Coulomb Drag in Double Layers with Correlated Disorder

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We study the effect of correlations between impurity potentials in different layers on the Coulomb drag in a double-layer electron system. It is found that for strongly correlated potentials the drag in the diffusive regime is substantially enhanced compared to what is conventionally predicted. The appropriate experimental conditions are discussed, and the new experiments are suggested.

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Over the past decade the frictional drag in double-layer two-dimensional electron systems has been a subject of extensive experimental [1] and theoretical [2–7] studies. This phenomenon is manifested in the appearance of a current I_2 or voltage V_2 in the "passive" layer 2 when an applied voltage V_1 causes a current I_1 to flow in the "active" layer 1. The strength of the drag is characterized by either transconductivity $\sigma_{21} = (I_2/V_1)_{V_2=0}$ or transresistivity $\rho_{21} = (V_2/I_1)_{I_2=0}$, which are related to each other as $\rho_{21} = -\sigma_{21}(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^{-1} \approx -\sigma_{21}\sigma_{11}^{-2}$, where σ_{ii} are the intralayer conductivities.

In the absence of tunneling (cf. Ref. [7]), the drag stems from interlayer momentum transfer mediated by inelastic scattering (mainly, Coulombic) of carriers in different layers. Hence, the drag provides a convenient tool for studying electron correlations in the coupled two-dimensional mesoscopic systems, including such important characteristics as the electron polarization function and the screened interlayer interaction.

In the conventional theory [2], the carriers in each layer are scattered by completely independent impurity potentials. As a result, the processes contributing to σ_{21} can be understood in terms of coupling between thermal density fluctuations in different layers. The phase space available to the thermal excitations is limited by energies $\omega \lesssim T$. Therefore, the drag effect rapidly vanishes with decreasing temperature. For instance, $\rho_{21} \propto T^2$ ($T^2 \ln T$) in a clean (dirty) normal metal [2] and $\rho_{21} \propto T^{4/3}$ for composite fermions in the double-layer system of electrons at the half-filled Landau level [3].

The picture of independent impurity potentials used in Refs. [2–7] is well justified in the case of the standard experimental geometry [1], where two delta-doped layers (DDLs) are situated on the outer sides of the double quantum well. The DDLs not only serve as the reservoirs supplying carriers but also introduce disorder in the form of a smooth random potential (SRP) of the ionized donors. Moreover, because of the efficient screening, the carriers in each quantum well experience only a SRP created by the nearest DDL.

Instead, one can consider an alternate geometry where a single DDL is located in the middle between the two electron layers, so that the SRPs in both layers are almost identical. This setup gives one an opportunity to study a new type of coherence in systems with spatially separated carriers. Previously, the interlayer coherent effects have been discussed in the context of a possibility of forming a superfluid ground state of composite fermions [5] or electron-hole pairs [6].

In the present paper we study the effect of correlations between the impurity potentials in different layers on the transresistivity. We focus our attention on the regime $\tau_g \gg \tau_{\rm tr}$, where τ_g is a characteristic time during which the carriers in the two layers propagate in highly correlated random potential landscapes, and $\tau_{\rm tr}$ is the transport scattering time in each layer. Under such conditions the drag effect appears to be strongly enhanced when compared to the uncorrelated situation because of a possibility for the carriers from different layers to maintain their coherent motion along almost perfectly superimposing planar trajectories. This property of the trajectories that bears a certain resemblance to weak localization in a single layer [8] provides strong correlations between the momenta of electrons (and, therefore, the currents) in different layers and gives rise to yet longer times during which these electrons experience mutual Coulomb scattering. As a result, the transresistivity demonstrates the following substantial enhancement:

$$\rho_{21}^{\rm corr} \simeq -\frac{\pi^4}{24}\frac{\hbar}{e^2}\frac{\ln(T\tau_g)}{(k_F d)^4(\kappa l)^2}\,, \qquad \tau_g^{-1} \ll T \ll \tau_{\rm tr}^{-1}\,, \tag{1}$$

$$\rho_{21}^{\rm corr} \simeq -\frac{\pi^4}{6}\frac{\hbar}{e^2}\frac{(T\tau_g)^2}{(k_F d)^4(\kappa l)^2}\,, \qquad T \ll \tau_g^{-1}\,, \tag{2}$$

Here, $l = v_F\tau_{\rm tr}$ is the electron mean-free path, k_F (v_F) is the Fermi momentum (velocity), d is the interlayer distance (throughout this paper we assume $l \gg d$), and κ is the Thomas-Fermi momentum. As long as the system remains in the diffusive regime, $T \ll \tau_{\rm tr}^{-1}$, Eqs. (1) and (2) yield the dominant contribution to ρ_{21} in the entire experimentally accessible temperature range. Below, we specify the experimental conditions necessary for the new effects to be observed.

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Let us start by discussing how the correlations between the impurity potentials in different layers affect the charge propagation. A signature of the onset of coherence in a diffusive motion of carriers is the appearance of singularities in the particle-hole and particle-particle propagators usually dubbed as diffusons and Cooperons. The equations for diffusons in the double-layer system can be written as follows:

$$\mathcal{D}^{ij}_{\mathbf{k}\mathbf{k}'}(\mathbf{q},\omega) = W^{ij}_{\mathbf{k}\mathbf{k}'} + \sum_{\mathbf{k}_1} W^{ij}_{\mathbf{k}\mathbf{k}_1} G^{Ri}_{\mathbf{k}_1+\mathbf{q}}(\epsilon+\omega) \times G^{Aj}_{\mathbf{k}_1}(\epsilon)\mathcal{D}^{ij}_{\mathbf{k}_1\mathbf{k}'}(\mathbf{q},\omega)\,. \quad (3)$$

In Eq. (3) the indices i, j label the layers, $\mathbf{k}$ and $\mathbf{k}'$ are the momenta of the incoming and outgoing electrons, $G^{R(A)i}_{\mathbf{k}}(\epsilon) = [\epsilon - \xi_{\mathbf{k}} \pm i/2\tau^{ii}]^{-1}$ is the impurity averaged retarded (advanced) electron Green function, and $W^{ij}_{\mathbf{k}\mathbf{k}'} = \langle u^i u^j\rangle_{\mathrm{imp}}$ are the elastic electron scattering probabilities. The finite $W^{ij}_{\mathbf{k}\mathbf{k}'}$ at $i \neq j$ stem from the correlations between the impurity potentials u^i in different layers. The total and the transport scattering times are given by the formulas

$$1/\tau^{ij} = \langle W^{ij}_{\mathbf{k}\mathbf{k}'}\rangle_{\mathbf{k}'}, \qquad 1/\tau^{ij}_{\mathrm{tr}} = \langle W^{ij}_{\mathbf{k}\mathbf{k}'}(1-\hat{\mathbf{k}}\hat{\mathbf{k}}')\rangle_{\mathbf{k}'},$$

where the symbol $\langle\cdots\rangle_{\mathbf{k}} = \nu_F\int_0^{2\pi}\cdots d\varphi_k$ stands for the angular integration over the Fermi surface, $\nu_F = m/2\pi$ is the single-spin density of states (assumed to be equal in both layers), and m is the effective electron mass.

By employing the formalism of Ref. [9] we solve Eq. (3) at $i \neq j$ and find a characteristic time

$$\tau_g = \frac{\tau^2}{\tau^{21}-\tau}\,, \quad (4)$$

where $1/\tau = [1/\tau^{11} + 1/\tau^{22}]/2$, which determines a crossover between the two distinct regimes. Namely, at $\tau^{ii}_{\mathrm{tr}} \gtrsim \tau_g$ there are no interlayer diffusons, and the system remains in the ballistic regime at all temperatures, as far as the interlayer elastic scattering is concerned. However, at $\tau_g \gg \tau^{ii}_{\mathrm{tr}}$ the solution of Eq. (3),

$$\mathcal{D}^{21}_{\mathbf{k}\mathbf{k}'}(\mathbf{q},\omega) \simeq \frac{1}{2\pi\nu_F\tau^2}\frac{\gamma_{\mathbf{k}}\gamma_{\mathbf{k}'}}{Dq^2 - i\omega + \tau_g^{-1}} + \mathcal{D}^{\mathrm{reg}}_{\mathbf{k}\mathbf{k}'}\,, \quad (5)$$

develops a quasi-diffuson pole. Here $D = v_F^2\tau_{\mathrm{tr}}/2$ is the diffusion coefficient, $\gamma_{\mathbf{k}} = 1 - i(\tau_{\mathrm{tr}} - \tau)\mathbf{q}\mathbf{k}/m$, and the regular part of the diffuson $\mathcal{D}^{\mathrm{reg}}_{\mathbf{k}\mathbf{k}'}$ coincides with its single-layer counterpart [9]. We neglect the difference between the interlayer and intralayer scattering times by putting $\tau^{ij} = \tau$ and $\tau^{ij}_{\mathrm{tr}} = \tau_{\mathrm{tr}}$ everywhere, except for the "gap" τ_g^{-1} in the first term of Eq. (5). For two identical SRPs, one has $\tau_g = \infty$, and the interlayer diffuson becomes indistinguishable from the intralayer one.

Similar conclusions can be drawn about the interlayer Cooperon $C^{21}_{\mathbf{k}\mathbf{k}'}(\mathbf{q},\omega)$ which is given by Eq. (5) with the inverse inelastic phase breaking time contributing to the total gap $\tau_g^{-1} + \tau_\varphi^{-1}$. Although at finite τ_g there are no true singularities in the interlayer diffusons and Cooperons for $\omega,\mathbf{q}\to 0$, at frequencies $\tau_g^{-1} \lesssim \omega \lesssim \tau_{\mathrm{tr}}^{-1}$ and momenta $\tau_{\mathrm{tr}}/\tau_g \lesssim (ql)^2 \lesssim 1$ the motion of carriers in the two layers remains highly correlated.

The origin of the interlayer decoherence rate τ_g^{-1} can be explained as follows. Consider two coherent electron waves propagating in slightly different random potentials ($u + \delta u$ and $u - \delta u$). After traveling over a distance of order of the SRP correlation length a, they pick up a random phase difference $\Delta\phi \sim (2\delta u)v_F^{-1}a$, which leads to the electron's phase diffusion with the diffusion coefficient $D_{ph} = (\Delta\phi)^2 v_F a^{-1}$ and causes a complete loss of phase coherence at a time scale $\tau_g \sim D_{ph}^{-1}$. It is important to realize that τ_g is intrinsically different from the regular transport scattering times.

Next, we set out to calculate the dc transconductivity given in terms of the retarded interlayer current-current correlator Π_{21} as $\sigma_{21} = ie^2(\hbar\Omega)^{-1}\Pi_{21}(\Omega)|_{\Omega\to 0}$. In the diagrammatic language, the new correlation effects are described by diagrams with two electron loops (each containing one current vertex), connected not only by the interlayer Coulomb interaction lines but also by the *impurity lines* combining into the interlayer diffusons and/or Cooperons. We find that, to any order in the interlayer interaction V_{21}, both the diffuson and the Cooperon diagrams for which the electron energy does not change its sign at the current vertices give no contribution to σ_{21} at zero external frequency $\Omega = 0$. A general proof can be obtained by extending the results of Ref. [8] for the Hartree corrections to the single-layer conductivity onto the double-layer case. Since the method of Ref. [8] accounts for the diagrams with an arbitrary number of impurity lines, its conclusions remain valid regardless of the presence of the gap τ_g^{-1} in diffusons and Cooperons.

Perturbative calculations confirm the above statement. To illustrate this, consider the diffuson diagrams which give rise to the well-known logarithmic conductivity corrections in the single-layer case [8]. Among those, only the diagrams consisting of two different electron loops (see Fig. 1a) have their counterparts that contribute to σ_{21}. We found that the logarithmic terms in these diagrams cancel against each other. Therefore, in the double-layer system, only the intralayer conductivities σ_{ii} acquire the logarithmic Hartree corrections due to the presence of the second layer.

Thus the only diagrams contributing to σ_{21} are those where the electron energies change their signs at the current vertices. It can be readily checked that no diffuson diagrams of this kind survive in the dc limit. However, the Cooperon diagrams do, so the first nontrivial contribution to σ_{21} arises in the second order in the interaction V_{21} (similar to the uncorrelated case [2]) and involves three Cooperons. It is, however, more instructive to sum up the entire interlayer Coulomb-Cooperon ladder (see Fig. 1b). Taking into account the smooth character of the donors' potentials, one arrives at the two diagrams depicted in

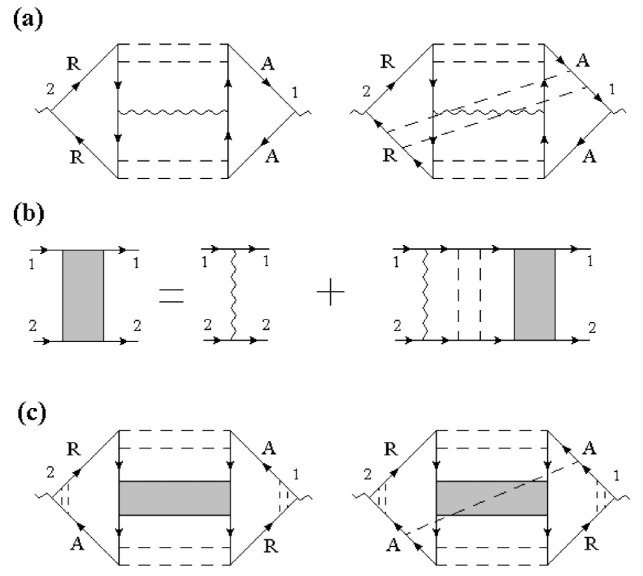

FIG. 1. Diffuson diagrams for transconductivity (a); interlayer Coulomb-Cooperon ladder (b); and two Cooperon diagrams which yield the main contribution to σ_{21} (c). Solid lines represent retarded (R) and advanced (A) electron Green functions, wavy lines and single dashed lines denote the screened Coulomb interaction and the impurity potential, respectively. Double dashed lines represent the diffusons, $\mathcal{D}^{ij}$, and Cooperons, C^{21}, and the layers are labeled by the numbers.

Fig. 1c. After the summation over electron frequencies and momenta, the contribution of these diagrams to the transconductivity reads

$$\sigma_{21}^{\rm corr} = \frac{4e^2}{\pi\hbar T}\int \frac{D(dq)}{Dq^2 + \tau_g^{-1} + \tau_\varphi^{-1}} \times \int_0^\infty \frac{d\omega}{\sinh^2\frac{\omega}{2T}} {\rm Im}\Psi_c(\mathbf{q},\omega){\rm Im}\Lambda_c(\mathbf{q}\cdot\omega)\,, \quad (6)$$

The quantities Ψ_c and Λ_c are given by

$$\Psi_c(\mathbf{q},\omega) = \psi\left(\frac{Dq^2 - i\omega + \tau_g^{-1} + \tau_\varphi^{-1}}{4\pi T} + \frac{1}{2}\right),$$

$$\Lambda_c(\mathbf{q},\omega) = 2\left[\ln\frac{\varepsilon_0}{T} + \lambda_{21}^{-1} - \Psi_c(\mathbf{q},\omega) + \psi(1/2)\right]^{-1},$$

where ψ is the digamma function, $\varepsilon_0 \propto \varepsilon_F$ is the upper energy cutoff, and $\lambda_{21} = (4\pi^2\nu_F)^{-1}\langle V_{21}(\mathbf{p} - \mathbf{p}')\rangle_{\mathbf{p},\mathbf{p}'}$ is the effective interaction constant. Here the screened interlayer Coulomb interaction has the form

$$V_{21}(\mathbf{p}) = \frac{1}{2\nu_F} \times \left[\frac{\kappa(1 + e^{-pd})}{p + \kappa(1 + e^{-pd})} - \frac{\kappa(1 - e^{-pd})}{p + \kappa(1 - e^{-pd})}\right].$$

Provided that screening is strong enough, $\kappa d \gg 1$, and that the interlayer Coulomb potential is sufficiently smooth, $k_F d \gg 1$, one finds $\lambda_{21} \simeq \pi(4k_F d\kappa d)^{-1}$.

Evaluation of the integrals in Eq. (6) yields

$$\rho_{21}^{\rm corr} \simeq -\frac{2\pi^2}{3}\frac{\hbar}{e^2}\frac{1}{(k_F l)^2[\lambda_{21}^{-1} + \ln(\varepsilon_0/T)]^2}\ln\frac{T\tau_\varphi\tau_g}{\tau_\varphi + \tau_g} \quad (7)$$

at $\tau_g^{-1} \ll T \ll \tau_{\rm tr}^{-1}$ (when the domain of $\mathbf{q}$ integration is effectively limited by $Dq^2 \lesssim T$), and

$$\rho_{21}^{\rm corr} \simeq -\frac{8\pi^2}{3}\frac{\hbar}{e^2}\frac{(T\tau_g)^2}{(k_F l)^2[\lambda_{21}^{-1} + \ln(\varepsilon_0\tau_g)]^2} \quad (8)$$

at lower temperatures. These equations constitute our main results. Under the realistic experimental conditions (see below), the value of λ_{21}^{-1} is sufficiently large for one to neglect the logarithmic terms in the denominators of Eqs. (7) and (8). Since, unlike $\tau_\varphi \propto T^{-1}$ [8], the interlayer decoherence time τ_g is temperature independent, at low enough temperatures one has $\tau_g \ll \tau_\varphi$, and Eqs. (7) and (8) reduce to Eqs. (1) and (2), respectively.

Now let us compare these results with the predictions of the standard theory [2]:

$$\rho_{21}^{\rm conv} = -\frac{\hbar}{e^2}\frac{\pi^2\zeta(3)}{16}\frac{1}{(k_F d)^2(\kappa d)^2}\left(\frac{T}{\varepsilon_F}\right)^2. \quad (9)$$

We observe that at $T \ll \tau_{\rm tr}^{-1}$ our result for the transresistivity exceeds the conventional one: in the interval $\tau_g^{-1} \ll T \ll \tau_{\rm tr}^{-1}$ the correlation effects lead to the weaker temperature dependence, while at $T \ll \tau_g^{-1}$ the prefactor of the T^2 dependence is $(\tau_g/\tau_{\rm tr})^2$ times greater in the case of correlated impurity potentials.

Two remarks are in order. Firstly, Eq. (6) bears a certain formal resemblance to the Maki-Thompson correction to the single-layer Drude conductivity [10], the effect of the temperature-independent gap τ_g^{-1} being somewhat reminiscent to that of a weak magnetic field [8]. A real perpendicular magnetic field leads to a suppression of the transresistivity by destroying the interlayer Cooperons. In a sufficiently strong field, one has to replace τ_g^{-1} by $\tau_H^{-1} = 4DeH/(\hbar c)$ in Eq. (1) at $\tau_g^{-1} \ll \tau_H^{-1} \ll T$ and, in Eq. (2) at $T, \tau_g^{-1} \ll \tau_H^{-1}$. This effect of the magnetic field can provide a discriminating test for the theory.

Secondly, it can be seen from Eq. (6) that the nontrivial contribution to σ_{21} arises in the second order in λ_{21} in which case one should replace ${\rm Im}\Lambda_c$ by $2\lambda_{21}^2\,{\rm Im}\Psi_c$. The resulting expression looks quite similar to the one obtained for the uncorrelated impurity potentials [2], since they both describe the second-order Coulomb interaction processes in terms of some effective bosonic modes. The difference is in the physical meaning of these modes, which are the (energy-integrated) interlayer Cooperons in the considered case versus the interlayer plasmons in the conventional situation. Also, due to interlayer coherence, the integrand in Eq. (6) lacks the q^2 factor which otherwise causes a rapid decay of $\rho_{21}^{\rm conv}$ with temperature.

In the preceding discussion we ignored the Aslamazov-Larkin–type diagrams [11] which contain two Λ_c ladders.

Indeed, being proportional to $T^2\lambda_{21}^2$, these terms are of the order of the ordinary contribution given by Eq. (9).

Now we discuss the experimental conditions under which the described theory applies. It can be easily seen that in the standard geometry the condition $\tau_g \gg \tau_{\rm tr}$ can never be satisfied as long as $\kappa d > 1$ and $k_F a > 1$ (here a is the distance from a DDL to the nearest quantum well). At $\kappa d < 1$ it requires $2(\kappa d)^2(k_F a)^2 \ll 1$ which becomes possible only at very small interlayer separations [12].

The situation is, however, different in the proposed geometry with a single DDL located *in between* the two quantum wells. Introducing a finite width of the DDL δ, we find that at $k_F d > 1$ and $(2\delta/d)^2 < (\kappa d)^{-1}$ the condition $\tau_g \gg \tau_{\rm tr}$ can be rewritten as $2(k_F\delta)^2 \ll \min[1, \kappa d]$. For $\kappa \sim 0.02$ Å^{-1}, $k_F \sim 0.015$ Å^{-1}, $\delta \sim 10$ Å, and $d \sim 400$ Å, the above criteria are fulfilled, and there exists a temperature interval where Eq. (1) holds. Note that it may be easier to observe this regime in dirty samples (yet with $l \gg d$). For instance, at $l \sim 5000$ Å (which implies $\tau_{\rm tr}^{-1} \sim 4$ K and $\tau_g^{-1} \sim 0.2$ K), Eq. (1) yields ρ_{21} of an order of a few mΩs within the entire temperature range $\tau_{\rm tr}^{-1} \gtrsim T \gtrsim \tau_g^{-1}$, whereas the conventional theory would predict a rapid decay of the transresistivity from $\rho_{21} \sim 1$ mΩ at $T \sim \tau_{\rm tr}^{-1}$ to $\rho_{21} \sim 3$ $\mu\Omega$ at $T \sim \tau_g^{-1}$.

Before concluding, we would like to comment on the zero-width approximation for the quantum wells used throughout the paper. As long as the width W of the wells remains much smaller than the interlayer separation, the parameters of our theory undergo only small changes of order $(2W/d)^2$. Although at $W \sim 100$ Å the zero-width approximation may no longer be capable of giving quantitatively accurate results, the finite W does not alter the temperature dependence of ρ_{21}, provided that the system remains symmetric under a reflection with respect to the center of the DDL. Conversely, any spatial asymmetry between the quantum wells measured by a relative shift δz in the positions of electron wave functions breaks the interlayer coherence and yields an extra contribution to τ_g^{-1} of order $(k_F\delta z)^2\tau_{\rm tr}^{-1}$.

In conclusion, we investigated the effect of correlations between impurity potentials in different layers of a double-layer electron system on the phenomenon of Coulomb drag. We found that for correlated potentials the low-temperature drag is substantially enhanced compared to the standard (uncorrelated) situation, whereas a weak magnetic field suppresses the effect.

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[12] We note that a natural roughness of the substrate may lead to a *correlated* interface roughness in both quantum wells, thanks to the long-range character of the deformation field. Then in very clean samples, where the interface roughness becomes the main scattering mechanism, one can expect some coherence of the kind described in this paper to occur even in the conventional geometry.