Phonon-mediated transresistivity in a double-layer composite Fermion system

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We consider the frictional drag in a double-layer system of two-dimensional electrons in the half-filled lowest Landau level. At sufficiently large interlayer separations the drag is dominated by the exchange of acoustic phonons and exhibits a variety of dependencies on temperature and distance between the layers. In all of the regimes, the phonon-mediated drag is found to be strongly enhanced compared to that in zero magnetic field.

In recent years, the problem of two-dimensional electrons in a double quantum well has attracted a lot of both experimental and theoretical attention. Apart from other interesting properties, this system offers a new opportunity for studying electron correlations by virtue of the phenomenon of the frictional drag. The latter manifests itself in a finite transresistivity defined as a voltage induced in an open circuit (passive) layer while a current is flowing through the other (active) one (see Ref. 1 and references therein).

In the absence of electron tunneling, the transresistivity which takes its origin in momentum transfer from the active to the passive layer can only result from the interlayer Coulomb coupling. Hence, being of the interaction origin, the transresistivity provides a sensitive probe for such important characteristics of the two-dimensional electron gas as the electron polarization operator and the dynamically screened interlayer coupling.

Detailed experimental studies of the frictional drag in semiconductor heterostructures revealed features associated with interlayer plasmons.[2] However, it was predicted that, despite its dominance at small interlayer distances d, the drag caused by the direct Coulomb coupling decays rapidly with d.[1] Since the experimentally measured drag showed a weaker distance dependence at large d, it was suggested that some other mechanism of momentum transfer ought to be at work.

As such, exchange of acoustic phonons was proposed as the most likely alternative.[2–4] The previous work was mainly concerned with the behavior at temperatures comparable to or higher than the Bloch-Gruneisen temperature $T_{BG}=2uk_F$, defined in terms of the Fermi momentum k_F and the speed of sound u, where the deformation potential electron-phonon coupling is known to become much more important than the piezoelectric one. At temperatures smaller than T_{BG} their relative strength reverts though.

According to the general argument,[1] the low-temperature transresistivity is controlled by the phase-space volume available for thermally excited bosonic modes of an appropriate kind emitted by the electrons in either of the layers and absorbed in the other one. Thus, by measuring the exponent n in the algebraic low-temperature dependence $\rho_{12}\sim T^n$ one can identify a mechanism responsible for the drag. Namely, the Coulomb drag yields $n=2$ while the phonon-mediated drag is predicted to give rise to substantially higher n values.[4]

Since at high enough temperatures the frictional drag of any origin is expected to undergo a crossover to a universal linear T dependence,[1] an observation of a maximum of $\rho_{12}(T)/T^2$ at a temperature of order T_{BG} provides an evidence in favor of the phonon mechanism.[2]

Recent drag measurements carried out in the quantum Hall regime[5] revealed features which are believed to be associated with a formation of compressible metal-like electron states at half-filling as well as other even-denominator filling fractions $\nu\sim 1/2p$ at the lowest Landau level. In the framework of the theory by Halperin, Lee, and Read[6] these states are described in terms of spinless fermionic quasiparticles dubbed composite fermions (CF's). In the mean field picture of the most studied $\nu=1/2$ quantum Hall state, the CF's experience zero effective field and occupy all the states inside some ostensible Fermi surface of the size $k_F=(4\pi n_e)^{1/2}$ given in terms of the electron density n_e.

Amongst other differences from the case of ordinary electrons in zero field, the compressible CF states were predicted to feature a peculiar Coulomb drag behavior $\rho_{12}\sim T^{4/3}$ characteristic of an anomalously slow relaxation of overdamped density fluctuations.[7,8]

Further work in this direction focused on the experimentally observed tendency of $\rho_{12}(T)$ to saturate at low T or even increase upon decreasing the driving current,[5] the features that were attributed to a possible interlayer CF pairing which might result in the formation of an incompressible (intrinsically double-layer) gapful electron state.[9]

However, on the general grounds, one expects that neither the direct Coulomb coupling nor the proposed pairing mechanism would still be relevant at large enough interlayer separations. Motivated by this expectation, in the present paper we consider the phonon-mediated drag in a widely separated double-layer CF system where the conventional second-order perturbative calculation is well justified.[1]

Under such conditions, the residual frictional drag provides an independent probe into coupling between CF's and acoustic phonons which complements the earlier insights gained from the experimental data on phonon-limited mobility,[10] phonon drag contribution to thermopower,[11] and electron-energy-loss rate via phonon emission[12] at $\nu=1/2$.

We start out by restricting our low-temperature analysis to the case of a purely piezoelectric phonon coupling described by the standard vertex

$$M_\lambda(\mathbf{Q}) = e h_{14} (A_\lambda / 2\rho u_\lambda Q)^{1/2},$$
$$A_l = \frac{9 q_z^2 q^4}{2 Q^6}, \quad A_{tr} = \frac{8 q_z^4 q^2 + q^6}{4 Q^6}, \tag{1}$$

where $\mathbf{Q} = (\mathbf{q}, q_z)$ is the three-dimensional (3D) phonon momentum, ρ is the bulk density of GaAs, u_λ is the speed of sound with polarization $\lambda = l, tr$, and h_{14} is the nonzero component of the piezoelectric tensor which relates the local electrostatic potential to the lattice displacement.

In the presence of both intralayer $[V_{11}(q) = 2\pi e^2/\epsilon_0 q]$ and interlayer $[V_{12}(q) = V_{11}(q) e^{-qd}]$ Coulomb potentials ($\epsilon_0 \approx 12.9$), the bare CF-phonon vertex (1) undergoes full dynamical screening[13] which is routinely described in terms of the intralayer CF density $\Pi_{00}(\omega, \mathbf{q})$ and current $\Pi_\perp(\omega, \mathbf{q})$ response functions.[6]

Notably, when computing physical observables one finds that the above response functions always appear in a particular combination which can be readily recognized as the irreducible electron density response function

$$\chi(\omega, \mathbf{q}) = \frac{\Pi_{00}}{1 - (4\pi)^2 \Pi_{00} \Pi_\perp} = \frac{q^2}{q^2 (dn_e/d\mu)^{-1} - (4\pi)^2 i\omega\sigma(q)}. \tag{2}$$

This form of the density response function characteristic of the compressible CF states is given in terms of the CF compressibility $dn_e/d\mu$ and the momentum-dependent conductivity $\sigma(q) = \frac{1}{2} k_F \min(l, 2/q)$, where l stands for the CF mean free path.[6] Throughout this paper we refer to the momenta q larger (smaller) than l^{-1} as the ballistic (diffusive) regime, respectively.

The dynamically screened CF-phonon coupling and the direct interlayer Coulomb interaction can be combined together into a total effective interlayer coupling

$$W_{12} = \frac{V_{12} + D_{12}}{[1 + \chi(V_{11} + D_{11})]^2 - \chi^2 (V_{12} + D_{12})^2}, \tag{3}$$

where the planar electron-phonon interaction function $D_{ij}(\omega, \mathbf{q})$ is computed with the use of the bulk phonon propagator $D_\lambda(\omega, \mathbf{Q}) = 2 u_\lambda Q/(\omega_+^2 - u_\lambda^2 Q^2)$ which depends on the three-dimensional momentum $\mathbf{Q} = (\mathbf{q}, q_z)$ and accounts for a finite phonon mean free path l_{ph} contained in $\omega_+ = \omega + i(u/2 l_{ph}) \mathrm{sgn}\omega$.

For ω close to $u_\lambda q$ the relevant values of q_z turn out to be small. Therefore one can only keep the contribution of the transverse phonons ($A_l \ll A_{tr} \approx 1/4$) with the velocity $u \approx 3 \times 10^5$ cm/s which for $qw \lesssim 1$ results in the expression

$$D_{ij}(\omega, \mathbf{q}) = \sum_\lambda \int \frac{dq_z}{2\pi} F_i(q_z) F_j(q_z) |M_\lambda(\mathbf{Q})|^2 \mathcal{D}_{ij}(\omega, \mathbf{Q})$$
$$\approx -\frac{(e h_{14})^2}{8 u^2 \rho \sqrt{q^2 - (\omega_+/u)^2}} \times [\delta_{ij} + (1 - \delta_{ij}) e^{-d\sqrt{q^2 - (\omega_+/u)^2}}], \tag{4}$$

where the form factor $F_i(q_z) = \int dz |\psi_i(z)|^2 e^{i q_z z}$ is determined by the wave function of the lowest occupied transverse electron subband $\psi(z) \sim z e^{-z/w}$.

A direct calculation of the transresistivity in the (lowest nontrivial) second order in the effective interlayer coupling (3) shows that it can be cast in the standard form[1]

$$\rho_{12} = \frac{1}{8\pi^2} \frac{h}{e^2} \frac{1}{T n_e^2} \int \frac{d\mathbf{q}}{(2\pi)^2} \int_0^\infty d\omega \times \left(\frac{q \,\mathrm{Im}\chi(\omega, \mathbf{q})}{\sinh(\omega/2T)} \right)^2 |W_{12}(\omega, \mathbf{q})|^2 \tag{5}$$

provided that one uses the response function $\chi(\omega, \mathbf{q})$ given by Eq. (2). The upper limit in the momentum integral is set by either the maximum span of the CF Fermi surface $2k_F$ or by the inverse width of the quantum well w^{-1}, which might be both equally restrictive ($2 k_F w \sim 1$) for the experimentally relevant parameter values.

Hereafter, in order to facilitate a direct comparison with the calculation of $\rho_{12}(T)$ in the zero field, we will refer to the expression $\epsilon(\omega, \mathbf{q}) = (dn_e/d\mu)(\chi^{-1} + V_{11}) = 1 + (dn_e/d\mu) \times \{V_{11}(q) + [i\omega\sigma(q)/q^2]\}$ as the effective dielectric function of a single-layer CF system.

The relative strength of the PE phonon coupling is controlled by a small parameter $\eta = h_{14}^2 \epsilon_0 / 16\pi u^2 \rho \sim 10^{-3}$. Therefore it proves convenient to divide the q integral onto the momenta smaller and greater than d^{-1}. In the former, the parameter η allows one to neglect D_{ij} in Eq. (5) altogether and, in this way, recover the pure Coulomb drag. In this range of momenta, the integral over frequencies receives its main contribution from $\omega \sim i q^2 (V_{11} - V_{12})/\sigma(q)$ corresponding to the overdamped density mode. The latter is a strong field counterpart of the ordinary acoustic plasmon which, as shown below, gives rise to an even stronger enhancement of the drag than in the case of zero field.[1]

Depending on the values of two-dimensionless parameters $\xi = k_F d$ and $\sigma = k_F l$, the contribution of the momenta $q \lesssim d^{-1}$ exhibits a number of crossovers. Provided that both σ and ξ are sufficiently large, Eq. (5) yields a variety of regimes characterized by different dependences on the dimensionless temperature $\tau = T/\Delta$ measured in units of the average Coulomb energy $\Delta = k_F e^2/\epsilon_0 \sim 100$ K:

$$\rho_{12}^C(T) \propto \begin{cases} (\tau\sigma/\xi)^2 \ln(\tau\sigma^3/\xi), & \tau < \xi/\sigma^3 \\ (\tau/\xi)^{4/3}, & \xi/\sigma^3 < \tau < 1/\xi^2 \\ \tau/\xi^2, & 1/\xi^2 < \tau \end{cases}$$

$$\text{or} \tag{6}$$

$$\begin{cases} (\tau\sigma/\xi)^2 \ln(\tau/\sigma\xi), & \tau < 1/\sigma\xi \\ \tau\sigma/\xi^3, & 1/\sigma\xi < \tau \end{cases}$$

for $l > d$ and $l < d$, respectively.

In the case of a long CF mean free path, the asymptotical low-T regimes $\rho_{12}^C \sim T^{4/3}$ and $T^2 \ln T$ were previously obtained under the assumptions of the ballistic and diffusive CF dynamics, respectively.[8] As we see now, upon increasing the interlayer separation beyond the CF mean free path the former regime becomes completely unattainable.

It is worthwhile mentioning that neither Eq. (6), nor its zero-field counterpart $\rho_{12}^{C,0} \sim \tau^2 \xi^{-4}$ (Ref. 1) depends on the mass of charge carriers (or, for that matter, the density of states). Equation (6) shows that the only reason behind the observed three orders of magnitude enhancement of the drag at $\nu = 1/2$ and $d \approx 300$ Å (Ref. 5) with respect to its value at zero field is that the Coulomb drag is normally measured at $T \ll \Delta$.

As follows from Eq. (6), when treated in the second-order perturbation theory the Coulomb drag at $\nu = 1/2$ can only account for a temperature dependence $\rho_{12}(T) \sim T^n$ with $n < 2$, except for a possible logarithmic enhancement.

Therefore, since the interlayer Coulomb correlations decay with d, one might conclude that the Coulomb drag alone could not explain a maximum in $\rho_{12}(T)/T^2$, should one be observed at sufficiently large separations.

However, a recent experiment[14] on the double-layer $\nu = 1/2$ system with $d \approx 5000$ Å did reveal such a maximum which, by analogy with the situation in zero field, calls for an alternate mechanism of the interlayer momentum transfer, possibly phonon exchange.

To this end, we first focus on the case of a short phonon mean free path $l_{ph} \sim 10$ μm inferred by the authors of Ref. 14 from their earlier data on zero-field transresistivity.[15] Then the phonon contribution to Eq. (5) assumes the form

$$\rho_{12}^{ph} \approx \frac{1}{16\pi^3} \frac{h}{e^2} \frac{\eta^2}{T n_e^2} \int_{1/d}^{2k_F} \frac{\sigma^2(q)\,dq}{q} \int_0^\infty \frac{\omega^2 d\omega}{\sinh^2 \dfrac{\omega}{2T}} \times \frac{e^{-2d\sqrt{[q^2-(\omega_+/u)^2]}}}{|\epsilon(\omega,\mathbf{q})|^4 \sqrt{[q^2-(\omega/u)^2]^2+(q/l_{ph})^2}}. \tag{7}$$

By introducing the parameter $\delta = u\epsilon_0/2\pi e^2 \approx 10^{-2}$ we present the results as follows:

$$\rho_{12}^{ph}(T) \propto \begin{cases} \tau^4 \sigma^2 \delta^{-2} (1+\sigma^2\delta^2)^{-2}, & \delta\xi^{-1} < \tau < \delta\sigma^{-1} \\ \tau^6 \delta^{-8}, & \max(\delta\xi^{-1}, \delta\sigma^{-1}) < \tau < \delta^2 \\ \tau^2, & \max(\delta\xi^{-1}, \delta\sigma^{-1}, \delta^2) < \tau < \delta \\ \tau\delta, & \delta < \tau. \end{cases} \tag{8}$$

All of the above regimes can only be accessible if there exist sizeable intervals of the dimensionless temperature τ set by the conditions $\xi^{-1} \ll \sigma^{-1} \ll \delta \ll 1$. With all the regimes present, the exponent n in the power-law temperature dependence of $\rho_{12}^{ph}(T)$ does not remain constant and increases as T gets smaller.

Yet another variety of regimes occurs in the case of a long phonon mean free path. In this case one must keep D_{ij} in the denominator of W_{12} in Eq. (5) in order to arrive at a finite result

$$\rho_{12}^{ph} \approx \frac{1}{16\pi^3} \frac{h}{e^2} \frac{\eta^2}{T n_e^2} \int_{1/d}^{2k_F} \frac{\sigma^2(q)\,dq}{q} \int_0^\infty \frac{\omega^2 d\omega}{\sinh^2 \dfrac{\omega}{2T}} \times \frac{e^{-2d\sqrt{q^2-(\omega/u)^2}}}{|\epsilon[\omega,\mathbf{q})(\epsilon(\omega,\mathbf{q})\sqrt{q^2-(\omega/u)^2} - 2\eta]|^2}. \tag{9}$$

With D_{ij} included, the integrand in the above expression develops a new weakly damped pole at $\omega \approx uq\sqrt{1-\eta^2}$ that corresponds to a coupled electron-phonon mode in the double-layer system. After expanding the denominator near the pole and carrying out the frequency integration over its vicinity, one obtains

$$\rho_{12}^{ph}(T) \propto \begin{cases} \tau^4 \sigma \delta^{-3} (1+\sigma^2\delta^2)^{-1}, & \delta\xi^{-1} < \tau < \delta\sigma^{-1} \\ \tau^5 \delta^{-6}, & \max(\delta\xi^{-1}, \delta\sigma^{-1}) < \tau < \delta^2 \\ \tau^3 \delta^{-2}, & \max(\delta\xi^{-1}, \delta\sigma^{-1}, \delta^2) < \tau < \delta \\ \tau, & \delta < \tau. \end{cases} \tag{10}$$

Unlike the Coulomb drag (6) that originates from small momenta $q \lesssim d^{-1}$, the integral (9 receives a negligible correction from the region where the dielectric function $\epsilon(\omega,\mathbf{q})$ approaches zero.

According to Eq. (10), the contribution of the coupled electron-phonon mode remains nearly independent of d as long as the condition $d\sqrt{q^2-(\omega/u)^2} \approx qd\eta \lesssim 1$ is satisfied for all momenta less than $2k_F$, which implies $\xi\eta < 1$.

Moreover, since the coupled electron-phonon mode can only occur provided that the denominator in Eq. (9) is not affected by the intrinsic phonon lifetime at all $q \gtrsim d^{-1}$, the additional conditions $l_{ph} \gg \eta^{-2} \max(d, (k_F\delta)^{-1}, d\sigma^{-1}\delta^{-1})$ have to be satisfied in order for Eq. (10) to hold.

A more detailed analysis shows that, similar to the situation in zero field,[4] Eqs. (7) and (9) receive their main contributions from virtual phonons that are associated with the real part of $D_{12}(\omega,\mathbf{q})$ and cause a non-Lorenzian deacy of the integrand in Eq. (9). Apart from other effects, this gives rise to a residual d dependence of ρ_{12}^{ph} in the limit $l_{ph} \to \infty$. The latter is, however, different from either $\propto \ln(l_{ph}/d)$ or $\propto \exp(-d/l_{ph})$ dependences exhibited by Eq. (7) at d smaller or greater than l_{ph}, respectively. It is for this reason that the asymptotical estimates (8) and (10) obtained for a short (long) phonon mean free path do not merge smoothly at $l_{ph} \sim d\eta^{-2}$.

In order to facilitate a comparison with the data from Ref. 14 we carried out a numerical evaluation of the transresistivity in the experimentally relevant range of parameters: $\xi \sim 10^0 - 10^2$, $\sigma \sim 10$, and $n_e = 1.5 \times 10^{11}$ cm^{-2}. For the phonon mean free path that can hardly be affected by the presence of the magnetic field we use the value $l_{ph} = 15$ μm extracted from the data taken at zero field.[15] Notably, the latter falls much too short of the threshold value d/η^2

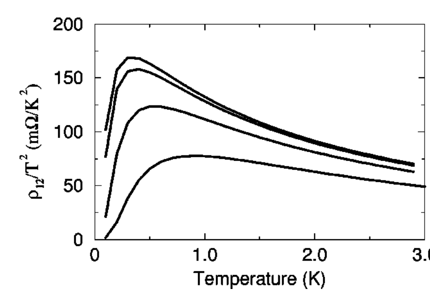

FIG. 1. Phonon-mediated transresistivity divided by T^2 versus temperature for different values of $\sigma = 5,10,15$, and 20 (in increasing order).

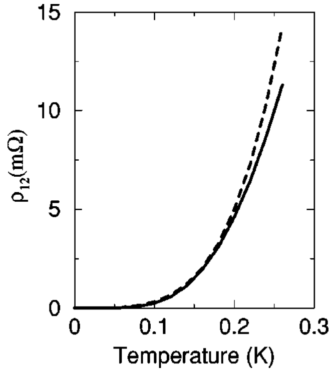

FIG. 2. $\rho_{12}(T)$ and a fit proportional to T^4 (dashed line) at low temperatures.

$\gtrsim 10^4$ μm, determining the onset of the regime of the coupled electron-phonon mode.

The numerical results show that, in accordance with Eq. (8), the peak value of the plot ρ_{12}^{ph}/T^2 shifts towards lower temperatures as the dimensionless CF conductivity σ increases while ξ remains constant (Fig. 1). Moreover, Fig. 2 reveals the T^4 behavior of the low-temperature transresistivity that should be contrasted against the best fit $\rho_{12}\sim T^{3.7}$ to the experimental data taken between 0.4 and 1.2 K.[14]

As follows from Eq. (8), the range of temperatures at which the T^4 behavior holds shortens (and eventually disappears) as σ increases past ξ. In either case, however, the exponent n in the asymptotical power law is substantially reduced compared to its values at zero field [$n=6$ or 5 (Ref. 4)].

It is worthwhile mentioning that the above mentioned downward shift of the position of the peak in Fig. 1 can easily outpower an extra factor of $\sqrt{2}$ which appears in the characteristic scale T_{BG} in the case of the spinless CF's. Interestingly enough, in Ref. 14 the peak's position was found to be roughly the same in both cases of electrons and CF's of equal densities.

As for the absolute value of the transresistivity, we find that in the CF case ρ_{12}^{ph} contains an extra factor of 2^8 which stems from the twice lower density of states and the presence of the filling factor ν in the CF dielectric function. The corresponding enhancement of the drag by two-three orders of magnitude compared to the case of zero field agrees with the data of Ref. 14 as well.

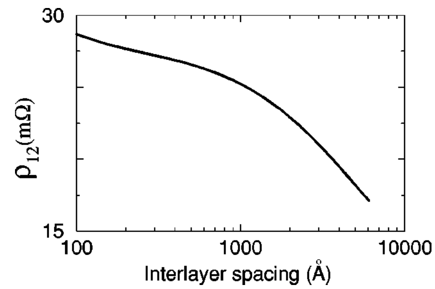

FIG. 3. ρ_{12} as a function of interlayer spacing d.

Finally, as a function of the interlayer spacing the calculated transresistivity approximately follows the same $\propto \log(l_{ph}/d)$ dependence as in the case of zero field (Fig. 3). However, the computed transresistivity turns out to be generally lower than the measured one. In the previously studied case of zero field a similar discrepancy between the theory and experiment was pointed out in Ref. 15 where it was attributed to a possible insufficiency of the random-phase approximation used to obtain the screened electron-phonon vertex.

To summarize, in our analysis of the frictional drag in the double-layer system of electrons in the half-filled Landau level we take into account the effective interlayer phonon-mediated coupling in addition to the direct Coulomb interaction. In agreement with the recent experiment,[14] we find that the phonon drag appears to be strongly enhanced compared to the case of ordinary electrons in zero field. Depending on several experimentally adjustable parameters, we obtain a variety of regimes that are all starkly different from their zero-field counterparts and reveal surprising features of the composite fermion ''pseudo-Fermi-liquid.''

[1] A.-P. Jauho and H. Smith, Phys. Rev. B **47**, 4420 (1993); L. Zheng and A.H. MacDonald, *ibid.* **48**, 8203 (1993); A. Kamenev and Y. Oreg, *ibid.* **52**, 7516 (1995); K. Flensberg, B.Y-K. Hu, A.-P. Jauho, and J.M. Kinaret, *ibid.* **52**, 14 796 (1995).

[2] T.J. Gramila *et al.*, Phys. Rev. Lett. **66**, 1216 (1991); Phys. Rev. B **47**, 12 957 (1993); Physica B **197**, 442 (1994); N.P.R. Hill *et al.*, Phys. Rev. Lett. **78**, 2204 (1997).

[3] H.C. Tso, P. Vasilopolous, and F.M. Peeters, Phys. Rev. Lett. **68**, 2516 (1992); C. Zhang and Y. Takahashi, J. Phys.: Condens. Matter **5**, 5009 (1993).

[4] M.C. Bonsager, K. Flensberg, B.Y-K. Hu, and A.H. MacDonald, Phys. Rev. B **57**, 7085 (1998).

[5] M.P. Lilly *et al.*, Phys. Rev. Lett. **80**, 1714 (1998).

[6] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[7] S. Sakhi, Phys. Rev. B **56**, 4098 (1997).

[8] I. Ussishkin and A. Stern, Phys. Rev. B **56**, 4013 (1997).

[9] I. Ussishkin and A. Stern, Phys. Rev. Lett. **81**, 3932 (1998).

[10] W. Kang *et al.*, Phys. Rev. Lett. **75**, 4106 (1995).

[11] B. Tieke *et al.*, Phys. Rev. Lett. **76**, 3630 (1996); Phys. Rev. B **58**, 2017 (1998).

[12] E. Chow *et al.*, Phys. Rev. Lett. **77**, 1143 (1996).

[13] D.V. Khveshchenko and M.Y. Reizer, Phys. Rev. Lett. **78**, 3531 (1997); Phys. Rev. B **56**, 15 822 (1998).

[14] S. Zelakiewicz *et al.*, cond-mat/9907396 (unpublished).

[15] H. Noh *et al.*, Phys. Rev. B **59**, 13 114 (1999).