Geometrical approach to bosonization of $D > 1$ dimensional (non)-Fermi liquids

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We discuss an approach to higher-dimensional bosonization of interacting fermions based on a picture of fluctuating Fermi surface. Compared with the linearized "constructive" approach this method allows an account of the Fermi surface curvature which leads to non-Gaussian terms in the bosonized Lagrangian. On the basis of this description we propose a procedure of calculating density response functions beyond the random-phase approximation (RPA). We also formulate a bosonic theory of the compressible metal-like state at the half-filled lowest Landau level and check that in the Gaussian approximation it reproduces RPA results of the gauge theory by Halperin, Lee, and Read.

I. INTRODUCTION

Recently there has been a lot of interest in possible scenarios of a breakdown of Landau-Fermi-liquid behavior in two- and three-dimensional strongly correlated systems. In particular, it was conjectured that it might happen in presence of long-ranged density-density or, even more likely, current-current interactions.[1–4] A system of nonrelativistic fermions coupled to an Abelian gauge field was argued to present such an example.[5]

However, a systematic investigation of these exciting possibilities still remains to be carried out. One essential reason is a lack of an adequate formalism capable of giving a proper description of singular interactions which may result in non-Fermi-liquid states. Although standard calculations in low-order perturbation theory can, in principle, reveal features suggesting a non-Fermi-liquid regime one certainly needs a more advanced technique to study a suspicious problem in greater detail.

Among possible improvements a renormalization group method was recently proposed and tested in the case of short-range interactions which do not destroy the Fermi liquid in $D > 1$ (except for an instability in the Cooper channel).[6] Another promising development was provided by the method of higher-dimensional bosonization. It was proposed in Ref. 7 in a manner close to the more recent Anderson picture of "tomographic projection"[8] where a D-dimensional space is considered as a set of essentially uncoupled one-dimensional "rays." In its original form the method of Ref. 7 was basically intended to reproduce fermion algebra and correlations along a given radial direction in momentum space. As an output a bosonic representation of free fermion correlation functions was obtained.[7]

More recently a method was proposed by Haldane[9] and then elaborated in Refs. 10 and 11 to treat couplings between different Fermi points in the bosonized theory. This generalization involves the construction of effective bosonic variables as sums over squat boxes ("patches") instead of radial rays in momentum space. In the framework of this description the main results of the Landau-

Fermi-liquid theory were reproduced.[10,11] However, when applied to less familiar problems such as the problem of two-dimensional nonrelativistic fermions coupled to gauge fields the method leads to results of uncertain status.[12] That is, the authors of Ref. 13 who studied the same problem by using a self-consistent diagrammatic approach argued that the results obtained in Ref. 12 can only be valid in the unphysical limit of a zero number of fermionic species.

An obvious shortcoming of the "constructive" approach to higher-dimensional bosonization is that it does not provide a complete account of the geometric curvature of the Fermi surface or of the parabolic corrections to the dispersion associated with it. Technically, this means that within this scheme one cannot treat properly transferred momenta tangential to the Fermi surface while these become more and more important as one gets closer to it.

For instance, if it turns out that at small energy transfer $\omega \to 0$ a given interaction vertex determines an average tangential transferred momentum to be $q_t \gg k_F(\frac{\omega}{\mu})^{1/2}$, then the term $\sim q^2$ which is due to a finite Fermi surface curvature cannot be neglected in typical denominators $\omega - \vec{v}_F\vec{q} + \frac{1}{2m}q^2$ which appear in the integral expressions for Green functions obtained via bosonization[10,11] (see also Ref. 14). In particular, omitting q_t in the problem of fermions with gauge interactions one can never get to the regime where the Migdal theorem stating the irrelevance of vertex corrections holds. On the contrary, it was argued in Refs. 15 and 13 that in the case of the problem of nonrelativistic fermions coupling to the gauge field where $q_t \sim k_F(\frac{\omega}{\mu})^{1/3} \gg k_F(\frac{\omega}{\mu})^{1/2}$ the Migdal theorem always holds in $D > 2$ [in the two-dimensional (2D) case it can only be valid in the limit of a large number of fermionic species.[13]]

We mention, in passing, that the general eikonal method applied to the 2D gauge problem in Ref. 16 is, in fact, not plagued with this flaw and provides a natural account of $\sim q^2$ terms. However, in Ref. 16 these terms were deliberately omitted to obtain an explicit form of the one-particle Green function. Thus the formula for the

one-particle Green function $G(\epsilon, p)$ obtained in Ref. 16 ceases to be valid in the very vicinity of the Fermi surface. On the other hand, including the above-mentioned terms $\sim q^2$ one ends up with an expression which agrees with the results of Ref. 13 in both regimes when either ϵ or $v_F\,(p - p_F)$ is much greater than the other.

Postponing a thorough revision of the eikonal results of Ref. 16 until another paper,[17] we shall present here a "geometrical" approach to $D > 1$ bosonization which allows a systematic account of tangential components of transferred momenta q_t. We shall also illustrate how the method works in the case of 2D density-density as well as current-density interactions.

II. NONLINEAR BOSONIZATION OF INTERACTING FERMIONS IN EXTERNAL FIELDS

Talking about bosonization one actually means a procedure of calculating (gauge invariant) response functions using functional integrals in terms of some bosonic variables. Remarkably, dealing with this problem one can avoid the subtle question about an explicit construction of the fermion operator in terms of those bosons. Although the corresponding formula was repeatedly conjectured in Refs. 7 and 9–11, the necessity to supplement the naive D-dimensional counterpart of the 1D relation $\psi \sim \exp i\phi$ by a complicated ordering operator makes this representation hardly useful in practice. Moreover, a calculation of the fermion Green function itself does not provide much physical insight in cases where some gauge symmetry is involved.

Nevertheless, a systematic approach to higher-dimensional bosonization can be developed in the framework of the general method of "coadjoint orbit quantization."[18] Adapting this general procedure to the case of interacting fermions one may choose a basis of coherent states

$$|\{g(\vec{p}, \vec{q})\}\rangle = \hat{g}|\text{vac}\rangle$$
$$= \exp\left(i \int d\vec{p}\,d\vec{q}\,g(\vec{p}, \vec{q})\hat{n}_{\vec{p}}(\vec{q})\right)\bigg|\text{vac}\rangle \quad (2.1)$$

created by elements $\hat{g}$ of the infinite group G from some vacuum state $|\text{vac}\rangle$. The group G is generated by fermion bilinear operators $\hat{n}_{\vec{p}}(\vec{q}) = \psi^\dagger(\vec{p} + \vec{q})\psi(\vec{p})$ obeying the algebra (referred to in the 1D case as W_∞)

$$[\hat{n}_{\vec{p}}(\vec{q}), \hat{n}_{\vec{p}'}(\vec{q}')] = \delta(\vec{p} - \vec{p}' - \vec{q}')\hat{n}_{\vec{p}'}(\vec{q} + \vec{q}')$$
$$- \delta(\vec{p}' - \vec{p} - \vec{q})\hat{n}_{\vec{p}}(\vec{q} + \vec{q}'). \quad (2.2)$$

We note that a similar choice of the basis of coherent states was made in Ref. 11. As we shall see below, in the Gaussian approximation the Lagrangian which appears in the functional integral over coherent states (2.1) [see (2.17)] is similar to the result obtained in Ref. 11 while there are extra (non-Gaussian) terms which provide a systematic account of the effects of the Fermi surface curvature.

The orbit of the group G associated with some refer-

ence ground state consists of elements

$$Q = \hat{g}|\text{vac}\rangle\langle\text{vac}|\hat{g}^{-1}. \quad (2.3)$$

The expectation value of the operator $\hat{A}$ taken over the coherent state (2.1) can be expressed in terms of Q as

$$\langle g|\hat{A}|g\rangle = \text{tr}\{Q\hat{A}\}, \quad (2.4)$$

where the trace stands for the integral over phase space $\text{tr} = \int d\vec{x}\,d\vec{q}$ and $\vec{x}$ is the Fourier transform of $\vec{p}$.

Under quantization by means of the functional integral the bosonic variable

$$w(\vec{p}, \vec{q}) = \langle g|\hat{n}_{\vec{p}}(\vec{q})|g\rangle = \text{tr}\{Q\hat{n}_{\vec{p}}(\vec{q})\} \quad (2.5)$$

parametrizing the orbit element (2.3) becomes a quantum field which can be identified with the partial Fourier transform of the quantum Wigner distribution function $\tilde{w}(\vec{p}, \vec{r}, t)$ (phase space density).

To write down the bosonic Lagrangian in terms of $w(\vec{p}, \vec{q})$ one has to find the analogue of the term $p\dot{q}$. It turns out that it can only be written in terms of a cocycle[18] by introducing a fictitious variable u such that $Q(\vec{p}, \vec{q}, u = 0) = Q(\vec{p}, \vec{q})$. Then the Lagrangian acquires the form

$$L = \langle g|i\partial_t - H|g\rangle$$
$$= i\int_0^\infty du\,\text{tr}(Q\{\partial_u Q, \partial_t Q\}_{\text{MB}}) - \text{tr}(HQ), \quad (2.6)$$

where $\{A, B\}_{\text{MB}}$ stands for the so-called Moyal bracket

$$\{A, B\}_{\text{MB}} = \frac{2}{\hbar}\sin\frac{\hbar}{2}\sum_i(\partial_{x_i}\partial_{q_i'} - \partial_{q_i}\partial_{x_i'})$$
$$\times A(\vec{x}, \vec{q})B(\vec{x}', \vec{q}')|_{\vec{x} = \vec{x}', \vec{q} = \vec{q}'}, \quad (2.7)$$

which amounts to the Poisson one in the semiclassical limit $\hbar \to 0$.

The functional integral for the theory (2.6) written in terms of $w(\vec{p}, \vec{q})$ provides an exact bosonization of the original fermion problem even in the case of a nonlinear bare fermion spectrum and/or nonlocal interactions in any dimension.[19,20] However, because of the overcompleteness of the basis of coherent states of $\mathcal{N}$ particles the variables $w(\vec{p}, \vec{q})$ have to be subjected to additional constraints

$$Q^2 = Q, \qquad \text{tr}\,Q = \mathcal{N}, \quad (2.8)$$

which make things too complicated. Nevertheless, this description can be used for a systematic derivation of corrections due to spectrum nonlinearity and/or nonlocality of interactions. In the lowest order in gradients the method is essentially equivalent to the "constructive" bosonization approach[9–11] where the right hand side of the commutation relations (2.1) is replaced by a c number

$$[\hat{n}_{\vec{p}}(\vec{q}), \hat{n}_{\vec{p}'}(\vec{q}')] \approx \frac{1}{2\pi}\delta(\vec{p} - \vec{p}')\delta(\vec{q} + \vec{q}')(\vec{q}\vec{\nabla}_{\vec{p}})n_{\vec{p}}^{(0)} \quad (2.9)$$

where $n_{\vec{p}}^{(0)} = \theta(k_F - p)$ is the bare Fermi distribution

function.

Depending on the details of the interaction, the approximate commutation relations (2.9) between $\hat{n}_{\vec{p}}(\vec{q})$ may become asymptotically correct in the sense of low-energy, small-angle scattering matrix elements of both sides of (2.9). However, the constructive method encounters the problem of an artificial low-energy cutoff $\Lambda \ll k_F$ which appears explicitly in the construction of oscillator-like bosonic variables $a_{\vec{n}}(\vec{q}) = \frac{1}{\sqrt{(\vec{n}\vec{q})}} \sum_{|p_F \vec{n} - \vec{p}| < \Lambda} \psi^\dagger(\vec{p} + \vec{q})\psi(\vec{p})$ defined as sums over patches of size Λ on the Fermi surface. Presumably, the whole procedure can only be trusted if this cutoff does not enter physical observables.

In contrast, such a problem simply does not occur in the formally exact method which we described above. Notice that it can be thought of as a hydrodynamical field theory which generalizes the phenomenological Landau theory including collisions between quasiparticles. However, to make this approach really work one needs a convenient parametrization of the phase space density in terms of some bosonic variables which resolve the constraints (2.8) imposed on the Wigner functions $w(\vec{p}, \vec{q})$.

It seems natural enough to formulate a purely bosonic description of interacting fermions in terms of the vector variable $\vec{k}_F(\vec{r}, t, \hat{s})$ which traces the shape of the $(D-1)$-dimensional Fermi surface parametrized by coordinates $\hat{s} = s_1, ..., s_{D-1}$ and varying from one space-time point to another. It is also consistent with the conjecture made in the course of the previous work[9,11] that all relevant low-energy processes can be described as fluctuations of the Fermi surface viewed as an extended dynamical object.

Notice that such a kind of description had already appeared in the theory of edge states of quantum Hall droplets where one treats edge states as capillary waves on the droplet boundary which plays the role of the Fermi surface in the case of a strong magnetic field.[21] It turns out that this description naturally occurs in the framework of the theory (2.6) if one considers Fermi surfaces with a sharp boundary and approximates $\tilde{w}(\vec{p}, \vec{r}, t)$ by a support function

$$\tilde{w}(\vec{p}, \vec{r}, t) \approx \theta(|\vec{k}_F(\vec{r}, t, \hat{s})| - p) \qquad (2.10)$$

which becomes appropriate in the long-wavelength limit.

One may also understand $\vec{k}_F(\vec{r}, t, \hat{s})$ as a sum $\vec{k}_F(\vec{r}, t, \hat{s}) = \vec{k}_{F_0}(\hat{s}) + \vec{\nabla}\phi(\vec{r}, t, \hat{s})$ where the first term corresponds to some reference shape of the Fermi surface (not necessarily circular) while the second one describes fluctuations around it.

Physically, the field $\phi(\vec{r}, t, \hat{s})$ has the meaning of the phase of the wave function of the wave packet created at some space-time point $x = (\vec{r}, t)$ with a momentum in the direction $\hat{s}$. Then the variable ϕ is defined modulo 2π which opens the possibility of nontrivial winding numbers along noncontractable contours on the Fermi surface (if any).

To proceed further one has to express the local density operator $\rho(\vec{r}, t)$ in terms of $\vec{k}_F(\vec{r}, t, \hat{s})$. Only for the sake of notational simplicity we shall concentrate on the 2D spinless case.

The wanted relation can be readily found on purely geometrical grounds since the total density is nothing but the area in momentum space enclosed by the curve $\vec{k}_F = \vec{k}_F(s)$ parametrized by a 2π-periodic variable s:

$$\rho = \frac{1}{2} \oint \partial_s \vec{k}_F \times \vec{k}_F \frac{ds}{(2\pi)^2}. \qquad (2.11)$$

Subtracting the constant term $\partial_s \vec{k}_{F_0} \times \vec{k}_{F_0}$ from the integrand in (2.12) we can write the remainder as

$$\rho_s = \frac{1}{2\pi}[|\partial_s \vec{k}_{F_0}|(\hat{n}_s \vec{\nabla})\phi + \tfrac{1}{2}\vec{\nabla}\partial_s \phi \times \vec{\nabla}\phi] \qquad (2.12)$$

so that $\rho(\vec{r}) = \rho_0 + \oint \frac{ds}{2\pi} \rho_s(\vec{r})$ where $\rho_0 = \frac{1}{2} \oint \frac{ds}{(2\pi)^2} \partial_s \vec{k}_{F_0} \times \vec{k}_{F_0}$ is the average density.

The quantity ρ_s associated with a point s on the Fermi surface plays a role similar to the oscillatorlike partial density operator $\sqrt{\vec{q}\vec{n}}a_{\vec{n}}(\vec{q})$ introduced in Refs. 9–11. It also follows from (2.10) that $\delta w(\vec{k}_{F_0}, \vec{q}) \approx \delta\rho_s(\vec{q}) \frac{\partial n^0(p)}{\partial \epsilon_p}$.

In the presence of some gauge vector potential $\vec{A}$ our $\vec{k}_F$ has to be modified as $\vec{k}_F \rightarrow \vec{k}_{F_0} + \vec{\nabla}\phi - \vec{A}$. This prescription is naturally consistent with the above-mentioned physical meaning of the field $\phi(\vec{r}, t, \hat{s})$ as the phase of the wave packet wave function. On switching to a scalar potential A_0 all time derivatives also become covariant: $\partial_t \phi \rightarrow D_0\phi = \partial_t \phi - A_0$. Concerning derivatives with respect to the parameter s labeling points of the Fermi surface, one can also include a new component of the gauge field A_s corresponding to a reparametrized invariance of the Fermi surface. However, in the following we shall leave this interesting possibility apart and simply fix the gauge by putting $A_s = 0$.

One can also see that the partial density ρ_s is a gauge noninvariant quantity while the total density ρ does not depend on the gauge field A_μ since $\partial_s \vec{A} = 0$ and $\oint \partial_s \vec{k} \frac{ds}{2\pi} = 0$.

The formula (2.12) allows a straightforward generalization to the case of arbitrary dimension D: $\rho = \frac{1}{D} \oint \epsilon_{\mu_1 \cdots \mu_D} \partial_{s_1} k_{\mu_1} \times \cdots \partial_{s_{D-1}} k_{\mu_{D-1}} \times k_{\mu_D}$.

The equation of motion for $\vec{k}_F(\vec{r}, t, \hat{s})$ in the presence of external electric $\vec{E} = -\partial_t \vec{A} - \vec{\nabla}A_0$ and magnetic $B = \vec{\nabla} \times \vec{A}$ fields can be derived as the Euler-Lagrange equation for the Lagrangian (2.6)

$$\partial_t w = \{H, w\}_{MB} \qquad (2.13)$$

within the approximation (2.10). In the lowest order in $\hbar$ (2.13) reads as the standard kinetic equation (in what follows we shall drop the subscript in the notation of $\vec{k}_F$):

$$\partial_t \vec{k} = (\vec{v}\vec{\nabla})\vec{k} + \vec{E} + \times \vec{v}B. \qquad (2.14)$$

To stress the parallel with the single particle equation of motion we introduced in (2.14) a generalized "Fermi velocity" $\vec{v}_s$ defined in terms of the second derivative of the Hamiltonian H with respect to $\delta\vec{k}$:

$$\epsilon_s = \frac{\delta H}{\delta\rho_s}, \qquad \delta\epsilon_s = \vec{v}_s \delta\vec{k}_s. \qquad (2.15)$$

To complete the scheme we also present a gauge invariant current [see (2.18)]

$$\vec{j} = \frac{\delta H}{\delta \vec{A}} = \times \int d\vec{r}' \oint \frac{ds'}{2\pi} f_{ss'}(\vec{r} - \vec{r}') \partial_{s'} \vec{k}_{s'}(\vec{r}') \quad (2.16)$$

satisfying the continuity equation $(\partial_t \rho + \vec{\nabla} \vec{J} = 0)$ provided the equation of motion (2.14) is fulfilled. It should be noticed that the definition (2.16) involves a spatial gradient rather than a time derivative which can be traced back to the fact that the bosonic field $\phi(\vec{r}, \hat{s})$ is chiral (those bosons propagate only in the direction $\hat{s}$). Actually, the relation (2.16) is a familiar one in the conventional Landau-Fermi-liquid theory.

Being estimated with functions $w(\vec{p}, \vec{q})$ given by (2.10) the Lagrangian (2.6) acquires the form

$$L = \int d\vec{r} \oint \frac{ds}{(2\pi)^2} [\tfrac{1}{2} D_0 \phi(\partial_s \vec{k} \times \vec{k})$$

$$+ \tfrac{1}{6} \partial_s \phi(\vec{k} \times \vec{E} - D_0 \phi B)] - H. \quad (2.17)$$

One can also check that the kinetic equation (2.14) follows from (2.17) as the Euler-Lagrange equation of motion. Following Ref. 9 we choose a simple form of the Hamiltonian which is quadratic in ρ_s,

$$H = \frac{1}{2} \int d\vec{r} \oint \frac{ds}{2\pi} \int d\vec{r}' \oint \frac{ds'}{2\pi} f_{ss'}(\vec{r} - \vec{r}') \rho_s(\vec{r}) \rho_{s'}(\vec{r}'),$$

$$(2.18)$$

where the diagonal part of the quadratic form $f_{ss'}(\vec{r} - \vec{r}') = v_F \delta(s - s') \delta(\vec{r} - \vec{r}') + \Gamma_{ss'}(\vec{r} - \vec{r}')$ includes the bare kinetic energy of fermions near the Fermi surface.

An important feature of the Lagrangian (2.17) and (2.18) is that it remains essentially non-Gaussian in terms of the fundamental field ϕ even in absence of interactions $(\Gamma_{ss'} = 0)$. Thus our theory is "geometrical" in the same sense as, say, the nonlinear σ model is.

It can be seen that the Gaussian part of (2.17) and (2.18) is essentially similar to the Lagrangian obtained in Ref. 11. Although non-Gaussian terms in (2.17) contain extra gradients they are not necessarily negligible even in the long-wavelength limit if the relation $q_t^2 \gtrsim k_F q_n$ holds between tangential and normal components of a typical transferred momentum $\vec{q}$. It follows from the preceding discussion that these terms are due to the finite geometric curvature of the Fermi surface. Being combined with quadratic terms they are supposed to reproduce the effects of the collision integral which is introduced in the phenomenological Landau theory to account for quasiparticle scattering.

Strictly speaking, the most complete account of the Fermi surface curvature also requires one to add to the Hamiltonian (2.18) terms $\sim v_F \rho_s^3$ to represent the ki-

netic energy of fermions with parabolic dispersion. It is worth recalling here that the exact bosonized theory of 1D free nonrelativistic fermions is given by the cubic bosonic Hamiltonian $H = \frac{1}{2}(\rho_R^2 + \rho_L^2) + \frac{1}{6k_F}(\rho_R^3 + \rho_L^3)$.[22] The present formalism is well suited to accommodate those cubic terms too. However, in the present paper we shall confine ourselves to the case of quadratic Hamiltonians of the form (2.18).

Also the Hamiltonian (2.18) has to be improved if one intends to include scattering processes corresponding to the Cooper channel.[10] Nevertheless, one could believe that the form (2.18) is sufficient to study the interesting case of a "strange metal" governed by interactions singular at small scattering angles.

Another feature of the Lagrangian (2.19) specific for two dimensions is the appearance of the famous Chern-Simons structure $\frac{\sigma_{xy}}{2} AdA$ where $AdA = \vec{A} \times \vec{E} - A_0 B$ with the coefficient given by the circulation (first Chern class)

$$\sigma_{xy} = \frac{1}{2} \oint \partial_s \phi \frac{ds}{(2\pi)^2}. \quad (2.19)$$

It seems natural to assume that in the case of a zero external magnetic field the winding number (2.19) must vanish so the Lagrangian (2.17) does not contain parity-odd terms.

On the contrary, one could interpret the case of the "twisted" Fermi surface characterized by the lowest nontrivial value of circulation $\oint \partial_s \phi \frac{ds}{2\pi} = 1$ giving $\sigma_{xy} = 1/4\pi$ (in absolute units of $\frac{e^2}{\hbar}$) as a proper effective description of the metal-like state at the half-filled lowest Landau level. We shall comment on this point further in Sec. IV.

Comparing the preceding discussion based on the Lagrangian formalism with the Hamiltonian approach applied in Refs. 9 and 10 one should make another remark here. According to the physical interpretation of the functional integral variable $\phi(\vec{r}, t, \hat{s})$ one may consider it in the Hamiltonian approach as an operator with fundamental equal-time commutation relations (compare with Ref. 21)

$$[\phi(\vec{r}, \hat{s}), (\vec{n}\vec{\nabla})\phi(\vec{r}', \hat{s}')] = 2\pi i \delta(\hat{s} - \hat{s}') \delta(\vec{r} - \vec{r}') \quad (2.20)$$

where $n_\mu(s) = \frac{1}{|\partial_s \vec{k}|^{D-1}} \epsilon_{\mu \nu_1 \cdots \nu_{D-1}} \partial_{s_1} k_{\nu_1} \times \cdots \times \partial_{s_{D-1}} k_{\nu_{D-1}}$ is a unit vector normal to the reference Fermi surface $\vec{k}_F = \vec{k}_{F_0}(s)$.

Notice that the commutation relations (2.20) are not canonical since the momentum variable conjugated to $\phi(\vec{r}, \hat{s})$ can be expressed as its gradient. It is consistent with the idea of the chiral nature of the boson field $\phi(\vec{r}, \hat{s})$ which obeys a first-order equation of motion.[9-11]

It is worth mentioning that our $(D > 1)$-dimensional local density operator ρ_s given by (2.12) is intrinsically nonlinear in terms of $\phi(\vec{r}, \hat{s})$. Consequently, the algebra of density operators ρ_s

$$[\rho_s(\vec{r}), \rho_{s'}(\vec{r}')] = \left\{1 + \frac{1}{2|\partial_s \vec{k}_{F_0}|}(\vec{n} \times \vec{\nabla})(\partial_s \phi - \phi \partial_s) + \frac{1}{4|\partial_s \vec{k}_{F_0}|^2}[(\partial_s \vec{\nabla}\phi)^2 + (\vec{\nabla}\phi)^2 \partial_s^2 - 2(\partial_s \vec{\nabla}\phi \vec{\nabla}\phi)\partial_s]\right\}$$

$$\times \delta(\hat{s} - \hat{s}')\frac{i(\vec{n}\vec{\nabla})}{2\pi}\delta(\vec{r} - \vec{r}') \qquad (2.21)$$

differs from the U(1) Kac-Moody algebra of oscillatorlike operators $a_{\vec{n}}(\vec{q})$ constructed in Ref. 10 by terms containing higher-order gradients.

III. DENSITY RESPONSE FUNCTION

The formalism of the preceding section enables a calculation of (gauge invariant) correlation functions such as the density response function without calculating first the (gauge noninvariant) one-particle Green functions. Keeping in the Lagrangian (2.17) only terms quadratic in ϕ one encounters the problem of diagonalization of a quadratic form. Then one obtains the density response function

$$K_0(\omega, \vec{q}) = \langle \rho(\omega, \vec{q})\rho(-\omega, -\vec{q})\rangle$$
$$= \oint \frac{ds}{2\pi} \oint \frac{ds'}{2\pi}(\vec{n}_s \vec{q})(\vec{n}_{s'}\vec{q})G^0_{ss'}(\omega, \vec{q}), \qquad (3.1)$$

where the correlator $G_{ss'}(\omega, \vec{q}) = \langle \phi_s(\omega, \vec{q})\phi_{s'}(-\omega, -\vec{q})\rangle$ is given by the inverse of the quadratic form

$$G^0_{ss'}(\omega, \vec{q}) = \{[(\vec{n}_s \vec{q})\omega - v_F(\vec{n}_s \vec{q})^2]\delta_{ss'}$$
$$- (\vec{n}_s \vec{q})\Gamma_{ss'}(q)(\vec{n}_{s'}\vec{q})\}^{-1}. \qquad (3.2)$$

In the absence of interactions and in the case of a circular Fermi surface (3.2) becomes diagonal in s space and (3.1) amounts to the long-wavelength approximation for the free fermion bubble

$$K_0(\omega, \vec{q}) = \Pi_0(\omega, \vec{q}) = \oint \frac{ds}{2\pi}\frac{(\vec{n}_s \vec{q})}{\omega - v_F(\vec{n}_s \vec{q})}. \qquad (3.3)$$

In the case of a rotationally invariant interaction $\Gamma_{ss'}(q) = V(q)$ (3.2) reproduces the results of the random-phase approximation (RPA). To see that one can simply expand the inverse operator into a series in powers of $V(q)$ which yields

$$K_{\text{RPA}}(\omega, \vec{q}) = \frac{\pi_0(\omega, q)}{1 + \Pi_0(\omega, \vec{q})V(q)}, \qquad (3.4)$$

where $\Pi_0(\omega, \vec{q})$ is given by (3.3). It is well known that in the case of long-ranged interactions the RPA compressibility $K(0, q \to 0) \sim V^{-1}(q)$ vanishes at $q \to 0$. Notice that in the Gaussian approximation equivalent to the RPA spectrum of the collective mode $\omega \sim qV^{1/2}(q)$ lying outside the particle-hole continuum remains undamped at zero temperature. However, this property of collective excitations can only hold in the case of a 1D Luttinger liquid where the dynamics of low-energy bosonic density modes is governed by an exactly quadratic Lagrangian.[23]

To proceed beyond the RPA one has to consider the nonlinear equation of motion (2.14) written in terms of $\phi(x)$,

$$\partial_t \phi_s(x) + \oint \frac{ds'}{(2\pi)}f_{ss'}(x - x')$$
$$\times [\vec{n}_{s'} + \tfrac{1}{2}\vec{\nabla}\partial_{s'}\phi_{s'}(x') \times]\vec{\nabla}\phi_{s'}(x') = 0. \qquad (3.5)$$

To compute the response function $G_{ss'}$ corresponding to Eq. (3.5) we apply a nonperturbative eikonal-type method similar to the one used in the context of the Navier-Stokes equation.[24] Similarly to the case of the one-particle fermion Green function studied in Ref. 16 the use of the eikonal method becomes possible due to the presence of a large term in (3.5) which contains a bare Fermi velocity v_F. Analogously to the case of advection of a passive scalar in the theory of turbulence[25] the method provides a consistent summation of infrared relevant terms in perturbation theory for the Lagrangian (2.17).

Using (3.5) one arrives at the equation for the Fourier transform $G(q, x) = \int(dq)e^{iq(x-x')}G(x, x')$ of the (translationally noninvariant) response function in an external field $\phi(x)$,

$$(\vec{n}_s \vec{\nabla})\left[\partial_t G_{ss'}(q, x)\right.$$
$$- \int(dp) \oint \frac{ds''}{(2\pi)}f_{ss''}(\vec{q} - \vec{p})e^{ipx}$$
$$\left.\times [\delta(p)\vec{n}_{s''} + \tfrac{1}{2}\vec{p}\partial_{s''}\phi_{s''}(p) \times]G_{s''s'}(q, x)\right] = \delta_{ss'}. \qquad (3.6)$$

According to Ref. 24 the response function can be written in Fradkin's integral form

$$G_{ss'}(q, x) = i\int_0^\infty d\nu \oint \frac{ds''}{2\pi}\langle s|e^{i\nu G_0^{-1}(q)}|s''\rangle$$
$$\times \langle s''|\exp[i\Psi(q, x)]|s'\rangle, \qquad (3.7)$$

where $G^0_{ss''}$ is given by (3.2). Then keeping the term of lowest order in $\vec{q}$ one obtains

$$\langle s|\Psi(q, x)|s'\rangle$$
$$= i\int_0^\nu d\nu' \int(dk)e^{ikx} \oint \frac{ds''}{2\pi}f_{ss''}(q - k)(\vec{k} \times \vec{q})$$
$$\times \partial_{s''}\phi_{s''}(k)\langle s''|e^{i\nu'G_0^{-1}(k)}|s'\rangle, \qquad (3.8)$$

where $(dk) = \frac{d\Omega d^2\vec{k}}{(2\pi)^3}$. All terms of higher orders in $\vec{q}$ (or, a typical scattering angle $\vartheta \sim q/k_F$) can be found by recursion.[16]

The consistency condition requires one to average (3.7) over fluctuations of $\phi(x)$. Then for the response function

$$G_{ss'}(q) = \langle \phi_s(q)\phi_{s'}(-q)\rangle = \int \mathcal{D}\phi G[q|\phi(x)]e^{\frac{i}{2}\int \phi K \phi}$$

we get the equation

$$G_{ss'}(q) = i \int_0^\infty d\nu \langle s|e^{i\nu G_0^{-1}(q)}|s'\rangle \exp\Big(i \int (dk) \oint_{s_1} (\vec{q}\,\vec{n}_s) \oint_{s_2} (\vec{q}\,\vec{n}_{s'})(\vec{k} \times \vec{q})^2 f_{ss_1}(q-k)f_{s's_2}(k-q)$$

$$\times \partial_{s_1}\partial_{s_2}G_{s_1 s_2}(k)\langle s_1|[(e^{i\nu' G_0^{-1}(k)} - 1)G_0^2(k) - i\nu' G_0(k)]|s_2\rangle\Big). \tag{3.9}$$

This equation can be treated using various approximations which are supposed to give corrections to the RPA expression (3.2). In particular, one could expect to find in this way a damping of collective excitations absent in the Gaussian theory. The simplest approximation would be to substitute $G_{s_1 s_2}(k)$ in the exponent in (3.9) by G_0 given by the formula (3.2).

It is also worth comparing Eq. (3.9) with the eikonal formula for $K(\omega, \vec{q})$ obtained in the original fermion representation:[16]

$$K(\omega, \vec{q}) = \int d^2\vec{p}\, n_{\vec{p}}^{(0)} \int_0^\infty d\nu\, e^{i\nu(\omega + \xi_{\vec{p}} - \xi_{\vec{p}+\vec{q}} + i\delta)} \exp\Big(i \int (dk)V(k)\Big[(1 - n_{\vec{p}+\vec{q}+\vec{k}}^{(0)})$$

$$\times \frac{1 - e^{i\nu(\xi_{\vec{p}+\vec{q}} - \xi_{\vec{p}+\vec{q}+\vec{k}} + \Omega)}}{\xi_{\vec{p}+\vec{q}} - \xi_{\vec{p}+\vec{q}+\vec{k}} + \Omega} - n_{\vec{p}+\vec{k}}^{(0)} \frac{1 - e^{i\nu(\xi_{\vec{p}} - \xi_{\vec{p}+\vec{k}} + \Omega)}}{\xi_{\vec{p}} - \xi_{\vec{p}+\vec{k}} + \Omega}\Big]^2\Big) + (\omega \to -\omega), \tag{3.10}$$

where $\xi(p)$ is an (in general, nonlinear) fermion dispersion. An obvious advantage of this formula as compared to (3.9) is that nonlinear terms in the fermion dispersion are already accounted for in (3.10). However, to see the effect of the Ward identities which guarantee a cancellation of self-energy versus vertex corrections in (3.10) one has to treat carefully a combination of three terms forming a complete square in the exponential factor. On the other hand, it automatically follows from the appearance of extra powers of q in the corresponding exponential factor in (3.9), as it should in a consistent hydrodynamics of interacting fermions.

IV. TOWARDS (NON-)FERMI-LIQUID THEORY OF THE HALF-FILLED LANDAU LEVEL

There exists much experimental evidence in favor of the compressible metal-like state at $\nu = 1/2$.[26] Experiments on geometric resonance in the antidot array as well as magnetic focusing show convincingly that quasiparticle excitations at half filling experience no magnetic field.

A theoretical explanation of the phenomenon proposed in Ref. 27 was based on Jain's idea of attaching a pair of fictitious flux quanta to each electron to compensate an external magnetic field.[28] Moreover, it was suggested to treat fractional quantum Hall effect (FQHE) states belonging to the sequence of fractions $\nu = \frac{N}{2N+1}$ converging towards $\nu = 1/2$ as integer quantum Hall effect (IQHE) states of quasiparticles occupying N Landau levels in the net field $B_{\text{eff}} = \frac{1}{2N+1}B_{\text{ext}}$.

In the framework of this picture the oscillating magnetoresistivity $\Delta R_{xx}(B, T)$ at $\nu \neq 1/2$, for example, can be treated as Shubnikov–de Haas (SdH) oscillations in the system of quasiparticles with some effective mass m^* placed in the field B_{eff}. In the physically relevant case of unscreened Coulomb interaction the theory[27] predicts, in particular, a weak (logarithmic) divergence of $m^* \sim \ln|\delta\nu|$ as $\delta\nu = |\nu - 1/2|$ approaches zero.

An extensive study of the effects of the transverse statistical gauge interaction carried out in Ref. 13 led to the conclusion that in many respects the system looks like a Fermi liquid except for the "marginal" change $\epsilon \to \Sigma(\epsilon) \sim \epsilon \ln \epsilon$ in its one-particle Green function. Another different feature is an extremely weak divergency of the $2k_F$ scattering amplitude[13] which presumably does not lead to a divergency of any physical polarizability.

These results imply that, in spite of a strong lowest-order renormalization of the (gauge noninvariant) fermion propagator, transverse gauge fluctuations have only a little effect on physical observables. In addition to the results of the self-consistent diagrammatic approach of Ref. 13 this conclusion was confirmed by a straightforward two-loop calculation of irreducible density and current polarizabilities which showed no sign of a divergent effective mass either.[29]

On the other hand, as opposed to earlier reported results, the most recent measurements of magnetoresistivity at relatively high temperatures on both electronlike[30] and holelike[31] systems revealed a strong dependence of m^* on ν extracted from the Fermi-liquid formula for the first harmonics of SdH oscillations,

$$\Delta R_{xx}(B, T) \sim \frac{Tm^*/B_{\text{eff}}}{\sinh(Tm^*/B_{\text{eff}})}e^{-m^*/B_{\text{eff}}\tau};$$

namely, the results obtained in Ref. 31 were best fitted by the function

$$m^*(\delta\nu) \sim \exp|\delta\nu|^{-3/2}$$

while the authors of Ref. 30 found a rather strong power-law behavior of $m^*(\delta\nu)$.

When contrasting these experimental observations with the theoretical conclusions[27,13] it seems that a strong dependence of the effective mass $m^*(\delta\nu)$ does not find an immediate explanation in the framework of the "marginal" Fermi-liquid behavior. Although one should be cautious about applying Fermi-liquid formulas to the analysis of data obtained in Refs. 30 and 31 (see, how-

ever, Ref. 32) it nevertheless gives enough motivation to search for an alternative description which uses no spurious gauge field at all (or, equivalently, the statistical gauge field is integrated out exactly) for an independent check of predictions of the gauge theory. It is quite likely that a possible candidate could be a sort of "nonlinear Landau-Fermi-liquid theory" which one can use to understand the drastic effective mass dependence on ν (for a related discussion see Ref. 33).

To make an attempt in this direction we propose the Lagrangian describing the $\nu = 1/2$ state in the form (2.19) where A_μ is now a sum of the statistical gauge field a_μ and the external electromagnetic potential $\vec{A}_{\text{ext}} = \frac{B_{\text{ext}}}{2}(y, -x)$. As we mentioned above this requires the field ϕ to have a nonzero winding number $\oint \frac{ds}{(2\pi)^2} \partial_s \phi = 1$.

Notice that, in comparison with the conventional Landau-Ginzburg-type description of the odd-denominator FQHE states given in terms of the phase field $\phi(\vec{r}, t)$, the conjectured theory of the even-denominator states involves an extra variable s along the boundary of an extended region in momentum space identified with the bare Fermi surface of transformed ("neutral")

fermions.

Varying the Lagrangian (2.17) with respect to a_0 one obtains an intrinsic operator relation between local fermion density and statistical flux,

$$\frac{1}{(2\pi)^2} \oint ds (\partial_s \phi \vec{\nabla} \times \vec{a} - \partial_s \vec{k} \times \vec{k}) = 0 \ . \tag{4.1}$$

Due to the local constraint (4.1) the external field $\vec{A}_{\text{ext}}$ is canceled out by the averaged flux and the bare Lagrangian of gauge field fluctuations is given by the Chern-Simons term and by the pairwise interaction $V(\vec{r} - \vec{r}')$ rewritten in terms of $\vec{a}$:

$$L_g = \frac{1}{2} \int d\vec{r} d\vec{r}' (\vec{\nabla} \times \vec{a})_{\vec{r}} V(\vec{r} - \vec{r}') (\vec{\nabla} \times \vec{a})_{\vec{r}'}$$

$$+ \frac{1}{8\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \ . \tag{4.2}$$

Intending to deal with gauge invariant quantities one can choose the gauge $\vec{\nabla} \vec{a} = 0$ and then integrate a_μ out to end up with the effective Lagrangian written solely in terms of densities,

$$L_{\text{eff}} = \oint \frac{ds}{4\pi^2} \rho_s \partial_t \phi_s - \frac{1}{2} \oint \frac{ds}{2\pi} \oint \frac{ds'}{2\pi} \left(\rho_s(\vec{r}) V(\vec{r} - \vec{r}') \rho_{s'}(\vec{r}') + i \vec{\nabla} \times \vec{j}_s(\vec{r}) \left\langle \vec{r} \left| \frac{1}{\vec{\nabla}^2} \right| \vec{r}' \right\rangle \rho_{s'}(\vec{r}') \right) , \tag{4.3}$$

where ρ_s and $\vec{j}_s$ are given by (2.12) and (2.16), respectively, and the induced ρ-j coupling is due to the statistical Chern-Simons interaction contained in (2.17).

It is worth mentioning here the possibility of an independent microscopic check of the validity of the effective theory (4.3); namely, at $\nu = 1/2$ one can choose to work with a basis of coherent states of $\mathcal{N}$ fermions on the lowest Landau level labeled by a set of $\mathcal{N}$ two-dimensional momenta $\{\vec{k}_i\}$:[34]

$$|\{\vec{k}\}\rangle = \det e^{i\vec{k}_i \vec{R}_j} \prod_{i<j} (z_i - z_j)^2 e^{-\frac{1}{4}|z_i|^2} , \tag{4.4}$$

where $\vec{R}_i$ is the center of Larmor's orbit operator of the ith electron. This choice of the basis is not accidental. The conventional (in our case symmetrical) Jastrow factor in (4.4) takes care of short-range correlations and provides a good variational energy. Due to the antisymmetry of the entire wave function all $\vec{k}_i$ have to be distinct; therefore the other factor is the Slater determinant of $\exp(i\vec{R}_i \vec{k}_j)$ which reflects an alleged metal-like behavior governed by long-range correlations. Then using a kind of collective field approximation[21] one can describe different patterns of occupied $\vec{k}_j$ states in terms of the Fermi momentum tracing the boundary of the filled region in $\vec{k}$ space.[34]

However, the basis of coherent states (4.4) is not, of course, orthonormal. One might expect that it is the

overlap between different states which causes the induced ρ-j coupling in the effective Lagrangian

$$L = \frac{\langle \{\vec{k}\} | (i\partial_t - H | \{\vec{k}\} \rangle}{\langle \{\vec{k}\} | \{\vec{k}\} \rangle} \ . \tag{4.5}$$

An explicit calculation of (4.5) which will provide a decisive check of the status of (4.3) remains to be done.

From (4.3) one reads off the bare interaction kernel in the form

$$\Gamma_{ss'}(q) = V(q) + \frac{v_F}{q^2} \vec{q} \times (\vec{n}_s - \vec{n}_{s'}) \ . \tag{4.6}$$

Similarly to the case of pure density-density interactions, a diagonalization procedure applied to the quadratic form with $\Gamma_{ss'}$ given by (4.6) leads to the RPA density and current response functions. In particular, (3.2) now yields

$$K_{\text{RPA}}(\omega, \vec{q}) = \frac{q^2 \Pi_0(\omega, \vec{q})}{k_F \Pi_0(\omega, \vec{q})[k_F \Pi_\perp(\omega, \vec{q}) + V(q)q^2] + q^2} , \tag{4.7}$$

where $\Pi_0(\omega, \vec{q})$ is given by (3.3) and the long-wavelength approximation for the current polarization operator is

$$\Pi_\perp(\omega, \vec{q}) = v_F^2 \oint \frac{ds}{2\pi} \frac{(\vec{n}_s \vec{q})(\vec{n}_s \times \vec{q})^2}{q^2 [\omega - v_F(\vec{n}_s \vec{q})]} - \frac{1}{2} v_F \ . \tag{4.8}$$

Note that the crossed current-density polarization oper-

ator $\Pi_H(\omega, \vec{q}) \sim \oint \frac{ds}{2\pi} \frac{(\vec{n}_s \vec{q})(\vec{n}_s \times \vec{q})}{q^2[\omega - v_F(\vec{n}_s \vec{q})]}$ vanishes in this approximation.

Substituting the well known asymptotics of free fermion polarizations $\Pi_{0,\perp}(\omega, \vec{q})$ at $\omega \ll v_F q$

$$\Pi_0(\omega, \vec{q}) = \frac{1}{2\pi v_F}\left(1 + i\frac{\omega}{v_F q}\right),$$

$$\Pi_\perp(\omega, \vec{q}) = \frac{v_F}{2\pi}\left(\frac{q^2}{k_F^2} + i\frac{\omega}{v_F q}\right) \qquad (4.9)$$

into (4.7) one readily reproduces the RPA compressibility $K_{\mathrm{RPA}}(0, \vec{q} \to 0) \sim 1/V(q)$ and the location of the pole of (4.7) for small q at $\omega \sim iV(q)q^3$ in agreement with Ref. 27. Notice that similar results can be obtained from a direct solution of the eigenvalue problem in the case of the induced current-current interaction[16]

$$(\vec{n}_s \vec{q})[\omega - v_F(\vec{n}_s \vec{q})]\phi_s$$

$$= \frac{\Gamma_\perp(q)}{q^2}(\vec{n}_s \times \vec{q}) \oint \frac{ds'}{2\pi}(\vec{n}_{s'} \times \vec{q})\phi_{s'}, \qquad (4.10)$$

where $\Gamma_\perp(\omega, \vec{q}) = \langle(\vec{\nabla} \times \vec{a})(\vec{\nabla} \times \vec{a})\rangle = (4\pi)^2 K_{\mathrm{RPA}}(\omega, \vec{q})$.

We plan to undertake an analysis of corrections to the RPA using the eikonal-type formula (3.9) in a separate paper.[17] Although we do not expect that at small ω, q the corrections to (4.7) alter the behavior found in the RPA, one might think that RPA becomes insufficient when calculating the $2k_F$ response (particularly if the RPA result contains a possible divergency[13]).

One remark is in order here. In the case of a zero q but finite ω the function $K(\omega, 0)$ has a pole located at the renormalized cyclotron frequency $\omega_c^* = B/m^*$ while according to the Kohn theorem it would have to occur at the bare one. It was proposed in Ref. 12 to improve this point by adding an extra Fermi-liquid interaction $f_{ss'} \sim F_1(\vec{n}_s \vec{n}_{s'})$ to the Chern-Simons gauge theory which allows one to restore the bare mass $\frac{1}{m_0} = \frac{1}{m^*} + F_1$ while using the RPA. However, it still does not seem to reflect the fact that by switching the interaction off one completely eliminates quasiparticle dispersion ($v_F \sim \frac{1}{m^*} \to 0$).

Moreover, this recipe may appear to lead to a double counting of the effects of the original interaction $V(q)$ if one goes beyond the RPA. To this end, it is conceivable that studying the problem (4.3) at small energies one should put the bare mass m_0 in (4.6) equal to infinity for consistency and assume that the kernel $\Gamma_{ss'}$ does have higher angular harmonics in contrast to the naive $\Gamma_{ss'}^0(q) = V(q)$ which is a pure s wave.

V. CONCLUSIONS

In the present paper we discussed a scheme of $D > 1$ bosonization of interacting fermions in external fields which is based on a geometrical picture of a fluctuating Fermi surface. The natural description can be done in terms of the vector field $\vec{k}_F(\vec{r}, \hat{s})$ tracing the space-time-dependent shape of the Fermi surface. We argue that it arises in a long-wavelength approximation for the formally exact bosonization scheme in terms of coherent states on quantum phase space.[19,20] Conceptually, this sort of description can be viewed as a generalization of the phenomenological Landau theory which leads to a quantum hydrodynamics incorporating small-angle scattering between quasiparticles around a nonflat Fermi surface.

In contrast to the constructive approach of Refs. 9–11 leading to a Gaussian bosonic theory the present method gives an intrinsically nonlinear one. It is this property which makes it possible to account for the effects of the Fermi surface curvature.

It should be mentioned here that in special cases a bare Fermi surface having flat faces may preserve them under renormalization. For instance, it was argued in Ref. 35 that a square Fermi surface is stable with respect to parallel face interactions. However, interactions between adjacent faces are likely to cause rounding of the square Fermi surface.[35]

The bosonic formalism also allows one to avoid the subtle problem of an explicit representation of the fermion operator in terms of bosons when calculating gauge invariant response functions.

A simple diagonalization of the quadratic form while neglecting nonlinear terms gives the usual RPA results. To go beyond the RPA we formulate an eikonal-type procedure leading to integral equations for the density response function which can be solved iteratively. In particular, one might expect to obtain damping of collective modes out of these equations.

In addition, we apply our approach to the compressible state of the half-filled lowest Landau level and formulate a nonlinear effective theory in terms of chiral bosons representing density fluctuations. In the Gaussian approximation the theory reproduces the results of the RPA in the gauge theory of the $\nu = 1/2$ state.[27] We intend to study effects of nonlinear terms on RPA results elsewhere.

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[1] P.W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); **65**, 2306 (1990); **66**, 3226 (1991).

[2] P.C. E. Stamp, Phys. Rev. Lett. **68**, 2180 (1992); J. Phys. (France) I **3**, 625 (1993).

[3] P. Bares and X.G. Wen, Phys. Rev. B **48**, 8636 (1993).

[4] D.V. Khveshchenko, R. Hlubina, and T.M. Rice, Phys. Rev. B **48**, 10 766 (1993).

[5] T. Holstein, R.E. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973); C.J. Pethick, G. Baym, and H. Monien, Nucl. Phys. **A498**, 313c (1989); M.Y. Reiser, Phys. Rev. B **39**,

1609 (1989); **40**, 11 571 (1989).

[6] R. Shankar, Physica A **177**, 530 (1991); Rev. Mod. Phys. **68**, 129 (1994).

[7] A. Luther, Phys. Rev. B **19**, 320 (1979).

[8] P.W. Anderson, Phys. Rev. Lett. **67**, 2092 (1991).

[9] F.D.M. Haldane, *Varenna lectures, Proceedings 1992*, edited by R. Schrieffer and R.A. Broglia (North-Holland, Amsterdam, 1994).

[10] A. Houghton and J.B. Marston, Phys. Rev. B **48**, 7790 (1993); A. Houghton, H.J. Kwon, and J.B. Marston, *ibid.* **50**, 1351 (1994).

[11] A.H. Castro Neto and E. Fradkin, Phys. Rev. Lett. **72**, 1393 (1994); Phys. Rev. B **49**, 10877 (1994).

[12] H.J. Kwon, A. Houghton, and J.B. Marston, Phys. Rev. Lett. **73**, 284 (1994).

[13] L.B. Ioffe, D. Lidsky, and B.L. Altshuler, Phys. Rev. Lett. **73**, 472 (1994); B.L. Altshuler, L.B. Ioffe, and A. Millis, Phys. Rev. B **50**, 14 048 (1994).

[14] C. Castellani, C. Di Castro, and W. Metzner, Phys. Rev. Lett. **72**, 316 (1994).

[15] J. Polchinski, Nucl. Phys. **B422**, 617 (1994).

[16] D.V. Khveshchenko and P.C.E. Stamp, Phys. Rev. Lett. **71**, 2118 (1993); Phys. Rev. B **49**, 5227 (1994).

[17] D.V. Khveshchenko (unpublished).

[18] B. Kostant, *Lectures in Modern Analysis and Applications*, Lecture Notes in Mathematics Vol. 170 (Springer-Verlag, Berlin, 1970), p. 87; A.A. Kirillov, *Elements of the Theory of Representations* (Springer-Verlag, Berlin, 1978); A. Perelomov, *Generalized Coherent States and their Applications* (Springer-Verlag, Berlin, 1986).

[19] S.R. Das, A. Dhar, G. Mandal, and S.R. Wadia, Mod. Phys. Lett. A **7**, 71 (1992); A. Dhar, G. Mandal, and S.R. Wadia, *ibid.* **7**, 3129 (1992).

[20] D.V. Khveshchenko, Phys. Rev. B **49**, 16 893 (1994).

[21] S. Iso, D. Karabali, and B. Sakita, Nucl. Phys. **B388**, 700 (1992); Phys. Lett. B **296**, 143 (1992).

[22] J. Avan and A. Jevicki, Phys. Lett. B **266**, 35 (1991).

[23] I.E. Dzyaloshinski and A.I. Larkin, Zh. Eksp. Teor. Fiz. **65**, 411 (1973) [Sov. Phys. JETP **38**, 202 (1974)].

[24] H.M. Fried and J. Tessendorf, J. Math. Phys. **25**, 1144 (1984).

[25] B.I. Shraiman and E.D. Siggia, Phys. Rev. E **49**, 2912 (1994).

[26] R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **70**, 2944 (1993); Solid State Commun. **90**, 71 (1994); D. Leadley, R. Nicholas, C. Foxon, and J. Harris, Phys. Rev. Lett. **72**, 1906 (1994); V. Goldman, B. Su, and J. Jain, *ibid.* **72**, 2065 (1994).

[27] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[28] J. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Rev. B **40**, 8079 (1989); **41**, 7653 (1990).

[29] Y.B. Kim, A. Furusaki, X.G. Wen, and P.A. Lee, Phys. Rev. B **50**, 17 917 (1994).

[30] R.R. Du, H.L. Stormer, D.C. Tsui, A. Yeh, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **73**, 3274 (1994).

[31] H.C. Manoharan, M. Shayegan, and S.J. Klepper, Phys. Rev. Lett. **73**, 3270 (1994).

[32] F. Pelzer, Phys. Rev. B **44**, 293 (1991).

[33] S.H. Simon and B.I. Halperin, Phys. Rev. B **48**, 17 368 (1993).

[34] F.D.M. Haldane (unpublished).

[35] A. Luther (unpublished).