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Piezoelectric electron-phonon interaction in impure semiconductors: Two-dimensional electrons versus composite fermions

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We study the effects of piezoelectric coupling between two-dimensional electrons and bulk phonons in cases of both zero and strong perpendicular magnetic fields, the latter corresponding to even-denominator filling fractions. In contrast to the case of coupling via the deformation potential, the leading contributions due to impurity-renormalized electron-phonon vertices are not exactly canceled by processes of inelastic electron-impurity scattering. Electron energy relaxation time, diffusion correction to the conductivity, and the phonon emission rate by hot electrons are computed for realistic GaAs/Al$_x$Ga$_{1-x}$As heterostructures. [S0163-1829(97)07147-6]

It is well known[1] that in impure metals the Coulomb interaction is drastically enhanced as a result of a singular renormalization of interaction vertices in the diffusive regime of small momentum and energy transfers ($q\ell<1$ and $\omega\tau<1$, where τ is the electron momentum relaxation time due to elastic electron-impurity scattering, and $\ell=v_F\tau$ is the electron mean free path). Earlier analysis of the effects of the deformation potential in impure metals[2] was based on the assumption that the electron-phonon interaction undergoes the same renormalization. It turned out, however, that the situation is essentially more complicated. It was shown in Ref. 3 that screening of Coulomb ionic and impurity potentials together with inelastic electron-impurity scattering leads to a precise cancellation of the impurity renormalization of the electron-phonon vertex. The underlying physics of this cancellation is a local electroneutrality of the system.

In this paper we reexamine the problem of piezoelectric electron-phonon interactions in crystalline semiconductors without an inversion center, such as GaAs. In piezoelectric crystals an elastic strain is accompanied by a macroscopic electric field, and the electron-phonon interaction results from coupling between electrons and the lattice polarization caused by the strain. However, the electroneutrality of the electron-ion system does not affect piezoelectric coupling, so one could expect its effects to be different from those of the deformation potential, particularly in the long-wavelength limit.

In a metal, inelastic electron-impurity scattering is determined by the same impurity potential $V_{e\text{-imp}}$ which stems from a local charge disturbance. In the case of a piezoelectric crystal, however, there is another potential source of inelastic scattering processes. That is, if the size of impurity atoms is different from that of host ones, then the lattice strain caused by this difference gives rise to a local lattice polarization. The latter results in an additional inelastic electron-impurity scattering which may become important in the long-wavelength limit.

One can argue, however, that in Si-doped GaAs/Al$_x$Ga$_{1-x}$As heterostructures, the inelastic electron-impurity scattering associated with the piezoelectric coupling can be safely ignored. Indeed, the covalent sizes of Al, Ga, As, and Si atoms are very close (1.18, 1.26, 1.20, and 1.19 Å, respectively). Therefore an additional perturbation induced by the impurity size appears to be much smaller than the effect of its Coulomb potential. Hence inelastic piezoelectric electron-impurity scattering is negligible compared to the elastic (Coulomb) one.

This important conclusion had been used implicitly in a recent theory of kinetic phenomena associated with piezoelectric electron-phonon interactions in the two-dimensional electron gas in a quantizing perpendicular magnetic field corresponding to filling factors equal to some of the even denominator fractions (EDF's) $\nu=n\pm 1/2q$.[4] The effective coupling of electrons to acoustic phonons at EDF's was derived in Ref. 4 by employing the composite fermion theory by Halperin, Lee, and Read.[5]

In Ref. 4 we found the temperature dependencies of a number of experimentally measured quantities, such as the phonon contribution to electronic mobility, phonon-drag thermopower, and phonon emission rate by hot electrons, which all appeared to be in qualitative agreement with the available experimental data. In the present paper we also calculate the electron energy relaxation time, the correction to the diffusion conductivity, and the hot-electron energy-loss rate for both cases of zero magnetic field and for EDF's. For the latter quantity, which was measured directly,[6] an agreement is found.

ELECTRON ENERGY RELAXATION TIME

In what follows we consider the range of temperatures below a few K, where the only important coupling is the piezoelectric one, and treat phonons as bulk acoustic modes coupled to a local electronic density by virtue of the bare vertex[7]

$$M_\lambda^{(0)}(\mathbf{Q})=eh_{14}(A_\lambda/2\rho u_\lambda Q)^{1/2},\quad A_l=\frac{9q_z^2q^4}{2Q^6},$$

$$A_{\rm tr}=\frac{8q_z^4q^2+q^6}{4Q^6} \tag{1}$$

where $\mathbf{Q}=(\mathbf{q},q_z)$ is the three-dimensional (3D) phonon momentum, ρ is the bulk mass density of GaAs, u_λ is a longitudinal (l) or a transverse (tr) sound velocity, and h_{14} is the only nonzero component of the piezoelectric tensor.

As a result of the Coulomb interaction in the two-dimensional electron gas (2DEG) the vertex (1) undergoes a dynamical screening which appears to be important at practically all scales,[8]

$$M_\lambda(\omega,\mathbf{Q})=\frac{M_\lambda^{(0)}(\mathbf{Q})}{\varepsilon(\omega,q)},\tag{2}$$

where $\varepsilon(\omega,q)=1+H(q)(2\pi e^2/\varepsilon_0 q)P_{00}^R(\omega,q)$ is the 2DEG dielectric function which depends on the (retarded) 2D scalar polarization $P_{00}^R(\omega,q)$ and the form factor of the quantum well $H(q)=\int\int dz dz'\xi^2(z)\xi^2(z')e^{-q|z-z'|}$ given in terms of the wave function of the lowest occupied quantum well subband $\xi(z)\sim ze^{-z/w}$.

In the diffusive regime $q\ell<1$, the dielectric function is given by the standard formula

$$\epsilon(\omega,q)=1+\frac{D\kappa q}{i\omega+Dq^2},\tag{3}$$

where $D=v_F^2\tau/2$ is the diffusion coefficient and $\kappa=2\pi e^2\nu_F/\varepsilon_0$ is the 2D Debye wave vector proportional to the two-spin electron density of states at the Fermi level $\nu_F=m/\pi$.

As follows from the above discussion, in the diffusive regime one should also renormalize the vertex (2) by an impurity ladder.[2] The kinetic equation which describes electron energy relaxation and accounts for such a renormalization was derived in Refs. 2 and 3. Generalizing it onto the case of different electronic (T) and lattice (Θ) temperatures, we obtain

$$\frac{\partial n(\epsilon)}{\partial t}=-\frac{1}{(2\pi)^4}\sum_\lambda\int d^3\mathbf{Q}\int d\omega|F(q_z)|^2|M_\lambda(\omega,\mathbf{Q})|^2 \times \mathrm{Im}D^R(\omega,\mathbf{Q})\mathrm{Re}\frac{1}{i\omega+Dq^2}R_{T,\Theta}(\epsilon,\omega),\tag{4}$$

where $R_{T,\Theta}(\epsilon,\omega)=\coth(\omega/2\Theta)[S(\epsilon+\omega)-S(\epsilon)]-S(\epsilon+\omega)S(\epsilon)+1$, $S(\epsilon)=2n(\epsilon)-1$, and $F(q_z)=\int dz\ e^{izq_z}\xi^2(z)$.

For thermal acoustic phonons with a dispersion $\Omega_\lambda(Q)=u_\lambda Q$, one can use the customary expression for the phonon propagator $\mathrm{Im}D^R(\omega,Q)=-\pi(\delta[\omega-\Omega_\lambda(Q)]-\delta[\omega+\Omega_\lambda(Q)])$.

Putting $T=\Theta$ and using the equilibrium form of the electron distribution function $S(\epsilon)=-\tanh(\epsilon/2T)$, we arrive at the following expression for the energy relaxation time:

$$\begin{aligned}1/\tau_\epsilon(T)&=-\frac{\delta}{\delta n(\epsilon)}\left.\frac{\partial n(\epsilon)}{\partial t}\right|_{\epsilon=0}\\&=\frac{2}{(2\pi)^3}\sum_\lambda\int d^3\mathbf{Q}|F(q_z)|^2|M_\lambda[\Omega_\lambda(Q),\mathbf{Q}]|^2\\&\quad\times\mathrm{Re}\frac{1}{i\Omega_\lambda(\mathbf{Q})+Dq^2}\{\coth[\Omega_\lambda(\mathbf{Q})/2T]\\&\quad-\tanh[\Omega_\lambda(\mathbf{Q})/2T]\}.\end{aligned}\tag{5}$$

This formula is valid at temperatures below $T_1=u/\ell$ which correspond to $q\ell\sim 1$, and characterizes the onset of the hydrodynamical regime (throughout this paper we neglect the difference between u_l and u_{tr}, while making rough estimates).

In typically used high-mobility samples ($\mu\sim 10^6$ cm^2/V s) with sheet electron concentration $n_e\sim 10^{11}$ cm^{-2} the applicability of Eq. (5) is limited to the regime of extremely low temperatures ($T<T_1\sim 5$ mK).

In this temperature range, Eq. (5) yields

$$1/\tau_\epsilon(T)=C_3\frac{(eh_{14})^2}{2\rho u_l^3}\frac{T^2}{D\kappa^2},\tag{6}$$

where the numerical factor $C_3=\frac{1}{4}\int_0^1 dx(9x^2(1-x^2)^2+(u_l/u_t)^3[8x^4(1-x^2)+(1-x^2)^3])\approx 1.35$ receives its main contribution from two transverse phonon modes which are slower than the longitudinal one ($u_l/u_{\mathrm{tr}}\approx 1.73$).

Since the width of the quantum well $w\sim(\kappa k_F^2)^{-1/3}$ is typically smaller than or of the order of $1/k_F$, the small-q response does not probe the structure of $\xi(z)$, and one can set the form factors $H(q)$ and $F(q_z)$ equal to unity.

We note, in passing, that in contrast to the conclusion drawn in Ref. 2, where the case of the unscreened deformation potential was considered, the temperature dependence of τ_ϵ given by Eq. (5) continues all the way down to $T=0$, and undergoes no additional crossover at $T\sim T_2=u^2/D$ defined in Ref. 2 from the condition $Dq^2\sim\Omega_\lambda(q)$. On the contrary, in our calculation of τ_ϵ given by Eq. (5), this condition does not appear at all, since the factor $(i\omega+Dq^2)^{-1}$ resulting from the impurity vertex correction is exactly canceled by an identical factor contained in $M_\lambda[\Omega_\lambda(Q),\mathbf{Q}]$ [see Eqs. (2) and (3)].

For comparison, in the clean regime $T_1<T<T_D=2uk_F\sim 10$ K (the latter condition facilitates that both in-plane $\mathbf{q}$ and out-of-plane q_z components of the phonon momentum $\mathbf{Q}$ are controlled by temperature rather than by the width of the quantum well w), we obtain the well-known result[7]

$$1/\tau_\epsilon(T)\sim\frac{(eh_{14})^2}{\rho u^4}\frac{T^3}{v_F\kappa^2}.\tag{7}$$

Equation (4) can be also used to determine the rate of energy transfer from the 2DEG to the lattice in the case of so-called ‘‘hot’’ electrons, with the effective temperature $T>\Theta$:

$$\begin{aligned}P(T,\Theta)&=-\int\epsilon\nu_F\frac{\partial n(\epsilon)}{\partial t}d\epsilon\\&=\sum_\lambda\int d^3\mathbf{Q}\ \Omega_\lambda(Q)\frac{\partial N[\Omega_\lambda(Q)]}{\partial t}.\end{aligned}\tag{8}$$

The detailed derivation of the kinetic equation for the phonon distribution function $N(\omega)$ can be found in Ref. 3.

In the strongly disordered regime, we obtain the result

$$P=-\frac{C_3}{\pi^2}\frac{(eh_{14})^2}{2\rho u_l^3}\frac{\nu}{D\kappa^2}\int_0^\infty\omega\ d\omega\int_{-\infty}^\infty\epsilon\ d\epsilon\ R_{T,\Theta}(\epsilon,\omega).\tag{9}$$

The frequency integral

$$\int_0^\infty \omega\, d\omega \int_{-\infty}^{\infty} \epsilon\, d\epsilon\, R_{T,\Theta}(\epsilon,\omega)$$

$$=2\int_0^\infty [N(\omega/2\Theta)-N(\omega/2T)]\omega^3 d\omega$$

$$=(2\pi^4/15)(\Theta^4-T^4)$$

yields an unusual behavior of the electron-energy-loss rate $P(T,\Theta)\sim T^4-\Theta^4$ discussed in Ref. 6, whereas in the clean limit $P(T,\Theta)$ follows the standard dependence ($\sim T^5-\Theta^5$).

As a reciprocal effect of the electron-phonon coupling, the impurity-renormalized matrix element (2) leads to attenuation of long-wavelength phonons:

$$1/\tau_{ph,\lambda}(u_\lambda Q)=\frac{|M^0_\lambda[\Omega_\lambda(Q),\mathbf{Q}]|^2}{2\pi e^2/\varepsilon_0 q}\mathrm{Im}\frac{1}{\varepsilon[q,\Omega_\lambda(Q)]}$$

$$=|M_\lambda[\Omega_\lambda(Q),\mathbf{Q}]|^2 \mathrm{Im} P^R_{00}[q,\Omega_\lambda(Q)]. \tag{10}$$

It can be readily seen that under the conditions of the diffusive regime $1/\tau_{\mathrm{ph},\lambda}(Q)$ remains constant, whereas in the clean limit it varies as a linear function of Q.

In all regimes the estimates made with the use of Eqs. (5) and (10) satisfy the relation

$$\frac{C_e(T)}{\tau_\epsilon(T)}\sim\frac{C_{\mathrm{ph}}(T)}{\tau_{\mathrm{ph}}(T)}, \tag{11}$$

where $C_{e,\mathrm{ph}}(T)$ is a 2DEG ($\sim T$) or a 3D phonon ($\sim T^3$) specific heat, which is implied by the energy balance equation (8).

Our Eq. (9) is in agreement with the results of Ref. 6, where the phonon emission rate was computed in the framework of a hydrodynamical model which postulates a constancy of the conductivity $\sigma_{xx}(q)$ in the strongly disordered regime. Compared to this phenomenological approach, our microscopic analysis allows one to calculate τ_ϵ and τ_{ph} directly, as well as to generalize the results onto the case of compressible states of electrons in quantizing magnetic fields.

CONDUCTIVITY CORRECTION

The correction to the conductivity from the piezoelectric electron-phonon interaction can be obtained by modifying the expression derived in Ref. 2:

$$\frac{\delta\sigma_{e\text{-ph}}}{\sigma_0}=-\frac{2}{(2\pi)^4}\sum_\lambda \mathrm{Im}\int d^3\mathbf{Q}$$

$$\times\int d\ \omega D^R(\omega,\mathbf{Q})|M_\lambda(\omega,\mathbf{Q})|^2|F(q_z)|^2$$

$$\times\left[\frac{Dq^2}{(i\omega+Dq^2)^3}\right]f\left(\frac{\omega}{T}\right), \tag{12}$$

where

$$f(\omega/T)=\tfrac{1}{2}\int_{-\infty}^{\infty} d\epsilon\ S(\epsilon+\omega)(\partial/\partial\epsilon)S(\epsilon)$$

$$=(\partial/\partial\omega)[\omega\ \coth(\omega/2T)].$$

In the range of temperatures $T_2<T<T_1$, Eq. (12) yields

$$\frac{\delta\sigma_{e\text{-ph}}}{\sigma_0}=-\frac{2C_1}{\pi^2}\frac{(eh_{14})^2}{2\rho u_l}\frac{1}{D^2\kappa^2}\ln\left(\frac{T_1}{T}\right), \tag{13}$$

where $C_1=\frac{1}{4}\int_0^1 dx(9x^2(1-x^2)+u_l/u_t[8x^4+(1-x^2)^2])\approx 1.22$.

At temperatures $T<T_2$ the conductivity correction $\delta\sigma_{e\text{-ph}}(T)$ ceases to diverge logarithmically, and shows only a quadratic deviation from its value at $T=T_2$. For the realistic parameter values one has $T_1/T_2=u/v_F\approx 20-30$ and, hence, the interval where Eq. (13) holds is quite narrow.

COMPOSITE FERMIONS

A comprehensive description of the physics of EDF's had been achieved in the framework of the theory of composite fermions (CF's), regarded as a new kind of fermionic quasiparticle forming a nearly Fermi-liquid state.[5]

The residual interactions of the CFs, as well as their interactions with charged impurities (remote ionized donors which are usually set back some 10^2 nm from the 2DEG), turn out to be essentially more singular than the conventional Coulomb potential. In the framework of the Chern-Simons theory of Ref. 5, these interactions are formally described as gauge forces representing local-density fluctuations.

The microscopic analysis of the coupling between composite fermions and bulk phonons was done in Ref. 4, where it was shown that in the presence of transverse gauge interactions the electron-phonon scattering is strongly enhanced compared to the zero-field case of (purely scalar) Coulomb interactions. Experimentally, the features attributed to the enhanced electron-phonon interaction were observed in both phonon-limited electron mobility[9] and phonon drag thermopower[10] at EDF's.

It was shown in Ref. 4 that instead of Eq. (2) the screened piezoelectric electron-phonon matrix element reads (hereafter we focus on the EDF state at $\nu=\frac{1}{2}$, which is the one best studied experimentally)

$$M^{\mathrm{cf}}_\lambda(\omega,\mathbf{Q})=M^{\mathrm{cf},s}_\lambda+M^{\mathrm{cf},v}_\lambda$$

$$=\frac{M_\lambda(\mathbf{Q})}{\varepsilon_{cf}(\omega,q)}\left[1+4i\pi H(q)\frac{\mathbf{v}\times\mathbf{q}}{q^2}P^R_{00}(\omega,q)\right], \tag{14}$$

where

$$\varepsilon_{\mathrm{cf}}=1+H(q)(2\pi e^2/\epsilon_0 q)P^R_{00}+H(q)^2(4\pi/q)^2 P^R_{00}P^R_\perp$$

.

The transverse component of the vector polarization is given by the formula $P^R_\perp(q,\omega)=\chi_{\mathrm{cf}}q^2+i\omega\sigma_{\mathrm{cf}}(q)$, where $\chi_{\mathrm{cf}}=1/48\pi^2\nu_{F,\mathrm{cf}}$ and $\sigma_{\mathrm{cf}}(q)$ equals $(\sqrt{2}/4\pi)k_F l_{\mathrm{cf}}$ if $ql_{\mathrm{cf}}<2$, and $\sqrt{2}k_F/(2\pi q)$ otherwise.

The diffusive regime, which is only accessible at ultralow temperatures in the zero-field case, now sets in at much higher temperatures ($T_{1,cf}\sim 1$ K), since the bare Drude conductivity at EDF's is typically two orders of magnitude lower than the zero field one.

One more remark is in order here. Recent experiments[11] demonstrated that in the typical parameter range the $\nu=\frac{1}{2}$

state retains its metallic conductivity down to $T\sim 20$ mK, and only becomes insulating when the amount of disorder effectively increases by a factor of 2, which can be achieved by applying a negative voltage to the side gate and thereby depleting the 2DEG. In view of that, we believe that in the interaction-dominated regime the CF metal obeys a conventional diffusive behavior, whereas effects such as a disorder-induced multifractality of single-particle wave functions becomes important only at excessively low temperatures.

Proceeding along the lines of the previous zero-field analysis, we arrive at the formula for the CF energy relaxation time:

$$1/\tau_{\epsilon,\mathrm{cf}}(T)=\frac{2}{(2\pi)^4}\sum_{\lambda}\int d^3\mathbf{Q}\int d\omega \times \mathrm{Im} D^R(\omega,\mathbf{Q})|F(q_z)|^2 \mathrm{Re}\left[\tau_{\mathrm{cf}}|M_{\lambda}^{\mathrm{cf},v}|^2 + \frac{|M_{\lambda}^{\mathrm{cf},s}|^2}{(Dq^2+i\omega)}\right][\coth(\omega/2T)-\tanh(\omega/2T)]. \tag{15}$$

In the disordered regime $T<T_{1,\mathrm{cf}}$ the resulting expression for $\tau_{\epsilon,\mathrm{cf}}$ can be cast in the form of Eq. (6), where the Debye screening momentum $\kappa_{\mathrm{cf}}=2\pi e^2\nu_{F,\mathrm{cf}}/\varepsilon_0$, and the diffusion coefficient $D_{1/2}=\sigma_{1/2}/\nu_{F,\mathrm{cf}}$ determined in terms of the physical conductivity $\sigma_{1/2}\approx(e^2/2h)^2/\sigma_{\mathrm{cf}}$,[5] contain the mean-field composite fermion density of states $\nu_{F,\mathrm{cf}}=m_{\mathrm{cf}}/2\pi$.[5]

However, Eq. (6) only holds provided the conductivity $\sigma_{1/2}$ is large compared to $\sigma_M=\epsilon_0 u/2\pi\sim 10^{-2}e^2/h$. In the opposite case, $\sigma_{1/2}<\sigma_M$, the result reads

$$1/\tau_{\epsilon,\mathrm{cf}}(T)=C_5\frac{(eh_{14})^2}{2\rho u_l^5}D_{1/2}T^2, \tag{16}$$

where the value of the numerical factor

$$C_5=\tfrac{1}{4}\int_0^1 dx\left[9x^2(1-x^2)^3+\left(\frac{u_l}{u_t}\right)^5 \times[8x^4(1-x^2)^2+(1-x^2)^4]\right) \approx 2.48$$

is again dominated by the transverse phonon modes.

In the clean regime $T_{1,\mathrm{cf}}<T<T_{D,\mathrm{cf}}=\sqrt{2}T_D$, one obtains

$$1/\tau_{\epsilon,\mathrm{cf}}(T)\sim\frac{(eh_{14})^2}{\rho u^2}\frac{T\sigma_{1/2}k_F}{\max[u^2/D_{1/2};D_{1/2}\kappa_{\mathrm{cf}}^2]}. \tag{17}$$

As opposed to the case of zero field described by Eqs. (6) and (7), the exponent in the power-law temperature dependence of $\tau_{\epsilon,\mathrm{cf}}$ reduces by 1 as T increases.

Equations (16) and (17) are also consistent with the energy balance relation (11), since the phonon relaxation rate $1/\tau_{\mathrm{ph}}(Q)$ due to scattering against CF's changes from a constant in the diffusive regime to the $1/Q$ behavior in the ballistic one.

In the diffusive regime both the zero field and the $\nu=\frac{1}{2}$ energy relaxation rates share the same T^2 behavior. The ratio of the prefactors can be estimated as

$$\frac{\tau_{\epsilon,\mathrm{cf}}^{-1}}{\tau_{\epsilon,\mathrm{cf}}^{-1}}\sim\left(\frac{\nu_F}{\nu_{F,\mathrm{cf}}}\right)\min\left[\frac{\sigma_0}{\sigma_{1/2}};\frac{\sigma_0\sigma_{1/2}}{\sigma_M^2}\right], \tag{18}$$

where σ_0 is the zero-field conductivity.

In pure samples exhibiting the ‘‘$\nu=\frac{1}{2}$ anomaly,’’ the numerical value of the right-hand side of Eq. (18) is substantially greater than unity, but reduced compared to that of the transport relaxation rates[4] which does not contain the extra factor $\nu_F/\nu_{F,\mathrm{cf}}=2m/m_{\mathrm{cf}}\approx 0.14$.

The CF conductivity correction differs from Eq. (12), due to the fact that the CF-phonon vertex (14) contains both scalar and vector parts, the latter making the dominant contribution:

$$\frac{\delta\sigma_{1/2}}{\sigma_{1/2}}=-\frac{\delta\sigma_{\mathrm{cf}}}{\sigma_{\mathrm{cf}}}=\frac{\tau_{cf}}{(2\pi)^4}\sum_{\lambda}\mathrm{Im}\int d^3\mathbf{Q}\int d\omega\, D^R(\omega,\mathbf{Q}) \times\left[\frac{|M_{\lambda}^{\mathrm{cf},v}|^2}{(Dq^2+i\omega)}+\frac{v_{F,\mathrm{cf}}^2 q^2|M_{\lambda}^{\mathrm{cf},s}|^2}{(Dq^2+i\omega)^3}\right]f\left(\frac{\omega}{T}\right). \tag{19}$$

The peculiar relation between the CF ‘‘quasiparticle conductivity’’ σ_{cf} and the physical current response functions[5] makes both the dc conductivity $\sigma_{1/2}$ and the resistivity $\rho\approx\sigma_{1/2}/(e^2/2h)^2$ increase as $T\rightarrow 0$.

At $T_{2,\mathrm{cf}}<T<T_{1,\mathrm{cf}}$ and $\sigma_{1/2}>\sigma_M$, we obtain

$$\frac{\delta\sigma_{1/2}}{\sigma_{1/2}}=\frac{C_1}{\pi^2}\frac{(eh_{14})^2}{\rho u_l}\left(\frac{2\epsilon_0}{e^2}\right)^2\ln\left(\frac{T_{1,cf}}{T}\right), \tag{20}$$

whereas at $\sigma_{1/2}<\sigma_M$ the result reads

$$\frac{\delta\sigma_{1/2}}{\sigma_{1/2}}=\frac{C_3}{\pi^2}\frac{(eh_{14})^2}{\rho u_l^3}\left(\frac{\sigma_{1/2}}{\sigma_{cf}}\right)\ln\left(\frac{T_{1,\mathrm{cf}}}{T}\right). \tag{21}$$

The logarithmic growth of correction (21) stops at $T=T_{2,\mathrm{cf}}=u^2/D_{1/2}\sim 0.1$ K and at lower temperatures $\delta\sigma_{1/2}$ shows only a $\sim T^2$ deviation from its value at $T=T_{2,\mathrm{cf}}$.

In either case described by Eqs. (20) or (21), the absolute value of the conductivity correction is small compared to the logarithmic term resulting from the residual (gauge) interactions of CF's.[12] The latter Altshuler-Aronov-type term, which was predicted to exist in a much wider range of temperatures (1 mK$<T<$1 K),[12] provides a reasonable account of the available data on the temperature dependence of the resistivity at EDF's obtained in Refs. 11 and 13.

At compressible EDF states the power $P_{\mathrm{cf}}(T,\Theta)$, carried out by piezoelectric phonons from hot electrons, appears to be strongly enhanced compared to that at zero field (P_0). Repeating our calculation, which led to Eq. (9), for $\nu=\frac{1}{2}$ we obtain

$$\frac{P_{1/2}}{P_0}=\min\left[\frac{\sigma_0}{\sigma_{1/2}};\frac{C_5}{C_3}\frac{\sigma_0\sigma_{1/2}}{\sigma_M^2}\right]. \tag{22}$$

Equation (22) interpolates between the limiting cases of $\sigma_{1/2}$, being much greater or much smaller than σ_M. Compared to Eq. (18), ratio (22) is free from an uncertainty related to a definition of the (strictly speaking, gauge-noninvariant) CF density of states. It can also be used as an estimate in the practically relevant case of $\sigma_{1/2}\sim\sigma_M \sim 10^{-2}e^2/h$, where it predicts an enhancement of the rate of phonon emission by two orders of magnitude, which is consistent with the experimental data from Ref. 6.

It is worth mentioning that after being expressed in terms of the physical conductivity, the energy-loss rate at EDF's contains absolutely no reference to CF's used in its derivation. This formulation enables one to make a direct link to the hydrodynamical model of Ref. 6, and to compare the enhancement of $P(T,\Theta)$ to that of the surface acoustic wave attenuation at $\nu=\frac{1}{2}$.

To conclude, in the present paper we elaborated on the unified treatment of the piezoelectric electron-phonon interaction in the disordered 2DEG in cases of both zero and strong magnetic fields, which was developed in Ref. 4. First, we pointed out an important difference between the impurity-assisted renormalizations of the electron-phonon vertices due to the deformation potential and piezoelectric coupling. Second, we showed that in high-mobility samples the diffusive regime can be probed at experimentally accessible temperatures only in the case of strong fields corresponding to compressible EDF states.

In the course of our comparative study of the zero-field case versus that of EDF's, we compute the phonon contribution to the electron energy relaxation time, the energy-loss rate by hot electrons, and the conductivity correction which results from quantum interference between electron-impurity and electron-phonon scattering. We observe that in the disordered regime the above quantities follow the same temperature dependences in both cases. Numerically, however, the results obtained for EDF's are all enhanced compared to those at zero field by the numerical factor related to the ratio of physical conductivities. Further experiments of the kind carried out in Ref. 6 would allow one to test this and other predictions of the theory.

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