A Symbolic Design Method for ETCS Hybrid Level 3 at Different Degrees of Accuracy

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Abstract

The *European Train Control System (Hybrid) Level 3* (ETCS Hybrid Level 3) allows for introducing *Virtual Subsections* (VSS) into existing railway infrastructures. These VSS work similarly to blocks in conventional block signaling but do not require installation or maintenance of trackside train detection. This added flexibility can be used to adapt a given railway network's (virtual) layout to the changing demands of new schedules. Automated methods are needed to properly use this flexibility and design such layouts on demand and avoid time-intensive manual labor. Recently, approaches inspired by design automation of electronic hardware have been proposed to address this need. But those methods – which are particularly well suited for inherently discrete problems in electronic design automation – have struggled with modeling continuous properties like train positions, time, and acceleration. This work proposes a *Mixed Integer Linear Programming* (MILP) formulation that, for the first time, can accurately model design problems for ETCS Hybrid Level 3 by including essential, continuous constraints, e.g., for train dynamics or braking curves. The formulation is designed to be flexible and extendable, allowing the user to include/exclude certain constraints or simplify the model as needed. By this, the user can decide whether he/she wants to quickly generate a less accurate solution or a more accurate one at the expense of higher runtimes – basically allowing him/her to trade-off accuracy and efficiency. A case study showcases the potential of the proposed approach and sketches examples to analyze which trade-offs are worthwhile and which simplifications can be safely made. The resulting tool and the benchmarks considered in this work are publicly available at `https://github.com/cda-tum/mtct` (as part of the *Munich Train Control Toolkit*, MTCT).

2012 ACM Subject Classification Applied computing → Transportation

Keywords and phrases ETCS, MILP, design automation, block signaling, virtual subsection

Digital Object Identifier 10.4230/OASIcs.ATMOS.2023.6

Supplementary Material *Software (Source Code)*: `https://github.com/cda-tum/mtct`
archived at `swh:1:dir:df97b4f2a6638ce93578147ee5d9220a63973f00`

1 Introduction

Although railway transportation has a long history, it also plays a vital role in the future of sustainable transportation. Unlike cars, trains cannot be operated on sight, and signaling systems are essential to prevent collisions. For this, many national train control systems have been implemented – about 40 in Europe alone [13].

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Due to the resulting compatibility issues, especially in times of increasing cross-border traffic, it was decided to harmonize these safety systems across Europe, namely in the *European Train Control System* (ETCS) [30]. Similar standardized systems exist in China (*Chinese Train Control System*, CTCS), North America (*Positive Train Control*, PTC), and even for metro lines (*Communication Based Train Control*, CBTC) [25, 29].

In addition to harmonization, these systems also aim at increasing throughput and, by this, increase demand for the entire train infrastructure. In fact, if building new tracks is not feasible, the increasing demand for rail transportation has to be tackled another way. To this end, new levels of train control systems have constantly been defined to allow for shorter train following times while maintaining the high safety requirements imposed [27]. *ETCS Hybrid Level 3* (ETCS HL3), for instance, defines *Virtual Subsections* (VSS) that introduce new blocks into an existing layout without requiring new hardware. In addition to positioning being determined via *Trackside Train Detection* (TTD) systems such as axle counters, the occupation of VSS is communicated live via a radio control center. This, in turn, allows for a much more fine-grained design of the train control system since the overhead does not limit the number of blocks for maintenance and installation of TTDs, and the block layout can be changed on demand as it is only virtual. In principle, such virtual layouts also allow for shorter headways and can, therefore, be used to increase the throughput of an existing network.

This potential gives rise to several new design tasks for ETCS HL3 systems, namely *verification* of HL3 layouts, *placement* of VSS, and *optimization* of train schedules using VSS [11]. Previous methods for designing ETCS L2 layouts did not have to consider dynamic block placement and are, therefore, aimed at more general performance indicators [14, 5, 18, 9, 23, 31]. Similarly, while train routing [15] and allocation [6, 3, 21, 4, 22] have been considered, these approaches are tailored for fixed layouts. There is also work on routing under ETCS Level 3, which uses so-called *moving blocks* and does not require *any* VSS.

Design tasks within ETCS HL3 have already been tackled using symbolic reasoning [32] and guided state-space search [26]. While these approaches seem promising for these tasks, they all make simplifying assumptions and do not model train movement – in particular acceleration and braking curves – accurately. This is partly due to the fundamental limitations of the methods used, as they are ill-fitted to model continuous properties directly.

Mixed Integer Linear Programming (MILP) is an alternative symbolic method that allows for modeling discrete values as well as continuous variables and is, therefore, a promising approach for solving design problems for ETCS HL3 accurately. MILP has already been used previously for routing within ETCS Level 3 with full moving blocks [28, 19], but no MILP approach exists that can automatically design ETCS HL3 layouts. This work introduces a comprehensive framework for solving ETCS HL3 design tasks using a symbolic MILP formulation that enables a designer of ETCS HL3 networks to model problems with varying degrees of accuracy. For the first time, this framework allows for accurate modeling of continuous properties for ETCS HL3 design tasks. Furthermore, since the expressiveness of MILP also subsumes previous formulations, the proposed MILP formulation can be simplified or extended with further constraints to model ETCS HL3 design tasks with varying degrees of accuracy.

It is to be expected that more detailed models are harder to solve and, thus, lead to longer solving times. The flexibility of the proposed framework allows for evaluating how model accuracy influences runtimes for solving instances of ETCS HL3 design tasks. Therefore, the impact of using more or less detailed models is evaluated on a range of benchmarks, including real-world examples, e.g., designing an ETCS HL3 layout for the S-Bahn Stammstrecke in Munich. The framework and the benchmarks are publicly available within the *Munich Train Control Toolkit* (MTCT) at `https://github.com/cda-tum/mtct`.

The remainder of this paper is structured as follows: Sec. 2 reviews the relevant background on block-signaling within ETCS and the design problem considered in this work. Sec. 3 then proposes the MILP formulation, starting with a minimal required encoding and successively introducing constraints that allow for more accurate models. The evaluation of the trade-off between runtime efficiency and model accuracy is discussed in Sec. 4. Finally Sec. 5 concludes this paper.

2 Block Signaling in ETCS HL3

ETCS HL3 builds upon principles of classic block signaling. This section provides the necessary background on block signaling within ETCS and what constraints exist on VSS placement.

2.1 Background

ETCS Level 1 (ETCS L1) separates a railway network in blocks. *Trackside Train Detection* (TTD), e.g., *Axle Counters* (AC), at the boundaries is used to determine if a train is present within a certain block. Hence they are also known as TTD sections. Conventional signals are used to show if the upcoming block is occupied or not. At distinct points, the signal state is transmitted to the train through *Eurobalises* (EB). A train always has to be able to come to a complete stop before the point to which it has received moving authority to, which is ensured by braking curves [12].

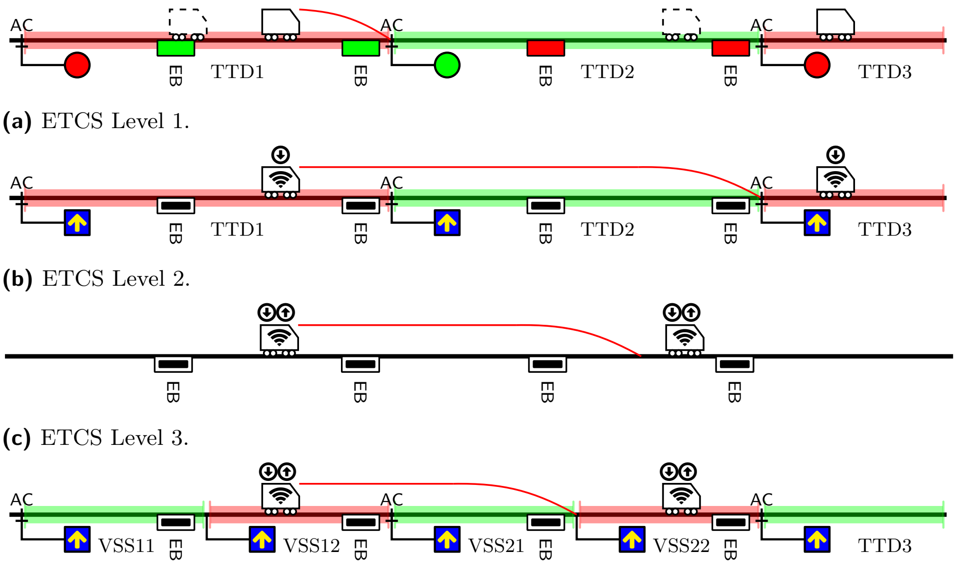
In *ETCS Level 2* (ETCS L2), trains communicate with the control system via the wireless system GSM-R. Balises are no longer used to transmit variable information, but fixed position data is employed instead. Still, TTDs are used to detect the status of blocks. Move authority is continuously transmitted to the trains.

With the introduction of *ETCS Level 3* (ETCS L3) the main principles change for the first time since the 19th century. The train itself reports the exact position and its integrity to the control system. TTDs are no longer needed to safely declare track segments as free. In theory, this allows movement in a so-called *moving block*, i.e., trains are cleared to drive all the way to the previous train (minus some safety buffer) without the necessity of any fixed blocks or correspondingly needed TTD systems.

Since the main principle of block signaling is not part of L3, it poses difficulties when implemented in practice [2]. Because of this, *ETCS Hybrid Level 3* (ETCS HL3) has been specified in [10] to overcome this issue, yet providing the advantages of Level 3 systems. Again the trains transmit their location and integrity to the control system, and TTDs are not necessary to clear sections. Hence, existing TTD sections can be divided into *Virtual Subsections* (VSS) without adding additional hardware. The principles of L2 remain the same, with the only exception that these new VSS are used instead of the old sections, which, again, allows shorter train following times than on low-level systems.

▶ **Example 1.** Consider two trains following one another on a straight section of track as shown in Fig. 1 for different ETCS Levels.

Fig. 1a shows how block-signaling works in ETCS Level 1. While the train on the right has already cleared TTD2, the following train has not yet received permission to enter TTD2 because the block was still occupied when the train last passed a balise. As soon as it hits the next balise (this time transmitting a green signal), the authority is updated so that the following train can enter TTD2.



(a) ETCS Level 1.

(b) ETCS Level 2.

(c) ETCS Level 3.

(d) ETCS Hybrid Level 3.

Figure 1 Schematic drawings of various ETCS levels.

This problem does not exist in ETCS Level 2 as the following train receives permission to enter TTD2 immediately after the train on the right has left that block in Fig. 1b.

In Fig. 1c, the flexibility of ETCS Level 3 (using moving block) is shown. The train on the left can follow the leading train as close as possible, only needing to keep the respective braking distance.

Fig. 1d shows a compromise between Fig. 1b and Fig. 1c by separating TTD2 into two virtual subsections. This allows the train on the left to follow more closely without requiring the installation of additional hardware.

2.2 Placing Virtual Subsections under ETCS HL3

In this work, we focus on the promising ETCS HL3, which allows for separating TTD sections into virtual subsections (VSS). Some TTD sections might not be separable into VSS, e.g., because of turnouts (where close section borders would not comply with flank protection) or constraints imposed by railway crossings or section breaks in overhead lines [17]; on others only a minimal VSS length is specified.

Since VSS do not require new trackside hardware, they can, in theory, be changed without changing the trackside hardware on a virtual level. Hence, adapting the block layout for a new schedule might, for the first time, be reasonable. Because of this, it is of great interest to consider precise timetables and block layouts jointly in the planning process.

Various design tasks arise within the abovementioned context (see also [32, 26, 11]). Since adding VSS to preexisting railway networks can increase their capacity, it is of interest to efficiently determine where to place them, also with respect to a new timetable, which might not be realizable on a given TTD layout. One wants to find a VSS layout under which the previously infeasible timetable can be accomplished. Or, one might want to tweak the timetable to reduce the travel time or headway to a minimum using a predefined number of VSS. Finally, one can also consider a mixed mode of passenger and freight services. In that case, a predefined schedule should be fulfilled while maximizing freight train throughput.

Exemplary, in this case study, we focus on generating layouts to realize predefined schedules. We are given (part) of a railway network under consideration together with a (macroscopic) timetable for various trains. This includes times when trains enter and leave the network and scheduled stops in between. The number of VSS sections that can be implemented in operation is limited by the efficiency of the used components. Hence, it is desirable to keep the number of VSS low. If the control system in operation can safely manage more sections, the remaining ones might be used to improve robustness. Overall this leads to the design task considered in this work: Given a railway network with TTD sections, a list of trains, and their respective (macroscopic) timetables, separate the TTD sections into a minimal number of VSS to make the timetable feasible and determine a respective (microscopic) routing/refinement.

3 Symbolic Formulation

In the literature, various models have been introduced which, in combination with corresponding reasoning engines or solvers, can be used to solve the design task described in Sec. 2.2 [32, 26]. In this work, we propose a method that allows for a trade-off between the accuracy and efficiency of such formulations. We start with a base formulation that contains only the most relevant details on an equivalent level to previous work (Sec. 3.1). Afterward, we describe how more realistic, continuous details like train dynamics (Sec. 3.2) and braking curves enforced by the train control system (Sec. 3.3) can be added to the base formulation. Finally, we consider how fixing the train routes a priori simplifies the described model (Sec. 3.4).

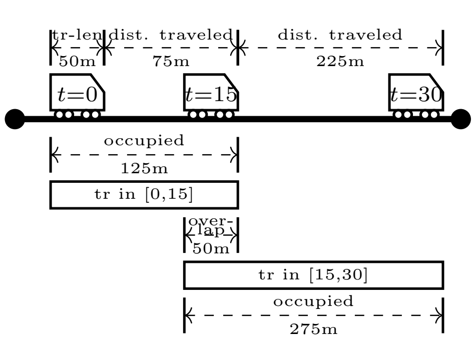
We keep the corresponding descriptions at a high level yet precise enough to allow for a discussion in the following sections. In particular, we omit constraints irrelevant to understanding the central ideas or whose logical form is easier to grasp the relevant concepts. They can easily be transformed into linear constraints using standard techniques (e.g., big-M). Readers interested in a more detailed treatment are referred to our open-source implementation available at `https://github.com/cda-tum/rail`, which includes some minor (yet efficient) additions to strengthen the relaxation.

3.1 Base Model

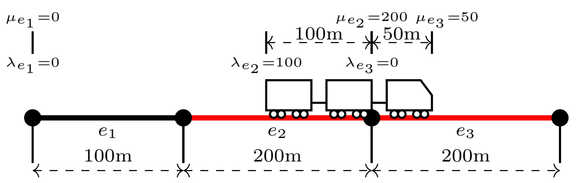
To state a MILP model, we first discretize the time horizon into intervals of a predefined length, say Δt. Train positions are then modeled over intervals instead of single time points. Assume that v_t and $v_{t+\Delta t}$ are the velocities of a train at the interval boundaries of $[t, t+\Delta t]$ and that the speed of the train within the interval can be determined by linearly interpolating between the initial and final velocity. Then, the traveled distance can be easily approximated as $\frac{v_t + v_{t+\Delta t}}{2} \cdot \Delta t$. In particular, during a given time interval, a train occupies not only the track corresponding to its length but also the track section it travels over. The intersection of occupied track sections at two adjacent intervals $[t-\Delta t, t]$ and $[t, t+\Delta t]$ leads to the exact position at time point t.

▶ **Example 2.** Consider Fig. 2a with a 50m-long train moving forward in time steps of 15 seconds. At $t = 0\text{s}$ the train stopped and is now accelerating to 10m/s at $t = 15\text{s}$ and 20m/s at $t = 30\text{s}$. By assuming that the velocity can be linearly interpolated (e.g., $v_{7.5\text{s}} = 5\text{m/s}$), the train moves 75m within $[0\text{s}, 15\text{s}]$ and 225m within $[15\text{s}, 30\text{s}]$. Hence, it occupies a total of $50\text{m} + 75\text{m} = 125\text{m}$ in the first interval and $50\text{m} + 225\text{m} = 275\text{m}$ in the latter. The overlap of the two occupations has a length of 50m, i.e., the same length as the train.

In the following paragraphs, we describe how this basic idea is implemented in the MILP model.



(a) Train's occupation.



(b) Train's position.

Figure 2 Modeling of continuous positions depending on routing choice.

Variables describing train positions. To model the abovementioned, we add the following variables to the model:

- $v_t^{tr} \in [0, v_{max}^{tr}]$: is the current speed of train tr (with maximal speed v_{max}^{tr}) at time t.
- $x_{t,e}^{tr} \in \{0,1\}$: indicates if train tr occupies edge e anytime within $[t, t+\Delta t]$.

A train does usually not occupy an entire edge but might only be present on parts of it. Thus, train positions cannot be modeled precisely using only binary (discrete) variables [2]. In a MILP setting, continuous variables can also be added. By doing so, we model the exact train position on an edge using variables for both the front and rear of the train:

- $\mu_{t,e}^{tr} \in [0, \text{len}(e)]$: front of tr on e measured from edge's start in interval $[t, t+\Delta t]$.
- $\lambda_{t,e}^{tr} \in [0, \text{len}(e)]$: rear of tr on e measured from edge's start in interval $[t, t+\Delta t]$.

The binary variables $x_{t,e}^{tr}$ indicate that a train is present on edge e. These can be inferred from $\mu_{t,e}^{tr}$ and $\lambda_{t,e}^{tr}$. If they are both equal to 0, the train is not present on the respective edge; otherwise, it is.

▶ **Example 3.** Consider the simple network shown in Fig. 2b with a 150m-long train located on edges e_2 and e_3. Both these edges are 200m long, but the train only occupies 100m and 50m, respectively. In this case, $\lambda_{e_2} = 100$m and $\mu_{e_2} = 200$m denote that the train occupies the second half of e_2. Similarly, $\lambda_{e_3} = 0$m and $\mu_{e_3} = 50$m. Because the train is not present anywhere on e_1, we have $\lambda_{e_1} = \mu_{e_1} = 0$m. This implies that $x_{e_2} = x_{e_3} = 1$, i.e. the train occupies edge e_2 and e_3. On the other hand, $x_{e_1} = 0$, i.e., the train does not occupy e_1.

Occupation and overlap. The length of the track section a train occupies during one timestep can be obtained by summing over the $(\mu_e - \lambda_e)$-differences over every edge e the train occupies. The overlap length is obtained by taking the difference of these lengths for two adjacent time steps (given that the train is present on the edge). Symbolically, these constraints are encoded in the MILP formulation as follows:

$$\sum_{e \in E} \left(\mu_{t,e}^{tr} - \lambda_{t,e}^{tr}\right) = \text{len}(tr) + \frac{v_t^{tr} + v_{t+\Delta t}^{tr}}{2} \cdot \Delta t \quad \forall t, tr \tag{1}$$

$$\sum_{e \in E} x_{t+\Delta t,e}^{tr} \left(\mu_{t,e}^{tr} - \lambda_{t+\Delta t,e}^{tr}\right) = \text{len}(tr) \quad \forall t, tr. \tag{2}$$

[2] This is already a departure point from previous symbolic approaches that encoded positions as binary variables indicating whether a train occupies an edge.

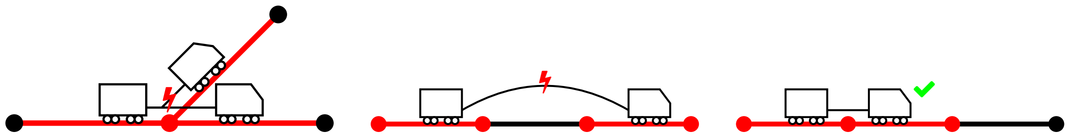

(a) Vertex degree too large (b) Train not connected (c) Valid train position

Figure 3 Ensuring trains position is valid.

Train integrity. The formulation above does not guarantee valid train positions without further constraints. For example, the situations depicted in Fig. 3a and Fig. 3b would be technically correct, as the length constraints are not violated.

Let $G = (V, E)$ be the graph representing a railway network, and for any vertex $v \in V$, let $\delta(v)$ denote the set of incident edges of v. Then the situation depicted in Fig. 3a can be avoided by imposing

$$\sum_{e \in \delta(v)} x_{t,e}^{tr} \leq 2 \quad \forall v \in V, \tag{3}$$

i.e., train tr can occupy at most two adjacent edges to any vertex during any given time t.

The situation in Fig. 3b can be avoided by utilizing the following observation: for any cycle-free subgraph $G' = (V', E')$ of a railway network, being connected is equivalent to $|E'| = |V'| - 1$. Since the edges a train occupies during one timestep form a subgraph of G, we can encode this constraint on the cardinalities of E' and V' as follows:

$$\sum_{e \in E} x_{t,e}^{tr} = \sum_{v \in V} \left(\bigvee_{e \in \delta(v)} x_{t,e}^{tr} \right) - 1 \quad \forall t, tr, \tag{4}$$

for all trains tr, times t. Assuming all cycles within the railway network are sufficiently large (i.e., longer than the train's length plus maximal possible braking distance), the subgraph induced by the edges the train occupies in one timestep is cycle-free by design.

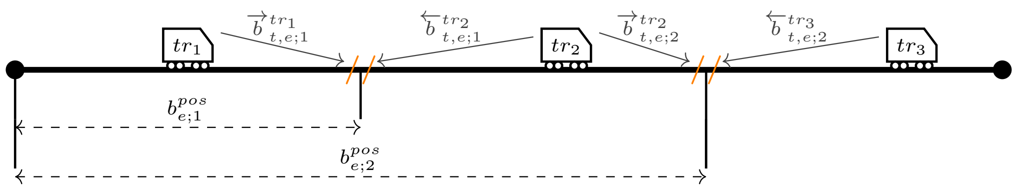
Note that more constraints are required to ensure realistic modeling of train movements (e.g., trains can only move in the direction of the engine). For brevity's sake, we omit these here.

Speed limit. While every train's maximal speed is directly included in the variable bounds, there might be a more restrictive speed limit on some railway tracks. Hence the following logical constraints need to be imposed:

$$x_{t,e}^{tr} = 1 \Rightarrow v_t^{tr} \leq v_{max}^e \text{ and } v_{t+\Delta t}^{tr} \leq v_{max}^e \quad \forall t, e, tr. \tag{5}$$

Timetable. Trains are affiliated with a train schedule. In our setting, a schedule essentially defines if a train needs to stop at stations (i.e., a subset of edges of the railway network) at certain times. This includes the possibility to include multiple parallel tracks (corresponding to different platforms). Assume that tr has to stop in station $S_i^{tr} \subset E$ during the time interval $[\underline{t}_i^{tr}, \overline{t}_i^{tr}]$. Obviously, this means that the train must have a speed of 0m/s during that time interval which is encoded by the constraint

$$v_t^{tr} = 0 \quad \forall t \in [\underline{t}_i^{tr}, \overline{t}_i^{tr}]. \tag{6}$$



tr_1
$\overrightarrow{b}^{tr_1}_{t,e;1}$
$\overleftarrow{b}^{tr_2}_{t,e;1}$
tr_2
$\overrightarrow{b}^{tr_2}_{t,e;2}$
$\overleftarrow{b}^{tr_3}_{t,e;2}$
tr_3
$b^{pos}_{e;1}$
$b^{pos}_{e;2}$

Figure 4 Including VSS constraints in the model.

Moreover, the train must be fully positioned within the station and cannot occupy any track outside the station, hence, we have

$$x^{tr}_{t,e} = 0 \quad \forall t \in [\underline{t}^{tr}_i, \overline{t}^{tr}_i],\, e \in E - S^{tr}_i \qquad \text{and} \qquad \sum_{e \in S^{tr}_i} x^{tr}_{t,e} \geq 1 \quad \forall t \in [\underline{t}^{tr}_i, \overline{t}^{tr}_i]. \tag{7}$$

VSS condition. Finally, we model constraints imposed by an ETCS HL3 control system. In previous work, VSS boundaries were solely modeled by binary variables on vertices. In this paper, we model VSS boundaries as continuous variables instead. Assume that we allow I_e VSS boundaries to be set on an edge e, introduce variables $b^{pos}_{e;i} \in [0, \text{len}(e)]$ for $0 \leq i < I_e$ denoting the positions of the respective VSS boundaries or 0 if they are not used. Hence, we can already formulate the objective of generating VSS layouts (using some small $\varepsilon > 0$, e.g., the minimal VSS block length) on a logic level as

$$\min \sum_{e \in E} \sum_{i \in I_e} [b^{pos}_{e;i} \geq \varepsilon] \qquad ([\cdot] \text{ denotes the Iverson bracket}). \tag{8}$$

Now, assume that n trains are present on one edge e at time t. Then they have to be separated by $n-1$ VSS boundaries. Put differently, there must be $n-1$ VSS boundaries that separate some trains' front from some other trains' rear. Let this be indicated by binary variables $\overrightarrow{b}^{tr}_{t,e;i}$ and $\overleftarrow{b}^{tr}_{t,e;i}$ respectively, then

$$\overrightarrow{b}^{tr}_{t,e;i} = 1 \Rightarrow \mu^t_{t,e} \leq b^{pos}_{e;i} \quad \text{and} \quad \overleftarrow{b}^{tr}_{t,e;i} = 1 \Rightarrow \lambda^t_{t,e} \geq b^{pos}_{e;i} \qquad \forall t, tr, e, i. \tag{9}$$

Finally, it has to be ensured that the correct number of $\overrightarrow{b}^{tr}_{e;i}$ and $\overleftarrow{b}^{tr}_{e;i}$ is 1, so that $n-1$ VSS boundaries are chosen.

▶ **Example 4.** Consider Fig. 4 with three trains on an edge. The trains are separated by two VSS boundaries, whose positions on the edge are given by the continuous variables $b^{pos}_{e;1}$ and $b^{pos}_{e;2}$. The first VSS boundary separates tr_1's front from tr_2's rear ($\overrightarrow{b}^{tr_1}_{t,e;1} = \overleftarrow{b}^{tr_2}_{t,e;1} = 1$). Similarly, the second VSS boundary separates tr_2 from tr_3 ($\overrightarrow{b}^{tr_2}_{t,e;2} = \overleftarrow{b}^{tr_3}_{t,e;2} = 1$). All other binary indicators are 0 in this case.

The formulation described here allows for modeling the problem of VSS layout generation in comparable accuracy as previous work [32, 26]. However, in the following sections, we show how MILP can be used for modeling additional aspects that are infeasible or hard to encode in previous formulations due to their discrete nature.

3.2 Train Dynamics

In the base MILP model, train movements are not constrained by realistic dynamics like acceleration and deceleration. For a more realistic model, we also need to encode these properties. In fact, given maximal accelerations a_{tr} and decelerations d_{tr}, these dynamics can be added to the base MILP model with few constraints:

$$v^{tr}_{t+\Delta t} \leq v^{tr}_t + \Delta t \cdot a_{tr} \quad \forall t, tr \qquad \text{and} \qquad v^{tr}_{t+\Delta t} \geq v^{tr}_t - \Delta t \cdot d_{tr} \quad \forall t, tr. \tag{10}$$

3.3 Braking Curves

As described in Sec. 2, train control systems (such as ETCS) ensure that the complete track section a train needs to come to a full stop is not occupied by any other train using braking curves. The base model has no restrictions on safety distances between trains.

In time interval $[t, t + \Delta t]$, the final velocity is given by $v_{t+\Delta t}^{tr}$ and the braking distance of tr can be approximated using

$$brakelen_t^{tr} = \frac{1}{2 \cdot d_{tr}} \cdot \left(v_{t+\Delta t}^{tr}\right)^2 \quad \forall t, tr. \tag{11}$$

This is not linear (in the variable $v_{t+\Delta t}^{tr}$ to be precise), which poses problems with the inclusion in MILPs. Since it is an equality constraint, the resulting feasible region is not even convex. However, some solvers can even solve these constraints within a mixed integer program to optimality by using spatial branching [1, 24]. The approximation is then included locally in the respective branched subproblems. Hence, it is possible to model the braking distance directly.

Alternatively, one can add Eq. (11) as a piecewise linear approximation globally to the problem formulation itself. Under the hood, this will add binary variables and linear constraints. In particular, the integer model remains linear and can be solved with any MILP-solver.

In either case, we add the braking distance to the length of the track section a train occupies by slightly adapting Eq. (1) and (2):

$$\sum_{e \in E} \left(\mu_{t,e}^{tr} - \lambda_{t,e}^{tr}\right) = \text{len}(tr) + \frac{v_t^{tr} + v_{t+\Delta t}^{tr}}{2} \cdot \Delta t + brakelen_{t+\Delta t}^{tr} \quad \forall t, tr. \tag{12}$$

$$\sum_{e \in E} x_{t+\Delta t,e}^{tr} \left(\mu_{t,e}^{tr} - \lambda_{t+\Delta t,e}^{tr}\right) = \text{len}(tr) + brakelen_t^{tr} \quad \forall t, tr.. \tag{13}$$

In some sense, we let the train length dynamically change throughout time depending on the speed. Since the train lengths attribute to the occupied track sections, no further constraints need to be added, as the base model already restricts these.

3.4 Fixed Routes

When designing a train schedule, a train's route (i.e., the exact tracks it uses) is sometimes already known a priori. If not, it might be possible to fix routes separately before placing VSS sections. In this case, the model significantly simplifies. If a train's route is known a priori, it is not necessary to model the exact location on every edge but can rather be modeled by single integer variables:

- $\mu_t^{tr} \in [0, \text{len(route)}]$: front of the train in interval $[t, t + \Delta t]$
- $\lambda_t^{tr} \in [0, \text{len(route)}]$: rear of the train in interval $[t, t + \Delta t]$

Since the position of the edges within a route is known, the corresponding indicator variables follow quickly. Variables corresponding to the exact position on every edge can even be removed entirely.

Additionally, Eq. (1) and (2) simplify to

$$\mu_t^{tr} - \lambda_t^{tr} = \text{len}(tr) + \frac{v_t^{tr} + v_{t+\Delta t}^{tr}}{2} \cdot \Delta t \quad \forall t, tr \tag{14}$$

$$\mu_t^{tr} - \lambda_{t+\Delta t}^{tr} = \text{len}(tr) \quad \forall t, tr. \tag{15}$$

Braking curves can be included analog to Eq. (12) and (13).

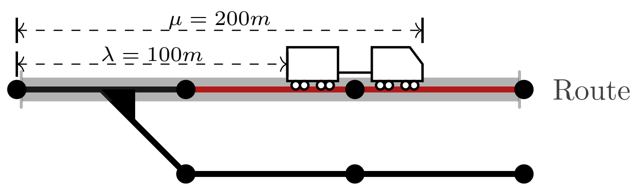


Figure 5 Modeling continuous train position on fixed routes.

▶ **Example 5.** Consider the network in Fig. 5 with a 100m-long train. While the train could potentially choose different routes, in theory, it is fixed to take the top track. Hence, $\lambda = 100m$ and $\mu = 200m$ uniquely define the train's position measured from the route's starting point.

4 Case Study

The symbolic formulation in its different modeling details presented in the previous section has been implemented in C++ and made publicly available as open-source implementation at `https://github.com/cda-tum/rail`. By that, a tool got available which allows the user to include/exclude certain constraints and, by that, decide whether he/she wants to quickly generate a less accurate solution or a more accurate solution at the expense of higher runtimes. This section now summarizes the results of a case study showcasing the potential of such an approach. To this end, we first outline the setup of the case study. Afterward, we summarize as well as discuss the correspondingly obtained results.

4.1 Setup

The basis of the case study was provided by the tool mentioned above (again, available at `https://github.com/cda-tum/rail`) based on the symbolic formulations presented in Sec. 3. This tool's initialization requires defining a temporal discretization, i.e., a value for the time interval length Δt. Choosing the correct value of Δt compromises computational time and accuracy. Within the national railway company of Germany, Deutsche Bahn AG (DB), Δt is usually chosen to be 15 seconds (cf. [22]). While optimizing Δt when using the presented model is potentially interesting, this is out of the scope of this paper. Hence, we follow DB's default and run all experiments with a temporal resolution of $\Delta t = 15s$.

As a computing device, we utilized an AMD Ryzen Threadripper PRO 5955WX system using a 4.0-4.5 GHz CPU (16 cores) and 128GB RAM running on Ubuntu 20.04. We use the C++ API of Gurobi version 10.0.1 [16] to solve the considered instances. A 1h hard timeout for solving each instance has been set.

We considered a range of sample train networks as benchmarks, including typical (simple) test cases (such as single tracks or stations) and real-world examples. More precisely:

- *Single track* considers only one line on which trains have to slow down towards the end. To enable the possibility of multiple trains following each other VSS have to be placed. We also consider a variant with a scheduled stop (i.e., station) in between.
- *Highspeed track* is a variant of the example above but considering a larger distance and faster trains. We consider variants with 2 and 5 trains following each other, respectively.
- *Simple 2-track station* is a basic station with two tracks that trains can approach from different directions.

- *Simple network* consists of trains that have to trespass a single track in opposite directions. For this, there is a bypass in the middle where trains do not have a scheduled stop.
- *Overtake* models the situation of faster trains overtaking slower trains at a bypass.
- *Stammstrecke* is a real-world inspired instance. For this, we model the Munich S-Bahn Stammstrecke between Pasing and Munich East using publicly available data on tracks [7], timetable [8], and technical data of the trains (DB BR 423) [20]. We reduced the train following times slightly to enforce the necessity of VSS.

Figures of the respective track plans and more details on the used timetables are provided in Appendix A. Furthermore, all benchmarks are available through the open-source implementation at `https://github.com/cda-tum/rail`.

For each benchmark, the design task reviewed in Sec. 2.2 has been solved using three different degrees of detail: The base model (Sec. 3.1), this model extended by considering train dynamics (Sec. 3.2), and this model further extended by considering braking curves directly without approximation (Sec. 3.3). Additionally, we considered two cases, one in which the routes had to be determined by the tool and another in which the routes had been fixed beforehand (Sec. 3.4).

The number of placed VSS (*#VSS*) is optimized for every benchmark instance. To assess the model's efficiency, we measured the runtime (t, in CPU seconds) for creating and solving each instance. Since the solver guarantees that the optimum number of VSS will be found (within the bounds of the model's detail), another criterion is needed to assess model accuracy. The model, including train dynamics and braking curves, is the most detailed and, thus, is, by definition, the most accurate. The less detailed models can generate solutions that, while valid within the level of detail of the formulation, cannot be mapped to reality directly. Specifically, taking the routes and VSS placements generated by less detailed models and checking them with the highest detail, it is often revealed that the routes cannot be run on the computed layout precisely as they were computed because block signaling constraints might be violated. When this safety constraint is violated, a train must halt and wait before getting the move authority. This leads to a delay (*Delay*) between the given schedule and the schedule that is possible on the computed layout in reality. We use this delay as a criterion to assess model accuracy in the following. By definition, the model including train dynamics and braking curves does not incur a delay (i.e., it is the most accurate of the considered model and serves as ground truth).

4.2 Results

The results of the conducted case study are summarized in Tab. 1. Tab. 1a provides results for instances in which the routes have *not* been fixed and, hence, have to be determined by the design method. Tab. 1b provide results for instances in which the routes have been fixed. The first columns denote the considered benchmarks as described above, while the following columns provide the number of VSS as well as the delay of the obtained solution and the runtime for each of the considered settings.

From the results, several interesting conclusions can be drawn. First, and most obviously, fixing the routes has a severe impact on the efficiency of the solution (comparing Tab. 1a and Tab. 1b). Fixing them beforehand reduces the search space substantially and, hence, yields substantially better runtimes. In the case of *Simple Network* it even allows one to complete the design task within the given time limit. At the same time, this sometimes is achieved at the expense of quality (as seen by the fact that some results obtained while considering fixed routes have a worse delay). This can easily be explained by the fact that having routes not fixed allows for a higher degree of freedom to find better solutions (causing the higher

Table 1 Experimental results.

(a) With routing included.

		Base Model			+ Train Dynamics			+ Braking Curves	
		t [s]	#VSS	Delay [s]	t [s]	#VSS	Delay [s]	t [s]	#VSS
Single Track	Without Station	53.8	7	30	53.5	8	30	290	9
	With Station	33.6	3	75	27.0	4	30	237	4
Highspeed Track	2 Trains	16.0	10	45	14.6	10	30	126	18
	5 Trains	443	10	60	604	10	30	1757	18
Simple 2-Track Station		6.5	1	15	9.2	1	0	8.5	1
Simple Network		>1h	n/a	n/a	>1h	n/a	n/a	>1h	n/a
Overtake		12.4	8	45	15.8	8	45	14.0	14
Stammstrecke	4 Trains	6.3	0	30	5.5	6	15	11.0	6
	8 Trains	17.8	0	60	15.0	14	30	68.1	14
	16 Trains	77.5	0	90	55.7	15	45	83.4	15

(b) With fixed routes.

		Base Model			+ Train Dynamics			+ Braking Curves	
		t [s]	#VSS	Delay [s]	t [s]	#VSS	Delay [s]	t [s]	#VSS
Single Track	Without Station	42.0	7	45	62.5	8	30	138	9
	With Station	15.7	3	60	15.7	4	45	38.3	4
Highspeed Track	2 Trains	19.0	10	45	16.0	10	30	83.8	18
	5 Trains	536	10	45	504	10	30	1610	18
Simple 2-Track Station		0.7	1	15	0.7	1	0	1.6	1
Simple Network		64.6	5	60	41.0	5	60	1433	6
Overtake		1.4	8	45	1.5	8	45	2.7	14
Stammstrecke	4 Trains	1.2	0	45	1.1	6	15	1.3	6
	8 Trains	2.9	0	60	2.7	14	30	3.6	14
	16 Trains	7.7	0	105	6.9	15	45	13.7	15

runtime but may yield slightly better results). However, since the best possible routes are always rather apparent in the considered benchmark, this effect only marginally affects the quality of the results.

Besides that, the symbolic formulation's level of detail obviously affects the accuracy of the results and the efficiency of the solving process. Here, one would expect that the base model (i.e., the most simplistic/abstract) model has the lowest accuracy but the best efficiency, while it is vice versa for the more detailed formulations. While this is generally true, interesting insights can be unveiled when checking out the numbers in detail. In fact, additionally considering train dynamics in the base model hardly degrades the efficiency (i.e., runtime) of the solving process (it even gets better sometimes). At the same time, the solution accuracy either improves or remains equivalent. Hence, when choosing between the base model and the base model plus train dynamics, the latter is the better option as it provides almost the same accuracy while being more efficient. This behavior can be explained by the fact that including train dynamics reduces the number of feasible train movements – and, thus, the search space – without adding too much complexity.

More complex is the trade-off when additionally considering braking curves. This substantially affects the efficiency and leads to increased runtimes which are factors (sometimes even magnitudes) higher than for the other models. At the same time, this provides the best accuracy of all models considered in this work. This clearly shows the potential offered by the proposed approach: The user can decide whether he/she wants a quick but less accurate solution; or whether he/she is willing to "invest" larger computation times to get more accurate solutions. The tool presented in this work allows the end user to do both.

5 Conclusions & Outlook

In this work, we considered symbolic formulations for automatically determining ETCS HL3 layouts. While previous work relied on discrete formulations, we explicitly proposed continuous formulations that are, e.g., essential to model concepts such as train dynamics or braking curves properly. To this end, we introduced a *Mixed Integer Linear Programming* (MILP) formulation. The resulting approach is flexible and allows users to explicitly include/exclude certain constraints or simplify the model as needed. By that, he/she can decide whether a fast but less accurate solution or a slow but more accurate solution should be generated.

A case study showcased the corresponding trade-off between accuracy and efficiency. While these confirmed some obvious expectations (the less precise the model, the more efficient the solving process and vice versa), some rather counter-intuitive insights are also unveiled. For example, additionally considering train dynamics makes the model more precise but hardly degrades (and sometimes even improves) the efficiency (i.e., runtime) of the solving process. In contrast, considering braking curves substantially affects the efficiency, i.e., leads to increased runtimes which are factors (sometimes even magnitudes) higher than for other models.

The proposed methods (whose implementations are also publicly available in open-source at `https://github.com/cda-tum/rail` as part of the *Munich Train Control Toolkit*, MTCT) allow the users to evaluate such trade-offs. Furthermore, the resulting methods/tool has been implemented in a modular and flexible fashion – allowing for easy integration of further aspects in the future, such as optimizing train schedules or maximizing the throughput of additional freight trains. Those developments are left for future work. Overall, the presented work provides a solid basis for the design of ETCS HL3 layouts at different degrees of accuracy.

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A Benchmarks

In the following we present the track layouts (all referenced figures can be found at the end of the appendix) used for the case study together with brief descriptions on the timetable. For detailed information on timetables, we refer to the sample instances provided through our open-source implementation.

Single Track

Single Track Without Station is a single track that is $15km$ long with no TTD. Two trains follow each other with a 1 minute headway.

The track layout of *Single Track With Station* is shown in Fig. 6. Three trains travel from A to B with 90 seconds separation. The first two trains have a scheduled stop at S_1.

Highspeed Track

Highspeed Track is a single track that is $50km$ long with no TTD. Two to five trains are following each other with a 1 to 2 minute headway while slowing down towards the end.

Simple 2-Track Station

The track layout is given in Fig. 7. Two trains are moving from left two right, one (longer) train from right to left. All three trains have a scheduled stop in S_1 at more or less the same time.

Simple Network

The track layout is given in Fig. 8. Two slow trains move from left to right and right to left respectively with two scheduled stops. Faster trains follow on the main line without stopping.

Overtake

The track layout is given in Fig. 9. Three trains enter at A and leave at B in the reverse order, hence, have to overtake each other. Additionally, the first train has a scheduled stop in S_1.

Stammstrecke

We model the Munich S-Bahn Stammstrecke between Pasing and Munich East using publicly available data on tracks [7]. The track layout is shown in Fig. 10. S-Bahn trains in Munich are mainly of type *DB BR 423* with technical data described in [20]. The timetable is inspired by [8] and consists of trains entering at Pasing, Laim or Donnersbergerbrücke traveling to Munich East and vice versa in the opposite direction. Depending on the instance, 2, 4, and 8 trains (per direction) were considered following each other approximately every 90 seconds.

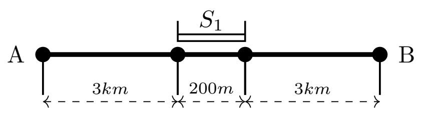

Figure 6 Tracks of *Single Track With Station.*

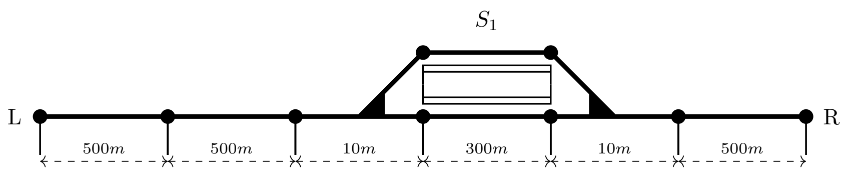

Figure 7 Tracks of *Simple 2-track station.*

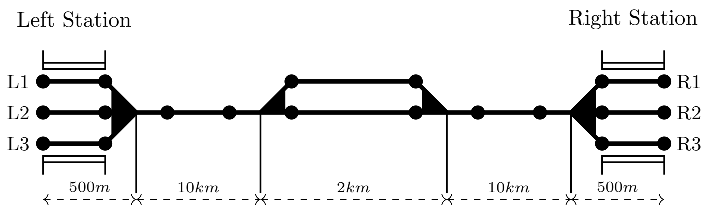

Figure 8 Simple Network.

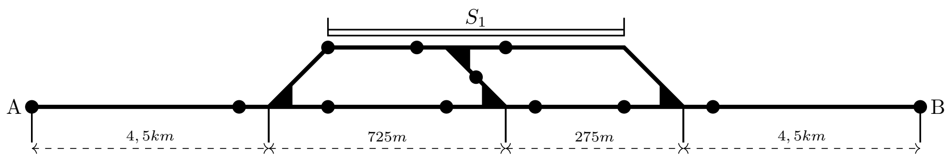

Figure 9 Overtake.

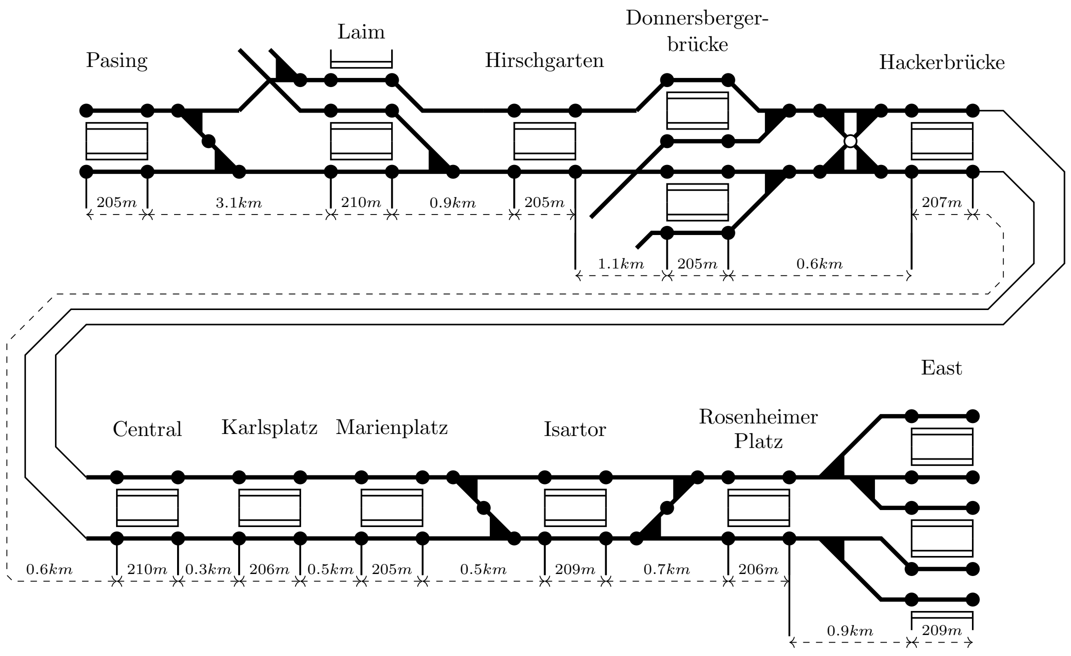

Figure 10 Stammstrecke (Munich S-Bahn).