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Research article

Official long-term and short-term strategies for preventing the spread of rumors

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Abstract: Recently, public security incidents caused by rumor spreading have frequently occurred, leading to public panic, social chaos and even casualties. Therefore, how governments establish strategies to restrain rumor spreading is important for judging their governance capacity. Herein, we consider one long-term strategy (education) and two short-term strategies (isolation and debunking) for officials to intervene in rumor spreading. To investigate the effects of these strategies, an improved rumor-spreading model and a series of mean-field equations are proposed. Through theoretical analysis, the effective thresholds of three rumor-prevention strategies are obtained, respectively. Finally, through simulation analysis, the effectiveness of these strategies in preventing rumor spreading is investigated. The results indicate that long-term and short-term strategies are effective in suppressing rumor spreading. The greater the efforts of governments to suppress rumors, the smaller the final rumor size. The study also shows that the three strategies are the best when applied simultaneously. The government can adopt corresponding measures to suppress rumor spreading effectively.

Keywords: rumor spreading; official control; intervention strategy; critical threshold

1. Introduction

Throughout the history of human development, rumors have always existed. Rumors are typically based on individual self-interest or psychological anxiety. Furthermore, they are a collective action by social groups to gain social cognition and a medium for expressing public opinions and a noninstitutionalized way of participation. With the iterative replacement of media, rumors seem to have wings, the scope of spread continues to expand and their influence gradually increases. In particular, in major emergencies such as the COVID-19 outbreak, the rapid spread of rumors has evolved into a serious problem in social management that cannot be ignored.

Based on the theory of infectious diseases, the quantitative studies of rumor spreading have made great progress, from the research on rumor spreading in the small-scale social network stage represented by the classic DK model [1] and MT model [2] to the large-scale social stage in which complex network theory is applied [3,4]. In recent years, with the enrichment of scientific theoretical knowledge, scholars have redefined the research to include population reclassification [5], multiplex environment [6,7], complex network structure [8,9], individual characteristics [10,11], media functions [12] and so on. Rumor spreading is an extremely complex process with several factors influencing the process. Integrating all the influencing factors and establishing a unified and effective rumor-spreading model is impossible. However, scholars have studied the rumor spreading process from various perspectives with different focuses, which can provide some reliable guidance for practice.

Compared with the unreasonable and ever-changing rumor spreading, it is difficult to block them, showing that "A lie can travel halfway around the world while the truth is putting on its shoes," which is determined as a difficult issue. These problems are of high concern to society and receive considerable academic attention [13,14]. Scholars have conducted considerable amount of theoretical research on controlling rumor spreading. Zhu and Wang added the people who did not care about rumors in the classic rumor propagation model and introduced a silence-forcing function to reduce the frequency of rumor propagation from the perspective of optimal control methods [15]. He et al. designed cost-effective strategies and determined the minimal cost of suppressing rumor propagation in social networks [16]. Huo and Zhang proposed a modified rumor propagation model and showed that people are more likely to believe the global rumor refutation information employed by official information than the local rumor refutation information [17]. Zhang et al. found that the quantity of official rumor refutation information played a positive but noncritical role in the rumor dissemination process, and it even exhibited a backfire effect on the rumor controlling process under certain conditions [18]. Tian and Ding considered debunking behavior in the rumor-spreading process and proposed an ILRDS model to describe the rumor dynamics in OSNs during emergencies [19]. Zhu et al. considered a rumor-spreading model with discontinuous control strategies and studied the conditions for the existence of positive equilibrium and its global asymptotic stability [20]. In addition, some scholars introduced the idea of considering the cases of infectious diseases into the rumor propagation model and applied the method of isolating and immunizing individuals to suppress rumors [21,22]. Some studies studied rumor propagation suppression strategies within two stages [23] or two communication contexts [24] to achieve a close to real-life situation.

When more factors are considered, rumor models become more complex. The complex models seem to be more relevant to real life. However, most of the aforementioned rumor management propagation models are proof of basic reproduction number or equilibrium point stability obtained in open systems. Although these research theories are significant, they are relatively obscure and difficult to explain in practical operations. Based on a study [25], there are two ways to suppress rumor spreading: blocking rumors from influential users and spreading the truth to clarify rumors. Blocking rumors serves similarly as an isolation strategy for infectious diseases; for instance, blocking Weibo accounts from the Internet so that users cannot publish information. While spreading the truth, for example, during the COVID-19 pandemic, official and nonofficial organizations have adopted corresponding rumor-refuting websites to inhibit rumor spreading. These methods were short-term measures taken in response to rumors. Based on an old Chinese saying, "Rumor ends with wise men." However, the question regarding how to be a wise person arises. In addition to individual innate factors, acquired education is important [26]. Raising the public to be wise is a long-term strategy for the government to deal with rumor spreading. Therefore, this study proposes an improved rumor propagation model considering long-term and short-term strategies. The propagation mechanism of this model can well reflect the education strategy, isolation strategy and debunking strategy (spreading the truth to clarify rumors). The mean-field equations are provided, and through mathematical analysis, the thresholds for suppressing rumor spreading under different strategies are obtained. Using the simulation method, the effectiveness of different strategies is discussed and verified, which provides a policy-making reference for the government to effectively and efficiently control rumors. The study is outlined as follows. Section 2 introduces the improved rumor-spreading model in detail. In Section 3, the critical thresholds for preventing the spread of rumors are obtained. In Section 4, the numerical study is performed to verify the theoretical analysis. Section 5 presents the conclusions.

2. Improved rumor-spreading model

Moreno et al. presented a classic classification method for the rumor propagation model [3] in which the crowd was categorized into three groups: ignorant (people who have not heard of the rumor, denoted by I), spreader (people who know the rumor and spread it, denoted by S) and stifler (people who know the rumor but do not spread it, denoted by R). We have adopted their notation but re-adjusted the propagation rules to be more realistic.

First, considering the implementation of the long-term education strategy of the government, it can be said that education makes people wiser and more rational. Therefore, we further divide the I group into two categories, namely the perceptual ignorant, denoted as I_g, and the rational ignorant, denoted as I_l . I_g tend to believe and spread rumors, that is, people who can switch from I to S with higher probabilities. Conversely, I_l .are reluctant to believe and spread rumors. They belong to the category that transform from I to S with lower probabilities. Second, considering the implementation of the short-term strategies of officials, we divide stifles into two categories, namely, stop-spreader and truth-spreader, denoted as R_s and R_t , respectively. R_s are those who do not actually see through the rumors and just do not spread them. The S who has implemented the isolation strategy will become a R_s. Knowing that the rumor is misinformation, the R_t will disseminate information against the rumor and may play a positive role in terminating the rumor spread.

In a closed system consisting of N individuals, each individual in the network is regarded as a vertex and the connection between people is regarded as an edge. The connection direction of the edges between the vertices is not considered. An abstract social network $G = (V, E)$ is obtained, where V is the set of all vertices representing individuals in the society and E is the set of all edges representing the connections between people. According to previous assumptions, we divide people into five groups, I_g, I_l, S, R_s and R_t, respectively. The transformation relationships between various groups are shown in Figure 1, in which parentheses indicate meeting a certain type of person.

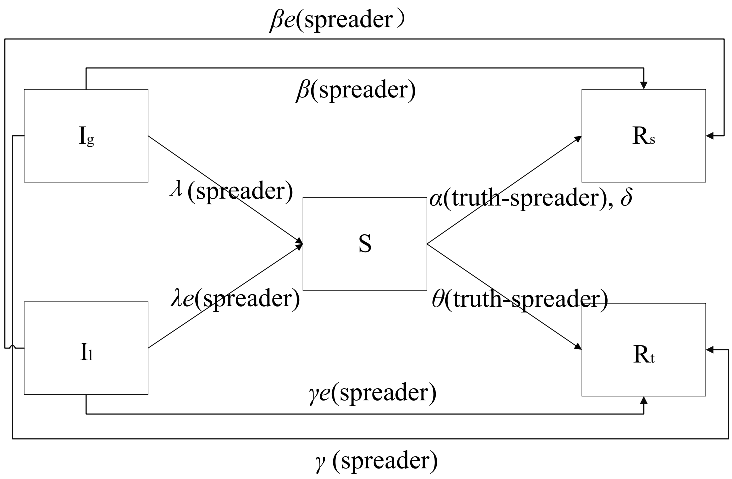


Figure 1. Rumor propagation process.

Combining Figure 1, we propose the following rules for rumor spreading:

① When a I_g contacts a S, three scenarios arise. The I_g who gets very interested in rumor spreading will become a S with probability λ; the I_g who has no interest in the rumor and is unable to discern the accuracy of the information will become a R_s with probability β; the I_g who has the relevant knowledge and can see through the rumor immediately will become a R_t with probability γ. Whichever of the aforementioned three situations occurs, the I_g is already aware of the rumor and is impossible to remain I. Thus, $\lambda+\beta+\gamma=1$.

② Similar to I_g, when a I_l contacts a S, the I_l may become a S with probability λ_e, a R_s with probability β_e, or a R_t with probability γ_e. We also have $\lambda_e+\beta_e+\gamma_e=1$. According to the hypothesis of the model, it is evident that $\lambda > \lambda_e$ and $\gamma < \gamma_e$.

③ When a S contacts a R_t, they may become a R_s with probability α or another R_t with probability θ. Here we argue that only the R_t has an influence the S. When the S contacts the R_t, he/she recognize the essence of the rumor and becomes a R_t. In addition, it is also possible that even after contacting the R_t, the S may still not figure out the truth and becomes confused. The S does not know whether to believe the rumor or not and gives up spreading the rumor and becomes a R_s.

④ The S will also spontaneously terminate the spread of a rumor with probability δ and become a R_s due to forgetting or losing interest, defined as the forgetting mechanism. The behavior of stopping the rumor from spreading due to the forgetting mechanism is not related to the falsehood of the rumor.

Moreover, $I_g(t)$, $I_l(t)$, $S(t)$, $R_s(t)$, and $R_t(t)$ denote the density of these five individuals at time t, respectively. In this case, based on the rumor-spreading process, the mean-field equations can be presented as follows:

$$\frac{dI_g(t)}{dt} = -(\lambda+\beta+\gamma)\bar{k}I_g(t)S(t) \tag{1}$$

$$\frac{dI_l(t)}{dt} = -(\lambda_e+\beta_e+\gamma_e)\bar{k}I_l(t)S(t) \tag{2}$$

$$\frac{dS(t)}{dt} = \lambda \bar{k} I_g(t)S(t) + \lambda_e \bar{k} I_l(t)S(t) - \alpha \bar{k} S(t)R_t(t) - \theta \bar{k} S(t)R_t(t) - \delta S(t) \tag{3}$$

$$\frac{dR_s(t)}{dt} = \beta \bar{k} I_g(t)S(t) + \beta_e \bar{k} I_l(t)S(t) + \alpha \bar{k} S(t)R_t(t) + \delta S(t) \tag{4}$$

$$\frac{dR_t(t)}{dt} = \gamma \bar{k} I_g(t)S(t) + \gamma_e \bar{k} I_l(t)S(t) + \theta \bar{k} S(t)R_t(t) \tag{5}$$

where $\bar{k}$ denotes the average degree of the network. The normalization condition is $I_g(t) + I_l(t) + S(t) + R_s(t) + R_t(t) = 1$.

3. Preventing effects of long-term and short-term strategies

3.1. Long-term education strategy

First, from the aforementioned condition $\lambda+\beta+\gamma=\lambda_e+\beta_e+\gamma_e=1$, we can obtain a relational expression between the I_g and I_l as $I_l(t) = \varepsilon I_g(t)$, where ε is a positive constant. We can divide Eq (2) by Eq (1) to obtain the following equations:

$$\frac{dI_l(t)}{dI_g(t)} = \frac{-(\lambda_e + \beta_e + \gamma_e)\bar{k} I_l(t)S(t)}{-(\lambda + \beta + \gamma)\bar{k} I_g(t)S(t)} = \frac{I_l(t)}{I_g(t)},$$

then

$$I_g(t)dI_l(t)\text{-}I_l(t)dI_g(t)=0 \Rightarrow \frac{I_g(t)dI_l(t)\text{-}I_l(t)dI_g(t)}{I_g^{\,2}(t)}=0 \Rightarrow \frac{d}{dt}\left(\frac{I_l(t)}{I_g(t)}\right) = 0.$$

The first derivative is zero, and the function is constant. There is $\frac{I_l(t)}{I_g(t)} = \varepsilon$, that is the above relationship is established. ε is called the education factor, which is the ratio of the density of I_l to the density of I_g. The bigger the ε, the better the official education strategy for the people.

The spreading process starts with a few people being informed of a rumor. Therefore, we assumed that the initial condition for spreading the rumor is $S(0) = \frac{1}{N} \approx 0$, $I_g(0) + I_l(0) = \frac{N-1}{N} \approx 1$, $R_s(0) = R_t(0) = 0$. From the relationship between I_g and I_l, it is easy to obtain $I_g(0) \approx \frac{1}{1+\varepsilon}$, $I_l(0) \approx \frac{\varepsilon}{1+\varepsilon}$. We can then divide Eqs (4) and (5) by Eq (1) to obtain the following equations:

$$\frac{dR_s(t)}{dI_g(t)} = \frac{\beta \bar{k} I_g(t)S(t) + \beta_e \bar{k} I_l(t)S(t) + \alpha \bar{k} S(t)R_t(t) + \delta S(t)}{-\bar{k} I_g(t)S(t)}$$

$$= -(\beta + \beta_e \varepsilon) - \alpha \frac{R_t(t)}{I_g(t)} - \frac{\delta}{\bar{k} I_g(t)} \tag{6}$$

$$\frac{dR_t(t)}{dI_g(t)}=\frac{\gamma\bar{k}I_g(t)S(t)+\gamma_e\bar{k}I_l(t)S(t)+\theta\bar{k}S(t)R_t(t)}{-\bar{k}I_g(t)S(t)}$$

$$=-(\gamma+\gamma_e\varepsilon)-\theta\frac{R_t(t)}{I_g(t)} \tag{7}$$

Given $x=I_g(t), y=R_s(t), z=R_t(t)$,

$$\frac{dy}{dx}=-(\beta+\beta_e\varepsilon)-\alpha\frac{z}{x}-\frac{\delta}{\bar{k}x} \tag{8}$$

$$\frac{dz}{dx}=-(\gamma+\gamma_e\varepsilon)-\theta\frac{z}{x} \tag{9}$$

First, we calculate the value of Eq (9). Let $z/x=u$ and $dz=xdu+udx$. Therefore,

$$\frac{dz}{dx}=\frac{xdu+udx}{dx}=-(\gamma+\gamma_e\varepsilon)-\theta u$$

$$\therefore xdu+udx=-(\gamma+\gamma_e\varepsilon)dx-\theta udx$$

$$\Rightarrow -\frac{1}{\theta+1}\ln[(\gamma+\gamma_e\varepsilon)+\theta u+u]=\ln C_1x$$

$$\Rightarrow \ln[(\gamma+\gamma_e\varepsilon)+\theta u+u]=\ln(C_1x)^{-(\theta+1)}.$$

Substituting the logarithm to base *e* on both sides, we get

$$(\gamma+\gamma_e\varepsilon)+\theta u+u=C_1^{-(\theta+1)}x^{-(\theta+1)}$$

$$\Rightarrow u=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}+\frac{C_2x^{-(\theta+1)}}{\theta+1}, C_2=C_1^{-(\theta+1)}$$

$$\therefore \frac{z}{x}=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}+\frac{C_2x^{-(\theta+1)}}{\theta+1}$$

$$\Rightarrow \mathrm{z}=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}x+\frac{C_2}{\theta+1}x^{-\theta}$$

$$\Rightarrow R_t(t)=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}I_g(t)+\frac{C_2}{\theta+1}I_g(t)^{-\theta} \tag{10}$$

Considering the initial condition $I_g(0)\approx\frac{1}{1+\varepsilon}$, $R_s(0)=0$, we put them into Eq (10) and obtain:

$$C_2=\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}}$$

$$\therefore R_t(t)=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}I_g(t)+\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g(t)^{-\theta} \tag{11}$$

Substituting Eq (11) into Eq (6), we obtain:

$$\frac{dR_s(t)}{dI_g(t)}=-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}-\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g(t)^{-(\theta+1)}-\frac{\delta}{\bar{k}I_g(t)}$$

$$\Rightarrow dR_s(t)=\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}-\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g(t)^{-(\theta+1)}-\frac{\delta}{\bar{k}I_g(t)}\right]dI_g(t)$$

$$\Rightarrow R_s(t)=\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g(t)+\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g(t)^{-\theta}-\frac{\delta}{\bar{k}}\ln I_g(t)+C_3 \quad (12)$$

Considering the initial condition $I_g(0)\approx\frac{1}{1+\varepsilon}$, $R_s(0)=0$, we put them into Eq (12) and obtain:

$$C_3=\frac{\beta+\beta_e\varepsilon}{1+\varepsilon}-\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)\theta}+\frac{\delta}{\bar{k}}\ln(\frac{1}{1+\varepsilon})$$

$$\therefore R_s(t)=\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g(t)+\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g(t)^{-\theta}-\frac{\delta}{\bar{k}}\ln I_g(t)+$$

$$\frac{\beta+\beta_e\varepsilon}{1+\varepsilon}-\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)\theta}+\frac{\delta}{\bar{k}}\ln(\frac{1}{1+\varepsilon}) \quad (13)$$

The end of rumor spreading is the condition when the density of S is 0. Except for the S, we consider the steady-state densities of other groups. Let us define $I_g=I_g(\infty)=\lim_{t\to\infty}I_g(\mathrm{t})$, $I_l=I_l(\infty)=\lim_{t\to\infty}I_l(\mathrm{t})$, $R_t=R_t(\infty)=\lim_{t\to\infty}R_t(\mathrm{t})$, $R_s=R_s(\infty)=\lim_{t\to\infty}R_s(\mathrm{t})$, which represent the final densities of I_g, I_l, R_s and R_t at the end of rumor spreading, respectively, and $R=R_s+R_t$. R is the final size of the rumor, which represents the level of the rumor influence. In the final equilibrium state, there are I_g, I_l, R_s and R_t left in the system. Therefore, $I_g+I_l=I_g+\varepsilon I_g=(1+\varepsilon)I_g=1-R_s-R_t=1-R$, and we can get $I_g=\frac{1-R}{1+\varepsilon}$ and $R=1-(1+\varepsilon)I_g$, and according to Eqs (11) and (13), we can obtain

$$I_g+I_l=I_g+\varepsilon I_l=1-R_s-R_t=1-\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g-\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g^{-\theta}$$

$$+\frac{\delta}{\bar{k}}\ln I_g-\frac{\beta+\beta_e\varepsilon}{1+\varepsilon}+\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)\theta}-\frac{\delta}{\bar{k}}\ln(\frac{1}{1+\varepsilon})+\frac{\gamma+\gamma_e\varepsilon}{\theta+1}I_g-\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g^{-\theta}$$

Therefore,

$$(1+\varepsilon)I_g=1+(\beta+\beta_e\varepsilon)I_g-\left[\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g$$

$$-\left[\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}+\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)^{\theta+1}(\theta+1)}\right]I_g^{-\theta}+\frac{\delta}{\bar{k}}\ln I_g-\frac{\beta+\beta_e\varepsilon}{1+\varepsilon}+\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)\theta}-\frac{\delta}{\bar{k}}\ln(\frac{1}{1+\varepsilon})$$

$$\Rightarrow 1-R=1+(\beta+\beta_e\varepsilon)\frac{1-R}{1+\varepsilon}-\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)}{\theta+1}\frac{1-R}{1+\varepsilon}-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}(\frac{1-R}{1+\varepsilon})^{-\theta}$$

$$+\frac{\delta}{\overline{k}}\ln(\frac{1-R}{1+\varepsilon})-\frac{\beta+\beta_e\varepsilon}{1+\varepsilon}+\alpha\frac{\gamma+\gamma_e\varepsilon}{(1+\varepsilon)\theta}-\frac{\delta}{\overline{k}}\ln(\frac{1}{1+\varepsilon})$$

$$\Rightarrow R=\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R+\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}-\frac{\delta}{\overline{k}}\ln(\frac{1-R}{1+\varepsilon})$$

$$-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}+\frac{\delta}{\overline{k}}\ln(\frac{1}{1+\varepsilon})$$

Given

$$f(R)=R-\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}$$

$$+\frac{\delta}{\overline{k}}\ln(\frac{1-R}{1+\varepsilon})+\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}-\frac{\delta}{\overline{k}}\ln(\frac{1}{1+\varepsilon})$$

Taking the derivative of $f(R)$ with respect to R, we have

$$f'(R)=1+\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)-(\beta+\beta_e\varepsilon)(\theta+1)}{(1+\varepsilon)(\theta+1)}-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}\left(1-R\right)^{-\theta-1}-\frac{\delta}{\overline{k}}\frac{1}{1-R}$$

$$f''(R)=-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{1+\varepsilon}\left(1-R\right)^{-\theta-2}-\frac{\delta}{\overline{k}}\left(1-R\right)^{-2}$$

$$\because -\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{1+\varepsilon}<0\,,\quad -\frac{\delta}{\overline{k}}<0$$

$$\therefore f''(R)<0$$

This implies $f(R)$ is a concave function on the interval [0, 1) and we can calculate $f(0)=0$

$$f'(0)=1+\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)-(\beta+\beta_e\varepsilon)(\theta+1)}{(1+\varepsilon)(\theta+1)}-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}\left(1-0\right)^{-\theta-1}-\frac{\delta}{\overline{k}}\frac{1}{1-0}$$

$$=\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}-\frac{\delta}{\overline{k}}$$

$$\lim_{R\to1-}f\left(R\right)=\lim_{R\to1-}\left[R-\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R-\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}\right.$$

$$\left.+\frac{\delta}{\overline{k}}\ln(\frac{1-R}{1+\varepsilon})+\frac{(\alpha+\theta)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta(\theta+1)}-\frac{\delta}{\overline{k}}\ln(\frac{1}{1+\varepsilon})\right]$$

$$=-\infty<0\,.$$

Thus, when $f'\left(0\right)>0,$ i.e., $\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}-\frac{\delta}{\overline{k}}>0$, there exists an $R^*\in(0,1)$ satisfying $f\left(R^*\right)=0$. Thus, it can be deduced that when the education factor satisfies the inequality $\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}>\frac{\delta}{\overline{k}}$, the rumor will spread. In addition, it can be said when satisfying the inequality

$$\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}<\frac{\delta}{\bar{k}} \tag{14}$$

the rumor cannot spread. Now we obtain the critical threshold of the education strategy.

3.2. Short-term isolation strategy

Based on the long-term strategy, we added $S\xrightarrow{\omega}R_s$ to the spreading mechanism, which indicates that the S turns into a R_s with probability ω, representing the official isolation strategy. ω is the mute rate. Therefore, the rumor-spreading mean-field equations (1)–(5) become:

$$\frac{dI_g(t)}{dt}=-(\lambda+\beta+\gamma)\bar{k}I_g(t)S(t) \tag{15}$$

$$\frac{dI_l(t)}{dt}=-(\lambda_e+\beta_e+\gamma_e)\bar{k}I_l(t)S(t) \tag{16}$$

$$\frac{dS(t)}{dt}=\lambda\bar{k}I_g(t)S(t)+\lambda_e\bar{k}I_l(t)S(t)-\alpha\bar{k}S(t)R_t(t)-\theta\bar{k}S(t)R_t(t)-\delta S(t)-\omega S(t) \tag{17}$$

$$\frac{dR_s(t)}{dt}=\beta\bar{k}I_g(t)S(t)+\beta_e\bar{k}I_l(t)S(t)+\alpha\bar{k}S(t)R_t(t)+\delta S(t)+\omega S(t) \tag{18}$$

$$\frac{dR_t(t)}{dt}=\gamma\bar{k}I_g(t)S(t)+\gamma_e\bar{k}I_l(t)S(t)+\theta\bar{k}S(t)R_t(t) \tag{19}$$

Similar to the previous algorithm, it is easy to determine that when $\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}>\frac{\omega+\delta}{\bar{k}}$, the rumor will spread. Conversely, when the mute rate satisfies the inequality

$$\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}<\frac{\omega+\delta}{\bar{k}}, \tag{20}$$

the rumor will not spread. Further, we obtain the critical threshold of the isolation strategy.

3.3. Short-term debunking strategy

The R_t can influence the S to stop spreading rumors by clarifying the truth. The time-lag effect is ignored and it is assumed that the official will adopt a debunking strategy at the initial stage of rumor spreading by having a few R_t in the system. Consequently, the proportion of R_t in the initial state can be set as p. p is called the initial truth-spreading proportion. The initial condition for mean-field equations (1) to (5) becomes $S(0)=\frac{1}{N}\approx 0$, $R_t(0)=p$, $R_s(0)=0$, $I_g(0)+I_l(0)=\frac{N-1-p}{N}\approx 1-p$, $I_g(0)\approx\frac{1-p}{1+\varepsilon}$, $I_l(0)\approx\frac{\varepsilon(1-p)}{1+\varepsilon}$. Substituting the new initial conditions into Eq (10), we obtain

$$C_2'=\frac{(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}}{(1+\varepsilon)^{\theta+1}}$$

$$\therefore R_t\left(t\right)=-\frac{\gamma+\gamma_e\varepsilon}{\theta+1}I_g\left(t\right)+\frac{\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g\left(t\right)^{-\theta} \tag{21}$$

Substituting Eq (21) into Eq (6), we get

$$R_s(t)=\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g(t)$$

$$+\alpha\frac{\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g(t)^{-\theta}-\frac{\delta}{k}\ln I_g(t)+C_3'$$

Considering the new initial condition $I_g(0)\approx\frac{1-p}{1+\varepsilon}$, $R_t(0)=0$,

$$C_3'=\frac{\theta(\beta+\beta_e\varepsilon)\left(1-p\right)-\alpha(\gamma+\gamma_e\varepsilon)\left(1-p\right)-\alpha p(1+\varepsilon)}{(1+\varepsilon)\theta}+\frac{\delta}{k}\ln(\frac{1-p}{1+\varepsilon})$$

$$\therefore R_s(t)=\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g(t)+\alpha\frac{\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g(t)^{-\theta}$$

$$-\frac{\delta}{k}\ln I_g(t)+\frac{\theta(\beta+\beta_e\varepsilon)\left(1-p\right)-\alpha(\gamma+\gamma_e\varepsilon)\left(1-p\right)-\alpha p(1+\varepsilon)}{(1+\varepsilon)\theta}+\frac{\delta}{k}\ln(\frac{1-p}{1+\varepsilon}) \tag{22}$$

With the new initial conditions and according to the previous algorithm, we obtain

$$I_g+I_l=I_g+\varepsilon I_g=1-R_s-R_t=1-\left[-(\beta+\beta_e\varepsilon)+\alpha\frac{\gamma+\gamma_e\varepsilon}{\theta+1}\right]I_g$$

$$-\alpha\frac{\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}}{(1+\varepsilon)^{\theta+1}\theta(\theta+1)}I_g^{\ -\theta}+\frac{\delta}{k}\ln I_g$$

$$-\frac{\theta(\beta+\beta_e\varepsilon)\left(1-p\right)-\alpha(\gamma+\gamma_e\varepsilon)\left(1-p\right)-\alpha p(1+\varepsilon)}{(1+\varepsilon)\theta}-\frac{\delta}{k}\ln(\frac{1-p}{1+\varepsilon})+\frac{\gamma+\gamma_e\varepsilon}{\theta+1}I_g$$

$$-\frac{\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}}{(1+\varepsilon)^{\theta+1}(\theta+1)}I_g^{\ -\theta}$$

$$\Rightarrow R=\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R$$

$$+\frac{(\alpha+\theta)\left[\left(\theta+1\right)p(1+\varepsilon)\left(1-p\right)^{\theta}+(\gamma+\gamma_e\varepsilon)\left(1-p\right)^{\theta+1}\right]}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}-\frac{\delta}{k}\ln(\frac{1-R}{1+\varepsilon})$$

$$-\frac{p(\beta+\beta_e\varepsilon)}{1+\varepsilon}-\frac{\alpha p}{\theta}+\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}-\frac{(\alpha-\alpha p)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta}+\frac{\delta}{k}\ln(\frac{1-p}{1+\varepsilon})$$

Given: $g\left(R\right)=R-\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R$

$$-\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}+\frac{\delta}{\bar{k}}\ln(\frac{1-R}{1+\varepsilon})$$

$$+\frac{p(\beta+\beta_e\varepsilon)}{1+\varepsilon}+\frac{\alpha p}{\theta}-\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}+\frac{(\alpha-\alpha p)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta}-\frac{\delta}{\bar{k}}\ln(\frac{1-p}{1+\varepsilon})$$

Taking the derivative of $g(R)$ with respect to R, we have

$$g'(R)=1+\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)-(\beta+\beta_e\varepsilon)(\theta+1)}{(1+\varepsilon)(\theta+1)}$$

$$-\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{(1+\varepsilon)(\theta+1)}(1-R)^{-\theta-1}-\frac{\delta}{\bar{k}}\frac{1}{1-R}$$

$$g''(R)=-\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{1+\varepsilon}(1-R)^{-\theta-2}-\frac{\delta}{\bar{k}}(1-R)^{-2}$$

$$\because -\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{1+\varepsilon}<0,\quad -\frac{\delta}{\bar{k}}<0$$

$$\therefore g''(R)<0$$

That is, $g(R)$ is also a concave function on the interval [0, 1), and we can calculate

$$g(p)=0$$

$$g'(p)=1+\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)-(\beta+\beta_e\varepsilon)(\theta+1)}{(1+\varepsilon)(\theta+1)}$$

$$-\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{(1+\varepsilon)(\theta+1)}(1-p)^{-\theta-1}-\frac{\delta}{\bar{k}}\frac{1}{1-p}$$

$$=1+\frac{-(\gamma+\gamma_e\varepsilon)-(\beta+\beta_e\varepsilon)-(\alpha+\theta)p(1+\varepsilon)(1-p)^{-1}}{(1+\varepsilon)}-\frac{\delta}{\bar{k}}(1-p)^{-1}$$

$$=\frac{\lambda+\lambda_e\varepsilon}{1+\varepsilon}-\frac{(\alpha+\theta)p\bar{k}+\delta}{\bar{k}(1-p)}\quad \lim_{R\to1-}g(R)=\lim_{R\to1-}\left\{R-\frac{(\beta+\beta_e\varepsilon)(\theta+1)-(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}R\right.$$

$$-\frac{(\alpha+\theta)\left[(\theta+1)p(1+\varepsilon)(1-p)^{\theta}+(\gamma+\gamma_e\varepsilon)(1-p)^{\theta+1}\right]}{(1+\varepsilon)\theta(\theta+1)}(1-R)^{-\theta}+\frac{\delta}{\bar{k}}\ln(\frac{1-R}{1+\varepsilon})$$

$$\left.+\frac{p(\beta+\beta_e\varepsilon)}{1+\varepsilon}+\frac{\alpha p}{\theta}-\frac{(\alpha-1)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)(\theta+1)}+\frac{(\alpha-\alpha p)(\gamma+\gamma_e\varepsilon)}{(1+\varepsilon)\theta}-\frac{\delta}{\bar{k}}\ln(\frac{1-p}{1+\varepsilon})\right\}$$

$$=-\infty<0$$

Thus, considering $g'(p) > 0$, that is $\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} - \frac{(\alpha+\theta)p\bar{k} + \delta}{\bar{k}(1-p)} > 0$, there exists a $R^{\varphi} \in (0,1)$ satisfying $g(R^{\varphi}) = 0$. Therefore, when $\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} > \frac{(\alpha+\theta)p\bar{k} + \delta}{\bar{k}(1-p)}$, the rumor will spread. Conversely, when the initial truth-spread proportion satisfies the inequality

$$\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} < \frac{(\alpha+\theta)p\bar{k} + \delta}{\bar{k}(1-p)} \tag{23}$$

the rumor will not spread. Moreover, we obtain the critical threshold of the debunking strategy.

Notably, the two short-term strategies can work simultaneously. Similar to the aforementioned mathematical analysis, we can obtain

$$\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} < \frac{\omega + \delta + (\alpha+\theta)p\bar{k}}{\bar{k}(1-p)} \tag{24}$$

when the rumor will not spread. Inequality (24) is the critical threshold when the two short-term (isolation and debunking) strategies work simultaneously.

3.4. Critical threshold analysis

Regarding the critical threshold inequalities (14), (20), (23) and (24), we find that the values at the left end of the inequalities are the same. Setting the function $h(\varepsilon) = \frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon}$ and taking the first derivative of $h(\varepsilon)$, we get $h'(\varepsilon) = \frac{\lambda_e - \lambda}{(1+\varepsilon)^2}$. As $\lambda_e < \lambda$, that is $h'(\varepsilon) < 0$, $h(\varepsilon)$ is a decreasing function. Further, with increasing education factor ε, the value of the function $h(\varepsilon)$ decreases. When the parameters in $\frac{\delta}{\bar{k}}$ do not change, inequality (14) is easier to be satisfied. This implies that with increasing education factors, rumors are easier to be controlled, which proves the effectiveness of the long-term strategy.

Further, we will explore the threshold inequalities (20), (23) and (24) related to short-term strategies. With the addition of the parameters ω and p, the values at the right end of the inequalities become larger and the inequalities are easier satisfied. This implies that the rumors can be easily controlled, which shows that both short-term strategies are effective. In addition, inequality (20) is transformed to obtain

$$\omega > \frac{\lambda + \lambda_e \varepsilon}{(1+\varepsilon)\bar{k}^{-1}} - \delta \tag{25}$$

and inequality (23) is transformed to obtain

$$p > \frac{\lambda + \lambda_e \varepsilon - \delta \bar{k}^{-1}(1+\varepsilon)}{\lambda + \lambda_e \varepsilon + (\alpha+\theta)(1+\varepsilon)} \tag{26}$$

Simultaneously, we can also separate ω and p from inequality (24) and obtain

$$\omega > \frac{\lambda + \lambda_e \varepsilon}{(1+\varepsilon)\bar{k}^{-1}} - \delta - \frac{\lambda + \lambda_e \varepsilon}{(1+\varepsilon)\bar{k}^{-1}} p - (\alpha+\theta)p\bar{k} \tag{27}$$

and

$$p > \frac{\lambda + \lambda_e \varepsilon - (\delta + \omega)\bar{k}^{-1}(1+\varepsilon)}{\lambda + \lambda_e \varepsilon + (\alpha + \theta)(1+\varepsilon)} \tag{28}$$

We can easily determine that the value of ω that satisfies inequality (27) is smaller than that in inequality (25), and the value of p that satisfies inequality (28) is smaller than that in inequality (26). Thus, when one short-term strategy is implemented, another short-term strategy requires no more effort to prevent rumor spreading. This indicates that the simultaneous implementation of two short-term strategies is better than the implementation of only one.

Moreover, in addition to the long-term and short-term strategies, based on the threshold inequalities, we can determine whether rumors can spread is also related to the parameters of propagation rate λ and λ_e, inhibition rate α and θ, forgetting rate δ and average network degree $\bar{k}$. We derived that the smaller the propagation rates λ and λ_e, the larger the inhibition rates α and θ as well as forgetting rate δ. Further, smaller the average network degree $\bar{k}$, the easier the establishment of the inequalities. Thus, the spread of rumors is more unfavorable. The law of the relationship between parameter variation and rumor propagation is consistent with the common sense of the public and insights from previous research.

4. Numerical simulations

We used simulations to investigate the effectiveness of these strategies in the modified rumor-spreading model. We conducted simulations in the context of a homogenous network and real-world Renren network. Renren (http://www.renren.com/) is a leading real-name social networking platform in China and has several users. Based on the findings of data mining techniques applied by Fu et al. [27,28], the Random Walk sampling method was used to simplify Renren. A small network was sampled from Renren, which retained the same network topology features as the original real social network Renren. Fu et al. showed that the Renren network exhibits scale-free characteristics [27,28]. We applied their sampled network data for analysis and constructed a schematic of Renren's sample network structure, as shown in Figure 2. A node represents a user, with a total of 9590 nodes in the network. The edge represents the friend relationship between users, with 89,873 edges on the network. The degree represents the number of friends (neighbors) of the node; the larger the node in the figure, the greater its degree. The sample data found that approximately 47%, 23%, 5% and 2% of nodes with <10, 10–20 and 20–50, 50–100 and >100 neighbors, respectively. The average degree of the Renren sample network is around 19. Based on the sample data, the degree distribution of Renren is fitted as $P_{\mathrm{Re}nren}(k) = \begin{cases} 0.098k^{-0.53} & k \le 20 \\ 10.969k^{-2.2} & k > 20 \end{cases}$. To maintain comparability, the size and average degree of the homogenous network are the same as those of the Renren sample network. Thus, in the homogenous network, we assume that the size of the network is $N = 9590$ and the average degree $\bar{k} = 19$.

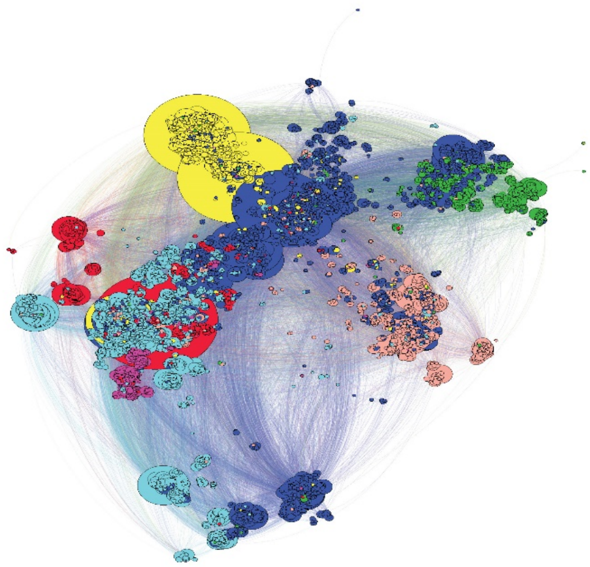
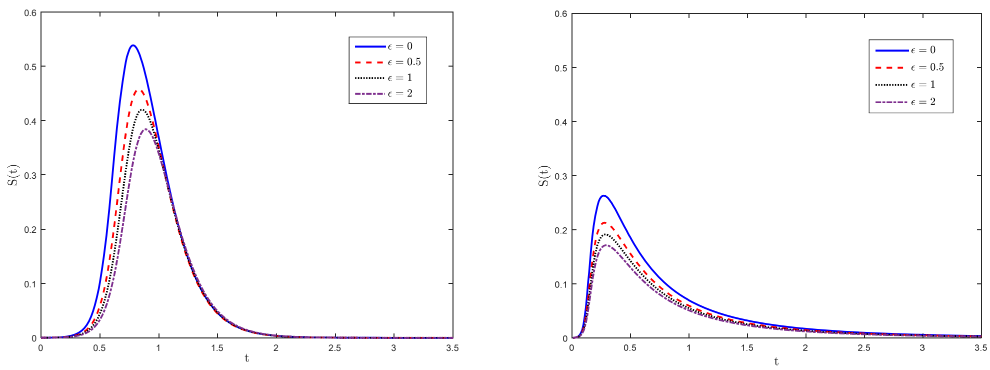


Figure 2. Schematic of Renren's sample network structure.

The Runge–Kutta method and finite difference method are used to simulate the characteristics of rumor threshold change and its propagation law in the homogenous and Renren sample networks, respectively. The program code of simulation example can be found in the Supplementary material. Furthermore, only one S is present in the initial condition. To eliminate the differences in degree values caused by the random selection of initial spreads, we adopted the method of simulating 50 times to calculate the average value for the simulation in the Renren network.

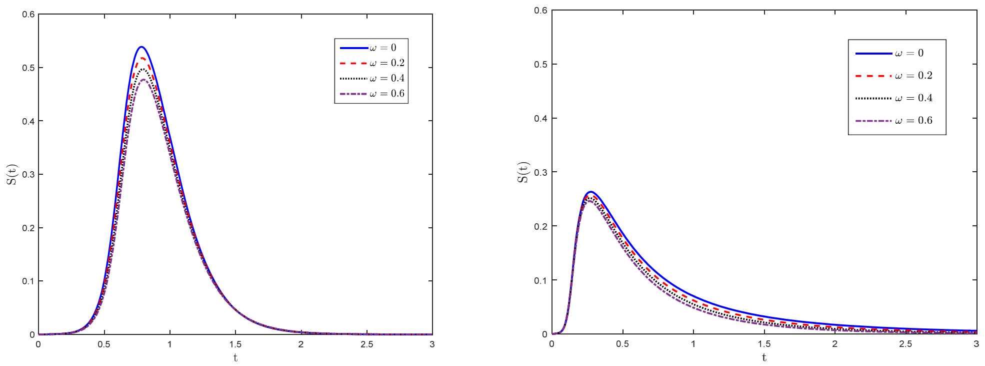


(a) In the homogenous network (b) In the Renren network

Figure 3. Variations in the density of the S over time (t) as a function of the education factor ε in the homogenous network (a) and Renren network (b). The values of the model parameters are λ=0.8 , β=γ=0.1 , λ_e=0.6 , β_e=γ_e=0.2 , α=θ=0.3 , δ=0.4 , ω=0 and $p=0$.

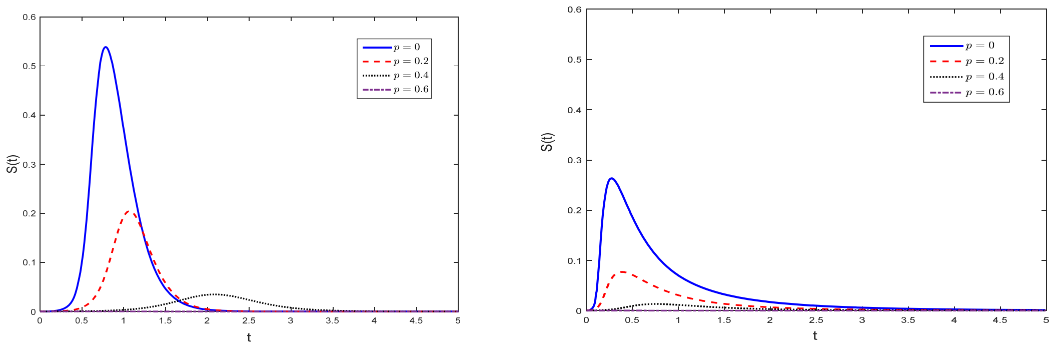
Figure 3 shows the density of the S changing over time under different education factors ε. The higher the education factor ε, the more the official invests in the long-term strategy for preventing the rumor from spreading. As shown in Figure 3, we can find that owing to differences in the structure of networks, the maximum density of S and the time required to reach the maximum density in homogenous and Renren networks are different. However, in homogenous or Renren networks, as the education factor ε increases, the peak of the density of S decreases. A negative correlation exists between the maximum rumor-spreading power and education factor, which reflects the effectiveness of the official education strategy in suppressing rumor spread.

Figure 4 shows the density of S variation over time under different mute rate ω. The higher the ω, the more official invests in the short-term isolation strategy for preventing rumor spread. As shown in Figure 3, owing to differences in the network structure, compared to Renren, the homogenous network is more conducive to spreading rumors. However, regardless of the network structure, from Figure 4, we can find that as the mute rate ω increases, the peak of the S density decreases, which reflects the effectiveness of the official isolation strategy.



(a) In the homogenous network (b) In the Renren network

Figure 4. Changes in the density of the S over time (t) as a function of the mute rate ω in the homogenous network (a) and Renren network (b). The values of the model parameters are λ=0.8, β=γ=0.1, λ_e=0.6, β_e=γ_e=0.2, α=θ=0.3, δ=0.4, ε=0 and $p=0$.

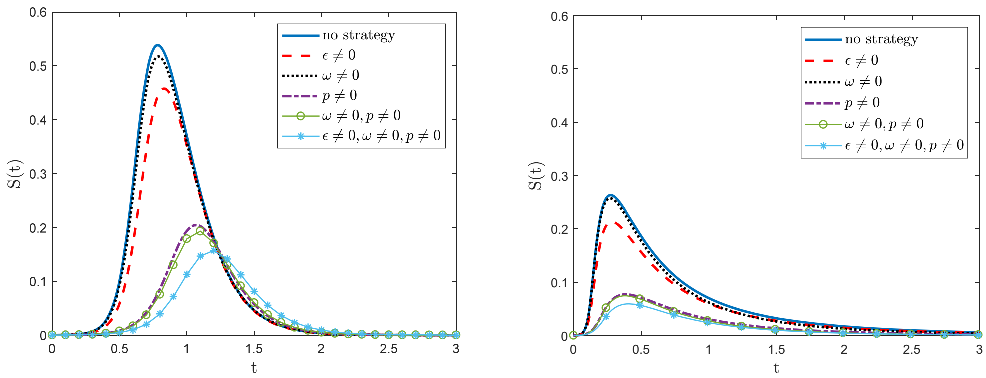


(a) In the homogenous network (b) In the Renren network

Figure 5. Changes in the density of the S over time (t) as a function of the initial truth-spreading proportion p in the homogenous network (a) and Renren network (b). The values of the model parameters are λ=0.8, β=γ=0.1, λ_e=0.6, β_e=γ_e=0.2, α=θ=0.3, δ=0.4, ε=0 and ω=0.

Figure 5 shows the density of S variation over time under different initial truth-spreading proportions p. From Figure 5, we can find that as the initial truth-spreading proportion p increases, the peak of the density of S decreases, which shows that the rumor spreading is gradually being controlled in homogenous and Renren networks. When the initial truth-spreading proportion p reaches a certain value, as shown in Figure 5(a), $p = 0.6$, the density of S is always zero; that is, the rumor has not spread at all. Integrating the corresponding parameter values from Figure 5 into inequality (28), we can get $p = 0.6 > 0.56$. Inequality (28) is established; therefore, there is only zero solution in the system, and the correctness of the theoretical analysis results for the debunking strategy has been verified. We cannot deduce a similar threshold inequality (28) in a heterogenous network, even theoretically. However, a similar conclusion through simulation in Figure 5 (b) has been found. In the Renren network when $p = 0.6$, the rumor still cannot be spread. In addition, we performed some simulations and obtained that if inequalities (14) and (20) can be satisfied by appropriate parameters, the rumor cannot spread. The simulated graph is a straight line with a value of 0, resembling the graph with $p = 0.6$ in Figure 5.

Figure 6(a),(b) illustrate the densities of S changing over time under different official strategies to deal with rumor spreading in homogenous and Renren networks, respectively. Regardless of the network structure, the density trend of S changes is similar. It can be seen from Figure 6(a),(b) that no matter which strategy the government adopts, the density of the S will be lower than that with no measures taken. This shows that these measures play a certain role in preventing and controlling rumor spreading. From the maximum rumor-spreading power in Figure 6(a),(b), it can be found that different strategies have different effects on suppressing the rumor. Results may change with different parameter values. However, the following two rules will not change with variations in the parameter value. First, the combination of the short-term isolation strategy and debunking strategy is better than any single short-term strategy. Second, the combined effect of the long-term and short-term strategies is the best. Figure 6(c),(d) illustrate the variation of the density of R ($R_s + R_t$) over time under different official strategies in homogenous and Renren networks, respectively. Regardless of the network structure, the trend of R density changes is similar to those in Figure 6(c),(d). We can find that the change in the density of R over time is from the initial minimum value of 0 or 0.2 and continues to rise. Finally, when the rumor spread ends, the stable point is at a constant fixed value. The difference between the initial minimum of R density (0 and 0.2) depends on whether the debunking strategy is implemented. The value in the final steady state is the level of the rumor's influence. It can be seen from Figure 6(c),(d) that implementing different strategies will weaken the level of the rumor's influence, and the two aforementioned rules still hold.



(a) Spreader in the homogenous network (b) Spreader in the Renren network

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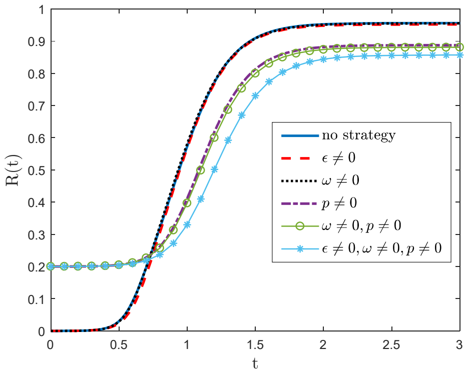
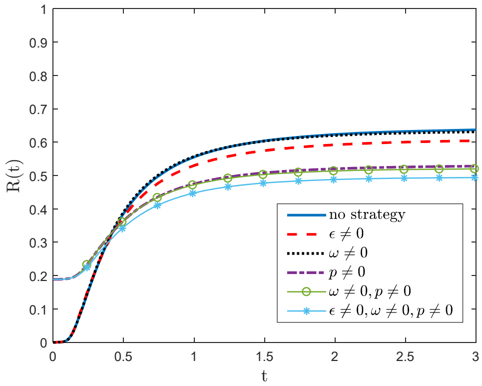


(c) Stifler in the homogenous network



(d) Stifler in the Renren network

Figure 6. Changes of the densities of the spreader in the homogenous network (a) and Renren network (b), and stifler in the homogenous network (c) and Renren network (d) over time (t) as a function of the official strategy for preventing the spread of rumors（"no strategy" represents adopting no strategy; "$\varepsilon \neq 0$ ($\varepsilon = 0.5$)," represents adopting only long-term education strategy; "$\omega \neq 0$ ($\omega = 2$)," represents adopting only short-term isolation strategy; "$p \neq 0$ ($p = 0.2$)" represents adopting only short-term debunking strategy; "$\omega \neq 0, p \neq 0$ ($\omega = p = 0.2$)" represents adopting short-term strategy; "$\varepsilon \neq 0, \omega \neq 0, p \neq 0$ ($\varepsilon = 0.5, \omega = p = 0.2$)" represents combining the long-term and short-term strategies simultaneously). The values of the model parameters are $\lambda = 0.8$, $\beta = \gamma = 0.1$, $\lambda_e = 0.6$, $\beta_e = \gamma_e = 0.2$, $\alpha = \theta = 0.3$ and $\delta = 0.4$.

5. Conclusions

In this paper, we considered the long-term education strategy and short-term isolation and debunking strategies for governments to deal with rumor spreading. Based on the SIR rumor-spreading model, we divided *I* and *R* into two groups with different characteristics and updated the transmission mechanism afterward. We established a modified rumor-spreading model and derived the mean-field equations. Through the mathematical analysis, the condition $\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} < \frac{\delta}{\bar{k}}$ is obtained for the education strategy. The conditions $\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} < \frac{\omega + \delta}{\bar{k}}$ and $\frac{\lambda + \lambda_e \varepsilon}{1+\varepsilon} < \frac{(\alpha + \theta) p \bar{k} + \delta}{\bar{k}(1-p)}$ are obtained for the isolation strategy and debunking strategy for controlling the spread of the rumor. If the government only implements short-term strategies, then for the aforementioned inequalities of ω and p, consider that the value of ε is 0. If the aforementioned inequality holds, rumors will not spread. Numerical simulations of the rumor-spreading model were performed in homogenous and heterogenous networks to verify the validity of the theoretical results.

Rumor spread and prevention is a complex process, of which our proposed model will be further improved from the following perspectives. First, we have ignored the time-lag effect in our model, while in reality, there must be a time delay in the debunking strategy. Therefore, it will be inevitable to consider the effect of time delay during rumor prevention. Second, although very little quantification of the parameters has been proposed in publications on rumor spreading to date, the values of all these parameters in our model need to find ways to quantify them. Finally, although the homogenous and

Renren networks can reflect the general law of rumor propagation, most real-life networks life have unique topologies. In particular, when implementing different strategies to prevent rumor spreading, the government spends variable financial budgets on nodes with different topological characteristics. Therefore, the economic cost of the government's response to rumor-spreading strategies should be considered in the real network and a response strategy with the minimum cost should be constructed.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. D. J. Daley, D. G. Kendall, Stochastic rumours, *IMA J. Appl. Math.*, **1** (1965), 42–55. https://doi.org/10.1093/imamat/1.1.42
2. D. P. Maki, M. Thompson, *Mathematical Models and Applications, with Emphasis on Social, Life, and Management Sciences*, Englewood Cliffs, New Jersey, Prentice-Hall, 1973.
3. Y. Moreno, M. Nekovee, A. F. Pacheco, Dynamics of rumor spreading in complex networks, *Phys. Rev. E*, **69** (2004), 066130. https://doi.org/10.1103/PhysRevE.69.066130
4. L. Zhu, H. Zhao, Dynamical behaviors and control measures of rumour-spreading model with consideration of network topology, *Int. J. Syst. Sci.*, **48** (2017), 2064–2078. https://doi.org/10.1080/00207721.2017.1312628
5. L. Zhao, J. Wang, Y. C. Chen, Q. Wang, J. Cheng, H. Cui, SIHR rumor spreading model in social networks, *Physica A*, **391** (2012), 2444–2453. https://doi.org/10.1016/j.physa.2011.12.008
6. S. Yu, Z. Yu, H. Jiang, S. Yang, The dynamics and control of 2I2SR rumor spreading models in multilingual online social networks, *Inf. Sci.*, **581** (2021), 18–41. https://doi.org/10.1016/j.ins.2021.08.096
7. Z. Dang, L. Li, W. Ni, R. Liu, H. Peng, Y. Yang, How does rumor spreading affect people inside and outside an institution, *Inf. Sci.*, **574** (2021), 377–393. https://doi.org/10.1016/j.ins.2021.05.085
8. W. Jing, Y. Li, X. Zhang, J. Zhang, Z. Jin, A rumor spreading pairwise model on weighted networks, *Physica A*, **585** (2022), 126451. https://doi.org/10.1016/j.physa.2021.126451
9. Z. Zhang, X. Mei, H. Jiang, X. Luo, Y. Xia, Dynamical analysis of Hyper-SIR rumor spreading model, *Appl. Math. Comput.*, **446** (2023), 127887. https://doi.org/10.1016/j.amc.2023.127887

10. X. Chen, N. Wang, Rumor spreading model considering rumor credibility, correlation and crowd classification based on personality, *Sci. Rep.*, **10** (2020), 5887. https://doi.org/10.1038/s41598-020-62585-9
11. A. I. E. Hosni, K. Li, S. Ahmad, Minimizing rumor influence in multiplex online social networks based on human individual and social behaviors, *Inf. Sci.*, **512** (2020), 1458–1480. https://doi.org/10.1016/j.ins.2019.10.063
12. Y. Cheng, L. Huo, L. Zhao, Dynamical behaviors and control measures of rumor-spreading model in consideration of the infected media and time delay, *Inf. Sci.*, **564** (2021), 237–253. https://doi.org/10.1016/j.ins.2021.02.047
13. C. Budak, D. Agrawal, A. E. Abbadi, Limiting the spread of misinformation in social networks, in *Proceedings of the 20th International Conference on World Wide Web*, Hyderabad, India, (2011), 665–674. https://doi.org/10.1145/1963405.1963499
14. R. M. Tripathy, A. Bagchi, S. Mehta, A study of rumor control strategies on social networks, in *ACM Conference on Information & Knowledge Management*, Canada, (2010), 1817–1820. https://doi.org/10.1145/1871437.1871737
15. L. Zhu, B. Wang, Stability analysis of a SAIR rumor spreading model with control strategies in online social networks, *Inf. Sci.*, **526** (2020), 1–19. https://doi.org/10.1016/j.ins.2020.03.076
16. Z. He, Z. Cai, J. Yu, X. Wang, Y. Sun, Y. Li, Cost-efficient strategies for restraining rumor spreading in mobile social networks, *IEEE Trans. Veh. Technol.*, **66** (2017), 2789–2800. https://doi.org/10.1109/TVT.2016.2585591
17. L. A. Huo, Y. Q. Zhang, Effect of global and local refutation mechanism on rumor propagation in heterogeneous network, *Mathematics*, **10** (2022), 586. https://doi.org/10.3390/math10040586
18. Y. Zhang, J. Xu, M. Nekovee, Z. Li, The impact of official rumor-refutation information on the dynamics of rumor spread, *Physica A*, **607** (2022), 128096. https://doi.org/10.1016/j.physa.2022.128096
19. Y. Tian, X. Ding, Rumor spreading model with considering debunking behavior in emergencies, *Appl. Math. Comput.*, **363** (2019), 124599. https://doi.org/10.1016/j.amc.2019.124599
20. H. Zhu, X. Zhang, Q. An, Global stability of a rumor spreading model with discontinuous control strategies, *Physica A*, **606** (2022), 128157. https://doi.org/10.1016/j.physa.2022.128157
21. A. Singh, Y. N. Singh, Nonlinear spread of rumor and inoculation strategies in the nodes with degree dependent tie strength in complex networks, *Acta Phys. Pol. B*, **44** (2013), 5–28. https://doi.org/10.5506/APhysPolB.44.5
22. Y. Cheng, L. Huo, L. Zhao, Stability analysis and optimal control of rumor spreading model under media coverage considering time delay and pulse vaccination, *Chaos, Solitons Fractals*, **157** (2022), 111931. https://doi.org/10.1016/j.chaos.2022.111931
23. M. Jiang, Q. Gao, J. Zhuang, Reciprocal spreading and debunking processes of online misinformation: A new rumor spreading–debunking model with a case study, *Physica A*, **565** (2021), 125572. https://doi.org/10.1016/j.physa.2020.125572
24. L. Zhu, M. Zhou, Z. Zhang, Dynamical analysis and control strategies of rumor spreading models in both homogeneous and heterogeneous networks, *J. Nonlinear Sci.*, **30** (2020), 2545–2576. https://doi.org/10.1007/s00332-020-09629-6
25. S Wen, J Jiang, Y Xiang, S. Yu, W. Zhou, W. Jia, To shut them up or to clarify: Restraining the spread of rumors in online social networks, *IEEE Trans. Parallel Distrib. Syst.*, **25** (2014), 3306–3316. https://doi.org/10.1109/TPDS.2013.2297115

26. K. Afassinou, Analysis of the impact of education rate on the rumor spreading mechanism, *Physica A*, **414** (2014), 43–52. https://doi.org/10.1016/j.physa.2014.07.041
27. F. Fu, X. Chen, L. Liu, L. Wang, Social dilemmas in an online social network: the structure and evolution of cooperation, *Phys. Lett. A*, **371** (2007), 58–64. https://doi.org/10.1016/j.physleta.2007.05.116
28. F. Fu, L. Liu, L. Wang, Empirical analysis of online social networks in the age of Web 2.0, *Physica A*, **387** (2008), 675–684. https://doi.org/10.1016/j.physa.2007.10.006