Control of Distributed Parameter Systems

July 23-27, 2007

Book of Abstracts

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CDPS 2007

IFAC Workshop on
CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

University of Namur (FUNDP)
Namur, Belgium
July 23-27, 2007

HTTP://WWW.FUNDP.AC.BE/SCIENCES/CDPS07/

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Relation between the growth of $\exp(At)$ and $\big((A+I)(A-I)^{-1}\big)^n$

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Abstract

Assume that A generates a bounded C_0-semigroup on the Hilbert space Z, and define the Cayley transform of A as $A_d := (A+I)(A-I)^{-1}$. We show that there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M \ln(n+1)$, $n \in \mathbb{N}$.

Keywords

Cayley transform, reciprocal systems, stability.

5.1 Introduction

Consider the abstract differential equation

$$\dot{z}(t) = Az(t), \qquad z(0) = z_0 \tag{5.1}$$

on the Hilbert space Z. A standard way of solving this differential equation is the Crank-Nicolson method. In this method the differential equation (5.1) is replaced by the difference equation

$$z_d(n+1) = (I + \Delta A/2)(I - \Delta A/2)^{-1} z_d(n), \qquad z_d(0) = z_0, \tag{5.2}$$

where Δ is the time step. We denote $(I + \Delta A/2)(I - \Delta A/2)^{-1}$ by A_d.

If Z is finite-dimensional, and thus A is a matrix, then it is easy to show that the solutions of (5.1) are bounded if and only if the solutions of (5.2) are bounded:

$$\sup_{t \geq 0} \|e^{At}\| =: M_c < \infty$$

if and only if

$$\sup_{n\in\mathbb{N}} \|(A_d)^n\| =: M_d < \infty.$$

However, the best estimates for M_d depend on M_c and the dimension of Z, see [2].

If Z is infinite-dimensional, then under the assumption that A and A^{-1} generate a bounded C_0-semigroup e^{At}, and $e^{A^{-1}t}$, respectively, the following estimate has been obtained,

$$M_d = \sup_{n\in N} \|(A_d)^n\| \leq 2e \cdot (M_c^2 + M_{c,-1}^2), \tag{5.3}$$

where $M_c = \sup_{t\geq 0} \|e^{At}\|$ and $M_{c,-1} = \sup_{t\geq 0} \|e^{A^{-1}t}\|$, see [1], [3], and [5]. Note that this estimate is independent of time step Δ.

However, at the moment it is unclear whether the boundedness of the semigroup generated by A implies the existence and the boundedness of the semigroup generated by A^{-1}. So we take another approach to study the behavior of $(A_d)^n$.

5.2 The growth of $(A_d)^n$

In [3] the following result is shown.

Theorem 5.2.1. *Let A generate a bounded C_0-semigroup on the Hilbert space Z, then there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M \ln(n+1)$ for $n \in \mathbb{N}$.*

The proof of [3] uses estimates on resolvents and contour integrals. We present a proof which is based on techniques from system theory. More precisely, we use Lyapunov equations to obtain the estimate. If the semigroup generated by A is exponentially stable, then for small n's the estimate in Theorem 5.2.1 can be improved. We remark that by posing an extra, nontrivial condition on the resolvent of A, one can prove boundedness of $(A_d)^n$, see [4].

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