Influence of Design Parameters on Cogging Torque in Permanent Magnet Machines

Z. Q. Zhu, *Member, IEEE* and David Howe

***Abstract*—The influence of various design parameters on the cogging torque developed by permanent magnet machines is investigated. It is shown that the slot and pole number combination has a significant effect on the cogging torque, and influences the optimal value of both skew angle and magnet arc, as well as determining the optimal number of auxiliary teeth/slots. A simple factor, which is proportional to the slot number and the pole number and inversely proportional to their smallest common multiple, has been introduced to indicate the "goodness" of the slot and pole number combination. In general, the higher the "goodness" factor the larger the cogging torque.**

***Index Terms*—Cogging torque, electrical machines, machine design, permanent magnet machines, speed ripple, torque, torque ripple.**

I. INTRODUCTION

COGGING torque results from the interaction of permanent magnet mmf harmonics and the airgap permeance harmonics due to slotting. It manifests itself by the tendency of a rotor to align in a number of stable positions even when the machine is unexcited, and results in a pulsating torque, which does not contribute to the net effective torque. However, since it can cause speed ripples and induce vibrations, particularly at light load and low speed, its reduction is usually a major design goal [1]–[4].

In the paper, the effect of the slot and pole number combination on the cogging torque is investigated, and its relationship with various other design parameters, such as the width of the stator slot openings, the magnet arc and the skew angle, as well as with design features such as auxiliary teeth and slots is considered, with respect to machines in which the magnets are mounted adjacent to the airgap.

II. ANALYSIS TECHNIQUES

Cogging torque is produced predominantly as a result of fringing fields in the magnet interpole and slot regions [5], a typical cogging torque waveform being shown in Fig. 1. It can be shown that in a motor having full pole-pitched magnets the instantaneous cogging torque is zero when a) the interpole axes align with the centers of teeth, and b) the interpole axes align with the centers of slots. However, since a permanent magnet rotor would tend to rotate to a position of maximum stored energy, case a) corresponds to a stable equilibrium position, since the leakage flux paths between the edges of north and south poles have minimum length, effectively only crossing the airgap, while case b) corresponds to an unstable equilibrium position, since the leakage flux paths include the slot openings. The positive and negative peaks of the cogging torque occur approximately when the interpole axes align with the edges of the slots.

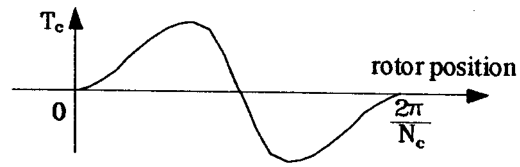

Fig. 1. Typical cogging torque waveform.

The electromagnetic torque can be calculated analytically or numerically in a variety of ways, such as by the Maxwell Stress and co-energy methods. However, they require very accurate global and local field solutions [5]–[7], particularly for the determination of cogging torque. In other words, a high level of mesh discretization is required in a finite element calculation, whilst a reliable physical model is essential to an analytical prediction. The authors have employed a variety of analytical techniques to predict the cogging torque in permanent magnet machine topologies in which the magnets are mounted adjacent to the airgap [8]. Most recently, they extended an analytical model to solve for the magnetic field distribution in the combined magnet/airgap/slot regions, albeit with the assumption of rectangular shaped slots. It provided a very reliable analysis tool for predicting the cogging torque, and underpins the investigation described in this paper. In general, it is capable of quantifying the effects of the following design parameters:

a) slot number and pole number combination, including auxiliary teeth and slots (optional);
b) slot opening width, airgap length, and magnet thickness;
c) magnet pole-arc to pole-pitch ratio;
d) magnetization distribution, which may range from regular to trapezoidal;
e) skewing of slots and/or magnets, and the stepped equivalent;
f) disposition of magnets.

Throughout, the calculations are for an internal rotor machine in which the radii of the stator bore, the rotor, and the rotor hub are 73.27 mm, 71.97, and 62.87 mm, respectively, the axial length is 95 mm and the stator slot openings are 3.8 mm. The magnets are bonded NdFeB with a remanence of 0.56 T. Typical analytically predicted, finite element calculated, and measured cogging torque waveforms are shown in Fig. 2.

Manuscript received November 9, 1997.

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Publisher Item Identifier S 0885-8969(00)11017-4.

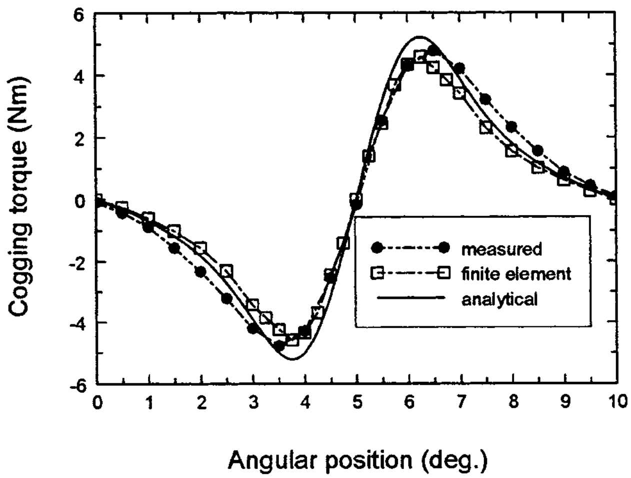

Fig. 2. Comparison of analytically/finite element predicted and measured cogging torque.

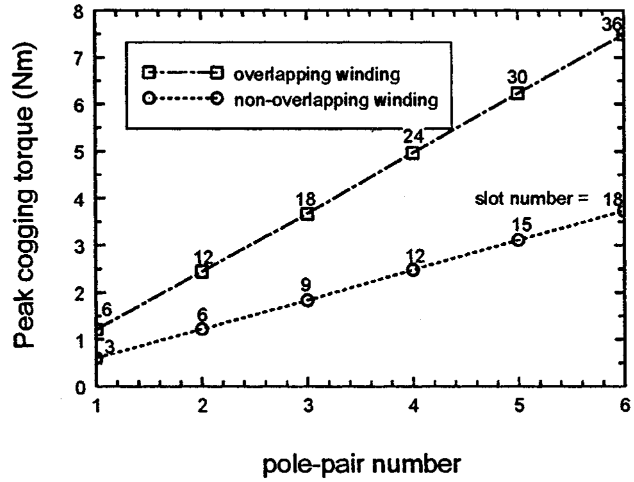

Fig. 3. Variation of cogging torque with slot and pole number combination.

III. Choice of Slot and Pole Number Combination

The cogging torque can be expressed in the general form:

$$T_c = \sum_{i=1,2,3,\cdots}^{\infty} K_{sk} T_i \sin i N_c \theta \tag{1}$$

where the fundamental order of the waveform, N_c, is the smallest common multiple between the slot number Q_s and the pole number $2p$; θ is the mechanical angle between the stator and the rotor, and K_{sk} is the skew factor given by:

$$K_{sk} = \frac{\sin(iN_c \pi \alpha_{sk}/Q_s)}{iN_c \pi \alpha_{sk}/Q_s} \tag{2}$$

where α_{sk} is the ratio of the total circumferential skew to the slot pitch.

In general, the larger the smallest common multiple N_c and the smaller the number of slots or poles, then the smaller will be the amplitude of the cogging torque. However, in order to aid the selection of Q_S and $2p$, the factor C_T is introduced to denote the "goodness" of slot and pole number combinations from the point of view of cogging torque, where:

$$C_T = \frac{2pQ_s}{N_c} \tag{3}$$

Although there is no formal basis for relating C_T to the amplitude of the cogging torque, it has been found that the larger the factor C_T the larger will be the cogging torque.

In order to demonstrate the influence of the slot and pole number combination on the cogging torque, it is assumed that the magnets have a pole-arc of 180° elec. and are fully radially magnetized throughout their volume. Fig. 3 shows the variation of the peak cogging torque for typical slot and pole number combinations which are used in brushless DC machines with overlapping and nonoverlapping stator windings, Fig. 4, for which the usual $Q_s/2p$ combinations are 3/2 and 6/2, respectively. However, other common combinations can be considered as simply being scaled by an integer factor k, as shown in Table I. It is obvious, from both Fig. 3 and Table I, that, due to the periodicity of the field, the cogging torque is proportional to the factor k, and that the level of cogging torque in motors with nonoverlapping stator windings (for which typical values of $Q_s/2p$ are 3/2, 6/4, 9/6 etc.) is usually about half of that in a similar motor having overlapping windings (for which typical values of $Q_s/2p$ are 6/2, 12/4, 18/6, etc.). This is due to the fact that when half of the edges of the north and south pole magnets in motors equipped with nonoverlapping windings face slot openings, the other edges face the centers of teeth (approximately), whereas

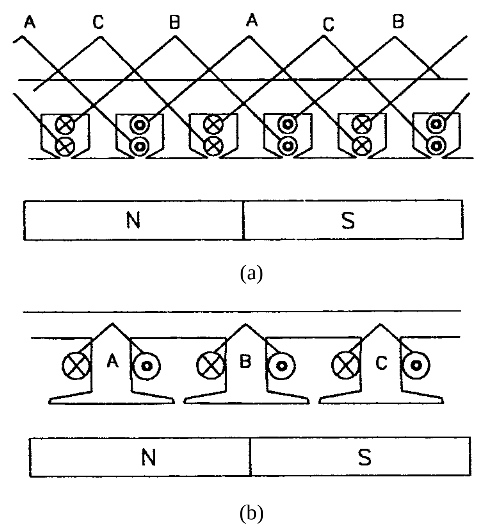

Fig. 4. Typical stator winding topologies in brushless permanent magnet motors. (a) Overflapping concentrated winding. (b) Nonoverlapping concentrated winding.

TABLE I
FACTOR C_T FOR TYPICAL BRUSHLESS PM DC MOTORS

non-overlapping winding	k	1	2	3	4	5	6
	$k\,(Q_s/2p)$	3/2	6/4	9/6	12/8	15/10	18/12
	N_c	6	12	18	24	30	36
	C_T	1	2	3	4	5	6
overlapping winding	k	1	2	3	4	5	6
	$k\,(Q_s/2p)$	6/2	12/4	18/6	24/8	30/10	36/12
	N_c	6	12	18	24	30	36
	C_T	2	4	6	8	10	12

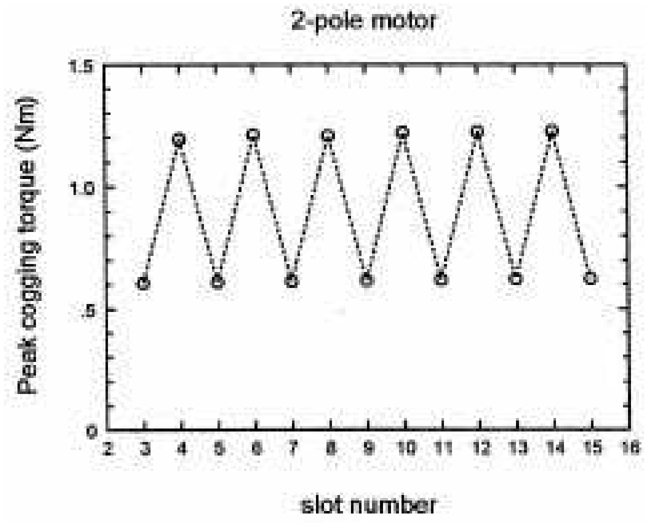

Fig. 5. Variation of peak cogging torque with slot number in 2-pole motor.

TABLE II
FACTOR C_T FOR TYPICAL 2-POLE BRUSHED PM MOTORS

Q_s	3	4	5	6	7	8	9	10	11	12	13	14	15
N_c	6	4	10	6	14	8	18	10	22	12	26	14	30
C_T	1	2	1	2	1	2	1	2	1	2	1	2	1

with overlapping windings all edges of magnets would face slot openings.

Fig. 5 shows the variation of the peak cogging torque with armature slot number for a 2-pole brushed permanent magnet motor, Table II shows the variation of the factor C_T with the number of slots. It will be noted that all motors with an odd number of slots exhibit essentially the same amplitude of cogging torque. Similarly, for motors having an even number of slots, although the cogging torque is then about twice the amplitude. Again, the reason for this is obvious, in that the edges of the north and south pole magnets in motors having an even slot number all have the same relative position with respect to the teeth, which is not the case for motors having an odd slot number. Thus, an odd number of slots is preferable for minimizing the cogging torque.

IV. USE OF AUXILIARY TEETH AND SLOTS

A knowledge of the influence of the slot and pole number combination makes it possible to reduce the cogging torque by introducing auxiliary teeth and/or slots [9], Figs. 6 and 7. However, the total number of slots should always be chosen so as to reduce the value of C_T.

V. OPTIMAL MAGNET POLE-ARC

The magnet arc is a particularly important parameter in regard to the level of cogging torque, and it has been found that, when magnet fringing is neglected, the optimum ratio of pole-arc to pole-pitch, α_p, for minimizing the fundamental component of

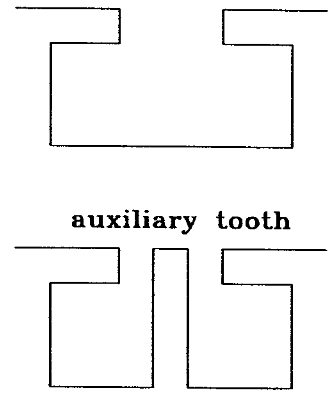

Fig. 6. Introduction of auxiliary teeth.

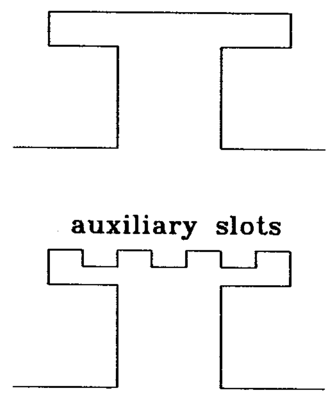

Fig. 7. Introduction of auxiliary slots.

cogging torque, for any combination of slot and pole number, is:

$$\alpha_p = \frac{N - k_1}{N}, \qquad k_1 = 0, 1, 2, \cdots, N \tag{4}$$

where $N = N_c/2p$. In practice, however, due to fringing of the magnet flux into the slots, the optimum value of α_p should be increased slightly by a small factor k_2, i.e.,

$$\alpha_p = \frac{N - k_1}{N} + k_2, \qquad k_1 = 1, 2, \cdots, N - 1 \tag{5}$$

where k_2 typically ranges from 0.01 to 0.03 depending on the airgap length, and k_1 is re-defined as $k_1 = 1, 2, \cdots, N - 1$, since $k_1 = N$ is unrealistic, while $k_1 = 0$, i.e. the magnets have a full pole-arc, is no longer an optimum for minimum cogging torque. Clearly, in order to maximize the airgap flux, and thereby the excitation torque, the optimal ratio of pole-arc to pole-pitch should be as high as possible. Hence, in practice $k_1 = 1$, i.e. $\alpha_p = (N-1)/N + k_2$, is usually the preferred value. Equation (5) shows that the optimal ratio of magnet pole-arc to pole-pitch depends on the slot and pole number combination. For example, for the most widely used combinations $Q_s/2p =$ 9/8, 6/4, 12/4 for 3-phase brushless permanent magnet motors the possible optimal ratios of pole-arc to pole-pitch are given in Table III, while the corresponding variations of the amplitude of the cogging torque with the pole-arc to pole-pitch ratio are shown in Fig. 8, assuming that the other motor design parameters for the different slot and pole number combinations

TABLE III
OPTIMAL RATIOS OF POLE-ARC TO POLE-PITCH ($k_2 = 0$)

magnet pole & stator slot number $Q_s/2p$	9/8	6/4	12/4
smallest common multiple, N_c, between $Q_s/2p$	72	12	12
fundamental order of cogging torque waveform (N_c)	72	12	12
C_T	1	2	4
k_1	$\alpha_p = (N - k_1)/N$		
1	0.889	0.667	0.667
2	0.778	0.333	0.333
3	0.667		
4	0.556		
5	0.444		
6	0.333		
7	0.222		
8	0.111		

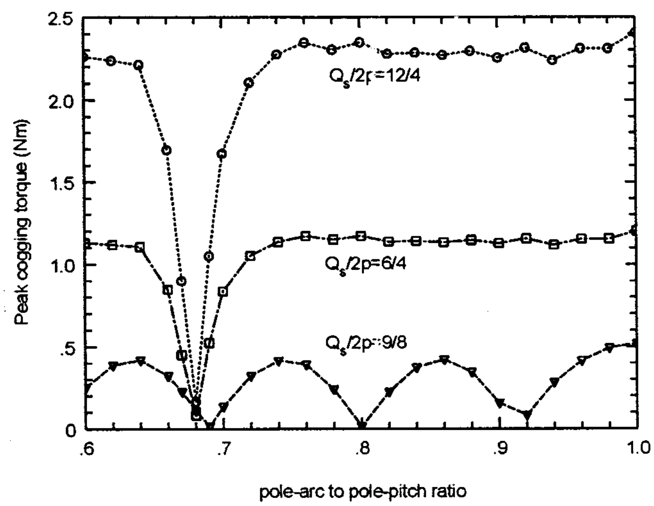

Fig. 8. Effect of pole-arc to pole-pitch ratio and slot and pole number combinations on amplitude of cogging torque.

remain constant. The corresponding cogging torque waveforms are shown in Fig. 9. It should be noted that in passing through the optimal values of pole-arc to pole-pitch ratio the cogging torque waveform undergoes a phase reversal. It should also be noted that since the results in Fig. 8 are derived from a 2-D analytical model which accounts for fringing flux, the optimal values of α_p are slightly higher than those given in Table III—which assumes that $k_2 = 0$. Obviously, the larger the value of $N = N_c/2p$ then the greater is the number of possible optimal ratios of pole-arc to pole-pitch, while, as described earlier, in general the higher the value of C_T the larger the cogging torque. Of course, the choice of slot and pole number combination and the magnet pole-arc to pole-pitch ratio also depends on other performance factors, such as the phase emf waveform, while the choice of the ratio of magnet pole-arc to pole-pitch sometimes also depends on the motor topology.

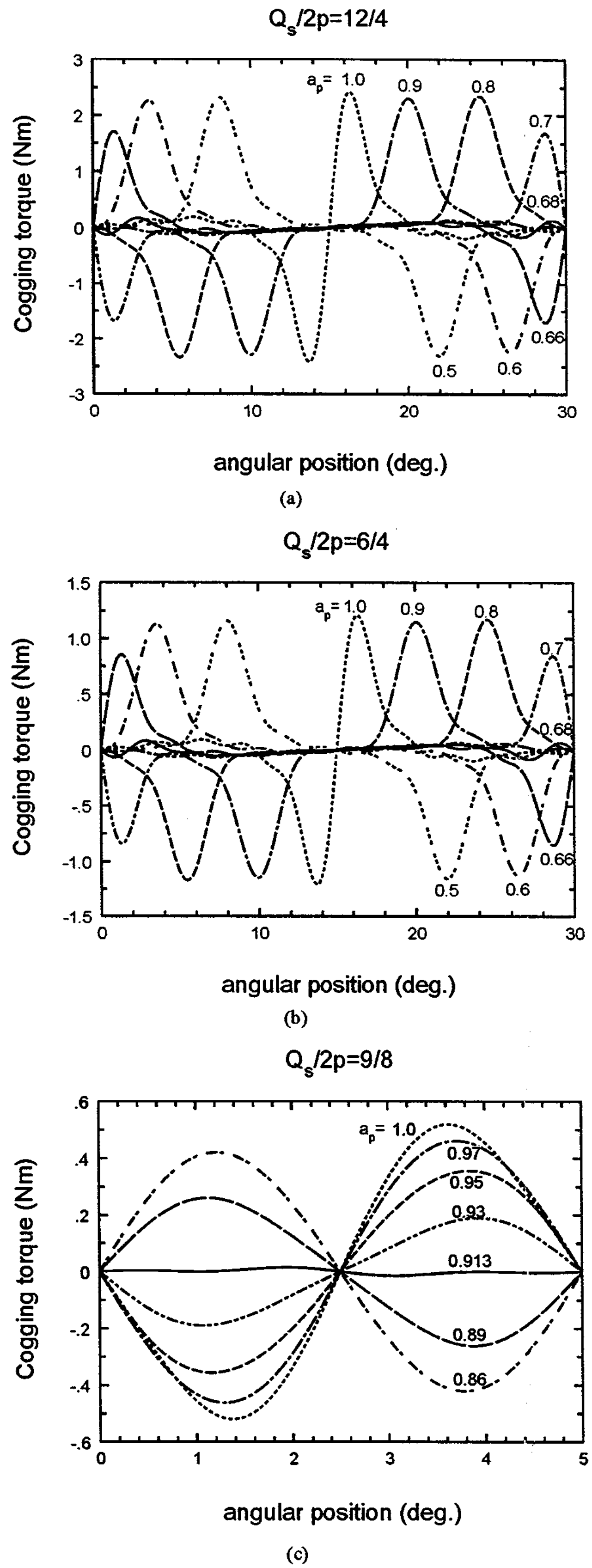

Fig. 9. Effect of pole-arc to pole-pitch ratio and combination of slot and pole numbers on cogging torque waveform.

VI. SKEWING

It is well-known that skewing either the magnets or the teeth can reduce the level of cogging torque, as will be obvious from (1) and (2). Although it is common practice to skew by one slot pitch, (1) and (2) indicate that this is not always essential, since the fundamental and harmonic orders of the cogging

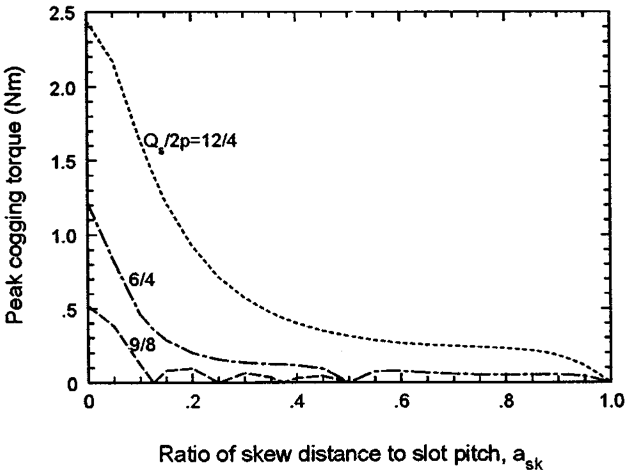

Fig. 10. Effect of skew on amplitude of cogging torque.

torque are integers of the smallest common multiple between the pole number and the slot number. As long as

$$\frac{\alpha_{sk} N_c}{Q_s} = \text{any integer} \tag{6}$$

i.e., if skewing is restricted to less than one slot pitch, the optimal skew which eliminates the cogging torque is:

$$\alpha_{sk} = \frac{kQ_s}{N_c} \qquad k = 1, 2, \cdots, \frac{N_c}{Q_s} \tag{7}$$

For example, for $Q_s/2p = 12/4$ the only optimal skew is one slot pitch, since $N_c/Q_s = 1$, i.e. $\alpha_{sk} = 1$. However, for $Q_s/2p = 6/4$ the optimal skew can be either one slot pitch or a half slot pitch, since $N_c/Q_s = 2$, i.e. $\alpha_{sk} = 0.5$ or 1. As for $Q_s/2p = 9/8$, there are eight possible optimal values of skew, since $N_c/Q_s = 8$, i.e. $\alpha_{sk} = 0.125$, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1.0. This is illustrated in Figs. 10 and 11, which show the effect of skewing on the amplitude and waveform of the cogging torque. It will be noted that the cogging torque reduces most rapidly when , α_{sk} is increased from 0 to 0.15.

VII. Stator Slot Opening

The stator slot opening can also have a significant effect on the level of cogging torque. For example, Fig. 12 shows the variation of the peak cogging torque with the stator slot opening for slot and pole number combinations $Q_s/2p = 9/8$, 6/4, 12/4, respectively. For all combinations, the cogging torque increases with the width of the slot openings, the cogging torque in a brushless motor with a nonoverlapping winding, being only about half that in an equivalent overlapping winding motor. Again, it is evident that a low value of C_T always results in a low cogging torque.

VIII. Conclusion

Certain design parameters can have a very significant effect on the cogging torque, in particular the slot and pole number combination. It also affects the choice of the optimal magnet pole-arc to pole-pitch ratio, and the optimal skew.

A simple factor $C_T = 2pQ_s/Nc$ has been introduced to indicate the "goodness" of a slot and pole number combination from the view point of cogging torque. It can, therefore, be used to aid the selection of an appropriate number of auxiliary teeth and slots. In general, the higher the value of C_T the larger

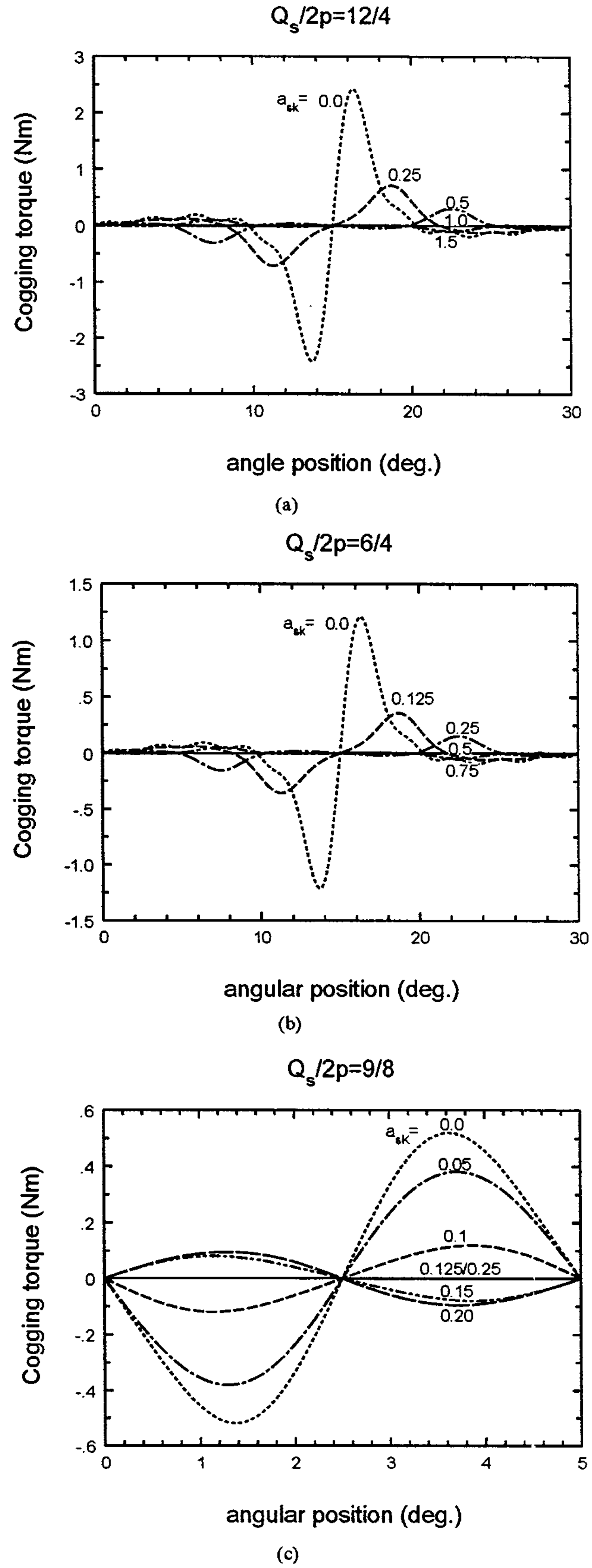

Fig. 11. Effect of skewing on cogging torque waveform.

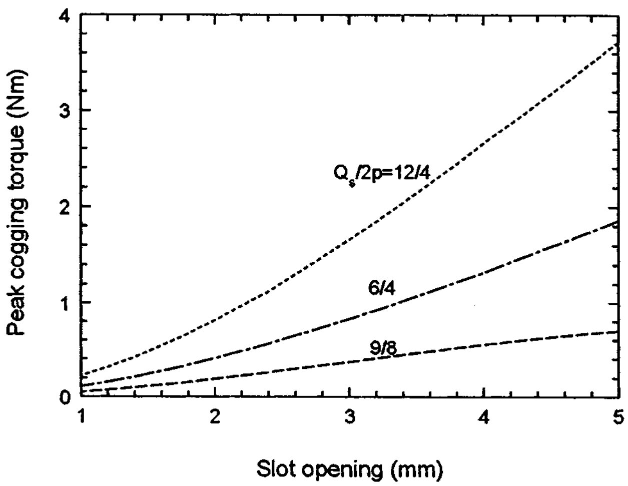

Fig. 12. Effect of stator slot opening on amplitude of cogging torque.

the cogging torque. Hence, brushless motors with overlapping stator windings will generally produce about twice the cogging torque as equivalent motors with nonoverlapping windings. An odd number of slots is preferred for 2-pole motors, since C_T is usually smaller than for an even number of slots.

It has been found that the larger the value of $N_c/2p$ then the greater is the number of possible optimal ratios of pole-arc to pole-pitch, while the larger the value of N_c/Q_s the greater the number of possible values of optimal skew.

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