Calculation method for sound insulation of lightweight enclosures at low frequencies

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Abstract. The objects of the study are lightweight enclosing structures (partitions) located between the premises of buildings. The surface density of lightweight fences lies in the range of $20 < \mu < 100$ kg/m². The elaboration of methods for calculating the sound insulation of lightweight enclosures, taking into account geometric and physical and mechanical parameters, is an urgent task for building acoustics. The studies are carried out on the basis of the theory of self-consistency of wave fields with account of the resonant and inertial sound transmission through the enclosures. The article presents the results of theoretical studies of sound insulation of lightweight enclosing structures in the frequency range from 50 Hz to 5000 Hz. It was proposed to divide the generalized frequency characteristic of the sound insulation of lightweight enclosing structures into two ranges, the boundary between which is determined by the geometric sizes of the fence. Based on the theory of self-consistency of wave fields, a method was developed for calculating the sound insulation of lightweight enclosing structures in the frequency range above the threshold frequency of sound field diffuseness in the plane of the enclosing structure and below the threshold frequency. The method enables calculating the sound insulation of lightweight enclosures, the threshold frequencies of the areas of resonant sound transmission, as well as the frequency characteristics of the coefficients of resonant and inertial sound transmission. The implementation of this method is considered on the example of a lightweight frame-sheathing partition with anti-resonance panels. The authors obtained theoretical frequency characteristics of the coefficients of sound transmission through a lightweight frame-sheathing partition and the frequency characteristics of the sound insulation of the partition in the calculated frequency range.

1. Introduction

The objects of the study are lightweight enclosing structures (partitions) located between the premises of buildings. The surface density of lightweight fences lies in the range of $20 < \mu < 100$ kg/m². In the high-level room there is a source of airborne noise, which generates a sound field in the volume of the room. Sound waves fall on a fence at various angles and excite an oscillatory motion in it. Vibrations of the

fence generate the emission of sound waves into an adjacent low-level room, where the sound field is also formed. At present, in the design and construction of residential, public and industrial buildings, lightweight enclosing structures are widely used: walls and partitions between rooms, as well as fences for noisy areas in industrial buildings. The most common design solutions for lightweight enclosures in construction are partitions made of sandwich panels and frame-sheathing partitions. Lightweight enclosures reduce the load on the supporting structures of buildings and decrease the material consumption of construction as a whole. It should be taken into account that the sound insulation of lightweight enclosures in many cases is lower than the sound insulation of massive fences. Therefore, when designing buildings, calculation methods should be used that allow determining the sound insulation of fences in various frequency ranges. Nowadays, there are no reliable theoretical methods for calculating the sound insulation of lightweight enclosing structures in the low-frequency sound range (f < 100 Hz). Existing calculation methods fail to take into account the geometric and physical-mechanical characteristics of fences. That is why the creation of methods for calculating the sound insulation of lightweight enclosing structures in the low-frequency sound range is an urgent task for building acoustics.

Numerous studies of the sound insulation of enclosing structures indicate the presence of significant discrepancies between theoretical and experimental values in the low-frequency region. The experimental values of sound insulation exceed the calculated values, as a rule, by 5 to 15 dB. A. London [1] and C.W. Kosten [2] assumed that the explanation of this effect necessitates taking into account the additional resistance of the enclosing structure. V.M.A. Peutz noted that the low-frequency improvement in sound insulation cannot be explained by the influence of the natural frequencies of the reverberation chambers, since the improvement occurs regardless of the size of the chambers [3]. W.A. Utley concluded that the main reason for the discrepancies between the theoretical and experimental results is the different measurement conditions in reverberation chambers [4]. A.C. Nilsson supposed that the main reason for the discrepancies between theoretical and experimental results is the absence of a diffuse sound field in the chambers [5]. M.J. Crocker, M. Battacharya, and A.J. Price applied the method of statistical energy analysis to calculate the sound insulation of enclosing structures [6], which had been previously developed by R.H. Lyon and G. Maidanik [7]. For a single-layer enclosing, the results of the theoretical calculation of sound insulation are in good agreement with the experimental results in the high-frequency range only. The authors explained the discrepancy in the field of low and medium frequencies by the presence of interaction between the natural vibration modes of the room and the partition. The method of statistical energy analysis was used by S.N. Ovsyannikov and O.V. Leliuga to take into account a structural sound transmission between rooms [8, 9].

Theoretical and experimental studies of sound insulation of lightweight enclosures made of sandwich panels are currently carried out by various scientists. The results of theoretical and experimental research of sound insulation of three-layer and five-layer (with additional lining in the middle of the medium layer) sandwich panels are presented by Y. Liu [10]. The conclusion is made about higher sound insulation values for five-layer sandwich panels compared to three-layer enclosures. In papers [11–13], a theoretical model was developed for sound transmission through sandwich panels with air gaps between the facings and the middle layer. In [14], the authors showed that the finite sizes of an enclosure affect its sound insulation, basically, in the low-frequency range. The elastic material of the middle layer makes a strong damping impact in the high-frequency range, improving the soundproofing properties of the sandwich panels. R.R. Wareing with co-authors studied the sound insulation of sandwich panels consisting of various facings [15]. Theoretical and experimental results of investigations are presented in this paper. Conclusions were made about three main factors affecting the soundproofing properties of sandwich panels: the density of the facing material, the thickness of the middle layer, and the method of fastening the facings and the middle layer. Theoretical and experimental studies of sound insulation of sandwich panels with an air gap located between two layers of elastic material of the middle layer were performed by S. Hwang et al. [16].

At present, the sound insulation of lightweight frame-sheathing partitions with various design solutions is also being intensively investigated. J.L. Davy researched the sound insulation of double leaf walls [17]. He revealed that the transmission of sound vibrations through the connecting elements between the fence sheathings must be taken into account only in the region above the resonant frequency "mass-elasticity-mass". In the field of lower frequencies, the transmission of sound vibrations between the fence sheathings occurs only through the air gap. In [18], J.L. Davy, referring to the results obtained by G. Maidanik et al., considered the emission of sound energy by a double enclosure, taking into account the contribution of resonant and non-resonant components. J. Legault and N. Atalla performed a comparative analysis of various theories of sound insulation of frame-sheathing partitions, taking into account the influence of rack profiles of the frame [19]. They confirmed that J.L. Davy's theoretical model, which takes into account the resonant response of the fence in the frequency range above the threshold frequency of wave coincidence, is the most complete. J. Poblet-Puig et al. carried out theoretical investigations of the sound insulation of frame-sheathing partitions with various shapes of rack-mount frame profiles [20]. Paper [21] presents the results of studies on optimizing the design solution of soundproof double enclosing

structures. J. Wang et al. researched the sound transmission through a frame-sheathing partition, taking into account the rack profiles of the frame [22]. The displacements of the skins during their vibrations were determined. In the articles [23] and [24], the influence of boundary conditions on the sound insulation of double enclosing structures were studied. S. Nakanishi investigated the soundproofing properties of various types of frame-sheathing partitions [25]. Partitions with a single and double frame, with single and double skins, with various options for filling the air gap with sound-absorbing material were considered. J.C.E. Wyngaert et al. developed a method for computer simulation of soundproofing frame-sheathing partitions, taking into account the deformation of the rack profiles of the frame [26]. In this case, the sound fields in the sound-transmitting and sound-receiving rooms are modeled as diffuse ones. A.A. Kochkin, N.A. Kochkin and I.L. Shubin developed new design solutions for soundproof lightweight partitions and linings with vibro-damping layers [27, 28]. J.C.E. Wyngaert, M. Schevenels, and E.P.B. Reynders elaborated a method for optimizing the parameters of the rack profiles of light frame-sheathing partitions [29, 30].

It follows from this review that at present there is no method in building acoustics for calculating the sound insulation of enclosing structures in the low-frequency sound range, which enables one taking into account geometric and physical-mechanical parameters.

The aim of this work is elaboration of method for calculating the sound insulation of lightweight enclosing structures in the low-frequency sound range. This method should take into account the geometric and physical-mechanical parameters of the enclosures. For this purpose, the following tasks are planned:

1) to investigate the resonant sound transmission in the low-frequency sound range theoretically, taking into account the self-consistency of wave fields;

2) to determine the threshold frequency, below which there are significant discrepancies in the results of theoretical and experimental studies of sound insulation of lightweight enclosures;

3) to obtain analytical expressions for calculating:

- threshold frequencies of the areas of resonant sound transmission through the lightweight enclosures;

- coefficients of both resonant and inertial transmission of sound through the enclosures;

- sound insulation of lightweight enclosures in the frequency ranges above and below the threshold frequency.

2. Method

Theoretical studies of sound insulation of lightweight building enclosures were carried out based on the theory of self-consistency of wave fields [31–34]. According to this theory, the sound transmission through enclosures has two components: the resonant sound transmission and the inertial sound transmission. The resonant sound transmission occurs during the natural vibrations of enclosing structure and is characterized by the resonant sound transmission coefficient, τ_r. Inertial sound transmission takes place during the inertial (forced) vibrations of the enclosure and is characterized by the inertial sound transmission coefficient, τ_i. The coefficient of sound transmission through a lightweight building enclosure is determined by the formula [31]:

$$\tau = \tau_r + \tau_i. \tag{1}$$

The theory of self-consistency of wave fields enables one calculating the sound insulation of single-layer and multi-layer lightweight building enclosures, taking into account their geometric sizes and physical and mechanical characteristics of materials. The method for calculating of sound insulation of three-layer sandwich panels is presented in [35–36].

3. Results and Discussion

A generalized frequency characteristic of sound insulation of lightweight building enclosures divided into two ranges is shown in Fig. 1:

1) the frequency range below the threshold frequency $\left(f < f_b \right)$. In this range, there are significant discrepancies in the results of theoretical and experimental studies of sound insulation (ΔR > 5 dB);

2) the frequency range above the threshold frequency $\left(f < f_b \right)$. In this range, there is a good agreement (ΔR = ± 2 dB) and a satisfactory agreement (ΔR = ± 5 dB) of the results of theoretical and experimental studies of sound insulation.

It should be noted that numerical values are not indicated on the coordinate axes in Fig. 1, because this graph shows only the qualitative dependence of the sound insulation of enclosure (R, dB) on the sound frequency (f, Hz).

The resonant sound transmission through the enclosure is determined by the degree of self-consistency of wave field of natural vibrations with the sound fields in the air volume of a high-level room and a low-level room.

For the case of incomplete spatial resonance, self-consistency of wave fields occurs along one of the sides of the enclosing structure, and there is no self-consistency of wave fields along the other side of the enclosure [31]:

$$m = m_0; \quad n \neq n_0; \quad m \neq m_0; \quad n = n_0, \tag{2}$$

where m and n are the numbers of projection lengths of free half-waves on sides a and b of the enclosure, respectively. In this case $m = m_0$, $n_0 = 1/2, \ 3/2, \ 5/2, \ ....$ or $n = n_0$, $m_0 = 1/2, \ 3/2, \ 5/2,$ ....

Self-consistency of the wave field of natural oscillations of the enclosing structure with sound fields in the air volume of the high-level room and low-level room is possible in the case when at least one half-wave of the sound field in the high-level room can be located along the smallest side of the enclosure:

$$n_0 > 1, \tag{3}$$

where n_0 is the number of projection lengths of half-waves of the sound field in the high-level room along the smallest side of the enclosure.

Taking into account condition (3), the frequency of self-consistency of wave fields in the high-level room and in the enclosure is determined as follows:
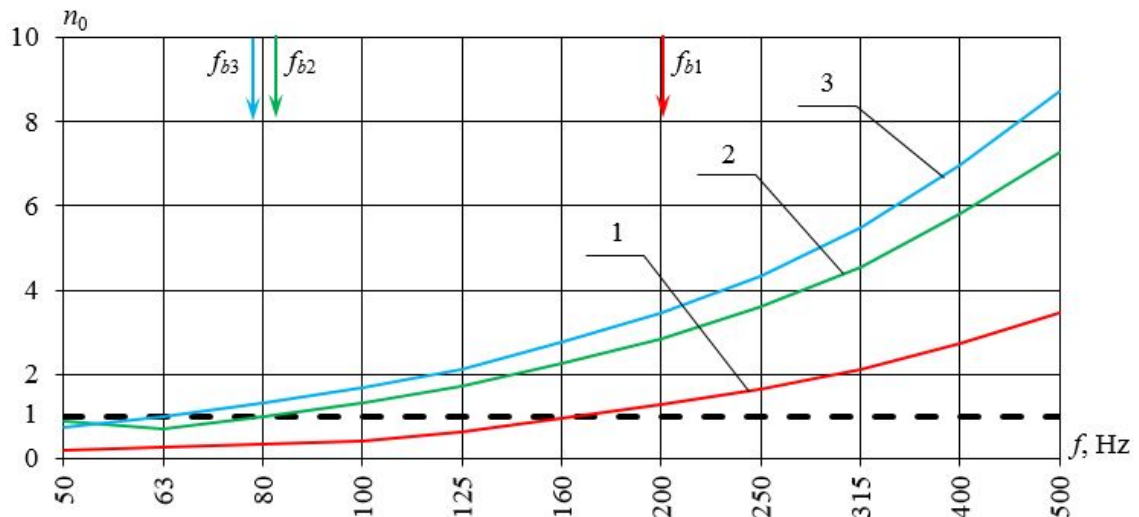
$$f_b > f \left(n_0 > 1 \right). \tag{4}$$
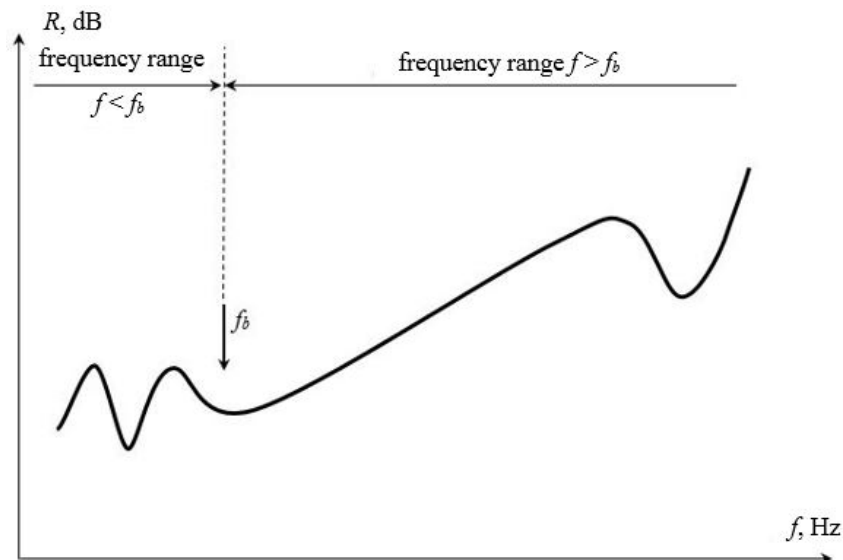
**Figure 1. The generalized frequency characteristic of sound insulation
of lightweight building enclosures divided into two frequency ranges.**

The frequency f_b is the threshold frequency of the diffuseness of the sound field in the plane of the building enclosure. The frequency characteristics of the number of lengths of the sound field half-wave projections in the high-level room along the smallest side of the enclosure, which were calculated for three lightweight frame-sheathing partitions with different geometric sizes (length a and height b), are plotted in Fig. 2.

Analysis of the data presented in Fig. 2 shows that the location of the threshold frequency f_b on the frequency scale is determined by the geometric sizes of the enclosures (length a and height b). In the frequency range $f \geq f_b$, the sound transmission consists of the resonant sound transmission (in the mode of natural oscillations of the enclosure) and the inertial sound transmission (in the mode of forced oscillations of the enclosure). In this frequency range, methods for calculating of sound insulation can be used that assume the diffuse nature of the sound fields in both high-level and low-level rooms. In the frequency range $f < f_b$, the oscillations of the enclosure are close to piston-like movements. Wherein, a sound transmits through the enclosure only in the inertial mode, there is no a resonant sound transmission $(\tau_r = 0)$.

Figure 2. The frequency characteristics of the number of lengths of projections of half-waves of the sound field in the high-level room along the smallest side of the enclosure for three light frame-sheathing partitions: $1 - a \times b = 2.0 \times 1.2$ m; $2 - a \times b = 4.2 \times 2.5$ m; $3 - a \times b = 6.0 \times 3.0$ m; f_b is the threshold frequency of the diffuseness of the sound field in the plane of the building enclosure.

Based on the results of the performed investigations, a method for calculating the sound insulation of lightweight building enclosures in the frequency ranges above the threshold frequency $\left(f \geq f_b \right)$ and below the threshold frequency $\left(f < f_b \right)$ was developed. The implementation of the calculation method is considered on the example of a lightweight frame-sheathing partition with anti-resonant panels [37]. This type of soundproofing enclosure was developed at the Department of Architecture of the Nizhny Novgorod State University of Architecture and Civil Engineering. The scheme of the constructive solution of the enclosure is shown in Fig. 3. A new type of lightweight partition has the following advantages over frame-sheathing partitions currently used in constructions: 1) anti-resonant panels are placed in the air gap between the partition sheathings. Thereby, the sound insulation of the enclosure is increased without increasing of its thickness; 2) using of sound-absorbing material (mineral wool) in the air gap between the skins is not required.
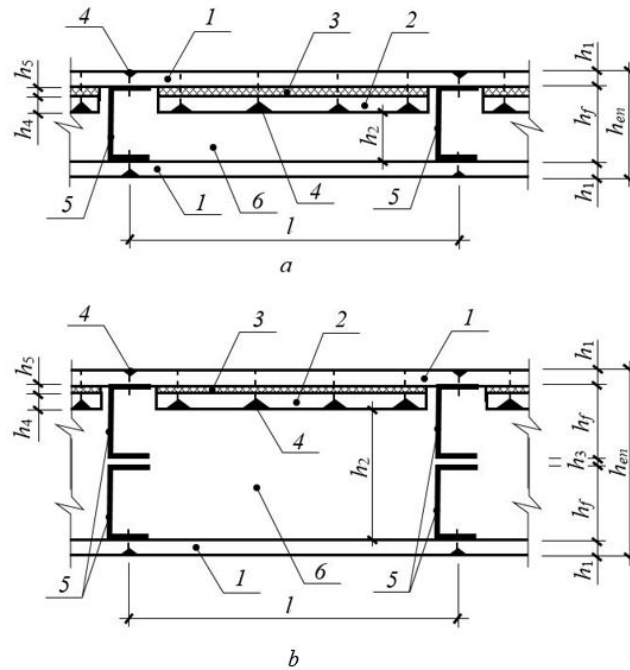
Figure 3. The scheme of a constructive solution for a framed sheathing partition with anti-resonant panels (hen is the total thickness of the enclosure): a is a partition with a single frame; b is a partition with a double frame: 1 is a sheathing with h_1-thickness; 2 is an anti-resonant panel with thickness $h_4 = h_1$; 3 is a continuous layer of elastic material with h_5-thickness; 4 are steel self-tapping screws; 5 are rack frame profiles; 6 is an air gap between sheathings with h_2-thickness.

The calculation is carried out in one-third octave frequency bands in the range from 50 Hz to 5000 Hz. The calculation method consists of six stages.

Stage No. 1. Determination of the threshold frequencies of the ranges of resonant sound transmission through a lightweight enclosure.

The threshold frequency of the range of incomplete spatial resonances (ISR) for a frame-sheathing partition is determined by the formula [31]:

$$f_{bmn_0} = \frac{c_0\sqrt{a^2 + 4b^2}}{4ab} + \Delta f_{bmn_0}, \tag{5}$$

where c_0 is the velocity of a sound wave in air, m/s; a and b are the length and the height of the enclosure, respectively, m; Δf_{bmn_0} is the positive frequency correction up to the nearest higher frequency of natural oscillations of the leaf sheathing, Hz.

Rack frame profiles form separate rectangular cells along the length of the partition. This leads to the appearance of additional conditions for self-consistency of wave fields within these cells. The length of the frame cell is assumed to be equal to the spacing of the rack profiles of the partition frame $(a_1 = l)$. The threshold frequency of the range of incomplete spatial resonances for a frame cell is calculated by the formula:

$$f_{bmn_0}(fc) = \frac{c_0\sqrt{a_1^2 + 4b_1^2}}{4a_1b_1} + \Delta f_{bmn_0}, \tag{6}$$

where a_1 and b_1 are the length and the height of the frame cell, respectively, m; Δf_{bmn_0} is the positive frequency correction up to the nearest higher frequency of natural oscillations of the leaf sheathing, Hz.

The threshold frequency of the range of total spatial resonances (TSR) for a frame-sheathing partition is determined by the formula [31]:

$$f_{bmn} = \frac{c_0^2}{2\pi}\sqrt{\frac{\mu}{D}}, \tag{7}$$

where μ is the surface density of the sheathing, kg/m²; D is the cylindrical stiffness of the sheathing, Pa·m³.

The resonant frequency of the system "mass-elasticity-mass" ("sheathing – air gap – sheathing") is determined by the well-known formula [31], which can be written for the case of identical sheathings in the following form:

$$f_{msm} = \frac{1}{2\pi}\sqrt{\frac{2E_0}{h_2\mu_1}}, \tag{8}$$

where E_0 is the dynamic modulus of elasticity of the air, Pa; μ_1 is the surface density of the sheathing, kg/m²; h_2 is the thickness of the air gap between the sheathings, m.

For an anti-resonant panel ("leaf sheathing – elastic layer – anti-resonant panel"), in the case of identical materials for leaf sheathings and anti-resonant panels, equation (8) takes on the form:

$$f_{msm(\mathrm{ARP})} = \frac{1}{2\pi}\sqrt{\frac{2E_2}{h_5\mu_1}}, \tag{9}$$

where E_2 is the dynamic modulus of elasticity of the material of the elastic layer located between the leaf sheathing and the anti-resonant panels, Pa; h_5 is the thickness of the elastic layer located between the sheathing and the anti-resonant panels, m.

Stage No. 2. Calculation of the resonant sound transmission coefficient.

The calculation is carried out in the frequency ranges of resonant sound transmission with different conditions of self-consistency of wave fields in the high-level room and low-level room with the wave field of natural oscillations of the partition: in the frequency ranges of simple spatial resonances (SSR), incomplete spatial resonances (ISR), total spatial resonances (TSR). Taking into account the change in the conditions for additional self-consistency of wave fields in the frame cells due to oscillations of the anti-resonant panels (ARP), the formula for calculating the resonant sound transmission coefficient is transformed as follows [31]:

$$\tau_r = \tau_{sr} + \tau_{1r}\tau_{2r}, \tag{10}$$

where τ_{sr} is the coefficient of resonant sound transmission through both sheathings with an elastic connection between them; τ_{1r} is the coefficient of resonant sound transmission through the sheathing located on the side of the high-level room; τ_{2r} is the coefficient of resonant sound transmission through the sheathing located on the side of the low-level room. Equation (10) is valid for the anti-resonant panel in the frequency range above the resonant frequency of the "mass-elasticity-mass" system $\left(f \geq f_{msm(\mathrm{ARP})}\right)$.

In the range $f < f_{msm(\mathrm{ARP})}$, equation (10) takes on the form:

$$\tau_r = \tau_{sr} + \tau_{1r}\tau_{2r(fc\,\mathrm{ARP})}, \tag{11}$$

where $\tau_{2r(fc\,\mathrm{ARP})}$ is the coefficient of resonant sound transmission through the sheathing located on the side of the low-level room, calculated for the frame cell (with geometric sizes $a_1 \times b_1$) taking into account the anti-resonant panel.

The coefficient of resonant sound transmission through both sheathings with an elastic connection between them is determined by the formula [31]:

$$\tau_{sr} = \frac{1}{\dfrac{\pi^2}{\rho_0^2 c_0^2}\dfrac{\mu^2 f^2}{A^2}\left(\dfrac{f^2}{f_{msm}^2}-1\right)^2 + 1}, \tag{12}$$

where $\rho_0 c_0$ is the characteristic air impedance; μ is the surface density of the partition, kg/m²; f is the current sound frequency, Hz; A is the characteristic of self-consistency of the wave field of natural oscillations of the sheathings with the sound fields in the high-level room and low-level room; f_{msm} is the resonant frequency of the "mass-elasticity-mass" system ("sheathing – air gap – sheathing"), Hz.

The coefficients of resonant sound transmission through the sheathings of the partition are calculated by the formulas [31]:

- in the frequency ranges of SSR and ISR $\left(f_{bm_0 n_0} < f < f_{bmn} \right)$:

$$\tau_{1r} = \frac{1}{\dfrac{1.15\pi^3}{8\rho_0^2 c_0^2 A^4} \mu_1^2 f^2 \eta_1 \cos^2 \theta_m + 1}, \quad \tau_{2r} = \frac{1}{\dfrac{1.15\pi^3}{8\rho_0^2 c_0^2 A^4} \mu_1^2 f^2 \eta_1 \cos^2 \theta_2 + 1}, \tag{13}$$

where $\rho_0 c_0$ is the characteristic air impedance; A is the characteristic of self-consistency of the wave field of natural oscillations of the sheathings with the sound fields in the high-level room and low-level room; μ_1 is the surface density of sheathing, kg/m²; f is the current sound frequency, Hz; η_1 is the coefficient of loss of the sheathing material; θ_m = 51.76° is the average angle of incidence of diffuse sound field waves on the partition from the high-level room; θ_2 is the angle of incidence of sound waves from the air gap onto the sheathing located on the side of the low-level room, degrees;

- in the frequency range of TSR $\left(f > f_{bmn} \right)$:

$$\tau_{1r} = \frac{1}{\dfrac{8\pi}{\rho_0^2 c_0^2} \mu_1^2 \dfrac{f^3}{f_{bmn}} \eta_1 \cos \theta_m \sqrt{1 - \dfrac{f_{bmn}}{f}} + 1},$$

$$\tau_{2r} = \frac{1}{\dfrac{8\pi}{\rho_0^2 c_0^2} \mu_1^2 \dfrac{f^3}{f_{bmn}} \eta_1 \cos \theta_2 \sqrt{1 - \dfrac{f_{bmn}}{f}} + 1}, \tag{14}$$

where $\rho_0 c_0$; μ_1, η_1, θ_m; f are the same as in formula (13); f_{bmn} is the threshold frequency of the TSR frequency range for sheathing of partition, Hz.

Stage No. 3. Calculation of the inertial sound transmission coefficient.

The calculation is carried out according to the formula [31]

$$\tau_i = \tau_{si} + \tau_{1i}\tau_{2i}, \tag{15}$$

where τ_{si} is the coefficient of inertial sound transmission through both sheathings with an elastic connection between them; τ_{1i} is the coefficient of inertial sound transmission through the sheathing located on the side of the high-level room; τ_{2i} is the coefficient of inertial sound transmission through the sheathing located on the side of the low-level room.

The coefficient of inertial sound transmission through both sheathings with an elastic connection between them is determined by the formula [31]:

$$\tau_{si} = \frac{1}{\dfrac{\pi^2}{\rho_0^2 c_0^2} \dfrac{\mu^2 f^2}{F_{1i}^2} \left(\dfrac{f^2}{f_{msm}^2} - 1 \right)^2 + 1}, \tag{16}$$

where $\rho_0 c_0$ is the characteristic air impedance; μ is the surface density of the partition, kg/m²; f is the current sound frequency, Hz; F_{1i} is the response function for the sheathing located on the side of the high-level room; f_{msm} is the resonant frequency of the "mass-elasticity-mass" system ("sheathing – air gap – sheathing"), Hz.
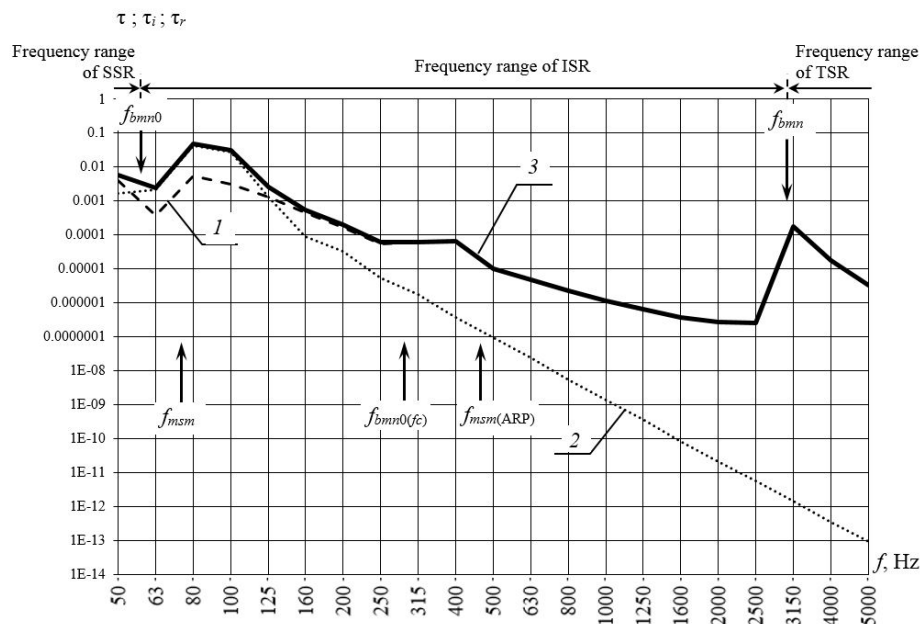
The coefficients of inertial transmission of sound through the sheathings of the partition are determined by the formulas [31]:

$$\tau_{1i} = \cfrac{1}{\cfrac{\pi^2}{\rho_0^2 c_0^2} \cfrac{\mu_1^2 f^2 \cos^2 \theta_m}{F_{1i}^2} + 1}, \quad \tau_{2i} = \cfrac{1}{\cfrac{\pi^2}{\rho_0^2 c_0^2} \cfrac{\mu_1^2 f^2 \cos^2 \theta_2}{F_{2i}^2} + 1}, \tag{17}$$

where μ_1 is the surface density of sheathing, kg/m²; F_{1i} is the response function for the sheathing located on the side of the high-level room; F_{2i} is the response function for the sheathing located on the side of the low-level room.

The frequency characteristics of the coefficients of sound transmission through a frame-sheathing partition with anti-resonant panels, which are calculated using the formulas (1), (10) – (17), are plotted in Fig. 4. Analysis of the data presented in Fig. 4 shows that in the field of low sound frequencies ($f \le 125$ Hz), the values of the coefficients of resonant and inertial sound transmission are comparable. Here, the inertial sound transmission predominates slightly over the resonant sound transmission. In the medium- and high-frequency sound ranges ($f \ge 160$ Hz), the predominant contribution to the sound transmission through the lightweight partition is made by the resonant component. In the frequency ranges located near resonant frequencies $\left(f_{msm}, \ f_{bmn_0}(f_c), \ f_{msm(\mathrm{ARP})}, \ f_{bmn} \right)$, there are rises of the frequency characteristics of the resonant sound transmission coefficient (τ_r) and the total sound transmission coefficient through the partition (τ). The most significant increase of resonant sound transmission through a lightweight partition is observed near the resonant frequency of the "mass-elasticity-mass" system $\left(f_{msm} \right)$ and near the threshold frequency of the TSR frequency range for sheathing of partition $\left(f_{bmn} \right)$.

Figure 4. The theoretical frequency characteristics of the coefficients of sound transmission through a lightweight frame-sheathing partition with anti-resonant panels, with a double frame ($a \times b$ = 4.2 × 2.5 m; thickness of the air gap between the sheathings is h_2 = 89 mm; sheathings are made by gypsum fiber sheets with thickness h_1 = 12.5 mm; spacing of rack profiles of the frame is a_1 = 0.6 m; cross-sectional height of rack profiles of the frame is h_f = 50 mm; distance between rack profiles of the double frame is h_3 = 5 mm; anti-resonant panels are made by gypsum fiber sheets with thickness h_4 = 12.5 mm; elastic layer between sheathing and anti-resonant panels is made by mats of polyester fiber with thickness h_5 = 12.5 mm; total thickness of the partition is hen = 130 mm): 1 – coefficient of resonant sound transmission through the partition (τ_r); 2 corresponds to the coefficient of inertial sound transmission through the partition (τ_i); 3 corresponds to the total coefficient of sound transmission through the partition (τ).

Stage No. 4. Calculation of the threshold frequency of the diffuseness of the sound field in the plane of the enclosure (*f_b*).

The calculation of the number of lengths of projections of half-waves of the sound field in the high-level room along the smallest side of the partition is performed:

$$n_0 = b\sqrt{\frac{4f^2}{c_0^2} - \frac{1}{a^2}},\tag{18}$$

where c_0 is the velocity of sound wave in air, m/s; a and b are the length and the height of the enclosure, respectively, m; f is the current sound frequency, Hz.

The nearest one-third octave sound frequency, for which condition (4) is satisfied, is the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f_b\right)$.

Stage No. 5. Calculation of the sound insulation of a lightweight partition in the frequency range above the threshold frequency of the diffuseness of the sound field in the plane of the enclosure (f ≥ f_b).

The calculation is executed according to the formula given in [31], taking into account the values of the resonant sound transmission coefficient and the inertial sound transmission coefficient found at the previous stages:

$$R = 10\lg\left(\frac{1}{\tau_r + \tau_i}\right),\tag{19}$$

where τ_r is the resonant sound transmission coefficient and τ_i is the inertial sound transmission coefficient.

Stage No. 6. Calculation of the sound insulation of a lightweight partition in the frequency range below the threshold frequency of the diffuseness of the sound field in the plane of the enclosure (f < f_b).

In the frequency range below the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f < f_b\right)$, there is no a resonant sound transmission, therefore $\tau_r = 0$. Taking this into account, formula (19) takes on the following form:

$$R = 10\lg\left(\frac{1}{\tau_i}\right).\tag{20}$$

The inertial sound transmission coefficient, τ_i, is calculated by formula (15), while the response functions of the partition's sheathings are taken to be $F_{1i} = F_{2i} = 1$.

Based on the results of the calculation, the frequency characteristic of the sound insulation of a frame-sheathing partition with anti-resonant panels is constructed. The frequency characteristics of the sound insulation of the lightweight partition under study, which are determined by formulas (19) and (20), are plotted in Fig. 5.
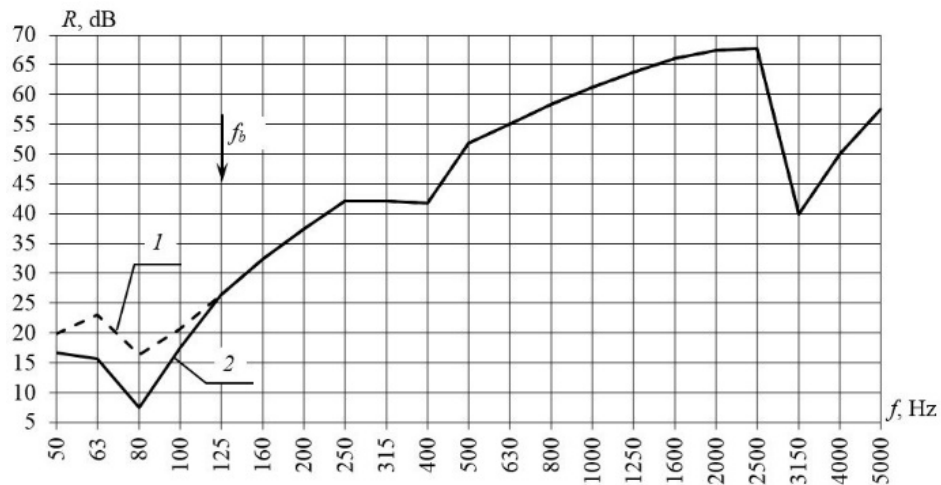
Figure 5. Theoretical frequency characteristics of sound insulation of a light frame-sheathing partition with anti-resonant panels, with a double frame ($a \times b$ = 2.2 × 2.65 m; the remaining parameters of the partition are similar to those given in the caption to Fig. 4): curve 1 corresponds to calculation of sound insulation in the entire frequency range is performed taking into account the resonant and inertial sound transmission (according to formula (19)); curve 2 corresponds to calculation of sound insulation in the frequency range below the threshold frequency ($f < f_b$) is performed taking into account only the inertial sound transmission (according to formula (20)); the calculation of sound insulation in the frequency range above the threshold frequency ($f \geq f_b$) was performed taking into account the resonant and inertial sound transmission (according to (19)).

Analysis of the presented data shows that in the frequency range $f < f_b$, the sound insulation of a light partition, which is calculated according to the proposed method with account only of the inertial sound transmission (graph 2), is 2–8 dB lower than the sound insulation calculated taking into account the resonant and inertial sound transmission (graph 1). In the frequency range above the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f \geq f_b \right)$, graphs 1 and 2 coincide.

4. Conclusions

Based on the results of the theoretical studies, the following conclusions can be drawn:

1. We proposed to divide the generalized frequency characteristic of the sound insulation of lightweight enclosing structures into two ranges, the boundary between which is found by the threshold frequency of the diffuseness of the sound field in the plane of the enclosure. The location of the threshold frequency on the frequency scale is determined by the geometric sizes of the fence (the length a and the height b).

2. In the frequency range above the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f \geq f_b \right)$, the sound transmission consists of the resonant sound transmission (in the mode of natural oscillations of the enclosure) and the inertial sound transmission (in the mode of forced oscillations of the enclosure). In this frequency range, methods for calculating the sound insulation can be used that assume the diffuse nature of the sound fields in high-level room and low-level room. In the frequency range below the threshold frequency $\left(f < f_b \right)$, the oscillations of the enclosure are close to piston-like movements, and the sound transmission occurs only in the inertial mode (there is no resonant sound transmission, i.e. $\tau_r = 0$).

3. Based on the theory of self-consistency of wave fields, we developed a method for calculating the sound insulation of lightweight building enclosures in the frequency range above the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f \geq f_b \right)$, and below the threshold frequency $\left(f < f_b \right)$. The method enables one calculating the sound insulation of lightweight enclosures, the threshold frequencies of the ranges of resonant sound transmission, and the frequency characteristics of the resonant and inertial sound transmission coefficients.

4. The implementation of the calculation method is considered on the example of a lightweight frame-sheathing partition with anti-resonant panels. The frequency characteristics of the resonant and inertial sound transmission coefficients, as well as the total coefficient of sound transmission through a light partition are presented. It was established that the most significant increase in the resonant sound transmission occurs near the resonant frequency of the "mass-elasticity-mass" system $\left(f_{msm} \right)$ and near the resonant frequency of the TSR range for the partition sheeting $\left(f_{bmn} \right)$.

5. The frequency characteristics of sound insulation of a lightweight frame-sheathing partition with anti-resonant panels are presented. It was established that in the range below the threshold frequency of the diffuseness of the sound field in the plane of the enclosure $\left(f < f_b \right)$ the sound insulation of a lightweight partition, which is calculated according to the proposed method considering only the inertial sound transmission, is 2–8 dB lower than the sound insulation calculated taking into account the resonant and inertial sound transmission.

6. The developed method can be applied to calculate the sound insulation of lightweight building enclosures in the frequency range from 50 Hz to 5000 Hz, including the low sound frequency range ($50 < f < 100$ Hz).

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