

Control of Distributed Parameter Systems

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Book of Abstracts

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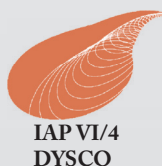
Hans Zwart

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Namur, Belgium

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Relation between the growth of $\exp(At)$ and $((A + I)(A - I)^{-1})^n$

Niels Besseling and Hans Zwart

University of Twente

P.O. Box 217

7500 AE, Enschede

The Netherlands.

`{n.c.besseling,h.j.zwart}@math.utwente.nl`

Abstract

Assume that A generates a bounded C_0 -semigroup on the Hilbert space Z , and define the Cayley transform of A as $A_d := (A + I)(A - I)^{-1}$. We show that there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M \ln(n + 1)$, $n \in \mathbb{N}$.

Keywords

Cayley transform, reciprocal systems, stability.

5.1 Introduction

Consider the abstract differential equation

$$\dot{z}(t) = Az(t), \quad z(0) = z_0 \tag{5.1}$$

on the Hilbert space Z . A standard way of solving this differential equation is the Crank-Nicolson method. In this method the differential equation (5.1) is replaced by the difference equation

$$z_d(n + 1) = (I + \Delta A/2)(I - \Delta A/2)^{-1}z_d(n), \quad z_d(0) = z_0, \tag{5.2}$$

where Δ is the time step. We denote $(I + \Delta A/2)(I - \Delta A/2)^{-1}$ by A_d .

If Z is finite-dimensional, and thus A is a matrix, then it is easy to show that the solutions of (5.1) are bounded if and only if the solutions of (5.2) are bounded:

$$\sup_{t \geq 0} \|e^{At}\| =: M_c < \infty$$

if and only if

$$\sup_{n \in \mathbb{N}} \|(A_d)^n\| =: M_d < \infty.$$

However, the best estimates for M_d depend on M_c and the dimension of Z , see [2].

If Z is infinite-dimensional, then under the assumption that A and A^{-1} generate a bounded C_0 -semigroup e^{At} , and $e^{A^{-1}t}$, respectively, the following estimate has been obtained,

$$M_d = \sup_{n \in \mathbb{N}} \|(A_d)^n\| \leq 2e \cdot (M_c^2 + M_{c,-1}^2), \quad (5.3)$$

where $M_c = \sup_{t \geq 0} \|e^{At}\|$ and $M_{c,-1} = \sup_{t \geq 0} \|e^{A^{-1}t}\|$, see [1], [3], and [5]. Note that this estimate is independent of time step Δ .

However, at the moment it is unclear whether the boundedness of the semigroup generated by A implies the existence and the boundedness of the semigroup generated by A^{-1} . So we take another approach to study the behavior of $(A_d)^n$.

5.2 The growth of $(A_d)^n$

In [3] the following result is shown.

Theorem 5.2.1. *Let A generate a bounded C_0 -semigroup on the Hilbert space Z , then there exists a constant $M > 0$ such that $\|(A_d)^n\| \leq M \ln(n + 1)$ for $n \in \mathbb{N}$.*

The proof of [3] uses estimates on resolvents and contour integrals. We present a proof which is based on techniques from system theory. More precisely, we use Lyapunov equations to obtain the estimate. If the semigroup generated by A is exponentially stable, then for small n 's the estimate in Theorem 5.2.1 can be improved. We remark that by posing an extra, nontrivial condition on the resolvent of A , one can prove boundedness of $(A_d)^n$, see [4].

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