

1 **Forecasting Change of the Magnetic Field using Core**
2 **Surface Flows and Ensemble Kalman Filtering**

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5 Accurate forecasting of the change of the Earth's internal magnetic field
6 over short intervals of time (e.g. less than five years) has many applications
7 for government, academic and commercial users. Forecasting can be achieved
8 by making a number of reasonable assumptions about how the main field in-
9 teracts with the flow in the liquid outer core. In particular, the magnetic field
10 can be considered to be entrained in the large scale flow along the core-mantle
11 boundary surface over short time periods, giving rise to measurable change
12 of the field at the Earth's surface. The observed change (or secular variation)
13 at or above the surface of the Earth can thus be inverted to produce flow
14 models; these can be used to propagate fluid parcels threaded by the field
15 forwards in time to forecast the non-linear change of the magnetic field. In
16 addition to prediction of field change by flow models, it would be advanta-
17 geous to include observations of the field from satellite measurements or ground-
18 based observatories. We therefore present a method using Ensemble Kalman
19 Filtering (EnKF) to produce an optimal assimilation between magnetic field
20 change as forecast from core flow models and direct observations of the field.
21 We show, by assuming a steady flow and assimilating field observations an-
22 nually, it is possible to produce a forecast over five years with less than 30nT
23 root mean square difference from the 'true' field – within an assumed error
24 budget. The EnKF method also allows sensitivity analysis of the field mod-
25 els to noise and uncertainty within the physical representation.

1. Introduction

26 The slow temporal variation of the Earth's magnetic field is termed 'secular variation'
27 (SV) and is related to advection and diffusion of the field within the liquid outer core.
28 Forecasting the short term change of the field in an accurate and timely fashion is of great
29 benefit to commercial users in areas such as mining, underground drilling and naviga-
30 tion, as well as for academic and civilian users, e.g. where access to real-time data may
31 not be available. The International Geomagnetic Reference Field (IGRF) model enjoys
32 widespread use for this purpose. The model is revised and updated every five years and
33 forecasts secular variation for the future five year period [*Macmillan and Maus, 2005*].
34 Methods for forecasting the magnetic field change have previously relied upon extrapola-
35 tion of ground-based observatory data and the forecasts can often be quite in error at the
36 end of their desired lifetime.

37 Recently, high resolution magnetic field models such as GRIMM [*Lesur et al., 2008*],
38 POMME [*Maus et al., 2006*] and xCHAOS [*Olsen and Manda, 2008*] have been developed
39 using data from the CHAMP, Ørsted and SAC-C satellite missions. These provide an
40 excellent description of the field, SV and secular acceleration (SA) over the period 1999–
41 2009. Detailed models of the large-scale surface core flows generating the observed SV
42 have been developed by a number of researchers [e.g. *Hulot et al., 2002; Holme and Olsen,*
43 *2006*]. If it is assumed on short time scales that advection by core flow of the magnetic
44 field dominates diffusion then, in a manner analogous to weather forecasting, the evolution
45 of the field can be forecast by propagation of the flow forwards in time.

46 Using this approach, *Maus et al.* [2008] generated SV from a series of flow models with
47 differing physical constraints to investigate how well the field could be hindcast compared
48 to the CM4 magnetic field model [*Sabaka et al.*, 2004]. They found the misfit between the
49 hindcast field from core flow models and the CM4 model to be less than 100nT root mean
50 square (RMS) difference after five years and up to 300nT after ten years. We improve
51 upon this result by using a different flow model inversion technique and employing an
52 Ensemble Kalman Filter.

53 The Ensemble Kalman Filter (EnKF) is a Monte-Carlo method for optimally combining
54 models of and observational information about a physical process by statistical represen-
55 tation of the associated uncertainties [*Evensen*, 1994]. It is extensively used in weather
56 and ocean dynamics forecasting to improve the accuracy of forecasts and to explore the
57 sensitivity of systems to minor perturbations [*Evensen et al.*, 2007]. Data assimilation in
58 geomagnetism is still in its infancy but has recently been investigated [*Fournier et al.*,
59 2007; *Kuang et al.*, 2008]. In this paper, we adapt the EnKF for magnetic field prediction
60 using a simple steady flow model and assuming a relatively noisy field model from lim-
61 ited satellite coverage and ground-based magnetic observations is available. This scenario
62 might occur at some point in the future where continuous satellite monitoring has ceased.

2. Methods

63 In the following we describe the methods used to derive a steady flow model that is used
64 for forecasting, the implementation of an EnKF model and the resulting improvements of
65 the field forecast using EnKF with assimilation compared to the forecast. We choose a
66 steady flow as experiments by *Maus et al.* [2008] found that hindcasts from a steady flow

67 model produced the best average long term fit to the CM4 field model and because it is
 68 the simplest assumption to make for a flow model. More complex flows (e.g. time-varying
 69 or different physical hypotheses) can be used if necessary.

2.1. Flow Modelling and Forecasting

70 Magnetic main field models are typically represented as a vector of spherical harmonic
 71 Gauss coefficients ($\mathbf{g} = [g_l^m; h_l^m]$). Secular variation of the field can be inverted for toroidal
 72 and poloidal flow using the linear relationship between SV and flow spherical harmonic
 73 coefficients. The relation is through the Gaunt/Elsasser matrix (\mathbf{H}) whose elements de-
 74 pend on the main field coefficients [*Whaler, 1986*] which change with time. In this study,
 75 the main field, SV and flow coefficients are truncated at degree and order $l_{max} = 14$, thus
 76 we have assumed that only large scale flows are responsible for the large scale SV. Note
 77 that we invert SV data directly (as explained below) rather than using spherical harmonic
 78 models ($\dot{\mathbf{g}}$) of SV.

79 With knowledge of the data covariance, we seek the flow ($\hat{\mathbf{m}}$) which can be obtained
 80 from the SV using the standard L_2 least-squares minimisation norm. We then apply an
 81 additional step using an iterative L_1 norm minimisation technique as described in *Beggan*
 82 *et al.* [2009]. The L_1 norm technique improves the fit of the flow to the SV data by
 83 iterative reweighting of the residual differences. The flow is regularized by imposition of
 84 the so-called ‘strong’ norm *a priori* conditions [*Bloxham, 1988*], with a damping parameter
 85 controlling fit to the data versus flow smoothness.

86 In our first experiment, a series of 25 monthly SV data sets, over the period 2001.9–
 87 2004.0, were generated from CHAMP satellite data using the ‘Virtual Observatory’

88 method of *Mandea and Olsen* [2006]. The SV data were inverted for a steady flow model
 89 [*Voorhies and Backus*, 1985], with a tangentially geostrophic flow constraint. This pro-
 90 duces a set of flow coefficients ($\hat{\mathbf{m}}_{SF}$) representing an ‘average’ flow over the period. The
 91 steady flow model was used to forecast the change of the magnetic field over the five
 92 year period from 2004.0 to 2009.0 and compared to the GRIMM, POMME and xCHAOS
 93 satellite field models.

The Gauss coefficients from the xCHAOS model for 2004.0 were used as the starting field model. The field was advected forward over successive months (k) for five years using the equation:

$$\mathbf{g}_{k+1} = \mathbf{g}_k + (\mathbf{H}_k \hat{\mathbf{m}}_{SF})/12 \quad (1)$$

with the \mathbf{H}_k matrix updated at every timestep using the main field coefficients forecast from the previous timestep, making the system non-linear. To evaluate the validity of this forecast, the RMS difference (or misfit) metric (\sqrt{dP}) to a satellite field model is calculated by:

$$dP = \sum_{l=1}^{l_{max}} \sum_{m=0}^l (l+1) [(\mathbf{g}_l^m)_{field} - (\mathbf{g}_l^m)_{forecast}]^2 \quad (2)$$

94 Figure 1 shows the misfit of the forecast from the flow model to the GRIMM, POMME
 95 and xCHAOS satellite field models. Note the GRIMM model spline coefficients extend to
 96 2006.5, while the POMME model is extrapolated beyond 2007.5 using constant SV.

97 We now show how to improve upon these results by employing an Ensemble Kalman
 98 Filter to assimilate field observations into forecasts from core flow models.

2.2. Data Assimilation in Ensemble Kalman Filtering

99 In an EnKF, the state of a dynamic process at any particular time can be represented
 100 as a vector in n -dimensional space, where n is the number of parameters in the system.
 101 The uncertainty of the process is represented by perturbing the inputs randomly by a
 102 known variance (with zero-mean) to produce an ‘ensemble’ of states – conceptually imag-
 103 ined as a ‘cloud’ of points in n -dimensional space. The evolution of the states though
 104 time is controlled by propagating the ensemble forward using model equations of the sys-
 105 tem behavior. When an observation is available, it can be optimally assimilated into the
 106 ensemble by applying the standard Kalman Filter equations [*Kalman*, 1960]. With a suf-
 107 ficiently large ensemble (determined through experimentation), the mean state represents
 108 the most likely value for the process at the time. The evolution of the ensemble can be
 109 explored by examining the ‘spread’ of the states about the mean.

A traditional Kalman Filter is implemented in two steps: (1) prediction of the evolution of the model state by dynamic equations believed to adequately represent the system and (2) assimilation of a measurement to correct any accumulated error from the model. At time k , the optimal blending of a forecast state (\mathbf{x}_k^f) and measurement (\mathbf{z}_k) to generate the assimilated state vector, \mathbf{x}_k^a , is through the so-called Kalman gain matrix (\mathbf{K}_k):

$$\mathbf{x}_k^a = \mathbf{x}_k^f + \mathbf{K}_k(\mathbf{z}_k - \mathbf{x}_k^f) \quad (3)$$

with

$$\mathbf{K}_k = \mathbf{P}_k^f(\mathbf{P}_k^f + \mathbf{Q})^{-1}. \quad (4)$$

where \mathbf{P}_k^f is the covariance of the model and \mathbf{Q} is the covariance of the data measurement. The balance between the error of the model and measurement controls the assimilation

step. When the Kalman gain matrix has been calculated, the covariance of the assimilated state vector is calculated as:

$$\mathbf{P}_k^a = (\mathbf{I} - \mathbf{K}_k)\mathbf{P}_k^f. \quad (5)$$

In the EnKF, \mathbf{x}_k^f is a model forecast with noise \mathbf{w}_k^f , and \mathbf{z}_k is a measurement with some associated measurement noise \mathbf{u}_k . The forecast, measurement and the newly assimilated estimate, \mathbf{x}_k^a , are related to the true state of the system, \mathbf{x}_k^t , by:

$$\mathbf{x}_k^f = \mathbf{x}_k^t + \mathbf{w}_k^f; \quad \mathbf{x}_k^a = \mathbf{x}_k^t + \mathbf{w}_k^a; \quad \mathbf{z}_k = \mathbf{x}_k^t + \mathbf{u}_k \quad (6)$$

110 with expectations (i.e. the mean of) $\overline{\mathbf{w}_k^f} = \overline{\mathbf{w}_k^a} = \overline{\mathbf{u}_k} = 0$, given a large enough ensemble.

111 If we consider the covariance of an assimilated ensemble, it can be shown [Evensen, 1994]:

$$\begin{aligned} \mathbf{P}^a &= \overline{(\mathbf{w}^a)^2} = \overline{(\mathbf{x}^a - \mathbf{x}^t)^2} \\ &= \left(\mathbf{I} - \frac{\mathbf{P}^f}{\mathbf{P}^f + \mathbf{Q}} \right) \mathbf{P}^f \\ &\quad + 2 \frac{\mathbf{P}^f}{\mathbf{P}^f + \mathbf{Q}} \left(\mathbf{I} - \frac{\mathbf{P}^f}{\mathbf{P}^f + \mathbf{Q}} \right) \overline{\mathbf{w}^f \mathbf{u}}. \end{aligned} \quad (7)$$

112 This leads to the key result of the EnKF: when the expectation $\overline{\mathbf{w}^f \mathbf{u}} = 0$, Equation 7 is
 113 equivalent to Equation 5. This occurs when a suitably large number of ensemble states
 114 are employed.

2.3. Practical Implementation

115 There are three stages required to implement the EnKF for this problem: (1) generation
 116 of the initial ensemble, (2) forecasting the change of the field by driving the field model
 117 with SV predicted by core flow models and (3) assimilation of measurements e.g. from a
 118 ‘true’ field model. Each of these stages is explained in detail below.

Initiating the Ensemble

119 The ensemble is initiated by generating a perturbed set of Gauss coefficients. The
 120 mean value of the initial ensemble is equal to the input coefficients of the field. This is
 121 implemented as follows:

122 1. An initial state vector, at time $k = 1$, is set to be a vector of Gauss coefficients from
 123 a field model (e.g. xCHAOS).

124 2. If a time series of flow models are available, rather than a single steady flow, the
 125 variability of the the flow model coefficients can be used as additional information. To
 126 generate the perturbation to the \mathbf{g}_l^m field coefficients, the standard deviation for each
 127 coefficient over the entire set of *flow* models is calculated (from the variability in each
 128 flow coefficient of $\hat{\mathbf{m}}$). However, with a single steady flow an alternative estimate of the
 129 variance must be made.

130 3. A matrix of normally distributed random numbers $N(0, 1)$ with size $[l_{max}(l_{max} + 2) \times$
 131 $n_{ensembles}]$ is created, where $n_{ensembles}$ is the number of ensemble states.

132 4. The matrix of random numbers is multiplied by the standard deviation of the flow
 133 coefficients to give a perturbed flow coefficient matrix.

134 5. The perturbed flow coefficient matrix is pre-multiplied by the \mathbf{H} matrix to produce
 135 a matrix of perturbed SV coefficients, correctly scaled to reflect the uncertainty in the
 136 flow models.

137 6. The perturbed SV coefficient matrix is then added to the initial state vector to
 138 produce an ensemble matrix (Ensemble₁).

139 Once the initial ensemble has been created, forecasting and assimilation can take place.

Driving the Ensemble Forecasts

140 The forecast (prediction) of the field is driven forwards by the summation of (1) the field
 141 coefficients and (2) the monthly SV coefficients from the flow model which are perturbed
 142 by a random matrix with zero mean and standard deviation computed from the variance
 143 of the flow over time. In addition, at each timestep, model noise is added to simulate the
 144 variance of the ensemble, forcing it to grow at each forecast iteration. The model noise is
 145 controlled by the size of the time-step (Δt), the standard deviation of the SV coefficients
 146 from the previous iteration, and a parameter ρ , which can be used to control the time
 147 correlation of the noise, if required [Evensen *et al.*, 2007].

148 1. The SV coefficients generated by the flow model for month k are calculated by
 149 multiplying the flow model coefficients by the \mathbf{H} matrix.

150 2. The monthly SV coefficients are perturbed by the standard deviation of the flow
 151 converted into an equivalent SV.

152 3. Model noise is simulated by multiplication of a matrix of random zero-mean
 153 normally-distribution numbers (of size $[l_{max}(l_{max} + 2) \times n_{ensembles}]$) with the square-root
 154 of the timestep $\sqrt{\Delta t}$ and ρ .

155 4. The matrix of perturbed SV coefficients and model noise are added to the ensemble
 156 from the previous timestep (Ensemble $_{k-1}$) to produce the forecast for the current ensemble
 157 (Ensemble $_k$).

158 These four steps are repeated until a measurement becomes available for assimilation
 159 into the ensemble.

Assimilation of Measurements into the Ensemble

160 Over time, the forecast field will begin to diverge from the actual field. To improve
 161 the forecast, data can be input into the ensemble to update (correct) it. The data have
 162 associated errors which are used to generate a perturbed data ensemble. These perturbed
 163 data are assimilated into the overall ensemble using the Kalman Filter algorithm.

164 1. Data, for example a set of Gauss coefficients (\mathbf{z}_k), are available with a certain (esti-
 165 mated or known) error for each coefficient.

166 2. A matrix of zero-mean Gaussian random numbers is generated and scaled with the
 167 data error.

168 3. The data are added to the matrix of scaled random numbers to produce a matrix of
 169 ‘perturbed data’, with mean equal to that of the data themselves.

170 4. Using Equation 3 the data perturbation matrix and the perturbed SV coefficients
 171 are optimally assimilated into the ensemble at this timestep.

172 The covariance matrices can be estimated from the ensemble and measurement errors
 173 [Evensen, 1994]. Note it is also possible to use non-synoptic (i.e. partial) measurements
 174 of the field in the assimilation step with an appropriate ‘observation’ operator. *Evensen*
 175 *et al.* [2007] outlines and demonstrates how to efficiently code and compute the matrix
 176 operations for the EnKF. The number of ensemble states was set to 1000 after experimen-
 177 tation, though it was found that any more than 500 is adequate. Typically, a measurement
 178 (i.e. Gauss coefficients from a field model) is assimilated every twelve months.

3. Applying the Ensemble Kalman Filter to Forecasting

179 In Figure 1, the steady flow model prediction slowly diverges from the main field mod-
 180 els over the time period. Assimilating actual field measurements would be expected to
 181 improve the fit of the predicted field to the ‘true’ field. Any improvement is dependent
 182 on the errors of the input measurement. For example, a poor measurement allocated an
 183 associated small estimated error will increase the RMS misfit of the ‘nowcast’. However,
 184 it is often difficult to correctly estimate the errors associated with each Gauss coefficient
 185 in a field model given that we do not have full knowledge of the field [*Langel et al.*, 1989].

186 The results of the forecast with data assimilation for the GRIMM and POMME (both
 187 extrapolated beyond 2006.5) and xCHAOS field models are shown in Figure 2. Each
 188 ensemble was initiated using the xCHAOS field model. Assimiliations of noisy measur-
 189 ements from the relevant field model are indicated by jumps in the curves. The solid black
 190 line represents the misfit (Equation 2) of the mean Gauss coefficients of the ensemble to
 191 the satellite field models, while the dashed lines are misfits of the Gauss coefficients one
 192 standard deviation above or below the mean. The middle and lower panels show that the
 193 mean ensemble (solid line) fits to better than 25nT for both the POMME and xCHAOS
 194 models over the entire period. Most of the misfit is from the difference between forecast
 195 and model at degrees $l = 1 - 4$.

196 From Equation 4 it should be clear that the calculation of the EnKF is sensitive to
 197 the estimates of input errors. Analysis of the factors affecting the forecast fidelity shows
 198 that the error associated with the assimilated Gauss coefficients is the major contributor.
 199 The error associated with the steady flow model coefficients is a secondary effect. In our

200 example, after experimentation, the error on each of the field model coefficients was set
 201 to $\mathbf{z}/(2 \cdot 10^3)$. For the largest coefficient (g_1^0) this is a relative error of 15nT, equivalent to
 202 approximately two years of SV. Larger errors than this produce forecasts that are worse
 203 than predictions from steady flow alone. In this case, increasing the size of the error
 204 estimate of the measurement by two approximately doubles the size of the misfit. A ten-
 205 fold increase in the measurement error results in a poor input field estimate causing a
 206 large divergence from the ‘true’ field (the misfit after five years rises to over 400nT).

4. Discussion and Conclusion

207 The EnKF allows exploration of the system under consideration through examination
 208 of the ‘spread’ of the ensemble. In Figure 2, the ensemble models -1σ away from the
 209 mean are a poorer match to the ‘true’ model, though the $+1\sigma$ model is usually better than
 210 the mean for the GRIMM and POMME comparisons. Another note-worthy point is that
 211 certain measurement assimilations have little or no effect. For example, for POMME at
 212 2008.0, the measurement assimilation barely alters the mean but does reduce the spread
 213 of the ensemble (the $\pm 1\sigma$ states become close to the mean).

214 With a steady flow model and annual data assimilations, the RMS difference between
 215 the forecast model and the ‘true’ field can be maintained at less than 30nT from 2004.0–
 216 2009.0 within assumed errors. This can result in a many-fold improvement e.g. compare
 217 the misfit of the forecast to xCHAOS in Figure 1 with the misfit of the mean forecast in
 218 Figure 2 (lower panel).

219 The use of the EnKF for this particular example is, perhaps, unnecessarily complicated.
220 However, the method can be readily adapted for more complex flow regimes models and
221 different data types.

222 In conclusion, we have demonstrated that forecasting of secular variation using a steady
223 core flow model can achieve an acceptable match to the actual field. We have adapted
224 the Ensemble Kalman Filter to improve forecasts and characterise their uncertainty by
225 propagating a large number of possible field models forward in time using core flow models
226 to control the evolution of the individual states. Optimal assimilation of measured data
227 into the ensemble produces an improvement in the fit of the forecast to the actual field.
228 Our approach thus offers a method to improve operational forecasting of the magnetic
229 field.

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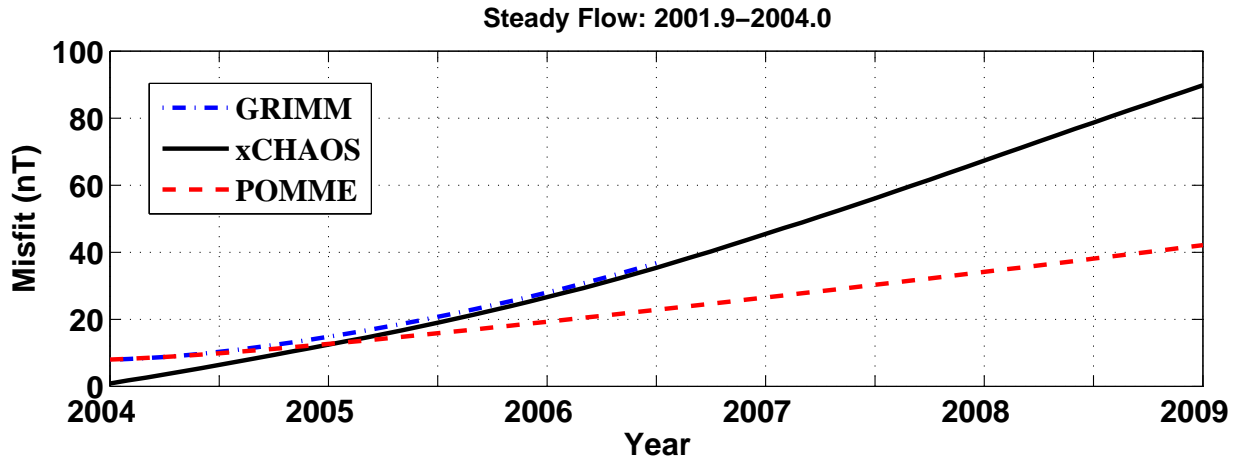


Figure 1. RMS difference (in nT) between the forecast field from a steady flow model generated from data over the period 2001.9–2004.0 and the GRIMM, POMME and xCHAOS satellite field models.

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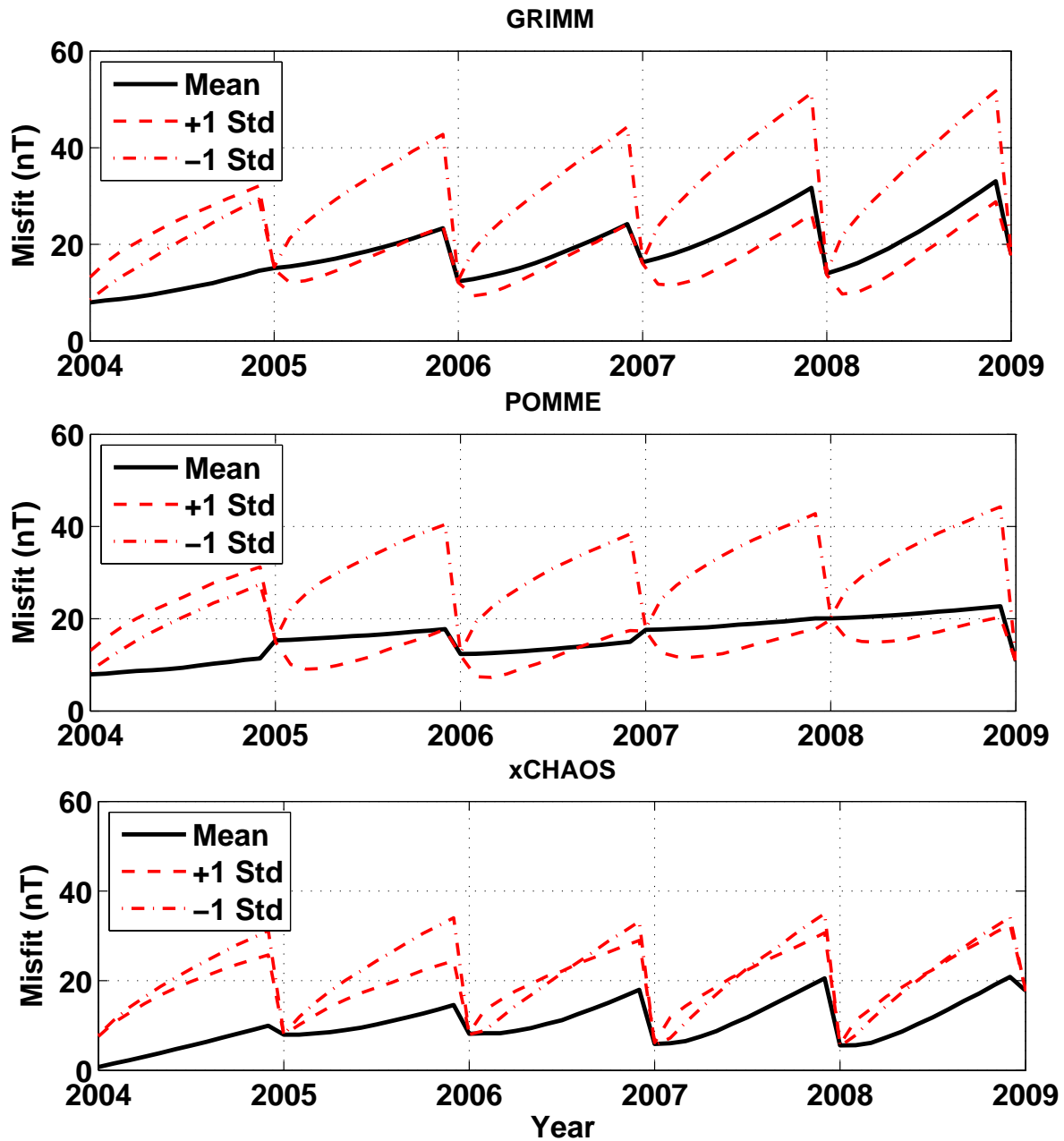


Figure 2. RMS difference (in nT) between a EnKF field forecast with annual assimilation derived from SV generated by a steady flow model from CHAMP satellite data over the period 2001.9–2004.0 and the GRIMM (top panel), POMME (middle panel) and xCHAOS (bottom panel) field models.