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Numerical treatment for mathematical model of farming awareness in crop pest management

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The most important factor for increasing crop production is pest and pathogen resistance, which has a major impact on global food security. Pest management also emphasizes the need for farming awareness. A high crop yield is ultimately achieved by protecting crops from pests and raising public awareness of the devastation caused by pests. In this research, we aim to investigate the intricate impacts of nonlinear delayed systems for managing crop pest management (CPM) supervised by Ordinary Differential Equations (ODEs). Our focus will be on highlighting the intricate and often unpredictable relationships that occur over time among crops, pests, strategies for rehabilitation, and environmental factors. The nonlinear delayed CPM model incorporated the four compartments: crop biomass density [B(t)], susceptible pest density [*S(t)*], infected pest density [*I(t)*], and population awareness level [*A(t)*]. The approximate solutions for the four compartments *B(t)*, *S(t)*, *I(t)*, and *A(t)* are determined by the implementation of sundry scenarios generated with the variation in crop biomass growth rate, rate of pest attacks, pest natural death rate, disease associated death rate and memory loss of aware people, by means of exploiting the strength of the Adams (ADS) and explicit Runge-Kutta (ERK) numerical solvers. Comparative analysis of the designed approach is carried out for the dynamic impacts of the nonlinear delayed CPM model in terms of numerical outcomes and simulations based on sundry scenarios.

KEYWORDS

non-linear delayed crop pest management model, public awareness, explicit Runge–Kutta method, Adams method, comparative analysis, approximate solutions, graphical illustrations

1. Introduction

In recent years, researchers have paid more attention to integrated pest management, and its use in the crop field has increased. This strategy emphasizes the implication of biological control factors to minimize the credence of pesticides. In agriculture, forest management, and population health, microbiological pesticides play a significant role in incorporated pest management. In the case of crops, biopesticides provide noticeable pest management dependability as part of incorporated pest management [1]. In North America and Europe, viruses are used as pest control agents against insect pests [2–4]. Agricultural-related awareness programs on radio, TV, mobile and other media might aid in disseminating agricultural knowledge among farmers and ranchers about the hazards of pesticide use on human health as well as the other linked environmental concerns [5–9]. Pesticide overuse is significantly associated with farmers' lack of pesticide knowledge, the impact of pesticide retail outlets, and inaccessibility to non-synthetic pest control methods, while the tendency to overuse reduces higher levels of the learning process in Integrated Pest Management [10]. As a result, farming awareness is essential to prevent crop losses having the least amount of detrimental side effects [11, 12]. Pesticide communication campaigns made it easier for farmers to understand the substantial risks pesticides pose to public health and the ecosystem, and to limit harmful consequences. Farmers primarily learned about pesticide use and hazards through oral communication [13].

Time delay is a key feature in both natural and manmade systems. Kuang provided an example to demonstrate the significance of time delay [14]. He claimed that animals require time to digest their food before moving on to other activities and reflexes. As a result, any species model with no temporal delay is at best an approximation [15]. Many systems as well as industrial plants, including biological systems, machining, metal forming, thermal acoustic systems and many others experience time delays [16–19]. Furthermore, dynamical systems including time delay exhibit far more complex behaviors than those without delay in time [20]. There are two key reasons for the presence of temporal delays in prey-predator systems [21, 22]. The first is the gestation period, and the second is the maturation period. As a result, incorporating delays into predator-prey model is essential for ensuring the realistic nature of these models and demonstrating how well the population dynamics of such models are influenced by previous relevant information. In fact, time delays have a significant impact on the overall characteristics of dynamic systems. Many publications in the literature have described the theory-based analysis of the prey-predator model involving time delay, such as hunting delay [23], dispersal delay [24], predator gestation period [25], as well as intra-specific competitive pressure generated feedback delay [26].

There is a lack of mathematical modeling on agricultural awareness to limit plant pests as well as diseases. Daudi et al. [27] proposed a dynamic model using the fractional derivative operator for maize growth as well as interactions with fall armyworms. They described the basic reproductive number, which was the average amount of newborns generated by a single female moth over the course of a lifespan. The resilience of the trivial equilibria, as well as the positive equilibria of the dynamical system, were investigated by Li et al. [28] and the threshold requirements for pest destruction and system permanence were determined.

TABLE 1 Parameters default values used for non-linear delayed CPM model [38].

Parameters	Value	Parameters	Value
a	0.2	φ	0.5
N	50	μ	0.6
δ	0.025	μ_1	0.12
c	0.1	l	0.025
β	0.05	d	0.015
A_0	0.2	υ	0.05
τ	1	-	-

Xiang et al. [29] explored the influence of MD controls on the dynamical behavior of the pest systems by adding the gestational delay and sex pheromones. First, the system's bounds, stability, as well as bifurcation were discussed. Second, by integrating the constraint violating function, an optimized control problem depending on sex pheromone and pesticides was reduced into an analogous optimized parameter decision issue. The bifurcation control of mosaic viruses fractional order infection models for Jatropha curcas with agricultural awareness and an executing delay was examined by Liu et al. [30] Hopf bifurcation generated by executing delay was explored for the unregulated system by examining the corresponding characteristic equation. They found that changing the fractional order had a considerable impact on bifurcation dynamics. Kumari et al. [31] employed the Integrated Pest Management (IPM) technique to create a mathematical model that used a combination of chemical and biological management. The feasibility of pest eradication and non-trivial equilibria state were examined, and the local stability of the pest eradication equilibria state was investigated further. Shi et al. [32] presented a unique population Smith framework with continual delay as well as impulsive phase adaptive control and examined how it may be used in pest management. The model's singularity was first qualitatively examined, and then the presence and uniqueness of order one periodical orbits were considered in order to calculate the frequency of chemical control implementation. A Filippov prey predator model incorporating time delay was introduced by Arafa et al. [33], where the delay time indicated the changes in the natural enemy's growth rate before discharging it to fatten up pests. The bifurcation parameter time delay was used to derive the threshold constraints for the stability of the equilibrium. Utilizing Hopf bifurcation, it was proven that whenever the time delay parameter crosses through specific critical levels, a periodical oscillation phenomenon arises. They also established the equation of slipping motion and addressed the sliding phase dynamics using the Filippov convex approach. Al Basir et al. [34] presented prey predator framework for assessing the impact of delay time in crop pest management utilizing agricultural awareness-based treatments. The authors indicated that the application of biological insecticides is proportionate to the pest population density on the plantation. The presence of steady states, as well as their stability, had been examined. Allen-Perkins and Estrada [35] constructed an epidemic model to explore disease transmission and control in planted agricultural farms as a long-term pest management

Abbreviations: IPM, Integrated Pest Management; ERK, Explicit Runge-Kutta; MD, Mating disruption; TV, Television; ODEs, Ordinary differential equations; HIV, Human immunodeficiency virus; COVID-19, coronavirus disease of 2019; ADS, Adams method; CPS, Crop pest management; $B(t)$, $S(t)$, $I(t)$, $A(t)$, Crop biomass density, Susceptible density, Infected density, Aware people density; B_0, S_0, I_0 and A_0, Initial conditions for B, S, I and A; *NDSolve*, Numerical solution of differential equations; α, Crop biomass growth rate; N, Maximum crop biomass percentage; δ, Rate of pest attacks; c, Pest natural death; β, Disease associated death rate; l, Aware individuals' activity level; d, Growth rate of aware individuals; υ, Memory loss of aware people; A_0, Awareness level from a widespread source; τ, Delay in time.

TABLE 2 Illustration of scenarios for the non-linear delayed CPM model.

Scenario 1 for the crop biomass growth rate		
	ADS solver	ERK solver
C-1	$a = 0.25$	$a = 0.25$
C-2	$a = 0.3$	$a = 0.3$
C-3	$a = 0.5$	$a = 0.5$
C-4	$a = 0.7$	$a = 0.7$
C-5	$a = 0.9$	$a = 0.9$
Scenario 2 for the rate of pest attacks		
	ADS solver	ERK solver
C-1	$\delta = 0.015$	$\delta = 0.015$
C-2	$\delta = 0.025$	$\delta = 0.025$
C-3	$\delta = 0.035$	$\delta = 0.035$
C-4	$\delta = 0.045$	$\delta = 0.045$
C-5	$\delta = 0.055$	$\delta = 0.055$
Scenario 3 for the pest natural death rate		
	ADS solver	ERK solver
C-1	$c = 0.15$	$c = 0.15$
C-2	$c = 0.25$	$c = 0.25$
C-3	$c = 0.33$	$c = 0.33$
C-4	$c = 0.45$	$c = 0.45$
C-5	$c = 0.55$	$c = 0.55$
Scenario 4 for disease associated death rate		
	ADS solver	ERK solver
C-1	$\beta = 0.01$	$\beta = 0.01$
C-2	$\beta = 0.02$	$\beta = 0.02$
C-3	$\beta = 0.03$	$\beta = 0.03$
C-4	$\beta = 0.04$	$\beta = 0.04$
C-5	$\beta = 0.05$	$\beta = 0.05$
Scenario 5 for memory loss of aware people		
	ADS solver	ERK solver
C-1	$\upsilon = 0.01$	$\upsilon = 0.01$
C-2	$\upsilon = 0.02$	$\upsilon = 0.02$
C-3	$\upsilon = 0.03$	$\upsilon = 0.03$
C-4	$\upsilon = 0.05$	$\upsilon = 0.05$
C-5	$\upsilon = 0.06$	$\upsilon = 0.06$

strategy. In an epidemiological susceptible, infectious and removed model, the model includes the mobility of aphids carrying a virus in an agricultural farm, the spatial dispersion of plants in a planted field, and the existence of "trapped crops." Abraha et al. [36] studied a mathematical model for crop pest control that took into account plant biomass, pests, and the impact of farmer awareness. The basic reproductive number and delays in time were used to determine the presence as well as stability of the equilibria. Whenever time delays approach critical values, stability transitions happen due to Hopf-bifurcation. The delayed system's cost-effectiveness was assessed using optimal control-theory. Rossini et al. [37] presented a mathematical framework for calculating the analytical solutions to the second variant of the distributed delay model. The researchers also investigated how the model behaved when it came to representing the population of insect pests in various environmental factors, particularly with respect to temperature. Al Basir et al. [38] proposed a mathematical model including delay to investigate the impact of public awareness on agricultural pest management using crop biomass, and pests. The basic reproductive number was used to determine the presence and stability conditions of the equilibria. The Hoph bifurcation analysis was performed at the epidemic equilibria with time delay as the bifurcation parameter.

Numerical approaches are frequently employed in science and engineering to solve mathematical problems for which exact solutions are difficult or impossible to grab. Only a limited number of differential equations can be solved analytically. There are several analytical methods to solve ordinary differential equations (ODEs). Although several ODEs have closed form solutions that can be obtained using renowned analytical methods, numerical methods must be evolved and applied to obtain numerical solutions of a differential equation under a predefined initial history. Many researchers used a variety of numerical methods to simulate the solution of mathematical models, acquiring results that were more accurate than those found in the literature, such as [39–43]. Researchers have recently focused their efforts on the numerical solutions of numerous mathematical models in the realm of epidemiology, such as the HIV model [44], COVID-19 [45], plant disease model [46], tuberculosis propagation model [47], computer virus transmission model [48]. Although the above mentioned techniques have high precision and consistency but they require considerable memory and long computational cost. Consequently, the procedures for this technique present noteworthy challenges that may be resolved in order to ensure that the solution is precise and consistent. Therefore, several efforts have been made by researchers to develop efficient techniques for solving linear and non-linear ODE systems [49–51]. The importance of numerical solutions is emphasized in the literature listed above. As a result of these considerations, the authors have decided to use the ADS (Adams) and ERK (explicit Runge–Kutta) numerical solvers to solve the delay differential system [52–56]. The Adams predictor-corrector approach [46, 57–60] is also a more efficient and straightforward numerical tool for solving delay differential systems.

In order to manage crop pests, insecticides or other preventative measures are frequently used. The emphasis is shifted to educating farmers about alternative techniques including rotation of crops, biological insect control, and cultural practices by incorporating agricultural knowledge into the pest management approach. Through integration, agricultural pest control may be approached holistically and sustainably [61]. To model and simulate the dynamics of agricultural pest populations while taking into account a variety of elements such as environmental conditions, insect life cycles, and farming practices, numerical analytic techniques such as the use of differential equations and optimization methods can be used. The model can offer

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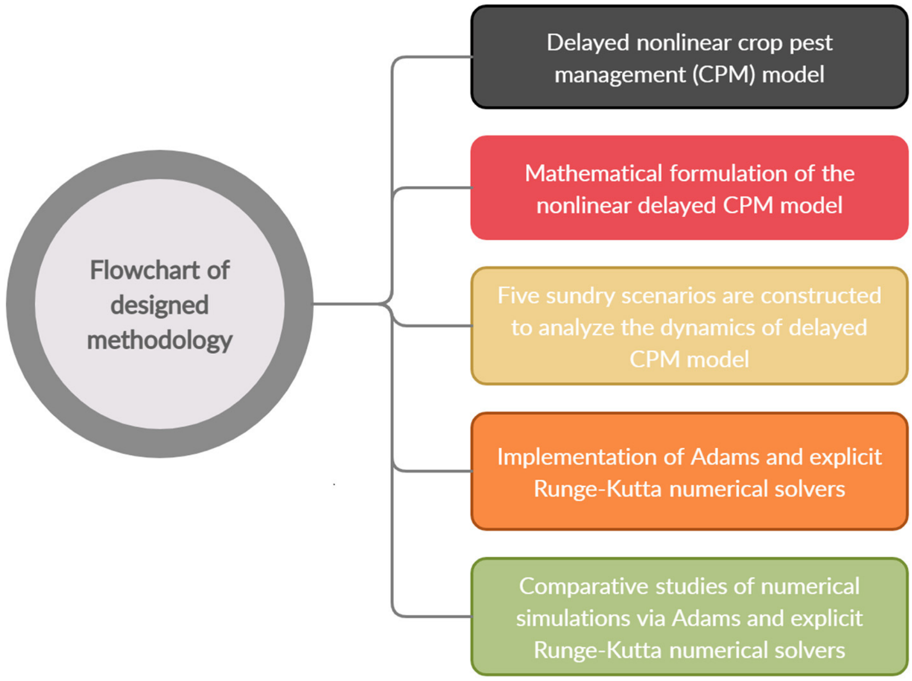

FIGURE 1
Flowchart of designed methodology for non-linear delayed CPM model.

insights to the population dynamic of pests, the effects of various management techniques, and the ideal time for putting control measures into place by using numerical analysis. By offering data-driven advice to farmers and decision-makers, this quantitative method improves the decision-making process [62]. In general, the notion is innovative since it addresses agricultural pest control by combining principles of farming awareness with numerical analytic methods. This multidisciplinary approach emphasizes eco-friendly and sustainable practices while also offering a quantitative foundation for analyzing and improving pest management tactics. Combining these factors helps researchers create agricultural pest management strategies that are both more practical and ecologically responsible. This research may help with the creation of efficient and long-lasting farming awareness campaigns, the adoption of integrated pest management techniques, and the alleviation of farmer difficulties brought on by crop pests. Time delay models really have the potential to increase complexity because of the intrinsic properties of temporal latencies and their impact on system dynamics. The implementation of numerical solutions for non-linear delayed systems may be challenging and computationally expensive. Researchers may require sophisticated software, outstanding durability computing devices, as well as expertise in both computational and mathematical modeling strategies. In this study, we used state-of-the-art numerical techniques like Adam (ADS) and explicit Runge–Kutta (ERK) to find the numerical solution of the non-linear delayed CPM model. The presented study has the following salient features:

- The dynamic impact of the non-linear delayed crop pest management (CPM) system supervised by ODEs is analyzed by incorporating awareness growth level.
- The approximate solutions for the four compartments $B(t)$, $S(t)$, $I(t)$, and $A(t)$ are determined by the implementation of sundry scenarios generated with the variation in crop biomass growth rate, rate of pest attacks, pest natural death rate, disease associated death rate and memory loss of aware people.
- The strength of the Adams and explicit Runge–Kutta numerical solvers are utilized to determine the approximate solutions for the non-linear delayed CPM model.
- Comparative analysis is carried out for the dynamic impact of the non-linear delayed CPM model in terms of numerical outcomes as well as graphical illustrations based on sundry scenarios.

The rest of the article's layout is as follows: In the second section, the non-linear delayed CPM model is formulated. The third section provides a detailed overview of the methodology. The fourth section provides the analysis, discussion, and graphical interpretation of approximate solutions. The fifth section presents the analysis-based conclusion.

2. Formulation of the mathematical model

The model [38] integrated agricultural biomass, pests, and the population's awareness. Crop biomass density, susceptible pest density, infected pest density, and aware people density are the four compartments incorporated in the model. Logistical evolution for the densities of crop biomass is assumed, since crop fields have a finite size (though it might be large), with a net growth rate a and N is carrying capacity. Pests that are susceptible to the crop are

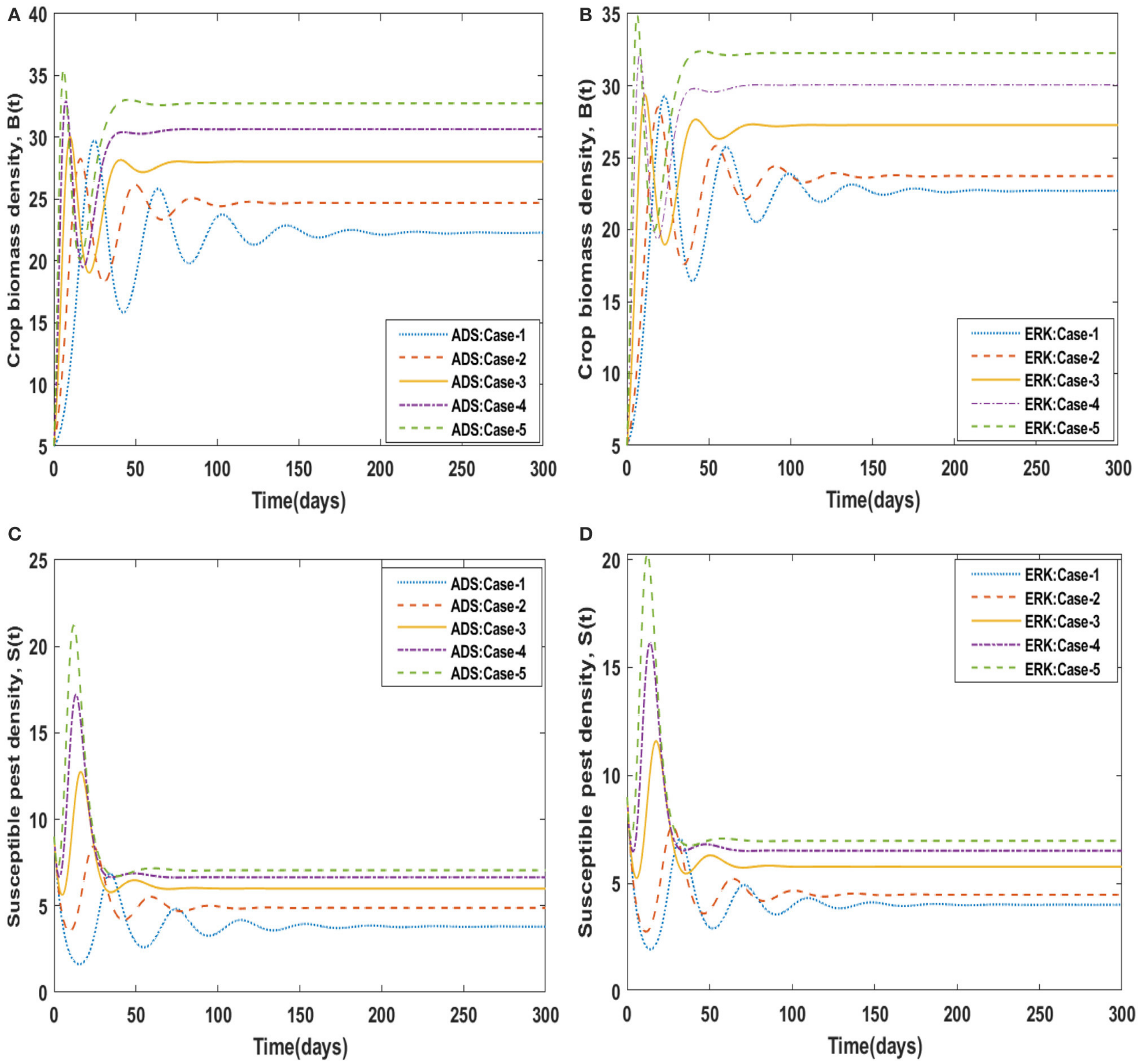

FIGURE 2
(A–D) Dynamics of *B(t), S(t)* for variation in α by ADS and ERK solvers.

attacked, significantly reducing the crop. Let λ represent the pest attack rate on crops [38].

$$\frac{dB}{dt} = aB\left(t\right)\left(1 - \frac{B\left(t\right)}{N}\right) - \delta B\left(t\right)S\left(t\right) - \varphi\delta B\left(t\right)I\left(t\right) \tag{1}$$

To take into consideration their interests as well, aware individuals may keep the crops under observation and, if properly trained, will squirt biopesticides or integrate them into fertilizer applications to manage the insect invasion. The massive term $lA(t)S(t)$ can be used to introduce the awareness action rate l, which results from deliberate human activities and control actions like the application of biopesticides [38].

$$\frac{dS}{dt} = \mu\delta B\left(t\right)S\left(t\right) - lA\left(t\right)S\left(t\right) - cS\left(t\right) \tag{2}$$

Pests that are infected can also harm the crop, $\phi\delta$, although at a much lesser rate $\phi < 1$. Here, c represents the pests' natural death rate, and the infection mortality rate β is a result of knowing human behavior, like the application of insecticides [38].

$$\frac{dI}{dt} = \mu_1\varphi\delta B\left(t\right)I\left(t\right) + lA\left(t\right)S\left(t\right) - \left(c+\beta\right)I\left(t\right) \tag{3}$$

μ and μ_1 represent the "conversion efficacy" of susceptible as well as infected pests, or how well the pests can use plant components. Since pests influenced by pesticides are less effective, $\mu > \mu_1$. Because of media initiatives and increased public awareness, farmers now have a higher level of awareness, which is denoted by A. Additionally, it is expected that the exposure of the resilient pests influences the rate at which local information is

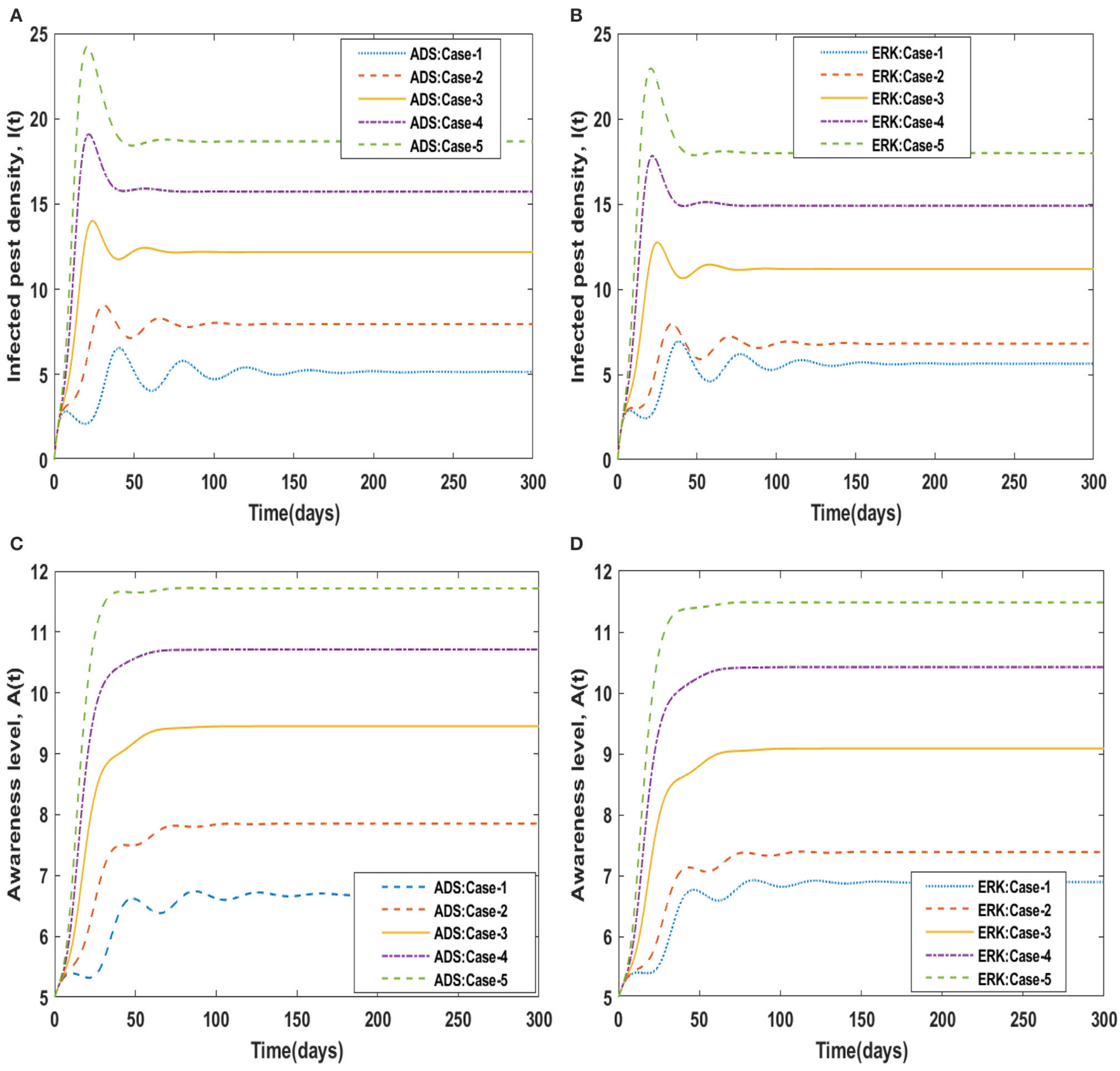

FIGURE 3
(A–D) Dynamics of *I(t), A(t)* for variation in α by ADS and ERK solvers.

increasing at a rate d. The loss of memory causes farmers' levels of consciousness to decline at a rate υ [38].

$$\frac{dA}{dt} = A_0 + d\,(S + I) - \upsilon A\,(t) \tag{4}$$

A delay in observing the number of pests or their activity might occur in a field. Typically, this prediction is produced by studying past incidences of pest prevalence. As a consequence, there are differences in the degree of awareness and the application of preventive countermeasures. Enforcement of such remedies is anticipated to be delayed. The number of pests present at time *(t-τ)* (or time $\tau > 0$ in some cases) will determine how intense the awareness campaigns are at time t.

The following modified mathematical model results from the abovementioned assumptions [38].

$$\begin{aligned}
\frac{dB}{dt} &= aB\,(t)\left(1 - \frac{B\,(t)}{N}\right) - \delta B\,(t)\,S\,(t) - \varphi\delta B\,(t)\,I\,(t)\,, \quad (5)\\
\frac{dS}{dt} &= \mu\delta B\,(t)\,S\,(t) - lA\,(t)\,S\,(t) - cS\,(t)\\
\frac{dI}{dt} &= \mu_1\varphi\delta B\,(t)\,I\,(t) + lA\,(t)\,S\,(t) - (c + \beta)\,I\,(t)\,,\\
\frac{dA}{dt} &= A_0 + d\,[S\,(t - \tau) + I\,(t - \tau)] - \upsilon A\,(t)\,,
\end{aligned}$$

and initial conditions are as:

$$B_0 > 0,\ S_0 > 0,\ I_0 > 0,\ A_0 > 0.$$

TABLE 3 Numerical solutions of non-linear delayed CPM model.

Time (Days)	ADS method: Case-1, scenario 1				ERK method: Case-1, scenario 1			
	B	*S*	*I*	*A*	*B*	*S*	*I*	*A*
0	5.0000	9.0000	0.0000	5.0000	5.0000	9.0000	0.0000	5.0000
30	26.1125	5.8948	3.9220	5.5834	23.3716	6.8244	5.0614	5.8591
60	25.0607	2.9136	4.0531	6.4258	25.7283	3.6210	4.6352	6.5924
90	20.9814	3.3823	5.2252	6.7278	22.5739	3.5602	5.4542	6.8884
120	21.3736	4.0253	5.4197	6.7014	21.973	4.0203	5.7923	6.9152
150	22.5170	3.9243	5.1375	6.6640	22.5754	4.1149	5.6882	6.8880
180	22.4914	3.7534	5.0876	6.6792	22.8188	4.0266	5.6053	6.8852
210	22.2127	3.7693	5.1551	6.6915	22.736	3.9931	5.6144	6.8919
240	22.2056	3.8118	5.1717	6.6890	22.6739	4.0055	5.6330	6.8939
270	22.2742	3.8102	5.1559	6.6859	22.6809	4.0143	5.6342	6.8929
300	22.2797	3.7995	5.1510	6.6864	22.6949	4.0132	5.6306	6.8924
Time (Days)	ADS method: Case-1, scenario 2				ERK method: Case-1, scenario 2			
	B	*S*	*I*	*A*	*B*	*S*	*I*	*A*
0	5.0000	9.0000	0.0000	5.0000	5.0000	9.0000	0.0000	5.0000
30	42.1137	1.4442	1.2169	4.9590	37.5666	2.4939	1.7529	5.0883
60	30.9961	4.2045	4.7578	5.9536	24.7972	3.0352	4.8875	6.4178
90	35.8779	2.8193	3.4942	6.0433	29.1146	3.9229	4.4357	6.2426
120	34.6532	3.2335	3.8803	6.0627	29.0943	3.2468	4.2783	6.3584
150	35.0477	3.0773	3.7627	6.0738	28.4761	3.5095	4.5016	6.3678
180	34.9494	3.1288	3.7958	6.0697	28.9109	3.4469	4.3903	6.3534
210	34.9740	3.1114	3.7869	6.0720	28.7450	3.4403	4.4240	6.3634
240	34.9693	3.1171	3.7891	6.0710	28.7786	3.4551	4.4215	6.3599
270	34.9697	3.1153	3.7886	6.0714	28.7857	3.4467	4.4174	6.3605
300	34.9700	3.1158	3.788	6.0713	28.7754	3.4496	4.4205	6.3607
Time (Days)	ADS method: Case-1, scenario 3				ERK method: Case-1, scenario 3			
	B	*S*	*I*	*A*	*B*	*S*	*I*	*A*
0	5.0000	9.0000	0.0000	5.0000	5.0000	9.0000	0.0000	5.0000
30	35.5348	3.2136	1.3479	4.8960	33.3640	3.5504	1.7029	5.0144
60	25.7824	2.3893	2.3939	5.8214	24.4191	2.2973	2.7589	5.9697
90	23.0944	3.4217	3.2682	5.9988	21.3008	3.46005	3.8654	6.1715
120	24.6101	3.7104	3.1689	5.9524	23.2995	3.8409	3.6906	6.0977
150	25.2161	3.5072	3.0193	5.9478	24.0785	3.5170	3.4583	6.0924
180	24.9855	3.4511	3.0346	5.9622	23.6254	3.4445	3.5082	6.1180
210	24.8747	3.4821	3.0615	5.9652	23.4641	3.5134	3.5601	6.1213
240	24.9097	3.4933	3.0603	5.9632	23.5660	3.5293	3.5490	6.1162
270	24.9307	3.4884	3.0556	5.9627	23.6013	3.5138	3.5377	6.1156
300	24.9257	3.4862	3.0556	5.9630	23.5785	3.5105	3.5402	6.1168

Each parameter used in the mathematical model is described in the nomenclature. The parameters' descriptions and default values are listed in Table 1 as per in Al Basir et al. [38]. These default parameter values are used to generate each scenario. The non-linear delayed CPM model by using numerical values can be mathematically defined for one of the cases as:

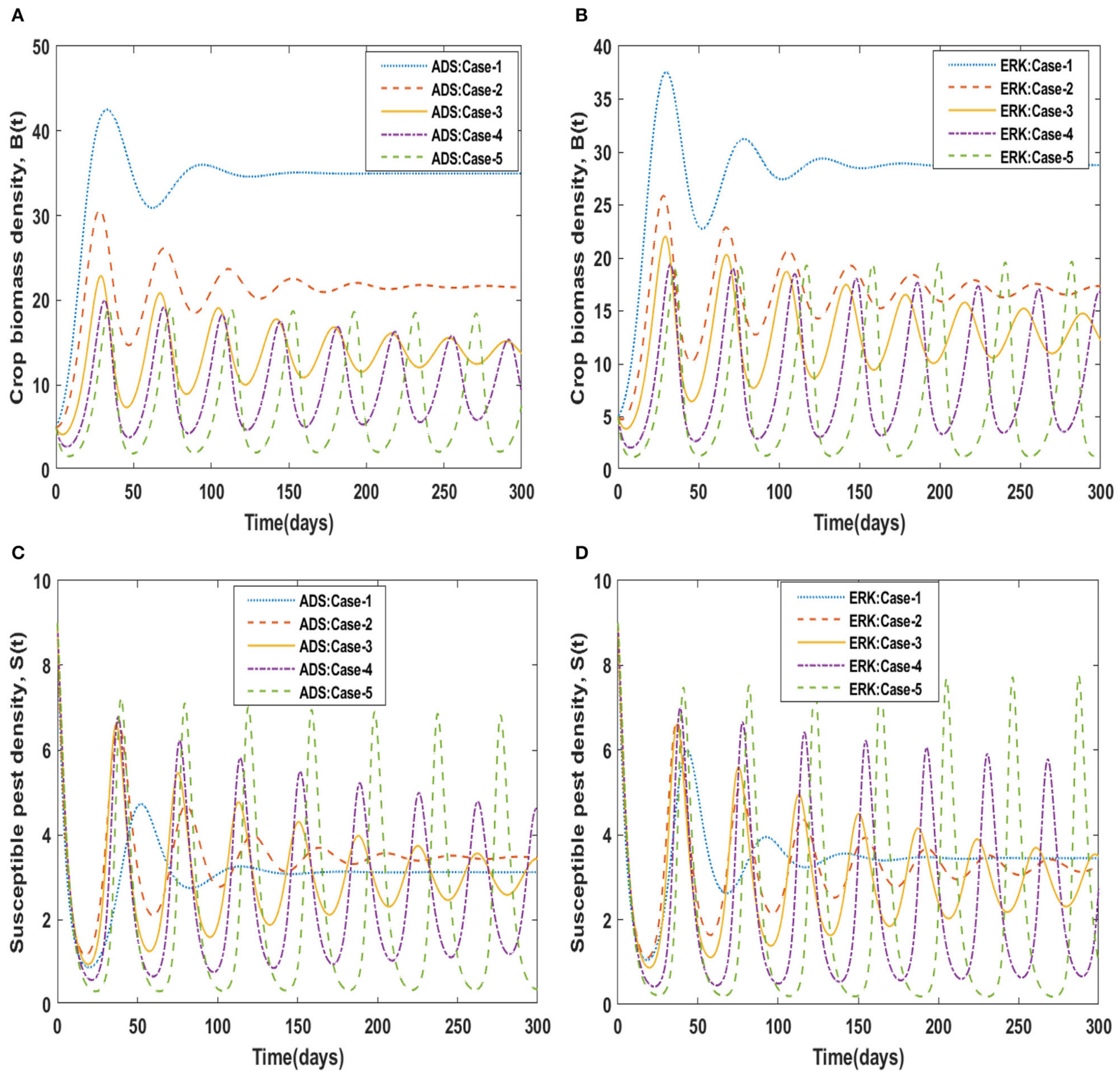

FIGURE 4
(A–D) Dynamics of *B(t), S(t)* for variation in δ by ADS and ERK solvers.

$$
\begin{aligned}
\frac{dB}{dt} &= 0.2B(t)\left(1-\frac{B(t)}{50}\right) - 0.025B(t)S(t) - 0.0125B(t)I(t), \\
\frac{dS}{dt} &= 0.015B(t)S(t) - 0.025A(t)S(t) - 0.01S(t) \qquad (6) \\
\frac{dI}{dt} &= 0.0015B(t)I(t) + 0.025A(t)S(t) - 0.15I(t), \\
\frac{dA}{dt} &= 0.2 + 0.015[S(t-1) + I(t-1)] - 0.6A(t),
\end{aligned}
$$

3. Methodology

This section includes a detailed presentation of the learning methodologies that are used to determine the approximate solutions of the non-linear delayed CPM model.

3.1. Adams method

A two-step process called the ADS numerical solver is used to solve an ODE [63, 64]. Initially, the predictive stage provides a rough approximation of the target outcome in order to utilize an explicit technique. The corrector step uses a different method, typically an implicit one, to speed up the previous approximation.

$$
\frac{dB}{dt} = H(t, B, S, I), \qquad B(t_0) = B_0 \qquad (7)
$$

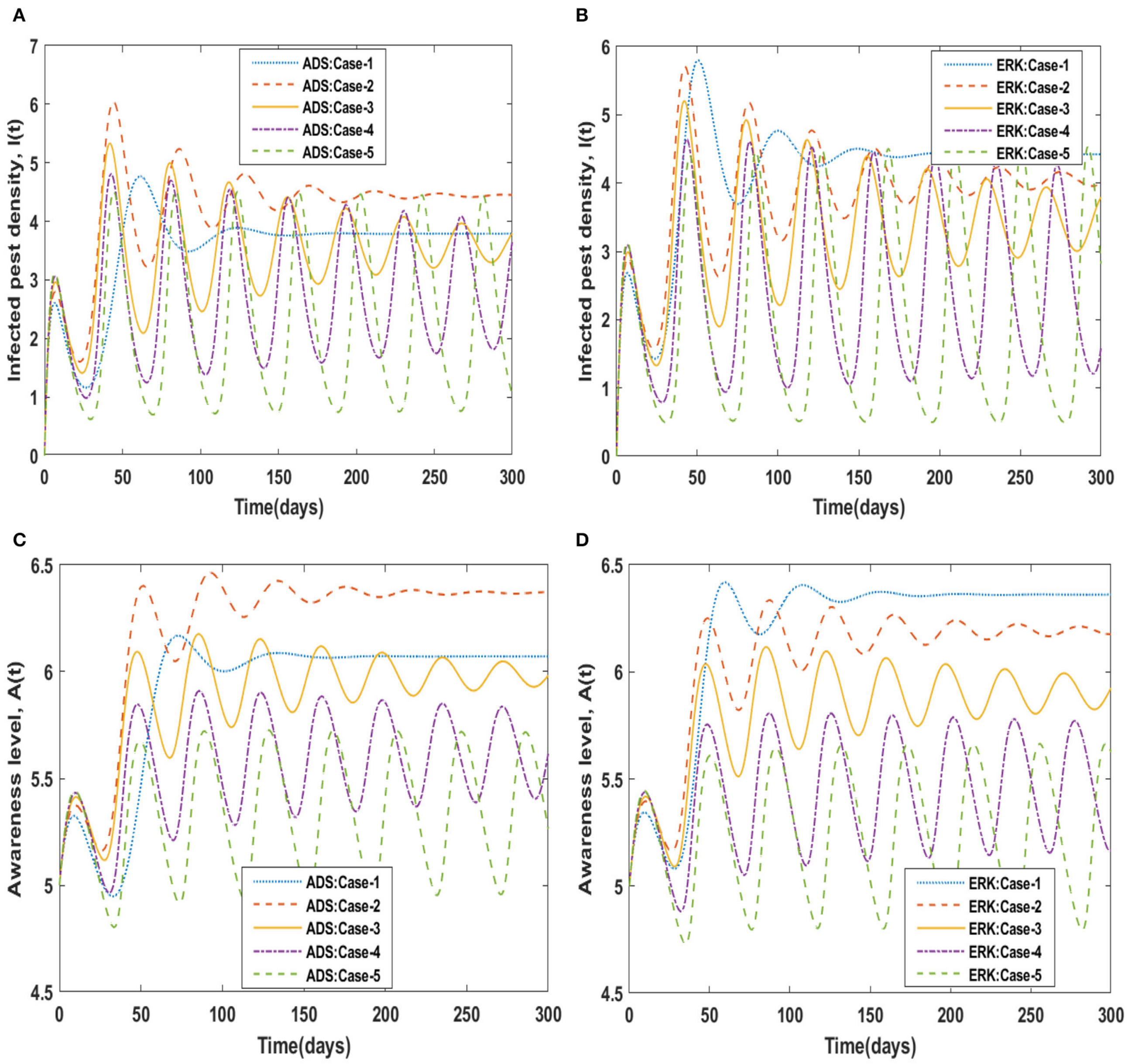

FIGURE 5
(A–D) Dynamics of *I(t), A(t)* for variation in δ by ADS and ERK solvers.

$$\frac{dS}{dt} = H(t, S, B, A), \qquad S(t_0) = S_0$$
$$\frac{dI}{dt} = H(t, I, B, S, A), \qquad I(t_0) = I_0$$
$$\frac{dA}{dt} = H(t, A, S, I), \qquad A(t_0) = A_0$$

For the very first equation in set (7) of the non-linear delayed CPM model, use the following formula to produce a two-step prediction solution:

$$B_{k+1} = B_k + \frac{6}{4}hH(t_k, B_k) - \frac{1}{2}hH\left(t_{k-1}, B_{k-1}\right), \tag{8}$$

Once the very first equation in the non-linear delayed CPM model has been evaluated, the following two step corrector formula is obtained:

$$B_{k+1} = B_k + \frac{1}{2}hH\left(t_{k+1}, B_{k+1}\right) + H(t_k, B_k). \tag{9}$$

Adams techniques may be used to solve a variety of initial value problems, including those involving delay differential equations and ODEs. They are capable of handling stiff as well as non-stiff systems. When compared to other numerical approaches, such as implicit methods, these techniques are computationally efficient. They can lead to faster computations since they require fewer function evaluations each step. Adams techniques contain stability constraints on the step-size and the ratio of step-size to time delay,

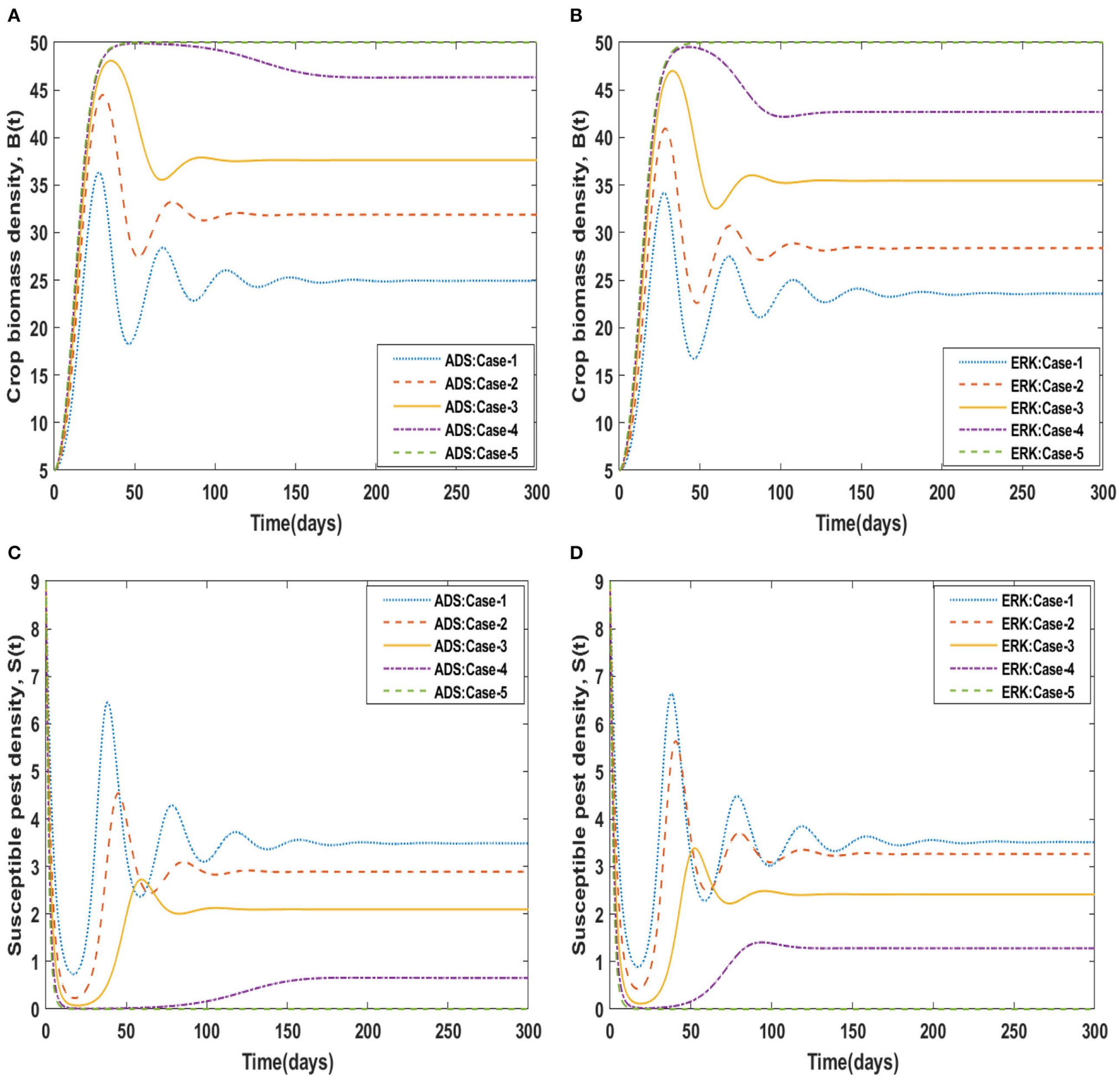

FIGURE 6
(A–D) Dynamics of *B(t), S(t)* for variation in *c* by ADS and ERK solvers.

making them conditionally stable. The approach could result in unstable solutions if these requirements are not satisfied. Adams techniques need a sufficient number of starting values to begin the iteration process since they are multi-step approaches. When starting quantities are difficult to get or need further calculations, this might be difficult [65].

3.2. Runge–Kutta method

The explicit Runge–Kutta (ERK) numerical solver can be used efficiently and comprehensively to solve ODEs [66]. C. Runge and M. W. Kutta introduced the Runge–Kutta methods in the early 1900s. As time went on, this approach played a significant part in the research of iterative approaches based on explicit and implicit assumptions that were used to solve ODEs using time discretization.

The generic form of ODE is considered as:

$$\frac{dy}{dt} = f\left(t, y\right), \tag{10}$$

A generic form of ERK method is defined as:

$$m_1 = f\left(t_n, y_n\right), \tag{11}$$

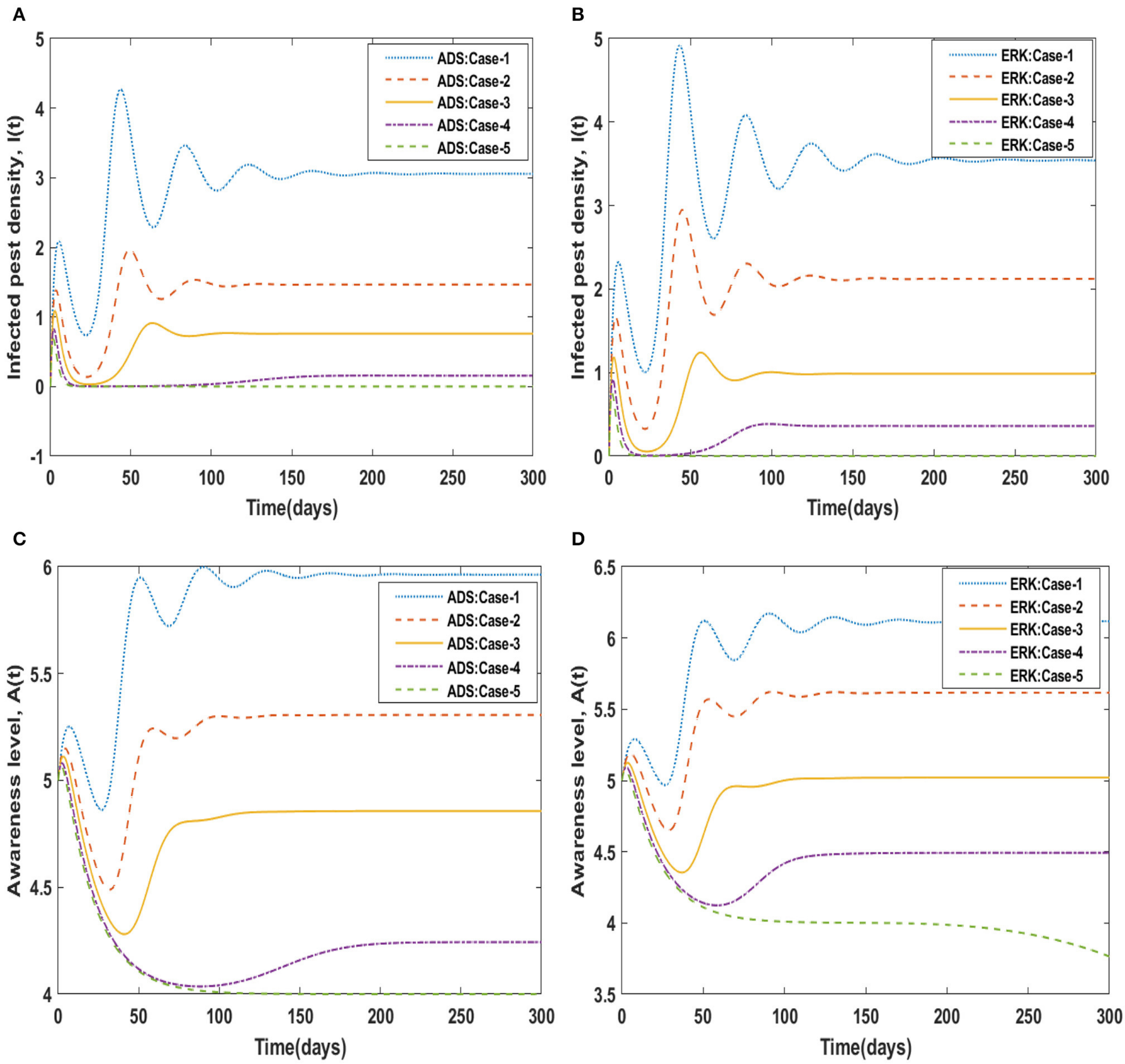

FIGURE 7
(A–D) Dynamics of *I(t)*, *A(t)* for variation in *c* by ADS and ERK solvers.

$$m_j = f\left(t_n + b_j h,\ y_n + h\sum_{i=1}^{j-1} c_{ji} m_i\right),\ j = 2, ..., l, \quad (12)$$

$$y_{n+1} = y_n + h\sum_{j=1}^{l} a_j m_j, \quad (13)$$

where the time interval is $h = \Delta t$, and y_n approximates $y(t_n)$.

The stability characteristics of Runge–Kutta techniques are well established. They can manage a variety of concerns, which includes stiff systems, without running into stability problems. Numerical simulations can be resilient and trustworthy thanks to this stability. These methods are adaptable and effective for dealing with delayed differential equations as well as regular differential equations, partially differential equations, and other forms of differential equations. They have broad applications in several fields of science and engineering [67].

4. Analysis and discussion

The approximate numerical solutions for compartments *B(t)*, *S(t)*, *I(t)*, and *A(t)* of the non-linear delayed CPM model are presented here in this section. The dynamics of the non-linear delayed CPM model are investigated for sundry scenarios each comprising of 1–5 cases by means of ADS and ERK numerical solvers with input points from 0 to

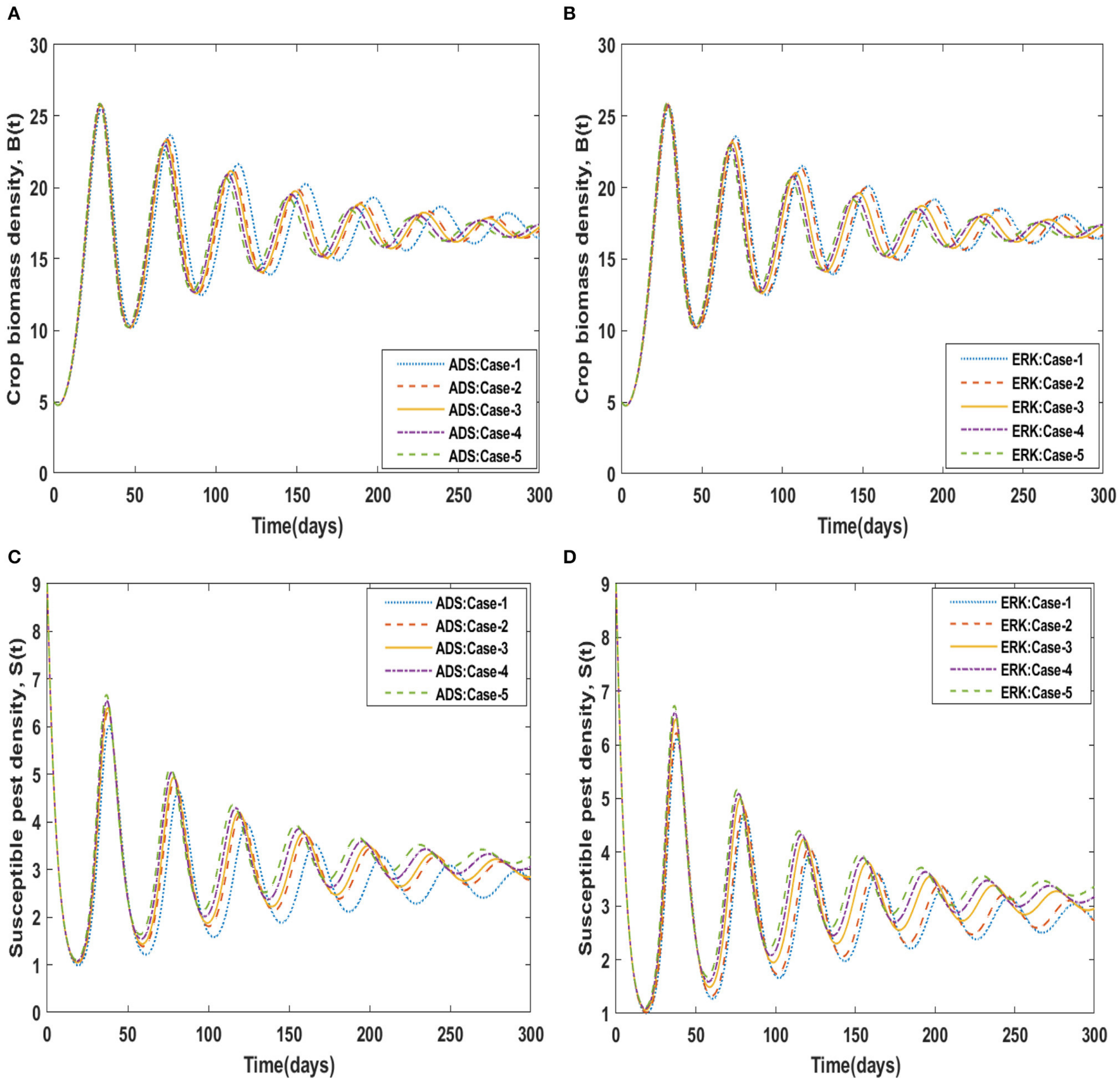

FIGURE 8
(A–D) Dynamics of *B(t), S(t)* for variation in β by ADS and ERK solvers.

300 and step size 0.5. The approximate solutions for the sundry scenarios with 1–5 cases of the non-linear delayed CPM model are computed by varying the crop biomass growth rate, rate of pest attacks, pest natural death, disease associated mortality rate and memory loss of aware people.as listed in Table 2. Figure 1 presented the flowchart of the designed methodology.

The dynamics of crop biomass density are shown in Figures 2A, B, respectively, using the ADS and ERK numerical solvers for the variation in crop biomass growth rate, i.e., a for the non-linear delayed CPM model. The crop biomass density has been found to increase as the value of a increases. Figures 2C, D for various values of a illustrate the effects of susceptible pest density. The graph shows that as the value of a increases, so does the density of pests that are susceptible. Figures 3A, B show how infected pests' behavior varies as the value of a changes. There is an increase in the density of infected pests for larger values of a. The effects of people's level of awareness for various values of a are depicted in Figures 3C, D. The graph illustrates how increasing the value of a raises the level of awareness. Table 3 presents the numerical results for the classes $B(t)$, $S(t)$, $I(t)$, and $A(t)$ for scenario 1, case-1 of the non-linear delayed CPM model. Using the strength of ADS and ERK numerical solvers for cases 1 to 5 of scenario 2, the dynamics of the non-linear delayed CPM model for the rate of pest attacks, i.e., δ, is investigated for all four classes $B(t)$, $S(t)$, $I(t)$, and $A(t)$ and graphically shown in Figures 4, 5 respectively.

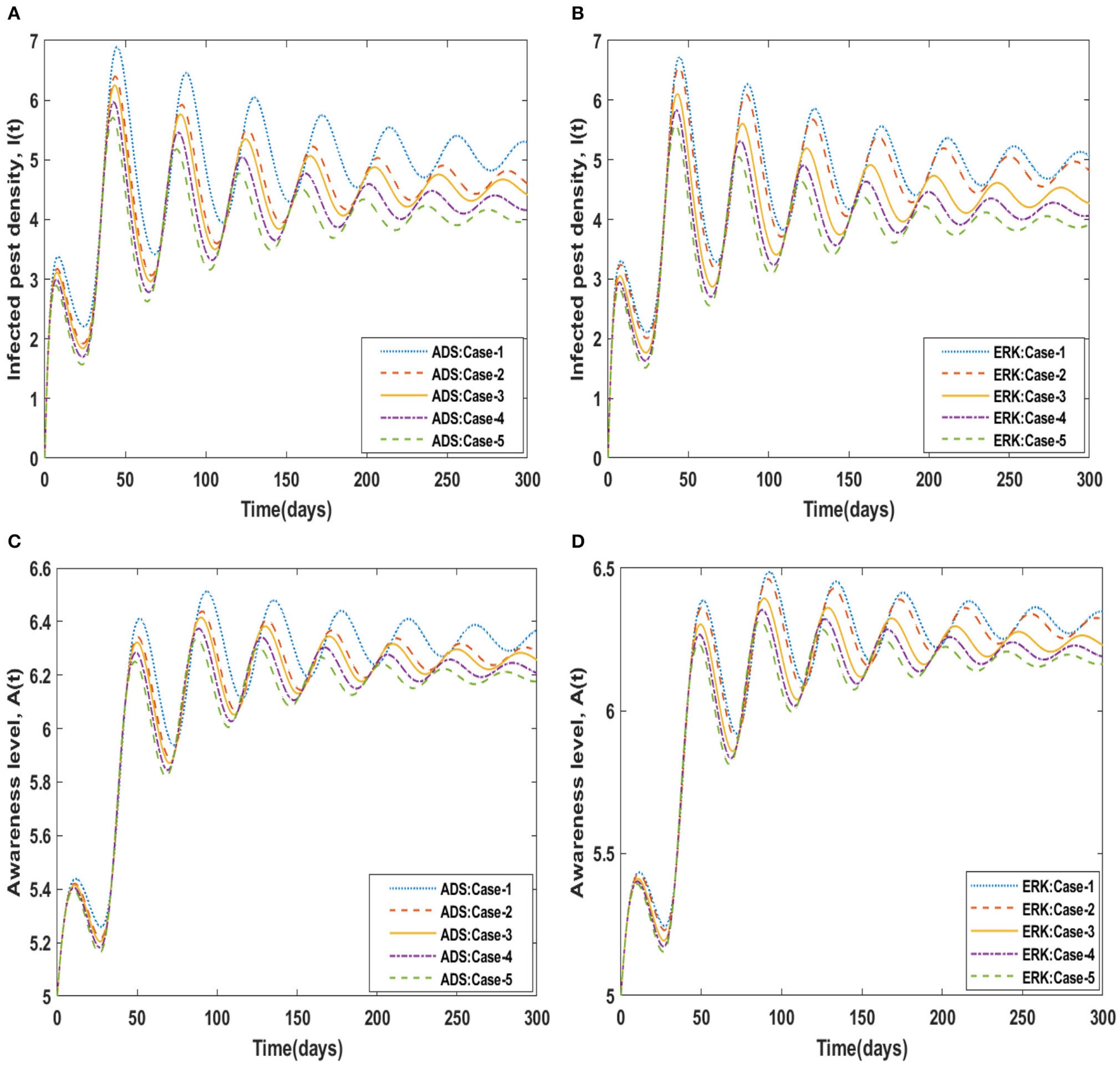

FIGURE 9
(A–D) Dynamics of *I(t)*, *A(t)* for variation in β by ADS and ERK solvers.

The numerical outcomes for the classes *B(t)*, *S(t)*, *I(t)*, and *A(t)* for scenario 2, case-1 of the non-linear delayed CPM model are shown in Table 3. Raising the value of δ increases the density of crop biomass, as presented in Figures 4A, B. For case-1 of pest attacks rate, the maximum value of *B(t)* is approximately between 5 to 45, oscillates from 0 to 150 days, and then maintains steady state behavior. The maximum value for case-2 is between 5–30 and it initially exhibits oscillating behavior in the range of 0 to 250 days before becoming stable in the range of 250 to 300 days. Cases 3 to 5, as depicted in Figures 4A, B, exhibit oscillations with varying amplitudes across the time interval. As the value of pest attacks i.e., δ expanded, the density of susceptible pests also increased, as seen in Figures 4C, D.

Susceptible pest density demonstrated oscillatory behavior from 0 to 150 days before returning to steady state behavior, whereas cases 2 to 5 exhibit oscillatory behavior with varying amplitudes from 0 to 300 days, as shown in Figures 4A, B. For compartment *I(t)* of the non-linear delayed CPM model, Figures 5A, B depict the effects of pest attacks rate. The graphs show that increasing the value of δ will result in decreasing the infected pest density. The impact of pest attacking rate, i.e., δ is also determined for compartment *A(t)* of the non-linear delayed CPM model. As seen in Figures 5C, D, the awareness level decreases as the value of pest attacks grow.

Similarly, for scenario 3 of non-linear delayed CPM model, the dynamics of the four compartments *B(t)*, *S(t)*, *I(t)*, and *A(t)*

TABLE 4 Numerical solutions of non-linear delayed CPM model.

Time (Days)	ADS method: Case-1, scenario 4				ERK method: Case-1, scenario 4			
	B	*S*	*I*	*A*	*B*	*S*	*I*	*A*
0	5.0000	9.0000	0.0000	5.0000	5.0000	9.0000	0.0000	5.0000
30	25.6352	2.7948	2.6511	5.2802	25.6454	2.9384	2.5856	5.2683
60	16.0485	1.2237	4.1627	6.2469	16.4980	1.2738	3.9219	6.2050
90	12.5031	3.1099	6.3705	6.4831	12.4801	2.9846	6.0962	6.4722
120	19.5105	3.6746	4.8757	6.1610	18.5848	3.9118	4.9691	6.1742
150	19.2204	2.0351	4.3368	6.2646	19.6539	2.2603	4.1730	6.2164
180	15.3228	2.38003	5.3581	6.4332	15.8217	2.3344	5.0070	6.3913
210	16.5595	3.2259	5.4430	6.3382	16.0291	3.1776	5.3515	6.3469
240	18.6317	2.7134	4.7792	6.2699	18.2773	2.9676	4.7624	6.2551
270	17.1624	2.407	4.9620	6.3510	17.611	2.5253	4.7201	6.3055
300	16.4349	2.7711	5.2948	6.3684	16.4975	2.7282	5.0572	6.3479
Time (Days)	**ADS method: Case-1, scenario 5**				**ERK method: Case-1, scenario 5**			
	B	*S*	*I*	*A*	*B*	*S*	*I*	*A*
0	5.0000	9.0000	0.0000	5.0000	5.0000	9.0000	0.0000	5.0000
30	33.4212	1.2608	1.7179	9.7847	32.7550	1.4497	1.7952	9.2354
60	26.8333	0.5218	3.3758	14.054	25.9310	0.6014	3.3390	12.9231
90	30.8679	1.2775	4.4344	15.9967	27.0746	1.4925	5.0180	14.4833
120	37.2066	0.5837	2.6639	17.3430	34.5850	0.9614	3.1909	15.3405
150	35.5515	0.6244	3.2161	18.1282	32.5484	0.77943	3.5441	15.9731
180	37.9167	0.5752	2.7510	18.5074	33.3100	0.9351	3.6511	16.1777
210	37.6625	0.5377	2.7981	18.7708	34.2309	0.81780	3.3647	16.3285
240	38.1941	0.5387	2.7157	18.8880	33.7867	0.8282	3.4894	16.4132
270	38.2294	0.5225	2.6998	18.9693	34.1173	0.8365	3.4408	16.4400
300	38.3414	0.5238	2.6863	19.0066	34.1219	0.8184	3.4242	16.4658

are explored by varying the value of pest natural death rate i.e., c, which is represented by c and graphically portrayed in Figures 6, 7 respectively. The numerical solutions for compartments $B(t)$, $S(t)$, $I(t)$, and $A(t)$ for scenario 3, case-1 of the non-linear delayed model are computed and listed in Table 3. The influence of the pest's natural death rate on crop biomass density using the ADS and ERK numerical solvers respectively, is shown in Figures 6A, B. The effects of the pest natural death rate on the density of susceptible pests are shown in Figures 6C, D. It is noticed that the number of susceptible pests reduced as c increased. Figures 7A, B demonstrated how the density of infected pests decreases as the value of c rises. In Figures 7C, D, the level of people's awareness can be analyzed. It is worth noting that the larger value of the natural pest's death rate i.e., c, causes a decrease in people's awareness.

The dynamical behavior of the four compartments $B(t)$, $S(t)$, $I(t)$, and $A(t)$ for cases 1 to 5 of scenario 4 with the variability in disease associated death rate, i.e., β is analyzed and graphically portrayed in Figures 8, 9 respectively. The numerical solutions for scenario 4, case-1 of the non-linear delayed CPM model are calculated for all four classes $B(t)$, $S(t)$, $I(t)$, and $A(t)$ and presented in Table 4. The dynamics of crop biomass density are portrayed in Figures 8A, B exploiting the potential of the ADS and ERK numerical solvers for the variability in the disease associated death rate, i.e., β. It has been found that the crop biomass density falls as the value of β increases. Figures 8C, D show the effect of disease associated death rate on the density of susceptible pests. It is evident from Figure that raising the value of β would lead to a rise in the density of susceptible pests. The behavior of infected pest density for the variation in disease associated death rate is shown in Figures 9A, B. As the value of β is raised, it may be observed that the density of infected pests will decrease. Figures 9C, D show the impact of disease associated death rate against awareness level in people. The graphical representation presented that increasing the value of β causes the awareness level to decrease.

Using the strength of ADS and ERK numerical solvers, the dynamics for memory loss of aware people, i.e., υ , are investigated for all four compartments $B(t)$, $S(t)$, $I(t)$, and $A(t)$ for scenario 5, cases 1 to 5 of the non-linear delayed CPM model. The numerical

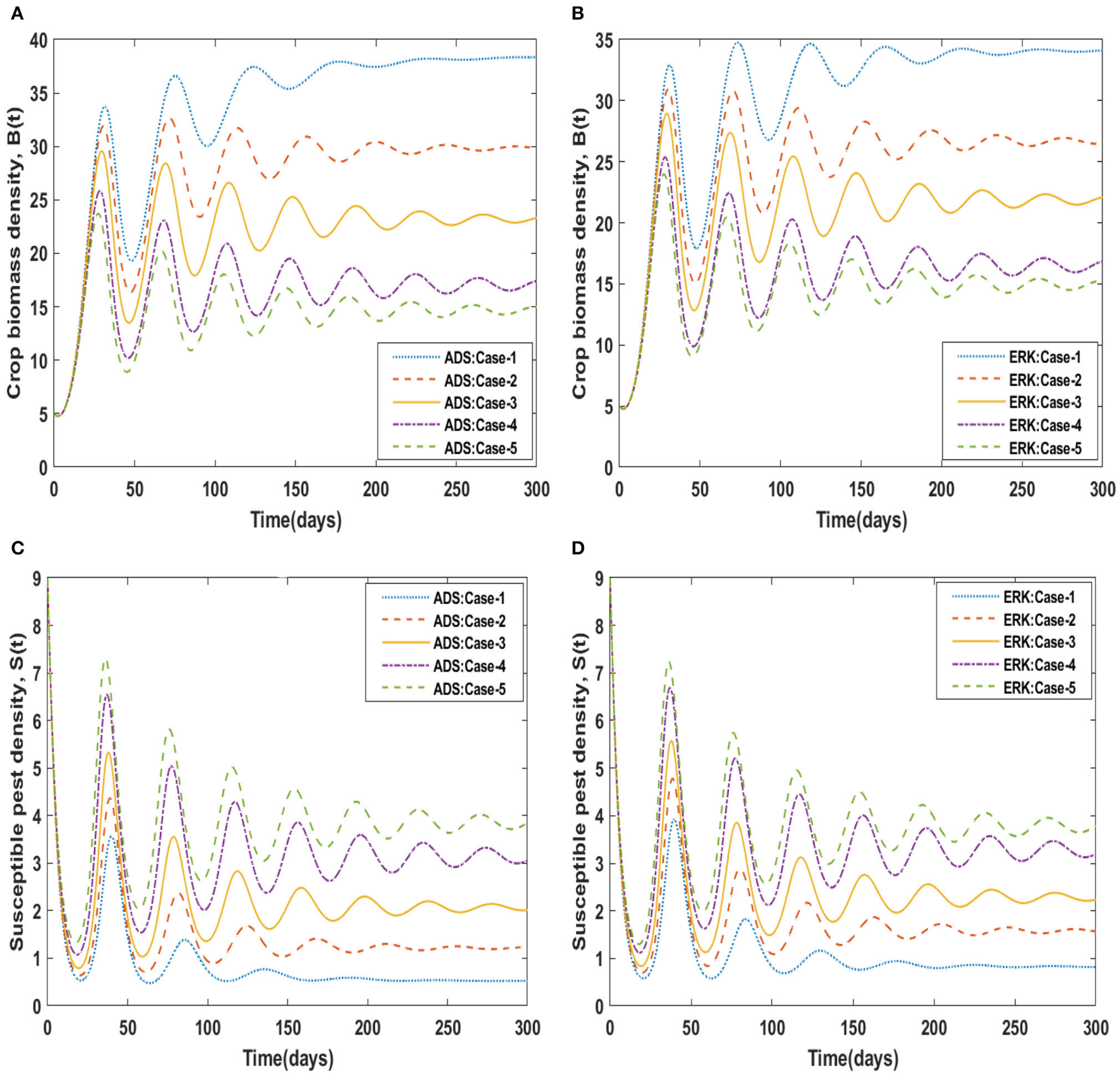

FIGURE 10
(A–D) Dynamics of *B(t)*, *S(t)* for variation in υ by ADS and ERK solvers.

outcomes of all four compartments *B(t)*, *S(t)*, *I(t)*, and *A(t)* for case-1 of scenario 5 are provided in Table 4. The behavior of crop biomass density for the varying values of υ is depicted in Figures 10A, B, and it can be observed that crop biomass density decreases for higher values of υ. Figures 10C, D illustrated how the density of susceptible pests increases as the value of υ increases. Figures 11A, B show the dynamics of infected pest density for the variation in memory loss of aware people, i.e., υ. One may witness that the density of infected pests increases continuously in the first three cases, for $\upsilon = 0.01, 0.02$, and 0.03, and then decreases again in the subsequent two cases, for $\upsilon = 0.04$, and 0.05, in the interval of 0 to 300 days. Consequently, the density of infected pests shows varied behavior for different values of υ. Figures 11C, D portrayed the effect of memory loss in aware people, i.e., υ on awareness level compartment *A(t)*. It is clearly noticed from Figures 11C, D that the awareness level is decreased as the value $\upsilon$is increased.

5. Conclusions

In this research, the numerical approximate solution of the non-linear delayed CPM system supervised by ODEs is investigated effectively to portray the dynamic impacts of unforeseen interactions between crops and pests, rehabilitation strategies, and environmental factors across time. Based on the presented model, the dynamic nature of crop biomass density

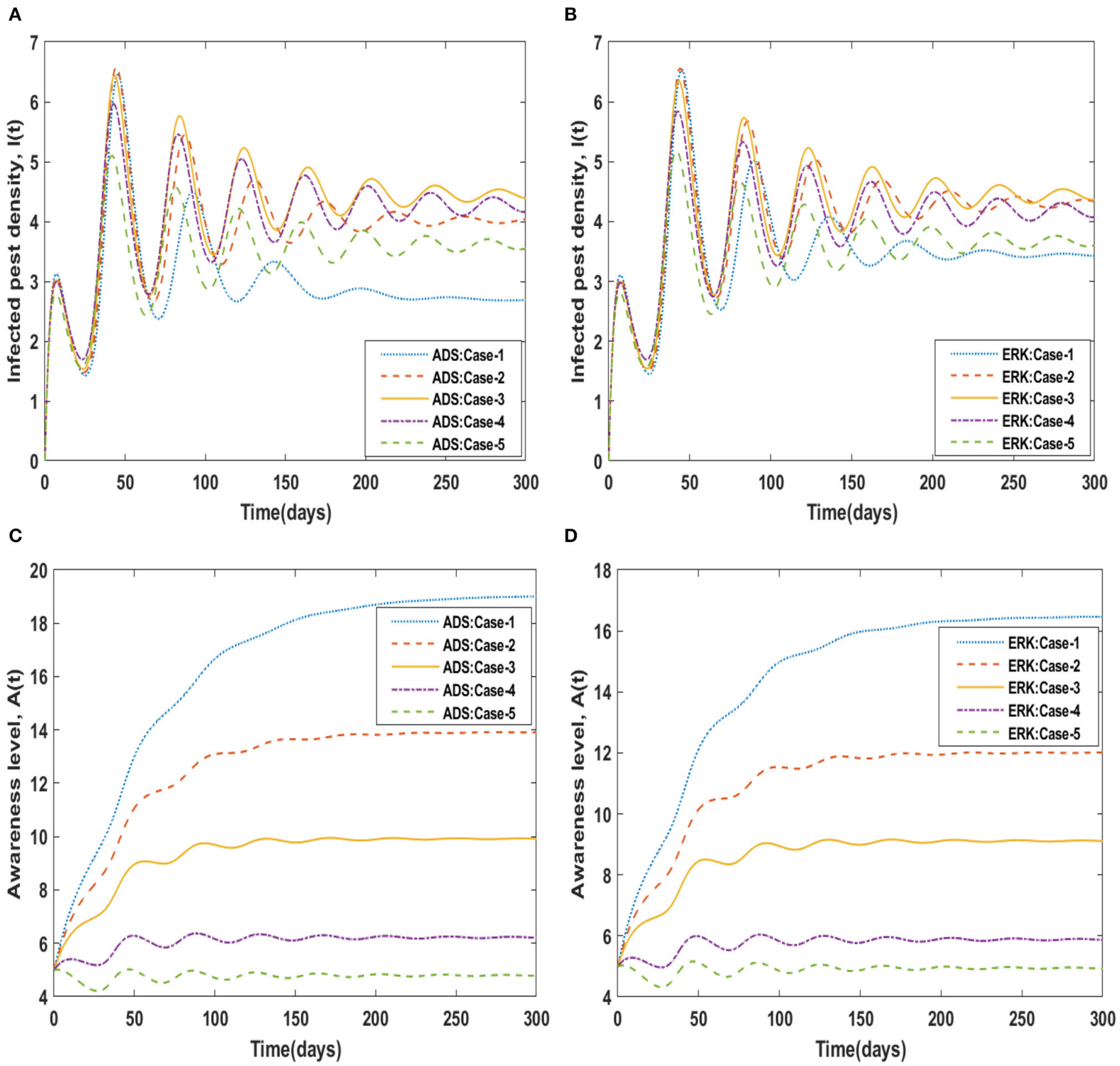

FIGURE 11
(A–D) Dynamics of *I(t)*, *A(t)* for variation in υ by ADS and ERK solvers.

[*B(t)*], susceptible pest density [*S(t)*], infected pest density [*I(t)*] and awareness level of the population [*A(t)*] may be forecasted effectively. Analysis based on the approximate numerical outcomes as well as graphic interpretations of the non-linear delayed CPM model is carried out by means of sundry scenarios by varying the different parameters utilized in the model. The approximate numerical solution of the non-linear delayed CPM model is computed by exploiting the state-of-the-art Adams (ADS) and explicit Runge–Kutta (ERK) numerical techniques. Compared with real-time models, delayed models exhibit greater realism because they take into account the interval between contact and infection. As delay affects processes along with dynamics, mathematically it impacts stability. This analysis can help to create predictive models for upcoming outbreaks and shed light on the efficacy of various pest management techniques. The numerical analysis that is being given makes it possible to optimize pest control tactics, analyze risks, educate people, and pursue continual improvement. It is essential for improving agricultural methods, reducing crop losses, and advancing environmentally friendly pest control strategies.

In the future, soft computing approaches based on artificial intelligence algorithms may be used to study the dynamics of epidemic models and other non-linear systems [68–73].

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

Conceptualization, methodology, and writing—review and editing: NA and MR. Software: AK. Validation: MR. Formal analysis: MS and MR. Investigation: IA. Writing—original draft preparation: NA. Visualization: NA and MS. Project administration: AK and IA. All authors have read and agreed to the published version of the manuscript.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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