Essays on Diversity and Selectivity

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ABSTRACT

ESTONIA BLACK: Essays on Diversity and Selectivity.
(Under the direction of Peter Norman)

This dissertation consists of two essays, both dealing with selectivity decisions and diversity.

Chapter 1 studies optimal admissions test design for a monopolistic profit-maximizing school in a signalling model with a notion of diversity that is correlated with an individual's ability to pay for education, but uncorrelated with productivity. Workers' willingness to pay for education is determined by firms' expectations of the productivity of educated and uneducated workers, which are in turn determined by the school's choice of admissions test and tuition. A more selective admissions test may increase the wage differential for educated workers, allowing the school to charge a higher tuition, but requires the school to admit fewer workers. At the same time, a higher tuition reduces attendance and dilutes the signalling value of education, as an uneducated worker becomes more likely to have passed the admissions test but been unable to afford education. This paper characterizes how the school may optimally discriminate in admissions and/or pricing under various policies restricting discrimination.

Chapter 2 demonstrates the impact that the involvement of individuals of diverse sexual or romantic orientations can have on the equilibria of simple marriage market models. I modify the model of Burdett and Coles (1997) to involve singles who are attracted only to individuals of their own gender and singles who are attracted to individuals of more than one gender, and show that this fundamentally altars the structure of any partial rational expectations equilibria. Singles can still be divided into finitely many classes such that individuals of the same class, gender, and orientation behave identically. However, the key characteristic of Burdett and Coles' equilibria, that marriages only occur between singles in the same class, does not hold in the presence of queer individuals.

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CHAPTER 1

SCHOOL SELECTIVITY DECISIONS AND DISCRIMINATION

1.1 Introduction

When education acts as a signal of productivity to firms, preferences for smarter educational peers can arise endogenously (MacLeod and Urquiola (2015)). The signalling benefits of smarter peers for a student are intuitive: education at a school with a more generally capable student body may make a worker appear more capable to potential employers.

What is less clear is how we should think about a school's incentives as they relate to the quality of its students or the signalling value of the education it offers. A school that can condition admission on information about a worker's ability level can influence the signal created by its degrees. But it is not necessarily obvious what a selective educational institution's goals are when deciding who it will allow to attend. In theoretical models, schools' goals are often characterized as either as either maximizing the number of students who will attend (eg. MacLeod and Urquiola (2015)), or attracting the highest quality students (eg. Epple, Romano, and Sieg (2006), Chade, Lewis, and Smith (2014), Fu (2006)). However, if a school perceives some value in both the quality and number of its students, an inherent tradeoff is introduced: at a certain point the school cannot improve the average quality of its student body without reducing its size.

The contribution of this paper is to formalize the tradeoff for a school that cares both about the signalling value of education to its students and the number of students it serves, and to outline the implications of the resulting incentives for educational discrimination and affirmative action.

I develop a model in which a single profit-maximizing school designs an admissions test and chooses a tuition. Admitted workers who can afford tuition choose whether or not to get educated and then enter into a competetive labor market, in which all workers are paid a wage equal to their expected productivity conditional on their education status, similar to the Spence (1973) signalling model. Thus the school faces a tradeoff between the signalling value of its education (which affects the tuition it can charge its students) and the number of workers it admits. I show that a threshold admissions rule will be optimal for the school. I then introduce a notion of worker type that is correlated with ability to pay tuition but not correlated with productivity, and characterize the school's incentives to discriminate based on this type in admissions and/or tuition.

I characterize the solution to the school's problem under various restrictions on how it can discriminate, showing that generally discrimination will take the form of charging a higher tuition to the 'richer' type of student or holding the 'poorer' type of student to a higher admissions standard. If firms are able to observe and condition wage on workers' types, they see an uneducated worker of the poorer type as more likely to be someone who passed the admissions test but could not afford tuition than an uneducated worker of the richer type who faced the same tuition and admissions rule. This dampens the signalling value of education with any given admissions threshold and tuition for the poorer type of workers, which can motivate the school to be more selective in admissions for that group. If firms are unable to observe or condition wage on workers' types, the school may benefit from admitting lower-quality students of the richer type at a high tuition, while holding the poorer type of worker to a higher admissions standard to make its whole student body look smarter.

The remainder of this paper proceeds as follows. Section 1.2 defines the basic model without diversity and establishes a result about the school's optimal admissions rule, which is useful in understanding the results in the rest of the paper. Section 1.3 introduces a type characteristic into the model and characterizes the school's optimal behavior under various restrictions when firms are able to observe and condition a worker's wage on their type. Section 1.4 characterizes the school's optimal behavior under the same restrictions when firms are unable to discriminate based on type.

1.2 The Model Without Diversity

1.2.1 The Environment

There is a unit mass of workers, each endowed with initial ability $x \in [\underline{x}, \overline{x}]$, representing their lifetime productivity. The ability of workers is distributed according to continuous cumulative distribution function F on $[\underline{x}, \overline{x}]$, with corresponding probability density function f such that $f(x) > 0$ for all $x \in [\underline{x}, \overline{x}]$. Each worker also has a wealth level y, independent of ability and distributed according to G (with probability density function g) on $\mathbb{R}^+$.[1]

While I refer to y as a worker's wealth level throughout, it may alternatively be viewed as a kind of credit constraint. What is important is that y represents the maximum amount of money a worker can spend on schooling before they enter the labor market, which is exogenous and varies between individuals; it is less important whether that money comes from an initial endowment or represents the amount a worker may borrow from future earnings.

There is an entity, which I refer to as the school, that can increase an ability x worker's productivity to ax for some $a > 1$ at cost $c(x) > 0$.

There is a competetive labor market, in which firms cannot directly observe workers' ability or wealth levels, but can observe whether or not a worker has been educated. I largely abstract away from the mechanisms within the labor market: firms form some shared beliefs about workers' productivity based only on their education status, and any worker entering the labor market receives a lifetime wage equal to their expected lifetime productivity given these beliefs.

1.2.2 Decisions and Timing

I model the market for education as a sequential game in which the school commits to some admissions test and tuition before workers decide whether to attend. The school announces a tuition $t \geq 0$ and an admissions test $s : [\underline{x}, \overline{x}] \to [0, 1]$, where $s(x)$ is the probability that a worker of ability x is admitted. Once announced, s and t are known to all workers and firms.

The admissions test s is then applied to all workers; an admitted worker with wealth $y \geq t$

[1] Throughout, I use the notation $\bar{H}(\cdot)$ to refer to the survival function $\bar{H}(\cdot) = 1 - H(\cdot)$ for any cumulative distribution function H.

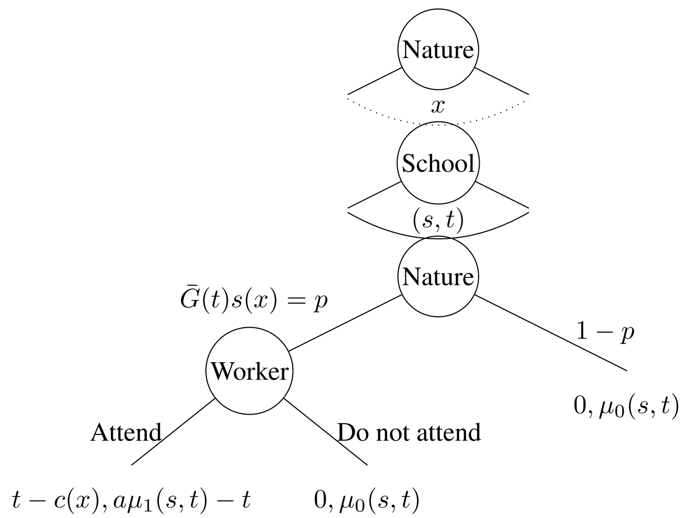

Figure 1.1: Extensive form for representative worker

chooses whether or not to pay tution t to attend the school and become educated. An admitted worker with wealth $y < t$ cannot afford to pay tuition. Workers who are not admitted or cannot afford tuition do not make an attendance decision, and cannot attend the school.

All workers then enter the labor market and receive a wage equal to their expected productivity, according to firms' beliefs. Firms observe a worker's education status, and are aware of the admissions rule and tuition announced by the school when formulating their expectations about worker's productivity. Given admissions test s and tuition t, write $\mu_0(s,t)$ and $\mu_1(s,t)$ for firms' expectations of the initial ability level of an uneducated and educated worker, respectively. Note that the total expected productivity of an educated worker is thus $a\mu_1(s,t)$.

A worker's payoff is their wage net of any tuition paid to the school. The school's payoff is its profit: total tuition extracted from workers who attend the school net of the total costs of educating those workers.

It would be equivalent to consider a model with a single representative worker, with ability level x and wealth level y drawn from F and G, in which the school seeks to maximize its expected profit from that one worker. The extensive form of that game with a representative worker is depicted in

Figure 1.1.

1.2.3 Strategies and Beliefs

A strategy for the school simply consists of a tuition $t \geq 0$ and an admissions test $s : [\underline{x}, \overline{x}] \to [0, 1]$. Denote the set of possible admissions tests by $S := [\underline{x}, \overline{x}]^{[0,1]}$.

A worker faced with an attendance decision is aware of the tuition and admissions test the school has chosen, and can condition their decision on that information. Each worker who is admitted to the school and has wealth $y \geq t$ faces the same attendance decision: regardless of a worker's ability or wealth levels, they will recieve a payoff of $a\mu_1(s,t) - t$ if they choose to attend the school, or $\mu_0(s,t)$ if they do not. Therefore, I assume that workers will follow a symmetric strategy profile, with admitted workers who can afford tuition all choosing to attend the school with the same probability. With that in mind, a worker strategy is a function $\sigma : S \times \mathbb{R}^+ \to [0, 1]$, where $\sigma(s,t)$ is the probability with which any worker will choose to attend the school if faced with an attendance decision.

When workers enter the labor market, firms are aware of the admissions rule and tuition chosen by the school, but cannot directly observe the ability levels of or strategy employed by workers. Given σ, for any (s,t) such that some workers will attend the school (i.e., $\sigma(s,t)\bar{G}(t)\int_{\underline{x}}^{\overline{x}} s(x)dF(x) > 0$), $\mu_1(s,t)$ is well-defined by Bayes' rule. For any $t > 0$, there will be some workers who cannot afford tuition and thus will not attend the school even if they are admitted, so $\mu_0(s,t)$ can be defined by Bayes' rule for all (s,t) and σ.

Definition 1.2.1. *Firm beliefs $\mu = (\mu_1, \mu_2)$ are consistent with worker strategy σ if $\mu_0(s,t)$ and $\mu_1(s,t)$ are determined by s, t, and σ using Bayes' rule wherever possible.*

That is, μ is consistent with σ if and only if

$$\mu_1(s,t) = \frac{\int_{\underline{x}}^{\overline{x}} xs(x)dF(x)}{\int_{\underline{x}}^{\overline{x}} s(x)dF(x)}$$

for all (s,t) such that $\sigma(s,t)\bar{G}(t)\int_{\underline{x}}^{\overline{x}} s(x)dF(x) > 0$, and

$$\mu_0(s,t) = \frac{\mathbb{E}(x) - \sigma(s,t)\bar{G}(t)\int_{\underline{x}}^{\overline{x}} xs(x)dF(x)}{1 - \sigma(s,t)\bar{G}(t)\int_{\underline{x}}^{\overline{x}} s(x)dF(x)}$$

for all (s,t).

1.2.4 Equilibrium Characterization

Given a worker strategy σ, the school's expected profit as a function of its choice of tuition and admissions rule can be written as

$$\Pi(s,t;\sigma) = \sigma(s,t)\bar{G}(t)\int_{\underline{x}}^{\overline{x}} (t - c(x))s(x)dF(x)$$

where $\sigma(s,t)\bar{G}(t)$ is the proportion of admitted workers who can afford tuition and will choose to attend the school.

Definition 1.2.2. *An equilibrium is an object $(s^*, t^*, \sigma^*, \mu)$, satisfying the following conditions:*

(i) σ^* is a best response to s^*, t^* for workers:

$$\sigma^*(s^*,t^*) \in \arg\max_{\sigma} \sigma(a\mu_1(s^*,t^*) - t^*) + (1-\sigma)\mu_0(s^*,t^*)$$

(ii) (s^*, t^*) is profit-maximizing for the school:

$$(s^*,t^*) \in \arg\max_{(s,t)} \Pi(s,t;\sigma^*)$$

(iii) μ is consistent with σ^*.

$(s^*, t^*, \sigma^*, \mu)$ is a subgame perfect equilibrium (SPE) if it is an equilibrium and

$$\sigma^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t) - t) + (1-\sigma)\mu_0(s,t)$$

for all admissions rules s and tuitions $t > 0$.

The additional constraint of subgame perfection rules out many equilibria in which a particular strategy is optimal for the school only because of a threat of non-optimal play by the workers should the school announce a different admissions test or tuition. However, many fairly trivial SPE are possible since firms can hold extreme and arbitrary beliefs when they are not well-defined by Bayes' rule. For example, if $a\underline{x} \leq \mathbb{E}(x)$, then a large number of subgame perfect equilibria exist in which the workers' strategy is to attend the school if and only if a particular admissions rule and tuition are announced.

In order to rule out such trivial equilibria, and to illustrate the incentives of the school when it has the most control over the signalling value of the education it provides, I restrict my analysis to equilibria in which workers use the following strategy:

$$\tilde{\sigma}(s,t) := \begin{cases} 1 & a\mathbb{E}(x|\text{admitted}; s) - t \geq \mathbb{E}(x|\text{not admitted or } y < t; s) \\ \phi(s,t) & \mathbb{E}(x) < a\mathbb{E}(x|\text{admitted}; s) - t < \mathbb{E}(x|\text{not admitted or } y < t; s) \\ 0 & \text{else} \end{cases}$$

where

$$\phi(s,t) := \frac{1}{\bar{G}(t)\int_{\underline{x}}^{\overline{x}} s(x)dF(x)} \frac{a\mathbb{E}(x|\text{admitted}; s) - t - \mathbb{E}(x)}{(a-1)\mathbb{E}(x|\text{admitted}; s) - t}$$

A detailed motivation for restriction to this worker strategy is given in Appendix A.1.

For any admissions rule s and tuition t such that $\tilde{\sigma}(s,t) \in (0,1)$, the admissions rule s' given by $s'(x) = \tilde{\sigma}(s,t)s(x)$ has the property that $\tilde{\sigma}(s,t) = 1$ and $\Pi(s',t;\tilde{\sigma}) = \Pi(s,t;\tilde{\sigma})$. Thus the school can earn at least as much profit by selecting a tution and admissions rule such that all workers with an education decision will choose to attend.

If workers use strategy $\tilde{\sigma}$, the school's problem can be formulated as choosing admissions rule s and tuition t to maximize profit subject to attendance constraint

$$a\mathbb{E}(x|\text{admitted}; s) - t \geq \mathbb{E}(x|\text{not admitted or } y < t; s) \tag{1.1}$$

which guarantees that all admitted workers who can afford tuition will choose to attend the school.

1.2.5 Threshold Admissions Rules

Assume that higher ability workers are not more costly to educate: $c'(x) \geq 0$ for all $x \in [\underline{x}, \overline{x}]$. Since admitting more productive workers would be conducive to a higher signalling value of education, it is natural to consider admissions tests that simply set a minimum ability level for admission.

Definition 1.2.3. *Admissions test s is a threshold admissions rule if there exists some $\tilde{x}$ such that $s(x) = 0$ almost everywhere on $[\underline{x}, \tilde{x})$ and $s(x) = 1$ almost everywhere on $(\tilde{x}, \overline{x}]$.*

In fact, if workers are following strategy $\tilde{\sigma}$ and the school has a best response that would induce some workers to attend the school, it must involve a threshold admissions rule:

Proposition 1.2.1. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium in which some workers attend the school with positive probability, then s^* is a threshold admissions rule.*

Intuitively, if s^* is not a threshold admissions rule, the school must be able to shift some probability of admission from lower ability levels to higher ability levels without changing the expected mass of workers who would attend the school. This would slacken the attendance constraint without increasing costs, creating an opportunity for the school to increase profits by adjusting tuition or the number of students admitted. The full proof of Proposition 1.2.1 is given in Appendix A.3.

Given that the school's best response to $\tilde{\sigma}$ is either to shutdown or to use a threshold admissions rule, the school's problem can again be reformulated as choosing an admissions threshold $\tilde{x} \in [\underline{x}, \overline{x}]$ and tuition $t > 0$ to solve

$$\max_{\tilde{x},t} \bar{G}(t) \int_{\tilde{x}}^{\overline{x}} (t - c(x)) dF(x)$$

subject to attendance constraint

$$a\mathbb{E}(x|x \geq \tilde{x}) - t \geq \mathbb{E}(x|x < \tilde{x} \text{ or } y < t) \tag{1.2}$$

1.3 The Model with Diversity

In this section, I introduce a notion of diversity among workers. I outline results about this model with diversity that are similar to those in section 1.2, and then examine the school's behavior under various restrictions on how it can discriminate in admissions and tuition.

Suppose that the unit mass of workers each also have a type characteristic $i \in \{1, 2\}$ independent of ability, but correlated with wealth. Let G_i be the cumulative distribution function of wealth among type i workers, with corresponding probability density function g_i. I assume that both wealth distributions display a non-decreasing hazard rate.

Let q_i be the proportion of workers who are type i. Further, suppose that type 1 workers are 'richer' than type 2 workers in the sense of the monotone likelihood ratio property: $\frac{g_1(y)}{g_2(y)} \geq \frac{g_1(y')}{g_2(y')}$ for all $y > y'$.

1.3.1 Decisions and Timing

The order of events is the same as that of the basic model. First, the school announces admissions rules $s = (s_1, s_2)$ and tuitions $t = (t_1, t_2)$, where $t_i > 0$ and $s_i : [\underline{x}, \overline{x}] \to [0, 1]$ for $i = 1, 2$.

Each admissions test s_i is then applied to all type i workers; an admitted type i worker with wealth $y \geq t_i$ may choose whether or not to attend the school at price t_i.

All workers then enter the labor market, receiving a wage equal to their expected productivity, according to firms' beliefs. Firms observe workers' type and education status, and are aware of each admissions rule and tuition announced by the school. Given admissions tests $s = (s_1, s_2)$ and tuitions $t = (t_1, t_2)$, write $\mu_0(s, t; i)$ and $\mu_1(s, t)$ for firm's expectations of the initial ability level of an uneducated and educated type i worker, respectively.

The school's and workers' payoffs are defined analogously to those in the model without diversity.

1.3.2 Strategies and Beliefs

A strategy for the school is a pair of tuitions $t \in \mathbb{R}^+ \times \mathbb{R}^+$ together with a pair of admissions tests $s : [\underline{x}, \overline{x}] \to [0, 1] \times [0, 1]$. A strategy for type i workers is a function $\sigma_i : (s, t) \mapsto \sigma_i(s, t) \in$

$[0,1]$, where $\sigma_i(s,t)$ is the probability with which an admitted type i worker with wealth $y > t_i$ will choose to attend the school.

When workers enter the labor market, firms observe their type and education status, and form expectations about their ability level based on the school's choice of admissions rules and tuition. In Section 1.4, I examine an alternate version of the model with diversity, in which firms are unable to observe worker type or prohibited from conditioning wage on type. I refer to the model in which firms can condition wage on type as the model with firm discrimination, and the model in which firms cannot condition wage on type as the model without firm discrimination.

Definition 1.3.1. *Firm beliefs $\mu = (\mu_0, \mu_1)$ are consistent with worker strategies $\sigma = (\sigma_1, \sigma_2)$ in the model with firm discrimination if $\mu_0(s,t;i)$ and $\mu_1(s,t;i)$ are determined by s_i, t_i, and σ_i using Bayes' rule wherever possible.*

That is, μ is consistent with σ if and only if

$$\mu_1(s,t;i) = \frac{\int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{\int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)}$$

for all (s,t) and i such that $\sigma_i(s,t)\bar{G}_i(t_i)\int_{\underline{x}}^{\overline{x}} s_i(x)dF(x) > 0$, and

$$\mu_0(s,t;i) = \frac{\mathbb{E}(x) - \sigma_i(s,t)\bar{G}_i(t_i)\int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{1 - \sigma_i(s,t)\bar{G}_i(t_i)\int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)}$$

for all (s,t) and i.

1.3.3 Equilibrium Characterization

Given worker strategies $\sigma = (\sigma_1, \sigma_2)$, the school's expected profit as a function of its choice of admissions rule can be written as

$$\Pi(s,t;\sigma) = \sum_{i=1}^{2} q_i \sigma_i(s,t)\bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(X)$$

where $\sigma_i(s,t)\bar{G}_i(t)$ is the proportion of admitted type i workers who are able to afford tuition and choose to attend the school.

Definition 1.3.2. *An equilibrium in the model with firm discrimination is an object* $(s^*, t^*, \sigma^*, \mu)$ *satisfying the following conditions:*

(i) σ_i^* *is a best response to* s^*, t^**:*

$$\sigma_i^*(s^*, t^*) \in \arg\max_{\sigma} \sigma(a\mu_1(s^*, t^*; i) - t_i^*) + (1-\sigma)\mu_0(s^*, t^*; i)$$

for $i = 1, 2$*.*

(ii) (s^*, t^*) *is profit maximizing for the school:*

$$(s^*, t^*) \in \arg\max_{(s,t)} \Pi(s, t; \sigma^*)$$

(iii) μ *is consistent with* σ^**.*

$(s^*, t^*, \sigma^*, \mu)$ *is a subgame perfect equilibrium if it is an equilibrium and*

$$\sigma_i^*(s, t) \in \arg\max_{\sigma} \sigma(a\mu_1(s, t; i) - t_i) + (1-\sigma)\mu_0(s, t; i)$$

for all s*,* $t > 0$*, and* $i \in \{1, 2\}$*.*

Let

$$v_i(s, t) := a\mathbb{E}(x|\text{admitted}; s, i) - t_i$$

For reasons similar to those laid out in Section 1.2.4 and Appendix A.1, I restrict my analysis to subgame perfect equilibria in which workers use the strategies $\tilde{\sigma} = (\tilde{\sigma}_1, \tilde{\sigma}_2)$ given by

$$\tilde{\sigma}_i := \begin{cases} 1 & v_i(s,t) \geq \mathbb{E}(x|\text{not admitted or } y < t_i; s, i) \\ \phi_i(s,t) & \mathbb{E}(x) < v_i(s,t) < \mathbb{E}(x|\text{not admitted or } y < t_i; s, i) \\ 0 & \text{else} \end{cases}$$

where

$$\phi_i(s,t) := \frac{1}{\bar{G}_i(t_i)\int_{\underline{x}}^{\overline{x}} s_i(x)dF(x)} \frac{v_i(s,t) - \mathbb{E}(x)}{v_i(s,t) - \mathbb{E}(x|\text{admitted}; s, i)}$$

When the school faces no additional restrictions on its admissions rules or tuitions, its profit maximization problem in the model with diversity is essentially to solve two separate versions of the profit maximization problem in the model without diversity. Thus it is optimal for the school to choose two threshold admissions rules under similar circumstances to those in which it would be optimal in the model without diversity.

Proposition 1.3.1. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium in which some workers of type i attend the school with positive probability, then s_i^* is a threshold admissions rule.*

The proof of Proposition 1.3.1 is identical to that of Proposition 1.2.1.

In the remainder of Section 1.3, I characterize potential equilibria in the model with firm discrimination when the school faces additional restrictions on how they vary admissions rules and tuition based on type. I refer to the practice of applying a different admissions rule to each type of worker (ie. $s_1 \neq s_2$) as admissions discrimination on the part of the school, and the practice of charging different tuitions to each type (ie. $t_1 \neq t_2$) as tuition discrimination.

1.3.4 No school discrimination

Suppose the school must apply the same admissions rule and tuition to each type of worker. For (s,t) such that $s_1 = s_2$ and $t_1 = t_2$, firms will likely have different expectations of the ability of an uneducated worker with $i = 1$ versus $i = 2$. If some workers attend the school with positive probability and some are not admitted, an uneducated type 2 worker is relatively more likely to be a worker who passed the admissions test of the school but was unable to afford tuition than an uneducated type 1 worker. However, since ability is independent of wealth and of i, $\mathbb{E}(x|\text{admitted}; s, i)$ will be the same for both types of worker if $s_1 = s_2$.

Definition 1.3.3. *A subgame perfect equilibrium with firm discrimination and no school discrimination is an object $(s^*, t^*, \sigma^*, \mu)$ such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t;i) - t_i) + (1-\sigma)\mu_0(s,t;i)$$

for all s*,* $t > 0$*, and* $i \in \{1,2\}$.

(ii)

$$(s^*,t^*) \in \arg\max_{\{(s,t)|s_1=s_2,t_1=t_2\}} \sum_{i=1}^{2} q_i\sigma_i^*(s,t)\bar{G}_i(t_i)\int_{\underline{x}}^{\overline{x}}(t_i - c(x))s_i(x)dF(x)$$

(iii) μ *is consistent with* σ^*.

If workers follow strategy profile $\tilde{\sigma}$, the school essentially faces an attendance constraint for each type of worker. Its best response will either be to shut down, to choose (s,t) such that $\tilde{\sigma}_i(s,t) = 1$ for $i = 1,2$ or to choose (s,t) such that all workers of one type facing an education decision will choose to attend the school, while all workers of the other type will choose not to attend the school.

Proposition 1.3.2. *If* $(s^*,t^*,\tilde{\sigma},\mu)$ *is a subgame perfect equilibrium with firm discrimination and no school discrimination in which some workers attend the school with positive probability, then* $s_1^* = s_2^*$ *is a threshold admissions rule.*

The proof of Proposition 1.3.2 is similar to that of Proposition 1.2.1. The main difference to consider is the possibility that the school might choose (s,t) such that only students of one type i will choose to attend. However, if the school were to choose such an (s,t) for which $s_1 = s_2$ is not a threshold admissions rule, it would be possible to shift some probability of admission to higher ability workers without changing the number of admitted workers, and without making attendance optimal for type $j \neq i$ workers. This would slacken the attendance constraint for type i workers, creating an opportunity for the school to increase profit.

Using Proposition 1.3.2, if $(s^*,t^*,\tilde{\sigma},\mu)$ is an SPE with firm discrimination and no school discrimination, then

$$\mu_1(s^*,t^*;1) = \mu_1(s^*,t^*;2) > \mathbb{E}(x)$$

Since a type 1 worker who does not face an education decision is less likely to have passed the school's admissions test than a type 2 worker who does not face an education decision,

$$\mathbb{E}(x|\text{not admitted or } y < t_1^*; s^*, 1) \leq \mathbb{E}(x|\text{not admitted or } y < t_2^*; s^*, 2)$$

The signalling value of an education must then be higher for a type 1 worker than a type 2 worker. If (s^*, t^*) satisfies the attendance constraint for type 2 workers, it must also satisfy the attendance constraint for type 1 workers. Thus, if $(s^*, t^*, \tilde{\sigma}, \mu)$ is an SPE with firm discrimination and no school discrimination in which only one type of worker attends the school, it is type 1 workers (the 'richer' type).

1.3.5 Tuition-only school discrimination

Suppose the school must apply the same admissions rule to each type of worker, but is not restricted from tuition discrimination. Again in this case, the expected ability of a worker without an education decision will vary based on type i, but the expected productivity of an admitted type i worker will be the same for $i = 1, 2$.

Definition 1.3.4. *A subgame perfect equilibrium with firm discrimination and tuition-only school discrimination is an object $(s^*, t^*, \sigma^*, \mu)$ such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t;i) - t_i) + (1-\sigma)\mu_0(s,t;i)$$

for all s, $t > 0$, and $i \in \{1, 2\}$.

(ii)

$$(s^*, t^*) \in \arg \max_{\{(s,t)|s_1 = s_2\}} \sum_{i=1}^{2} q_i \sigma_i^*(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(x)$$

(iii) μ is consistent with σ^*.

If workers use strategy profile $\tilde{\sigma}$, the school's best response when restricted to tuition-only discrimination is again to use a threshold admissions rule.

Proposition 1.3.3. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with firm discrimination and* *tuition-only school discrimination in which some workers attend the school with positive probability, then $s_1^* = s_2^*$ is a threshold admissions rule.*

The proof of Proposition 1.3.3 is similar to that of Proposition 1.2.1.

Naturally, if the school engages in tuition-only discrimination, it will charge a higher tuition to the 'richer' type of students.

Proposition 1.3.4. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with firm discrimination in which some workers of both types attend the school, then $t_1^* \geq t_2^*$.*

A full proof of Proposition 1.3.4 is given in Appendix A.3. Intuitively, type 1 students will always be willing to pay more for education than type 2 students when faced with the same admissions threshold. Furthermore, the MLRP and monotone hazard rate assumptions on G_1 and G_2 guarantee that if $t_1 < t_2$, the marginal revenue of raising t_1 is greater than that of raising t_2.

1.3.6 Admissions-only discrimination

Suppose the school must charge the same tuition to each type of worker, but is not restricted from admissions discrimination. In this case, both the expected productivity of educated and uneducated workers may vary based on type.

Definition 1.3.5. *A subgame perfect equilibrium with firm discrimination and admissions-only school discrimination is an object $(s^*, t^*, \sigma^*, \mu)$ such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t;i) - t_i) + (1-\sigma)\mu_0(s,t;i)$$

for all s, $t > 0$, and $i \in \{1, 2\}$.

(ii)

$$(s^*, t^*) \in \arg \max_{\{(s,t)|t_1=t_2\}} \sum_{i=1}^{2} q_i \sigma_i^*(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(x)$$

(iii) μ is consistent with σ^*.

Again, if workers use strategy $\tilde{\sigma}$ and firm beliefs are consistent with $\tilde{\sigma}$, the school's best response involves threshold admissions rules.

Proposition 1.3.5. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with firm discrimination and admissions-only school discrimination in which some workers of type i attend the school with positive probability, then s_i^* is a threshold admissions rule.*

The proof of Proposition 1.3.5 is similar to that of Proposition 1.2.1.

For all $\tilde{x} \in (\underline{x}, \overline{x})$, each type of worker has a maximum tuition $\bar{t}_i(\tilde{x})$ which they would be willing to pay if the admissions threshold for type i is $\tilde{x}$, determined by their attendance constraint

$$a\mathbb{E}(x|x > \tilde{x}) - t \geq \mathbb{E}(x|x < \exists \text{ or } y < t; i)$$

In particular, $\bar{t}_1(\tilde{x}) > \bar{t}_2(\tilde{x})$ for all $\tilde{x}$, since the signalling value of an education with the same tuition and admissions rule is greater for type 1 workers than type 2 workers. In the special case where c is constant on $[\underline{x}, \overline{x}]$, this leads to the conclusion that the school will apply a weakly lower admissions threshold to the 'richer' type of student.

Proposition 1.3.6. *If $c'(x) = 0$ for all $x \in [\underline{x}, \overline{x}]$ and $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with firm discrimination and admissions-only school discrimination in which some workers of both types attend the school with positive probability, where $\tilde{x}_1$ is the admissions threshold corresponding to s_1^* and $\tilde{x}_2$ is the admissions threshold corresponding to s_2^*, then $\tilde{x}_2 \geq \tilde{x}_1$.*

Essentially, if c is constant and $\tilde{x}_1 > \tilde{x}_2$ with both attendance constraints satisfied, the school could earn strictly higher profit by either applying admissions threshold x_2 to both types of student (if $t_1 = t_2 \geq c(x_2)$) or shutting down completely (else).

1.4 Case without firm discrimination

Suppose that firms are either unable to observe worker type, or prohibited from wage discrimination. Then firm's expectations of the ability of an educated or uneducated worker do not depend on the worker's type, but do depend on the admissions rule and tuition applied to *both* types.

1.4.1 Equilibrium Characterization

For any $s = (s_1, s_2)$ and $t = (t_1, t_2)$, let $\mu_1(s, t)$ denote firms' expectation of the initial ability of an educated worker, and $\mu_0(s, t)$ denote firms' expectation of the ability of an uneducated worker.

Definition 1.4.1. *Firm beliefs $\mu = (\mu_0, \mu_1)$ are consistent with worker strategy profile $\sigma = (\sigma_1, \sigma_2)$ in the model without firm discrimination if μ_0 and μ_1 are determined by σ, s, and t using Bayes' rule wherever possible.*

That is, μ is consistent with σ if and only if

$$\mu_1(s,t) = \frac{\sum_{i=1}^{2} q_i \sigma_i(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{\sum_{i=1}^{2} q_i \sigma_i(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)}$$

for all s, t such that $\sum_{i=1}^{2} q_i \sigma_i(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x) > 0$, and

$$\mu_0(s,t) = \frac{\mathbb{E}(x) - \sum_{i=1}^{2} q_i \sigma_i(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{1 - \sum_{i=1}^{2} q_i \sigma_i(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)}$$

for all s, t.

I define equilibria in the model without firm discrimination in the way that natural coincides with equilibria in the models without it:

Definition 1.4.2. *A subgame perfect equilibrium in the model without firm discrimination is an object $(s^*, t^*, \sigma^*, \mu)$ such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t) - t_i) + (1-\sigma)\mu_0(s,t)$$

for all s, $t > 0$, and $i \in \{1, 2\}$.

(ii)

$$(s^*, t^*) \in \arg\max_{(s,t)} \Pi(s,t;\sigma^*) = \sum_{i=1}^{2} q_i \sigma_i^*(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(x)$$

(iii) μ is consistent with σ^*.

I restrict my analysis of equilibria in the model without firm discrimination to cases in which workers use the particular strategy profile $\tilde{\sigma} = (\tilde{\sigma}_1, \tilde{\sigma}_2)$, which is defined fully in Appendix A.2, and has the following properties:

(i) $\tilde{\sigma}_1(s,t) = \tilde{\sigma}_2(s,t) = 1$ if and only if

$$a\mathbb{E}(x|\text{admitted}; s) - \frac{\mathbb{E}(x) - \sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{1 - \sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)} \geq \max\{t_i\}_{i=1}^{2} \tag{1.3}$$

(ii) $\tilde{\sigma}_i(s,t) = 1$ and $\tilde{\sigma}_{-i}(s,t) = 0$ if and only if

$$t_i \leq a\mathbb{E}(x|\text{admitted}; s, i) - \frac{\mathbb{E}(x) - q_i \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{1 - q_i \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)} \leq t_{-i} \tag{1.4}$$

and not (1.3)

(iii) $\tilde{\sigma}_1(s,t) = \tilde{\sigma}_2(s,t) = 0$ if and only if

$$a\mathbb{E}(x|\text{admitted}; s) - \mathbb{E}(x) \leq \min\{t_i\}_{i=1}^{2} \tag{1.5}$$

and not (1.3) and not (1.4).

(iv) For any s, t such that $\tilde{\sigma}_i(s,t) \in (0,1)$ for some i, there exists s' such that $\tilde{\sigma}_{-i}(s',t) = \tilde{\sigma}_{-i}(s,t)$, $\tilde{\sigma}_i(s',t) \in \{0,1\}$ and

$$\Pi(s', t; \tilde{\sigma}) = \Pi(s, t; \tilde{\sigma})$$

If the school faces no restrictions on its admissions rules or tuition, and workers use strategy profile $\tilde{\sigma}$, it will be optimal for the school to employ threshold admissions rules:

Proposition 1.4.1. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium in the model without firm discrimination such that some type i workers will attend the school with positive probability, then s_i^* is a threshold admissions rule.*

The proof of Proposition 1.4.1 is similar to that of Proposition 1.2.1.

Proposition 1.4.2. *If F has a non-decreasing hazard rate and $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium in the model without firm discrimination in which the school makes a positive profit,* *then $t_1 \geq t_2$, and the admissions threshold is weakly higher for type 2 workers.*

Intuitively, the school may benefit from extracting more money from the richer type of worker, while holding the poorer type of worker to a higher admissions standard in order to make all of their students seem smarter. The proof of Proposition 1.4.2 is detailed in Appendix A.3.

In the remainder of Section 1.4, I characterize potential equilibria in the model without firm discrimination when the school faces various additional restrictions on how it can vary its admissions test and/or tuition based on type.

1.4.2 No school discrimination

Suppose the school must apply the same admissions rule and tuition to each type of worker. For any (s, t) such that $s_1 = s_2$ and $t_1 = t_2$, all workers who face an education decision perceive the same value and face the same cost of education, regardless of type.

Definition 1.4.3. *A subgame perfet equilibrium with no firm discrimination and no school discrimination is an object $(s^*, t^*, \sigma^*, \mu)$ such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t) - t_i) + (1-\sigma)\mu_0(s,t)$$

for all s, $t > 0$, and $i \in \{1, 2\}$.

(ii)

$$(s^*, t^*) \in \arg \max_{\{(s,t)|s_1=s_2,t_1=t_2\}} \sum_{i=1}^{2} q_i \sigma_i^*(s,t)\bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x))s_i(x)dF(x)$$

(iii) μ is consistent with σ^*

If workers follow strategy $\tilde{\sigma}$, then the school is essentially facing the same problem as in the model without diversity. Thus Proposition 1.2.1 has the following corollary:

Corollary 1.4.1. *If $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with no firm discrimination and no school discrimination in which some students attend the school with positive probability, then $s_1^* = s_2^*$ is a threshold admissions rule.*

1.4.3 Tuition-only school discrimination

Suppose the school must apply the same admissions rule to each type of worker, but is not restricted from tuition discrimination.

Definition 1.4.4. *A subgame perfect equilibrium without firm discrimination and with tuition-only school discrimination is an object* $(s^*, t^*, \sigma^*, \mu)$ *such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t) - t_i) + (1-\sigma)\mu_0(s,t)$$

for all s, $t > 0$, *and* $i \in \{1,2\}$.

(ii)

$$(s^*, t^*) \in \arg \max_{\{(s,t)|s_1=s_2\}} \sum_{i=1}^{2} q_i \sigma_i^*(s,t) \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(x)$$

(iii) μ *is consistent with* σ^*.

If workers use strategy profile $\tilde{\sigma}$, the school's best response when retricted to tuition-only discrimination is to use a threshold admissions rule.

Proposition 1.4.3. *If* $(s^*, t^*, \tilde{\sigma}, \mu)$ *is a subgame perfect equilibrium without firm discrimination and with tuition-only school discrimination such that some students attend the school with positive probability, then* $s_1^* = s_2^*$ *is a threshold admissions rule.*

The proof of Proposition 1.4.3 is similar to that of Proposition 1.2.1. Naturally, if the school engages in tuition-only discrimination, it will charge a higher tuition to the 'richer' type of students:

Proposition 1.4.4. *If* $(s^*, t^*, \tilde{\sigma}, \mu)$ *is a subgame perfect equilibrium with no firm discrimination and with tuition-only school discrimination such that some students attend the school with positive probability, then* $t_1^* \geq t_2^*$.

Because type 1 students are richer than type students in the sense of the monotone likelihood ratio property and both wealth distributions have a non-decreasing hazard rate, if $t_1 < t_2$, the marginal profit of raising t_1 will always be higher than that of t_2. Thus, if the school can benefit from tuition-only discrimination, it will charge a higher tuition to students it knows are more likely to be able to pay. A full proof of Proposition 1.4.4 is given in Appendix A.3.

1.4.4 Admissions-only school discrimination

Suppose the school is permitted to apply different admissions rules s_1 and s_2 to each type of worker, but must charge the same tuition to all students.

Definition 1.4.5. *A subgame perfect equilibrium with no firm discrimination and admissions-only school discrimination is an object* $(s^*, t^*, \sigma^*, \mu)$ *such that*

(i)

$$\sigma_i^*(s,t) \in \arg\max_{\sigma} \sigma(a\mu_1(s,t) - t_i) + (1-\sigma)\mu_0(s,t)$$

for all s, $t > 0$, *and* $i \in \{1,2\}$.

(ii)

$$(s^*, t^*) \in \arg \max_{\{(s,t)|t_1=t_2\}} \sum_{i=1}^{2} q_i \sigma_i^*(s,t)\bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} (t_i - c(x)) s_i(x) dF(x)$$

(iii) μ *is consistent with* σ^*.

If workers use strategy profile $\tilde{\sigma}$, the school essentially faces one attendance constraint. It is optimal for the school to employ a threshold admissions rule:

Proposition 1.4.5. *If* $(s^*, t^*, \tilde{\sigma}, \mu)$ *is a subgame perfect equilibrium with no firm discrimination and admissions-only school discrimination in which some workers attend the school with positive probability, then* s_i^* *is a threshold admissions rule for all* $i \in \{1,2\}$.

The proof of Proposition 1.4.5 is similar to that of Proposition 1.2.1.

Furthermore, the school has no incentive to actually vary its admissions rules:

Proposition 1.4.6. *If* $(s^*, t^*, \tilde{\sigma}, \mu)$ *is a subgame perfect equilibrium with no firm discrimination and admissions-only school discrimination in which the school makes a positive profit, then* $s_1^* = s_2^*$.

The proof of Proposition 1.4.6 is detailed in Appendix A.3. Thus the school will not discriminate in the model with no firm discrimination if it is restricted to admissions-only discrimination.

1.5 Conclusion

In this paper I have studied a monopolistic profit-maximizing school designing an admissions test. The admissions test chosen by the school affects the signalling value of its degrees, which affects the tuition potential students are willing to pay for education. The tuition the school charges also effects not only the number of workers who will be able to attend the school, but also the signalling value of education (via the expected ability of uneducated workers). I show that the school's optimal admissions test involves an admissions threshold.

I then apply the basic structure of this model to an environment in which workers are distinguished by a type characteristic that is correlated with ability to pay tuition but not correlated with productivity in the labor market. I characterize the school's optimal admissions thresholds and tuition with and without restrictions on whether they can set different thresholds and/or tuitions for different types of worker, both in a model in which firms are able to observe and discriminate based on type and in a model in which they are not.

There are several areas for further study. An obvious potential restriction on the school that is not considered in this paper is a quota on the diversity of its students. Competition between multiple educational institutions might result in different schools catering to workers of different ability levels, resulting in a more complex relationship between a school's admissions test, the signalling value of its degrees, and the number of students it educates. Finally, a similar model in which the school or schools are able to designate separate programs with different admissions standards (e.g. an 'honors' degree program) may provide insight into diversity levels in programs of different calibers within the same institution.

CHAPTER 2

QUEER MARRIAGE AND CLASS

2.1 Introduction

Since Becker's seminal 1973 paper on marriage as a matching process, and subsequent papers which extend his analysis to introduce search frictions (eg. Burdett and Coles (1997); Mortensen and Pissarides (1999)), economic marriage market models have been used empirically to study topics including online dating Hitsch, Hortaçsu, and Ariely (2010), mate selection in speed dating Fisman, Iyengar, Kamenica, and Simonson (2006), and the interactions between household formation and inequality Fernández, Guner, and Knowles (2005). In the language of the vast majority of marriage market literature, a man and a woman bump into each other and then decide whether or not to enter into a relationship together. One could imagine that same-sex couples form in two separate, one-sided marriage markets that operate in roughly the same way as the one described in economic literature. But people with no strict preferences over the gender of a potential partner cannot be trivially accounted for in this framework.

In Burdett and Coles (1997), unmarried men and women occasionally contact singles of the opposite gender, and upon observing their objective level of impressiveness, or 'pizazz', decide whether or not to propose marriage. They show that in an equilibrium in which all singles are rational except that they believe the current distribution of pizazz among each gender will remain constant throughout time, men and women are partitioned into finitely many 'classes' (half-open pizazz intervals), and singles will only marry each other upon meeting if they are in the same class. In this paper I modify the model of Burdett and Coles (1997) to allow for the existence of singles of different orientations.

Rather than unmarried individuals only contacting singles of the opposite gender, the singles one meets on the market might be of any orientation or gender. Throughout it is assumed that a potential partner's orientation and gender are immediately observable upon contact. In this environment, people with no preferences over the gender of a potential partner have a distinct advantage, in that they are more likely to contact another single with whom they share a mutual attraction. I show that, as in Burdett and Coles (1997), equilibrium behavior can be characterized by partitions of each type of individual into classes, but unlike in the model that only includes straight individuals, singles who are not in the same class will get married.

The paper is organized as follows. In Section 2.2 I outline the basic framework of the model. In Section 2.3, the notion of a partial rational expectations equilibrium in this model is defined, and the unique equilibrium for a given collection of constant distributions of pizazz for singles of different orientations within the market. In Sections 2.4, conditions and characteristics of possible steady state equilibria are explored.

2.2 Model Framework

Suppose that a large population of singles participate in a marriage market. For simplicity, half of these singles are men, and half are women. Each individual can be characterized by their gender, m or w, an orientation $j \in \{s, g, b\}$ and a pizazz level $x \in \mathbb{R}$, which represents an objective measure of impressiveness that will determine the flow utility that a potential partner would recieve from marrying them. In this model, singles will search for potential partners, and occasionally succeed in contacting another single. The singles that they meet, however, may or may not be suitable candidates for marriage; for instance, a straight woman might run into another woman during the process of her search. Regardless of how much pizazz this contacted woman posesses, the straight woman will not consider marrying her, and both will have to continue searching. If the same straight woman happens to contact a straight man, however, each will weigh the other's level of pizazz against their prospective value from continued search, and decide whether to propose marriage. They will only get married if both parties choose to propose. Otherwise, they both continue their searches.

An individual's orientation defines a restriction on the gender of singles with which they would be willing to consider marriage. Singles of orientation s can only form marriages with individuals of the opposite gender. Singles of orientation g can only form marriages with singles of the same gender. Singles of orientation b do not face any particular restrictions as to the gender of their potential spouses. Two singles are 'compatible' with each other if each individual's gender and orientation do not preclude them from marrying someone of the other's gender. For example, a man of orientation b is compatible with a woman of orientation s or b, or a man of orientation g or b, but not with a woman of orientation g or a man of orientation s. If two singles get married, they permanently exit the market, and each receives a flow utility from marriage equal to their spouse's pizazz level. All individuals receive zero flow utility while single.

Following Burdett and Coles (1997), let singles meet others according to a poisson process with parameter α, where α is assumed to be independent of the number of participating singles. Upon meeting, singles observe each other's gender, orientation, and pizazz level. If two singles who meet are not compatible with each other, they seperate and continue searching. If they are compatible with each other, each individual will decide whether or not to propose marriage. If both propose, they get married. If not, they seperate and continue searching. Assume that individuals who get married must permanently leave the market (i.e., marriages last forever and there is no search while married). Let individuals live according to an exponential random variable with parameter $\delta > 0$, so that individuals die at rate δ, and assume that widowed individuals do not return to the market. All individuals discount at rate $r > 0$.

Let β be the arrival rate of new singles, so that the number of new singles entering the market in any time interval dt is βdt. Any new individual entering the market has equal probability of being a man or a woman, has orientation s with probability p, orientation g with probability q, and orientation b with probability $1 - p - q$. Assume that a new single's pizazz level is independent of their gender and orientation, and let $F(z)$ be the probability that any new single has pizazz no greater than z. Assume that F is twice differentiable and strictly increasing on support $[\underline{x}, \tilde{x}]$.

Without a clone assumption, the distribution of pizazz among singles of a particular orientation in the market at a given time may not equal F. Let $G_j(\cdot, t)$ for $j \in \{s, g, b\}$ denote the distribution of pizazz among singles of orientation j in the market at time t. If a searcher meets a single individual of type j in the market at time t, $G_j(z, t)$ is the probability of that individual having pizazz no greater than z. The proportion of individuals of each orientation may also vary across time. Let $\tilde{p}(t)$ and $\tilde{q}(t)$ denote the proportion of singles who are type s and g at time t, respectively.

2.3 Stationary Environment

The analysis in this section is based on the concept of a partial rational expectations equilibrium, in which all agents are rational, except that they believe they exist in a stationary environment. Accordingly, the equilibrium results in this section must hold in any steady-state equilibrium, if one exists, but are less useful for considering a marriage market outside of a steady state.

Assume that all singles believe that the market can be characterized by G_s, G_g, G_b, where G_j is continuous with support $[\underline{x}, \tilde{x}]$ for $j = s, g, b$, and

$$G_s(z,t) = G_s(z) \text{ for all } z \text{ and for all } t; \tag{R1}$$

$$G_g(z,t) = G_g(z) \text{ for all } z \text{ and for all } t; \tag{R2}$$

$$G_b(z,t) = G_b(z) \text{ for all } z \text{ and for all } t; \tag{R3}$$

$$\tilde{p}(t) = \tilde{p} \text{ for all } t; \tag{R4}$$

$$\tilde{q}(t) = \tilde{q} \text{ for all } t. \tag{R5}$$

Conditions (R1)-(R3) require that all singles believe that the distribution of pizazz among all groups they face today will remain constant throughout time. Likewise, conditions (R4) and (R5) require that singles believe the number of singles in the market belonging to each orientation will not change. Thus, they believe they are operating in a stationary environment; when determining their expected value of continued search, they assume that their prospects will remain the same forever. Following (Burdett and Coles 1997, p. 6), conditional on beliefs (R1)-(R5) a partial rational expectations equilibrium (PREE) "requires that all agents use utility-maximizing strategies, given the behavior of other agents."

Let $G_j(\cdot|x)$ denote the distribution of pizazz among singles of type $j \in \{s, g, b\}$ who would propose upon meeting a single of pizazz x with whom they were compatible. Let $\alpha_{jk}(x)$ denote the arrival rate of proposals faced by a single of type j and pizazz x from singles of type k.

Let $U_j(x)$ denote the expected lifetime utility of an individual with orientation j and pizazz level x. Then

$$\begin{aligned}(1+rdt)U_s(x) = (1-\delta dt)\Big[&\alpha_{ss}(x)dt\mathbb{E}\big(\max\{\tilde{x}, U_s(x)\}|x \text{ and offer from s}\big)\\ &+ \alpha_{sb}(x)dt\mathbb{E}\big(\max\{\tilde{x}, U_s(x)\}|x \text{ and offer from b}\big)\\ &+ \big(1-\alpha_{ss}(x)dt - \alpha_{sb}(x)dt\big)U_s(x)\Big]\end{aligned}$$

Rearranging and letting $dt \to 0$,

$$\begin{aligned}(r+\delta)U_s(x) = \alpha_{ss}(x)\mathbb{E}\big(&\max\{\tilde{x}-U_s(x), 0\}|x \text{ and offer from s}\big)\\ &+ \alpha_{sb}(x)\mathbb{E}\big(\max\{\tilde{x}-U_s(x), 0\}|x \text{ and offer from b}\big)\end{aligned}$$

Thus, the optimal strategy is characterize by a reservation value, $R_s(x) = U_s(x)$, defined by

$$R_s(x) = \frac{1}{r+\delta}\left(\alpha_{ss}(x)\int_{R_s(x)}^{\tilde{x}}[1-G_s(\tilde{x}|x)]d\tilde{x} + \alpha_{sb}(x)\int_{R_s(x)}^{\tilde{x}}[1-G_b(\tilde{x}|x)]d\tilde{x}\right)$$

Similarly,

$$R_g(x) = \frac{1}{r+\delta}\left(\alpha_{gg}(x)\int_{R_g(x)}^{\tilde{x}}[1-G_g(\tilde{x}|x)]d\tilde{x} + \alpha_{gb}(x)\int_{R_g(x)}^{\tilde{x}}[1-G_b(\tilde{x}|x)]d\tilde{x}\right)$$

and

$$\begin{aligned}R_b(x) =& \frac{1}{r+\delta}\Bigg(\alpha_{bs}(x)\int_{R_b(x)}^{\tilde{x}}[1-G_s(\tilde{x}|x)]d\tilde{x}\\ &+ \alpha_{bg}(x)\int_{R_b(x)}^{\tilde{x}}[1-G_g(\tilde{x}|x)]d\tilde{x}\\ &+ \alpha_{bb}(x)\int_{R_b(x)}^{\tilde{x}}[1-G_b(\tilde{x}|x)]d\tilde{x}\Bigg)\end{aligned}$$

In a PREE, $R_j(\cdot)$ must be nondecreasing: anyone willing to propose to a single of a particular gender and orientation j with pizazz x is also willing to propose to a single of that gender and orientation with pizazz $x' > x$. So $U_j(x') \geq U_j(x)$, and thus $R_j(x') \geq R_j(x)$.

Further, equilibrium implies that type b individuals must be the pickiest. That is, $R_b(x) \geq R_j(x)$ for all x and for all $j \in \{s, g, b\}$. Consider $\alpha_{gg}(x)$ and $\alpha_{bg}(x)$. The arrival rate of proposals from type g individuals to a single of pizazz x with whom they are compatible is $\frac{\alpha}{2}\tilde{q}\Pr(R_g(\tilde{x}) \leq x|g)$: the instantaneous probability of meeting someone of the same gender, who is type g, and who (given that they are type g) has pizazz low enough to be willing to propose to someone of pizazz x. Because the reservation match strategy of type g individuals is irrespective of the orientation of their prospective partner, this arrival rate is the same for a type b individual of pizazz x as for a type g individual of the same pizazz level: $\alpha_{gg}(x) = \frac{\alpha}{2}\tilde{q}\Pr(R_g(\tilde{x}) \leq x|g) = \alpha_{bg}(x)$. Similarly, $\alpha_{ss}(x) = \alpha_{bs}(x)$. But in the arrival rate of proposals from singles of orientation b, type b individuals have an advantage. While a type s (g) individual has to be lucky enough to meet a type b individual of the opposite (same) gender with low enough standards, a type b individual is compatible with any other type b individual:

$$\begin{aligned}\alpha_{sb}(x) = \alpha_{gb}(x) = \frac{\alpha}{2}(1 - \tilde{p} - \tilde{q})\Pr(R_b(\tilde{x}) \leq x|b) \\ \leq \alpha(1 - \tilde{p} - \tilde{q})\Pr(R_b(\tilde{x}) \leq x|b) = \alpha_{bb}(x)\end{aligned}$$

In particular, individuals of orientation b can expect to get proposals from other individuals of orientation b twice as often as a type s or g individual of the same pizazz level would. Thus

$$\begin{aligned}&\alpha_{ss}(x)\int_z^{\tilde{x}}[1 - G_s(\tilde{x}|x)]d\tilde{x} + \alpha_{sb}(x)\int_z^{\tilde{x}}[1 - G_b(\tilde{x}|x)]d\tilde{x} \\ &\qquad \leq \alpha_{bs}(x)\int_z^{\tilde{x}}[1 - G_s(\tilde{x}|x)]d\tilde{x} + \alpha_{bb}(x)\int_z^{\tilde{x}}[1 - G_b(\tilde{x}|x)]d\tilde{x} \\ &\qquad \leq \alpha_{bs}(x)\int_z^{\tilde{x}}[1 - G_s(\tilde{x}|x)]d\tilde{x} + \alpha_{bg}(x)\int_z^{\tilde{x}}[1 - G_g(\tilde{x}|x)]d\tilde{x} \\ &\qquad + \alpha_{bb}(x)\int_z^{\tilde{x}}[1 - G_b(\tilde{x}|x)]d\tilde{x}\end{aligned}$$

for all $x \in [\underline{x}, \tilde{x}]$ and for all $z \leq \tilde{x}$, and so $R_s(x) \leq R_b(x)$ for all pizazz levels x. Similarly, $R_g(x) \leq R_b(x)$ for all x.

As in Burdett and Coles (1997), market participants of each type can be partitioned into a finite number of distinct classes, with individuals in the same type and class sharing the same equilibrium behavior. Unlike in Burdett and Coles, the addition of overlapping orientations will preclude a meaningful definition of classes such that singles will only marry others within the same class.

Proposition 2.3.1. *Given $\{G_j\}_{j\in\{s,g,b\}}$ and $\tilde{p}, \tilde{q}$, a PREE implies the existence of a unique collection of three partitions, $(\{y_s(n)\}_{n=0}^{J_s}, \{y_g(n)\}_{n=0}^{J_g}, \{y_b(n)\}_{n=0}^{J_b})$ with $y_j(0) = \tilde{x}$ and $y_j(J_j) \leq \underline{x}$ such that a single of type j and pizazz $x \in [y_j(n), y_j(n-1))$ proposes to an individual of type k with pizazz $x' \in [y_k(m), y_k(m-1))$ upon meeting if and only if they are compatible and $R_j(y_j(n)) \leq y_k(m)$.*

Proof.

Lemma 2.3.1. $R_j(\tilde{x}) < x$, $\forall j \in \{s, g, b\}$

Proof. Any compatible individual will propose if they encounter a single with pizazz $\tilde{x}$. Therefore, $\alpha_{bs}(\tilde{x}) = \alpha\frac{\tilde{p}}{2}$, $\alpha_{bg} = \alpha\frac{\tilde{q}}{2}$, and $\alpha_{bb} = \alpha(1-\tilde{p}-\tilde{q})$, and $G_j(\cdot|\tilde{x}) = G_j(\cdot)$ for $j = s, g, b$. So

$$\begin{aligned} R_b(\tilde{x}) =& \frac{\alpha}{r+\delta}\Bigg(\frac{\tilde{p}}{2}\int_{R_b(\tilde{x})}^{\tilde{x}}[1-G_s(\tilde{x})]d\tilde{x} + \frac{\tilde{q}}{2}\int_{R_b(\tilde{x})}^{\tilde{x}}[1-G_g(\tilde{x})]d\tilde{x} \\ &+ (1-\tilde{p}-\tilde{q})\int_{R_b(\tilde{x})}^{\tilde{x}}[1-G_b(\tilde{x})]d\tilde{x}\Bigg) \end{aligned}$$

Clearly, $R_b(x) \geq \tilde{x}$ does not satisfy this equation, so $R_b(x) < \tilde{x}$. Since type b individuals must be weakly pickier than type s and g individuals of the same pizazz level, $R_g(\tilde{x}) \leq R_b(\tilde{x}) < \tilde{x}$ and $R_s(\tilde{x}) \leq R_b(\tilde{x}) < \tilde{x}$ □

Noting also that $\alpha_{ss}(\tilde{x}) = \alpha\frac{\tilde{p}}{2}$, $\alpha_{gg}(\tilde{x}) = \frac{\tilde{q}}{2}$, and $\alpha_{sb}(\tilde{x}) = \alpha_{gb}(\tilde{x}) = \alpha\frac{1-\tilde{p}-\tilde{q}}{2}$,

$$R_s(\tilde{x}) = \frac{\alpha}{r+\delta}\left(\frac{\tilde{p}}{2}\int_{R_s(\tilde{x})}^{\tilde{x}}[1-G_s(\tilde{x})]d\tilde{x} + \frac{1-\tilde{p}-\tilde{q}}{2}\int_{R_s(\tilde{x})}^{\tilde{x}}[1-G_b(\tilde{x})]d\tilde{x}\right)$$

and

$$R_g(\tilde{x}) = \frac{\alpha}{r+\delta}\left(\frac{\tilde{q}}{2}\int_{R_g(\tilde{x})}^{\tilde{x}}[1-G_g(\tilde{x})]d\tilde{x} + \frac{1-\tilde{p}-\tilde{q}}{2}\int_{R_g(\tilde{x})}^{\tilde{x}}[1-G_b(\tilde{x})]d\tilde{x}\right)$$

Let $y_j(0) = \tilde{x}$ and $y_j(1) = R_b(\tilde{x})$ for $j = s, g, b$.

Lemma 2.3.2. *Singles of orientation j with pizazz $x \in [y_j(1), y_j(0))$ have $R_j(x) = R_j(\tilde{x})$.*

Proof. Consider a type s individual with pizazz $x \in [y_s(1), \tilde{x}) = [R_b(\tilde{x}), \tilde{x})$. Since $x \geq R_b(\tilde{x}) \geq R_s(\tilde{x})$, any compatible single will propose to this individual upon meeting. Thus, $\alpha_{ss}(x) = \alpha\frac{\tilde{p}}{2}$, $\alpha_{sb}(x) = \alpha\frac{1-\tilde{p}-\tilde{q}}{2}$, $G_s(\cdot|x) = G_s(\cdot)$, and $G_b(\cdot|x) = G_s(\cdot)$. Therefore, $R_s(x) = R_s(\tilde{x})$. A similar argument shows this result for $j = g, b$. □

The inductive process to define the remaining elements of the partition can be thought of as follows: Given $(\{y_s(n)\}_{n=0}^{n_s}, \{y_g(n)\}_{n=0}^{n_g}, \{y_b(n)\}_{n=0}^{n_b})$ such that $\{n : y_j(n) \geq y \text{ and } n > n_j\} = \emptyset, \forall j \in \{s, g, b\}$, where $y := \min\{y_s(n_s), y_g(n_g), y_b(n_b)\}$, and given

$$A(y) = \{(j, R_j(y_j(n))) : n \leq n_j, j \in \{s, g, b\}\},$$

let

$$B(y) := \{(j, \lim_{\epsilon \to 0} R_j(y - \epsilon)) : y_j(n_j) = y\}.$$

Now, let $y' := \max_{j=s,g,b}\{x : (j, x) \in A(y) \cup B(y) \text{ and } x < y\}$. Let, $y_j(n_j + 1) = y'$ for all j such that $(k, y') \in A(y) \cup B(y)$ for some k compatible with j. For j such that $(k, y') \notin A(y) \cup B(y)$ for all k compatible with j, we will have $y_j(n_j + 1) < y'$, so this process can now be repeated.

Essentially, given any class cut-off y and knowledge of reservation values for all singles with pizazz higher than y, the next highest partition element for any orientation will be the maximum of the reservation values in that set which are lower than the y together with the reservation values of singles with pizazz just below y. This next highest cut-off, y' will be an element of the partitions corresponding only to the orientations j for which there exists some k compatible with j and some $x \in [\underline{x}, \tilde{x}]$ such that $R_k(x) = y'$. Note that, since type b individuals are compatible with individuals of every orientation, this next highest cut-off y' will always be an element of $\{y_b(n)\}_{n=1}^{J_b}$. That is, $\{y_j(n)\}_{n=1}^{J_j} \subseteq \{y_b(n)\}_{n=1}^{J_b}$ for all j.

Lemma 2.3.3. *Given $y_j(n)$ defined in this way, any individual of orientation j and pizazz $x \in [y_j(n), y_j(n-1))$ will have reservation value $R_j(x) = R_j(y_j(n))$.*

Proof. Taking Lemma 2.3.2 as a basis step, suppose that $y_j(n)$ is found by this inductive process, and that this result holds for all k and m such that $y_k(m) > y_j(m)$. Now, let $x \in [y_j(n), y_j(n-1))$. By construction, $\nexists(k, x')$ such that k is compatible with j and $R_k(x') \in (y_j(n), y_j(n-1)) \supseteq (y_j(n), x]$. That is, there is no one compatible with singles of orientation j who would propose to someone of pizazz x and would not propose to someone of pizazz $y_j(n)$. Since type j individuals of pizazz x and pizazz $y_j(n)$ get the same offers, $\alpha_{jk}(x) = \alpha_{jk}(y_j(n))$ and $G_k(\cdot|x) = G_k(\cdot|y_j(n))$ for all k compatible with j, so $R_j(x) = R_j(y_j(n))$. □

Finiteness of the partitions follows from a simple contradiction argument if $\underline{x} > 0$.

Finally, consider a single of type j and pizazz $x \in [y_j(n), y_j(n-1))$ and a single of type k with pizazz $x' \in [y_k(m), y_k(m-1))$. The reservation match strategy and Lemma 2.3.3 imply that if j and k are compatible and $R_j(y_j(n)) \leq y_k(m)$, then $R_j(x) = R_j(y_j(n)) \leq y_k(m) \leq x'$ and individual of type j will propose upon meeting. Conversely, if the individual with orientation j and pizazz x would propose upon meeting, then j and k are compatible and $R_j(y_j(n)) = R_j(x) \leq x'$. Since j and k are compatible, by construction $R_j(x'') \notin (y_k(m), y_k(m-1)), \forall x'' \in [\underline{x}, \tilde{x}]$, so $R_j(y_j(n)) \leq x'$ and $R_j(y_j(n)) \notin (y_k(m), x']$, and therefore $R_j(y_j(n)) \leq y_k(m)$. □

Consider a type s individual with pizazz $x = R_b(\tilde{x}) - \epsilon$ for ϵ arbitrarily small. Then $x > R_s(\tilde{x})$, so any compatible type s single would propose to this indivudal upon meeting. Thus $\alpha_{ss}(x) = \alpha\frac{\tilde{p}}{2}$ and $G_s(\cdot|x) = G_s(\cdot)$. Since $x < R_b(\tilde{x})$, type b individuals of the highest class would not propose to this individual, but since x is just under this cutoff, any compatible type b individual with pizazz less than $y_b(1) = R_b(\tilde{x})$ would propose. So $\alpha_{sb}(x) = \alpha\frac{1-\tilde{p}-\tilde{q}}{2}G_b(y_b(1))$ So this individual's reservation value is

$$
\begin{aligned}
R_s(x) &= \frac{\alpha}{r+\delta}\left(\frac{\tilde{p}}{2}\int_{R_s(x)}^{\tilde{x}}[1-G_s(\tilde{x})]d\tilde{x} + \frac{1-\tilde{p}-\tilde{q}}{2}G_b(y_b(1))\int_{R_s(x)}^{y_b(1)}[1-G_b(\tilde{x}|x)]d\tilde{x}\right) \\
&= \frac{\alpha}{r+\delta}\left(\frac{\tilde{p}}{2}\int_{R_s(x)}^{\tilde{x}}[1-G_s(\tilde{x})]d\tilde{x} + \frac{1-\tilde{p}-\tilde{q}}{2}\int_{R_s(x)}^{y_b(1)}[G_b(y_b(1))-G_b(\tilde{x})]d\tilde{x}\right)
\end{aligned}
$$

This will be the reservation value for any type s individual in the second class, $[y_b(2), y_b(1))$.

In general, we can write the reservation value for all members of a class, $\phi_j(n) = R_j(y_j(n))$ as follows:

$$\begin{aligned}\phi_s(n) =& \frac{\alpha}{r+\delta}\Bigg(\frac{\tilde{p}}{2}\int_{\phi_s(n)}^{c_{ss}(n)}[G_s(c_{ss}(n)) - G_s(\tilde{x})]d\tilde{x} \\ &+ \frac{1-\tilde{p}-\tilde{q}}{2}\int_{\phi_s(n)}^{c_{sb}(n)}[G_b(c_{sb}(n)) - G_b(\tilde{x})]d\tilde{x}\Bigg)\end{aligned}$$

$$\begin{aligned}\phi_g(n) =& \frac{\alpha}{r+\delta}\Bigg(\frac{\tilde{q}}{2}\int_{\phi_g(n)}^{c_{gg}(n)}[G_g(c_{gg}(n)) - G_g(\tilde{x})]d\tilde{x} \\ &+ \frac{1-\tilde{p}-\tilde{q}}{2}\int_{\phi_g(n)}^{c_{gb}(n)}[G_b(c_{gb}(n)) - G_b(\tilde{x})]d\tilde{x}\Bigg)\end{aligned}$$

and

$$\begin{aligned}\phi_b(n) =& \frac{\alpha}{r+\delta}\Bigg(\frac{\tilde{p}}{2}\int_{\phi_b(n)}^{c_{bs}(n)}[G_s(c_{bs}(n)) - G_s(\tilde{x})]d\tilde{x} \\ &+ \frac{\tilde{q}}{2}\int_{\phi_b(n)}^{c_{bg}(n)}[G_g(c_{bg}(n)) - G_g(\tilde{x})]d\tilde{x} \\ &+ (1-\tilde{p}-\tilde{q})\int_{\phi_b(n)}^{c_{bb}(n)}[G_b(c_{bb}(n)) - G_b(\tilde{x})]d\tilde{x}\Bigg),\end{aligned}$$

where

$$c_{jk}(n) := \min_{m\in\mathbb{N}}\{y_k(m) : R_k(y_k(m)) > y_j(n)\} = \sup\{x : R_k(x) \leq y_j(n)\}$$

Let C_j be the set of types compatible with j. That is, $C_s = \{s, b\}$, $C_g = \{g, b\}$, and $C_b = \{s, g, b\}$. Then, for all $j \in \{s, g, b\}$ and all $n \geq 1$,

$$y_j(n) = \max_{k\in C_j, m\in\mathbb{N}}\{\phi_k(m) : \phi_k(m) < y_j(n-1)\}$$

2.4 Steady State Equilibria

A given $(G_s, G_g, G_b, \tilde{p}, \tilde{q})$ imply a unique partition as in Proposition 2.3.1. This partition implies a unique distribution of singles flowing out of the market through marriage. Together with the number N of singles in the market, this would imply an outflow rate of singles of each type exiting the market through marriage or death. Given (F, β, p, q), a steady state equilibrium is $(G_s, G_g, G_b, N, \tilde{p}, \tilde{q})$ such that the strategies are consistent with a PREE, and there is balanced flow:

$\forall n, \forall [z_1, z_2) \subseteq [y_s(n), y_s(n-1))$,

$$\tilde{p}N(\alpha\lambda_{sn} + \delta)[G_s(z_2) - G_s(z_1)] = p\beta[F(z_2) - F(z_1)]$$

$\forall n, \forall [z_1, z_2) \subseteq [y_g(n), y_g(n-1))$,

$$\tilde{q}N(\alpha\lambda_{gn} + \delta)[G_g(z_2) - G_g(z_1)] = q\beta[F(z_2) - F(z_1)]$$

and $\forall n, \forall [z_1, z_2) \subseteq [y_b(n), y_b(n-1))$,

$$(1 - \tilde{p} - \tilde{q})N(\alpha\lambda_{bn} + \delta)[G_b(z_2) - G_b(z_1)] = (1 - p - q)\beta[F(z_2) - F(z_1)]$$

where λ_{jn} denotes the probability that a meeting between an individual of type j and class n and another individual results in a marriage:

$$\lambda_{sn} = \frac{\tilde{p}}{2}\big[G_s(c_{ss}(n)) - G_s(\phi_s(n))\big] + \frac{1 - \tilde{p} - \tilde{q}}{2}\big[G_b(c_{sb}(n)) - G_b(\phi_s(n))\big]$$

$$\lambda_{gn} = \frac{\tilde{q}}{2}\big[G_g(c_{gg}(n)) - G_g(\phi_g(n))\big] + \frac{1 - \tilde{p} - \tilde{q}}{2}\big[G_b(c_{gb}(n)) - G_b(\phi_g(n))\big]$$

and

$$\lambda_{bn} = \frac{\tilde{p}}{2}\big[G_s(c_{bs}(n)) - G_s(\phi_b(n))\big] + \frac{\tilde{q}}{2}\big[G_g(c_{bg}(n)) - G_g(\phi_b(n))\big]$$
$$+ (1 - \tilde{p} - \tilde{q})\big[G_b(c_{bb}(n)) - G_b(\phi_b(n))\big]$$

As in Burdett and Coles (1997), these balanced flow conditions imply that within a type and class, the steady state density function is a rescaled version of the entry density function F; Since F is differentiable,

$\forall n, \forall z_2 \in [y_s(n), y_s(n-1))$,

$$G'_s(z_2) = \frac{p\beta}{\tilde{p}N(\alpha\lambda_{sn} + \delta)} F'(z_2)$$

$\forall n, \forall z_2 \in [y_g(n), y_g(n-1))$,

$$G'_g(z_2) = \frac{q\beta}{\tilde{q}N(\alpha\lambda_{gn} + \delta)} F'(z_2)$$

and $\forall n, \forall z_2 \in [y_b(n), y_b(n-1))$,

$$G'_b(z_2) = \frac{(1-p-q)\beta}{(1-\tilde{p}-\tilde{q})N(\alpha\lambda_{bn} + \delta)}$$

The problem of characterizing a steady state equilibrium is made difficult in this model by the fact that individuals do not marry only within their own class, but in fact might marry someone of a higher or lower class than their own, or someone of a different orientation with classes that are not in one-to-one correspondance with their own type's classes.

By splitting the integrals in the expression for a class's reservation value, $\phi_j(n)$ into sums of within-class integrals, $\phi_j(n)$ can be expressed in terms of the exogenous distribution of pizazz and proportions of orientations, rather than the endogenous $G_j(\cdot)$ and $\tilde{p}, \tilde{q}$. Similarly writing λ_{jn} as sums of within-class probability masses would aid in identifying the steady-state class cutoffs for a given (F, β, p, q). For instance,

$$y_b(1) = \frac{\alpha\lambda_{b1}}{r+\delta}\int_{y_b(1)}^{\tilde{x}} \frac{1-F(x)}{1-F(y_b(1))}dx$$

However, as the number of classes of potential partners between an individual's reservation pizazz level and the maximum pizazz level of someone who would be willing to marry that individual grows, the task of simplifying their reservation value and identifying it in terms of exogenous variables and distributions becomes more algebraically complex. I have not yet been able to directly or inductively identify the steady-state class partitions, and so the question of the existence of a steady state remains open.

2.5 Conclusion

This paper provides an example of how the existence of queer people is relevant to the structure of equilibria in economic models which study the formation of romantic partnerships. While I did not reach an explicit solution for steady state equilibria in this model, it is clear that the presence of singles of various orientations in the marriage market fundamentally alters the class structure of Burdett and Coles (1997). Thus, the restriction to straight individuals, an implicit assumption which often goes unremarked upon in marriage market literature, is not without loss of generality.

This suggests several avenues for future research. In this model, because individuals leave the market upon marriage, singles are indifferent between potential partners of the same pizazz level and different orientations. However, with a possibility of continued search after marriage, a preference for partners who are attracted to only one gender might arise. As individuals with attraction to more than one gender are more likely to meet someone with whom they are compatible, they may be perceived to present a higher risk of unfaithfulness. This may result in higher reservation values for the pizazz of a potential partner with that orientation, attenuating their natural advantage in the marriage market. This would also suggest the relevance of a model in which a single's orientation is not immediately or perfectly observable upon contact, as those who would be percieved as riskier matches may have incentive to hide their true type.

APPENDIX A

APPENDIX OF CHAPTER 1

A.1 Details from Section 1.2.4

The class of subgame perfect equilibria described in Section 1.2.4 includes many trivial equilibria in which the school's incentives do not reflect the signalling value of education. For example, suppose that $a\underline{x} \leq \mathbb{E}(x)$. Then for all (s, t) such that

(i) $\int_{\underline{x}}^{\overline{x}} (t - c(x))s(x)dF(x) \geq 0$ and

(ii) $t \leq a\mathbb{E}(x|\text{admitted}; s) - \mathbb{E}(x|\text{not admitted or } y < t; s)$,

there exists a subgame perfect equilibrium (s, t, σ, μ), where

$$\sigma(s', t') = \begin{cases} 1 & (s', t') = (s, t) \\ 0 & \text{else} \end{cases}$$

and $\mu_1(s', t') \leq \frac{\mathbb{E}(x)+t}{a}$ for all $(s', t') \neq (s, t)$.

(i) ensures that the school will make a non-negative profit if they announce (s, t) and all admitted workers who can afford tuition choose to attend, and (ii) ensures that the payoff of educated workers is not less than that of uneducated workers if all workers with an education decision choose to attend. Essentially, if $a\underline{x} < \mathbb{E}(x)$, it is always possible to set firm beliefs which are consistent with no workers getting educated if the school chooses a particular strategy, and which also justify workers choosing not to get educated if the school chooses that strategy. So for any (s, t) satisfying (i) and (ii), there are consistent beliefs which allow for an SPE in which workers force the school to choose (s, t) by refusing to attend if they announce any other admissions rule or tuition.

These equilibria arise because firms may hold arbitrary and extreme beliefs when their expectations of workers' productivity are not well-defined by Bayes' rule. One additional restriction on the 'reasonability' of beliefs which could eliminate such equilibria is a notion of trembling hand consistency.

Definition A.1.1. *Firm beliefs $\mu = (\mu_0, \mu_1)$ are trembling hand consistent with worker strategy*

Table A.1: Values of $\sigma^*(s,t)$ in a trembling hand consistent SPE

	$v(s,t) > \mathbb{E}(x \mid \text{not admitted or } y < t; s)$	$v(s,t) = \mathbb{E}(x \mid \text{not admitted or } y < t; s)$	$v(s,t) < \mathbb{E}(x \mid \text{not admitted or } y < t; s)$
$v(s,t) > \mathbb{E}(x)$	$\sigma^*(s,t) = 1$	$\sigma^*(s,t) = 1$	$\sigma^*(s,t) = \frac{1}{p(s)\bar{G}(t)} \frac{v(s,t)-\mathbb{E}(x)}{v(s,t)-\mu_1(s,t)}$
$v(s,t) = \mathbb{E}(x)$	$\sigma^*(s,t) = 1$	$\sigma^*(s,t) \in [0,1]$	$\sigma^*(s,t) = 0$
$v(s,t) < \mathbb{E}(x)$	$\sigma^*(s,t) \in \{0, 1, \frac{1}{p(s)\bar{G}(t)} \frac{v(s,t)-\mathbb{E}(x)}{v(s,t)-\mu_1(s,t)}\}$	$\sigma^*(s,t) = 0$	$\sigma^*(s,t) = 0$

$\sigma : S \times \mathbb{R}^+ \to [0,1]$ *if* $\mu = \lim_{\epsilon \to 0} \mu(\epsilon)$*, where for all* $\epsilon \in (0,1)$ $\mu(\epsilon)$ *is the unique firm belief consistent with the perturbed worker strategy* σ_ϵ *given by*

$$\sigma_\epsilon(s,t) = \sigma(s,t)(1-\epsilon) + (1-\sigma(s,t))\epsilon$$

That is, μ is trembling hand consistent with σ if it is the limit as ϵ approaches 0 of the beliefs consistent with a perturbed strategy in which workers with admissions decisions follow σ but have probability ϵ of 'accidentally' deviating.

Suppose that $(s^*, t^*, \sigma^*, \mu)$ is a subgame perfect equilibrium, and that μ is trembling hand consistent with σ^*. First, note that $\mu_1(s,t) = \mathbb{E}(x \mid \text{admitted}; s)$ for all admissions rules $s : [\underline{x}, \overline{x}] \to [0,1]$ and tuition $t > 0$ such that some students are admitted and can afford tuition with positive probability. Additionally, letting $p(s) := \int_{\underline{x}}^{\overline{x}} s(x) dF(x)$,

$$\mu_0(s,t) = \begin{cases} \mathbb{E}(x \mid \text{not admitted or } y < t; s) & \sigma^*(s,t) = 1 \\ \frac{\mathbb{E}(x) - \sigma^*(s,t)p(s)\bar{G}(t)\mathbb{E}(x \mid \text{admitted}; s)}{1 - \sigma^*(s,t)p(s)\bar{G}(t)} & \sigma^*(s,t) \in (0,1) \\ \mathbb{E}(x) & \sigma^*(s,t) = 0 \end{cases}$$

for all s, t.

Given this formulation of firm beliefs, worker optimality of σ^* for all s, t identifies the value of $\sigma^*(s,t)$ for most admissions rules and tuition. This identification can be determined based on the value of education to a worker,

$$v(s,t) := a\mathbb{E}(x \mid \text{admitted}; s) - t$$

in comparison to $\mathbb{E}(x)$ and $\mathbb{E}(x \mid \text{not admitted or } y < t; s)$. Table A.1 lists the possible values of $\sigma^*(s,t)$ in any trembling hand consistent SPE for all cases of that comparison. The value of $\sigma^*(s,t)$ is not pinned down by the requirements of trembling hand consistency or subgame perfection only if $\mu_1(s,t) > \mathbb{E}(x) > v(s,t) > \mathbb{E}(x|\text{not admitted or } y < t; s)$ or $\mu_1(s,t) = \mathbb{E}(x) = v(s,t) = \mathbb{E}(x|\text{not admitted or } y < t; s)$. In the former case, $\sigma^*(s,t) = 0$, $\sigma^*(s,t) = 1$, or a unique $\sigma^*(s,t) \in (0,1)$ could be supported in a trembling hand consistent SPE; in the latter, any value of $\sigma^*(s,t)$ could be supported. In my analysis I assume that workers use the strategy $\tilde{\sigma}$ given by

$$\tilde{\sigma}(s,t) = \begin{cases} 1 & v(s,t) \geq \mathbb{E}(x|\text{not admitted or } y < t; s) \\ \frac{1}{p(s)\bar{G}(t)} \frac{v(s,t) - \mathbb{E}(x)}{v(s,t) - \mathbb{E}(x|\text{admitted}; s)} & \mathbb{E}(x) < v(s,t) < \mathbb{E}(x|\text{not admitted or } y < t; s) \\ 0 & \text{else} \end{cases}$$

which is equivalent to restricting to trembling hand consistent SPE, and assuming that whenever possible, all workers with an education decision would choose to attend the school. This worker strategy gives the school maximum control over the signalling value of education.

A.2 Details from Section 1.4.1

The notion of trembling hand consistency can easily be extended into the model without firm discrimination, and is again helpful in ruling out trivial subgame perfect equilibria:

Definition A.2.1. *Firm beliefs $\mu = (\mu_0, \mu_1)$ are trembling hand consistent with worker strategy $\sigma = (\sigma_1, \sigma_2)$ in the model without firm discrimination if $\mu = \lim_{\epsilon \to 0} \mu(\epsilon)$, where for all $\epsilon \in (0,1)$ $\mu(\epsilon)$ is the unique firm belief consistent with the perturbed worker strategy σ_ϵ given by*

$$\sigma_{\epsilon_i} = \sigma_i(s,t)(1-\epsilon) + (1 - \sigma_i(s,t))\epsilon$$

Denote the mass of type i students who are admitted and can afford tuition by $n_i(s,t) := q_i \bar{G}_i(t_i) \int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)$ for all admissions rules s, tuition t, and $i = 1, 2$. Also denote the expected initial ability of an admitted type i worker by $m_i(s) := \frac{\int_{\underline{x}}^{\overline{x}} x s_i(x) dF(x)}{\int_{\underline{x}}^{\overline{x}} s_i(x) dF(x)}$ for all admissions rules s.

Finally, let

$$\tilde{w}(p_1, p_2; s, t) := \begin{cases} \frac{\sum_{i=1}^{2} n_i(s,t) m_i(s)}{\sum_{i=1}^{2} n_i(s,t)} - \mathbb{E}(x) & p_1 = p_2 = 0 \\ a\frac{\sum_{i=1}^{2} p_i n_i(s,t) m_i(s)}{\sum_{i=1}^{2} p_i n_i(s,t)} - \frac{\mathbb{E}(x) - \sum_{i=1}^{2} p_i n_i(s,t) m_i(s)}{1 - \sum_{i=1}^{2} p_i n_i(s,t)} & \text{else} \end{cases}$$

for all $(p_1, p_2) \in [0, 1] \times [0, 1]$ and all admissions rules s and tuitions t such that $n_i(s, t) > 0$ for some i. That is, $\tilde{w}(p_1, p_2; s, t)$ is the value of education to a worker if the school announces (s, t) and firm beliefs are trembling hand consistent with a worker strategy σ such that $\sigma_i(s, t) = p_i$ for $i = 1, 2$.

Now, if $(s^*, t^*, \sigma^*, \mu)$ is a subgame perfect equilibrium in the model without firm discrimination, and μ is trembling hand consistent with σ^*, then for all (s, t) such that $n_i(s, t) > 0$, $\sigma_i^*(s, t) = 1$ if $\tilde{w}(1, \sigma_{-i}^*(s, t); s, t) > t_i$, and $\sigma_i^*(s, t) = 0$ if $\tilde{w}(0, \sigma_{-i}^*; s, t) < t_i$. I will restrict my analysis to potential subgame perfect equilibria in which workers follow a particular strategy profile $\tilde{\sigma}$ which has the characteristic that it could be supported in a subgame perfect equilibria along with firm beliefs which are consistent with $\tilde{\sigma}$, and can be constructed as follows:

Suppose $t = (t_1, t_2)$, for some $t_1 = t_2 > 0$. Since workers of both types facing an education decision would receive the same value from education and face the same tuition, I make the simplifying assumption that in this case $\tilde{\sigma}_1(s, t) = \tilde{\sigma}_2(s, t)$. In particular, assuming all students with an education decision will choose to attend the school wherever possible,

$$\tilde{\sigma}_i(s, t) = \begin{cases} 1 & \tilde{w}(1, 1; s, t) \geq t_i \\ \frac{a\frac{\sum_{j=1}^{2} n_j(s,t) m_j(s)}{\sum_{j=1}^{2} n_j(s,t)} - \mathbb{E}(x) - t_i}{(a-1)\sum_{j=1}^{2} n_j(s,t) m_j(s) - t_i \sum_{j=1}^{2} n_j(s,t)} & \tilde{w}(1, 1; s, t) < t_i < \tilde{w}(0, 0; s, t) \\ 0 & \text{else} \end{cases}$$

Suppose instead that $t_1 > t_2$. Let $\tilde{\sigma}_1(s, t) = \tilde{\sigma}_2(s, t) = 1$ if

$$\tilde{w}(1, 1; s, t) \geq t_1 > t_2 \tag{A.1}$$

Else, if

$$t_1 \geq \tilde{w}(0, 1; s, t) \geq t_2 \tag{A.2}$$

let $\sigma_1(s,t) = 0$ and $\sigma_2(s,t) = 1$. Otherwise, if

$$\tilde{w}(0, 0; s, t) \leq t_2 < t_1 \tag{A.3}$$

let $\tilde{\sigma}_1(s,t) = \tilde{\sigma}_2(s,t) = 0$. If none of the above hold, and $\tilde{\sigma}_i(s,t)$ is a best response to (s,t) for $i = 1, 2$ when firm beliefs are trembling hand consistent with $\tilde{\sigma}$, then $\tilde{\sigma}_i \in (0, 1)$ for some i. In particular, if (A.2) does not hold, then either

$$t_2 > \tilde{w}(0, 1; s, t) \tag{A.4a}$$

or

$$t_1 < \tilde{w}(0, 1; s, t) \tag{A.4b}$$

If (A.4a) and not (A.1) and not (A.3), let $\tilde{\sigma}_1(s,t) = 0$ and $\tilde{\sigma}_2(s,t) = \frac{1}{n_2(s,t)} \frac{am_2(s) - \mathbb{E}(x) - t_2}{(a-1)m_2(s) - t_2}$, the unique $p_2 \in (0, 1)$ such that $\tilde{w}(0, p_2; s, t) = t_2$. If (A.4b) and not (A.1) and not (A.3), let $\tilde{\sigma}_2(s,t) = 1$, and suppose $\tilde{\sigma}_1(s,t) \in (0, 1)$. The condition $\tilde{w}(\tilde{\sigma}_1(s,t), 1) = t_1$ defines a quadratic equation in $\tilde{\sigma}_1(s,t) \in (0, 1)$, with solution

$$\tilde{\sigma}_1(s,t) = \begin{cases} \frac{n_2(s,t)(1 - n_2(s,t))(\tilde{w}(0,1);s,t) - t_1}{n_1(s,t)((1 - n_2(s,t))(\tilde{w}(0,1;s,t) - t_1) + t_1 + m_1(s) - am_2(s))} & (a-1)m_1(s,t) = t_1 \\ \frac{-B(s,t) + \sqrt{B(s,t)^2 + 4n_2(s,t)(1 - n_2(s,t))((a-1)m_1(s) - t_1)(\tilde{w}(0,1;s,t) - t_1)}}{2n_1(s,t)((a-1)m_1(s) - t_1)} & \text{else} \end{cases}$$

where $B_1(s,t) := (1 - n_2(s,t))((a-1)m_1(s) - \tilde{w}(0, 1; s, t)) + t_1 + m_1(s) - am_2(s)$.

The construction of $\tilde{\sigma}_i(s,t)$ for $i = 1, 2$ when $t_2 > t_1$ is symmetric to that of $\tilde{\sigma}_i(s,t)$ when $t_1 > t_2$.

Similarly to the model without diversity, the school cannot strictly benefit from selecting an admissions rule and tuition such that workers faced with an education decision will randomize:

For any (s,t) such that $\tilde{\sigma}_i(s,t) \in (0, 1)$ and $\tilde{\sigma}_{-i}(s,t) \in \{0, 1\}$ for some i, $\tilde{\sigma}_i(s,t)$ is inversely proportional to $n_i(s,t)$. Thus the school could adjust its admissions rule to s' given by $s'_i(x) = s_i(x)\tilde{\sigma}_i(s,t)$ and $s'_{-i}(x) = s_{-i}(x)$, and educate the same number of students from each type with the same distribution of abilities under (s', t) as they would under (s,t), but have $\tilde{\sigma}_i(s',t) = 1$. Similarly, if $t_1 = t_2$ and $\tilde{\sigma}_1(s,t) = \tilde{\sigma}_2(s,t) \in (0,1)$, then $\tilde{\sigma}_i(s,t)$ is inversely proportional to $\sum_{i=1}^{2} n_i(s,t)$ for all i. The school could reduce the number of students of each type it admits without changing the distribution of ability among its admitted workers in such a way that the same number of workers would attend the school and all workers with education decisions would attend for sure. Thus any school strategy in which some workers would randomize under $\tilde{\sigma}$ is weakly dominated by a strategy such that no workers randomize.

A.3 Proofs

A.3.1 Proofs for Section 1.2

Proof of Proposition 1.2.1. It is first useful to prove that the school's best response to $\tilde{\sigma}$ involves a deterministic admissions test

Lemma A.3.1. *If (s^*, t^*) solves $\max_{s,t} \Pi(s,t;\tilde{\sigma})$ and $\tilde{\sigma}(s^*,t^*) > 0$, then $s^*(x) \in \{0,1\}$ almost everywhere on $[\underline{x}, \overline{x}]$.*

Recall that when workers use strategy $\tilde{\sigma}$, the school's problem can be written as

$$\max_{s,t} \Pi(s,t) := \bar{G}(t) \int_{\underline{x}}^{\overline{x}} (t - c(x)) s(x) dF(x)$$

subject to attendance constraint (1.1).

Suppose that (s^*, t^*) satisfies the attendance constraint and there exists some set $A \subseteq [\underline{x}, \overline{x}]$ such that $s^*(A) \subseteq (0,1)$ and $\int_A f(x)dx > 0$. Since $0 < \int_A s^*(x) f(x) dx < \int_A f(x)dx$ and F is continuous, there exists some $\tilde{x} \in [\underline{x}, \overline{x}]$ such that $\int_{A'} f(x))dx = \int_A s^*(x) f(x) dx$, where

$A' := [\tilde{x}, \overline{x}] \cap A$. Consider the alternative admissions rule s given by

$$s(x) = \begin{cases} 1 & x \in A' \\ 0 & x \in A \setminus A' \\ s^*(x) & \text{else} \end{cases}$$

Now,

$$\begin{aligned} t^* &\leq a\mathbb{E}(x|\text{admitted}; s^*) - \mathbb{E}(x|\text{not admitted or } y < t^*; s^*) \\ &< a\mathbb{E}(x|\text{admitted}; s) - \mathbb{E}(x|\text{not admitted or } y < t^*; s) \end{aligned}$$

so the school's attendance constraint is satisfied and nonbinding at (s, t^*).

If $\int_{A'} c(x)dF(x) < \int_A c(x)s^*(x)dF(x)$, (s, t^*) brings the school the same revenue as (s^*, t^*) with strictly lower costs, so $\Pi(s, t^*) > \Pi(s^*, t^*)$.

Suppose instead that $\int_{A'} c(x)dF(x) = \int_A c(x)s^*(x)dF(x)$. Then c must be constant on A, and $\Pi(s, t^*) = \Pi(s^*, t^*)$. Denote $c_A := c(x)$ for all $x \in A$. If $\frac{\partial \Pi(s,t^*)}{\partial t} \neq 0$, then we are done. Otherwise, if $c_A \neq t^*$, the school could strictly increase profits without violating the attendance constraint by charging tuition t^* and modifying admissions rule s to admit slightly more or fewer workers whose ability levels fall within A. Finally, if $\frac{\partial \Pi(s,t^*)}{\partial t} = 0$ and $c_A = t^*$, then there exists $\epsilon > 0$ such the admissions constraint is slack at (s_ϵ, t^*), where the admissions rule s_ϵ is given by

$$s_\epsilon(x) = \begin{cases} 1 & x \in [\tilde{x} - \epsilon, \tilde{x}] \\ s(x) & \text{else} \end{cases}$$

Now, $\Pi(s_\epsilon, t^*) = \Pi(s, t^*) = \Pi(s^*, t^*)$, and

$$\begin{aligned}
\frac{\partial \Pi(s_\epsilon, t^*)}{\partial t} &= \bar{G}(t^*) \int_{\underline{x}}^{\overline{x}} s_\epsilon(x) dF(x) - g(t^*) \int_{\underline{x}}^{\overline{x}} (t^* - c(x)) s_\epsilon(x) dF(x) \\
&= \frac{\partial \Pi(s, t^*)}{\partial t} + \int_{\tilde{x}-\epsilon}^{\tilde{x}} f(x) dx \\
&= \int_{\tilde{x}-\epsilon}^{\tilde{x}} f(x) dx \\
&> 0
\end{aligned}$$

Thus, there exists some $t' > t$ such that (s_ϵ, t') satisfies the attendance constraint and $\Pi(s_\epsilon, t') > \Pi(s, t)$.

With Lemma A.3.1 in mind, suppose that (s^*, t^*) solves $\max_{s,t} \Pi(s, t; \tilde{\sigma})$ and that s^* is not a threshold admissions rule. Then there exist $A \subset [\underline{x}, \overline{x}]$ and $B \subset [\underline{x}, \overline{x}]$ such that

1. $x > x'$ for all $x \in A$ and $x' \in B$
2. $s^*(A) = \{0\}$ and $s^*(B) = \{1\}$
3. $\int_A f(x)dx > 0$ and $\int_B f(x)dx > 0$

Since F is continuous, there exist some $A' \subset A$ and $B' \subset B$ such that $\int_{A'} f(x)dx = \int_{B'} s^*(x)dF(x) = \int_{B'} f(x)dx$. Consider the alternative admissions rule s given by

$$s(x) = \begin{cases} 1 & x \in A' \\ 0 & x \in B' \\ s^*(x) & \text{else} \end{cases}$$

Now,

$$\begin{aligned}
t^* &\leq a\mathbb{E}(x|\text{admitted}; s^*) - \mathbb{E}(x|\text{not admitted or } y < t^*; s^*) \\
&< a\mathbb{E}(x|\text{admitted}; s) - \mathbb{E}(x|\text{not admitted or } y < t^*; s)
\end{aligned}$$

so the school's attendance constraint is satisfied and nonbinding at (s, t^*). The rest of the proof is similar to the proof of Lemma A.3.1. $\square$

A.3.2 Proofs for Section 1.3

Proof of Proposition 1.3.4. Suppose that $(s^*, t^*, \tilde{\sigma}, \mu)$ is a subgame perfect equilibrium with firm discrimination and tuition-only school discrimination in which some workers of both types attend the school with positive probability. By Proposition 1.3.3, s^* is a threshold admissions rule; let $\tilde{x}^*$ be the threshold set by s^*.

Let $\tilde{w}_i(\tilde{x}, t_i) := a\mathbb{E}(x|x > \tilde{x}) - \mathbb{E}(x|x < \tilde{x} \text{ or } y < t_i; i)$ and $n_i(\tilde{x}, t) = \bar{F}(\tilde{x})\bar{G}_i(t_i)$ for all $i \in \{1, 2\}$, $\tilde{x} \in [\underline{x}, \overline{x}]$, and $t \in \mathbb{R}^{+^2}$.

$\tilde{x}^*, t^*$ must solve

$$\max_{\tilde{x},t} \sum_{i=1}^{2} q_i(1 - G_i(t_i)) \int_{\tilde{x}}^{\overline{x}} (t_i - c(x))dF(x)$$

subject to attendance constraints $\tilde{w}_i(\tilde{x}, t_i) \geq t_i$ for $i = 1, 2$.

The corresponding Lagrangean function

$$L(\tilde{x}, t) = \sum_{i=1}^{2} \left(q_i(1 - G_i(t_i)) \int_{\tilde{x}}^{\overline{x}} (t_i - c(x))dF(x) + \lambda_i(\tilde{w}_i(\tilde{x}, t_i) - t_i) \right)$$

yields first order conditions

$$\begin{aligned} 0 &= \frac{\partial L(\tilde{x}, t)}{\partial t_i} \\ &= q_i \left(\bar{G}_i(t_i)\bar{F}(\tilde{x}) - g_i(t_i) \int_{\tilde{x}}^{\overline{x}} (t_i - c(x))dF(x) \right) \\ &\quad - \lambda_i \left(1 + \frac{g(t_i)}{(1 - n_i(\tilde{x}, t))^2} (\mathbb{E}(x|x \geq \tilde{x}) - \mathbb{E}(x)) \right) \end{aligned}$$

for $i = 1, 2$.

By the monotone likelihood ratio assumption, $\frac{\bar{G}_1(t)}{g_1(t)} \geq \frac{\bar{G}_2(t)}{g_2(t)}$ for all $t > 0$. Since each wealth distribution also has a nondecreasing hazard rate, if $t_1 < t_2$, $\frac{\bar{G}_1(t_1)}{g_1(t_1)} \geq \frac{\bar{G}_2(t_2)}{g_2(t_2)}$.

Suppose the attendance constraint for type 1 does not bind (ie. $\lambda_1 = 0$). Then $t_1^* - \frac{\bar{G}_1(t_1^*)}{g_1(t_1^*)} =$

$\mathbb{E}(c(x)|x \geq \tilde{x}^*)$ and $t_2^* - \frac{\bar{G}_2(t_2^*)}{g_1(t_2^*)} \leq \mathbb{E}(c(x)|x \geq \tilde{x}^*)$ (with equality only if $\lambda_2 = 0$). Thus,

$$t_1^* - t_2^* \geq \frac{\bar{G}_1(t_1^*)}{g_1(t_1^*)} - \frac{\bar{G}_2(t_2^*)}{g_2(t_2^*)}$$

which requires that $t_1^* \geq t_2^*$ (with equality only if the attendance constraint for type 2 also does not bind).

Suppose instead that the attendance constraint for type 1 binds. Then $t_1^* = \tilde{w}_1(\tilde{x}^*, t_1^*) > \tilde{w}_2(\tilde{x}^*, t_1^*)$, so in order for the attendance constraint for type 2 to be satisfied, it must be that $t_2^* < t_1^*$. □

A.3.3 Proofs for Section 1.4

Proof of Proposition 1.4.2. Suppose workers use the strategy profile $\tilde{\sigma}$ and firm beliefs μ are consistent with $\tilde{\sigma}$ in the model without firm discrimination. With Proposition 1.4.1 in mind, the school's problem can be reformulated as choosing thresholds $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ and tuitions $t = (t_1, t_2)$ to solve

$$\max_{\tilde{x},t} \sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\tilde{x}_i}^{\overline{x}} (t_i - c(x)) dF(x)$$

subject to attendance constraints

$$a\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t_i) - t_j \leq \mathbb{E}(x|x < \tilde{x}_i \text{ or } y < t_i)$$

for $j = 1, 2$. Letting $n(\tilde{x}, t) = \sum_{i=1}^{2} q_i \bar{G}_i(t_i) \bar{F}(\tilde{x}_i)$,

$$\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t_i) = \frac{\sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\tilde{x}_i}^{\overline{x}} x dF(x)}{n(\tilde{x}, t)}$$

and

$$\mathbb{E}(x|x < \tilde{x}_i \text{ or } y < t_i) = \frac{\mathbb{E}(x) - n(\tilde{x}, t)\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t_i)}{1 - n(\tilde{x}, t)}$$

The corresponding Lagrangean function

$$L(\tilde{x},t)=\sum_{j=1}^{2}\Bigg(q_j\bar{G}_j(t_j)\int_{\tilde{x}_j}^{\overline{x}}(t_j-c(x))dF(x)$$
$$+\lambda_j\Big((a+\frac{n(\tilde{x},t)}{1-n(\tilde{x},t)})\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\frac{\mathbb{E}(x)}{1-n(\tilde{x},t)}-t_j\Big)\Bigg)$$

yields first order conditions

$$0=\frac{\partial L}{\partial\tilde{x}_j}=q_j\bar{G}_j(t_j)f(\tilde{x}_j)\Bigg(c(\tilde{x}_j)-t_j$$
$$+(\lambda_1+\lambda_2)\Big((\frac{a}{n(\tilde{x},t)}-\frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\tilde{x}_j)$$
$$-\frac{\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\mathbb{E}(x)}{(1-n(\tilde{x},t))^2}\Big)\Bigg)$$

and

$$0=\frac{\partial L}{\partial t_j}=q_jg_j(t_j)\bar{F}(\tilde{x}_j)\Bigg(\frac{\bar{G}_j(t_j)}{g_j(t_j)}+\mathbb{E}(c(x)|x\geq\tilde{x}_j)-t_j-\frac{\lambda_j}{q_jg_j(t_j)\bar{F}(\tilde{x}_j)}$$
$$-(\lambda_1+\lambda_2)\Big((\frac{a}{n(\tilde{x},t)}-\frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x>\tilde{x}_j)-\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i))$$
$$-\frac{\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\mathbb{E}(x)}{(1-n(\tilde{x},t))^2}\Big)\Bigg)$$

for $j=1,2$.

If neither constraint binds, $t_1=c(\tilde{x}_1)$ and $t_2=c(\tilde{x}_2)$, which implies that $t_i>t_j$ only if $\tilde{x}_i<\tilde{x}_j$. If at least one constraint binds, the first order conditions with respect to $\tilde{x}_1$ and $\tilde{x}_2$ yield

$$\frac{t_1-c(\tilde{x}_1)}{t_2-c(\tilde{x}_2)}=\frac{(\frac{a}{n(\tilde{x},t)}+\frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\tilde{x}_1)-\frac{\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\mathbb{E}(x)}{(1-n(\tilde{x},t))^2}}{(\frac{a}{n(\tilde{x},t)}+\frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\tilde{x}_2)-\frac{\mathbb{E}(x|x\geq\tilde{x}_i\text{ and }y\geq t_i)-\mathbb{E}(x)}{(1-n(\tilde{x},t))^2}}$$

which likewise implies $t_i>t_j$ if and only if $\tilde{x}_i<\tilde{x}_j$.

Now, if neither constraint binds, all four FOCs yield

$$(c(\tilde{x}_1) - \mathbb{E}(c(x)|x > \tilde{x}_1)) - (c(\tilde{x}_2) - \mathbb{E}(c(x)|x > \tilde{x}_2)) = \frac{\bar{G}_1(t_1)}{g_1(t_1)} - \frac{\bar{G}_2(t_2)}{g_2(t_2)}$$

which cannot be true if $t_1 < t_2$ and $\tilde{x}_1 > \tilde{x}_2$.

If only the attendance constraint for type 2 workers binds, all four FOCs yield

$$\begin{aligned}
&\frac{\bar{G}_1(t_1)}{g_1(t_1)} - \frac{\bar{G}_2(t_2)}{g_2(t_2)} + \frac{\lambda_2}{q_2 g_2(t_2)\bar{F}(\tilde{x}_2)} \\
&= \mathbb{E}(c(\tilde{x}_1) - c(x)|x \geq \tilde{x}_1) - \mathbb{E}(c(\tilde{x}_2) - c(x)|x \geq \tilde{x}_2) \\
&+ \lambda_2(\frac{a}{n(\tilde{x},t)} + \frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x - \tilde{x}_1|x \geq \tilde{x}_1) - \mathbb{E}(x - \tilde{x}_2|x \geq \tilde{x}_2))
\end{aligned}$$

which cannot be true if $t_1 \leq t_2$ and $\tilde{x}_1 \geq \tilde{x}_2$, if F has a nondecreasing hazard rate, since $\mathbb{E}(x - \tilde{x}|x \geq \tilde{x})$ is nonincreasing in $\tilde{x}$. If only the constraint for type 1 workers binds, $t_1 > t_2$ is trivial, and thus also $x_1 < x_2$. If both constraints bind at the solution to the school's problem, $t_1 = t_2$, so $\tilde{x}_1 = \tilde{x}_2$. □

Proof of Proposition 1.4.4. With Proposition 1.4.3 in mind, if workers are using the strategy profile $\tilde{\sigma}$, the school's problem can be reformulated as choosing admissions threshold $\tilde{x}$ and tutions $t = (t_1, t_2)$ to solve

$$\max_{\tilde{x},t} \sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\tilde{x}}^{\overline{x}} (t_i - c(x))dF(x)$$

subject to attendance constraints

$$a\mathbb{E}(x|x > \tilde{x}) - t_j \geq \mathbb{E}(x|x < \tilde{x} \text{ or } y < t_i)$$

for $j = 1, 2$. Here, the expected productivity of an educated worker depends only on the threshold chosen by the school, and not the tuitions charged because

$$\begin{aligned}
\mathbb{E}(x|x \geq \tilde{x} \text{ and } y \geq t_i) &= \frac{\sum_{i=1}^{2} q_i \bar{G}_i(t_i) \int_{\tilde{x}}^{\overline{x}} x dF(x)}{(q_1 \bar{G}_1(t_1) + q_2 \bar{G}_2(t_2))\bar{F}(\tilde{x})} \\
&= \frac{\int_{\tilde{x}}^{\overline{x}} x dF(x)}{\bar{F}(\tilde{x})} \\
&= \mathbb{E}(x|x \geq \tilde{x})
\end{aligned}$$

for all $t, \tilde{x}$. Letting $n(\tilde{x}, t) := \bar{F}(\tilde{x}) \sum_{i=1}^{2} q_i \bar{G}_i(t_i)$, the school's Lagrangean function can be written as

$$\begin{aligned}
L(\tilde{x}, t) = \sum_{i=1}^{2} \Bigg(& q_i \bar{G}_i(t_i) \int_{\tilde{x}}^{\overline{x}} (t_i - c(x)) dF(x) \\
& + \lambda_i \Big(\big(a + \frac{n(\tilde{x}, t)}{1 - n(\tilde{x}, t)}\big) \mathbb{E}(x|x \geq \tilde{x}) - \frac{\mathbb{E}(x)}{1 - n(\tilde{x}, t)} - t_i \Big) \Bigg)
\end{aligned}$$

Which gives first order conditions

$$\begin{aligned}
0 = \frac{\partial L}{\partial t_i} = q_i g_i(t_i) \bar{F}(\tilde{x}) \Bigg(& \frac{\bar{G}_i(t_i)}{g_i(t_i)} + \mathbb{E}(c(\tilde{x})|\tilde{x} > x) - t_1 \\
& - \lambda_i \Big(\frac{1}{q_i g_i(t_i) \bar{F}(\tilde{x})} + \frac{\mathbb{E}(x|x \geq \tilde{x}) - \mathbb{E}(x)}{(1 - n(\tilde{x}, t))^2} \Big) \\
& - \lambda_{-i} \frac{\mathbb{E}(x|x \geq \tilde{x}) - \mathbb{E}(x)}{(1 - n(\tilde{x}, t))^2} \Bigg)
\end{aligned}$$

for $i = 1, 2$

If neither constraint binds, and $\lambda_1 = \lambda_2 = 0$,

$$\frac{\bar{G}_1(t_1)}{g_1(t_1)} - \frac{\bar{G}_2(t_2)}{g_2(t_2)} = t_1 - t_2$$

By the monotone likelihood ratio assumption, $\frac{\bar{G}_1(t)}{g_1(t)} \geq \frac{\bar{G}_2(t)}{g_2(t)}$ for all $t > 0$. Since each wealth distribution also has a nondecreasing hazard rate, if $t_1 < t_2$, $\frac{\bar{G}_1(t_1)}{g_1(t_1)} \geq \frac{\bar{G}_2(t_2)}{g_2(t_2)}$. Thus if neither constraint binds, $t_1 \geq t_2$.

If only the attendance constraint for type 2 students binds ($\lambda_1 = 0, \lambda_2 > 0$), $t_1 < t_2$, but

$$t_1 - t_2 = \frac{\bar{G}_1(t_1)}{g_1(t_1)} - \frac{\bar{G}_2(t_2)}{g_2(t_2)} + \frac{\lambda_2}{q_2 g_2(t_2)\bar{F}(\tilde{x})} \geq \frac{\lambda_2}{q_2 g_2(t_2)\bar{F}(\tilde{x})} > 0$$

So if the attendance constraint for type 2 binds, so does the attendance constraint for type 1.

If only the attendance constraint for type 1 student binds, $t_1 > t_2$ is trivial. □

Proof of Proposition 1.4.6. With Proposition 1.4.5 in mind, if workers are using strategy profile $\tilde{\sigma}$, the school's problem can be reformulated as selecting admissions thresholds $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ and tuition t to solve

$$\max_{\tilde{x},t} \Pi(\tilde{x}, t) := \sum_{i=1}^{2} q_i \bar{G}_i(t) \int_{\tilde{x}_i}^{\overline{x}} (t - c(x)) dF(x)$$

subject to attendance constraint

$$a\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - t \geq \mathbb{E}(x|x < \tilde{x}_i \text{ or } y < t)$$

Let $n(\tilde{x}, t) := \sum_{i=1}^{2} q_i \bar{G}_i(t)\bar{F}(\tilde{x}_i)$. Then

$$\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) = \frac{\sum_{i=1}^{2} q_i \bar{G}_i(t) \int_{\tilde{x}_i}^{\overline{x}} x dF(x)}{n(\tilde{x}, t)}$$

and

$$\mathbb{E}(x|x < \tilde{x}_i \text{ or } y < t) = \frac{\mathbb{E}(x) - \sum_{i=1}^{2} q_i \bar{G}_i(t) \int_{\tilde{x}_i}^{\overline{x}} x dF(x)}{1 - n(\tilde{x}, t)}$$

The school's Lagrangean function can be written as

$$\begin{aligned} L(\tilde{x}, t) = &\sum_{i=1}^{2} q_i \bar{G}_i(t) \int_{\tilde{x}_i}^{\overline{x}} (t - c(x)) dF(x) \\ &+ \lambda \left((a + \frac{n(\tilde{x}, t)}{1 - n(\tilde{x}, t)}) \mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - \frac{\mathbb{E}(x)}{1 - n(\tilde{x}, t)} - t \right) \end{aligned}$$

Which gives first order conditions

$$0 = \frac{\partial L}{\partial \tilde{x}_j} = q_j \bar{G}_j(t) f(\tilde{x}_j) \Bigg(c(\tilde{x}_j) - t$$

$$+ \lambda \Big((\frac{a}{n(\tilde{x},t)} + \frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - x_j)$$

$$- \frac{\mathbb{E}(x|x \geq x_i \text{ and } y \geq t) - \mathbb{E}(x)}{(1-n(\tilde{x},t))^2} \Big) \Bigg)$$

for $j = 1, 2$. Clearly, if $\lambda = 0$, $c(\tilde{x}_1) = t = c(\tilde{x}_2)$ so c is constant between x_1 and x_2, or $\tilde{x}_1 = \tilde{x}_2$. In the former case, it is possible for the school to slightly decrease the higher admissions threshold $\tilde{x}_i$ to some $\tilde{x}_i - \epsilon$ such that profit is unchanged, the attendance constraint is still satisfied, but $\frac{\partial \Pi((x_i - \epsilon, x_{-i}), t)}{\partial t} > 0$ If instead the attendance constraint binds, and $\lambda > 0$, combining the FOCs for $\tilde{x}_1$ and $\tilde{x}_2$ yields

$$\frac{t - c(\tilde{x}_1)}{t - c(\tilde{x}_2)} = \frac{(\frac{a}{n(\tilde{x},t)} + \frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - \tilde{x}_1) - \frac{\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - \mathbb{E}(x)}{(1-n(\tilde{x},t))^2}}{(\frac{a}{n(\tilde{x},t)} + \frac{1}{1-n(\tilde{x},t)})(\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - \tilde{x}_2) - \frac{\mathbb{E}(x|x \geq \tilde{x}_i \text{ and } y \geq t) - \mathbb{E}(x)}{(1-n(\tilde{x},t))^2}}$$

which would also imply $x_1 = x_2$, since c is weakly decreasing. □

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