Comment on "Electron Spectral Function and Algebraic Spin Liquid for the Normal State of Underdoped High T_c Superconductors"

In a recent Letter [1], Rantner and Wen made a theoretical prediction of the power-law behavior of the electron spectral function in the pseudogap phase of underdoped cuprates, reminiscent of that in the one-dimensional Luttinger liquid. This conclusion was drawn on the basis of the following (somewhat heuristic) propositions: (i) the properties of the pseudogap phase are described by the slave-boson QED_3-like effective theory formulated in terms of neutral spinons ψ, charged holons, and a gauge field A_μ; (ii) the propagator of physical electrons G_e can be computed as a simple product of the spinon (G_s) and the holon (G_h) ones; (iii) provided that the holons are (nearly) condensed ($G_h \approx$ const), G_e becomes proportional to the gauge-invariant spinon amplitude [here $x_\mu = (t, \mathbf{r})$ is a position vector in the three-dimensional space-time]

$$G_s(x) = \left\langle \psi(x) \exp\left(i \int_\Gamma A_\mu(z)dz^\mu\right) \overline{\psi}(0) \right\rangle \qquad (1)$$

with the contour Γ chosen as a straight line between the end points; (iv) the amplitude (1) decays algebraically, $G_s(x) \sim 1/|x|^{2+\eta}$, and the anomalous exponent η is *positive*. It turns out, however, that the value of η quoted in Ref. [1] and later derived in the original (the only one available at the time of submitting this Comment) version of Ref. [2] had a wrong sign, as was pointed out in Ref. [3] where instead a *negative* value, $\eta = -32/3\pi^2 N$, was obtained (the number N of fermion species in cuprates is $N = 2$).

In fact, thus far no gauge-invariant alternative to Eq. (1) that would exhibit a power-law decay with a positive η has ever been constructed, and it remains unknown if such a function can exist at all in the pure massless QED_3. In turn, the negative value of η disqualifies Eq. (1) introduced in Ref. [1] from being a viable candidate to the role of the gauge-invariant spinon propagator (much less the physical electron one), because instead of the anticipated *suppression* (as in other examples of doped Mott insulators governed by strong electron correlations), the amplitude (1) manifests *enhancement* as compared to the mean-field ($N = \infty$) result.

The argument appears to be particularly compelling in the limit of zero doping, in which case, while preventing the electrons' spatial motion by making $G_e(x)$ vanish for any $\mathbf{r} \neq \mathbf{0}$, the holon factor $G_h(x)$ does not affect the amplitude $G_e(t, \mathbf{0})$ which is directly related to the *physical* electron density of states (DOS) proportional to $\mathrm{Im} \int dt e^{i\epsilon t} G_e(t, \mathbf{0}) \sim |\epsilon|^{1+\eta}$. Therefore, a negative η would have given rise to a sublinear DOS which is increased with respect to the mean-field "V-shaped" one.

Thus, taken at its face value, the negative η invalidates the main prediction of Ref. [1] regarding the Luttinger-like behavior of the electron spectral function $A(\epsilon, \mathbf{p}) \sim \mathrm{Im} G_e(\epsilon, \mathbf{p})$ which, under the above assumptions, was identified in Ref. [1] with the Fourier transform of Eq. (1). Moreover, once the holon factor $G_h(x)$ becomes nontrivial as well, the electron spectral function [now given by a convolution of the Fourier transforms of $G_s(x)$ and $G_h(x)$] can no longer feature a simple algebraic behavior, unless *both* functions decay as power law. In the absence of any evidence suggesting otherwise, however, the possibility of such a behavior for the holons seems to be even more remote than for the spinons.

Furthermore, unless proven wrong, the absence of a physically sensible alternative to Eq. (1) in massless QED_3 may indicate a need for a revision of some of the above propositions which the work of Ref. [1] was based upon. Indeed, albeit obtained in the framework of a perturbative $1/N$ expansion, the unphysical behavior of $A(\epsilon, \mathbf{p})$ derived from Eq. (1) would have manifested itself at all energies/temperatures above a characteristic scale associated with such *nonperturbative* effects as spinon chiral symmetry breaking, holon condensation, and/or gauge field instantons. At still lower energies/temperatures, however, any of the above mechanisms may generate a finite spinon and/or gauge field gap, thereby drastically altering the power-law decay of Eq. (1), consistent with the anticipated onset of such intrinsic instabilities of the pseudogap phase as antiferromagnetism, superconductivity, and/or stripe order.

To conclude, despite its strong intellectual appeal, the QED_3 theory of underdoped cuprates has not yet provided a firm theoretical support for the Letter's prediction of the Luttinger-like or *algebraic* (which must be distinguished from both a *generic* non-Fermi liquid, characterized by a mere absence of the quasiparticle peak, and a *virtually* spin-charge separated Fermi liquid which has a small coherent peak at low energies) behavior of the electron spectral function, thus still leaving unsubstantiated the claim of its satisfactory agreement with the normal state photoemission data which was made in Ref. [1] on the basis of the original erroneous evaluation of Eq. (1).

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[2] W. Rantner and X.-G. Wen, cond-mat/0105540 (v1).
[3] D. V. Khveshchenko, Phys. Rev. B **65**, 235111 (2002).