INSIDER TRADING RIGHTS: SHORT-TERMISM AND TRADING CONSTRAINTS

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To my nonnas Maria and Pupatella, and to my best friend Mariasole.
You are the fixed points I always converge to.

ABSTRACT

Giovanni Cocco: Insider Trading Rights: Short-Termism and Trading Constraints
(Under the direction of Jesse Davis, Paolo Fulghieri and Jacob Sagi)

I investigate the effect of trading restrictions on price efficiency in a model with short-term shareholders. I show that insider trading constraints set a floor on prices. More importantly, as trade is the mechanism that transfers information into the price, the market maker's pricing function has non-constant sensitivity to order flows, with selling orders impacting the price less than positive orders. These two effects decrease price volatility. Prices will be more efficient in good states of the world but lose their informativeness in bad states of the world. Trading constraints will emerge in equilibrium when shareholders have preferences for short-term market prices. In such a case, an increase in cash flow volatility can translate into more severe limitations, while the impact of noise variance depends on the short-term shareholder's preferences.

Keywords: Insider trading rights, short-selling constraints, short-termism

JEL Classification: G12, G14, G30, G34

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CHAPTER 1: INTRODUCTION

Stock market prices are useful in aggregating information about company fundamentals. One of the sources of this information comes from insiders whose trades contain privately available news about the company. However, firms sometimes restrict insiders such as company managers and directors on how they can trade on their information. On the one hand, the regulator establishes rules to prevent exploiting the average investor from the insiders by restricting trades on material non-public available information. Companies, on the other hand, may implement insider trading policies that states limit to managers' and directors' trades, such as short-selling constraints and vesting periods, as well as faced consequences if the manager or the director violates those policies, such as exclusion from equity compensation plans, fines and loss of employment. These company-imposed restrictions may help prevent the revelation of information through trading and thus reducing price movements. In this paper, I begin by analyzing the equilibrium impact of unilateral minimum trading restrictions (short-selling constraints and a minimum buying level) in a canonical model à la Kyle (1985). The constraints put a floor on price distribution and reduce its volatility. More importantly, prices will better reflect fundamentals in good states of the world, while their efficiency decreases in economic downturns.

With this in mind, I analyze the setting under which such trading restrictions arise in equilibrium. Share- holders that only care about the fundamental value of the firm is indifferent to trading restrictions. This result may not be correct for shareholders (or other firm stakeholders) who value the stock's price beyond its fundamental value implications. I define "short-term shareholders" to be those whose valuation depends on prices in a way that departs from fundamentals. When such departure is concave in price, causing risk aversion to interim volatility, insider trading will be prohibited entirely. Selling restrictions can exist whenever short-term shareholders benefit more from an active insider's positive price impact than they lose when the market suspects that the insider is not trading because of bad news.[1]

Trading constraints affect the sensitivity of prices to order flow. While in the classical model à la Kyle (1985), prices are linear in the order flows, in the presence of trading constraints, price is less sensitive to negative order flow. This constraint will enhance trading liquidity in the market. The more constrained the insider is, the more liquid the market is. The insider will submit order flows that are no longer linear to respond to the pricing function's curvature. With the constraint, the prices are bounded below. Nevertheless, the price in a constrained trading regime will be lower than the one in the unconstrained equilibrium for some order flow. The reduced price will benefit short-term shareholders willing to either sell or buy large positions in the company but may negatively impact small shareholders who want to liquidate their holdings. Furthermore, the more short-term shareholders dislike unfavorable price realization relative to fundamentals-based shareholders, the more constraints the firm will impose. Cash flow volatility affects the severity of the constraints when short-term shareholders are risk-averse over interim price realizations, whereas noise variance has an ambiguous effect on the optimal constraint. Firms specify insider trading constraints in Insider Trading Policies. Most of these restrictions

[1] Shareholders may care about market prices beyond their implications for fundamentals. Shareholders that have to liquidate their position in the short-run would like high prices independently of company values. To raise capital to finance expiring investment opportunities, financiers tend to look at past and current market performances. Suppliers and customers may look at prices as a quick indicator of firm health. High prices deter hostile takeovers. These reasons boil down to how the firm is perceived and how these perceptions impact the cost of equity and, ultimately, the firm's cost of capital.

concern short- selling and vesting periods. Moreover, the SEC mandates company disclosure of any limits to managers' ability to offset losses on securities received as compensation.

In my model, a firm hires a manager to perform a task whose cash flows are independent of his ability. Once the firm hires him, the manager learns about these cash flows and may trade company shares subject to constraints. These constraints could be specified explicitly in a contract between the manager and the company (e.g., short-selling restrictions or vesting periods) or via tacit understanding among shareholders and executives about the consequences of unwanted behavior (e.g., reputation costs, demotion, or loss of employment). The market for company shares is characterized by some noise traders, whose order flows are independent of the firm's cash flow and a competitive, risk-neutral market maker. The latter observes the totality of order flows submitted by the noise trader and the manager (whose objective is to maximize his trading profit given the restrictions on trading) and set prices equal to the expected value of company cash flow, given the information contained in the order flow, as competition drives liquidity-provision premia to zero.

Acknowledging the trading restrictions, the market maker will ponder that he is facing a constrained insider for some order flows. As a consequence, the pricing function shows a lower sensitivity to negative versus positive order flows. The information coming from trading gets incorporated faster from higher-order flows. In turn, the insider will try to invert the curvature of prices. The solution to the best response problem has not a closed-form solution. However, I show that a quadratic approximation of the price function can characterize the market maker's equilibrium strategy and insider almost entirely within two standard deviations of the order flows. I used this approximation to characterize price behavior in equilibrium and market features such as liquidity and price discovery.

Two main features characterize the pricing function. Upon observing low order flows, the market maker will assign a higher probability of facing a constrained insider. Consequently, prices stabilize above a particular value, capturing some insiders' inability to dispose of their shares freely. Thus, prices are allowed to move in a smaller range of value, and the differential sensitivity to the order flows makes prices less volatile than the unconstrained equilibrium. On the opposite side, when the market maker observes a high order flow, he realizes that more of it can come from the unconstrained insider, and he will respond by changing the sensitivity to order flows. However, the average sensitivity to order flows is lower than the unconstrained equilibrium, fostering liquidity in the market. However, the standard tradeoff of market microstructure arises. The improved liquidity comes at the expense of reduced average price efficiency. The different sensitivity to order flow disrupts stock prices' responsiveness to information, making them less reliable as indicators of future cash flows. Moreover, in bad states of the world where negative news is prevalent and regularly concealed, stock prices will be less informative than in good states of the world. The more stringent the constraint, the less informative prices will be in downturns, and the more informative they will be in the promising market environment.

Having characterized the behavior of prices under different levels of constraints, I identify under which condition a firm is willing to impose these constraints. The firm in my model cares about a shareholder whose preferences are for the shares' secondary market price rather than the firm's fundamental value. Moreover, the shareholder observes ex-post the strategy of the insider, in line with rules of the Security and Exchange Commission (SEC).[2] The firm problem is thus to specify a level of constraints that will induce a price distribution that

[2] Section 16(a) and 23(a) of the Security Exchange Act of 1934. More information on https://www.sec.gov/about/forms/form4data.pdf

maximizes shareholders' utility for the interim price of the firm rather than the value of the company. The proposed constraint depends on these preferences. A (price) risk-neutral shareholder is indifferent between trading regimes and may allow trading from an insider on company shares. Instead, a (price) risk-averse shareholder imposes full trading restrictions, not allowing the insider to trade by any means. Allowing for any trading is a mean-preserving spread of a lottery for the expected value of the company. Thus a regime of no trading second-order stochastic dominates all the other regimes, constrained or unconstrained. However, doing so, the firm limits all the utility the shareholder can receive from positive outcomes of the distribution. Thus, I specify a different set of preferences, characterized by a downside-risk parameter. This parameter changes the weights in the expected utility of the shareholder. As this parameter changes from 0 to 1, the firm weights more and more negative outcomes, thus make it willing to impose more restrictions. A parametrization close to 0 indicates a more risk-seeking behavior of the shareholder toward positive outcomes. However, as long as there is a curvature to the shareholder utility function, the constrained equilibrium dominates the unconstrained equilibrium. The volatility of cash flow impacts the optimal constraints only if the shareholder is risk-averse. As long as the shareholder is risk-seeking in gains, an increase in the volatility of cash flow reduces the utility of the risk-averse shareholder; thus, the firm will impose more stringent constraints on the insider's action space. Instead, an increase in the market's noise variance has an ambiguous effect on the level of constraints. On one side, when the down-side risk parameter is small enough, the firm tends to ride the market for a price appreciation. Instead, when it has a high downside risk parameter, the firm will put further constraints to protect against prices' negative realization.

These considerations have clear testable implications. The principal/predominant shareholder in a company matters in determining the limitation of trading for insiders. If the shareholder is risk-neutral (she can diversify away the risk coming from the price of the firm), such as, for example, a large fund, we should expect less trading constraints and a price capable of reacting in the same facet to bad and good states of the world. When the predominant shareholder is risk-averse and invests most of her wealth in the company in the short- run, such as a founder, we should expect a full shut down of trading and no sensitivity at all of the prices to order flows. For example, in 2010, even before going public, a manager of Facebook was fired because he bought shares of the company, in breach of the Insider Trading Policy. The same year Facebook was discussing with Goldman a future IPO.

The main framework of this model is the market microstructure model of Kyle (1985). With its trading, the insider improves price efficiency by reducing the ex-post volatility of cash flows. In fact, the higher the cash flow volatility, the lower the demand or flow value, and hence the higher the market liquidity. The existence of linear equilibrium in the model is robust to other distributional assumptions that satisfy given properties (Bagnoli et al. (2001)). Moreover, Huddart et al. (2001) show that the insider would use a mixed strategy and give up some profit if they have to disclose the transaction.

Although in other market-making models such as Glosten and Milgrom (1985), researchers may characterize the equilibrium in the presence of short-selling constraints (Diamond and Verrecchia (1987)), this is not the case with models à la Kyle, except for Carre et al. (2019). In their model, insiders are subjected to regulatory penalties independently of the directions of the order flows. Instead, in the paper, I present a new methodology to study the imposition of directional constraints in models à la Kyle (1985). Specifically, I characterize the asymptotic behavior of the price function that best-responds to the insider's strategy when facing trading constraints. Moreover, I show that a quadratic function can approximate this best response within two standard deviations around the mean of the order flows. This approximation simplifies what is otherwise a highly complex and non-linear model and clarifies the pricing function's behavior. Moreover, this methodology is robust to further iterations of the convergence algorithm.

This paper assumes the perspective of shareholders of the company (or the Board of Directors) or the firm itself aims to maintain market prices at a sustainable level, thus rewarding those actions that will not depress prices. Graham et al. (2005) present some evidence in this regard, where the authors find that managers prefer to take actions that would adversely affect long-term value to smooth earnings and that managers would prefer to reveal information to reduce risk and raise stock prices. Moreover, limits to trading are associated with short-term price performances. Vesting equity correlates with managerial decisions to improve short-term price and maximize equity selling proceeds (Edmans et al. (2018)). Bolton et al. (2006) show that an optimal contract may foster market prices but reduce the company's long-term value. In my paper, the manager focuses on short-termism for two reasons: on the one hand, the manager wants to increase his trading profit; on the other hand, the shareholders want to guarantee a high enough price. As a result, management compensation relies on the price, in comparison to models where the price impact of contracts is exogenous (Bebchuk and Stole (1993); Edmans (2009)). A theoretical analysis of the relationship between market prices and short-termism is present in Piccolo (2019), but, differently from my model, his manager and speculator are two separate entities.

In the feedback effects literature [3], two papers may be considered similar and complementary to mine. Bebchuk and Fershtman (1994) consider an investment decision problem and granting trading realign share- holders and manager risk-taking. Their model is complementary to mine, as it analyzes a moral hazard problem, while I focus more on the asymmetric information. Nevertheless, they do not characterize unilateral constraints while only focusing on full trading or no trading rights.

The inability to trade on negative news in my model has some similarities with the model proposed by Edmans et al. (2015). In their model, the speculator will refrain from selling, as his decision will communicate to the managers information about future states of the world, thus advising them to disinvest and avoid losses. This effect results in bad news being incorporated slowly into prices. I find a similar pattern for news incorporation in my model, except now it is in a company's best interest to delay bad news, and cashflows are exogenous to managerial decisions.

The paper also addresses the insider trading literature about the predictability of insiders' order flow on the stock return. Early work (Lorie and Niederhoffer (1968)) provides evidence of predictability of insider purchase but not insider sales. Jeng et al. (2003) estimate that insider purchases earn an abnormal return of more than 6% per year, while sales do not earn significant abnormal returns. Practitioners agree that usually, sales are associated with liquidity trading motives. Thus their predictive power is different from purchase.[4] My paper offers another, and to my knowledge, unique and alternative explanation that is not at odds with common beliefs. Insiders could sell for liquidity related reasons when this decision has nothing to do with

[3] For a review of the literature in feedback effects, see Bond et al. (2012)

[4] Peter Lynch, former manager of Magellan Fund at Fidelity, was noted as saying that "insiders might sell their shares for any number of reasons, but they buy them for only one: they think the price will rise."

company cashflows (i.e., after a sale, share prices will go up), but as long as the market interprets their sale as carrying negative news about the company's fundamentals, the company will prevent it.

The paper proceeds as follows. Section 2 describes the model, characterizes the financial market equilibrium, and discusses the assumptions. In Section 3, I analyze the trading game and introduce some techniques to shed lights on the pricing function. In Section 4, I characterize the contracting stage and study different preferences specifications for the shareholder and show that the results are qualitative unchanged. Then I discuss the minimization problem associated with the cost of imposing trading restrictions. In Section 5, I summarize the main results of the paper and offer some empirical prediction.

CHAPTER 2: THE MODEL

A shareholder (she) needs a manager (he) to operate a firm with only one project. The firm compensates the risk-neutral manager (also the insider or the Insider) with wage and trading rights on his company's shares, all specified in a contract $w(p, x, v)$. If allowed, the manager can submit orders x for the company share on the market anonymously. Without loss of generality, I set his reservation utility to zero, and the market for managerial skills is perfectly competitive. The market features a noise trader who submits order flow $n \sim N(0, \sigma_n^2)$, and a perfectly competitive market maker observes total order flows $q = x + n$. Although managers can submit their trades anonymously, the regulator requires the ex-post disclosures of the orders to the shareholders (in compliance with SEC Rule 10b).

The shareholder has some preference over the short-term price of the company, net of any cost to compensate the manager. Her preferences with respect to the distribution of the share price are:

$$U^{\mathscr{W}}(p) = E_w[h(p)] \qquad (1)$$

$h(\cdot)$ is a function that focuses on prices resulting by imposing a contract W. As will soon be made clear, if $h(\cdot)$ is affine in interim price, p, then the shareholder cares only about the firm's cash flow fundamentals. While shareholders' preferences consider only the price of the stock, I can extend the model to include both short-term and long-term shareholders. Short-term emphasis may arise from the presence of various capital market frictions, such as is the need to raise money for the company in the immediate future, so that the price needs to be high or not volatile in order for funding providers to finance the business.

2.1. Information and Trading

The cashflow of the company per share v follows a distribution $N(v_0, \sigma_v^2)$. Upon signing the contract $w(p, x, v)$, the manager knows v with certainty, but the shareholders do not. Then he submits order $x: v \to \mathbb{R}$ with the final goal of optimizing his trading profit and compensation, and ex-post discloses the trade to the shareholder. As the shareholder is unaware of the company's value, she will refrain from trading, and suffering losses due to asymmetric information. Independently of the company's cash flow, a noise trader submits order flow n due to exogenous shock in his/her income.

On the secondary market, identical risk-neutral market makers compete à la Bertrand. Thus, a representative market maker observes total order flows $q = x + n$ and sets a price p according to Bayes' rule to break even in expectation. As the market maker only filters information about fundamentals when setting the price, if $h(p)$ is affine in p, then, like the market maker, the shareholder is only concerned about the fundamental value of the firm.

2.2. Contracts

The shareholder proposes an ("implicit or explicit") contract to the manager in the following way. The contract W will specify w, an ex-ante or ex-post "payment or punishment" plan, that can potentially be written contingent on the share price p after the round of trading, the order flow of the manager x and the value of the company v once this is known. Notice that the contract could specify a payment at the

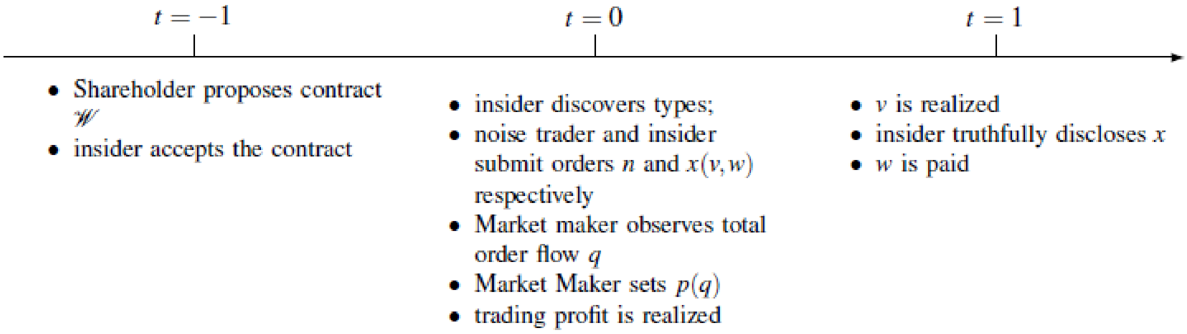


Figure 1. Timeline of the Model

beginning of the game, a payment later in the game, trading constraints and termination (in this case one can consider termination as negative amount w). The shareholder will have to commit to the contract and cannot renegotiate it. I will focus on types of contracts that are characterized by $\bar{x}$, which is a lower bound on insider trading. For instance, if $\bar{x} = 0$, the insider may not sell shares short. I am using the term contract in a broad sense. We could think of it as an explicit as well as an implicit agreement between shareholders and managers to not take actions that will negatively impact the price of the company.

2.3. Model Timeline, Benchmark Scenario and Equilibrium Definition

As shown in Figure 1, the game unfolds as follow:

1. At t=-1, the shareholder proposes a contract to the perspective insider to manage the single project of the firm

2. At t=0, upon signing of the contract, manager/insider discovers the cashflows v and submits order flow x, if the contract allows. In the meanwhile, noise traders submit order flow n. The market maker observes q, sets a price p, shares are exchanged, and trading profits are realized

3. At t=1, the value of the firm is realized, and the manager truthfully discloses x to the shareholder, and the payoffs of the contract are realized

When a contract is implementable, it will specify some payoff the firm will pay at the end of the game. In this environment, we suppose the shareholder can commit to a contract whose payoff will at the end depend on the realized value of p, x and v. The shareholder's problem is to

$$\max_{\mathscr{W}} \quad U^{\mathscr{W}}(p) \qquad (objective) \tag{2}$$

$$s.t. \quad E(w|x) + xE[v-p|x] \geq 0 \qquad PC \tag{3}$$

$$x \quad \in arg\max_{x} \quad E(w|x) + xE[v-p|x] \qquad IC \tag{4}$$

$$p = E[v|q] \qquad MM \tag{5}$$

Notice that we impose that the shareholder can commit to some payoff structure: once trading occurs, the price is set, and after which v becomes common knowledge, the manager gets (pays) w. Notice that x depends on w, and w may depend on both x and p. The optimal insider trading strategy depends on the contract. The participation constraint (3) of the insider has one source of uncertainty: the realization of noise trader order flow. This order flow impacts the price and, consequently, the "wage" of the insider. A feature of the above problem is that the payoff specified by the contract needs to be limited to the company's value. One could interpret an extremely negative w to be the loss in reputation for the insider following a price plunge, or any loss of future compensation deriving from the company firing the manager for bad stock market performances. If the cost to implement the contract is independent of the level of w, then the shareholder can impose unbounded punishment that is never experienced in equilibrium.[5]

Given this interpretation, it is incentive compatible to set the actual payoff from the

[5] As the range of vis unbounded as normally distributed, also trading profits could potentially be unbounded. To deter trading associated with potential negative realization of v, the shareholder has to set the punishment to -∞. Setting the punishment to a constant will not deter some extreme trading. This will generate a jump in the trading strategy of the insider, as shown in the appendix of Carre et al. (2019).

contract to be $w = -\infty$ if $x < \bar{x}$, 0 otherwise, where $\bar{x}$ is a level of transactions that the insider is allowed to trade. Moreover, as x will be a sufficient statistic to understand if the manager adhered to the contract, I define the following equilibrium

Definition 1. Equilibrium Definition

An equilibrium is a triple $x(v), p(q)$ *and* W such that:

1. $x(v)$ maximizes insider payoff given pricing rule p and contract w
2. W maximizes shareholders' payoff given the induced trading and pricing rule
3. $p(q)$ is such that the Market maker breaks even, that is $p(q) = E[v|q]$

In a baseline model with no contracting, I obtain the same solution as in Kyle (1985).

Proposition 2 (The no contract environment). *In an environment when no contract is enforceable, i.e.* $W = \{\emptyset\}$, *the strategy of the insider and the market maker are:*

$$x(v) = \frac{\sigma_n}{\sigma_v}(v - v_0)$$
$$p(q) = E(v|q) = v_0 + \frac{\sigma_v}{2\sigma_n}q$$

the insider profit from trading and the variance of the price distribution are respectively:

$$xE(v - p|x) = \frac{\sigma_n}{\sigma_v}\frac{(v - v_0)^2}{2}$$
$$var(v|p) = \frac{1}{2}\sigma_v^2$$

If the shareholder has no ex-ante information on v, the ex-post price given noise traders' order flow n will be:

$$\hat{p} = v_0 + \frac{(v - v_0)}{2} + \frac{\sigma_v}{2\sigma_n}n$$

Proof. As in Kyle (1985), with $\Sigma = \sigma_v^2$.

The second term in the last expression captures the information that the order flows reveal when an insider following the optimal strategy. This term is positive if $v > v_0$. However, in expectation (i.e., from the ex-ante shareholder's perspective), this term is equal to zero. This condition may suggest that the shareholder would be willing to write a contract to avoid the revelation of information in the case of realization of cash flows below the prior, thus imposing selling constraints to the insider's strategy. The no-contract case serves as a benchmark for the full model, and it is always implementable. The insider camouflages his order flow with the noise trader. In fact, the $E[x] = 0$ and $Var(x) = \sigma_v^2$- The market maker will observe total order flow $q \sim N(0{,}2\sigma_n^2)$.

CHAPTER 3: THE TRADING GAME

I solve for the equilibrium strategies by backward induction. As the shareholder observes the insider ex- post's order flow, she can condition the payoff of the contract on it. I then restrict my attention to contracts that specify a minimum order flow $\bar{x}$ and a level of punishment $w = -\infty$ whenever the insider trades below $\bar{x}$. Thus, the contract space is restricted to pick $\bar{x}$. With a cut-off $\bar{x} = 0$, the shareholder implements selling constraints that could emerge due to rights' vesting periods or for the signal that could be usually associated with negative order flow.

Under this contract, in equilibrium, all insiders that observe small enough will trade the same quantity. This contract is incentive compatible as long as the insider can offset losses of trading the sub-optimal quantity with gains in free trading space, and the shareholder will never punish the insider on the equilibrium path. I show the more general unfolding to the problem, but given the intractable form of the solution, I will show numerically how a linear strategy in price will be dominated by a quadratic one and show how the quadratic strategy could approximate the Bayes' rule strategy.

3.1. Equilibrium Strategies in the Trading Game

3.1.1. The Market Maker Problem

Suppose that the Market Maker believes that the insider is using a strategy $x(v): v \rightarrow \mathbb{R}$ with $x'(\cdot) \geq 0$ (a monotone, non-decreasing strategy), the joint distribution of q and v after observing order flow q is given by:

$$f(q,v)=\phi(\frac{(v-v_0)}{\sigma_v})\phi(\frac{q-x(v)}{\sigma_n})=\frac{e^{\left(-\frac{(q-x(v))^2}{2\sigma_n^2}-\frac{(v-v_0)^2}{2\sigma_v^2}\right)}}{2\pi\sigma_n\sigma_v} \tag{6}$$

as $n = q - x$ and independent from v. By applying Bayes' rule:

$$f(v|q)=\frac{1}{F_q(q)}e^{(-\frac{(q-x(v))^2}{2\sigma_n^2}-\frac{(v-v_0)^2}{2\sigma_v^2})} \tag{7}$$

Where

$$F_{(q)}(q)=\int_{-\infty}^{+\infty} e^{-\frac{(q-x(v'))^2}{2\sigma_n^2}-\frac{(v'-v_0)^2}{2\sigma_v^2}}dv' \tag{8}$$

Since market maker is competitive, he sets the price $p = E[v|q]$ and so:

$$p(q)=\frac{g_q(q)}{F_{(q)}(q)} \tag{9}$$

Where

$$g_q(q)=\int_{-\infty}^{+\infty} v'e^{-\frac{(q-x(v'))^2}{2\sigma_n^2}-\frac{(v'-v_0)^2}{2\sigma_v^2}}dv' \tag{10}$$

Equation (9) pins down $p(q)$ given $x(v)$. If the market maker (rightfully) conjectures that for some cutoff $\mathbf{v}$ the strategy of the insider will be

$$x(v)=\begin{cases} f(v) & v\geq \mathbf{v} \\ \bar{x} & v<\mathbf{v} \end{cases}$$

(9) becomes

$$p(q)=\frac{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)\int_{-\infty}^{\mathbf{v}} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{\mathbf{v}}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v)}{\sigma_n}\right)\mathrm{d}v}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)\Phi\left(\frac{\mathbf{v}-v_0}{\sigma_v}\right)+\int_{\mathbf{v}}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v')}{\sigma_n}\right)\mathrm{d}v'} \tag{11}$$

Lemma 3. *A horizontal asymptote characterizes the pricing function used by the insider in the constrained equilibrium at* $E[v|v < v]$*, which is*

$$\lim_{q \to -\infty} p(q) = E[v|v < v] \tag{12}$$

Proof. See appendix A.1.

Imposing the trading constraint, the shareholder can guarantee a price that will be bounded below. When the market maker observes a low order flow, he assigns more and more probability to a constrained insider. Thus, at the limit, the price will converge to a conditional expectation of $v < v$.

3.1.2. Insider's Problem

Given the induced constraint from the contract, the insider will only submit constraints orders, that is $x(v) \geq \bar{x}$. If insider believes that the market maker will set price $p(q)$, then the insider will solve the following problem[6]:

$$x(v) := \arg\max_{x \geq \bar{x}} \quad x(v - E[p(x+n)|x]) \tag{13}$$

$$s.t. \quad p(x+n) = E[v|q] \tag{14}$$

When the constraint on quantity is not binding, x(v) is such that it solves

$$(v - \bar{p}) - \bar{p}'x = 0 \tag{15}$$

[6] After signing the contract the insider knows the value v of the company. His profit from trading is

$$\begin{aligned} E[x(v - p(q))|x] &= xE[v - p(x+n)|x] \\ &= x(v - E[p(x+n)|x]) \end{aligned}$$

Where

$$\bar{p} = E[p(x+n)|x]$$

$$\bar{p}' = \frac{\partial}{\partial x} E[p(x+n)|x]$$

Moreover, the Second-order condition is satisfied whenever:

$$-x\bar{p}'' - 2\bar{p}' < 0 \tag{16}$$

Where

$$\bar{p}'' = \frac{\partial^2}{\partial x^2} E[p(x+n)|x]$$

3.1.3. Fixed Point Analysis

Proposition 4. *In the trading game, there exists at least one equilibrium where*

• *the strategy of the insider is characterized by a cut-off* $\nu = f^{-1}(\bar{x})$

$$x(v) = \begin{cases} f(v) & v \geq \nu \\ \bar{x} & v < \nu \end{cases}$$

with f(x) to be non-decreasing and continuous function.

• *The market maker strategy is right-continuous and increasing in* $q \in \mathbb{R}$

Proof. Given equations (11), (15) and (16), an equilibrium in the trading game consists of two strategies $p(q)$ and $x(v)$ such that (11) and (15) are satisfied for all v, and (16) is verified. It is easy to see that whenever x is 0 (the insider is not trading), equation (16) is satisfied if and only if the filtering of $p(q)$ with respect to n is strictly increasing with respect to x. Notice that the problem is a system of integrodifferential equations.

I first start by analyzing the property of the optimal $x(v)$ whenever the short-selling constraint is not binding. Assuming p is non-decreasing in the order flow, then if (15) binds

$$x = \frac{v - \bar{p}}{\bar{p}'} \tag{17}$$

By deriving (15) with respect to v when the constraint is not binding

$$\frac{\partial}{\partial v}[(\text{v-}\bar{p})\text{-}\bar{p}\text{'x}] = 0$$
$$1 - \bar{p}'\frac{\partial x}{\partial v} - \frac{\partial x}{v}(\bar{p}''x + \bar{p}') = 0$$
$$\frac{\partial x}{\partial v} = \frac{1}{\bar{p}''x + 2\bar{p}'} > 0 \tag{18}$$

Where the last inequality holds if and only if (16) holds. Unsurprisingly, (18) is telling us that $x(v)$ is an increasing function of v, confirming that, confirming that the better the firm's prospects are, the more the insider is willing to buy. I define with $\nu := \ sup\{v|x(v) \ = \ \bar{x}\}$. That is ν is the cutoff level on v, such that the insider is indifferent between selling $\bar{x}$ or a slightly higher amount. Thus, $x(v)$ is a continuous and non-decreasing function in v.

The second statement in the Proposition is trivial, given Bayes' rule and continuity of the order flow space, given the support of the noise and cash-flow distribution.

The major challenge comes into evaluating the second derivative, In fact:

$$\frac{\partial^2 x}{\partial v^2} = -\frac{x\bar{p}^{(3)} + 3\bar{p}''}{(x\bar{p}'' + 2\bar{p}')^3} \tag{19}$$

Now, since the denominator of (19) is always positive for (16), one must characterize the numerator's sign. When p is linear in q (i.e., the market maker uses an Ordinary Least Squared (OLS) estimator to set prices), one can easily see that the numerator is zero and the optimal strategy for the insider is linear in v. Thus, the insider's conjecture about the strategy of the market maker feeds into the convexity of the function following the sign and magnitude of the second and third derivatives of p.

3.2. Linear Strategy for the Market Maker

As previous literature has focused on linear strategy as equilibrium for the unconstrained problem, I restrict the market maker to use this functional form for the pricing strategy. To ease exposure, I set $\bar{x}$= 0, but the results hold. In a constrained problem with $\bar{x}$= 0, I analyze a linear strategy for the market maker and insider, and I argue a more flexible strategy dominates this strategy. This exercise sheds some light on the insider's behavior and the market maker in the context where trading can only be positive. The market maker set price

$$p(q) = \mu + \lambda q \tag{20}$$

Under zero-profit conditions, this will imply that the market maker uses a linear projection as the best predictor of price given the observed quantity. Recall that when both v and q follow a Normal distribution, the linear projection coincides with the conditional expectation of v given q. Instead, when one of the two variables is not Gaussian, the linear projection of v onto q is the minimum variance linear predictor of v given q.

3.2.1. The Insider Problem

The discussion follows Kyle (1985), but one must consider an additional constraint of the order space to be non-negative, $\bar{x}$ = 0.

If the insider anticipates that the market maker will set price $p(q) = \mu + \lambda q$, he solves the following problem:

$$\max_{x \geq 0} \quad x(v - \mu - \lambda x) \tag{21}$$

leading to the strategy:

$$x(v) = \begin{cases} \frac{v-\mu}{2\lambda} & v \geq \mu \\ 0 & v < \mu \end{cases} \tag{22}$$

In a standard fashion, for v ≥ μ I have that

$$\begin{cases} \alpha = & -\mu\beta \\ \beta = & \frac{1}{2\lambda} \end{cases} \tag{23}$$

Moreover

$$\nu = \mu \tag{24}$$

3.2.2. The Market Maker Strategy when $\bar{x} = 0$

Suppose that the market maker hypothesizes that the manager will submit orders according to the following strategy

$$x(v) = \begin{cases} \alpha + \beta v & v \geq \nu \\ 0 & v < \nu \end{cases} \tag{25}$$

Using the estimation procedure I have that

$$\begin{aligned} p =& E[v|q] \\ p =& E[v] + \frac{cov(v,q)}{var(q)}(q - E[q]) \\ p =& \left(v_0 - \frac{cov(v,q)}{var(q)}(\frac{\beta\sigma_v e^{-\frac{(\nu-v_0)^2}{2\sigma_v^2}}}{\sqrt{2\pi}} + (\alpha+\beta v_0)(1-\Phi\left(\frac{\nu-v_0}{\sigma_v}\right))) \right) + \frac{cov(v,q)}{var(q)}q \\ p =& \mu + \lambda q \end{aligned} \tag{26}$$

3.2.3. Equilibrium Analysis

To find the equilibrium coefficients ν, α, β, μ, λ, one needs to solve the following system of equations:

$$\nu = \mu \tag{27}$$

$$\alpha = -\mu\beta \tag{28}$$

$$\beta = \frac{1}{2\lambda} \tag{29}$$

$$\mu = v_0 - \lambda\left(\frac{\beta\sigma_v e^{-\frac{(\nu-v_0)^2}{2\sigma_v^2}}}{\sqrt{2\pi}} + (\alpha+\beta v_0)(1-\Phi\left(\frac{\nu-v_0}{\sigma_v}\right))\right) \tag{30}$$

$$\lambda = \frac{cov(v,q)}{var(q)} \tag{31}$$

By (27), (28), and (29) , one can use (30) to show

$$(\nu - vo) = -\frac{\sigma_v e^{-\frac{(\nu-v_0)^2}{2\sigma_v}}}{\sqrt{2\pi}\left(1+\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)\right)} \tag{32}$$

$$\frac{\partial\nu}{\partial v_0} = 1 \tag{33}$$

$$\frac{\partial\nu}{\partial\sigma_v} = -\frac{e^{-\frac{(\nu-v_0)^2}{2\sigma_v}}}{\sqrt{2\pi}\left(1+\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)\right)} \tag{34}$$

From (32), ν is smaller than v_0. Moreover, while the cutoff increases with the prior of the distribution, this also decreases with the standard deviation. It is worth noticing that now the insider will like to buy in a situation he was willing to sell without the selling constraints. For the indifferent type ν, this will imply that the expected price is lower under the constrained regime than the unconstrained one.[7]

[7] Notice that the opposite will be true if there was a buying constraint. (32) will be exactly the same but with a positive sign this time. Now if before the type ν was buying is now willing to non-sell, meaning that all else equal for this type, the expected price should be higher.

In this scenario, the insider will trade more aggressively (higher beta) to compensate for the profit he would have to give up if the signal is negative. The market maker will lower λ, as now more order flows are less correlated with v, making the market for the share more liquid, which will turn feedback into the insider strategy. In fact, with both v and n distributed as a standard Normal distribution, $\nu \approx -0.27$ and $\lambda \approx 0.44$ and $\beta = 1/2\lambda$. However, now the mean of q is approximately 0.55 β. The price is still unbiased in expectation. Order flows have a lower impact on price compared to the standard case, fostering liquidity, and so reducing the speed of adjustment of price to information, which is in line with Diamond and Verrecchia (1987).[8] Compared to the situation in Proposition (2), the insider can no longer perfectly trade by mimicking the distribution of noise. The semi-separation induced by the constraint does not allow him to mimic the noise trader's variance, as he cannot sell, and it impacts the expectation of his order flow.

This linear pricing function is not optimal for the Market Maker, as he would be better off using Bayes' rule. I now study the function's asymptotic behavior that would result from Bayes' rule if the insider were to use a linear strategy in his order flow.

3.2.4. Asymptotic Behavior of the Real Conditional Expectation

In order to break even, the Market Maker should set a price that is indeed different from the linear price schedule used in this section:

$$p(q) = E(v|q) \tag{35}$$

Now suppose that the insider is using the following strategy:

[8] From Diamond and Verrecchia (1987): "Constraints eliminate some informative trades, but do not bias prices upward. Prohibiting traders from shorting reduces the adjustment speed of prices to private information, especially to bad news".

$$x(v) = \begin{cases} 0 & \text{if } v < v \\ f(v) & \text{if } v \geq v \end{cases} \tag{36}$$

with f (v) = 0 and f '(v) ≥0, that is continuous and non-decreasing strategy. W.l.g., assume [v,n]~ $N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right))$, I have the following:

$$p(q) = \frac{\int_{-\infty}^{v} v * \phi(v)\phi(q)dv + \int_{v}^{+\infty} v * \phi(v)\phi(q - f(v))dv}{\int_{-\infty}^{v} \phi(v)\phi(q)dv + \int_{v}^{+\infty} \phi(v)\phi(q - f(v))dv} \tag{37}$$

Now, I notice that the first terms in the numerator and denominator depend only on the insider's strategy for v, and it is constant to the conditional expectation/probability of v being lower than v observing q. The first term in the numerator will be always equal to:

$$\int_{-\infty}^{v} v * \phi(v)\phi(q)dv = -\frac{e^{-\frac{v^2}{2} - \frac{q^2}{2}}}{2\pi} \tag{38}$$

This term, first of all is negative, due to the fact that the expectation is taken on the negative part of the distribution of v. In general this will be lower than the unconditional expectation of v.

Let us focus our attention to the following set of strategy for the insider. The insider is using a linear strategy such as:

$$x(v) = \begin{cases} 0 & v < v \\ \beta(v - v) & v \geq v \end{cases} \tag{39}$$

with β > 0. This makes the expression in 37 closed form:

$$p(q) = \frac{\beta e^{\frac{q^2}{2}} \left((\beta v + q)\Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right) - \beta\sqrt{\beta^2+1}\frac{e^{-\frac{(v-\beta q)^2}{2(\beta^2+1)}}}{\sqrt{2\pi}} \right)}{(\beta^2+1)\left(e^{\frac{q^2}{2}}\Phi\left(\frac{\beta q - v}{\sqrt{\beta^2+1}}\right) + \sqrt{\beta^2+1}\Phi(v) e^{\frac{(\beta v+q)^2}{2(\beta^2+1)}} \right)} \tag{40}$$

Following Lemma 3, one can show that

$$\lim_{q\to-\infty} p(q) = \frac{\beta}{1+\beta^2}\frac{e^{-\frac{v^2}{2}}}{\Phi(v)}\left(0 - \frac{\sqrt{\frac{2}{\pi}}}{\beta} - \sqrt{\frac{2}{\pi}}\beta\right) = \frac{-\sqrt{\frac{2}{\pi}}e^{-\frac{v^2}{2}}}{\Phi(v)} = E[v|v < v] \tag{41}$$

and more importantly

$$\lim_{q\to+\infty} p'(q) = \frac{\beta}{1+\beta^2} \tag{42}$$

I plot the above pricing function in Figure 2, given the result of the analysis with a market maker using OLS, that were $\nu = -0.27$ and $\beta = 1.13$.

If one considers two standard normal distribution and the unconstrained equilibrium, the term in (42) is precisely the market maker strategy’s coefficient. This result is generalizable to a situation with different mean and variance.

3.2.5. Goodness to Fit

Figure 2, shows that a linear approximation fails to capture the curvature in the conditional expectation that a selling constraint will induce. To understand the goodness to fit of the above function, given the distribution of order flows, I simulate order flows and price reactions according to equations (25) and (26). To smooth out noise in conditional expectation, I numerically solve for the Bayes’ rule price function and

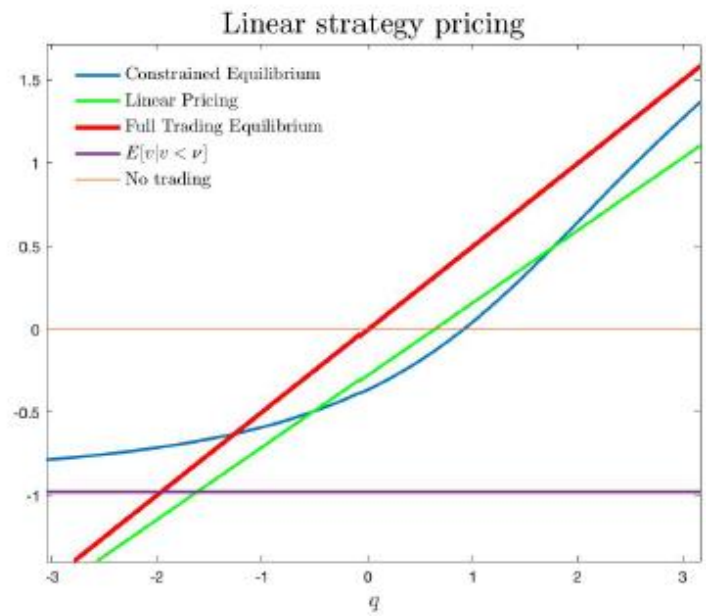


Figure 2. Asymptomatic Behavior of Conditional Expecation

interpolate and weight the observation. I consider different values for both σ_n and σ_v, while w.l.g. I fix v_0 at zero. A regression of the fitted price given OLS versus the Bayes' rule pricing function results in a R-squared of 93% on average, a insignificant intercept and a slope not significantly different from 1. The average pricing error is close to zero. These results are encouraging, but the conditional expectation function's shape suggests some curvature in the pricing function, thus the need for some polynomial approximation for it.

3.3. Quadratic Approximation of Market Maker Strategy

I now guess that the strategy used by the market maker is:

$$p(q) = \mu + \lambda_1 q + \lambda_2 q^2 \tag{43}$$

Using OLS, this predictor should still satisfy the assumption of minimum variance. Notice that the function is no longer monotone and has a minimum (or a maximum) at

$$q^{min} = -\frac{\lambda_1}{2\lambda_2} \tag{44}$$

$$p^{min} = \mu - \frac{\lambda_1^2}{4\lambda_2} \tag{45}$$

I claim the point with the above coordinates is the minimum of the pricing function, which simply means that $\lambda_2 > 0$. Being careful, one should interpret the minimum price as the price that the market maker should be setting for any order that is indeed less than q^{min}, following Lemma 3.

3.3.1. Insider Strategy and Market Maker Response

Under the above price function, assuming a positive λ1, the insider chooses x(v) to solve the following problem:

$$\max_{x \geq 0} \quad x(v - \mu - \lambda_2\sigma_n^2 - \lambda_1 x - \lambda_2 x^2)$$

The solution for the FOC is[9]:

$$x(v) = \begin{cases} \sqrt{\frac{1}{3\lambda_2}v - \frac{\mu+\lambda_2\sigma_n^2}{3\lambda_2} + \left(\frac{\lambda_1}{3\lambda_2}\right)^2} - \frac{\lambda_1}{3\lambda_2} & if \quad v \geq \mu + \lambda_2\sigma_n^2 \\ 0 & otherwise \end{cases}$$

I need to make sure that x(v) is positive and real. Thus, one need v to be big enough. Notice that x now follows a half normal. Given the not close form of the Chi-distribution mean and variance, I simulate the model.

Defining the following:

[9] For the problem to have a solution, one need to impose some restriction on the parameters. In particular, the positive solution to the FOC will guarantee a maximum for both λ s positive. While for λ2 negative, I need λ1 to be high enough. The solution of the unconstrained problem gives only local minimum and maximum, centered around the value of -λ1/3λ2 while imposing x non-negative, one can pick only one solution.

$$\beta = \frac{1}{3\lambda_2}$$
$$\alpha_1 = -\frac{\mu + \lambda_2\sigma_n^2}{3\lambda_2}$$
$$\alpha_2 = -\frac{\lambda_1}{3\lambda_2}$$
$$v = \mu + \lambda_2\sigma_n^2$$

The Market Maker observes q and uses OLS according to (43).

3.3.2. Simulations

To close the model, I perform a convergence loop. I generate normal distribution observations for v and n, both with mean zero, w.l.g. I used a starting point for the coefficient in x. I then compute x and q and I performed a Multivariate OLS to fit the price. The usual formulas for MOLS apply here.

After computed the lambdas, I used the functional form of x to compute the coefficients and generated the new x. I compute the difference in norm on the coefficients for the OLS regression regression and reiterate the process until its absolute value will converge to zero. I performed the simulation on a varying sample from 1000 to 1000000 observations and with multiple starting points. The estimation is not sensible to sample size, but starting points must be far enough from zero, otherwise, the resulting matrix for OLS will be singular, and estimation will be biased.

No matter the starting point, lambdas are always positive, that guarantees a maximum in the insider problem and a minimum price.

The market maker strategy can be approximated by the following coefficient, when $\bar{x} = 0$

$$\mu = -0.3\sigma_n$$
$$\lambda_1 = 0.35\frac{\sigma_v}{\sigma_n}$$
$$\lambda_2 = 0.054\frac{\sigma_v}{\sigma_n^2}$$

The strategy of the insider is thus:

$$\beta = 6.12\frac{\sigma_n^2}{\sigma_v}$$

$$\alpha_1 = 1.5\sigma_n^2$$

$$\alpha_2 = -2.13\sigma_n$$

$$\nu = -0.244\sigma_v$$

3.3.3. Some Graphic Analysis for Asymptotic Behavior

The major problem with a quadratic approximation is the impossibility to have a closed form of the integral for the conditional expectation, differently under the assumption of a linear strategy.

I numerically solve for the strategy of the Market Maker assuming that $f(v) = \sqrt{\beta v + \gamma} + \alpha$. The above strategy results from a conjecture of the insider to use a quadratic price approximation, given some sign conditions on {β, γ, α}. The function does not differ so much to the conditional expectation given a linear f(v). One could plot the numerical approximation and see that the conditional expectation is very similar to the one under the linear and the root strategy.

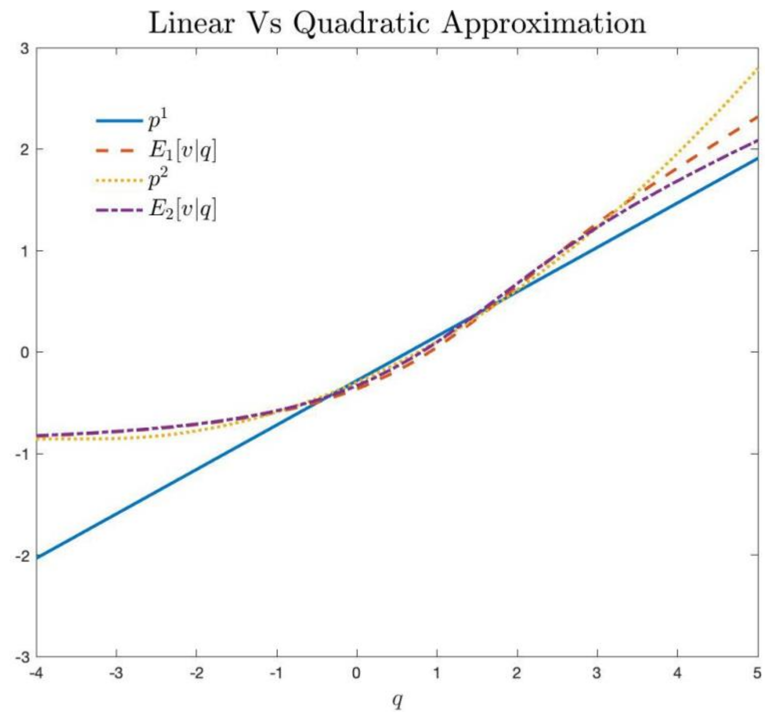


Figure 3. Quadratic Approximation and Comparison

With the “solution” parameter, one could plot the above function again and notice the very close resemblance to assuming a linear strategy. Moreover, most of the errors come from observations of q that are for more than two standard deviations from the average. Thus at least 95% of the time, the Market Maker is indeed pricing according to Zero profit condition. The second-order approximation is accurate in the lower part of the distribution of order flows (where the probability of facing an insider is low, so order flow lack informativeness). However, the Market Maker overvalues positive order flows. To further investigate the goodness of approximation, I solve the integral numerically and interpolate the conditional expectation for some simulated order flows. The OLS regression between the approximated price and conditional expectation shows an R-squared of 99%, with a slope coefficient of 1, very high significance, and an intercept of zero.

Although the approximation is still somewhat far to perfectly match the function suggested by Bayes’ rule, the shape is still similar. This observation would suggest that the market maker's behavior converges to some curved function, potentially linear for q → +∞, but the behavior of the insider is somewhat converging to a concave function. Starting from this consideration, I perform the fixed-point algorithm on simulated observations. These simulation results are qualitatively like the above approximation, both for the insider strategy and the market-maker one.

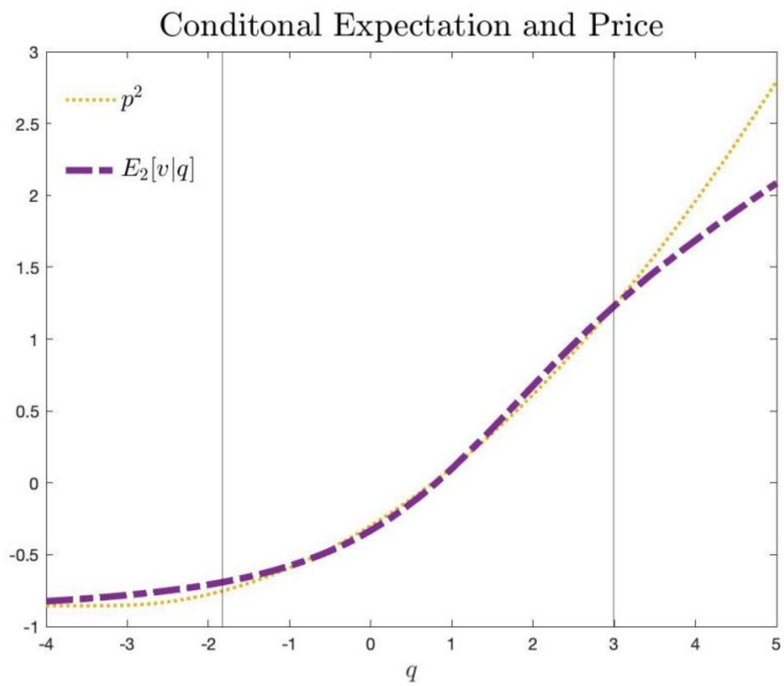


Figure 4. Square Strategy and Conditional Expectations

3.4. Analysis of the distribution of the posterior price

Without trading from an insider, the market maker has no means to infer the company's value from the order flow. Therefore, the price of the company will be the expected value of the cash flow.

On the other hand, allowing the full possibility of trading convey some information to the market, as the insider will shed his order flow according to 2, and the posterior distribution of the price will follow a Normal distribution with mean v_0 and variance $1/2\sigma_v^2$. In that situation, the noise traders provide camouflage to the insider. His order flow has (ex-ante) the same mean and variance as those of the noise trader. Thus, the market maker understands that only "half" of the order flows provides news about company fundamentals.

With trading constraints, the posterior expected value of the price is still matching the prior expected value. However, the price variance is now further decreased. The market maker understands that only a fraction of the order now contains information, as in Diamond and Verrecchia (1987). More negative order flows convey less information, as the probability of an unconstrained insider being present is low. In our approximation, the variance of the price is about 26% the variance of the cash flow, while the variance of the order flow from the insider is only 45% of the variance of the noise trader, and they cannot use the whole support of the distribution to mimic the noise traders' order flow. The insider's inability to perfectly camouflaging the order flow has thus two different effects on price. The first one is on their level. Prices will converge to a lower bound that is in line with the company's expected value under selling constraint to the insider.

Moreover, the ex-post distribution is less dispersed, given this bound. We can further notice the following. While the constrained price will be higher than the unconstrained price for some negative order flows, for the market maker to break-even in equilibrium, he must set an equal or lower price for more positive order flow. Thus, while buying the stock will be cheaper, all else equal, traders selling the stock will either more proceeds if the order flow is shallow or a worse deal for small selling order flows.

3.5. Trading Constraints

To simplify the discussions, I analyzed the trading constraints as selling constraints. However, the shareholder could set any sort of $\bar{x}$ as a constraint in the model. While it is easier to interpret a selling constraints ($\bar{x} \leq 0$) as an initial share grant at t=-1 and a subsequent short-selling constraint imposition, which will result in a minimum trade of $\bar{x}$, a buying constraint ($\bar{x} > 0$) can be interpret as stock vesting with the cliff at t=1.

I can specify equilibrium for different trading constraints. I define the following:

Definition 5 (Constrained Trading Equilibrium). An equilibrium in the constrained trading game is such that:

• The insider chooses $x(v) \geq \bar{x}$ to maximize his expected utility,

• The market maker sets price $p^C(q)$ according to Zero Profit condition, considering the constraint $\bar{x}$

• The Shareholder sets $\bar{x}$ such that $w = -\infty\ \forall x < \bar{x}$ and to maximize her utility

Definition 6 (No Trading equilibrium). The No Trading Equilibrium has $w = -\infty\ \forall x \neq 0$, $x(v) = 0 \forall v$ and $p^{NT}(q) = v_0 \forall q$.

Definition 7 (Full Trading equilibrium). The Full Trading Equilibrium has $w = 0\ \forall x$, and $x(v)$ and $p^T(q)$ as in Proposition 2

Definition 5 is a specific restatement of Definition 1, where I specify the constrained trading strategy and the amount of punishment. The No Trading Equilibrium is intuitive: the insider is not allowed to trade. Thus, all variability in order flows comes from the noise trader; the price will not reveal any information. In the Full Trading Equilibrium, the insider can mimic the noise trader perfectly. Thus, only half of the information coming from order flows is incorporated into the price, as the classical model of Kyle (1985). In a constrained equilibrium, the shareholder can pick $\bar{x}$. The resulting equilibrium will thus be in between the other two, as the insider will only partially mimic the noise trader. By Bayes' rule, the

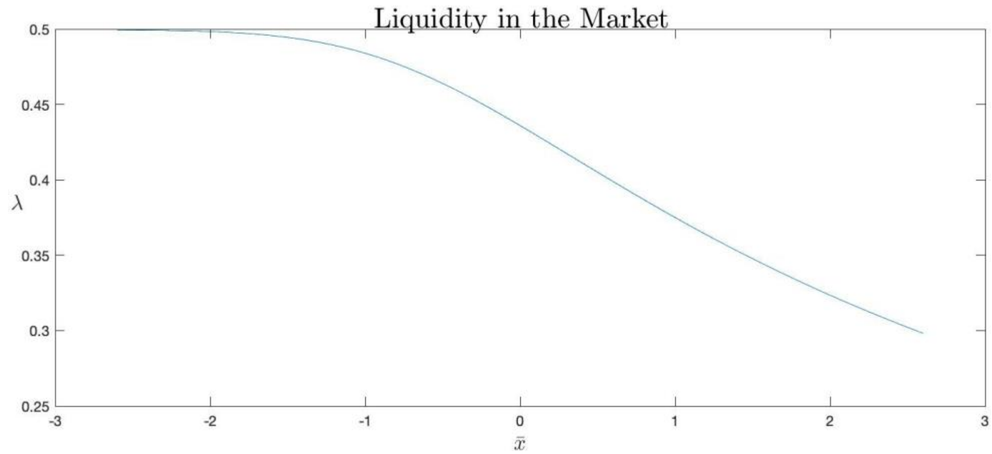


Figure 5. Liquidity

market maker will always match the expected price with the prior of the distribution, independently on the imposed constraints and the variance of the price is decreasing in the amount of constraint imposed. Lastly, as $\bar{x} \rightarrow +\infty$, the price process of the constrained equilibrium converges in distribution to the No Trading Equilibrium, and, as $\bar{x} \rightarrow -\infty$, the price process of the constrained equilibrium converges in distribution to the Full Trading Equilibrium. My tie-breaking assumption is that if $\bar{x}$ approaches $-\infty$, the Full Trading is the preferred equilibrium (being both the noise and the cash flow normally distributed, I will consider this cut-off to be 3 standard deviation around the mean of order flows), while as $\bar{x}$ approaches $+\infty$, I pick the No Trading equilibrium.

3.6. Price efficiency and liquidity

To study the impact of the constraints on standard measures of market functioning, the second-order approximation methodology highlighted in the previous subsection allows for a visual comparison of the Trading game equilibrium characteristics.

As the insider is now constrained, he can only mimic the behavior of the noise trader (as in the unconstrained equilibrium), up to some fraction of the variance of the noise trader, and the more stringent the constraints become, the further his ability to match moments of the noise trader distribution decreases. Consequently, the market maker realizes that less order flows now are not carrying all the information, so the average impact of the order flow, measured as the inverse of Kyle's lambda, is increasing. Kyle's lambda measures, however, the first-order impact of order flows on price. As order flows convey less information when the constraint tights, the market maker reduces the average sensitivity of order flows.

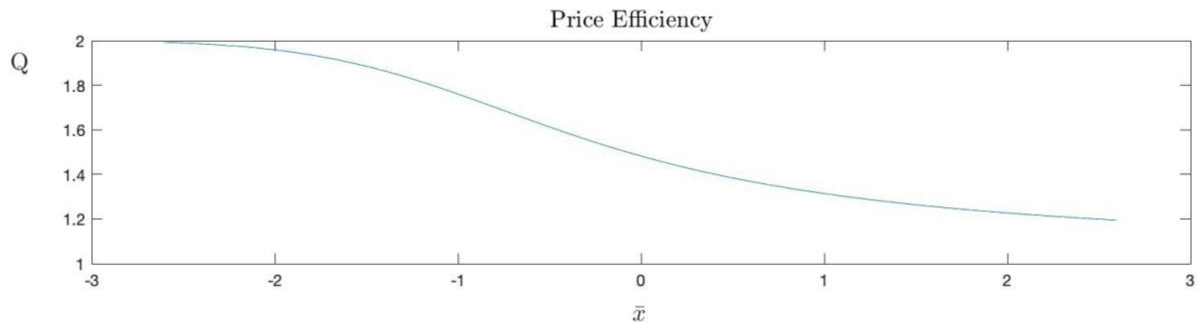


Figure 6. Price Efficiency

If I look at Q as one of the standard measures of price efficiency:

$$Q = Var(v|p)^{-1}$$

This measures prices' average predictive power. Not surprisingly, when the market becomes more liquid, less information is embedded in the price. Thus the price's capability to reflect fundamentals worsens.

However, I can compute the following Absolute Pricing Error APE($\bar{x}$, v)

$$APE(\bar{x}, v) = E_{\bar{x}}[|v - p(q)||v]$$

this measure considers the absolute distance from v of p(q) given the imposition of a trading constraint under $\bar{x}$. Notice, that without constraint

$$APE(FT, v) = \frac{|v - v_0|}{2}$$

,while for a contract that grants no trading rights

$$APE(NT, v) = |v - v_0|$$

Notice that for v and −v, the two measures are equivalent. In these two trading regimes, the APE and thus market efficiency (and the pricing error) are linear in the state's absolute value. This is not the case if I look instead at the pricing functions resulting in the constraints' imposition. While, on average, APE is above the one in the full trading equilibrium, there is a tremendous disparity between good or bad states of the world. In fact, as v goes to $-\infty$ the APE converges to the one under the No Trading Regime. Instead, if the state of the world is good, the APE is concave, and for the extreme value of v, the APE is indeed lower than the one under Full Trading. This difference highlights the primary result of the paper. Under unilateral trading constraints, the pricing function has two main characteristics: the sensitivity to order flows is non-constant, and the price efficiency is non-linearly state dependent. In bad states of the world, less information is incorporated into prices due to binding trading constraints. Thus, prices do not reflect fundamentals. Instead, in good states of the world, the prices maintain a similar level of informativeness as under the Full Trading regimes, while they become even more informative for excellent states of the world. This result is in line with empirical patterns that speak about the ability of insiders' purchases to predict positive abnormal returns while confirming that sales have less predictive power, as they are associated with liquidity shocks or the desire to disinvest, but because of the implicit constraints on trading. In bad states of the world, insiders will not trade, and as a result, the price will not adjust to negative information. This model has no room for liquidity trading from the insider. However, I could expect that the same results will hold as long as the firm can observe the insider's transaction, with the caveat, the contract will now specify both regions of the state space and order flows where the insider is allowed to trade. This result will be further accentuated if, on top of liquidity trading, there are effects on cashflows following the punishment of the insider.

The next section will characterize the optimal level of $\bar{x}$, under different specification of the shareholder's utility.

CHAPTER 4: SHAREHOLDER'S PROBLEM

In this section, I will study various utility specifications for the shareholder and when the different regimes of trading will emerge in equilibrium. The shareholder has some preferences for the price of the company's interim price, rather than for fundamentals.[10]

If shareholders do not allow the insider to trade, order flows will not contain any information, the price of the company will be equal to its expected value. The price will not reveal any information, and stakeholders

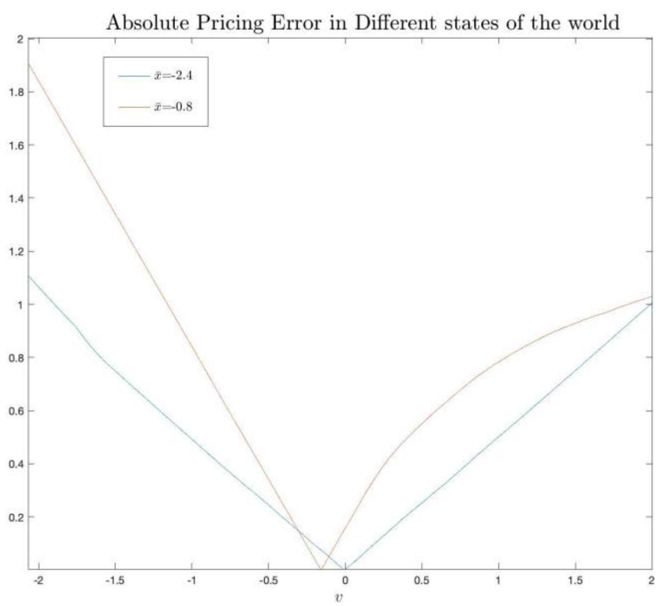


Figure 7. Absolute Pricing Error

[10] With this in mind, one should see the risk-neutral pricing the market maker is doing as facing a risk-neutral marginal uninformed investor, independent from the shareholders preferences towards price. As in the model, the shareholder is not trading directly, or only trading when she receives a liquidity shock, one should see the insider as the marginal trader the market maker faces.

cannot base further decisions on it. The distribution of the company price will collapse to a degenerated point. [11]

The contract specifying full trading allows for each order flow to carry some information. As a consequence, price is somewhat impacted by the order flow, and the posterior variance is half of the prior variance. However, in this case, the price could reveal lousy information on time, and if the shareholders care about the firm's price, this contract will not be beneficial for them.

A contract generated by trading constraints will allow the shareholder to benefit from good information while limiting the downside risk by imposing a lower bound to the price distribution. On average, the price will be lower for the upper part of the distribution of cash flows but much higher for the lower part. By reducing the trading, the shareholders buy protection against too low value of the company. Low prices are thus less informative of the actual value of the company.

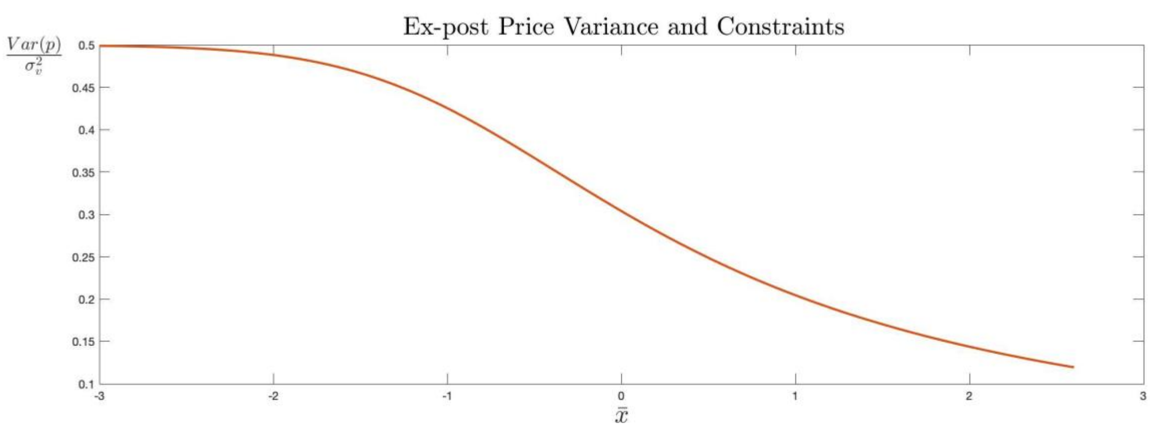


Figure 8. Associated Decrease in Variance as $\bar{x}\rightarrow\infty$

[11] Not allowing for the possibility of the insider to trade comes with a complete separation between ownership and control, and shareholders may see the value of the company impacted by agency frictions. Indirect costs may be associated with having a completely uninformative price, that cannot be useful to stakeholder of the company. Other direct costs can be cost associated to actual compensation costs, or litigation fees for imposing the punishment. While I acknowledge these considerations, the modeling of them is left for future research as they are not the main objective of this paper.

4.1. Risk-Neutral Shareholder

A risk neutral shareholder has preferences given by the utility function:

$$U^{\mathscr{W}}(p) = E_{\mathscr{W}}[p]$$

Given that the market maker uses Bayes' rule to update prices after observing order flow q, the ex-ante expectation over prices is not different from the prior of the distribution, thus for all order flow q and all types of trading constraints[12]

$$E[p(q)] = v_0$$

Thus, the shareholder is indifferent between enforcing or not trading constraints. This result should not be surprising as a risk-neutral shareholder acts as if she can diversify her holding in the company. However, when I interpret the shareholder as the company itself, to minimize its cost of capital, the argument in favor of a risk-neutral shareholder is faulty and cannot be applied generally.

4.2. Risk Averse Shareholder

In a situation where the shareholder has mean-variance preferences with a given degree of risk aversion, the problem will result in the minimization of the price variance. Figure 8 plots the variance of price, normalized by the variance of the cash flow, as the constraint gets tighter and tighter. From the figure, the shareholder will prefer to fully restrict trading (recall that p^{C} ^{d} p^{NT}, as $\bar{x} \rightarrow +\infty$). $+\infty$). In this situation the shareholder has symmetric preferences for both the $\longrightarrow$ adverse and favorable realization of the price. The shareholder suffers as much as insiders lick lousy information into the order flow, and if positive information arrives on the market. This

[12] In models à la Kyle (1985), the price process is a martingale with respect to the previous period order flow. In a two-period model, the expectation after trading in the first period, is simply the realized price. Thus a short-term oriented shareholder could potentially impact the price by allowing the insider to trade a total amount over the time of the game, as long as the market maker cannot anticipate the trading patterns correctly (thus with the addition of noise) of the insider. The result could differ, but I leave the discussion of a multi-period or continuous model for future research.

situation may apply to a shareholder who invests most of her wealth in the company over the short run. She cannot diversify away her holdings in the firm. Thus, she will prefer a stable price.

4.3. Asymmetric Utility

It is not surprising that a risk-averse shareholder will prefer no trading compared to a restricted equilibrium where only some trades are allowed. Given that the market maker applies Bayes' rule, any price process can be seen as a mean preserving spread of a lottery with a degenerated outcome at v_0. Thus, the lowest variance (No Trading) equilibrium will always second-order stochastically dominate the Constrained Trading Equilibrium and the Full Trading Equilibrium. However, while preferences characterized by risk aversion are useful and captures those dislike for risk that an individual may face, companies will appreciate upswing movements and fear negative swings in their market price. To capture these preferences for upside potentials, I will use preferences that have the following characteristics

Definition 8 (Short-term price utility function). An expected utility function

$$U^{\mathscr{W}}(p) = E_{\tilde{x}}[g(p)[(1-\omega)\mathbb{1}_{\{p-v_0\geq 0\}} + \omega\mathbb{1}_{\{p-v_0<0\}}]]$$

is a short-term utility function if:

- $g:\mathbb{R}\to\mathbb{R}$, $g(v_0)=0$ and $g'(p)\geq 0$
- $\omega\in[0,1]$

The short-term price utility function is characterized by a non-decreasing function centered around the prior of the cash flow distribution v_0, and a parameter ω is capturing the shareholder's preferences for downside risk. This parameter governs the expected utility by assigning relative weights to the realization of prices. However, I do not restrict the function to have a specific concavity or convexity. Examples of this kind of function are loss aversion functions or exogenous reference point functions. v_0 can be interpreted as a rescaling factor. This implies that all realizations of p north of the company's expected value carry more utility of realization south of the expected value. I believe these preferences can better describe firms' attitudes towards their price relative to classical mean-variance or risk-averse preferences. Companies will fear a price depreciation and enjoy the benefits of a better market price (think, for example, to a better cost of capital or a more profitable exit strategy for a large shareholder such as a fund). As this class of utility function has an asymmetric treatment of losses compared to gains, it encompasses prospect theory, as well as other non-classical preferences, but at the same time collapse to more standard preferences such as risk- neutrality (g(p) = $p - v_0$) or mean-variance (g(p) = $(p - v0) - \gamma(p - v_0)^2$) for ω = 1/2. The expectation is taken by considering then imposed constrained $\bar{x}$. Notice that the utility associated to the no-trading equilibrium is zero for all values of ω and all function g(p). Thus, the constrained or full trading equilibrium is preferred to the unconstrained one whenever

$$U^{\mathscr{W}}(p) \geq U^{NT}(p)$$

$$Prob_{\bar{x}}(p \geq v_0)(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0] + (1 - Prob_{\bar{x}}(p \geq v_0))\omega E_{\bar{x}}[g(p)|p \leq v_0] \geq 0$$

$$-\frac{(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0]}{\omega E_{\bar{x}}[g(p)|p \leq v_0]} \geq \frac{1 - Prob_{\bar{x}}(p \geq v_0)}{Prob_{\bar{x}}(p \geq v_0)}$$

$$\frac{(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0]}{\omega |E_{\bar{x}}[g(p)|p \leq v_0]|} \geq \frac{1 - Prob_{\bar{x}}(p \geq v_0)}{Prob_{\bar{x}}(p \geq v_0)} \quad (46)$$

Equation (46) is self-explanatory. The insider will prefer to allow for some trading if the ratio between the positive and negative utility is at least as good as the inverse odds ratio of the price distribution. Given normality, the odds ratio for the full trading equilibrium is equal to 1 w.p. 1, while it is not defined for the No Trading Equilibrium. The more constraints the shareholder adds, the higher is the odds ratio. The behavior of the denominator mostly dominates the left-hand side of the Equation. As $\bar{x}$ increases, prices result more and more bounded downward, and the denominator of the LHS decreases (and with the negative sign, thus the term is increasing). The effect of the constraints on the denominator is two-fold: on one side, imposing more and more constraints makes the price more sensitive to order flows and thus increase more, but the probability of observing some unconstrained order flows decreases, decreasing the expectation. The final effect will, however, depend on the concavity of the function g(·). The magnifying effect is given by ω. A lower ω increases the LHS, making the constrained equilibrium always preferred to the No trading. Notice that I may happen that the term of the LHS and RHS of the equations may meet only at $\bar{x} \to +\infty$, as $\omega \to 1$. When g(·) is a symmetric function, for the unconstrained equilibrium (Full trading), the LHS is simply $\frac{1-\omega}{\omega}$, implying that the full trading equilibrium is preferred to the non trading equilibrium any time $\omega \in \left[0, \frac{1}{2}\right]$.

Before further developing property of the equilibrium with the following utility function, let us consider some examples.

Example 9. Consider the following function

$$g(p) = (p - v_0)$$

For any $\omega = \frac{1}{2}$, the utility is simply a rescaling of risk neutrality. This utility form allows us to compute:

$$U^{NT}(p) = 0$$
$$U^{T}(p) = \frac{\sigma_v(1-2w)}{2\sqrt{\pi}}$$

Easy to see that the Full Trading Equilibrium is preferred to the No Trading Equilibrium whenever $\omega < \frac{1}{2}$. Given the linearity of the function, which is kinked for $p = v_0$, the solution is not smooth. Maximizing over the $\bar{x}$ space results in the Full Trading equilibrium for all $\omega < \frac{1}{2}$, while as soon as $\omega > \frac{1}{2}$, the preferred contract has No Trading Rights. When the contract restricts trading, the constrained price function lies below the unconstrained one in some positive quadrant area and above it in some area of the negative quadrant. As a consequence, an "upside risk lover" shareholder ($\omega < \frac{1}{2}$), does not want to sacrifice upside potentials. Whenever the loss matters more than the gains ($\omega > \frac{1}{2}$), the shareholder picks the No Trading Equilibrium, as it will ensure a higher price in state of the world she cares the most.

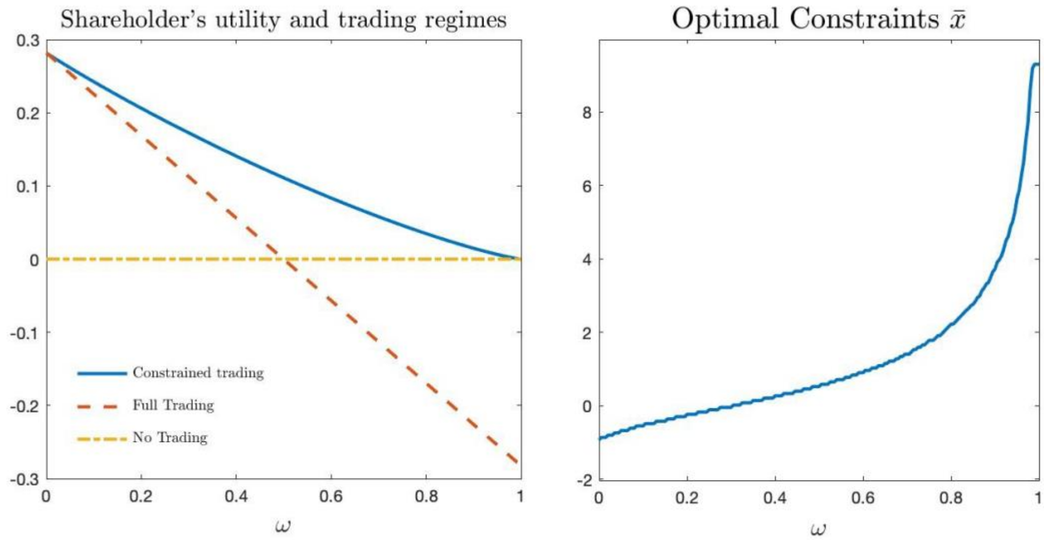
Example 10. Now instead consider

$$g(p) = (p - v_0)^3$$

Again

$$U^{NT}(p) = 0$$
$$U^{T}(p) = \frac{\sigma_v^3(1-2w)}{2\sqrt{\pi}}$$

This time however, I can show that a constrained equilibrium is indeed always preferred to the unconstrained one, and converges to the full constrained equilibrium for $\omega \to 1$. This utility specification is



$$g(p) = (p - v_0)^3$$

Figure 9. Optimal Constraints

capturing the asymmetric behavior of prices around its mean. This insider will always prefer a right-skewed distribution to a left-skewed one. This utility specification seems to best capture the company's behavior that wishes to maintain a substantial level of price. Moreover, this utility specification makes the shareholder willing to propose more and more restricted contract as $\omega \to 1$, and the constrained trading contact always dominates the full trading contract. Figure 9 plots the optimal constraints with the above utility specification. Not surprising is that a very downside risk shareholder will pose tighter constraints. However, even the most upside risk lover shareholder ($\omega = 0$) prefers to impose some constraints. This because the shareholder wants to ensure price to remain somewhat "high" in a negative state of the world, and at the same time allow the insider to trade freely (and transfer information into the price) in a good state of the world.

Example 11. To conclude, consider now

$$g(p) = 1 - e^{-(p - v_0)}$$

With constant absolute risk aversion (of 1)

$$U^{NT}(p) = 0$$
$$U^{T}(p) = \frac{1}{2}\left(e^{\frac{\sigma_v^2}{4}}\left((1-2w)\left(2\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)-1\right)-1\right)+1\right)$$

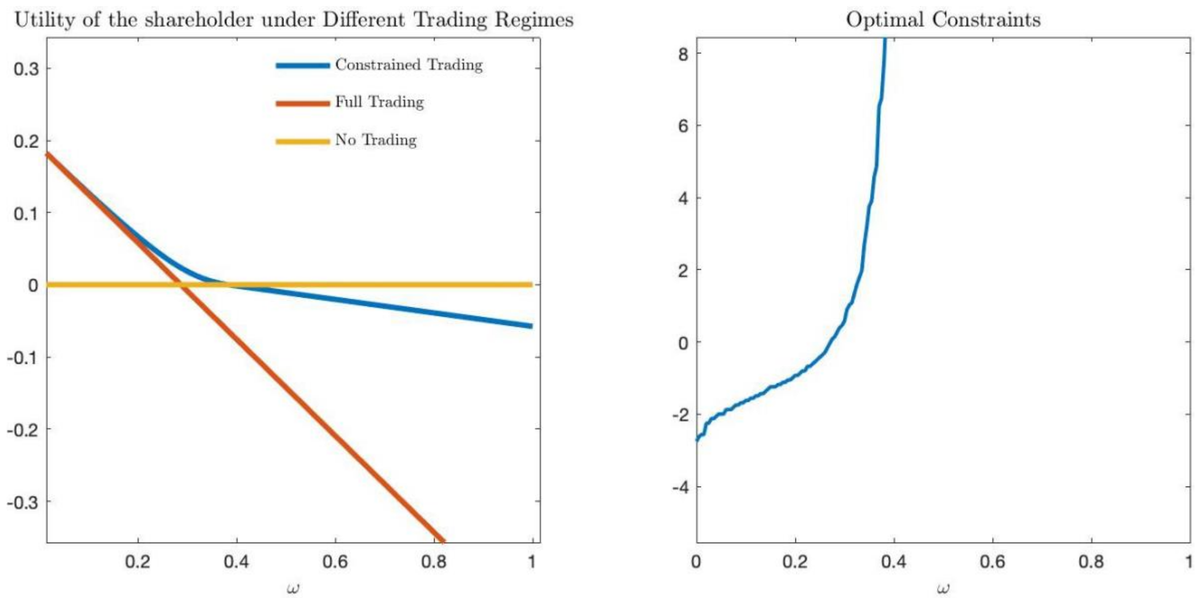


Figure 10. CARA Short-term Utility

Thus for $w \leq \frac{1-2e^{\frac{\sigma_v^2}{4}}\left(1-\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)\right)}{2e^{\frac{\sigma_v^2}{4}}\left(2\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)-1\right)}$, the Full Trading regimes dominates the Non-Trading regime. It is easy to see that as σ_v is associated with a decrease in this cut-off. The more the asset is volatile, the "sooner" the shareholder wants to shut down full trading. However, it may still have some value to impose some constraints without limiting trading itself. Reverting to simulations, there exists a level of ω for which the shareholder prefers to impose some constraints on the insider. Recall that, given CARA, and thus risk- aversion, all the price processes induced by the constraint are mean-preserving spread of the price process associated to $\bar{x} = +\infty$ (that is equal in distribution to the non-trading equilibrium). From the figure, I can see there are 2 regions in the ω space. In the first region the constraint moves from finite values of $\bar{x}$, where the constrained equilibrium is unique and always dominates the Full Trading. As ω → 0.5, $\bar{x}$ → +∞, and for the tie-breaking condition, for ω >0.5, the only equilibrium is the No-Trading Equilibrium.

The above examples illustrate an explicit feature of the model. Curvature in the utility function is fundamental to have the existence of an equilibrium in the constrained space. Without such curvature, the shareholder will prefer either the Full Trading or the No Trading equilibrium. As the utility gets some curvature, the shareholder finds it optimal to impose some constraints. Notice that a relatively high constraint is needed as soon as some curvature is imposed on the insider strategy. Moreover, as the shareholder becomes more and more downside averse, $\bar{x}$ increases, but while there exists some ω for which it is better to shut down trading if the utility captures risk aversion completely, this is not the case for preferences that instead look to skewness.

Lemma 12. *If g(·) is concave (the shareholder is risk averse), the No-Trading equilibrium dominates the Constrained Trading equilibrium for ω ≥ 1 . That is* $U^{W}(p) \leq U^{NT}(p)$ *for all* $\omega \geq \frac{1}{2}$.

Proof. See Appendix A.2

Corollary 13 (Comparative Statics on the optimal constraints). *The optimal amount of constraint* $\bar{x}$ *chosen by the shareholder is:*

- is increasing increasing in ω
- is increasing in σ_v if g(·) shows risk-aversion characteristics
- is increasing (decreasing) in σ_n for ω > (<)ω_0, where ω_0 is the level of ω for which $\bar{x}$ = 0 is the solution of the problem

Proof. See Appendix A.3

The above Lemma and Corollary states interesting facts about the equilibrium. First of all, it is worth notice that even if the shareholder is price risk-averse, as long as there is a higher appreciation for upside gains ($\omega < \frac{1}{2}$.), she is still willing to impose allow for some trading. This is because upside gains are weighted more and more. However, for the same level of ω, as the project's riskiness increases, she will limit the insider's action space more and more. The impact of σ_n is ambiguous. While the shareholder is willing to ride the market's noise for positive realization, she wants to limit the trading when the noise volatility results in a more probable low order flow. In that case, the shareholder wants to make prices insensitive to the order flows. Thus, she will impose more stringent constraints.

4.4. Who is Benefiting from the Constrained Equilibrium

As the insider cannot extract the totality of his advantageous information, and the market maker is reverting any loss from trading with the insider to the noise trader, the constraints are indeed protecting the noise trader. This consideration is the rationale behind the various insider trading regulations. This regulation serves the purpose of protecting the average market participant from the exploitation of the insider. Consequently, if the regulator's sole objective is to protect the noise trader, we should expect a complete shutdown of trades from the insiders. Nevertheless, to convict someone for the exploitation of Material Nonpublic in- formation is usually a lengthy and costly process, financed by taxpayers, the outcome of the investigation is not sure ex-ante. A complete shutdown of all the trading activities comes with associated agency problems and more expensive contracts for managers (Roulstone (2003)). Moreover, the SEC itself acknowledge the benefit for investors of observing insiders transactions[13] about the future value of companies.

[13] "Before investing, investors may wish to research insider ownership to consider the extent of insiders' economic stake in the success of the company, as reflected both in outright ownership and transactions (such as equity swaps) that may hedge the economic risk of that ownership. Many investors believe that reports of insiders' purchases and sales of company securities can provide useful information as to insiders' views of the performance or prospects of the company. Of course, insiders may sell company securities for any number of reasons, including for liquidity and diversification purposes." (https://www.sec.gov/ files/forms-3-4-5.pdf)

Companies could potentially impose these constraints to cater to their own needs rather than average market participants. As a manager can be fired after bad performances, the company could fire the insider after revealing information to the market. In the previous section, I established a relationship between the preferences for the company's interim price and the resulting constraints. However, these preferences are for the most influential shareholder in the company and may not represent one of the other shareholders. Once the firm imposes trading constraints to the insider, the market maker will react by setting a floor on the price distribution. As a consequence, compared to the Full trading equilibrium, to recover losses made in bad states of the world, where the insider is constrained, and the share is over-priced. The market maker has to underprice good states of the world. This underpricing is beneficial to a shareholder wishing either to sell a large stake of the company or to buy shares of the company, independently of the long position. However, the induced convexity of the pricing function implies that for some small and negative q, the price is lower than the unconstrained case. The proceeds from selling a small number of shares are thus lower or equal to the unconstrained case. The constraint, in that sense, will penalize a small shareholder willing to sell.

CHAPTER 5: CONCLUSIONS AND EMPIRICAL PREDICTIONS

In this paper, I analyze the role of myopic preferences of the shareholders and how these can impact the manager's incentives via the imposition of selling constraints. While shareholders should focus on developing contracts that incentivize the company's long-term value, when stakeholders of the company can make decisions based on market price, shareholders would like to postpone the revelation of negative information in the future. Imposing constraints on insiders' trading activity may help in achieving a less volatile and higher price. Imposing selling constraints on one side reduces informativeness of the order flows, but at the same time also reduces the price variance. The resulting price function is characterized by a lower bound. For very high order flows, the price becomes more reactive to them. For intermediate order flow, the price's behavior is convex, intending to revert the concavity of order flows induced by the selling constraint. As a consequence, the informativeness of price depends on their level. Higher prices are, on average, very informative, and as a consequence, markets are an efficient mechanism to communicate the firm's quality to stakeholders. On the opposite side, low-quality firms will be pooled together, and the resulting price conveys worse information, as now stakeholders cannot correctly infer the firm's quality. If those stakeholders are providers of capital, the terms they will impose on financing are the same, and some firms will benefit from being pooled together. Trading constraints may arise in equilibrium due to the variability of the company's cash flow. The more the company cash flow is variable, the higher is the probability of lower cashflows. By imposing more stringent trading constraints, the company's market price will not drop so much. At the same time, the firm can protect itself against swings in market conditions. The paper highlights the role of shareholder preferences for the company's interim price, trading restrictions, and ultimately price efficiency. When the most influential or significant shareholder of the firm is a well- diversified agent, such as a fund, we should expect fewer constraints imposed on insiders, and prices will contain the same informativeness independently of the state of the world. We should expect more trading for these companies' insiders, as well as fewer restrictions disclosed in Insider trading policies. If instead, the predominant shareholder is a founder, as in the example of Facebook presented in the Introduction, one should expect not more stringent constraints, as this shareholder has no means for diversification. Thus she will be more sensitive to variation in prices of the company. An intermediate situation and a valid interpretation of the coefficient ω is the urge to raise capital from financiers that have access to the firm's past price. More expiring investment opportunities should be preceded by periods of limits to trade, as it already happens around corporate events.

I am abstracting from compensation oriented situations, imposing trading restrictions on the manager; he should demand extra compensation to account for the forgone profit from trading. Empirical evidence in these directions is provided by Roulstone (2003). Moreover, on one side, trading restrictions may reduce the wedge in objectives between insiders and shareholders, but at the same time comes with indirect costs, and thus there is a tradeoff between value maximizing and short-term price behavior.

Although not explicitly, this paper speaks to managerial compensation and incentives. Fixing the volatility of cash flow, a more short-term oriented company should provide managers with fewer share grants so that short-selling constraints are more stringent or have more temporal restriction on trading. Moreover, the benefit of introducing selling constraints should increase as the company's cashflows become more volatile. In conclusion, granted trading rights should depend on cash flow volatility and the short-term investment opportunity of the firm.

APPENDIX A: PROOFS

A.1. Proof of Lemma 3

Proof.

$$
\begin{aligned}
&\lim_{q\to-\infty}\frac{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)\int_{-\infty}^{V} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{V}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v)}{\sigma_n}\right)\mathrm{d}v}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)\Phi\left(\frac{V-v_0}{\sigma_v}\right)+\int_{V}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v')}{\sigma_n}\right)\mathrm{d}v'} &=\\
&\lim_{q\to-\infty}\frac{\int_{-\infty}^{V} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{V}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{V-v_0}{\sigma_v}\right)+\int_{V}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v'} &=\\
&\frac{\int_{-\infty}^{V} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\lim_{q\to-\infty}\int_{V}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{V-v_0}{\sigma_v}\right)+\lim_{q\to-\infty}\int_{V}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v'} &=\\
&\frac{\int_{-\infty}^{V} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{V}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\lim_{q\to-\infty}\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{V-v_0}{\sigma_v}\right)+\int_{V}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\lim_{q\to-\infty}\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\underline{x}}{\sigma_n}\right)}\mathrm{d}v'} &=\\
&\frac{\int_{-\infty}^{V} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v}{\Phi\left(\frac{V-v_0}{\sigma_v}\right)} &=\\
&E[v|v<V]
\end{aligned}
$$

the second to last equality hold true for Dominated Convergence Theorem, while the last equality is comingfrom applying the limit and recalling that $f(v) \geq \overline{x}$.

A.2. Proof of Lemma 12

Proof. For $\omega = 0.5$, Suppose Not. There exists $\bar{x}$ such that $U^{\bar{X}}(p) > U^{NT}(p)$, or

$$
\begin{aligned}
&E(g(p)) > 0 \\
&g(E(p)) \geq E(g(p)) > 0 \quad \text{by Jensen Inequality} \\
&0 \geq E(g(p)) > 0 \quad \text{by definition of } g(\cdot) \text{ and Bayes rule}
\end{aligned}
$$

thus contradicting the statement. The proof for ω > 0.5 follows from the fact that the optimal constraint is always increasing in ω from equation (46) and the tie-breaking condition.

A.3. Proof of Corollary 13

Proof. I can rewrite Equation (46) as:

$$\frac{Prob_{\bar{x}}(p \geq v_0)}{1 - Prob_{\bar{x}}(p \geq v_0)} \frac{E_{\bar{x}}[g(p)|p \geq v_0]}{|E_{\bar{x}}[g(p)|p \leq v_0]|} \geq \frac{\omega}{1-\omega} \tag{47}$$

The odd ratio of the price distribution $\frac{Prob_{\bar{x}}(p \geq v_0)}{1 - Prob_{\bar{x}}(p \geq v_0)}$ is strictly decreasing in $\bar{x}$, as the more constrained is the insider, the less information will be given embedded in trading, thus lowering the probability of meeting price above the prior. At the same time, the RHS of the equation is increasing in ω. By construction, as long as g(·) is increasing, the ratio of expectations is increasing (the denominator decreases faster than the numerator due to normality and a Bayesian Market maker). This would imply that, for the inequality to be satisfied, $\bar{x}. \rightarrow +\infty$, as $\omega \rightarrow 1$. Unfortunately, the impossibility of having a close form solution for both the pricing function and the generality imposed on the utility function makes it difficult to characterize the behavior of the optimal constraint. However, if g(·) is characterized by risk aversion, an increase in σv, the cash flows' standard deviation, will, all else equal, be characterized by a decrease in utility, and as a consequence, an increase in the constraints for all level of ω, as long $\bar{x}$ is finite. This results is immediate if we consider Equation (47) and the impact on the utility of the variance.

The last parameter if the model is σ_n, the standard deviation of noise. Again, in the following specification, we cannot fully characterize the behavior of the company, at least formally, but a common pattern seems to emerge. In particular, as long as the solution for $\bar{x}$ is internal for level of ω, the behavior of the optimal constraint varies if the ω is below or above a cut-off ω_0, associated with the optimal level of the constraint to be $\bar{x} = 0$. In particular for level of $\omega < \omega_0$, the shareholder will find beneficial to relax the constraint, in order to bet with the market for an increase in price. For level of $\omega > \omega_0$, instead, the shareholder will impose further constraints as σ_n increases, suggesting now a desire to protect the price from unexpected swings in the market.

APPENDIX B. ASYMPTOTIC BEHAVIOR OF THE PRICING FUNCTION IN 40

I check the behavior of the function in 40 for q that goes to -∞ and +∞. Rewriting the above equation as

$$p(q) = \frac{\beta}{1+\beta^2} \frac{\left((\beta v + q) - \beta \frac{\sqrt{\beta^2+1}}{\sqrt{2\pi}} e^{-\frac{(v-\beta q)^2}{2(\beta^2+1)}} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1} \right)}{1+\sqrt{\beta^2+1}\Phi(v) e^{\frac{(\beta v+q)^2}{2(\beta^2+1)} - \frac{q^2}{2}} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}$$

$$= \frac{\beta}{1+\beta^2} \frac{(\beta v + q) - \beta\sqrt{\beta^2+1}\phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{1+\Phi(v) e^{\frac{v^2}{2}} \beta\sqrt{\beta^2+1} e^{-\frac{(v-\beta q)^2}{2(\beta^2+1)}} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}$$

$$= \frac{\beta}{1+\beta^2} \frac{e^{-\frac{v^2}{2}}}{\Phi(v)} \frac{\beta v + q - \beta\sqrt{\beta^2+1}\phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{\frac{e^{-\frac{v^2}{2}}}{\Phi(v)} + \beta\sqrt{\beta^2+1} e^{-\frac{(v-\beta q)^2}{2(\beta^2+1)}} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}} \tag{48}$$

Now recall that $e^{\left(-\frac{(v-\beta q)^2}{2(\beta+1)}\right)} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1} \rightarrow +\infty$ as $q \rightarrow -\infty$. As a consequence, if I expand the equation and take the limit for q to minus infinity, the first term is equal to zero. Thus, by taking the limit ofthe whole expression I have that:

$$\lim_{q \rightarrow -\infty} p(q) = \frac{\beta}{1+\beta^2} \frac{e^{-\frac{v^2}{2}}}{\Phi(v)} \left(0 - \frac{\sqrt{\frac{2}{\pi}}}{\beta} - \sqrt{\frac{2}{\pi}}\beta \right)$$

$$= \frac{-\sqrt{\frac{2}{\pi}} e^{-\frac{v^2}{2}}}{\Phi(v)} = E[v|v < v] \tag{49}$$

Equation (49) is a constant and it is equal to the expectation of v to be lower than ν. The function has an horizontal asymptote at the above value. I now consider the part of the expression that depends on q of (48):

$$g(q) = \frac{\beta v + q - \beta\sqrt{\beta^2+1}\phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{\frac{e^{-\frac{v^2}{2}}}{\Phi(v)} + \beta\sqrt{\beta^2+1} e^{-\frac{(v-\beta q)^2}{2(\beta^2+1)}} \Phi\left(-\frac{v-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}} \tag{50}$$

One can characterize the derivatives of this term at q→ +∞. This derivatives is

$$g'(q) = \frac{\beta^2}{4e^{-\frac{v^2}{2}}\Phi(v)^{-1}\pi\left(\sqrt{\beta^2+1}e^{\frac{v^2}{2}}\Phi(v)+e^{\frac{(v-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2}$$
$$+\frac{\sqrt{2\pi}\beta^2 e^{\frac{(v-\beta q)^2}{2(\beta^2+1)}}(\beta q - v)\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)+\pi\sqrt{\beta^2+1}e^{\frac{(v-\beta q)^2}{\beta^2+1}}2\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)^2}{4e^{-\frac{v^2}{2}}\Phi(v)^{-1}\pi\sqrt{\beta^2+1}\left(\sqrt{\beta^2+1}e^{\frac{v^2}{2}}\Phi(v)+e^{\frac{(v-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2}$$
$$+\frac{\pi e^{\frac{(v-\beta q)^2}{2(\beta^2+1)}}\left(\beta\left(-\beta v^2+\beta\left(q^2+1\right)+\left(\beta^2-1\right)vq\right)+1\right)2\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)+\sqrt{2\pi}\beta\sqrt{\beta^2+1}(\beta v+q)}{4\pi\sqrt{\beta^2+1}\left(\sqrt{\beta^2+1}e^{\frac{v^2}{2}}\Phi(v)+e^{\frac{(v-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{v-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2}$$

I show that

$$\lim_{q\to-\infty} g'(q) = 0$$

and more interestingly:

$$\lim_{q\to+\infty} g'(q) = e^{\frac{v^2}{2}}\Phi(v) \tag{51}$$

Combining this with the expression for p(q) I get a result:

$$\lim_{q\to+\infty} p'(q) = \frac{\beta}{1+\beta^2} \tag{52}$$

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