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## Design and implementation of adaptive neuro-fuzzy inference system for the control of an uncertain ball and beam apparatus

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### ABSTRACT

Controlling an uncertain mechatronic system is challenging and crucial for its automation. In this regard, several control-strategies are developed to handle such systems. However, these control-strategies are complex to design, and require in-depth knowledge of the system and its dynamics. In this study, we are testing the performance of a rather simple control-strategy (Adaptive Neuro-Fuzzy Inference System) using an uncertain Ball and Beam System. The custom-designed apparatus utilizes image processing technique to acquire the position of the ball on the beam. Then, desired position is achieved by controlling the beam angle using Adaptive Neuro-Fuzzy and PID control. We are showing that adaptive neuro-fuzzy control can effectively handle the system uncertainties, which traditional controllers (i.e., PID) cannot handle.

## 1. Introduction

Control systems play a crucial role in the design and automation of mechatronic systems. In this regard, numerous control-strategies have been proposed. Systems with uncertainties can easily jeopardize a control-strategy if the nature of the uncertainties are not taken into account while designing. These uncertainties can arise due to imperfection in design, measurement errors, time-varying properties, higher-order dynamics, and non-linearities in a system [1]. To deal with such systems, researchers have proposed several control-strategies. For example; H-infinity loop shaping and parametric estimation can make the system insensitive to uncertainties [2], adaptive control is suitable to handle

the changes in time-varying systems [3]; and Lyapunov technique is better to control non-linear systems [4], etc. Although, these techniques are very successful; yet, they require mathematical models of the systems. Obtaining an admissible mathematical model is viable for a simple system, but this is not always possible for complex cases. To bridge this gap, non-model based control-strategies like neural and fuzzy techniques were later developed [5] and implemented in a number of control applications like, cruise control [6-9], industrial processes [10-12], robotics [13-14] and ball and beam/plate systems [15-19], etc.

Notwithstanding the advancements in control literature, the classical Proportional-Integral-Derivative (PID) control remained as a widely used control-strategy

in the industry [3]. Since, PID can handle numerous applications without a need of mathematical model. As a result, researchers have often compared the performance of different control-strategies with classic PID. Keshmiri et al. [20] have compared classic PID (non-model-based control), and hybrid PID and Linear Quadratic Regulator (combination of model and non-model-based control). They have used ball and beam system for the comparison and reported a superior performance of model-based control-strategy. Similarly, Choudhary [21] has reported a better performance of fractional order PID over classic PID controller by simulating the ball and beam system.

Studies have shown that fuzzy control can outperform PID control in a number of applications. Perez et al. [8] have studied the throttle/brake control to improve the comfort level for passengers during acceleration/deceleration of a car. The results showed improved performance with the neuro-fuzzy controller (Adaptive Neuro-Fuzzy Inference System, ANFIS) compared with manually tuned control (typically used in the industries). Munyaneza et al. [7] concluded that PD control produces small rise time (in cruise-control system), yet it creates a high percentage of overshoot, resulting in overall poor performance as compared with fuzzy control. Similarly, Dawood et al. [6] have compared PID, fuzzy logic, and genetic-algorithm for the cruise control application. They reported fuzzy control as best in handling the overshoot, settling time, and steady-state error. Furthermore, it has been shown that fuzzy control can outperform PID in controlling a non-linear flow process [10-12]. A step further, Safwan et al. [14] noted that fuzzy-based PID controller was better in trajectory tracking as compared with a fuzzy logic controller itself (tested on non-holonomic mobile robot). However, their results were based on simulations only.

In this study, we tested the effectiveness of ANFIS to control a real-time system, i.e., ball and beam, similar to the earlier studies [15-17]. A step further, we used different balls to tune and test the controller; hence, introducing an uncertainty in the system. For a comparison, we choose classic PID controller. We speculated that PID control will fail, and ANFIS will handle the uncertainty. Since, neuro-fuzzy control works in a manner our brain performs, and humans are best in handling the uncertain systems [22]. Additionally, ANFIS combines a set of fuzzy rules that have learning capability to realize a nonlinear behaviour.

In the subsequent sections, we are explaining the working principle of ANFIS and Ball and Beam System. Then the designs of ball and beam apparatus and both

control-strategies used in this study are presented. Finally, we are presenting the experimental design, presenting the results, and concluding this work.

### 1.1 Adaptive Neuro-Fuzzy Inference System (ANFIS)

Fuzzy control or Fuzzy Inference System (FIS) uses linguistic rules to control a system [5, 23]. These rules are represented in the form of ‘If-then’ paradigm called knowledge-base (or rule-base). However, inputs and outputs of a physical system are real values; hence, incompatible with the knowledge-base. Therefore, fuzzification is used to make the inputs compatible with the knowledge-base. Finally, the control decisions made by FIS are converted back to real values using defuzzification. There are different methods to infer the linguistic rules for FIS. A human can directly define the rules by using his/her experience (heuristics). For example, a driver can intuitively state the rules for implementing a cruise control in a car. Alternatively, numerical data (inputs and outputs) are collected while a human expert controls the system in real-time or using simulations [16]. Then, input-output mapping is used to establish the heuristic rules for FIS. In this regard, several techniques have been proposed; Neuro-Fuzzy Control (e.g. ANFIS) is one such method. ANFIS is a hybrid approach proposed by Jang [24]. It is a FIS with adaptive nodes (neural network). The parameters (weights) of these nodes are adjusted to minimize the error in measurement through training. The architecture of ANFIS is further explained in context of this study in the design of ANFIS.

### 1.2 Ball and Beam System

The Ball and Beam system is simple; yet, it is non-linear, under-actuated, and inherently unstable system. It consists of a ball placed on the beam. Ball is only allowed to move along the axis of the beam. The position of the ball and beam ( $x$ ) is controlled by changing the angle of beam ( $\theta$ ) as shown in Fig. 1. The dynamics of ball and beam can be represented as a function of beam angle ( $\theta$ ) and acceleration due to gravity ( $g$ ) using Eq. 1 [25].

$$g \sin \theta = d^2x/dt^2 \quad (1)$$

## 2. Methodology

### 2.1 Hardware Setup

We used a 50 cm channel bar (made of aluminium) to create the beam. The beam was coupled with a servo motor (Tower Pro, MG945R), having a response time of 60°s-1. The motor was controlled by pulse-width-modulation (PWM) generated by a microcontroller (Microchip, PIC16F873A). The position of the ball on the beam was obtained from images taken by a USB webcam.

The webcam was placed at a distance of 80 cm from the ball and beam assembly. A white screen was placed behind the beam to facilitate image processing, as shown in Fig. 2.

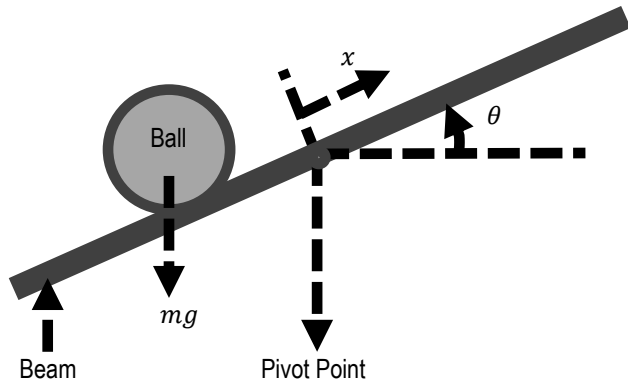


Fig. 1. Illustration of ball and beam system

We used a joystick to manually control the position of ball on the beam (i.e., to implement human-in-the-loop control).

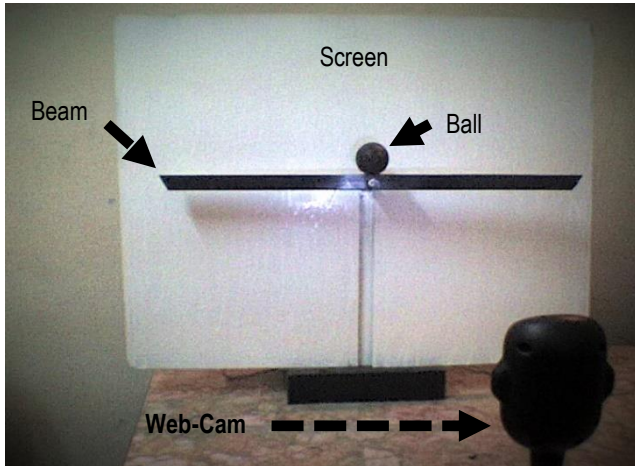


Fig. 2. Hardware implementation of Ball and Beam system

The image processing and control-strategies were implemented in Simulink (using Normal Mode), and sampling time ( $T_s$ ) was maintained at 0.16 s approximately. The block diagram depicting our setup is given in Fig. 3.

### 2.2 Image Processing

The steps involved in acquiring the position ( $x$ ) of ball on the beam are elaborated in **Error! Reference source not found.** First, we converted the RGB image to Binary image. Then the boundary of the beam was traced, and its edges were identified. Finally, the position of the ball was located across the pivot point (where, pivot point is at the centre of the beam). The velocity of the ball was computed by dividing the difference in position (in two samples) by sampling

time, i.e.,  $v = \Delta x / \Delta t$ .

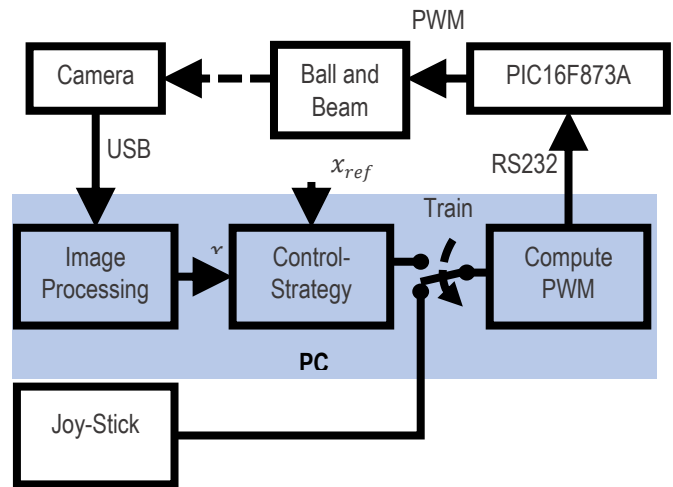


Fig. 3. Implementation of control strategies

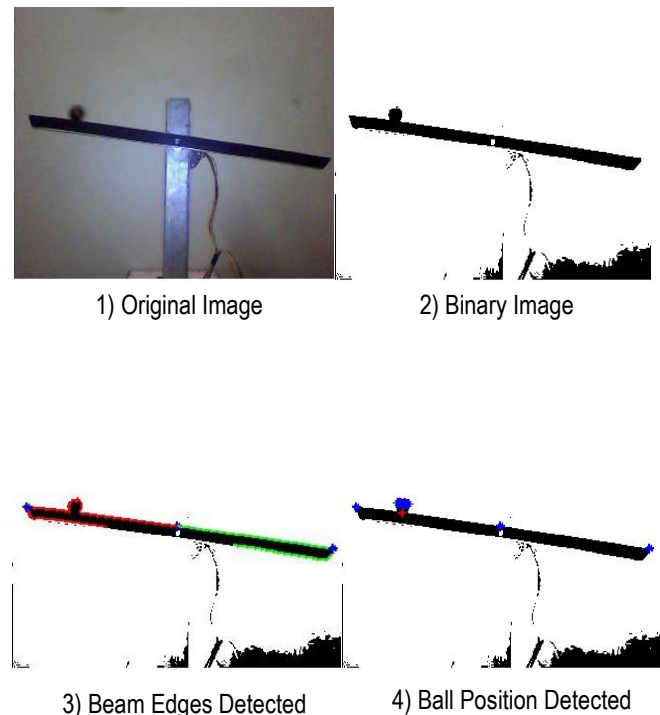


Fig. 4. Steps to acquire the position of a ball on the beam using image processing

### 2.3 Controller Design

The aim of closed-loop control was to keep the ball at desired position ( $x = 0$  cm), and velocity ( $v = 0$  cm/s) under uncertainty. To better judge the performance of ANFIS we opted for PID-control as a reference control-strategy. The details of each control-strategy are discussed below.

#### 2.3.1 Design of ANFIS

We considered the position ( $x$ ) and velocity ( $v$ ) of the ball as input variables and defined three linguistic-states: Negative (-ve), Desired (D) and Positive (+ve) for each

input variable. Where,  $x$  to the right of the pivot point, and  $v$  towards the right edge were considered as positive (see **Error! Reference source not found.**). Moreover, Gaussian membership functions were used to represent each linguistic state. Gaussian membership function is used because of its smoothness and suitability in small rule-base designs [26-27].

The angle of beam ( $\theta$ ) was output variable. It has seven linguistic states; high negative ( $-ve^{High}$ ), negative ( $-ve$ ), low negative ( $-ve^{Low}$ ), zero (Z), low positive ( $+ve^{Low}$ ), positive ( $+ve$ ), and high positive ( $+ve^{High}$ ). Where,  $\theta$  was positive in counter clockwise direction, as shown in **Error! Reference source not found.**. The output membership functions were constant values (Sugeno, type-3 [24], [28]).

We defined rule-base using heuristics to control the ball on the beam, as follows.

- R1: If  $x$  is D, And  $v$  is D, Then  $\theta$  is Z.
- R2: If  $x$  is  $-ve$ , And  $v$  is D, Then  $\theta$  is  $+ve^{Low}$ .
- R3: If  $x$  is  $+ve$ , And  $v$  is D, Then  $\theta$  is  $-ve^{Low}$ .
- R4: If  $x$  is  $-ve$ , And  $v$  is  $+ve$ , Then  $\theta$  is Z.
- R5: If  $x$  is  $+ve$ , And  $v$  is  $-ve$ , Then  $\theta$  is Z.
- R6: If  $x$  is  $-ve$ , And  $v$  is  $-ve$ , Then  $\theta$  is  $+ve^{High}$ .
- R7: If  $x$  is  $+ve$ , And  $v$  is  $+ve$ , Then  $\theta$  is  $-ve^{High}$ .
- R8: If  $x$  is D, And  $v$  is  $+ve$ , Then  $\theta$  is  $-ve$ .
- R9: If  $x$  is D, And  $v$  is  $-ve$ , Then  $\theta$  is  $+ve$ .

where, And method = product ( $\Pi$ ).

The weights ( $W_i$ ) assigned to each linguistic-rule were adjusted by training (Neural-Network, Back-Propagation technique). To get the training data, an expert participant controlled the ball on the beam with the joystick, and real-time data ( $x$ ,  $v$ , and  $\theta$ ) were recorded. The fuzzy-output was defuzzified using a weighted average rule. **Error! Reference source not found.** shows the architecture of ANFIS used in this study.

### 2.3.2 Design of PID Controller

PID is a widely used control-strategy in various real-time applications. Its simplicity and ability to work without the system model proved to be a great incentive for control engineers. The PID uses the difference ( $e(t)$ ) between the desired and actual output of the system to decide the controller gain. Difference of actual and desired position of the ball is used to calculate the error  $e(t)$ . Equation (2) shows the mathematical form of PID

control in continuous time; where,  $K_p$ ,  $K_i$ , and  $K_d$  represent the gains of proportional, integral and derivative terms, respectively.

For discrete-time approximation of Eq. 2, we used backward difference rule (for derivation), and right-side rule (for integration) [16]. The discrete-time approximation thus obtained is given by Eq. 3 (where,  $T_s$  is the sampling time,  $n$  is the number of the sample, and  $I_g(n)$  is the discrete-time integration as given by equation (4).

$$K_p e(t) + K_d de(t)/dt + K_i \int e(t) \quad (2)$$

$$-K_p x(n) + K_d \frac{(x(n)-x(n-1))}{T_s} + K_i I_g(n) \quad (3)$$

$$I_g(n) = I_g(n-1) - T_s(0 - x(n)) \quad (4)$$

Finally, the gains of PID controller were adjusted using Ziegler-Nichols tuning method. The final values for each gain were  $K_p=1.7$ ,  $K_i=1.1$ , and  $K_d=0.7$ .

### 2.4 Experimental Design and Procedures

In order to evaluate the relative performance of the controllers (ANFIS and PID), we placed the ball at 6 different initial positions ( $x_{init} = -20, -10, -5, +5, +10, +20$  cm). Moreover, to add a level of uncertainty we used three different balls during experiments (see **Error! Reference source not found.**). Different ball materials provide different friction coefficients to roll on the beam; therefore, ball material can be considered as an uncertainty. Hence, there were 18 different initial states (i.e., 6  $x_{init}$  x 3 Balls). Each initial state was repeated 4 times making 72 trials. During each trial, we noted if the controller was able to stabilize the ball at  $x = 0$  cm. In case of success, settling time ( $t_{ss}$ ) was measured from the response, and results were computed. It is important to mention that the maximum tilt angle ( $\theta$ ) that our system could achieve to balance the ball was  $\pm 15^\circ$ .

**Table 1**

Photovoltaic input parameter

Ball-Type	Material	Weight (g)	Diameter (cm)
1	Plastic	3	4.0
2	Rubber	80	3.0
3	Rubber	50	1.5

## 3. Results and Discussion

We observed a clear difference in the performance of two control-strategies (ANFIS and PID), as shown in Table 2 and Table 3. The key results are as follows. The failure rate of PID was higher as compared to ANFIS control (see Table 2).

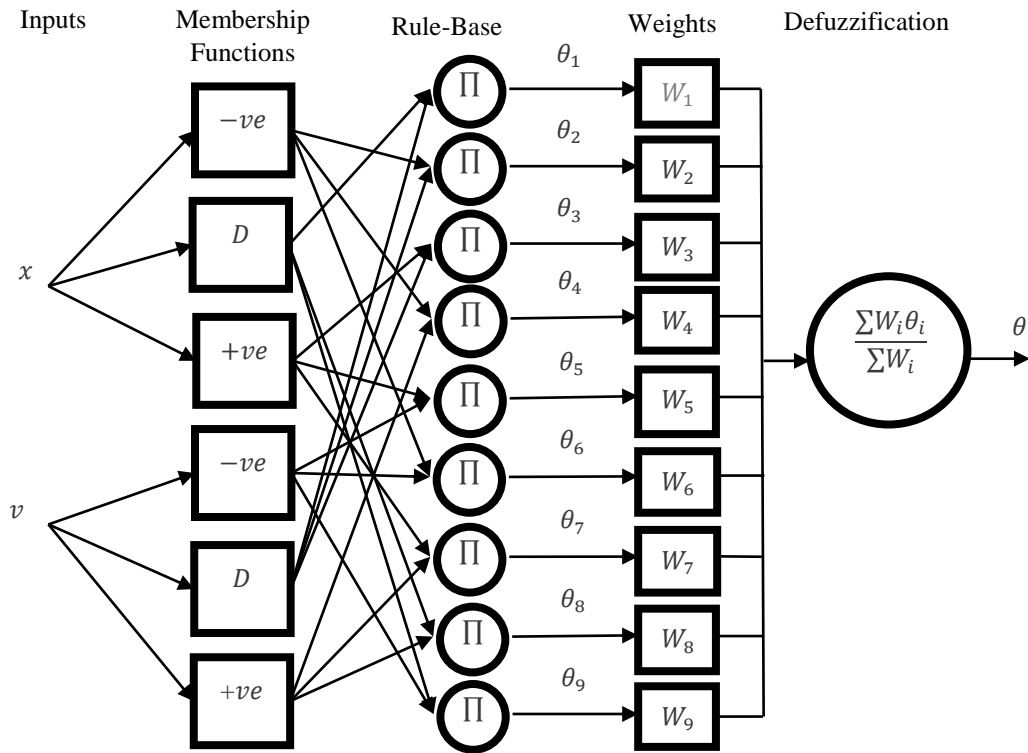


Fig. 5. Model of ANFIS

Particularly, PID controller gave a poor performance with Ball-Type 2 and 3. There was no effect of control-strategy on settling time ( $t_{ss}$ ), considering the successful cases only, as shown in Table 3.

Table 2

Comparison of success rate (%)

Control Strategy	Ball-Type	$x_{init}$ (cm)		
		$\pm 20$	$\pm 10$	$\pm 5$
ANFIS	1	62.5	87.5	100
	2	62.5	100	62.5
	3	62.5	87.5	37.5
PID	1	87.5	62.5	87.5
	2	25	25	0
	3	0	0	0

Our results showed that the performance of both controllers was alike with Ball-Type 1, (i.e., the controllers did not encounter uncertainty). This result is not in agreement with earlier studies, which reported a superior performance of fuzzy control over PID [6-8], [10-13]. The contradiction in results between this study and earlier ones might arise due to the difference in systems used for the comparison of control-strategies. Furthermore, the results in [10-13], were based on simulation; while, testing control-strategies on a real system is important to completely establish their effectiveness. Finally, as the tilt angle of the beam was

limited in our setup (i.e.,  $\pm 15^\circ$ ); therefore, the effect of nonlinearity was reduced. While the strength of the PID controller is renowned in handling a linear system.

Table 3

Comparison of settling time (seconds)

Control Strategy	Ball-Type	$x_{init}$ (cm)		
		$\pm 20$	$\pm 10$	$\pm 5$
ANFIS	1	7.1 $\pm$ 2.0	4.1 $\pm$ 1.0	5.4 $\pm$ 2.3
	2	6.7 $\pm$ 2.3	4.2 $\pm$ 2.0	3.7 $\pm$ 1.0
	3	6.8 $\pm$ 2.6	3.3 $\pm$ 0.7	3.3 $\pm$ 1.5
PID	1	6.3 $\pm$ 2.0	7.9 $\pm$ 2.2	5.7 $\pm$ 2.0
	2	5.8 $\pm$ 0.1	7.5 $\pm$ 2.5	-
	3	-	-	-

On the other hand, ANFIS outperformed PID controller when Ball-Type 2 and 3 were used (i.e., controllers encountered uncertainty). However, success rate was lower for shorter  $x_{init}$  (See Table 2). This is because, for shorter  $x_{init}$ , sudden surge in the control input deviated the ball more from the equilibrium point. As we have mentioned earlier, we used Ball-Type 1 (made of plastic) for tuning of PID gains and ANFIS weights. On the other hand, we used Ball-Type 2 and 3 besides Ball-Type 1 during the evaluation. Ball-Type 2 and 3 were made of rubber material, which has a high

friction coefficient compared with plastic. Hence, this added uncertainty in the system.

The poor performance of PID controller suggested that the level of uncertainty was considerable. As expected, the PID controller failed to balance the ball under uncertainty. This was expected as the conventional PID controller is not suitable to handle the uncertainties. On the other hand, ANFIS performed reasonably well under a similar uncertainty level.

It is important to highlight that the scope of this study is limited, as disturbances and noise factor are not considered. Also, the beam surfaces of different materials are not taken into account which in turn produce different frictional effect for the ball to roll on the beam. Therefore, further work is indeed required to clearly establish the limits of neuro-fuzzy control in handling the uncertainties. In future, we are planning more extensive comparisons to clearly highlight the limits of neuro-fuzzy control.

#### 4. Conclusion

This paper presented the comparison of two well-known control-strategies, Adaptive neuro-fuzzy inference system (ANFIS) and PID control, for the real-time Ball and Beam apparatus in the presence of uncertainties. Position of the ball on the beam is acquired through image processing which in turn used to find the error signal. The weights ( $W_i$ ) assigned to each linguistic-rule of the fuzzy system are adjusted by training (Neural-Network, Back-Propagation technique). The efficiency of the ANFIS is tested on the real time ball-beam system for 6 different initial positions of the ball on the beam. Also balls of different materials has been considered as uncertainty to evaluate the efficacy of the ANFIS control. We concluded that ANFIS can effectively control an uncertain system with minimum settling time and better success rate as compare to traditional controllers like PID.

In future studies, we will compare the performance ANFIS with other robust control techniques to further elaborate its effectiveness.

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