Non-Fermi-liquid behavior in two dimensions due to long-ranged current-current interactions

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We examine the low-energy behavior of two-dimensional electrons with current-current interactions. The interactions are long range, cannot be screened, and result in anomalies in perturbation theory. In the Hartree-Fock theory, we find an instability of the Landau Fermi-liquid state resulting in "fermion-condensate" states with a high degeneracy of levels at the Fermi energy. This degeneracy may be removed to give a Fermi-liquid state with vanishing Fermi velocity. As an example, we investigate further the Fermi-liquid state with the quasiparticle spectrum $\epsilon(p) \propto (p-p_F)^{3/2}$ proposed by Halperin *et al.* Application of the recent Haldane scheme of bosonization in higher dimensions to this state leads to coupling between bosons, which has the form required for a breakdown of the Landau Fermi-liquid behavior. A generalized form of the Luttinger liquid is obtained.

I. INTRODUCTION

Recent developments in the theory of strongly correlated systems put forward an exciting hypothesis about the existence of non-Landau Fermi-liquid ground states of interacting fermions in dimensions $D > 1$. In a series of papers, Anderson[1] argued that these systems remain metals down to zero temperature, but demonstrate some sort of the "Luttinger-liquid" behavior which is familiar in $D = 1$. In one dimension this behavior occurs at arbitrarily weak short-ranged interactions.

Anderson's proposal is based on the presence of a singularity in the scattering amplitudes at small transferred momenta. Although this proposal has been disputed in the case of short-ranged potentials in $D = 2$ (see Refs. 2 and 3), one could expect that it definitely takes place for singular (in general, retarded) long-ranged interactions. This point has been stressed in a recent paper by Bares and Wen[4] using an approach different to that applied here. Some promising results were obtained in Ref. 5 for the interaction function $f(\mathbf{p},\mathbf{p}') \sim \frac{\mathbf{p}(\mathbf{p}-\mathbf{p}')}{(\mathbf{p}-\mathbf{p}')^2}$ discussed in Ref. 1.

A physically relevant example of a singular interaction is provided by the magnetic (Ampère) force which occurs between charged electric currents. The four-fermion vertex function describing this interaction has the form (see Fig. 1)

$$\Gamma^0(\mathbf{p},\mathbf{p}';\mathbf{q},\omega) = \frac{g}{m^2}\frac{\frac{(\mathbf{pq})(\mathbf{p}'\mathbf{q})}{q^2} - \mathbf{pp}'}{q^2 - c^{-2}\omega^2}\ , \qquad (1.1)$$

where m is the mass of the fermions, c is the speed of light, and g is a measure of the strength of the interaction. Note that for electronic frequencies and momenta one can neglect in the denominator the term $c^{-2}\omega^2$ relative to q^2. From now on, we will use this approximation. It was first observed by Holstein, Norton, and Pincus[6] that in ordinary metals in $D = 3$ the electromagnetic interaction leads to a drastic renormalization of the Fermi-liquid spectrum at temperatures lower than 10^{-6} K (see also Ref. 7). Within perturbation theory, the lowest-order contribution to the one-particle self-energy has the form $\Sigma^0(\epsilon) \sim -e^2(\frac{v_F}{c})^2\epsilon|\ln\frac{\epsilon}{\epsilon_F}|$ (v_F is the Fermi velocity of the fermions) which is usually negligible because of the small value of the ratio $\frac{v_F}{c}$. However, at $T < 10^{-6}$ K this contribution could essentially destroy the conventional Landau-Fermi quasiparticle picture.

The problem of the ground state of fermions interacting by means of magnetic forces appears in its whole complexity in the modern gauge theories of strong correlation in $D = 2$ (see Refs. 8–12). The velocity of the collective mode of spin chirality fluctuations which is the counterpart of the transverse photon in the electromagnetic case is of the order v_F and the coupling constant $g \sim 1$. In lowest-order perturbation theory, it was shown by Lee[13] that in $D = 2$ the effective gauge interaction

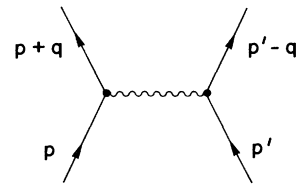

FIG. 1. The two-particle scattering vertex. The Fock channel corresponds to the choice $\mathbf{p}' = \mathbf{p}+\mathbf{q}$. The Cooper channel corresponds to $\mathbf{p}' = -\mathbf{p}$.

mediated by this mode leads to the self-energy of the form

$$\Sigma^0(\epsilon) \sim -g\epsilon^{2/3}\mathrm{sgn}\epsilon. \tag{1.2}$$

Baym and co-workers[14] used this result to propose an instability of the bare Fermi surface. Naively it could take place because in some region of momenta around the bare Fermi surface the group velocity of the fermions $v_g(p) = \frac{d\epsilon}{dp}$ becomes negative and in order to minimize the energy the distribution of electrons must be changed (see Fig. 2).

Still another application of the model of spinless fermions coupled to a gauge field was proposed recently by Halperin, Lee, and Read.[15] Halperin, Lee, and Read suggest that if the lowest Landau level of the $D = 2$ electron gas is half filled, it can be described precisely by that model. The investigation of the quantum Hall effect at half filling seems to be at present the most promising way to check the various theoretical predictions for fermions with magnetic interactions.

In this paper we investigate the possible candidates for the ground state of $D = 2$ fermions interacting with current-current interactions. As an illustration of how divergent these interactions are, we show in Sec. II that in the screened Hartree-Fock approximation (SHFA), the so-called "fermion condensate"[16,17] state is formed. In this state, electrons from a finite interval of momenta around the noninteracting Fermi surface have the same energy. However, residual interactions beyond the SHFA should lift this degeneracy of the single particle spectrum. Thus one is forced to use more powerful techniques to investigate the problem. To this end, a variant of Haldane's bosonization scheme in higher dimensions[18] is presented in Sec. III. In Sec. IV we demonstrate how this scheme works, applying it straightforwardly to the problem at hand. We derive the effective bosonized Hamiltonian and diagonalize it. It turns out that a necessary condition for the breakdown of the Landau Fermi-liquid behavior recently proposed by Haldane is satisfied and a generalized Luttinger liquid occurs. Section V contains a calculation of the momentum distribution function n_p in this state. In Sec. VI we discuss the recent proposal by Halperin, Lee, and Read[15] that the fermions form a Landau Fermi-liquid state with a spectrum $\epsilon(p) \propto (p-p_F)^{3/2}$. Applying an approximate bosonization to this state we show that it is unstable towards a generalized Luttinger-liquid behavior. The thermodynamic functions of such a state are discussed briefly. In Sec. VII, we summarize our results and discuss the possibilities of further development. The details of the mean field calculation for the model problem introduced in Sec. II are given in Appendix A. Appendix B contains the discussion of the problem of fermions with current-current interactions in $D = 3$.

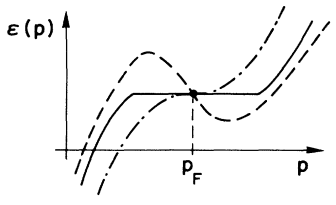

FIG. 2. The single particle spectrum in the vicinity of the Fermi momentum p_F in various approximations: dashed line: the result of the first-order perturbation theory; full line: the plateau obtained in the Hartree-Fock approximation, dashed-dotted line: the dispersion proposed in Ref. 15.

II. HARTREE-FOCK APPROXIMATION

Khodel' and Shaginyan[16] demonstrated that the momentum distribution n_p calculated in Hartree-Fock theory does not necessarily lead to the Fermi step function. They showed that for sufficiently long-ranged interactions, n_p can deviate substantially from the free particle distribution, losing any trace of the noninteracting Fermi surface. They constructed the energy functional

$$E[n_p] = \int \epsilon(p) n_p d^2p + \frac{1}{2\Omega}\int\int f(p,p') n_p n_{p'} d^2p d^2p', \tag{2.1}$$

where $\epsilon(p)$ is the bare spectrum of the fermions, n_p their distribution function, $f(p,p')$ the Landau interaction function, and Ω the volume of the system. The electron distribution was found by solving the extremum equation

$$\frac{\delta E[n_p]}{\delta n_p} = 0, \tag{2.2}$$

with the constraint $0 \le n_p \le 1$. For model long-range potentials, Khodel' and Shaginyan showed that the minimum was obtained for an electron distribution similar to that shown in Fig. 3. The extremum equation (2.2) is obeyed for $p_1 < p < p_2$ and since it coincides with the definition of the quasiparticle energy, there is a plateau in the single particle spectrum between p_1 and p_2. This is the so-called "fermion condensate" state.[19]

Nozières[17] investigated this possibility further and noticed that in the Hartree-Fock approximation the interaction contribution comes from the Fock term and $f(p,p') = -\Gamma^0(\mathbf{p},\mathbf{p}';\mathbf{p}-\mathbf{p}')$. Since the "fermion condensate" state occurs only if $f(p,p') > 0$, the Hamiltonian will provide attraction in the Cooper channel and lead to a competing pairing instability. Moreover, Nozières argued that screening of the interaction would prohibit the occurrence of the singularity.

None of these arguments apply in our case, however. To show this, let us investigate the "screened" vertex

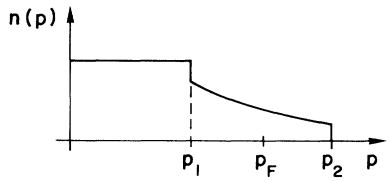

FIG. 3. The distribution function found within the Hartree-Fock approximation for the solvable 2D interaction $\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q}) = -g\frac{\mathbf{p}\cdot\mathbf{p}'q^2-(\mathbf{pq})(\mathbf{p}'\mathbf{q})}{|\mathbf{p}||\mathbf{p}'|q^2}$.

$\Gamma(\mathbf{p},\mathbf{p}';\mathbf{p}-\mathbf{p}')$. Until fermions remain in a normal metal state the interaction (1.1) is not screened at $\omega = 0$ and there is only "dynamical screening" (see Ref. 14):

$$\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q},\omega) = \mathrm{Re}\left(\frac{g}{m^2}\frac{\frac{(\mathbf{pq})(\mathbf{p}'\mathbf{q})}{q^2} - \mathbf{pp}'}{q^2 + i\lambda(q)|\omega|}\right), \qquad (2.3)$$

where $\lambda(q) = \gamma/q$ is the Landau damping. We believe this is a general feature of the magnetic current-current interaction in any normal metal-like state (i.e., a phase connected to the high-temperature metallic phase without intervening phase transitions to, e.g., a superconducting or insulating state; not necessarily a Landau Fermi liquid). Moreover, the interaction vertex (2.3) changes sign after an inversion of one of the momenta of the scattering fermions (see Fig. 1): $\Gamma(\mathbf{p},-\mathbf{p}';\mathbf{q}) = -\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q})$. This means that $f(p,p') > 0$ and the interaction is at the same time repulsive in the Cooper channel, unlike in the usual charge-charge interaction case.

Literally, the retarded interaction (2.3) cannot be treated within the time-independent Hartree-Fock theory. However, as an illustrative example we consider a less singular ("screened") version of the vertex (1.1)

$$\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q}) = g\frac{\frac{(\mathbf{pq})(\mathbf{p}'\mathbf{q})}{q^2} - \mathbf{pp}'}{|\mathbf{p}||\mathbf{p}'|} \qquad (2.4)$$

(where we have chosen $m = 1$), which is exactly solvable. At $g > g_c = \frac{6}{\pi}$ there exists a nontrivial solution of Eqs. (3) and (4) which has the form $n(p) = \frac{1}{2\pi g}\frac{p_0^2+3p^2}{p^2}$, $p_1 = \frac{p_0}{\sqrt{2\pi g/3-1}} < p < p_2 = p_0/\sqrt{3}$, $p_0 = p_F\sqrt{\frac{2\pi g}{4+\ln(\frac{2\pi g-3}{9})}}$ (see Fig. 3). Obviously, this solution demonstrates a divergent density of states at the bare Fermi surface. The details of this calculation are presented in Appendix A.

Note that the angular structure of the vertex function Eq. (2.4) is identical to that of the current-current interaction and does not result in the superconducting instability for any angular momentum pairing state. To show this we evaluate $\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q})$ in the Cooper channel for $|\mathbf{p}| = |\mathbf{p}'| = p_F$. The vertex function depends only on the angle θ between $\mathbf{p}$ and $\mathbf{p}'$, $\Gamma(\theta) \sim 1 + \cos\theta$. There are only three nonvanishing angular harmonics[20] for this vertex function: $\Gamma_m = \int_0^{2\pi}\Gamma(\theta)\exp(im\theta) \sim 2\delta_{m,0} + \delta_{m,1} + \delta_{m,-1}$, all of which are repulsive.

Summarizing, in addition to the breakdown of the lowest-order perturbation theory reported earlier, we have found within SHFA an instability of the Fermi-liquid state towards the "fermion condensate" state. However, as noticed by Nozières,[17] the intrinsic degeneracy of this state makes it hardly possible to be realized. The residual interactions which were not included in the Hartree-Fock approximation will lift the degeneracy and there should be no singularity in the resulting density of states. Despite this unphysical feature of the SHFA solution we believe that the nontrivial solution for $n(\mathbf{p})$ is not accidental and signals a breakdown of the Landau Fermi-liquid behavior. The investigation of this possibility is the subject of the rest of the paper.

III. BOSONIZATION IN HIGHER DIMENSIONS

In this section we describe a method proposed by Haldane[18] as a possible generalization of the $D = 1$ bosonization to higher dimensions. First, a toy model of interacting fermions in $D = 2$ is introduced and its solution using bosonization is sketched. Further, the model is readily generalized to describe more realistic systems of fermions. Bosonization of the latter model turns out to be equivalent to that proposed by Haldane. In the next section, we apply the present method to the problem of fermions with current-current interactions.

Let us investigate the system of fermions in $D = 2$ with the spectrum $\epsilon(\mathbf{p}) = v(|p_x| + |p_y|)$. Analogously to the case in $D = 1$, there are four different branches of the spectrum, one in each quadrant of the $\mathbf{p}$ plane (see Fig. 4). Let us construct four different density operators $R_i(\mathbf{q}) = \sum c^\dagger{}_i(\mathbf{p}+\mathbf{q})c_i(\mathbf{p})$ where i denotes the four quadrants and $\mathbf{p}$ and $\mathbf{p}+\mathbf{q}$ lie in the quadrant i. With the kinetic energy

$$H_{\mathrm{kin}} = \sum_{\mathbf{p}}\epsilon(p)c^\dagger_{\mathbf{p}}c_{\mathbf{p}},$$

one has $[H_{\mathrm{kin}}, R_i(\mathbf{q})] = \mathbf{v_i}\cdot\mathbf{q}R_i(\mathbf{q})$. The commutation relations of the density operators are $[R_i(\mathbf{q}), R_j(-\mathbf{q}')] = -\delta_{i,j}\delta_{\mathbf{q},\mathbf{q}'}\frac{\Lambda\Omega}{(2\pi)^2}\mathbf{n_i}\cdot\mathbf{q}$, where Λ denotes the length of the Fermi surface in each of the quadrants (see Fig. 4) . This means one can bosonize the kinetic energy of this two-dimensional (2D) problem. Introducing $\rho_i(\mathbf{q}) = 2\pi\sqrt{\frac{v_F}{\Lambda\Omega}}R_i(\mathbf{q})$ one obtains $H_{\mathrm{kin}} = \sum\rho_i(\mathbf{q})\rho_i(-\mathbf{q})$. The potential energy

$$U = \frac{1}{2\Omega}\sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} g(\mathbf{p},\mathbf{p}',\mathbf{q})c^\dagger_{\mathbf{p}+\mathbf{q}}c^\dagger_{\mathbf{p}'-\mathbf{q}}c_{\mathbf{p}'}c_{\mathbf{p}}$$

can be easily bosonized, if we neglect the large momentum and "around the corner" scatterings. The total bosonized Hamiltonian reads

$$H = \sum_{\mathbf{q}}\sum_{i,j}\left(\delta_{i,j} + \frac{\Lambda g_{i,j}(\mathbf{q})}{v_F}\right)\rho_i(\mathbf{q})\rho_j(-\mathbf{q}). \qquad (3.1)$$

A similar problem was studied recently by Houghton and Marston[21] using the renormalization group method. However, in their analysis they replaced the sides of the Fermi surface by four Fermi points.

Below we shall follow the Haldane bosonization method in higher dimensions.[18] This scheme can be recognized as a treatment of nonmetallic fixed points within

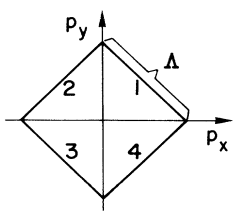

FIG. 4. The line of constant energy for the energy spectrum $\epsilon(\mathbf{p}) = |p_x| + |p_y|$.

the renormalization procedure when one integrates out all the modes lying far from the Fermi surface. The remainder states have momenta $|\mathbf{p} - p_F\mathbf{n}| < \Lambda$ and can be considered as belonging to "patches" of size Λ which cover all the Fermi surface. In D dimensions the total number of "patches" is of order $N \sim (\frac{\Lambda}{p_F})^{-D+1}$.

We can rederive the results of Haldane taking the limit of a large number of branches in our toy model (see Fig. 5). The detailed form of the spectrum far away from the Fermi surface is irrelevant for the low-energy physics and the two formulations are equivalent. Formally, increasing the number of the branches leads to a vanishing Λ. (This corresponds to taking smaller "patches" around the Fermi surface.) Equation (3.1) still holds with obvious changes in the definition of $g_{i,j}$ and the range of summation over i, j. The crucial difference with respect to the case $D = 1$ is that after the renormalization the interaction function acquires an additional factor of order Λ. Then for nonsingular interactions the ratio $\frac{\Lambda\Gamma}{v_F}$ goes to zero as $\Lambda \to 0$ and one ends up with a Fermi-liquid fixed point. One has to proceed with singular interactions to reach nontrivial fixed points (if any).[18]

In general, the momentum conservation law and the phase space arguments allow for three different types of low-energy scattering processes between two electrons in $D = 2$, namely, the scattering in the Hartree, Fock, and Cooper channels (see, e.g., Ref. 22). These are described by $\mathbf{p} \to \mathbf{p} + \mathbf{q}$, $\mathbf{p}' \to \mathbf{p}' - \mathbf{q}$ (Hartree or forward scattering channel); $\mathbf{p} \to \mathbf{p}' + \mathbf{q}$, $\mathbf{p}' \to \mathbf{p} - \mathbf{q}$ (Fock or exchange scattering); and $\mathbf{p} \to \mathbf{p}'$, $-\mathbf{p} + \mathbf{q} \to -\mathbf{p}' + \mathbf{q}$ (Cooper channel), where $\mathbf{p}$ and $\mathbf{p}'$ are arbitrary vectors on the Fermi surface and $q \ll p_F$. The momentum transfers in these three processes are $\mathbf{q}$, $\mathbf{p}' - \mathbf{p} + \mathbf{q}$, and $\mathbf{p}' - \mathbf{p}$, respectively. The Haldane bosonization scheme is applicable to the case when the low-energy physics is dominated by the scatterings in the Hartree channel only. Thus the model Eq. (3.1) is a direct generalization of the exactly solvable Tomonaga-Luttinger model in $D = 1$. Note that for the scattering in the Hartree channel the number of electrons in a given patch remains constant and the Haldane model Eq. (3.1) is invariant under a local $U(1)$ gauge transformation defined in the reciprocal space for the points on the Fermi surface.[18]

Specializing to the case of current-current interactions, it is obvious that the scattering in the Hartree channel should be relevant at low energies. On the contrary, we can argue that the scattering in the Cooper channel might be irrelevant. First of all, the less singular version of the bare interaction Eq. (2.4) is nonattractive in all angular momentum scatterings in the Cooper channel. Moreover, for the short-range interactions the Kohn-Luttinger mechanism of superconductivity in a high angular momentum channel[20] does not apply straightforwardly in $D = 2$, as shown, e.g., in Ref. 23. (However, a two loop calculation does lead to superconductivity in this case, see Ref. 24.) For the case of long-range interactions, the pairing instability does not seem likely if the renormalization of the bare singular vertex yields only subdominant (less singular) terms, however, it remains an open question whether it is indeed so. In the following section we will argue that the exchange scattering processes are less important than those in the Hartree channel. Thus it seems plausible to assume that the Haldane Hamiltonian Eq. (3.1) describes all relevant low-energy processes for the spinless fermions with current-current interactions. We emphasize, however, that the possibility of the occurrence of various symmetry breaking states [e.g., flux density wave states at $q = 2k_F$, the study of which is in progress (Ref. 25), superconductivity, etc.] should be investigated in more detail.

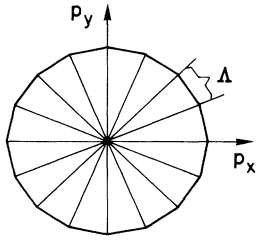

FIG. 5. The line of constant energy for the energy spectrum with many branches.

IV. BOSONIZATION OF FERMIONS WITH CURRENT-CURRENT INTERACTIONS

In this section we apply Haldane's method to the problem of fermions interacting via current-current forces. Following Haldane's prescription one has to find first the effective Hamiltonian for the renormalized quasiparticles

$$H = \sum_{\mathbf{p}} \epsilon(\mathbf{p}) c^\dagger_{\mathbf{p}} c_{\mathbf{p}} + \frac{1}{2\Omega} \times \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \Gamma(\mathbf{p},\mathbf{p}';\mathbf{q},\omega) c^\dagger_{\mathbf{p}+\mathbf{q}} c^\dagger_{\mathbf{p}'-\mathbf{q}} c_{\mathbf{p}'} c_{\mathbf{p}}, \quad (4.1)$$

where $\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q},\omega)$ is the renormalized vertex function corresponding to the bare vertex function defined in Eq. (2.3), the transferred energy is $\omega = \frac{1}{2}\{[\epsilon(\mathbf{p}+\mathbf{q}) - \epsilon(\mathbf{p})] + [\epsilon(\mathbf{p}') - \epsilon(\mathbf{p}'-\mathbf{q})]\}$, and $\epsilon(p)$ is the renormalized quasiparticle spectrum. However, a complete perturbative analysis of the problem involved in this renormalization has not been performed yet. In particular, attempts at a consistent calculation of the fermion self-energy in the Migdal approximation (neglecting any vertex corrections) reproduce the result given by Eq. (1.2) (see Ref. 26). However, a dramatic renormalization of the spectrum at $\epsilon \ll g^3$ implied by Eq. (1.2) requires that one includes the vertex corrections to satisfy the Ward identity $\Gamma(\epsilon) = 1 - \frac{\partial\Sigma(\epsilon)}{\partial\epsilon}|_{\epsilon\to 0}$, where ϵ refers to the transferred energy.

Leaving the problem of the self-consistent renormalization aside for the time being we will use in this section the bare vertex and the bare spectrum. It turns out to be sufficient to illustrate the substantial deviations from the Landau Fermi liquid resulting from the retarded current-current interaction Eq. (2.3). In Sec. VI we will compare the results of this section with those using the ansatz for the renormalized spectrum proposed in Ref. 11 and argue

that to some extent our main conclusions are insensitive to the form of the quasiparticle spectrum.

Proceeding with Haldane's bosonization of the Hamiltonian Eq. (4.1) we first introduce the density operators $R_\alpha(\mathbf{q}) = \sum_{\mathbf{p}\in\Lambda_\alpha} c^\dagger(\mathbf{p}+\mathbf{q})c(\mathbf{p})$ which obey the commutation relations $[R_\alpha(\mathbf{q}), R_\beta(-\mathbf{q}')] = -\delta_{\alpha,\beta}\delta_{\mathbf{q},\mathbf{q}'}\frac{\Lambda\Omega}{(2\pi)^2}\mathbf{n}_\alpha\cdot\mathbf{q}$. Then the interaction part can be written in a straightforward way in the bosonized form:

$$H_{\rm int} = \frac{1}{2\Omega}\sum_{\mathbf{q}}\sum_{\alpha,\beta} V_{\alpha,\beta}(\mathbf{q})R_\alpha(\mathbf{q})R_\beta(-\mathbf{q}), \qquad (4.2)$$

where we have defined

$$V_{\alpha,\beta}(\mathbf{q}) = \int_{\mathbf{p}\in\Lambda_\alpha}\frac{d^2\mathbf{p}}{\Lambda^2}\int_{\mathbf{p}'\in\Lambda_\beta}\frac{d^2\mathbf{p}'}{\Lambda^2}\Gamma(\mathbf{p},\mathbf{p}';\mathbf{q},\omega). \qquad (4.3)$$

Again we assumed that the large momentum and "around the corner" scatterings are negligible, analogously to the discussion in Sec. IV. Denoting the angles between $\mathbf{q}$ and the patches Λ_α and Λ_β as α and β and assuming $q \ll \Lambda \ll p_F$, one obtains using Eq. (2.3)

$$V_{\alpha,\beta}(\mathbf{q}) \approx -\frac{g}{m^2}\frac{p_F^2 q^2\sin\alpha\sin\beta}{q^4+\gamma^2 v_F^2(\cos\alpha+\cos\beta)^2}. \qquad (4.4)$$

The renormalized interaction $V_{\alpha,\beta}(\mathbf{q})$ demonstrates at $\Lambda\to 0$ the following remarkable property:

$$\sum_\beta V_{\alpha,\beta}(\mathbf{q})F(\alpha,\beta) = \frac{gv_F}{\gamma}\frac{p_F}{\Lambda}|\sin\alpha|[F(\alpha,-\alpha^*)-F(\alpha,\alpha^*)],$$

where $F(\alpha,\beta)$ is an arbitrary function and the following notation is used: α^* is the patch opposite to the patch α and $-\alpha$ is the mirror image of α with respect to a given $\mathbf{q}$. Thus as $\Lambda\to 0$, a given patch α is coupled only to two other patches, whose positions are given by the position of the patch α and the direction of $\mathbf{q}$. [Notice, however, that this is not the case for any other vertex from the family Eq. (2.3) with $\lambda(q)\sim q^{-\eta}$ for $\eta\neq 1$.] Hence the interaction part of the Hamiltonian is

$$H_{\rm int} = \frac{1}{4}\frac{(2\pi)^2 v_F}{\Omega\Lambda}\sum_{\mathbf{q}}\sum_{\alpha}\delta|\sin\alpha|[R_\alpha(\mathbf{q})-R_{-\alpha}(\mathbf{q})]\times[R_{\alpha^*}(-\mathbf{q})-R_{-\alpha^*}(-\mathbf{q})], \qquad (4.5)$$

where we have introduced a dimensionless coupling constant $\delta = \frac{gp_F}{2\gamma}$. Since the spectrum around p_F is linear, the kinetic energy can be bosonized as in Sec. III and we can write

$$H_{\rm kin} \approx \frac{(2\pi)^2 v_F}{\Omega\Lambda}\sum_{\mathbf{q}}\sum_{\alpha} R_\alpha(\mathbf{q})R_\alpha(-\mathbf{q}).$$

Comparison with Eq. (4.5) shows that both the kinetic and interaction energy are proportional to Λ^{-1}, i.e., they are equally relevant in the limit $\Lambda\to 0$ and the Haldane criterion for the breakdown of the Fermi-liquid behavior $\frac{\Lambda\Gamma(p,p';q)}{v_F(p)}|_{\Lambda\to 0}\to$const is satisfied. This is a consequence of the singular nature of the vertex Eq. (5). The total effective Hamiltonian reads

$$H = \frac{1}{4}\frac{(2\pi)^2 v_F}{\Omega\Lambda}\sum_{\alpha}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}>0}\Big([R_\alpha(\mathbf{q})R_\alpha(\mathbf{q})+R_{-\alpha}(\mathbf{q})R_{-\alpha}(-\mathbf{q})+R_{\alpha^*}(-\mathbf{q})R_{\alpha^*}(\mathbf{q})+R_{-\alpha^*}(-\mathbf{q})R_{-\alpha^*}(\mathbf{q})] + \delta|\sin\alpha|\{[R_\alpha(\mathbf{q})-R_{-\alpha}(\mathbf{q})][R_{\alpha^*}(-\mathbf{q})-R_{-\alpha^*}(-\mathbf{q})]+\text{H.c.}\}\Big). \qquad (4.6)$$

Note that for each given $\mathbf{q}$, there are sets of four coupled patches (see Fig. 6). Remarkably, this pattern resembles the concept of the "tomographic Luttinger liquid" discussed by Anderson.[1] Actually the effective "$D=1$ character" of the current-current interaction in the scaling limit enables us to perform the bosonization procedure in a simplified form originally proposed by Luther.[27]

Now we can show that the exchange scatterings are indeed irrelevant. In order to do that, we notice that for spinless fermions any exchange scattering $\mathbf{p}\to\mathbf{p}'-\mathbf{q}$, $\mathbf{p}'\to\mathbf{p}+\mathbf{q}$ with small $\mathbf{q}$ described by the matrix element $V(\mathbf{p},\mathbf{p}',\mathbf{Q})$ (where the transferred momentum $\mathbf{Q}=\mathbf{p}'-\mathbf{p}-\mathbf{q}$) can be thought of as a scattering in the Hartree channel $\mathbf{p}\to\mathbf{p}+\mathbf{q}$, $\mathbf{p}'\to\mathbf{p}'-\mathbf{q}$ with the matrix element $-V(\mathbf{p},\mathbf{p}',\mathbf{Q})$. Thus the patches α and β (containing the vectors $\mathbf{p}$ and $\mathbf{p}'$, respectively) are coupled in the exchange channel by

$$V_{\alpha,\beta}(\mathbf{q}) \approx -\frac{g}{m^2}\frac{\mathbf{p}_\alpha\cdot\mathbf{Q}\,\mathbf{p}_\beta\cdot\mathbf{Q}-\mathbf{p}_\alpha\cdot\mathbf{p}_\beta\,Q^2}{Q^4+\gamma^2 v_F^2\left[\mathbf{n}_\mathbf{Q}\cdot(\mathbf{p}_0+\mathbf{p}_0')\right]^2},$$

where $\mathbf{n}_\mathbf{Q}$, $\mathbf{p}_0$, and $\mathbf{p}_0'$ are unit vectors in the directions $\mathbf{Q}$, $\mathbf{p}$, and $\mathbf{p}'$, respectively. [Note that the transferred energy in the exchange channel is $\mathbf{Q}\cdot(\mathbf{p}+\mathbf{p}')/(2m)$.] The square bracket in the expression for $V_{\alpha,\beta}(\mathbf{q})$ gives $(1/|\mathbf{Q}|)[p'^2-p^2-\mathbf{q}\cdot(\mathbf{p}+\mathbf{p}')]$. After averaging over the patches, $\langle p'^2-p^2\rangle = 0$ and the square bracket becomes $-(1/|\mathbf{Q}|)\mathbf{q}\cdot(\mathbf{p}+\mathbf{p}')$, i.e., it vanishes only for $\cos\alpha+\cos\beta=0$, as in the Hartree channel. Thus the pattern of the coupled patches is again given by Fig. 6. However, in this case $\mathbf{Q}\approx\mathbf{p}'-\mathbf{p}$ and $V_{\alpha,\beta}(\mathbf{q})$ is, in general, not singular. Thus in the limit $\Lambda\to 0$, the exchange processes are irrelevant. Note that the exchange processes were ruled out essentially because of the retarded nature of the interaction: the scatterings are relevant, if both the transferred momentum and the transferred energy are small, which is true for forward scattering only.

The Hamiltonian Eq. (4.6) can be diagonalized in the following way: let us introduce the symmetric and antisymmetric combination of the pair of the operators $R_{\alpha,\mathbf{q}}$ and $R_{-\alpha,\mathbf{q}}$, $S_{\alpha,\mathbf{q}} = \frac{1}{\sqrt{2}}(R_{\alpha,\mathbf{q}}+R_{-\alpha,\mathbf{q}})$ and $A_{\alpha,\mathbf{q}} = \frac{1}{\sqrt{2}}(R_{\alpha,\mathbf{q}}-R_{-\alpha,\mathbf{q}})$. Similarly, let $S_{\alpha^*,\mathbf{q}} = \frac{1}{\sqrt{2}}(R_{\alpha^*,\mathbf{q}}+R_{-\alpha^*,\mathbf{q}})$ and $A_{\alpha^*,\mathbf{q}} = \frac{1}{\sqrt{2}}(R_{\alpha^*,\mathbf{q}}-R_{-\alpha^*,\mathbf{q}})$. In terms of these new operators, the Hamiltonian is

$$H = \frac{1}{4}\frac{(2\pi)^2 v_F}{\Omega\Lambda}\sum_{\alpha}\sum_{(\mathbf{n}_\alpha\cdot\mathbf{q}>0}([S_\alpha(\mathbf{q})S_\alpha(-\mathbf{q}) + S_{\alpha^*}(-\mathbf{q})S_{\alpha^*}(\mathbf{q})] + \{A_\alpha(\mathbf{q})A_\alpha(-\mathbf{q}) + A_{\alpha^*}(-\mathbf{q})A_{\alpha^*}(\mathbf{q}) + 2\delta|\sin\alpha|[A_\alpha(\mathbf{q})A_{\alpha^*}(-\mathbf{q}) + A_\alpha(-\mathbf{q})A_{\alpha^*}(\mathbf{q})]\}). \quad (4.7)$$

The symmetric combinations describe free bosons, while the antisymmetric part of the Hamiltonian is not diagonal. However, being quadratic it can be diagonalized by means of the Bogoliubov transformation mixing $A_{\alpha,\mathbf{q}}$ (which is a "creation operator" if the patch α is roughly parallel to $\mathbf{q}$, i.e., if $\mathbf{v}_\alpha\cdot\mathbf{q} > 0$) with $A_{\alpha^*,\mathbf{q}}$ (which is an "annihilation operator" if $\mathbf{v}_\alpha\cdot\mathbf{q} > 0$). The resulting Hamiltonian is

$$H = E(0) + \tfrac{1}{2}\sum_{\alpha,\lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}>0}\omega_{\alpha,\lambda}(\mathbf{q})a^\dagger_{\alpha,\lambda}(\mathbf{q})a_{\alpha,\lambda}(-\mathbf{q}), \quad (4.8)$$

where $E(0)$ is the ground state energy, $a_{\alpha,\lambda}(\mathbf{q})$ are true boson operators, $[a^\dagger_{\alpha,\lambda}(\mathbf{q}), a_{\beta,\lambda}(-\mathbf{q}')] \approx -\delta_{\alpha,\beta}\delta_{\mathbf{q},\mathbf{q}'}$, the index λ numerates the two eigenstates corresponding to a given $\mathbf{q}$ and α, and the spectrum of the bosons is $\omega_{\alpha,\lambda}(\mathbf{q}) = \mathbf{v}_{\boldsymbol{\alpha}}\cdot\mathbf{q}E_{\alpha,\lambda}(\mathbf{q})$. For $\lambda = 1$, $E_{\alpha,\lambda}(\mathbf{q}) = 1$ and for $\lambda = 2$, $E_{\alpha,\lambda}(\mathbf{q}) = \sqrt{1-(2\delta)^2\sin^2\alpha}$. Note that it is the fact that nearly opposite patches (with respect to $\mathbf{q}$) are coupled to each other which leads to the necessity of the Bogoliubov transformation for the bosons. The coupling between patches approximately parallel to each other (i.e., if $\mathrm{sgn}\mathbf{v}_\alpha\cdot\mathbf{q} = \mathrm{sgn}\mathbf{v}_\beta\cdot\mathbf{q}$ for patches α and β) is taken care of by constructing the corresponding symmetric and antisymmetric combinations of the original operators. In complete analogy to the case $D = 1$, for spinless fermions without coupling of the nearly opposite patches, there would be only minor deviations of the resulting state from the Landau Fermi-liquid behavior.

Thus we have shown that the fermions with current-current interactions cannot stay in the Landau Fermi-liquid state, if the hypothetical quasiparticle spectrum is taken to be identical to the bare spectrum.

V. DISTRIBUTION FUNCTION OF THE FERMIONS AT $T = 0$

To demonstrate the non-Landau Fermi-liquid behavior of fermions described by the bosonized Hamiltonian Eq. (4.6), we will calculate in this section the momentum distribution function $n(\mathbf{p})$ at zero temperature. The calculation presented here is a straightforward generalization to higher dimensions of the method used by Mattis and Lieb[28] in $D = 1$.

The momentum distribution function along the direction α is given by the integral $n(\mathbf{p}) = \int d^2\mathbf{x}\, e^{i\mathbf{p}\cdot\mathbf{x}}\langle G|\psi^\dagger_\alpha(\mathbf{x})\psi_\alpha(0)|G\rangle$, where $|G\rangle$ is the ground state of the interacting system described by the bosonized Hamiltonian Eq. (4.6). Following Mattis and Lieb, we notice that $|G\rangle = e^{-iS}|0\rangle$, i.e., $|G\rangle$ can be obtained from the noninteracting ground state $|0\rangle$ by applying to it that canonical transformation e^{-iS}, which leads from Eq. (4.7) to Eq. (4.8). The momentum distribution function then reads

$$n(\mathbf{p}) = \int d^2\mathbf{x}\, e^{i\mathbf{p}\cdot\mathbf{x}}\langle 0|e^{iS}\psi^\dagger_\alpha(\mathbf{x})e^{-iS}e^{iS}\psi_\alpha(0)e^{-iS}|0\rangle. \quad (5.1)$$

One verifies easily that the Bogoliubov transformation diagonalizing Eq. (4.7) can be realized as the canonical transformation $e^{iS}He^{-iS}$ with

$$S = \frac{i}{4}\frac{(2\pi)^2}{\Omega\Lambda}\sum_\beta\sum_{\mathbf{q}}\frac{\phi_\beta(\mathbf{q})}{\mathbf{n}_\beta\cdot\mathbf{q}}A_\beta(\mathbf{q})A_{\beta^*}(-\mathbf{q}) \quad (5.2)$$

and $\tanh 2\phi_\beta(\mathbf{q}) = -2\delta\sin\beta$. The next step is to evaluate $\tilde{\psi}(\mathbf{x}) = e^{iS}\psi_\alpha(\mathbf{x})e^{-iS}$. This is done in a simple way by solving the "equation of motion" for $f_\sigma(\mathbf{x}) = e^{i\sigma S}\psi_\alpha(\mathbf{x})e^{-i\sigma S}$ with the "time" σ.[28] One obtains

$$\tilde{\psi}_\alpha(\mathbf{x}) = \exp\left(\frac{(2\pi)^2}{\sqrt{2}\Omega\Lambda}\sum_{\mathbf{q}}\frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{\mathbf{n}_\alpha\cdot\mathbf{q}}\{A_\alpha(-\mathbf{q})[\cosh\phi_\alpha(\mathbf{q}) - 1] + A_{\alpha^*}(-\mathbf{q})\sinh\phi_\alpha(\mathbf{q})\}\right)\psi_\alpha(\mathbf{x}). \quad (5.3)$$

A similar formula for the electron operator was found by Haldane.[18] Using the property $[A_\alpha(\mathbf{q}), A_{\alpha^*}(\mathbf{q}')] = 0$ we can write the exponential in Eq. (5.3) as $\tilde{\psi}_\alpha(\mathbf{x}) = W(\mathbf{x})R(\mathbf{x})\psi_\alpha(\mathbf{x})$ where $W(\mathbf{x})$ contains operators from patch α,

$$W(\mathbf{x}) = \exp\left(\frac{(2\pi)^2}{\sqrt{2}\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{\cosh\phi_\alpha(\mathbf{q}) - 1}{\mathbf{n}_\alpha\cdot\mathbf{q}}[A_\alpha(-\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{x}} - A_\alpha(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{x}}]\right)$$

and $R(\mathbf{x})$ contains operators from patch α^*,

$$R(\mathbf{x}) = \exp\left(\frac{(2\pi)^2}{\sqrt{2}\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{\sinh\phi_\alpha(\mathbf{q})}{\mathbf{n}_\alpha\cdot\mathbf{q}}[A_{\alpha^*}(-\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{x}} - A_{\alpha^*}(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{x}}]\right).$$

It follows that $\langle G|\psi^\dagger_\alpha(\mathbf{x})\psi_\alpha(0)|G\rangle = \langle 0|\psi^\dagger_\alpha(\mathbf{x})R^{-1}(\mathbf{x})W^{-1}(\mathbf{x})W(0)R(0)\psi_\alpha(0)|0\rangle$. This expectation value factorizes due to the decoupling of different patches in the noninteracting case and one can write $\langle G|\psi^\dagger_\alpha(\mathbf{x})\psi_\alpha(0)|G\rangle = I_1(\mathbf{x})I_2(\mathbf{x})$, where $I_1(\mathbf{x}) = \langle 0|\psi^\dagger_\alpha(\mathbf{x})W^{-1}(\mathbf{x})W(0)\psi_\alpha(0)|0\rangle$ and $I_2(\mathbf{x}) = \langle 0|R^{-1}(\mathbf{x})R(0)|0\rangle$. To evaluate the matrix elements $I_1(\mathbf{x})$ and $I_2(\mathbf{x})$ it is convenient (see Ref. 28) to use the identities $W^{-1}(\mathbf{x})W(0) = W_-W_+Z_1(\mathbf{x})$ and $R^{-1}(\mathbf{x})R(0) = R_-R_+Z_2(\mathbf{x})$, where

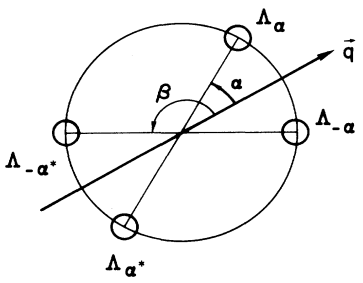

FIG. 6. Interacting patches for the magnetic interaction case. The patch Λ_α couples to patches satisfying the condition $\cos\alpha+\cos\beta=0$, i.e., to the patches Λ_{α^*} and $\Lambda_{-\alpha^*}$. These latter patches are coupled to the patch $\Lambda_{-\alpha}$, also. As a result, the four patches Λ_α, $\Lambda_{-\alpha}$, Λ_{α^*}, and $\Lambda_{-\alpha^*}$ are coupled together.

$$Z_1(\mathbf{x})=\exp\left(\frac{(2\pi)^2}{2\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{[\cosh\phi_\alpha(\mathbf{q})-1]^2}{\mathbf{n}_\alpha\cdot\mathbf{q}}(e^{i\mathbf{q}\cdot\mathbf{x}}-1)\right), \tag{5.4}$$

$$Z_2(\mathbf{x})=\exp\left(\frac{(2\pi)^2}{2\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{\sinh^2\phi_\alpha(\mathbf{q})}{\mathbf{n}_\alpha\cdot\mathbf{q}}(e^{-i\mathbf{q}\cdot\mathbf{x}}-1)\right), \tag{5.5}$$

and the index + (−) denotes that part of the operators W or R which contains only annihilation (creation) operators A. Thus $R_+|0\rangle=0$ and $W_+|0\rangle=0$ and we have $I_1(\mathbf{x})=Z_1(\mathbf{x})\langle 0|W_-^{-1}\psi_\alpha^\dagger(\mathbf{x})W_-W_+\psi_\alpha(\mathbf{x})W_+^{-1}|0\rangle$ and $I_2(\mathbf{x})=Z_2(\mathbf{x})$. Let us define $F_\alpha^\dagger(\mathbf{x})=W_-^{-1}\psi_\alpha^\dagger(\mathbf{x})W_-$ and $F_\alpha(0)=W_+\psi_\alpha(0)W_+^{-1}$. One easily verifies that $F_\alpha^\dagger(\mathbf{x})=\psi_\alpha^\dagger(\mathbf{x})\sqrt{Z_0(\mathbf{x})}$ and $F_\alpha(0)=\psi_\alpha(0)\sqrt{Z_0(\mathbf{x})}$ where

$$Z_0(\mathbf{x})=\exp\left(\frac{(2\pi)^2}{2\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{2[\cosh\phi_\alpha(\mathbf{q})-1]}{\mathbf{n}_\alpha\cdot\mathbf{q}}(e^{i\mathbf{q}\cdot\mathbf{x}}-1)\right). \tag{5.6}$$

The momentum distribution of the fermions then is [see Eq. (5.1)]

$$n(\mathbf{p})=\int d^2\mathbf{x}\,e^{i\mathbf{p}\cdot\mathbf{x}}e^{-Q_\alpha(\mathbf{x})}\langle 0|\psi_\alpha^\dagger(\mathbf{x})\psi_\alpha(0)|0\rangle, \tag{5.7}$$

where we have introduced $e^{-Q_\alpha(\mathbf{x})}=Z_0(\mathbf{x})Z_1(\mathbf{x})Z_2(\mathbf{x})$. Using Eqs. (5.4)–(5.6) we obtain

$$Q_\alpha(\mathbf{x})=\exp\left(\frac{(2\pi)^2}{2\Omega\Lambda}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}}\frac{1-\cos\mathbf{q}\cdot\mathbf{x}}{\mathbf{n}_\alpha\cdot\mathbf{q}}[\cosh 2\phi_\alpha(\mathbf{q})-1]\right)$$

or, replacing the sums by the integrals and denoting the component of $\mathbf{q}$ parallel (perpendicular) to the patch α as $q_\parallel$ ($q_\perp$),

$$Q_\alpha(\mathbf{x})=\exp\left(\frac{1}{2\Lambda}\int_0^{\Lambda/2}\frac{dq_\parallel}{q_\parallel}\int_{-\Lambda/2}^{\Lambda/2}dq_\perp(1-\cos\mathbf{q}\cdot\mathbf{x})[\cosh 2\phi_\alpha(\mathbf{q})-1]\right). \tag{5.8}$$

Using the fact that $\langle 0|\psi_\alpha^\dagger(\mathbf{x})\psi_\alpha(0)|0\rangle=\frac{1}{\Omega}\sum_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{x}}n_0(\mathbf{k})$, where $n_0(\mathbf{k})$ is the free fermion distribution function, we have

$$n(\mathbf{p})=\int_{|\mathbf{k}|<p_F}d^2\mathbf{k}F_\alpha(\mathbf{p}-\mathbf{k}),$$
$$F_\alpha(\boldsymbol{\kappa})=\int\frac{d^2\mathbf{x}}{(2\pi)^2}e^{i\boldsymbol{\kappa}\cdot\mathbf{x}}e^{-Q_\alpha(\mathbf{x})}. \tag{5.9}$$

The momentum distribution is given by Eqs. (5.8) and (5.9), which are a natural generalization of the corresponding formulas in $D=1$ (see Ref. 28). We remind the reader that Eqs. (5.8) and (5.9) determine the distribution of the fermions for $\mathbf{p}$ from the patch α. Since $\Lambda\ll p_F$, the Fermi surface is flat within the patch and the integral over $k_\perp$ gives $\frac{2}{x_\perp}\sin\frac{\Lambda x_\perp}{2}$, i.e., the dominant contribution to $n(\mathbf{p})$ comes from $x_\perp<1/\Lambda$, where $Q_\alpha(\mathbf{x})\approx Q_\alpha(x_\parallel,0)$. Using this approximation, the integration over $x_\perp$ can be performed and we have

$$n(p)=\text{const}-\int_0^{p-p_F}d\kappa\int_{-\infty}^{\infty}\frac{dx_\parallel}{2\pi}e^{i\kappa x_\parallel}e^{-Q(x_\parallel,0)} \tag{5.10}$$

in complete analogy to the $D=1$ case. Note, however, that $Q(x_\parallel,0)$ is given by Eq. (5.8) which differs from the corresponding formula in the paper by Mattis and Lieb.[28] Since $\tanh 2\phi_\alpha(\mathbf{q})=-2\delta\sin\alpha$, for small δ one obtains $\cosh 2\phi_\alpha(\mathbf{q})-1\approx 2\delta^2\sin^2\alpha=2\delta^2\sin^2 q_\perp^2/(q_\parallel^2+q_\perp^2)$ and the $q_\perp$ integration leads to

$$Q(x_\parallel,0)=\delta^2\int_0^r\frac{dt}{t}(1-\cos t)\left(1-\frac{t}{r}\arctan\frac{r}{t}\right),$$

where $r=\Lambda x_\parallel/2$. For $r\gg 1$ we have $Q(x_\parallel,0)=\delta^2\ln r$ and

$$n(p)=\text{const}-c\left(\frac{p-p_F}{\Lambda}\right)^{\delta^2}\text{sgn}(p-p_F), \tag{5.11}$$

with the constant $c=\frac{2^{\delta^2}I(\delta)}{\pi\delta^2}$, $I(\delta)=\int_0^\infty dt\,t^{-\delta^2}\cos t$. Thus we have shown that there is no discontinuity in $n(\mathbf{p})$ at the Fermi surface. The momentum distribution exhibits a nonuniversal power-law behavior in the vicinity of p_F, the power being proportional to the square of the coupling constant.

One can compute the jump Δn_p of the distribution function on the Fermi surface for a finite system with the linear dimension L: $\Delta n\sim e^{2\delta^2\ln(\Delta p/\Lambda)}$, where $\Delta p\sim(2\pi)/L$ is the spacing of the p levels. Identifying Δn with the wave-function renormalization Z, we obtain $Z\sim e^{-\delta^2\ln N}$, the formula proposed by Anderson[1]

as a result of the long-range "statistical interaction" $\Gamma(\mathbf{p},\mathbf{p}') \sim \delta^{\frac{\mathbf{p}(\mathbf{p}-\mathbf{p}')}{p^2-p'^2}}$. Notice that in our case of spinless fermions the exponent δ^2 in Eq. (5.11) coincides with the coefficient in the exponentially small Z factor while in the case of spin one-half fermions one would not expect the same equality.

VI. THE RENORMALIZED QUASIPARTICLE SPECTRUM

Recently, Halperin, Lee, and Read[15] proposed a self-consistent description of fermions in $D = 2$ subject to the current-current interactions in the context of the gauge description of the half-integer filled Hall effect. Halperin, Lee, and Read conjectured that the spectrum contains one-particle excitations with a strongly renormalized dispersion $\epsilon(p) \sim (p-p_F)^{3/2}$ which corresponds to a vanishing Fermi velocity (see Fig. 2). They also assumed that the resulting state is an extension of the Landau Fermi-liquid state to the case of quasiparticles with divergent mass.

Intuitively, the ansatz $\epsilon(p) \sim (p-p_F)^{3/2}$ seems to be plausible: on the one hand, it removes the degeneracy of the Hartree-Fock approximation, on the other hand it still reflects the tendency of the interactions (clearly seen in the perturbation theory and in HFA) to make the quasiparticles heavier (see Fig. 2).

Halperin, Lee, and Read determined the renormalized spectrum in the following way: they calculated the self-energy of the fermion in the lowest order of perturbation theory, where in the internal electron line they assumed a quasiparticle pole at the energy $\epsilon(p)$. They showed that the ansatz $\epsilon(p) \sim (p-p_F)^{3/2}$ and the corresponding change of the electron mass in the interaction vertex Eq. (2.3) lead to a self-energy of the same order as $\epsilon(p)$. In view of that the solution thus obtained was argued to be self-consistent.

In this section we will generalize the results of Sec. IV to the case where the quasiparticle spectrum is $\epsilon(p) = v_\xi(p)(|\mathbf{p}|-p_F)$. Here $v_\xi(p) = v_F|\frac{|\mathbf{p}|-p_F}{p_F}|^\xi$, the spectrum of Halperin, Lee, and Read[15] corresponds to $\xi = 1/2$, and the bare spectrum corresponds to $\xi = 0$.

Repeating the discussion before Eq. (4.4) we obtain the renormalized interaction

$$V_{\alpha,\beta}(\mathbf{q}) \approx -\frac{g}{m^2}\left(\frac{\Lambda}{p_F}\right)^{2\xi} \times \frac{p_F^2 q^2 \sin\alpha \sin\beta}{q^4 + (\frac{\Lambda}{p_F})^{2\xi}\gamma^2 v_F^2(\cos\alpha+\cos\beta)^2}, \tag{6.1}$$

which differs from Eq. (4.4) because of the renormalizations of the coupling constant (due to the mass renormalization) and that of the transferred energy. The resulting bosonized form of the interaction energy is thus that of Eq. (4.5) multiplied by $(\frac{\Lambda}{p_F})^\xi$.

In the present case, it is impossible to bosonize the kinetic energy because of the nonlinearity of the quasiparticle spectrum. However, making the crude approximation $[H_{\rm kin}, R_\alpha(\mathbf{q})] \approx (\frac{\Lambda}{p_F})^\xi \mathbf{v}_\alpha \cdot \mathbf{q} R_\alpha(\mathbf{q})$ we can write

$$H_{\rm kin} \approx \frac{(2\pi)^2 v_F}{\Omega\Lambda}\left(\frac{\Lambda}{p_F}\right)^\xi \sum_{\mathbf{q}}\sum_{\alpha} R_\alpha(\mathbf{q})R_\alpha(-\mathbf{q}).$$

Exactly as in the case of the bare spectrum, both the kinetic and the potential energy are proportional to $\Lambda^{\xi-1}$, and Haldane's criterion for the "marginal" breakdown of the Fermi-liquid behavior is satisfied for all ξ, i.e., for the whole class of quasiparticle spectra $\epsilon(p) = v_\xi(p)(|\mathbf{p}|-p_F)$ with power-law decaying effective masses there exists a kind of balance between the kinetic and interaction energy. The total effective Hamiltonian is now

$$H = \frac{1}{4}\frac{(2\pi)^2 v_F}{\Omega\Lambda}\sum_{\alpha}\sum_{\mathbf{n}_\alpha\cdot\mathbf{q}>0}\left(\frac{\Lambda}{p_F}\right)^\xi \{[S_\alpha(\mathbf{q})S_\alpha(-\mathbf{q}) + S_{\alpha^*}(-\mathbf{q})S_{\alpha^*}(\mathbf{q})] + \{A_\alpha(\mathbf{q})A_\alpha(-\mathbf{q}) + A_{\alpha^*}(-\mathbf{q})A_{\alpha^*}(\mathbf{q}) + 2\delta|\sin\alpha|[A_\alpha(\mathbf{q})A_{\alpha^*}(\mathbf{q}) + A_\alpha(-\mathbf{q})A_{\alpha^*}(\mathbf{q})]\}), \tag{6.2}$$

which is just Eq. (4.7) scaled by $(\frac{\Lambda}{p_F})^\xi$. Thus the diagonalized Hamiltonian is given by Eq. (4.8) where the boson spectrum is $\omega_{\alpha,\lambda}(\mathbf{q}) = (\frac{\Lambda}{p_F})^\xi \mathbf{v}_\alpha \cdot \mathbf{q} E_{\alpha,\lambda}(\mathbf{q})$.

Let us calculate now the energy as a function of the temperature T:

$$E(T) = E(0) + \frac{p_F}{\Lambda}\int d\alpha \sum_{\mathbf{q},\lambda}\frac{\omega_{\alpha,\lambda}(\mathbf{q})}{\exp(\omega_{\alpha,\lambda}(\mathbf{q})/T-1)}. \tag{6.3}$$

The sum over $\mathbf{q}$ is over a region with the size Λ. In fact, our approximate consideration can be done only for temperatures T of the same order as the characteristic fermion energy $\epsilon(\Lambda)$. This leads to $c_V \sim T^{1/(1+\xi)}$, which has to be compared with the result of the perturbation theory, where the dominant contribution to the specific heat $c_V \sim T^{2/3}$ comes from the density collective mode (as opposed to the regular term $\sim T$ provided by the particle-hole continuum). If we require that the renormalized spectrum gives the same c_V, we are led back to the Halperin-Lee-Read spectrum with $\xi = 1/2$. This property of the renormalized spectrum $\epsilon(p) \propto (p-p_F)^{3/2}$ was noted in Ref. 15.

Thus the fermions with current-current interactions cannot stay in the Landau Fermi-liquid state, if the hypothetical quasiparticle spectrum is taken to be $\epsilon(p) = v_\xi(p)(|\mathbf{p}|-p_F)$ with $v_\xi(p) = v_F|\frac{|\mathbf{p}|-p_F}{p_F}|^\xi$. For $\xi = 0$, one encounters the case studied by Haldane[18] and the procedure is rather well understood. In the case $\xi \neq 0$, the bosonization is impossible, and we have to introduce drastic approximations to make the problem tractable. Obviously, the thermodynamic properties depend crucially on the form of the renormalized spectrum.

Strictly speaking, the approximation used in this section is not sufficient to calculate the distribution function $n(p)$ reliably for any ξ different from zero. Within this approximation $n(p)$ simply does not depend on ξ.

To improve the calculation one should take into account that the nonlinearity of the quasiparticle spectrum actually implies the boson spectrum to be of the form $\omega_{\alpha,\lambda}(\mathbf{q}) \sim (\mathbf{v}_\alpha \cdot \mathbf{q})^{1+\xi}$. Although the straightforward Haldane bosonization is not any more applicable, we could speculate that the exact fermion distribution function does have a powerlike behavior in the vicinity of p_F governed by some universal (coupling independent) exponent. Notice that such a behavior occurs in the strong coupling limit of the $D = 1$ Tomonaga-Luttinger model.[31]

VII. CONCLUSIONS

In this paper we undertook an attempt to analyze the effect of the long-ranged current-current interaction on the spectrum of $D = 2$ fermions. This investigation was performed in the framework of the approach developed by Haldane which provides criteria for non-Landau Fermi-liquid behavior in $D > 1$. The important ingredient in the Haldane renormalization scheme is the relative scaling of the kinetic and potential energy terms. If the latter is negligible compared to the former, Landau Fermi-liquid behavior results. If both scale together, then Haldane's method applies and gives non-Landau Fermi-liquid behavior. Remarkably, we find here that the retarded current-current interactions fulfill this criterion for the application of Haldane's method. A second criterion, to make this method tractable, is that only a finite number of rays (patches) on the Fermi surface are coupled to each other. In Anderson's original scheme, each ray coupled only to itself leading to the "tomographic Luttinger-liquid" behavior. In the present case, we find coupling between four rays which can also be diagonalized by similar methods and leads to similar Luttinger-liquid-type behavior. Examples of this behavior are the fact that the singularity in $n(\mathbf{p})$ is power law rather than discontinuity; also the one-particle Green's function does not have a pole indicated by the vanishing Z factor.

In the analysis presented above we suppressed the spin for reasons of simplicity. The generalization to include the spin variable within the Haldane procedure is straightforward: The first term in Eq. (3.1) is diagonal in spin space while the second term in general is not. The diagonalization of the Hamiltonian leads therefore automatically to different spectra for spin-singlet and spin-triplet bosons, manifesting the spin charge separation which occurs in Luttinger liquids. This is a common feature of our results and other approaches which lead to Luttinger-liquid-like behavior.[1,18]

To demonstrate the effects of these singularities we used initially as an input the bare linear fermion spectrum and then could apply Haldane's bosonization scheme in a straightforward way. However, there are important renormalizations of the quasiparticle spectrum and if we follow the Halperin-Lee-Read approach,[15] the quasiparticle spectrum is strongly nonlinear in the vicinity of p_F which leads to difficulties with Haldane's procedure. Nonetheless, if we use a crude approximation to bosonize the kinetic energy, the results obtained with the linear spectrum remain valid. This is a remarkable feature of our results which encourages us to believe in the general validity of our results. However, the question of the proper renormalization of the quasiparticle spectrum needs further study. Moreover, in the present study we assumed that only the Hartree (forward) scattering channel is relevant at low energies resulting in a "Tomonaga-Luttinger" reduction of the gauge model. We gave qualitative arguments for the validity of this reduction, but the scattering processes in the Fock and Cooper channels should be studied in more detail.[29]

We conclude with the analysis of fermions with current-current interactions in $D = 3$. As shown in Appendix B, applying the Halperin-Lee-Read method for constructing the renormalized spectrum, we find that the mass diverges logarithmically in the vicinity of p_F. Repeating the calculation of Sec. IV we show that in $D = 3$, the kinetic energy dominates over the potential energy in the limit $\Lambda \to 0$ and we conclude that the fermions stay in a Landau Fermi-liquid state, with a divergent quasiparticle mass, however. The fact that the interaction Eq. (2.3) leads to the breakdown of the Landau Fermi-liquid behavior in $D = 2$ and its survival in $D = 3$ is in qualitative agreement with the results of Bares and Wen[4] which state that for every spatial dimension D there exists a critical strength of interaction η [denoting the power of the transferred momentum q in the interaction vertex: $\Gamma(q) \sim \frac{1}{q^\eta}$]. Bares and Wen find the critical value $\eta = 2D-2$. However, note that this formula should not be taken too literally in our case, since it was derived for density-density interactions.

It is interesting to compare our results with those known in relativistic QED with a negligible density of fermions. In $D = 2$ we obtained that at finite fermion density, the one-particle Green's function has a vanishing Z factor for the pole which would correspond to coherent fermion propagation while in the vacuum case ($p_F = 0$) studied in the framework of 2D QED, there is just a finite renormalization. In 3D we did not observe a vanishing Z factor, although there is a logarithmic spectrum renormalization in the energy range $|\epsilon - \epsilon_F| < \epsilon_F e^{-1/r}/r$, where $r \sim \frac{p_F e^2}{mc^2}$ is a small number. It should be compared with the well known result for the infrared asymptotics of the vacuum fermion Green function in 3D relativistic QED first obtained by Landau, Abrikosov, and Khalatnikov,[30] who found that the interaction mediated by undamped propagating photons strengthens the singularity of the fermion Green function on the mass shell: $G(\epsilon, p) \sim (\epsilon^2 - p^2c^2 - m^2c^4)^{-(1+3g/8\pi^2)}$, since the exponent becomes greater than one. We assign the physical origin of these differences with respect to the vacuum renormalization to the overdamped character of the effective current-current interaction governed by the fermion polarization.

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APPENDIX A: THE FERMION-CONDENSATE STATE

The model vertex Eq. (2.4) leads to the interaction function

$$f(p,p') = -\Gamma^0(\mathbf{p},\mathbf{p}';\mathbf{p}-\mathbf{p}') = g\frac{(\mathbf{p}\times\mathbf{p}')^2}{|\mathbf{p}||\mathbf{p}'|(\mathbf{p}-\mathbf{p}')^2}.$$

Inserting this interaction function into the energy functional Eq. (2.1) and taking the variation with respect to $n_{\mathbf{p}}$ [Eq. (2.2)] we have

$$0 = p^2 - p_0^2 + g\int d^2\mathbf{p}'\frac{pp'\sin^2\theta}{p^2+p'^2-2pp'\cos\theta}n_{\mathbf{p}'}, \quad \text{(A1)}$$

where we have taken the quasiparticle spectrum to be $\epsilon(p) = p^2$, p_0^2 is the chemical potential, and θ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. We will assume that the momentum distribution is rotationally symmetric, $n_{\mathbf{p}} = n(p)$. Then the angular part of the integral over $\mathbf{p}'$ can be performed, yielding

$$0 = p^2 - p_0^2 + \pi gp\int_p^\infty dp' n(p') + \frac{\pi g}{p}\int_0^p dp' p'^2 n(p').$$

Multiplying by p and differentiating twice with respect to p one obtains

$$n(p) = \frac{1}{2\pi g}\frac{p_0^2+3p^2}{p^2}. \quad \text{(A2)}$$

For $p\to 0$ $n(p)$ diverges and for $p\to\infty$ the number of particles $N\sim\int^\infty dp\,pn(p)\to\infty$. Thus the solution Eq. (A2) cannot hold for very small and very large momenta. This means there has to exist a region of momenta $p_1 < p < p_2$, where the true fermion distribution is given by Eq. (A2) and outside of which it acquires its limiting values 0 or 1. Inserting the ansatz $n(p) = 1$ for $0 < p < p_1$, $n(p) = 0$ for $p > p_2$, and Eq. (A2) for $p_1 < p < p_2$ into Eq. (A1) yields $p_1 = p_0/\sqrt{2\pi g/3-1}$ and $p_2 = p_0/\sqrt{3}$. Calculating the total number of electrons one finds $p_0 = p_F\sqrt{\frac{2\pi g}{4+\ln(\frac{2\pi g-3}{9})}}$, where p_F is the Fermi momentum of an equal number of noninteracting electrons.

APPENDIX B: FERMIONS WITH CURRENT-CURRENT INTERACTIONS IN $D = 3$

We begin by repeating the calculation of Sec. IV in the case $D = 3$. Again we will use in Eq. (4.1) the bare spectrum. Formulas (4.2)–(4.4) hold with obvious changes. The pattern of interacting patches is determined from $\cos\alpha+\cos\beta = 0$. Now, a given patch is coupled to a ring of "opposite" patches. (See Fig. 7. The ring of patches

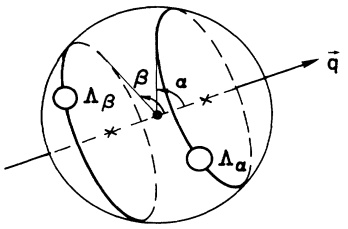

FIG. 7. Interacting patches for the magnetic interaction case in $D = 3$.

in $D = 3$ arises from the two patches α^* and $-\alpha^*$ in $D = 2$.) Repeating the calculation leading to Eq. (4.5) we obtain

$$H_{\rm int}\sim\frac{v_F}{\Omega\Lambda}\sum_{\mathbf{q}}\sum_{\alpha,\beta}f(\alpha,\beta,\mathbf{q})R_\alpha(\mathbf{q})R_\beta(-\mathbf{q}), \quad \text{(B1)}$$

where the sum over β is over the ring corresponding to the given α and $\mathbf{q}$; $f(\alpha,\beta,\mathbf{q})$ are numerical factors of the order unity. On the other hand, the kinetic energy reads

$$H_{\rm kin}\sim\frac{v_F}{\Omega\Lambda^2}\sum_{\mathbf{q}}\sum_{\alpha}R_\alpha(\mathbf{q})R_\alpha(-\mathbf{q}). \quad \text{(B2)}$$

Thus in the limit $\Lambda\to 0$ the kinetic energy term dominates and the system flows to the free fermion fixed point. Since the spectrum renormalization does not change this conclusion on the level of accuracy of Sec. VI, we conclude that the fermions stay in the Landau Fermi-liquid state.

Next we use the method of Halperin, Lee, and Read to estimate the quasiparticle spectrum in $D = 3$. To this end, let us repeat briefly the argument in $D = 2$ (see Ref. 15): the self-energy of an electron with the momentum p evaluated on the mass shell is

$$\Sigma(p,\epsilon(p))\sim\frac{\epsilon(p)^{2/3}}{m^*}. \quad \text{(B3)}$$

Equation (B3) was obtained from the lowest-order perturbation theory by replacing the bare energy in the fermion Green's function by the renormalized dispersion $\epsilon(p)$ and the bare mass in the interaction vertex Eq. (2.3) by the renormalized mass m^*. Using the definition

$$\epsilon(p) = \frac{1}{m^*}(p-p_F) \quad \text{(B4)}$$

and requiring $\Sigma(p,\epsilon(p))\sim\epsilon(p)$ (this is the self-consistency condition) we obtain a closed set of equations for $\epsilon(p)$ with the solution $\epsilon(p)\sim(p-p_F)^{3/2}$ mentioned in the text.

In $D = 3$, instead of Eq. (B3) we have (see Ref. 7)

$$\Sigma(p,\epsilon(p))\sim\frac{1}{m^*}\epsilon(p)\ln[\epsilon(p)]. \quad \text{(B5)}$$

Solving Eqs. (B4) and (B5) together with the self-consistency condition, we have finally

$$\epsilon(p)\sim\frac{(p-p_F)}{|\ln|p-p_F||}, \quad \text{(B6)}$$

i.e., within our approximation, the fermions are in a Landau Fermi-liquid state with a divergent mass.

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