards. The usage of elevon deflection with different magnitudes is a key characteristic of the Daisy Chain method.

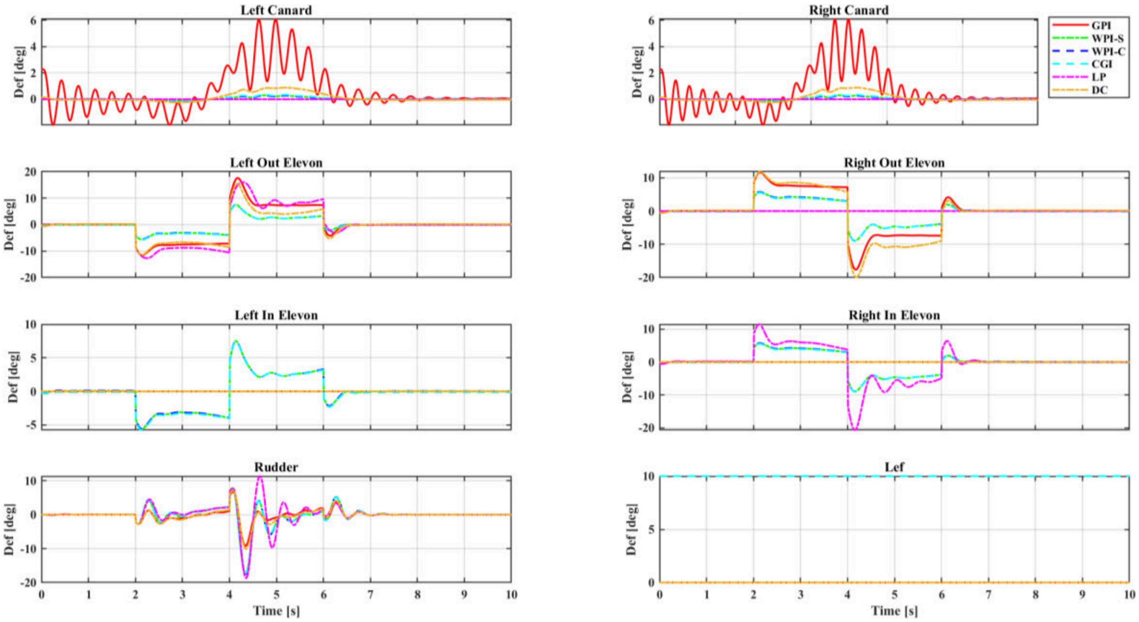

Figure 26: Scenario 1: Comparison of Closed-Loop Control Surface Deflections for Each Allocation Approach.

In Figure 27, it can be observed that all control allocation methods can achieve the controller-commanded desired effect, although there are some differences in how each method achieves the desired effect.

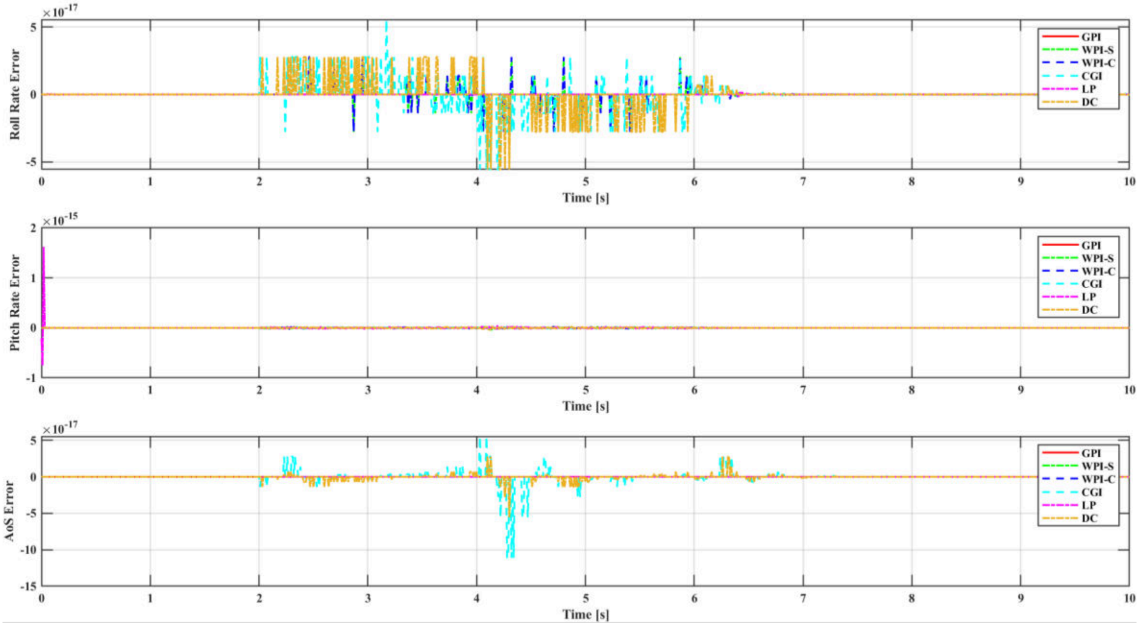

Figure 27: Scenario 1: Comparison of Closed-Loop Moment Error for Each Allocation Approach.

In Figure 28, the performance of each control allocation method is compared in terms of the Euclidean-Norm. The Euclidean-Norm is a commonly used performance metric that quantifies the magnitude of control surface deflection induced by the control allocation method. This metric provides a measure of the drag coefficient that acts upon the aircraft during maneuvers. The results show that the Cascaded Generalized-Inverse approach induces the highest drag coefficient due to leading-edge flap deflection during maneuvering. On the other hand, the Ganged Pseudo-Inverse, Daisy Chain method, and Linear Programming methods result in similar drag coefficients. However, the Ganged Pseudo-Inverse method exhibits oscillations, which may negatively affect aircraft stability and control. The Weighted

Pseudo-Inverse methods exhibit the same Euclidean-Norm, which is the lowest among all methods. Therefore, it can be concluded that the Weighted Pseudo-Inverse method provides the best performance in terms of drag reduction.

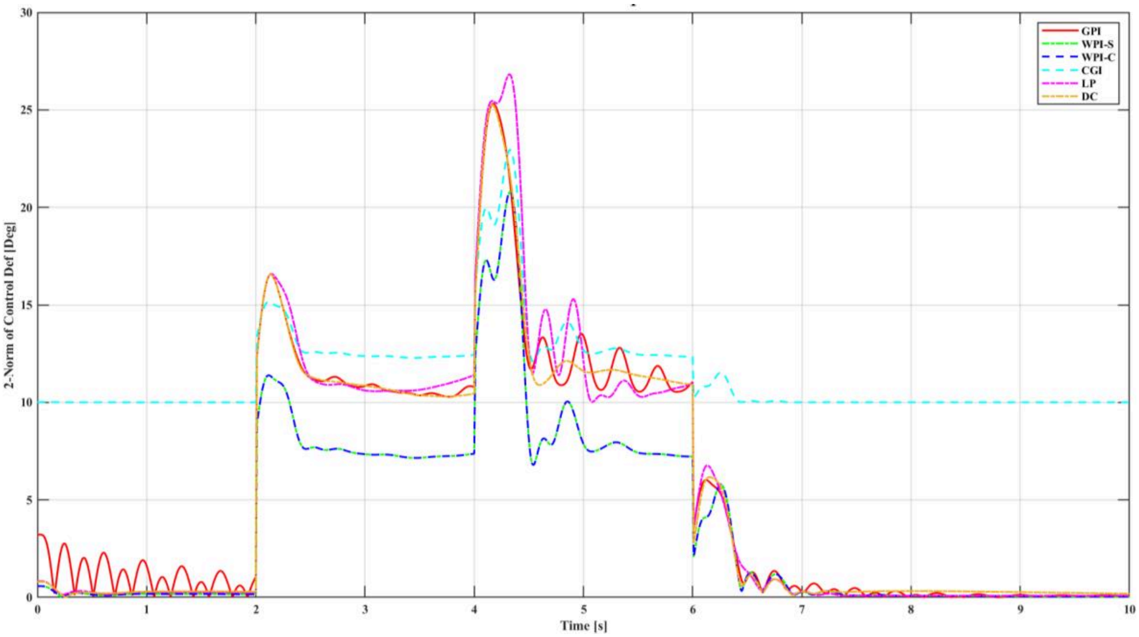

Figure 28: Scenario 1: Comparison of Closed-Loop Euclidian Norm for Each Allocation Approach.

In the closed-loop analysis, the performance of the control allocation method is evaluated based on the aircraft states. Analyzing aircraft states for each allocation method is important as it shows how well the controller performs with the suggested control allocation method. Figure 29 provides a comparison of aircraft states for all allocation methodologies. The responses of all methods are smooth and well, and they have similar behaviors except for Linear Programming. Although all control allocation methods use similar control surfaces, Linear Programming uses different kinds of controller surface deflection for the same maneuver. Additionally, there are oscillations in the Ganged Pseudo-Inverse method.

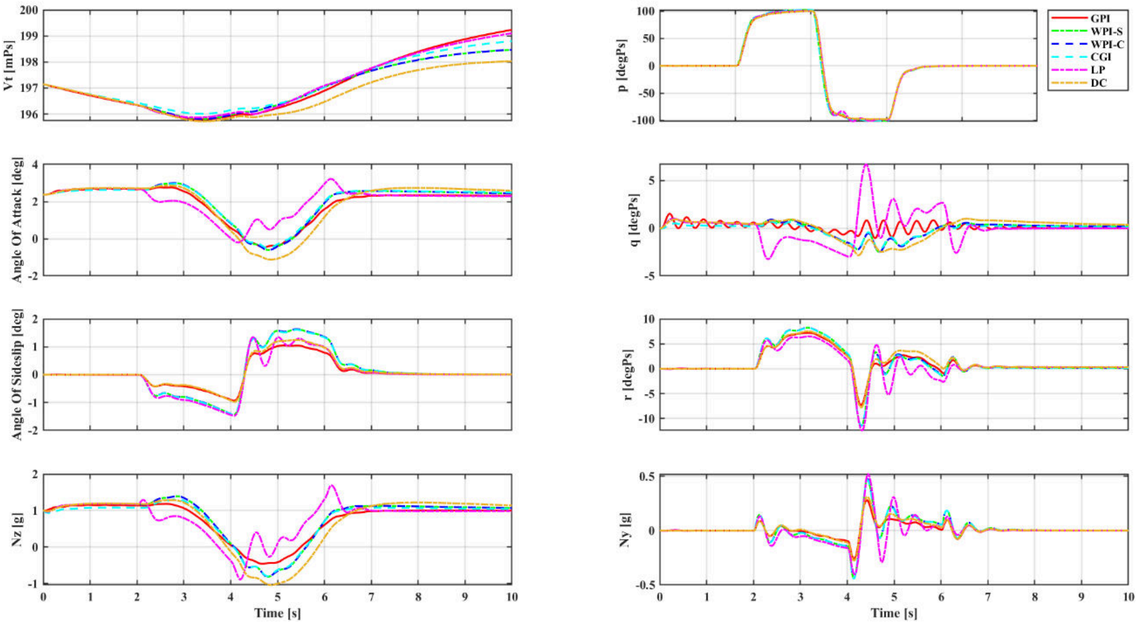

Figure 29: Scenario 1: Comparison of Closed-Loop Nonlinear Aircraft States Comparison

B. Scenario 2: Half-Cuban Eight

In this section, control allocation methodologies are compared using the open-loop and closed-loop analysis for the half-Cuban Eight maneuver. Firstly, open-loop analysis results are given, and then closed-loop analysis results are presented. Also, performance analysis is presented.

1. Scenario 2 - Open Loop Analysis Result

This section describes the open-loop analysis conducted on each control allocation method using the same moment inputs for a representative maneuver shown in Figure 6, and the pilot stick force inputs shown in Figure 7. The control effector's positions and the desired versus obtained moments for each allocation method are plotted and compared. The final step involves combining all of these figures into a single figure to compare the methods with each other.

As previously stated, the Ganged Pseudo-Inverse method employs primary control effectors, which for this study were selected as the canard, outboard elevon, and rudder for the ADMIRE aircraft model. Figure 30 shows that the Ganged Pseudo-Inverse method utilizes a positive angle deflection (control surface edge down) for both the right and left canard to produce a positive pitch moment along the pitch axis. Additionally, this method produces a positive roll moment by employing a positive deflection (control surface edge down) on the left outboard elevon and a negative deflection (control surface edge up) on the right elevon. To generate a positive yaw moment, the Ganged Pseudo-Inverse method employs a negative deflection (control surface edge right) on the rudders.

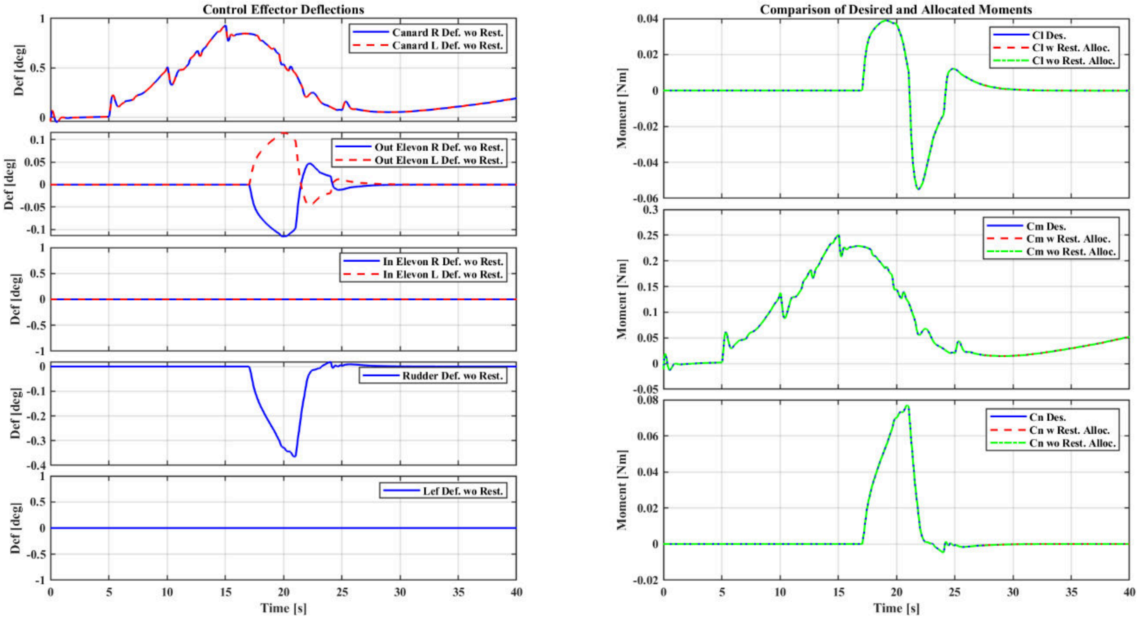

Figure 30: Scenario 2: Open-Loop Results for Ganged Pseudo-Inverse Approach.

In Figures 31 and 32, the results for the clipped and scaled Weighted Pseudo-Inverse methods are presented. These figures indicate that at 20 seconds of the maneuver, the canard is symmetrically deflected, which enables the elevons to generate a pitch moment for the half-Cuban Eight maneuver. After 20 seconds, there are little asymmetry in canard deflection. So, canard support the elevons to generate roll moment. The leading-edge flap provides the rest of the required pitch moment, as shown in both figures. Additionally, inboard- and outboard-elevons produce roll moment by deflecting asymmetrically, while only rudder deflection is utilized for the yaw moment.

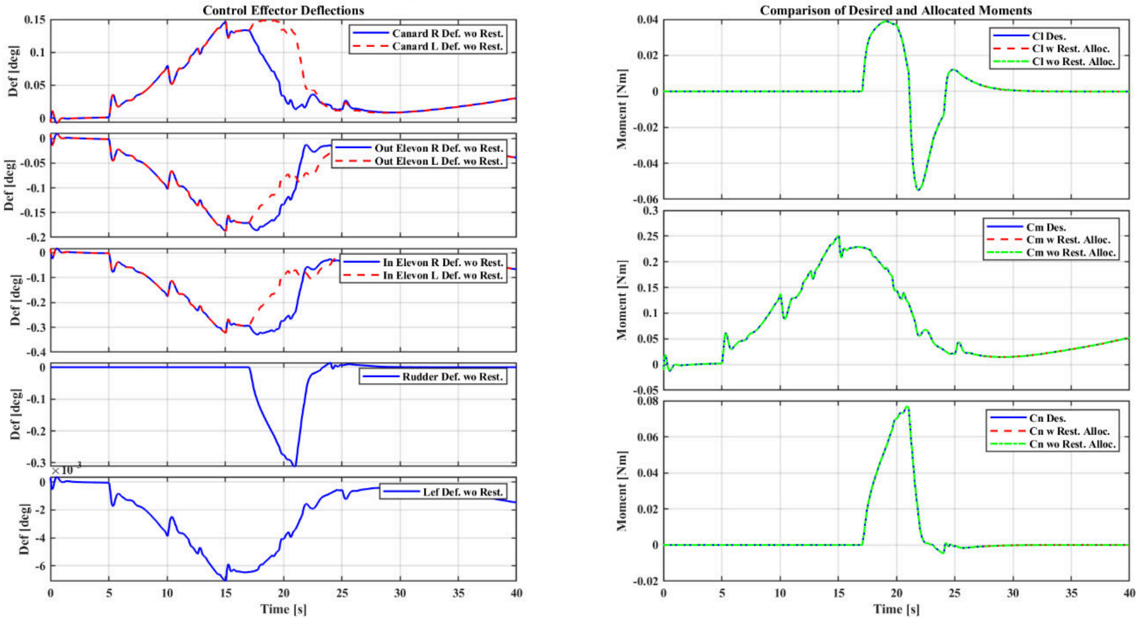

Figure 31: Scenario 2: Open-Loop Results for Weighted Pseudo-Inverse (Scaled) Approach.

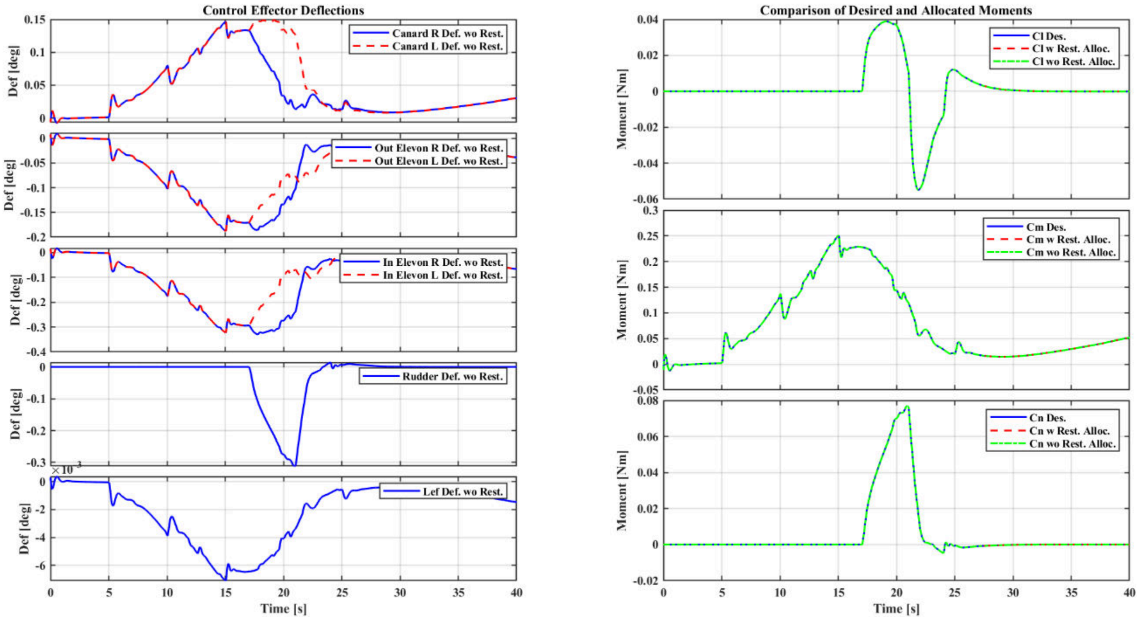

Figure 32: Scenario 2: Open-Loop Results for Weighted Pseudo-Inverse (Clipped) Approach.

The cascaded generalized inverse uses the Moore-Penrose pseudo-inverse to find control surface deflections until one or more of the control effectors exceeds the position limits. The control effectors that exceed their limits are saturated and removed from the control effectiveness matrix, and the remaining moments are calculated. The cascaded generalized inverse method employs the same pseudo-inverse approach as the Weighted Pseudo-Inverse to calculate control deflections for the desired moment. This is evident in Figure 33, where the control surface deflections for both methods are the same.

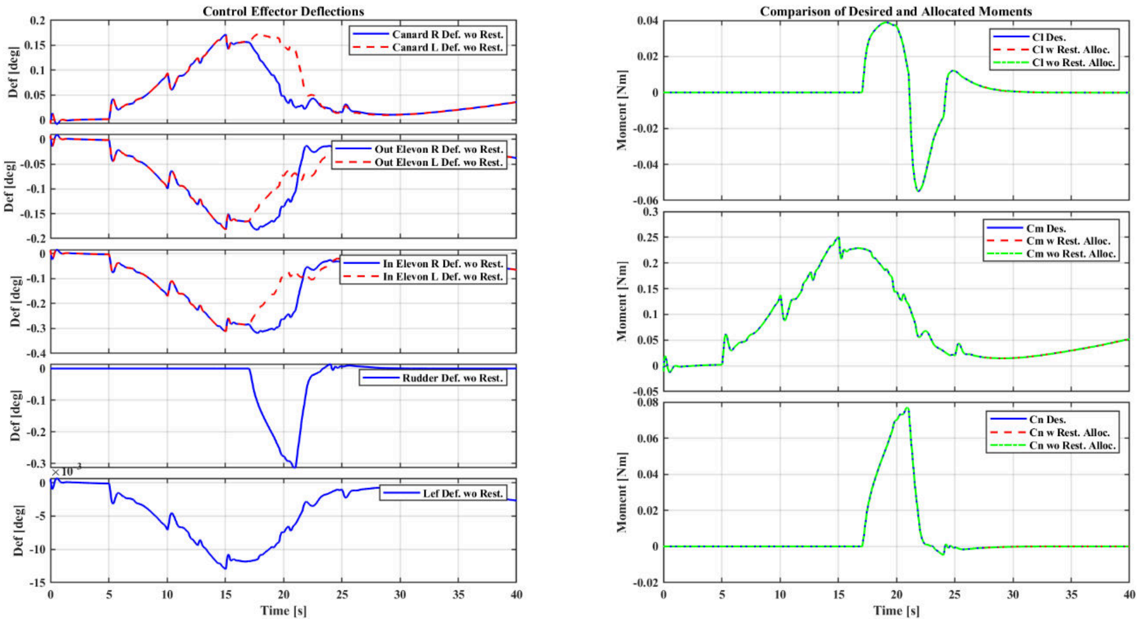

Figure 33: Scenario 2: Open-Loop Results for the Cascading Generalized-Inverse Approach.

In Figure 34, it can be observed that the Daisy Chain method generates pitch moment using only symmetrical canards, while asymmetrical outboard elevon deflection is used to generate the roll moment. Additionally, rudder deflection is used to produce a yaw moment.

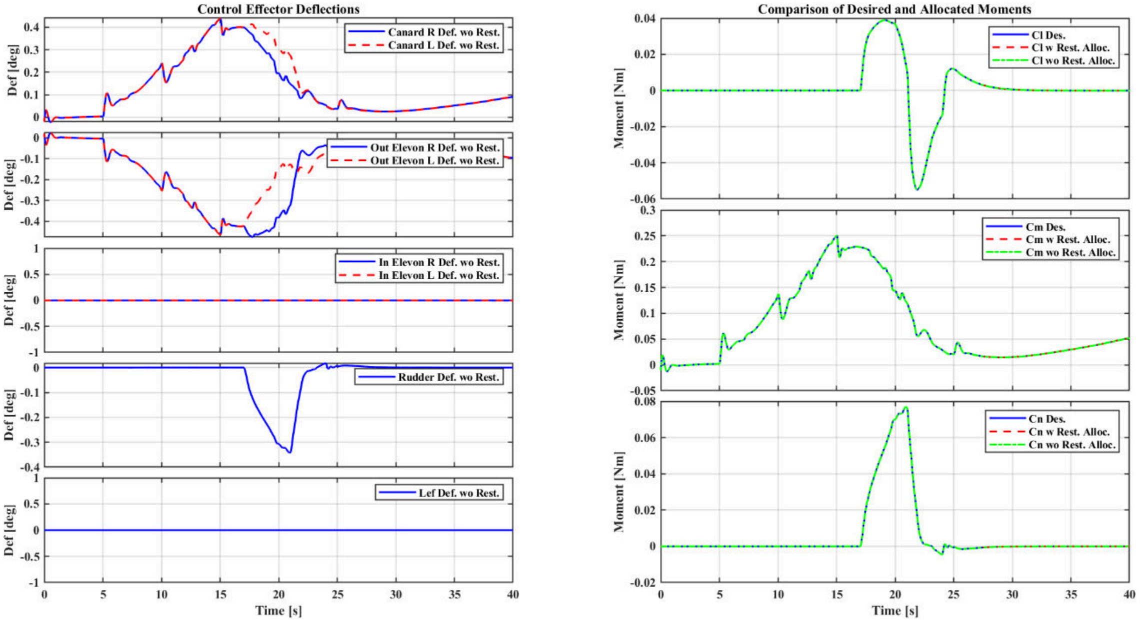

Figure 34: Scenario 2: Open-Loop Results for the Daisy Chain Approach.

In Figure 35, the result of the Linear Programming method for the half-Cuban Eight maneuver is presented. It can be observed that Linear Programming employs symmetrical deflection of inboard elevons to generate pitch moment, and asymmetrical deflection of the right and left inboard elevons to generate roll moment. Furthermore, for the yaw axis, rudder deflection is utilized. Notably, unlike the other methods investigated in this study, Linear Programming does not employ canards and outboard-elevons as primary control surfaces.

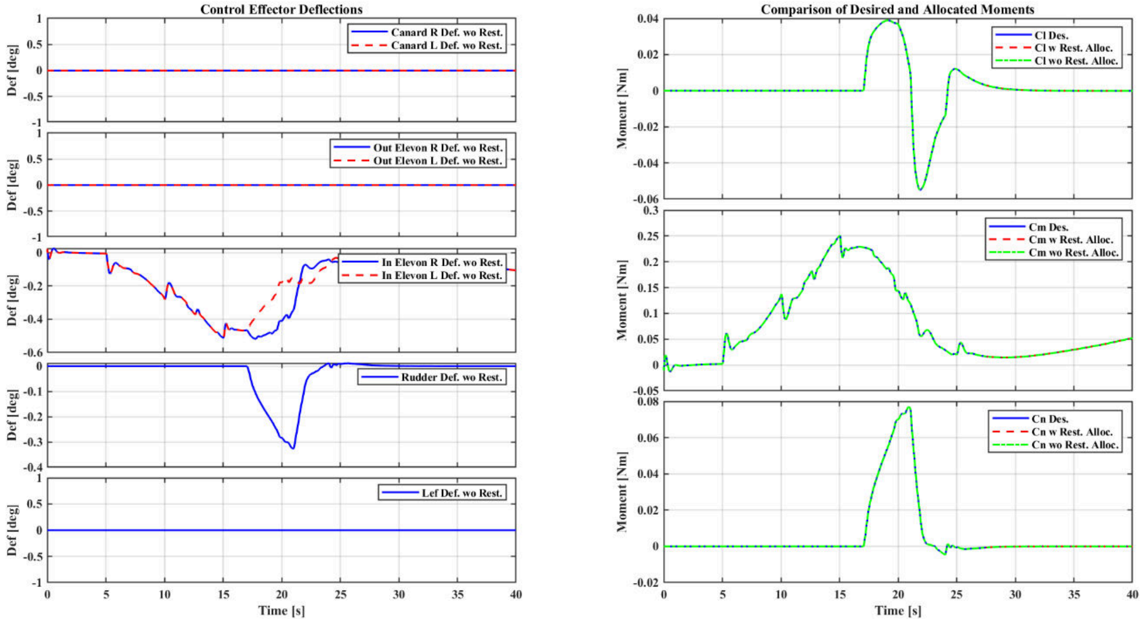

Figure 35: Scenario 2: Open-Loop Results for the Linear Programming Approach.

Figure 36 shows a comparison between the desired moments and obtained moments for all control allocation methods. It can be observed that all methods perform well in terms of obtaining the desired moment.

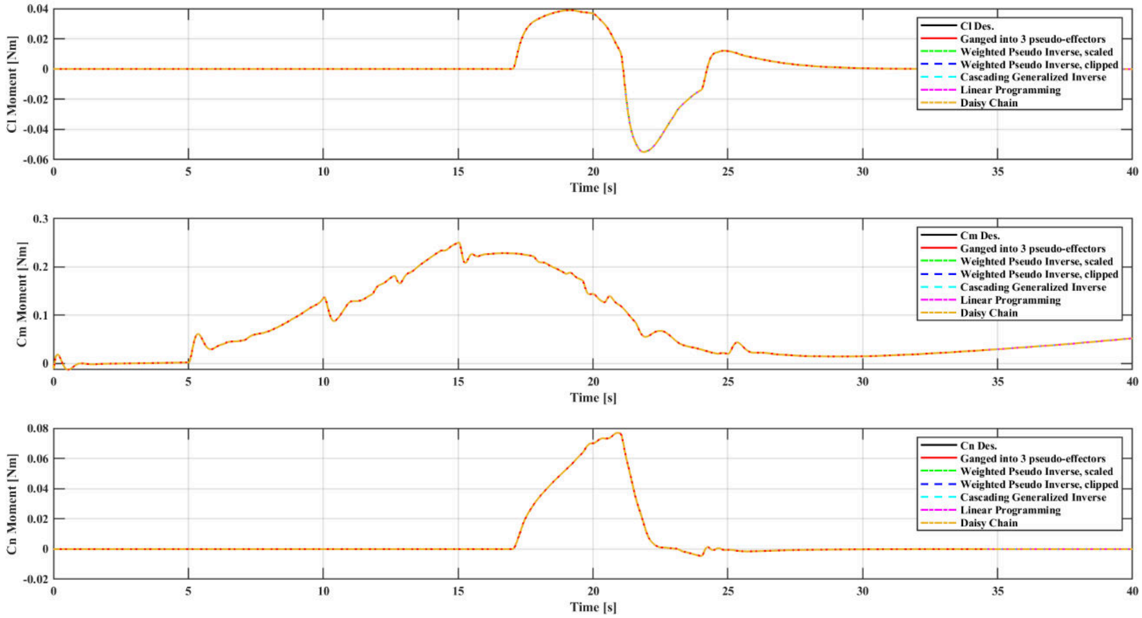

Figure 36: Scenario 2: Comparison of Open-Loop Desired/Obtained Moments for Each Allocation Approach.

Figure 37 presents the control surface deflections for each control allocation method. It can be observed that the Ganged Pseudo-Inverse method primarily employs canards, while the Daisy Chain method uses elevons and rudders. Additionally, the Cascaded Generalized Inverse and Linear Programming methods use right and left elevons for maximum deflection, with the left being predominantly used by the Cascaded Generalized Inverse.

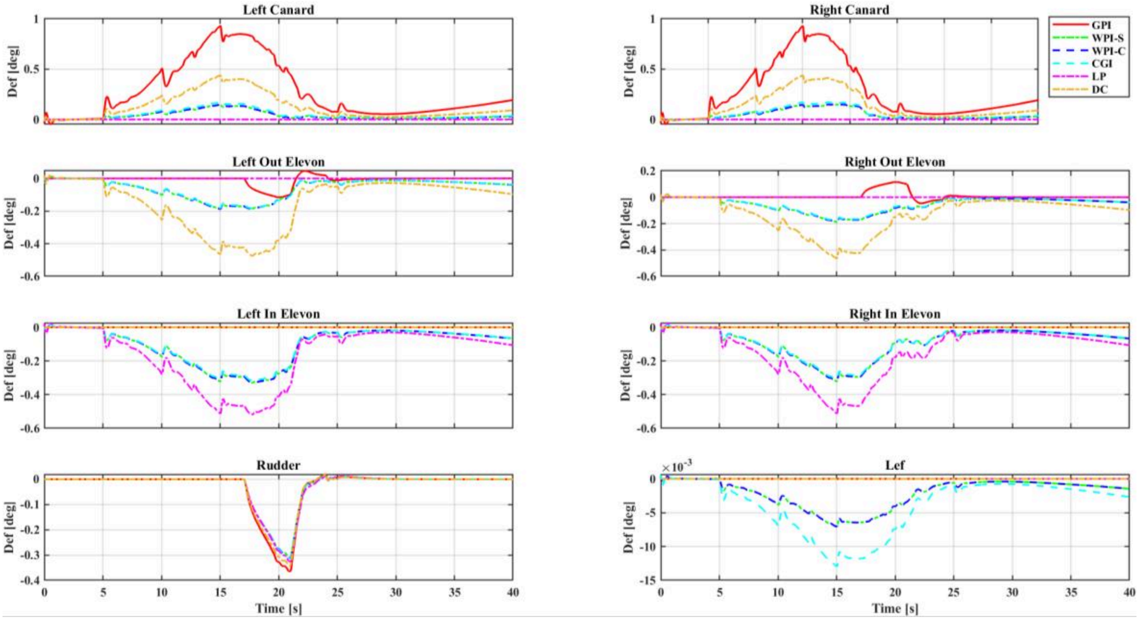

Figure 37: Scenario 2: Comparison of Open-Loop Control Surface Deflections for Each Allocation Approach.

Figure 38 displays the total execution time for 30 seconds of simulation time for each control allocation method. The Clipped Weighted Pseudo-Inverse method exhibits the best performance in terms of execution time, whereas Linear Programming has the highest execution time and thus the worst performance. The Cascaded Generalized Inverse, Ganged Pseudo-Inverse, and Daisy Chain methods have better performance than Linear Programming concerning execution time.

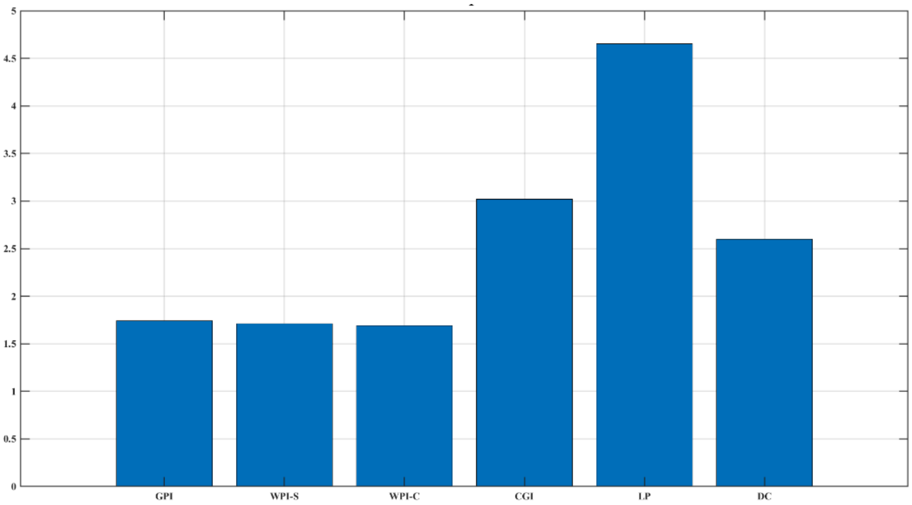

Figure 38: Scenario 2: Comparison of Open-Loop Execution Time for Each Allocation Approach.

2. *Scenario 2 - Closed-Loop Analysis Result*

In the closed-loop analysis, each control allocation method is analyzed for given inputs as depicted in Figure 7. Figure 39 demonstrates that the Ganged Pseudo-Inverse approach is unable to provide the desired pitch rate in the closed-loop analysis. Additionally, the rudder, left canard, and right canard reaches their rate and position limits due to the use of primary control surfaces for each axis in the Ganged Pseudo-Inverse method. The primary control surfaces deflect too quickly, resulting in their deflection hitting the position limit of the actuator, rendering the controller unable to control the pitch axis. Therefore, it can be concluded that the Ganged Pseudo Inverse method is not suitable for this maneuver as this method is unable to provide the desired moments.

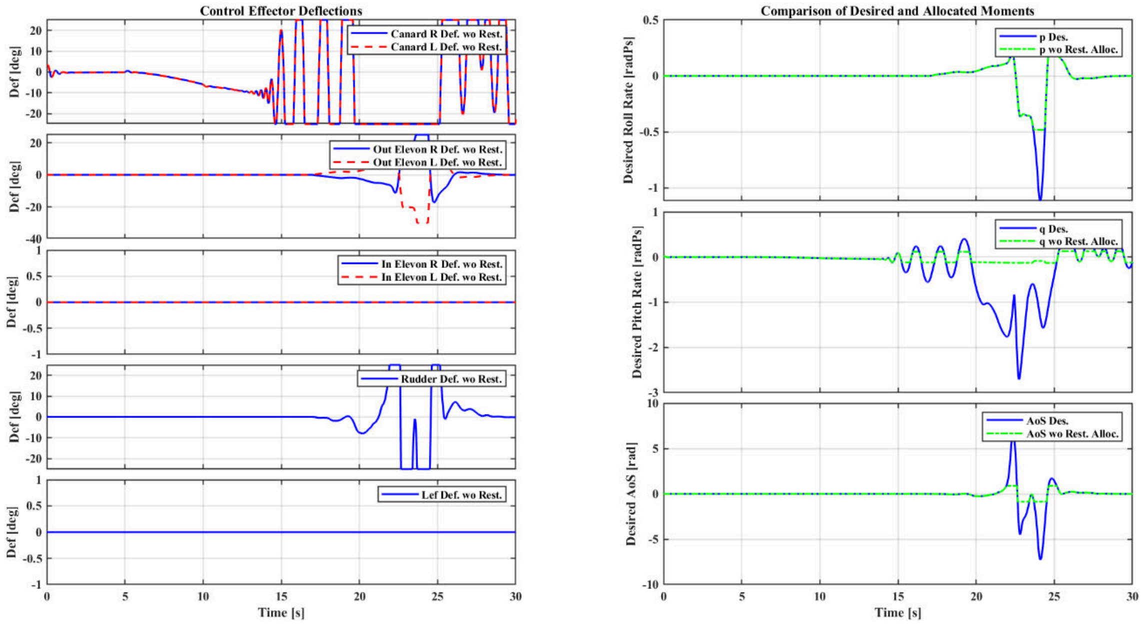

Figure 39: Scenario 2: Closed-Loop Results for Ganged Pseudo-Inverse Approach.

Figures 40-41 illustrate the control surface deflection results for the Clipped and Scaled Weighted Pseudo Inverse methods for the same inputs in closed-loop analysis. As shown in the figures, the canard deflects symmetrically to generate pitch moment, and the leading-edge flap remains at zero deflection as all pitch moments are generated from the symmetric canard deflection. Additionally, the inboard- and outboard elevons deflect asymmetrically to generate roll moment, while the yaw moment is due to rudder deflection. Overall, both Clipped and Scaled Weighted Pseudo Inverse methods perform well in generating the desired moments in closed-loop analysis.

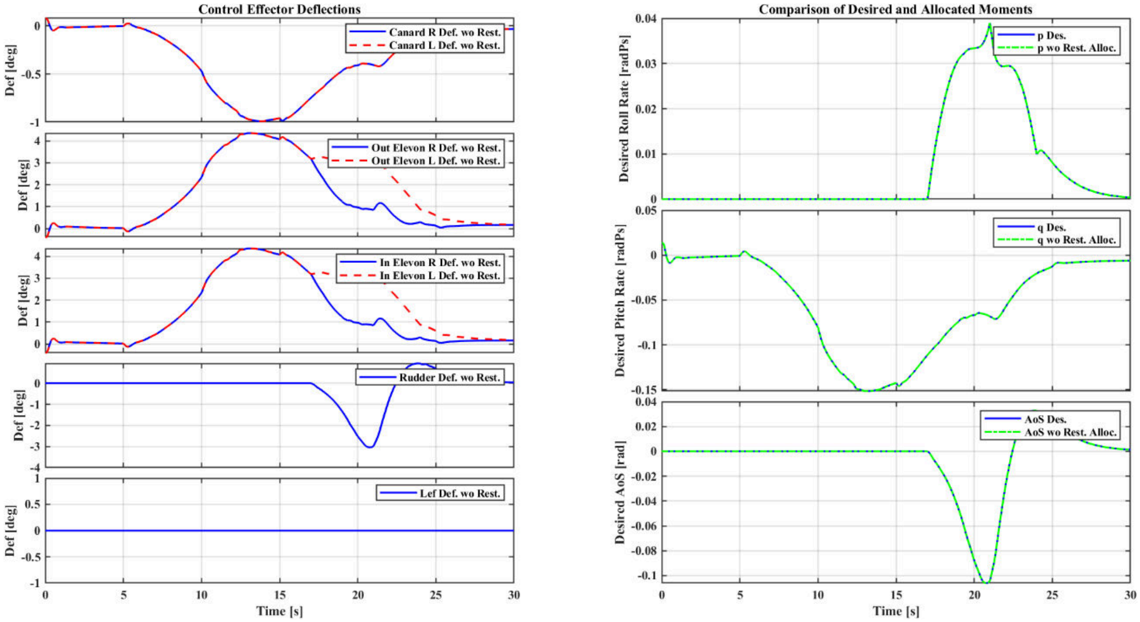

Figure 40: Scenario 2: Closed-Loop Results for Weighted Pseudo-Inverse (Scaled) Approach.

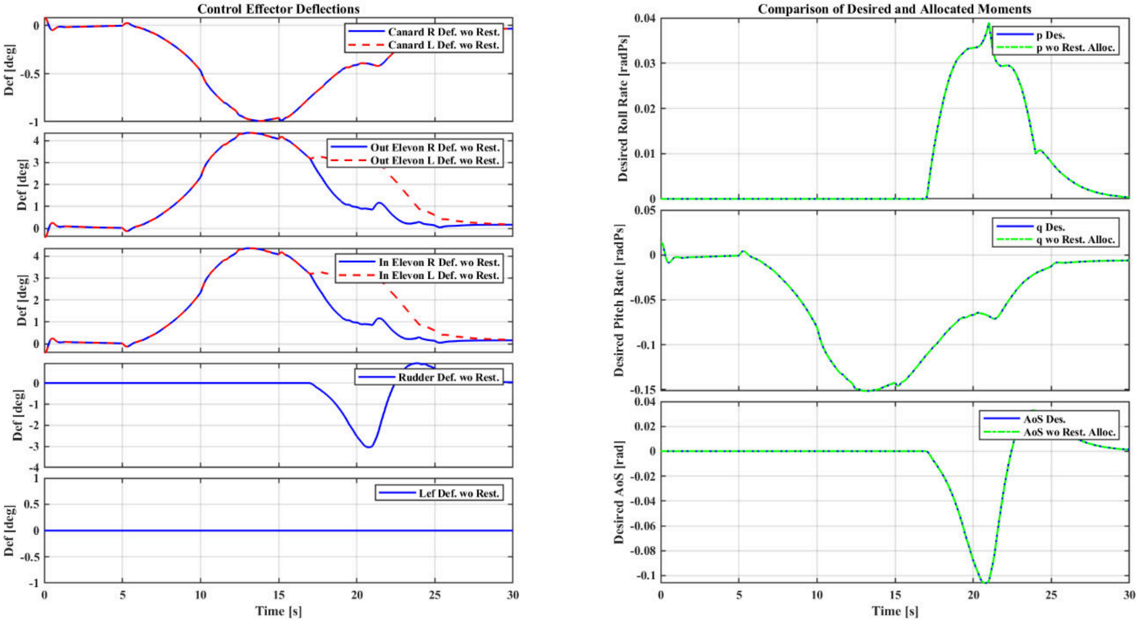

Figure 41: Scenario 2: Closed-Loop Results for Weighted Pseudo-Inverse (Clipped) Approach.

As seen in Figure 42, canards are used to generate the pitch moment, asymmetrical elevon deflections are used to generate the roll moment, and the rudder is used to generate the yaw moment in a cascaded pseudo-inverse approach. However, a leading edge flap is not used in this method.

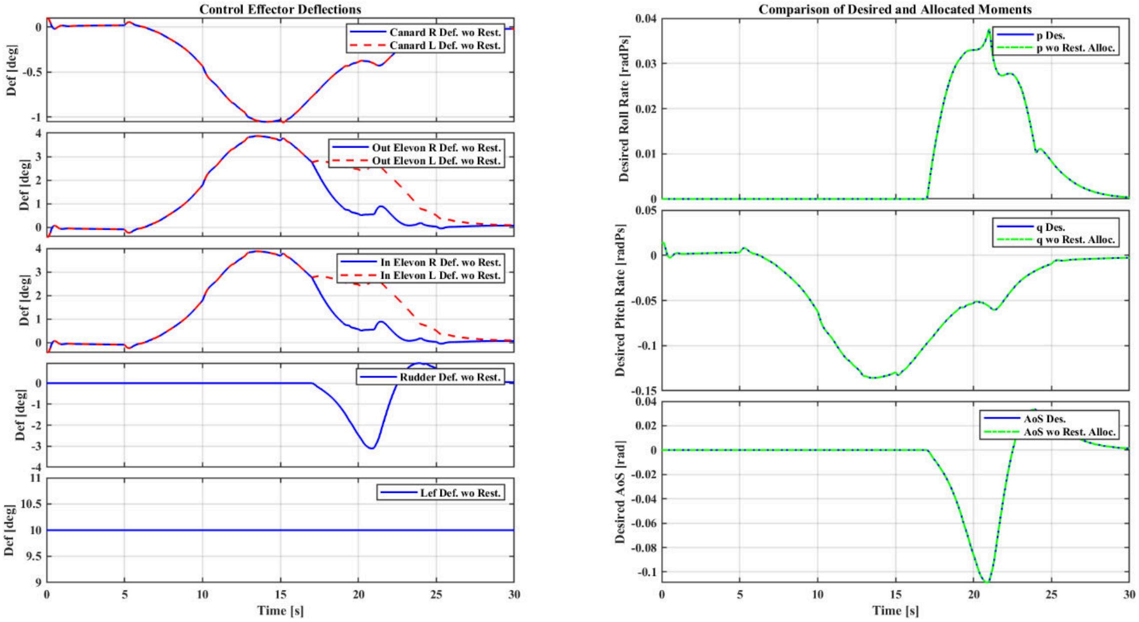

Figure 42: Scenario 2: Closed-Loop Results for Cascaded Generalized-Inverse Approach.

Figure 43 shows that, similar to the Cascaded Generalized Inverse approach, the Daisy Chain method uses canards to generate pitch moment, asymmetrical elevon deflections to generate roll moment, and a rudder to generate yaw moment.

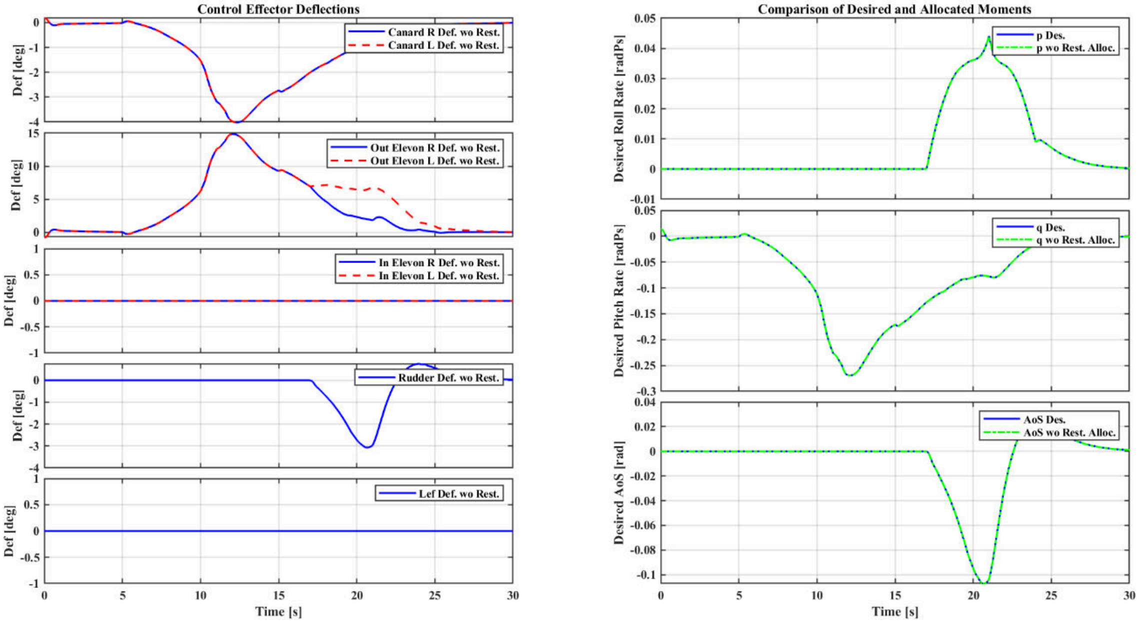

Figure 43: Scenario 2: Closed-Loop Results for Daisy Chain Approach.

In Figure 44, the results obtained using the Linear Programming approach for the given inputs are presented. As shown in the figure, elevons deflect asymmetrically to generate both pitch and roll moment, while rudder deflection is utilized to generate yaw moment.

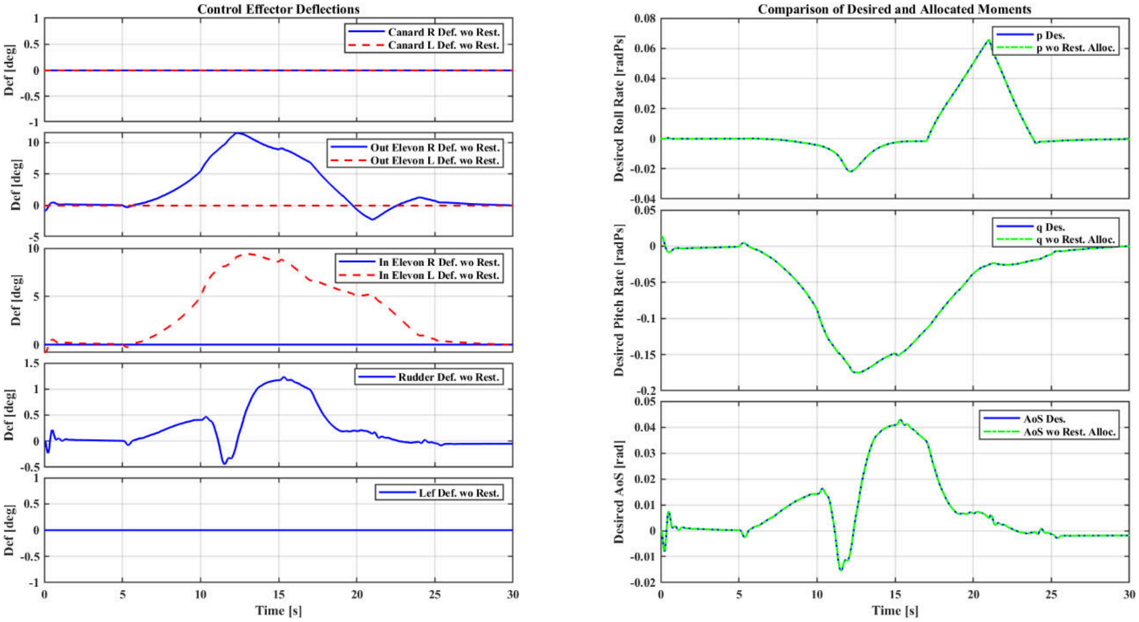

Figure 44: Scenario 2: Closed-Loop Results for Linear Programming Approach.

In Figure 45, a comparison of control allocation methods for providing desired effects is presented. It can be observed that the roll rate and sideslip angle commands from the controller are similar for all control allocation methods, except for the Linear Programming. Linear Programming provides the desired effect on both the pitch rate and angle of the sideslip, while the other methods show different performance in the pitch axis. The Weighted Pseudo-Inverse, Cascaded Generalized Inverse, and Daisy Chain methods demand similar sideslip angles from the controller. However, the Daisy Chain approach requires a higher pitch rate demand from the controller compared to the other methods. The Ganged Pseudo-Inverse method is not considered due to its inability to provide the desired moments for this maneuver.

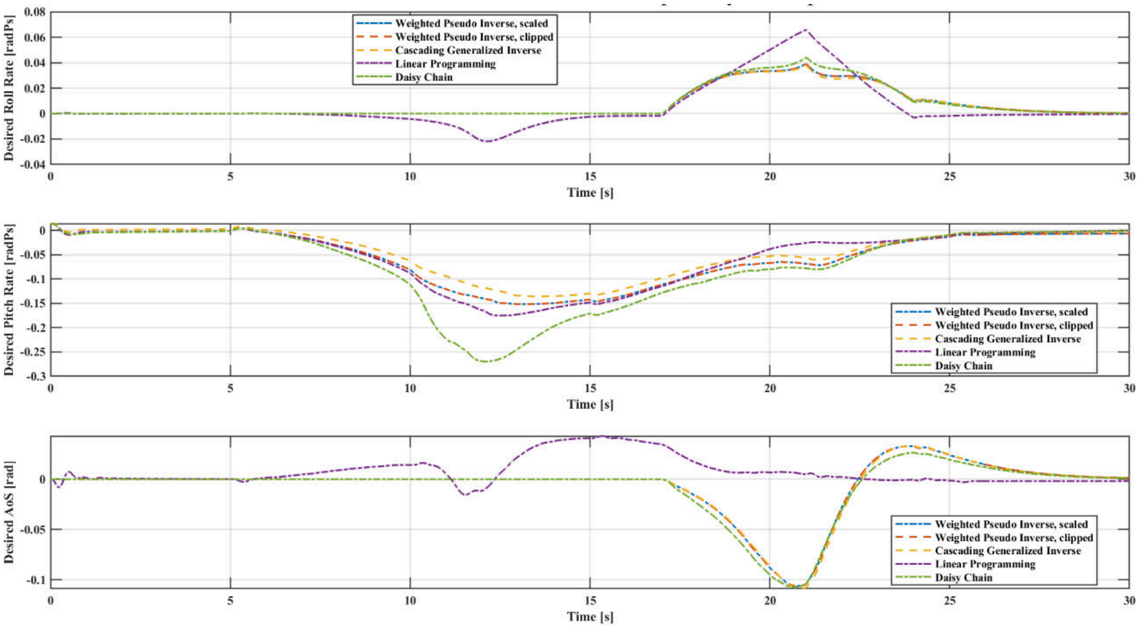

Figure 45: Scenario 2: Comparison of Closed-Loop Desired/Obtained Rates for Each Allocation Approach.

In Figure 46, it can be observed that the Weighted Pseudo-Inverse and Cascaded Generalized-Inverse methods have similar magnitudes of control surface deflections. On the other hand, the Daisy Chain method has maximum deflection for both left and right canards, indicating that it relies heavily on primary control surfaces to achieve the desired effect. Additionally, the Daisy Chain method has maximum deflection for outboard elevons and rudder. This trend is similar to what was observed in pitch rate demand. The Linear Programming method, however, exhibits maximum deflection and oscillatory behavior for inboard elevons.

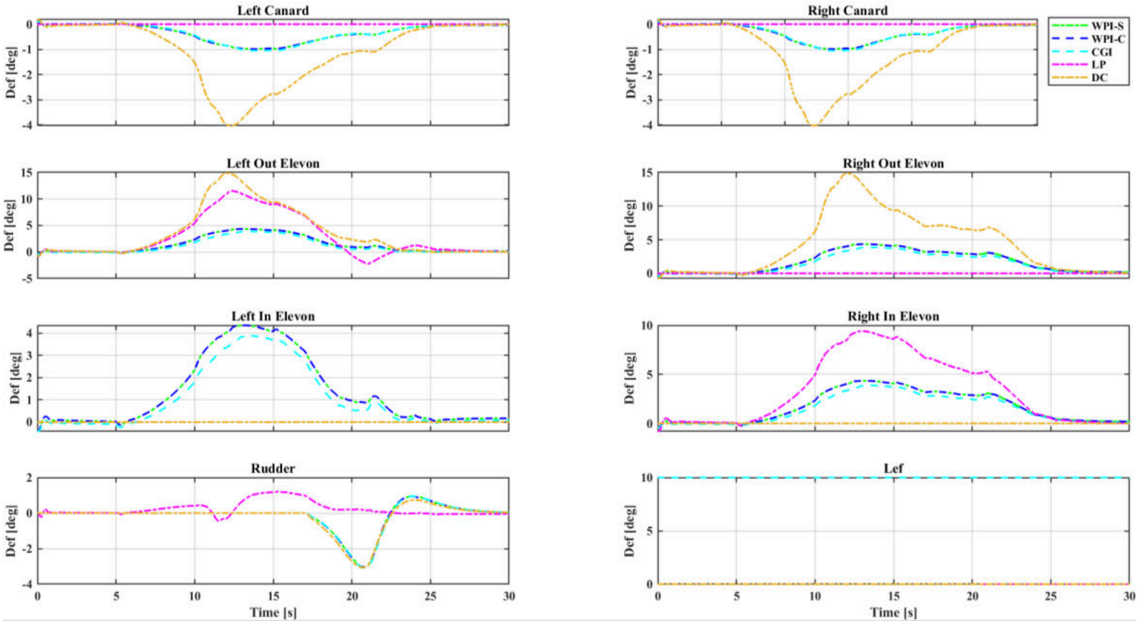

Figure 46: Scenario 2: Comparison of Closed-Loop Control Surface Deflections for Each Allocation Approach.

In Figure 47, the difference between the controller commanded desired effect and the effect obtained from the allocation methods is plotted. All methods can provide the desired effect except for the Ganged Pseudo Inverse method. Although the Daisy Chain method gives slightly different results than the other methods, its magnitude is small and can be considered negligible.

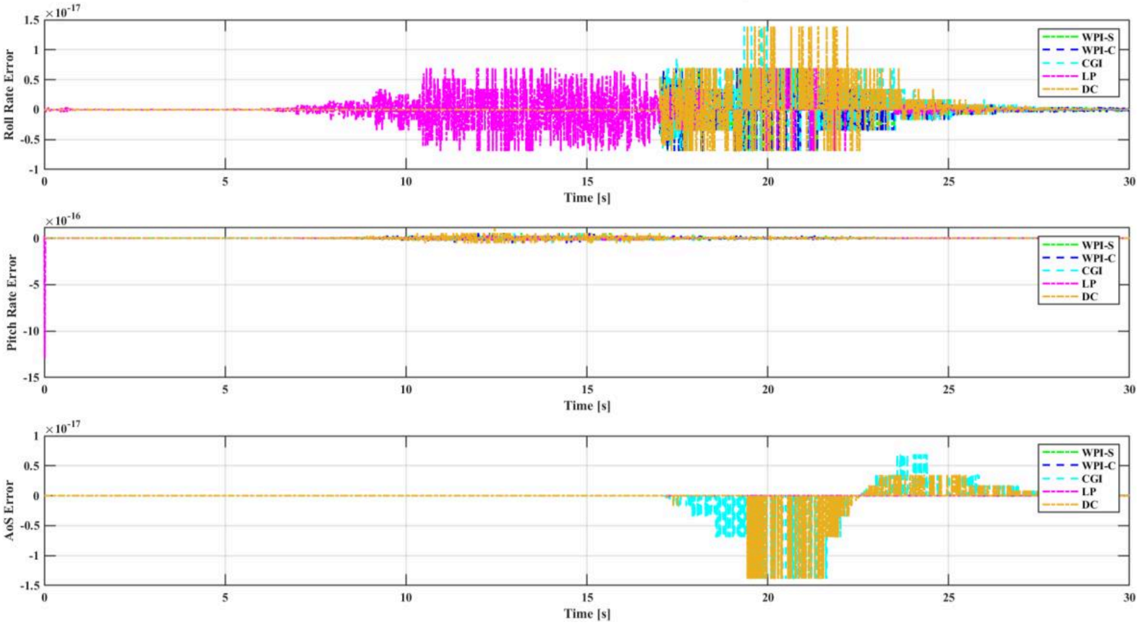

Figure 47: Scenario 2: Comparison of Closed-Loop Moment Error for Each Allocation Approach.

In Figure 48, the Euclidean-Norm for each control allocation method is compared. As seen in this figure, Ganged Pseudo Inverse method creates the highest norm among the control allocation methods, indicating the highest control effort. Also, the Daisy Chain method has the second-highest norm. The norm of the Linear Programming method is higher than the Weighted Pseudo-Inverse method as seen in the figure. The Cascaded Generalized-Inverse method creates more norm than the Weighted Pseudo-Inverse method but less than the Linear Programming and Ganged Pseudo-Inverse methods.

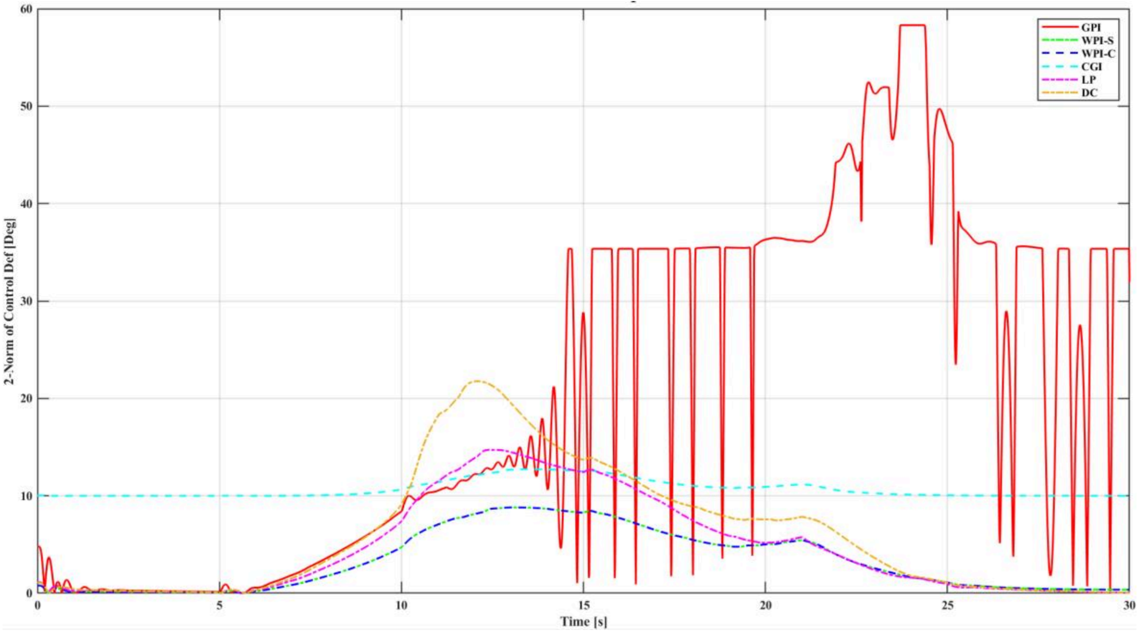

Figure 48: Scenario 2: Comparison of Closed-Loop Euclidian Norm for Each Allocation Approach.

Figure 49 presents a comprehensive comparison of the aircraft states for all control allocation methods. It can be observed that the Weighted Pseudo Inverse, Daisy Chain, Cascaded Generalized Inverse, and Linear Programming methods provide very smooth and well-behaved state responses. This indicates that these control allocation methods are capable of achieving the desired effect on the aircraft while maintaining stable and predictable behavior.

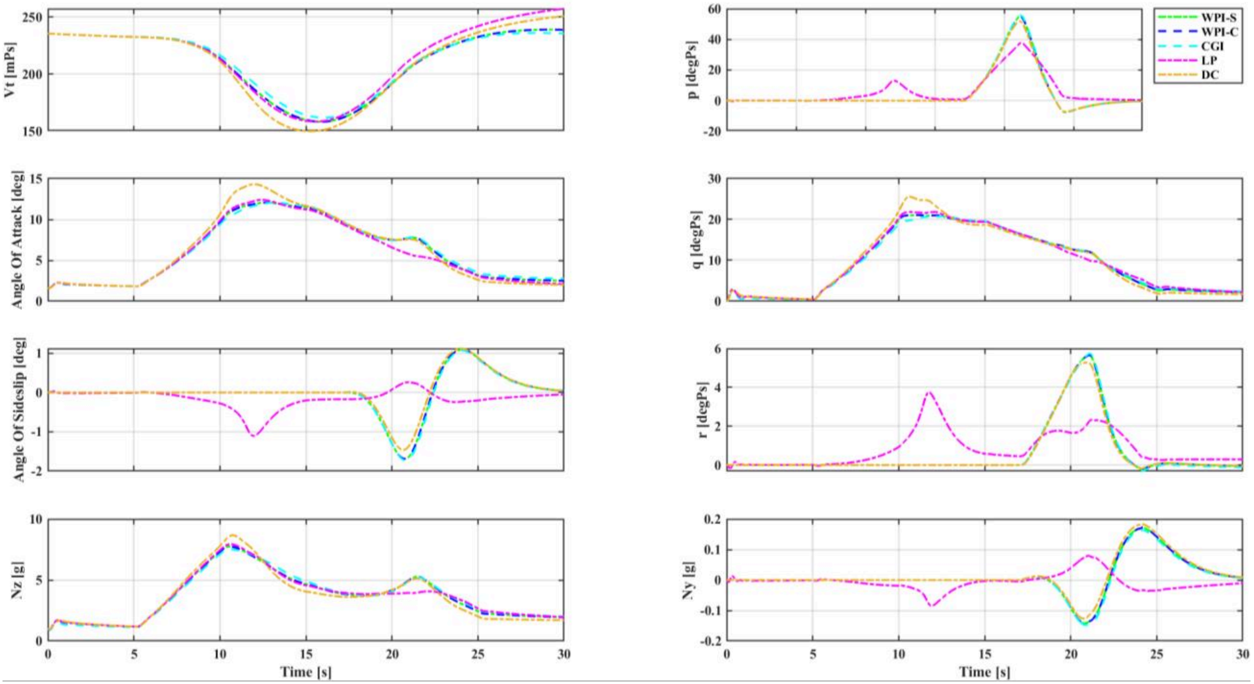

Figure 49: Scenario 2: Comparison of Closed-Loop Nonlinear Aircraft States Comparison

VI. Conclusion

The primary objective of this research is to implement the most common control allocation methods that are currently available in the literature and to perform a full comparison of a realistic scenario. The approaches implemented in this study are Ganged Pseudo-Inverse, Weighted Pseudo-Inverse (including scaled and clipped variants), Cascaded Generalized-Inverse, Daisy Chain, and Linear Programming approaches. Their capabilities under two specific maneuver scenarios, Lateral Doublet Stick Input and Half-Cuban Eight, in ADMIRE aircraft model, are compared using the Euclidean-Norm as a performance metric. Euclidean-Norm shows how much the control allocation method deflects aircraft control surfaces for a given desired moment. The control surface deflections are a direct expression of how much drag acts upon the aircraft.

In terms of Euclidean-Norm, the control allocation approaches for the first scenario (Lateral Doublet Stick Input) can be ranked in the increasing order of performance (1:best performing approach) as follows: 1) Weighted Pseudo-Inverse, 2) Cascaded Generalized-Inverse, 3) Daisy Chain, 4) Ganged Pseudo Inverse, and finally 5) Linear Programming. The performance of two different Weighted Pseudo-Inverse methods is the same. The rank of the control allocation approaches for the second scenario (half-Cuban Eight) in the increasing order of performance are as follows: 1) Weighted Pseudo-Inverse, 2) Cascaded Generalized-Inverse, 3) Linear Programming, 4) Daisy Chain, and 5) Ganged Pseudo Inverse. In terms of execution time, the control allocation approaches for both scenarios can be ranked as follows from fastest to slowest: 1) Weighted Pseudo-Inverse, 2) Ganged Pseudo Inverse, 3) Daisy Chain, 4) Cascaded Generalized-Inverse, and 5) Linear Programming. This means that the Weighted Pseudo-Inverse method is the fastest among all the approaches, while the Linear Programming method is the slowest. It is important to consider the execution time when selecting a control allocation approach, especially for real-time applications where speed is critical.

Some of the key findings of this study are as follows:

- Even though the Weighted Pseudo-Inverse performs best for two scenarios, it's not fair to say that this approach performs the best compared to other approaches since the controller of ADMIRE model was developed using the Weight Pseudo-Inverse method.

- Cascaded Generalized-Inverse performs better than other approaches except for Weighted Pseudo-Inverse for two scenarios.
- The performance of Cascaded Generalized-Inverse is almost the same as the performance of Weighted Pseudo-Inverse. The only difference is the result of leading-edge flap deflection where 10deg is deflected during the Cascaded Generalized Inverse method.
- Weighted Pseudo-Inverse performs best from the execution time-wise.
- As seen from the given results, performance of all control allocation methods is same in open-loop analysis since attainable moment can be obtained by all control allocation methods. However, in the case of closed loop analyses, their performance starts to differentiate because of the controller performance.
- Considering controller allocation and controller design simultaneously is crucial. Both of these factors can have a significant impact on the performance of the control system, so it's important to take them into account together.

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