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Simulating analogue holography in flexible Dirac metals

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Abstract – We explore an apparent holography-like relationship between the bulk and boundary properties of non-interacting massive Dirac fermions living on a flexible surface, such as a sheet of graphene. We demonstrate that the boundary correlations can mimic those normally found in the system of one-dimensional interacting fermions, a specific form of such phantom interaction being determined by the bulk geometry. This geometrical interpretation of the boundary interaction effects offers a new insight into the possible origin of the previously reported examples of the so-called generalized holographic correspondence and suggests potential ways of testing analogue holography in the experiment.

The past few years have shown a tremendous activity in the emerging area of broadly defined holographic correspondence. Originally, the notion of holography was put forward with regard to the conjectured (and, quite possibly, exact) relationship between certain highly symmetric relativistic $4d$ gauge field and $5d$ string theories, in which context it is commonly referred to as "AdS/CFT duality" [1].

More recently, the holographic conjecture was further extended by abandoning much of the original stringent symmetry conditions in the hope of still capturing some key aspects of the underlying correspondence. Specifically, it was speculated that it might also be applicable to a broad variety of non-symmetric and/or non-relativistic systems which, incidentally, would be of interest to condensed-matter physics.

Among other things, such a drastic ("non-AdS/non-CFT") generalization was motivated by the growing realization that the limited set of the classic "AdS black brane" geometries utilized in the early work [2] appears to be much too restrictive, thus allowing access to, essentially, just one specific type of all the possible non-Fermi-liquid (NFL) compressible states of fermions in d spatial dimensions.

Namely, the historic Reissner-Nordstrom (RN) solutions to the coupled Einstein-Maxwell equations which asymptotically approach the AdS_{d+2} and $AdS_2 \times R^d$ geometries in the ultraviolet (UV) and infrared (IR) limits, respectively, were shown to invariably result in the behavior dubbed as "semi-local criticality" [2]. This particular regime is characterized by the fermion propagator $G(\omega, k)$ demonstrating a non-trivial frequency, yet mundane momentum, dependence, as manifested by the self-energy $\Sigma(\omega, k) \sim \omega^{\nu_k}$, where ν_k is a regular function with no singularities at the putative Fermi surface(s).

Superficially, this "generalized marginal Fermi liquid" bears a certain resemblance to that found in some of the heavy-fermion compounds. However, it turns out to be plagued with such starkly spurious features as multiple Fermi surfaces or non-vanishing zero temperature entropy and, therefore, could only describe some intermediate, rather than the true asymptotic IR, regime.

Regardless of the physical relevance of the above scenario, though, it would be interesting to find potential gravity duals for other types of both the documented and the suspected NFL states of correlated fermions. Of particular interest are those "strange" Fermi (non-relativistic) and Dirac (pseudo-relativistic) metals where both the frequency and momentum dependence of the fermion propagator would appear to be markedly non-trivial.

The recent efforts in that direction produced a number of prospective geometries, including the Schroedinger, Lifshitz, and, especially, hyperscaling violating ones. Such metrics were found among the "electron star" solutions of the minimal Einstein-Maxwell theory with back-reaction of the fermionic matter included, as well as those of the alternate dilaton, massive vector field, and Horava gravity theories [3].

While the use of such geometries can greatly expand the list of potentially attainable boundary NFL theories, it still does not clarify the status of the generalized holographic conjecture itself. Arguably, though, establishing its true status would seem to be far more important than continuing to apply it to the ever increasing number of model geometries with physically obscure boundary duals. It is also quite likely that clarifying this central issue may not be possible without understanding *why* some holography-like relationship should be expected in the first place (*i.e.*, what physical principles would demand that).

In this paper, we make a step in that direction by demonstrating that a certain form of the apparent bulk-to-boundary correspondence can indeed be common for the systems in question, regardless of the presence of any extended (super) symmetries or a lack thereof. To that end, we show that the physical edges of the system of non-interacting $2d$ Dirac fermions propagating in curved spaces can exhibit those non-trivial properties that would be typically attributed to the effects of certain $1d$ interactions. In contrast to the aforementioned "semi-local" scenario [2], though, we show that in all these cases it is the momentum, rather than the frequency, dependence of the propagator $G(\omega, k)$ of the $1d$ boundary fermions that gets affected.

It might be tempting to view this form of correspondence as yet another aspect of the general Einstein's equivalence principle, according to which the effects of a curved metric can be described as a certain interaction. Besides, this observation suggests that, barring all the practical challenges, it might be possible to observe some signatures of such a relationship in custom-deformed flakes of $2d$ Dirac metals (*e.g.*, graphene) grown on commensurate substrates (*e.g.*, $h-BN$) which endow the bulk fermions with a finite mass via hybridization [4].

The generally covariant action describing the kinematics of massive $(d+1)$-dimensional Dirac-like electronic excitations at zero temperature and density propagating in a curved geometrical background reads (hereafter the Fermi velocity is chosen to be unity)

$$S = \int \mathrm{d}r\mathrm{d}t\mathrm{d}^d x\sqrt{|\det\hat{g}|}\bar{\psi}\gamma^a e^\mu_a \times \left(i\partial_\mu + \frac{i}{8}\omega^{bc}_\mu[\gamma_b,\gamma_c] + A_\mu - m\right)\psi. \qquad (1)$$

In the case of a flexible membrane ($d=1$), such as a strained sheet of graphene, the vielbein e^μ_a determining the induced metric $g_{\mu\nu} = e^\mu_a e^b_\nu \eta_{ab}$, vector potential A_μ, and spin connection ω^{ab}_μ can be expressed in terms of the local lattice displacement and its derivatives [5].

In what follows, we consider a class of static rotationally invariant diagonal metrics represented by the interval

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + q(r)\mathrm{d}\vec{x}^2, \qquad (2)$$

where $r \le R$ is the ("holographic") radial coordinate, and perform the Wick rotation of the time variable (measured in the laboratory frame), $t \to i\tau$, thereby switching to the Euclidean signature.

According to the standard holographic prescription [1], the (retarded) boundary propagator $G(\omega, k)$, where $k = |\vec{k}|$, of a spin-s probe particle subject to the bulk metric (2) could be obtained by finding a zero-energy solution of the radial wave equation with some effective potential $V(r,\omega,k)$ (see eq. (7) below),

$$\frac{\partial^2\psi(r,\omega,k)}{\partial r^2} = V(r,\omega,k)\psi(r,\omega,k), \qquad (3)$$

that satisfies the in-falling boundary condition [2] at the IR cutoff $r=a$. Then, expanding the thus-obtained solution in the opposite (UV) regime, *i.e.*, near the boundary at $r = R \gg a$, over the functions $\psi_\pm(r,\omega,k)$ which are chosen as normalizable and non-normalizable, respectively, one can read off the boundary propagator as the reflection coefficient for the incident radial wave

$$G(\omega,k) = \frac{\psi_+(r,\omega,k)}{\psi_-(r,\omega,k)}|_{r\to R}. \qquad (4)$$

Despite its rather specialized construction (which is neither unique, nor non-debatable [1,2]), the propagator (4) shares its singular dependence on ω and k (if any) with that of the conventional Green function of the boundary problem for the Sturm-Liouville equation (3),

$$G(r,r',\omega,k) = \frac{\theta(r-r')\psi_+(r)\psi_-(r') + (r\leftrightarrow r')}{\psi_-\frac{\mathrm{d}\psi_+}{\mathrm{d}r} - \psi_+\frac{\mathrm{d}\psi_-}{\mathrm{d}r}}, \qquad (5)$$

when the arguments r and r' simultaneously approach the boundary, $r, r' \to R$. Notably, apart from the standard matrix-valued prefactor, eq. (5) yields the ordinary propagator of massive $2d$ Dirac fermions in a curved bulk geometry. Being directly measurable by tunneling, photoemission, and other techniques, this function could then provide a tangible probe for an underlying bulk-to-boundary correspondence (if any) in the experimental simulations of analogue holography.

Although eq. (3) cannot be solved for generic gravitational backgrounds, in the regime $m(\tau, x) \gg 1$ one can resort to the semiclassical approach [6] and choose

$$\psi_\pm(r,\omega,k) \sim \frac{1}{V^{1/4}(r,\omega,k)} e^{\mp\int_r^R \mathrm{d}r'\sqrt{V(r',\omega,k)}}, \qquad (6)$$

where r_t is the turning point defined as $V(r_t)=0$. It is worth mentioning that the condition of applicability of the semiclassical approach, $mR \gg 1$, would be readily satisfied in a typical micron-size sample of graphene deposited on a $h-BN$ substrate.

Moreover, to the leading order in $mR \gg 1$, the semiclassical effective potential in (3) appears to be independent of the probe's spin,

$$V(r,\omega,k) = g(r)\left(m^2 + \frac{k^2}{q(r)} + \frac{\omega^2}{f(r)}\right) + \ldots, \qquad (7)$$

where the dots stand for the subdominant s-dependent terms [6].

Thus, the leading asymptotic large-scale decay in the space-time domain

$$G(\tau, x) \sim \exp(-S_0(\tau, x)) \tag{8}$$

of both boundary propagators (4) and (5) for a field of any spin is governed by the purely classical action

$$S(\tau, x) = m \int \mathrm{d}r \sqrt{g(r) + f(r)(\mathrm{d}\tau/\mathrm{d}r)^2 + q(r)(\mathrm{d}\vec{x}/\mathrm{d}r)^2} \tag{9}$$

computed along the extremal path between the points $(0, 0, R)$ and (τ, x, R).

After being evaluated upon such a geodesic trajectory, eq. (9) yields

$$S_0(\tau, x) = 2m^2 \int_{r_t}^{R} \mathrm{d}r \frac{\sqrt{g}(r)}{M(r)}, \tag{10}$$

where $M(r) = \sqrt{m^2 - k^2/q(r) - \omega^2/f(r)}$ and the factor of two accounts for the particle's radial excursion from R down to r_t and back.

In eq. (10) the values of the conserved canonical momenta ω and k which must be determined from the equations of motion are

$$x = k \int_{r_t}^{R} \frac{\mathrm{d}r\sqrt{g}(r)}{q(r)M(r)}, \qquad \tau = \omega \int_{r_t}^{R} \frac{\mathrm{d}r\sqrt{g}(r)}{f(r)M(r)}. \tag{11}$$

As the simplest $2d$ geometry, we first consider a flat circle of radius R with the line element

$$\mathrm{d}l^2_{flat} = \mathrm{d}r^2 + r^2\mathrm{d}\phi^2, \tag{12}$$

its natural embedding into the $3d$ Euclidean space-time being given by the interval $\mathrm{d}s^2 = \mathrm{d}\tau^2 + \mathrm{d}l^2$.

Switching from the in-plane angular variable ϕ to a (compactified) boundary coordinate $x = R\phi$ we obtain the momenta $k = m\cos(x/2R)$ and $\omega = m\tau/\sqrt{\tau^2 + 4R^2\sin^2(x/2R)}$. For the minimal path (chord) connecting two points on the circular boundary of radius R, eq. (10) then yields

$$S_{flat}(\tau, x) = m\sqrt{\tau^2 + 4R^2\sin^2(x/2R)} \tag{13}$$

which is characteristic of a non-interacting massive field. The corresponding dynamical critical exponent is $z = 1$, as suggested by the relative scaling between the spatial and temporal coordinates, $\tau \sim x^z$.

Next, we consider a surface of revolution (SOR) described by the line element

$$\mathrm{d}l^2_{sor} = \mathrm{d}r^2 \left[1 + \left(\frac{\partial h(r)}{\partial r}\right)^2\right] + r^2\mathrm{d}\phi^2, \tag{14}$$

where $h(r)$ is the vertical displacement out of the x-y plane.

For a $2d$ sheet shaped as a funnel, $h(r) \sim (R/r)^\eta$ for $r \geq a$, the "warp factor" $g(r) \sim 1/r^{2\eta+2}$ diverges at small r. One then obtains

$$S_{sor}(\tau, x) = m\sqrt{\tau^2 + (Rx^\eta)^{2/(\eta+1)}} \tag{15}$$

which reveals an unconventional behavior of the edge propagator (8) as a function of the distance along the edge. In contrast, its temporal dependence remains trivial, thus implying the "holographic" value of the dynamical critical exponent $z_{hol} = \eta/(\eta + 1)$.

It is instructive to compare the asymptotic (15) with the propagator of $1d$ fermions interacting via a pairwise potential $U(x) \sim 1/x^\sigma$ with $\sigma < 1$. The latter can be evaluated with the use of the standard bozonization technique [7]. To leading approximation, the chiral (left/right moving) components of that propagator read

$$G^{\pm}_{bos}(\tau, x) \sim \exp\left[-\int \frac{\mathrm{d}k}{2\pi}\frac{2 + U_k}{\epsilon_k}(1 - e^{\pm ikx - i\epsilon_k t})\right], \tag{16}$$

where $\epsilon_k = k\sqrt{1 + U_k}$ is the $1d$ plasmon dispersion, suggesting the dynamical exponent $z_{bos} = (1 + \sigma)/2$.

Matching the large-x asymptotics, one finds that eq. (15) mimics the spatial decay of the propagator (16), provided that $\eta = (1 - \sigma)/(1 + \sigma)$. However, comparing the long-τ asymptotics we find them to be incompatible, as the former suggests $z_{hol} = (1 - \sigma)/2$, in contrast with the above z_{bos} for all $\sigma \neq 0$.

In fact, the long-time behavior in the boundary theory would not be readily recoverable with the use of any bulk metric with a constant $f(r)$ (we expound on this point below). By contrast, in the "semi-local" AdS_2 regime [2,6] the counterpart of eq. (15), $S_{s\text{-}l}(\tau, x) = \sqrt{(1 - \nu_0)^2(\ln\tau/a)^2 + m^2x^2}$, manifests a predominantly temporal character of the NFL correlations in that case.

Another instructive example is provided by the line element

$$\mathrm{d}l^2_{log} = \mathrm{d}r^2 + R^2\exp(-2(r/R)^\lambda)\mathrm{d}\phi^2. \tag{17}$$

For $\lambda = 1$ eq. (17) represents a $2d$ surface of constant negative (Gaussian) curvature known as "Beltrami trumpet" which, using the parametrization $\rho = R\ln(R/r)$, can be transformed into the "Lobachevsky plane", $\mathrm{d}l^2 = \mathrm{d}\rho^2/\rho^2 + \rho^2\mathrm{d}\phi^2$. Notably, its conformally flat embedding into the physical $3d$ space-time was also invoked in the recent discussions of the possibility of observing the analogue Unruh-Hawking effect in graphene [8].

By computing (10) one obtains

$$S_{log}(\tau, x) = m\sqrt{\tau^2 + R^2(\ln x/a)^{2/\lambda}}. \tag{18}$$

For $\lambda = 1$ and at large x the propagator (8) then decays algebraically, $G(0, x) \sim 1/x^{mR}$ which is reminiscent of the behavior found in the $1d$ Luttinger liquids [7].

In contrast, for $\lambda \neq 1$ eq. (18) yields a variety of stretched/compressed exponential asymptotics which decay faster (for $\lambda < 1$) or slower (for $\lambda > 1$) than any power law. For instance, by choosing $\lambda = 2/3$ one can simulate a faster-than-algebraic spatial decay, $G(0,x) \sim \exp(-\mathrm{const}\ln^{3/2} x)$, in the $1d$ Coulomb gas ($\sigma = 1$) which is indicative of the incipient formation of a $1d$ charge density wave [7].

For other values of λ eq. (18) reproduces the behavior in the boundary theory governed by the interaction $U(x) \sim (\ln x)^{(2/\lambda)-3}/x$. Although the physical origin of such a bare potential would not be immediately clear, multiplicative logarithmic factors do routinely emerge in those effective $1d$ couplings that are associated with various marginally (ir)relevant two-particle operators [7].

Turning now to the metrics with $f(r) \neq \mathrm{const}$, one well-known example is provided by the so-called BTZ solution [9]. In the UV limit, the non-rotating BTZ metric approaches the AdS_3 one,

$$\mathrm{d}s^2_{AdS} = (\mathrm{d}\tau^2 + \mathrm{d}x^2)r^2 + \frac{\mathrm{d}r^2}{r^2} \tag{19}$$

and, correspondingly, eq. (10) produces the expressly Lorentz-invariant result

$$S_{AdS}(\tau, x) = 2mR\ln\left(\frac{\sqrt{\tau^2+x^2}}{a} + \sqrt{\frac{\tau^2+x^2}{a^2}+1}\right) \tag{20}$$

which is in full agreement with the exact zero-temperature boundary propagator (in the general case of a rotating BTZ solution the two chiral sectors have different temperatures [9])

$$G_{AdS}(\tau, x) \sim \frac{1}{(x-i\tau)^{2\Delta_+}(x+i\tau)^{2\Delta_-}}. \tag{21}$$

Notably, the exact left/right dimensions $\Delta_\pm = mR/2 + 1/2 \pm 1/4$ of the boundary fermion operator satisfy the condition $\Delta_+ + \Delta_- > 1$. Therefore, the corresponding boundary theory cannot be obtained from any short-ranged repulsive interaction $U(x) \sim \delta(x)$, in which case the corresponding Luttinger parameter would be restricted to the interval $1/2 \leq K \leq 1$ [7], thereby imposing the upper/lower bounds on the total conformal dimension, $1/2 \leq \Delta_+ + \Delta_- = \frac{1}{4}(K + 1/K) \leq 5/8$, contrary to the above.

Nevertheless, a power-law interaction potential $U(x) \sim 1/x^\sigma$ with $\sigma < 1$ makes the Luttinger parameter K momentum dependent and can drive it all the way down to zero for $k \to 0$, thereby raising the above upper bound. However, as already shown, this interaction results in a different, non-algebraic, x-dependence. Therefore, the boundary conformal field theory dual to the BTZ solution does not appear to have a microscopic realization in terms of any of the aforementioned pairwise potentials.

In general, while a metric with $f(r) \neq \mathrm{const}$ may not be readily attainable in the lab, it might still be possible to practically construct its conformal equivalent known as the Zermelo optical metric. More specifically, under the parametrization $r = a\coth\rho/R$ the conformally flat BTZ solution reads

$$\mathrm{d}s^2_{BTZ} = \frac{1}{\sinh^2\rho/R}\left(\left(\frac{a}{R}\right)^2 \mathrm{d}\tau^2 + \mathrm{d}\rho^2 + a^2\cosh^2\left(\frac{\rho}{R}\right)\mathrm{d}\phi^2\right) \tag{22}$$

and the corresponding optical interval is given by the expression in the brackets, its spatial part being readily identifiable as a hyperbolic pseudosphere whose practical realization has also been envisioned in the context of graphene [8]. However, it is worth mentioning that, contrary to the massless case, the propagator of massive Dirac fermions would not remain invariant under the conformal transformation relating the two metrics.

The above analysis can be extended in a number of ways. For one, the observed correspondence between the large-scale asymptotics of the two-point correlation functions in the bulk and boundary theories can be extended to the other physical observables of geometrical origin, one such example being entanglement entropy which would be naturally associated with the area of a minimal surface [1].

Also, a non-trivial background metric can be complemented by various patterns of the vector potential A_μ which for rotationally invariant configurations amounts to substituting the frequency and momentum in eq. (7) with $\omega - A_t(r)$ and $k - A_\phi(r)$. In the case of a graphene sheet, this (pseudo) electromagnetic potential would represent both elastic strain and the extrinsic curvature associated with pentagonal/heptagonal and other localized structural defects [5].

Taken at their face value, our results demonstrate that certain interaction-like features can indeed emerge at the boundary of even a non-interacting bulk theory, provided that the latter is defined in a curved space. Moreover, the phantom force between the one-dimensional boundary fermions needed to reproduce the behavior of the bulk propagator represents a strong interaction, in agreement with the general lore of the holographic correspondence, even in the case of a short-ranged potential that completely destroys the $1d$ Fermi liquid. This observation suggests that some form of a holographic relation might, in fact, be quite robust and hold regardless of whether or not the system in question is highly symmetrical, as per the original AdS/CFT conjecture, or even Lorentz invariant.

It remains to be seen whether or not the reported generic form of the bulk-to-boundary relationship could indeed account for any of the circumstantial evidence that was argued to support the condensed-matter applications of the generalized holographic conjecture [1–3,6]. However, these findings could help one to better assess the potential significance of such evidence, given that essentially all of the earlier results (with the exception of those precious few that pertain to the classic RN solutions at zero fermion density and/or mass) were obtained by solving eq. (3) numerically and often amount to a mere visual resemblance between the plots of such numerical solutions and those of some selectively chosen experimental data on the underdoped cuprates or heavy-fermion materials.

On the experimental side, such planar Dirac metals as graphene could, in principle, provide a playground for simulating the proposed "non-AdS_3/non-CFT_2" analogue holography. The availability of flexible graphene devices and advances in the stress engineering techniques may allow for an experimental study of the properties of simulated $1d$ strongly coupled systems and their dependence on the bulk geometry.

To that end, a proper choice of the substrate is instrumental for not only opening a bulk gap, but also for suppressing various edge magnetization effects which can further complicate matters. Conceivably, the boundary correlations could be probed with such relevant experimental techniques as time-of-flight, edge tunneling, and local capacitance measurements.

Lastly, some form of the bulk-boundary duality could also be anticipated in the properties of planar ($2d$) massless Dirac fermions residing on the surfaces of gapped $3d$ topological insulators which are subject to internal stress. However, apart from the obvious complexity of engineering curved $3d$ spaces, the $2d$ boundary systems are also known to be much less likely to exhibit any effects of the interactions, whether of a real or phantom (holographic) nature, thus making their experimental detection even more intricate.

To summarize, we report on a robust holography-like correspondence between the properties of free massive fermions in a curved bulk ($2d$) space and strongly interacting boundary ($1d$) fermions. Our results urge one to exercise caution when inspecting any observations believed to be supportive of the generalized holographic conjecture. Alternatively, one can choose to view them as actually exposing some generic type of the bulk-to-boundary correspondence —albeit by far simpler than any kind of relationship that can stem from a string-theoretical embedding, thereby questioning the need of invoking the latter in the first place. We also argue that the already available "massive Dirac metals" could offer a viable experimental playground for studying various aspects of the holographic phenomena, thereby making it possible to simulate and study this interesting behavior in the lab.

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