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Control of Quarter-Car Active Suspension System Based on Optimized Fuzzy Linear Quadratic Regulator Control Method

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Abstract: Vehicle suspension systems, which affect driving performance and passenger comfort, are actively researched with the development of technology and the insufficient quality of passive suspension systems. This paper establishes the suspension model of a quarter of the car and active control is realized. The suspension model was created using the Lagrange–Euler method. LQR, fuzzy logic control (FLC), and fuzzy-LQR control algorithms were developed and applied to the suspension system for active control. The purpose of these controllers is to improve car handling and passenger comfort. Undesirable vibrations occur in passive suspension systems. These vibrations should be reduced using the proposed control methods and a robust system should be developed. To enhance the performance of the fuzzy logic control (FLC) and fuzzy-LQR control methods, the optimal values of the coefficients of the points where the feet of the member functions touch are calculated using the particle swarm optimization (PSO) algorithm. Then, the designed controllers were simulated in the computer environment. The success of the control performance of the applied methods concerning the passive suspension system was compared in percentages. The results are presented and evaluated graphically and numerically. Using the integral time-weighted absolute error (ITAE) criterion, the methods were compared with each other and with the studies in the literature. As a result, it was found that the proposed control method (fuzzy-LQR) is about 84.2% more successful in body motion, 90% in car acceleration, 84.5% in suspension deflection, and 86.7% in tire deflection compared to the studies in the literature. All these results show that the car's ride comfort has been significantly improved.

Keywords: active control; LQR; fuzzy logic control (FLC); fuzzy-LQR control; particle swarm optimization algorithm (PSO); quarter-car suspension system

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1. Introduction

Vehicle suspension systems, which have a decisive influence on a vehicle's ride comfort and handling characteristics, have been researched from the past to the present with the development of technology [1–4]. Suspension systems have the important task of supporting the vehicle's weight, keeping the tires in contact with the ground, and isolating the disturbing influences on the chassis. D'Amato and Viassolo proposed a fuzzy logic control method for the ASS using internal and external control loops [5]. A fuzzy logic controller is applied to the model of a quarter, where the inner loop controls the nonlinear hydraulic actuator to monitor the desired actuation force. In contrast, the outer loop is calculated by GA-based optimization. Foda proposed a fuzzy logic control method for controlling the suspension system of a small car and applied it in the simulation environment [6]. Results were obtained for three different road types. The body handling and the responses in the suspension working area exhibited good damping characteristics for different road conditions. Al-Holou et al. [7] proposed a fuzzy logic control method with a neural network for ASSs. The method was applied to the model of a quarter car in the simulation environment and contrasted with the methods of a passive system, SMC, SMFLC, and SMNNNFLC. The simulation results show that the proposed SMNNNFLC method provides higher ride comfort and quality than the other methods mentioned in the article. Sam et al. [8] proposed a proportional-integral sliding mode control method for the ASS and applied it to a quarter-car model in a simulation environment. This study compared the controller's performance with the linear quadratic controller (LQR) and the existing passive suspension system. It was shown that the proposed method is more robust compared to the linear quadratic controller method and the passive suspension system. The constrained H∞ control of active suspensions, an LMI approach, was proposed by Chen and Guo [9]. In a simulation environment for the LMI-optimized H∞ control of a 2-DOF-constrained suspension system, good handling, constraining the suspension stroke, and avoiding actuator saturation were among the issues investigated. The result of this study was that close contact of the wheels with the road was ensured, and the best possible ride comfort was achieved when the suspension shocks and control inputs were kept within the specified limits.

Chen et al. [10] applied the constrained H∞ control to the half-car model in a simulation environment using multicriteria LMI optimization. Positive results were obtained in improving ride comfort. Yagız et al. [11] proposed a robust fuzzy sliding mode controller for the half-car model with an ASS. The magnitudes of trunk displacement and pitching motion were reduced and the resonance peak caused by the car body was eliminated. It was also found that the magnitudes of trunk acceleration and pitching motion were reduced over a wide range of frequencies, significantly improving ride comfort. Gao et al. [12] investigated the problem of robust sampling data control for indeterminate active car suspension systems on a quarter-car model. Using an input-lag approach, the active car suspension system was transformed into a continuous-time system with state delay using sampling measurements, and polytopic parameter uncertainty was used to characterize the actual uncertain situation. Salem and Aly proposed a fuzzy logic control method for the ASS of a quarter car and applied it in a simulation environment [13]. This method was compared with the PID control method in two different types of railroads. The ride comfort was improved by reducing the body acceleration caused by the car body during road disturbances caused by smooth roads and real road bumps. Lin and Lian studied the intelligent control of ASSs in a simulation environment [14]. This study addressed the problem of selecting learning rate and weight distribution parameters that affect the proposed self-organizing fuzzy controller (SOFC) performance in engineering problems. For this reason, a hybrid self-organizing fuzzy and radial basis functional neural network controller (HSFRBNC) was proposed in this study. SOFC and HSFRBNC methods were compared, and HSFRBNC was found to provide better control performance in improving the suspension system lifetime and ride comfort of a car. Sun et al. [15] applied H∞ control for active car suspension systems in the finite frequency domain with time domain constraints to the model of a quarter car in a simulation environment. In this study, using the generalized Kalman–Yakubovich–Popov (KYP) lemma, a state feedback controller was designed in linear matrix inequality (LMI) optimization in H∞-norm, leading to controlled output without distortion. Reduced results were obtained in a certain frequency band to improve driving comfort. Li et al. [16] proposed a reliable fuzzy H∞ control method for actuator-delayed and faulty ASSs and applied it in a simulation environment. A passive system and an FLC method were contrasted with the approach. The simulation results showed that the designed reliable fuzzy controller could provide better suspension performance in the presence of changes in sprung and unsprung mass, delay, and actuator failure.

Sun et al. [17] proposed an adaptive rollback control strategy for car suspensions with rigid conditions. The proposed controller considers suspension spacing, dynamic tire loads, and actuator saturation as time boundary conditions. In the presence of parameter uncertainties, the method was applied to the half-car model in a simulation environment in the study to stabilize vehicle performance and improve ride comfort. Sun et al. [18] proposed a limited adaptive back-stepping control strategy for ASSs to achieve multi-purpose control. In addition to improving ride comfort, the time-domain constraints required for active suspension control were also guaranteed throughout the time domain and the Lyapunov barrier function was used in this study. The method was applied to the model of a quarter vehicle. Deshpande et al. [19] proposed a disturbance observer-based sliding mode control method for ASSs and implemented it in a simulation and experimental environment. In the study, three different types of road profiles and load variations were tried and verified. The results were compared with those of passive suspension systems, significantly improving mass displacement and acceleration performance. To shed light on the design of suspension control systems in the existing literature, Tseng and Hrovat offered a literature study [20] in which they shared their observations on the most recent hardware implementations of active and semi-ASSs. Wang et al. [21] proposed a static output feedback control strategy based on the variable substitution method and linear matrix inequality for ASSs with limited information. The main feature of the proposed method was that it provided a pre-assignment of the controller structure. This study, which used a half-vehicle model, aimed to improve ride comfort and balance simultaneously. The validity of the designed controller for different track profiles was shown by numerical examples, taking into account other boundary conditions such as suspension preload, actuator saturation, controller-dependent information, etc. Simulation results showed that the optimized controller with static output feedback achieved better suspension performance than a suitable controller with static output feedback. Palanisamy and Karuppan proposed a fuzzy logic control method for controlling ASSs and applied it to a quarter-vehicle model in a simulation environment [22]. The system was compared with a passive and a PID-controlled system. The FLC method was applied to three road profiles and positive results were obtained.

Zhao et al. [23] proposed an adaptive neural network control method for the ASS with actuator saturation and applied it in a simulation environment. A YSA observer was used for state prediction using the system's measured input and output data. The study optimized the feedback control parameters using PSO based on the state observer. The method was compared with the passive state suspension system, neural sliding mode control (SMCNN), and traditional neural network (TNN) control methods. In the simulation results, body vibration was effectively suppressed, the values of the proposed controller RMS were improved under different road conditions, and the body displacement results were obtained at a lower value than the SMCNN controller and TNN controller. Pan et al. [24] proposed an adaptive control method for ASSs to improve the vertical dynamic performance of the car in the presence of parameter uncertainties, disturbing inputs, and non-ideal actuators. Comparative simulations were performed. Compared with existing control methods, the presented control method was adaptive to parameter uncertainties, and it was shown to reduce the effects of non-ideal actuators in the simulation environment. Wen et al. [25] proposed a fuzzy control method for uncertain active car suspension systems with a dynamic sliding mode approach. They applied it to a quarter-car model in a simulation environment. This study used a T-S fuzzy method to consider the change in masses. The sliding mode control parameters were linearly generated and a dynamic fuzzy term was used to construct the sliding mode controller. It was found that the method significantly reduced the spring-mass acceleration compared to passive systems and a lower body acceleration was achieved during road disturbances.

Senthil Kumar et al. [26] proposed an Adaptive Neuro-Fuzzy Inference System (ANFIS) controller for a hydraulically actuated active suspension and applied it to a semi-auto model in a simulation environment. The proposed method was compared with passive and PID control methods for a sinusoidal trajectory profile. As a result of the simulation, it was found that the body displacement and tilt angle of the ANFIS-controlled ASS were significantly lower than those of the PID-controlled suspension system. Zhou et al. [27] proposed an optimal sliding mode control (OSMC) for an ASS based on a genetic algorithm and implemented it in a simulation environment. From the simulation results, the comprehensive performance index (RMS) of the OSMC controller was reduced by 12.73% and 33.4% compared to the SMC controller and passive system, respectively. Nagarkar et al. [28] implemented GA-based multicriteria optimized fuzzy logic and PID control of the active quarter-car nonlinear suspension system in a simulation environment. It was shown that the active FLC control system based on GA minimized the frequency-weighted RMS acceleration and VDV compared with PID and passive suspension systems, thus improving ride comfort. Li et al. [29] proposed an adaptive event-driven fuzzy control method for active vehicle suspension systems with uncertainties. They applied it to the model of a quarter car in the simulation environment. The T-S fuzzy model was used for suspension system uncertainties. The purpose of this method was to save communication resources. As a result of the simulation, a reliable fuzzy controller design approach based on the adaptive event-driven scheme was provided, which can effectively reduce the transmission load at all frequencies.

Youness and Lobusov proposed a networked control scheme for the ASS of a complete car model [30]. The study investigated two networked control systems. PID and LQR control methods were used. They proposed a networked control system for the ASS using the CAN network model. In a networked system with low network speed, the results show that the performance of the LQR method was better than that of PID. Liu et al. [31] proposed an adaptive neural network (NN) as a control method for a four-car model with time-varying vertical displacement and speed limit and an ASS with unknown car mass. The unknown car mass was inserted into the simulation environment using the NN used in the method. Wang et al. [32] proposed and experimentally applied a new method for active suspension systems using model-independent finite-time motion control methods. In this method, the vertical dynamics of the suspension system, including the external disturbances, were estimated by the time delay estimation. The estimated error was compensated using the integral sliding mode control method. The experimental results showed that the method performed adequately as long as the vertical displacement of the sprung mass and the vertical car acceleration reduced the RMS error compared with the traditional suspension system methods. Lin et al. [33] proposed the fuzzy sliding mode control method with a proportional differential sliding mode observer for an ASS. It was shown that the fuzzy sliding mode control method in the simulation environment could improve the ride comfort, operational stability, and driving safety of the car, and reduce energy consumption.

Wang et al. [34] proposed a fuzzy sliding mode-based anti-disruptive control for a 7-DOF active car suspension system. An extended state observer was used to estimate all disturbances in the system. The fuzzy sliding mode control for active distortion suppression (FSM-ADRC) was compared with existing active distortion suppression methods (ADRC and SM-ADRC). This comparison showed that the proposed method performed better. Li et al. [35] proposed an adaptive optimal control with neural networks and output feedback for ASSs and implemented it in a simulation environment. In this study, NNs were used to identify unknown nonlinear states, and an NN state observer was used to predict unmeasurable states. The proposed optimal control algorithm ensured the boundedness of the signals of the controlled system, and the power of the control input and the amplitude of the vertical displacement was minimized. Nichiţelea and Unguritu proposed an adaptive harmonic control for the active suspension of a quarter of a car and applied it in a simulation environment [36]. The method was compared with the classical PI controller, the H∞ controller, and the model predictive control method. The comparison results showed that the harmonic controller performed better than control algorithms for some performance criteria. Using the least squares method, Nagarkar et al. [37] optimized and implemented a passive suspension system as an active FLC system. This study compared passive systems, traditional FLC, optimized passive systems, and optimized FLC methods. The optimized passive suspension system obtained 30% and 27% lower RMS acceleration and VDV values than the original. The governor force was reduced by 38% compared to the original FLC system. VDV and RMS acceleration and maximum acceleration were also reduced by 32%, 28%, and 24%, respectively.

Yuvapriya et al. [38] proposed an LQR control method using the BAT algorithm for car suspension system control, which was applied in an experimental environment on different road profiles. The study also compared the effectiveness of the method with the GWO algorithm. Regarding the road profile with bumps, the percentage reduction in the RMS value of trunk acceleration for the proposed BA-adjusted LQR was 64.6%, while the GWO-adjusted LQR was 37.5%. Similarly, for the impulse path profile, the BA-adjusted LQR resulted in a 43% reduction and the GWO-adjusted LQR resulted in a 25.2% reduction in the RMS value of hull acceleration compared to the conventional LQR. The proposed control method increased the ride comfort despite the rough road profile. Manna et al. [39] proposed an LQR control method using the active suspension system's ant colony optimization (ACO). They applied it to the model of a quarter car in an experimental environment. The proposed method was experimentally compared with the classical LQR and model predictive control (MPC) methods in three track profiles. The results showed that the proposed method significantly reduced the acceleration of the body due to uneven road profiles compared to the classical tuned LQR and model predictive control (MPC). It was also shown to improve car handling and occupant comfort significantly. Ma and Li proposed an adaptive fault-tolerant fuzzy control method for active seat suspension systems with full state constraints [40]. The method included electromagnetic actuator failures, constraints on the car seat, active suspension and wheel displacement, vertical vibration velocities, and current. In control design, fuzzy logic systems approximate unknown nonlinear dynamics because the systems have dynamic properties such as complexity and spring nonlinearity. A new adaptive fuzzy FTC method used an adaptive recursive backstepping design algorithm with barrier Lyapunov functions. When the electromagnetic actuator failed, the proposed method ensured that all vertical vibration states were stable. Kozek et al. [41] presented a neural algorithm based on reinforced learning to optimize ASSs' linear quadratic controller (LQR). They applied it to the system of a quarter car in a simulation environment. The method was compared with the passive and classical LQR control methods. The method seemed to increase user comfort by 67% compared to the passive system and 14% compared to the non-optimized LQR. Consequently, the comparison of these studies is provided in Table 1 to better comprehend and compare the studies in the literature.

In this paper, the suspension model of a quarter-car is created, and active control is implemented. The suspension model is created using the Lagrange–Euler method. LQR, fuzzy logic control (FLC), and fuzzy-LQR control algorithms are developed and applied to the suspension system for active control. The purpose of these controllers is to improve car handling and passenger comfort. Undesirable vibrations occur in passive suspension systems. These vibrations should be reduced using the proposed control methods and a robust system should be developed. The optimal values of the coefficients of the contact points of the membership functions are obtained using the particle swarm optimization (PSO) algorithm to enhance the performance of the FLC and fuzzy-LQR control methods. Following that, the computer environment is used to simulate the designed controllers. The success of the control performance of the applied methods concerning the passive suspension system is compared in percentages. The results are presented and evaluated graphically and numerically. The methods are contrasted against one another and the research in the literature using the integral time-weighted absolute error (ITAE) criterion. As a result, it is found that the proposed control method (fuzzy-LQR) is about 84.2% more successful in body motion, 90% in car acceleration, 84.5% in suspension deflection, and 86.7% in tire deflection compared to the studies in the literature. All these results show that the car's ride comfort is significantly improved.

Table 1. The comparison table of the existing studies.

References	Proposed Control Methods	Results/Findings/Limitations
D'Amato, ve Viassolo [5] (2000)	Fuzzy logic control (FLC)	Regarding the open loop, passenger comfort for small/medium road disturbances was improved by 58%, while the maximum bump amplitude for which the suspension limits cannot be reached was increased by 18%.
Foda [6] (2000)	FLC	Both body ride and suspension workspace responses exhibited good damping characteristics for different road inputs.
Al-Holou et al. [7] (2002)	Sliding mode neural network inference fuzzy logic control (SMNNNFLC)	The simulation results showed that the proposed SMNNNFLC provided higher ride comfort and handling quality than the other methods presented in the paper.
Sam et al. [8] (2004)	Proportional-integral sliding mode control	The method was shown to be more robust than the benchmarked method.
Chen and Guo [9] (2005)	Constrained H∞ control	As a result of this study, when the suspension strokes and control inputs were kept within the defined limits, tight contact of the wheels with the road was ensured and the best possible ride comfort was achieved.
Yagız et al. [11] (2008)	Fuzzy sliding-mode control (fuzzy-SMC)	It was seen that the magnitudes of the body displacement and pitching motion were reduced and the resonance peak caused by the vehicle body was eliminated. It was also seen that the magnitudes of the body and pitching motion acceleration were reduced in a wide frequency range, thus providing a significant improvement in driving comfort.
Salem and Aly [13] (2009)	FLC	Improved driving comfort by reducing body acceleration caused by the vehicle body in road disturbances
Lin and Lian [14] (2010)	Hybrid self-organizing fuzzy and radial basis function neural network controller (HSFRBNC)	The challenge of finding suitable parameters for SOFC design were eliminated. It was shown to improve the service life of the suspension claim and the ride comfort of a car.
Zhao et al. [25] (2016)	Adaptive neural network control	In the simulation results, the vehicle body vibrations were effectively suppressed, and RMS values were improved.
Palanisamy and Karuppan [22] (2016)	FLC	Positive results were obtained in terms of applicability in ASSs.
Wen et al. [25] (2017)	Fuzzy control with dynamic sliding mode approach	It was observed that the spring-mass acceleration under road disturbance was significantly reduced, and lower body acceleration was achieved.
Nichiţelea and Unguritu [36] (2022)	Adaptive harmonic control	Using the proposed method, better results were obtained in some performance criteria compared to other control algorithms.
Yuvapriya et al. [38] (2022)	BAT/GWO-LQR	In the case of the bump road profile, the percentage reduction in the RMS value of the body acceleration was 64.6% for the proposed BA-tuned LQR and 37.5% for the GWO-tuned LQR. Similarly, for the pulse path profile, the reduction in the RMS value of the body acceleration compared to the conventional LQR was 43% for the BA-tuned LQR and 25.2% for the GWO-tuned LQR.

The following is a concise list of the study's main contributions.

a. By utilizing the particle swarm optimization (PSO) algorithm, obtaining the weights of the points where the membership functions of the fuzzy-LQR control method touch, and applying this control method to the ASS, the system gains the ability to make its own decision to optimally determine the critically important Q and R parameters in the control of the system. This is the first major contribution of our study.

b. Using the passive state of the ASS, LQR control method, PSO-based fuzzy logic control (FLC), and PSO-based fuzzy-LQR control methods together, and comparing the obtained results is the second important contribution of this study.
c. Comparing the methods used with each other using the ITAE performance index and comparing the method showing the most successful performance among the methods with the studies in the literature is another contribution of the study.

The most important contribution of this study is to combine the self-determination capability of the fuzzy logic control method with the advantage of the LQR control method, which is one of the optimal control methods, and to bring it to the literature in the control of ASSs. As a result of the reasons and research given above, it was concluded that it would be beneficial to publish the study since it is thought that it is original and will contribute to the literature. The authors believe that this work is novel theoretically as a result. The remaining sections of this paper are the following: Section 2 presents the mathematical model of the suspension system. Section 3 describes the design of the LQR, PSO-based FLC, and PSO-based fuzzy-LQR control methods. Section 4 reviews numerical simulation results. The study's numerical and graphical findings are presented in this section. At the end of this chapter, a comparison table with the literature, each other, and interpretation is also provided. Section 5 concludes by summarizing the findings. This section analyzes and interprets the article's findings. Additionally, tips for enhancing the method are provided at the end of this section, along with details about upcoming research studies involving the method.

2. Mathematical Model of the Suspension System

The mathematical model that will be utilized to control the system was obtained using the Lagrange–Euler approach. The quarter-car model is shown in Figure 1. Our quarter-car model has two degrees of freedom. In this model, the car is designed symmetrically and is divided into four parts. The model includes only vertical vibration motion without considering the pitch and roll motion of the chassis and wheel. The model of the suspension system is given in the following equations.

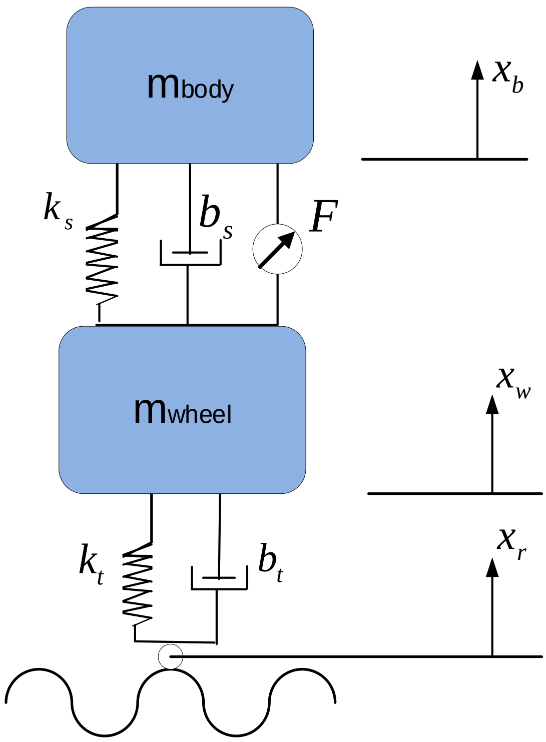


Figure 1. Quarter-car model.

Here m_b is the mass of a quarter of the car body, m_w is the mass of the wheel assembly, k_s is the spring coefficient of the suspension system, b_s is the damper coefficient of the suspension system, k_t is the spring coefficient of the tire, and b_t is the damper coefficient of the tire. In addition, while the state variables show the displacement (x_w) movements of the body (x_b) and the wheel assembly, the input variables x_r show the road roughness (distortion) and F the control force exerted by the active element applied between the body and the wheel.

$$m_b\ddot{x}_b + b_s(\dot{x}_b - \dot{x}_w) + k_s(x_b - x_w) - F_s = 0 \tag{1}$$

$$m_w\ddot{x}_w + b_t(\dot{x}_w - \dot{x}_r) + k_t(x_w - x_r) - b_s(\dot{x}_b - \dot{x}_w) - k_s(x_b - x_w) - F_s = 0 \tag{2}$$

$$\ddot{x}_b = -\frac{1}{m_b}(b_s(\dot{x}_b - \dot{x}_w) + k_s(x_b - x_w) - F_s) \tag{3}$$

$$\ddot{x}_w = -\frac{1}{m_w}(b_t(\dot{x}_w - \dot{x}_r) + k_t(x_w - x_r) - b_s(\dot{x}_b - \dot{x}_w) - k_s(x_b - x_w) - F_s) \tag{4}$$

$$x_1 = x_b; x_2 = x_w; x_3 = \dot{x}_b; x_4 = \dot{x}_w; \tag{5}$$

$$\begin{gathered}\dot{x}_1 = x_2 \\ \dot{x}_2 = -\tfrac{1}{m_b}(b_s(x_3 - x_4) + k_s(x_1 - x_2) - F_s) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\tfrac{1}{m_w}(b_t(x_4 - \dot{x}_r) + k_t(x_2 - x_r) - b_s(x_3 - x_4) - k_s(x_1 - x_2) - F_s)\end{gathered} \tag{6}$$

The dynamic model of the linearly moving system is constructed in the state space form $\dot{x} = Ax + Bu$ as follows.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b}\\ 0 & 0 & 0 & 1\\ \frac{k_s}{m_w} & \frac{b_s}{m_w} & \frac{-k_s-k_t}{m_w} & -\frac{b_s}{m_w}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & \frac{1}{m_b}\\ 0 & 0\\ \frac{k_t}{m_w} & -\frac{1}{m_w}\end{bmatrix} \tag{7}$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 0 & -1 & 0\\ -\frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & \frac{1}{m_b}\end{bmatrix} \tag{8}$$

The dynamics of the hydraulic actuator used in active suspension control are shown with the transfer function given in Equation (9).

$$H_{act}(s) = \frac{1}{1/60s + 1} \tag{9}$$

The active suspension road disturbance is modeled as a 5 cm high road bump using Equation (10) for the input signal.

$$r(t) = 0.025 \times [1 - \cos(8\pi t)] \tag{10}$$

Road disturbances with different characteristics, quality of sensing sensors required for feedback, limitations on available control force, and body acceleration are some factors that affect passenger comfort, represented by $a_{b,}$ and suspension deflection, represented by $s_{d.}$ This article presents the control algorithms applied to the quarter-car active suspension model in a closed-loop system with the measurement of body acceleration $\ddot{x}_b$ as feedback.

3. Controller Designs for the Suspension System

These controllers designed for the suspension system aim to increase car handling and passenger comfort. In addition, it is aimed to reduce the vibrations that occur in passive suspension systems by using the proposed control methods and designing a robust system. While the controller was being designed, the equations obtained for the system's control were prepared in the previous section. The system's control aims to design a controller that achieves minimum error value by increasing car handling and passenger comfort. The suspension system is controlled using linear quadratic regulator (LQR), PSO-based fuzzy logic control (FLC), and PSO-based fuzzy linear quadratic regulator (fuzzy-LQR) control methods. The most important problem during a system's control is determining the optimal control law that minimizes the determined performance index under various economic and safety constraints. The linear quadratic regulator (LQR) control method, which is one of the optimal control methods, is a method that uses the state-space model of the system [42]. It is a type of controller based on the principle of complete state feedback. The main objective of optimal control is to obtain control signals that result in a system satisfying certain physical constraints while overfulfilling (maximizing or minimizing) a selected performance criterion or cost function. The LQR control method is widely used because it is simple, optimal, and robust [43,44]. The input of the LQR control method is $u = -K \times x$, where K represents the feedback control input and x represents the states of the system. Control input is chosen to minimize the following cost function, which is determined using state space equations.

$$J = \frac{1}{2}\int_0^t \left(x^T(t)Qx + u^T Ru\right)dt \tag{11}$$

The purpose of the control here is to minimize the integral of the squared power index. The matrices Q and R are the weight matrices here. Q is a positive semidefinite symmetric matrix and R is a positive definite matrix ($Q \geq 0, R > 0$). In optimal control, the control vector u, x, indicates the system states and Equation (11) is quadratic concerning both *x(t)* and *u(t)*. The optimal feedback input K is determined using the following equation.

$$K = R^{-1}B^T P \tag{12}$$

Here, B represents the input matrix of the system. The positive definite matrix p value is determined using the Riccati equation. Here A stands for the state matrix.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{13}$$

Another ASS control method is the fuzzy logic control (FLC) method. The FLC algorithm is a method invented by Zadeh [45]. An FLC algorithm consists of five stages. In the first stage, the input variables are converted into a fuzzy set and, in the second stage, rule tables, membership functions, and a rule base are formed. This rule base consists of a set of IF-THEN rules derived from the verbal statements of experts with knowledge of the system. In the third stage, the inference mechanism performs the inference process of these rules on the system and provides fuzzy outputs. In the fourth stage, the membership functions and the range of fuzzy sets are defined in the database. In the last stage, defuzzification, a fuzzy set is converted into a net value for output. In this paper, the boundary values of the membership functions of the FLC algorithm were obtained using particle swarm optimization (PSO), one of the swarm-based optimization algorithms [46,47]. The mean square error (MSE) was chosen as the PSO algorithm's goal function to reduce system-related faults. The Mamdani inference approach was applied to the FLC method that was used for system control. For all input and output values used, triangle-type membership functions are also preferred. The particle swarm optimization flowchart (PSO-FLC) used to fit the membership functions of the FLC method proposed for suspension system control is shown in Figure 2.

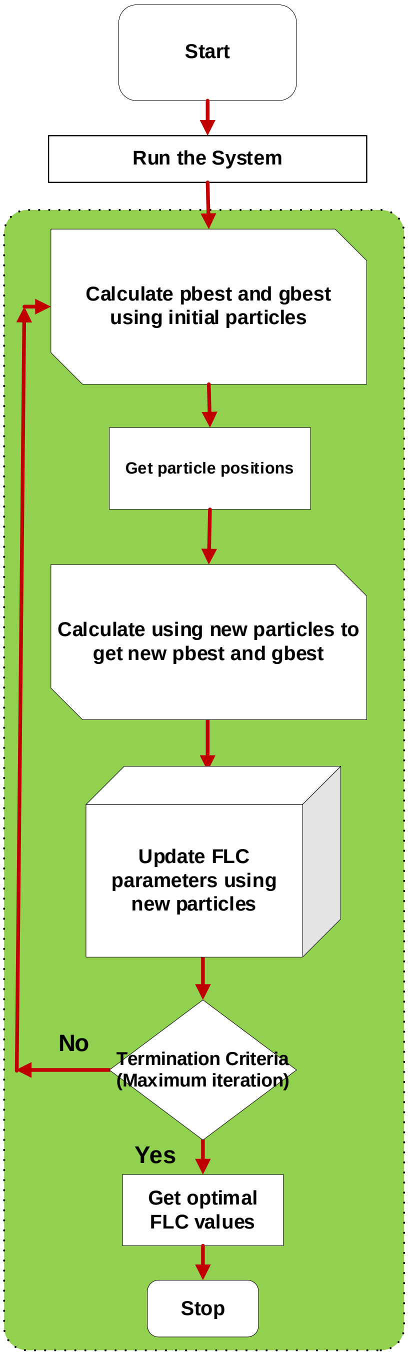


Figure 2. Particle swarm optimization (PSO-based FLC) flowchart used to adjust the membership functions of the FLC method.

The controller uses the error (e) and the rate of change of the error ($\dot{e}$) as input values. The membership functions and the control table obtained using triangular membership functions and PSO-FLC are given below. Figures 3 and 4 show the membership functions defined for the system's input values e and $\dot{e}$. Figure 5 shows the membership functions defined for the F (force) output value.

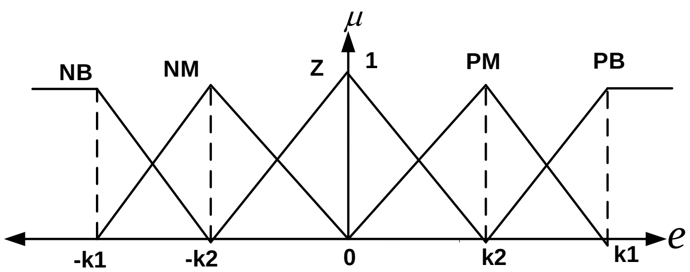


Figure 3. Optimized FLC membership functions defined for the input value ***e***.

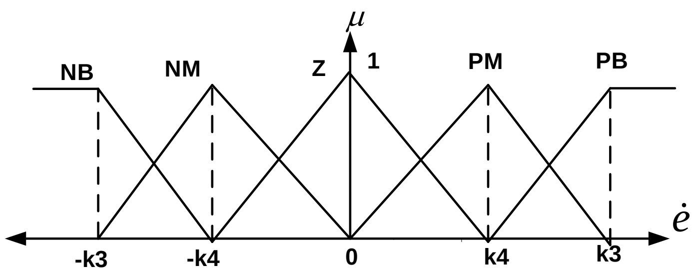


Figure 4. Optimized FLC membership functions defined for the input value $\dot{\boldsymbol{e}}$.

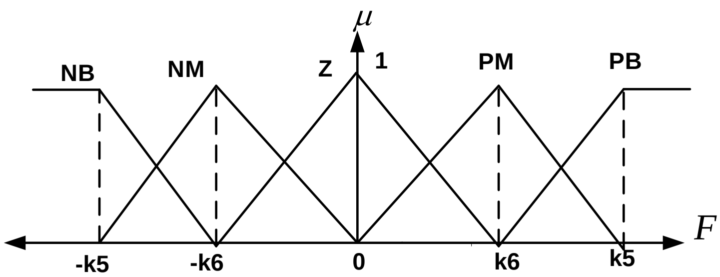


Figure 5. Optimized FLC membership functions defined for the output value ***F***.

For the PSO algorithm, the number of particles was set to 50 and the number of iterations to 80. The limits of the membership functions used in the FLC method optimized with the PSO algorithm are given in Table 2. Table 3 shows the control table created for the FLC control method. The fuzzy control variables e, e, and F compose the control table given in Table 3; e, $\dot{e}$, F indicate the error, error change, and power, respectively. NB, NM, Z, PM, and PB stand for negative large, negative medium, zero, positive medium, and positive large.

Table 2. The limit values of FLC membership functions.

k_1	k_2	k_3	k_4	k_5	k_6
0.92	0.46	0.78	0.36	1.24	0.83

Table 3. The rule table created for FLC.

F	**NB**	**NM**	**Z**	**PM**	**PB**
NB	NB	NB	NB	NM	Z
NM	NB	NB	NM	Z	PM
Z	NB	NS	Z	PM	PB
PM	NM	Z	PM	PB	PB
PB	Z	PM	PB	PB	PB

The fuzzy linear quadratic regulator (fuzzy-LQR) control method, which is another recommended method for active control of the suspension system, is a control method that shares the advantageous aspects of the linear quadratic controller (LQR) and the fuzzy control method. Here, an attempt is made to utilize the advantages of the LQR and FLC methods by combining them. Obtaining the Q and R matrices that affect the performance of the LQR control method with fuzzy logic causes the proposed method (fuzzy-LQR) to become a dynamic structure that depends on variable road conditions, thus increasing passenger comfort and achieving a realistic performance of the method. The fuzzy-LQR controller, which takes the error and its derivative as input to the controller, uses the fuzzy controller rules to change the $K = [k_1, k_2, k_3]$ expressions of the state feedback gain. The boundary values of the membership functions of the fuzzy-LQR control algorithm were obtained using particle swarm optimization (PSO), one of the swarm-based optimization algorithms. For the PSO algorithm, the mean square error (MSE) was set as the objective function to minimize the errors resulting from the system's operation. The controller uses the error (e) and the rate of change of the error ($\dot{e}$) as input values. The fuzzy-LQR control method uses the Mamdani method and triangular membership functions. The membership functions and the rule table created for fuzzy-LQR are given below. For the PSO algorithm, the number of particles is 75 and the number of iterations is 130. Figures 6 and 7 shows the membership functions and limits for the system's input values e and $\dot{e}$. The membership functions and bounds for the F (force) output value is shown in Figure 8.

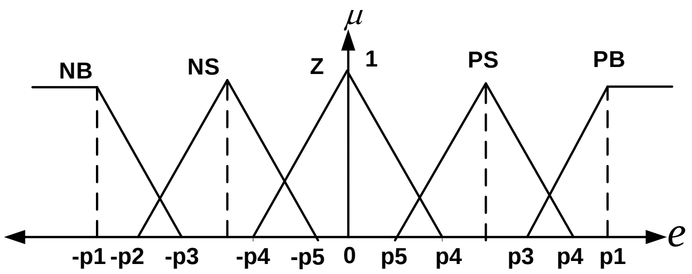


Figure 6. Fuzzy-LQR membership functions defined for the input value e.

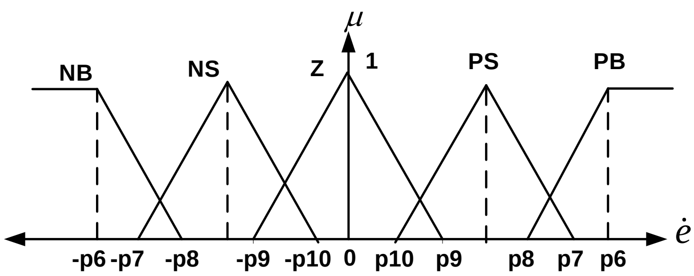


Figure 7. Fuzzy-LQR membership functions defined for the input value $\dot{e}$.

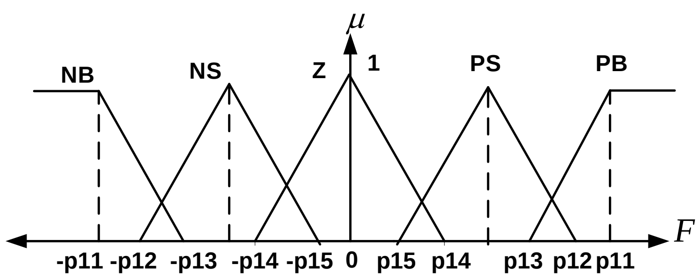


Figure 8. Fuzzy-LQR membership functions defined for the output value F.

Table 4 shows the boundary values of the membership functions used in the fuzzy-LQR control method and optimized with the PSO algorithm. Table 5 shows the rule table created for the fuzzy-LQR control method. In the table, the fuzzy control variables e, $\dot{e}$, F, and F indicate the error, error variation, and power, respectively. The values given in the table, NB, NS, Z, PS, and PB, represent negative large, negative small, zero, positive small, and positive large, respectively.

Table 4. The limit values of fuzzy-LQR membership functions.

p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
1.8	1.4	0.83	0.42	0.23	1.63	1.34	0.92
p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	
0.64	0.36	1.42	1.1	0.74	0.33	0.18	

Table 5. The rule table created for fuzzy-LQR.

F	NB	NS	Z	PS	PB
NB	NB	NS	PB	NB	PS
NS	NS	NB	NS	PS	Z
Z	Z	NB	NB	PB	PB
PS	NB	NS	PS	PB	PS
PB	Z	Z	PB	PS	PB

4. Numerical Simulation Results

In this section, controllers for the suspension of a quarter car are designed, and the results of the simulations that have been tried on the system are given. The control algorithms developed and applied for the system are LQR, PSO-based FLC, and PSO-based fuzzy-LQR methods. Active control of the system was performed using these methods, and the obtained results were presented with graphs and tables. The main objective in controlling the car suspension system is to minimize the impact of the disturbance caused by the road entrance on passenger comfort. The initial parameters of the system are determined as follows: Matlab/Simulink package program was used in this study. The physical parameters of the car suspension system are m_b = 300 kg, m_w = 60 kg, k_s = 16,000 N/m, b_s = 1000 Ns/m, k_t = 190,000 N/m, and b_t = 100 Ns/m. The initial value of the position of the car suspension system is assumed to be x = 0 m. The simulation time of the system is assumed to be 2.5 s. The road entrance is shown in Figure 9. This driveway is a 5 cm high mound of earth.

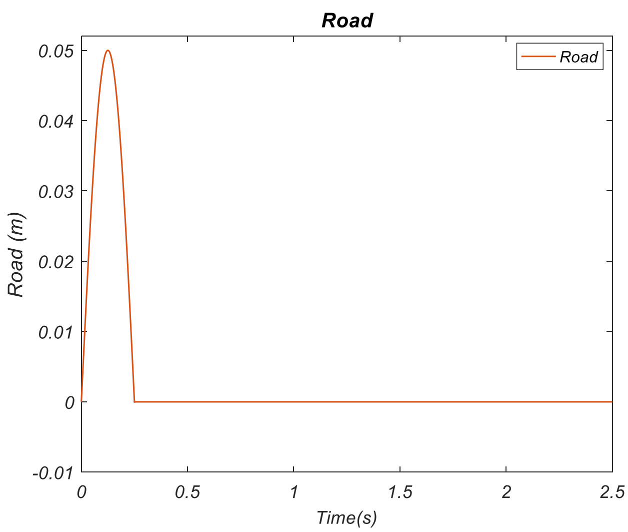


Figure 9. The road input.

This study presents the graphs resulting from the passive state of the suspension system and the active control of the LQR, PSO-based FLC, and PSO-based fuzzy-LQR methods. Figures 10 and 11 show the graphs obtained as a result of the motion of the car body and the acceleration of the car body, which are the parameters affecting passenger comfort, and the passive state and active control with the LQR, PSO-based fuzzy, and PSO-based fuzzy-LQR control methods.

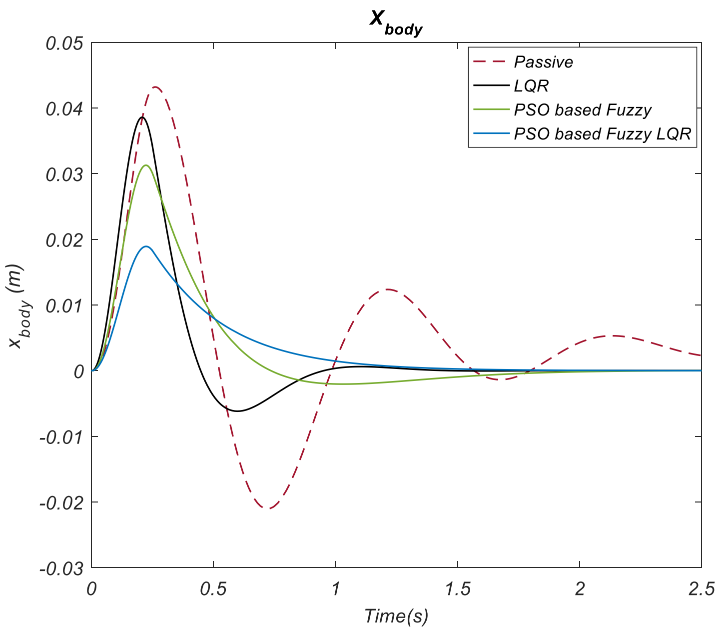


Figure 10. The response to the body motion using the methods described.

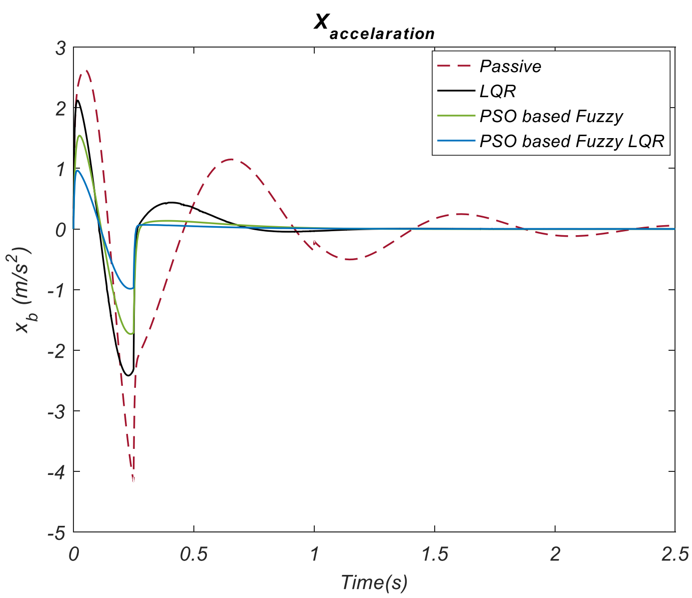


Figure 11. The response to the body acceleration using the methods described.

By looking at Figure 10, the plot of the vertical deflection of the car body as a function of time, it is clear that all the methods used significantly reduce the vibration amplitude and the seat time compared to the passive suspension system. It can be seen that the smallest change in vibration amplitude is in the PSO-based fuzzy-LQR control (0.018~0 m range) and the highest amplitude is in the LQR control (0.038~0.005 m). The acceleration acting on the car due to the poor road entrances can reach dimensions that disturb the passengers. Therefore, reducing car acceleration amplitudes is important for car ride comfort. By examining the acceleration curve of the car body, it can be seen that the PSO-based fuzzy-LQR control type (in the range of 1~1 m/s^2) attenuates the acceleration amplitudes much more successfully. Similarly, the highest amplitude is observed for the LQR control type (about 2.1~2.4 m). It is also observed that the LQR control type causes undesirable amplitude increases. The suspension deflection and tire deflection resulting from the methods used are shown in Figures 12 and 13.

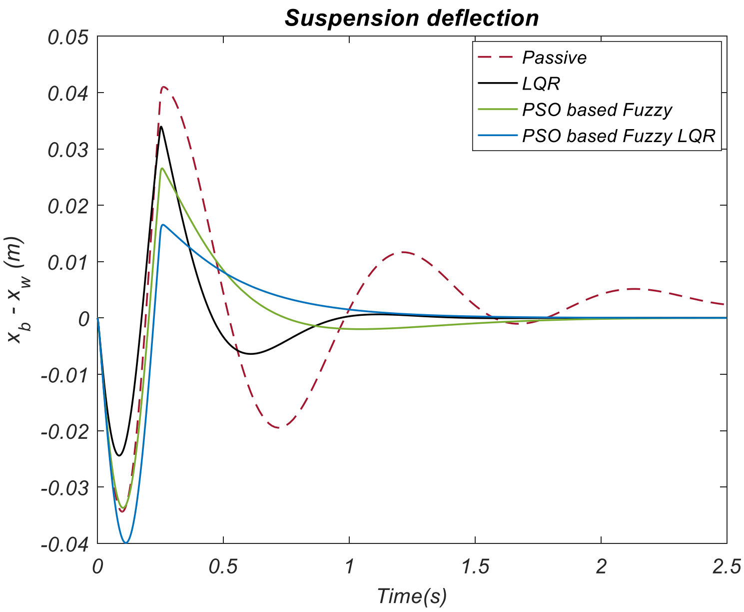


Figure 12. The response to the suspension deflection using the methods described.

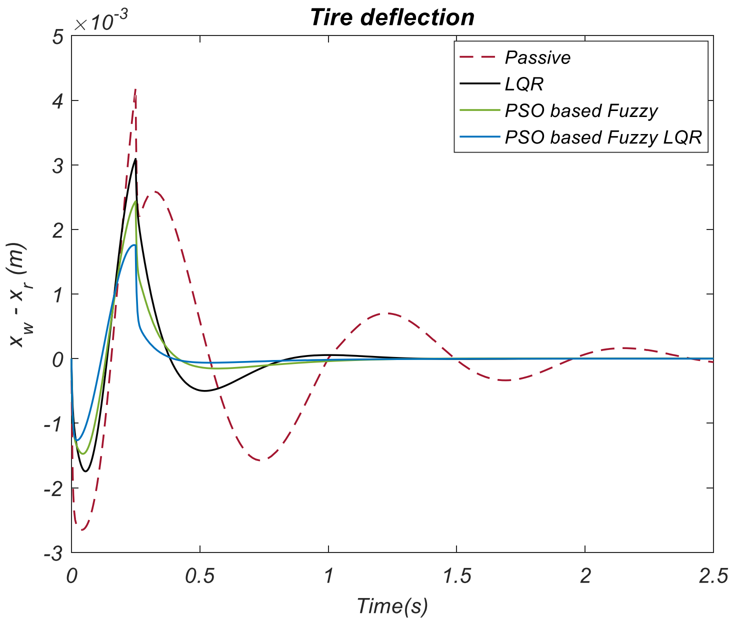


Figure 13. The response to the tire deflection using the methods described.

Another criterion for the success of the methods is whether the methods used in car suspension systems cause the problem of suspension constriction. The suspension deflection curve in Figure 12 shows that the suspension range has reached zero, i.e., the initial position, for all three controller types and thus does not narrow. It can be seen that the PSO-based fuzzy-LQR (in the range of about 0.018~0 m) occurs at a much lower amplitude in the control mode. Similarly, the highest amplitude is observed in LQR control (about

0.033~0.005 m). The graph of the tire deflection in Figure 13 shows that the tire deflection occurs in all three types of controllers. It can be seen that the PSO-based fuzzy-LQR (about −0.0012~0.0018 m) occurs at a much lower amplitude in the control type. Similarly, it can be seen that the highest tire deviation occurs in the LQR control type (about 0.0018~0.003 m). Figure 14 shows the control forces that result from active control.

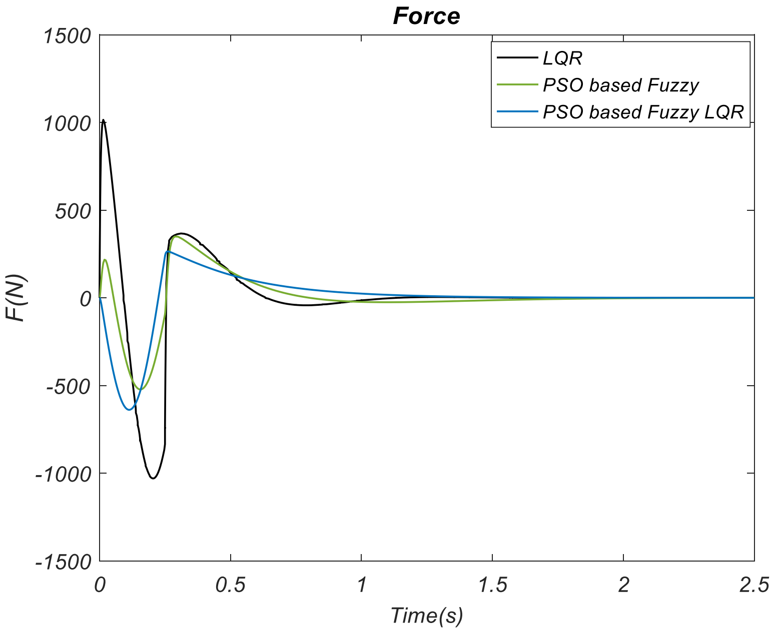


Figure 14. The response to the force using the methods described.

In car vibration reduction studies, the fact that the control force required by a controller is low is accepted as an indicator of the successful operation of that controller. The fuzzy-LQR controller requires less power, as seen in Figure 13 when looking at the change in the controller forces required by the motor over time. It can be seen that it ranges from about −520 to 250 N for the PSO-based fuzzy-LQR controller. Similarly, it can be seen that the highest force requirement is for LQR control (about −1000~1000 N). The previous graphs show that the fuzzy-LQR controller provides better control with less force requirement. Table 6 provides a comparison table for the body motion, car acceleration, suspension deflection, tire deflection, and required force parameters of the methods used in the study according to the performance index of the integral time-weighted absolute error (ITAE).

$$ITAE = \int tedt \tag{14}$$

Table 6. Comparisons of applied control methods using performance criteria.

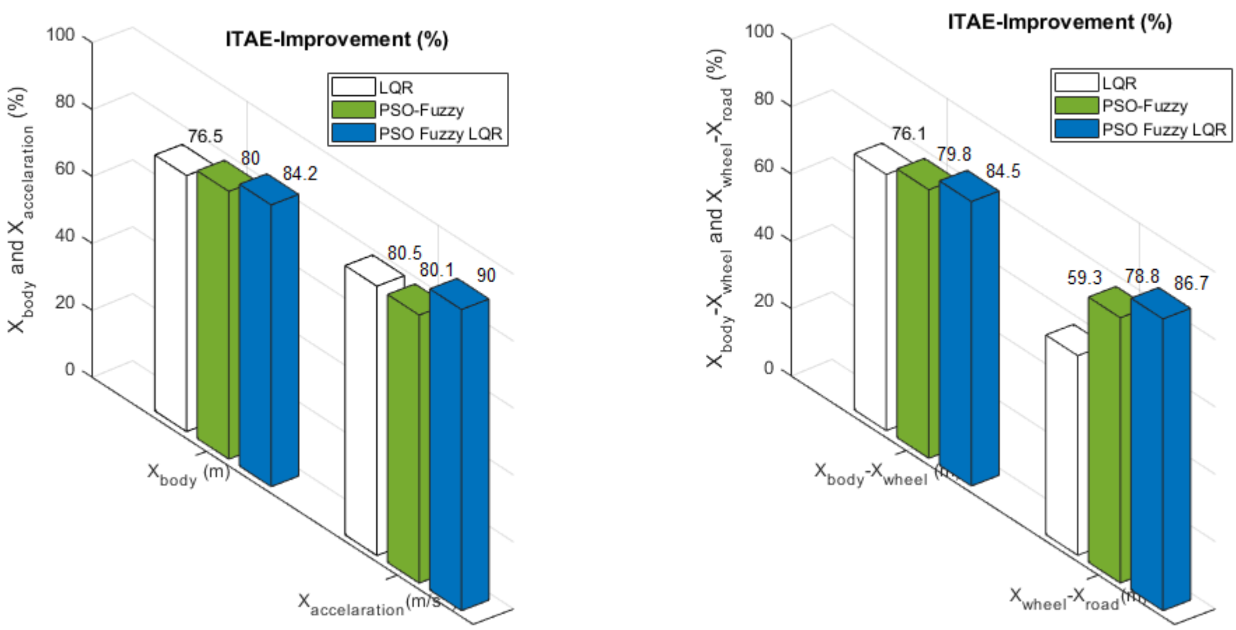
Performance Criteria (ITAE)	Passive	LQR	PSO-Based FLC	PSO-Based Fuzzy-LQR
Body travel $(X_{body})(\mathrm{m})$	0.01914	0.00450	0.00380	0.00302
Body acceleration $(\ddot{X}_{body})$ $(\mathrm{m/s^2})$	0.75410	0.14680	0.15020	0.07545
Suspension deflection $(X_{body} - X_{wheel})(\mathrm{m})$	0.01801	0.00430	0.00363	0.00280
Tire deflection $(X_{wheel} - X_{road})(\mathrm{m})$	0.00113	0.00046	0.00024	0.00015
Actuator $(F)(\mathrm{N})$	-	67.16	68.46	59.29

In Table 6, the values that have the best error performance are shown in bold. According to the car body motion results obtained using the performance criteria in Table 4, the fuzzy-LQR control method has the lowest error performance with a value of 0.00302 m.

The LQR control approach (0.00450 m) showed the worst error performance. According to the ITAE criteria, the error performance of the PSO-based FLC control method (0.00380 m) was relatively better than that of the LQR control method. For the results of the acceleration error of the car body, another parameter listed in the table, the PSO-based fuzzy-LQR control method, showed the lowest error performance with a value of 0.007545 m/s^2. The LQR control approach (0.14680 m/s^2) showed the worst error performance. According to the ITAE criteria, the PSO-based FLC method performed relatively better than the LQR control method in the acceleration error of the car body (0.15020 m/s^2). According to the results of suspension span error, another parameter listed in the table, the PSO-based fuzzy-LQR control method, showed the lowest error performance and was 0.00280 m. The LQR control approach (0.00430 m) showed the worst error performance.

According to the ITAE criteria, the error performance of the PSO-based FLC method (0.00363 m) was relatively better than that of the LQR control method. According to the tire deflection error results, another parameter in the table, the PSO-based fuzzy-LQR control method, showed the lowest error performance and its value was 0.00015 m. The LQR control approach (0.00046 m) showed the worst error performance. According to the ITAE criteria, the error performance of the PSO-based FLC method in tire deformation (0.00024 m) was relatively better than that of the LQR control method. According to the error results of another given parameter, the force to be applied, the PSO-based fuzzy-LQR control method showed the lowest error performance, and its value was 59.29 N. The LQR control approach (67.16 N) showed the worst error performance. According to the ITAE criteria, the error performance of the PSO-based FLC method in tire deformation (68.46 N) was relatively better than that of the LQR control method. The percentage improvement performance of the applied methods compared to the passive system was determined using Equation (15). Figure 15 shows the percentage improvement graphs of the methods compared to the passive system.

$$Improvement(\%) = \left| \frac{passive - active}{passive} \right| \times 100 \tag{15}$$



(**a**) (**b**)

Figure 15. The percentage performance comparisons of (**a**) the X body position and X body acceleration and (**b**) suspension and tire deflection.

According to the graphs in Figure 15, the body performance increased by 84.2%, car acceleration increased by 90%, suspension deflection increased by 84.5%, and tire deflection increased by 86.7% compared to the passive system by using the PSO-based fuzzy-LQR control method. As shown in Figure 15, maximum performance improvements were obtained for all parameters using the PSO-based fuzzy-LQR method, one of the three control methods we used. Additionally, and perhaps most significantly, Table 7 contrasts the performance standards of the fuzzy-LQR control method, one of our suggested approaches, with those of the research in the literature. This table demonstrates how effective our method is in comparison to other methods.

Table 7. The performance comparison of the proposed control method with the literature methods.

Performance Criteria (ITAE)	PSO-Based Fuzzy-LQR	Harmonic [36]	H-Infinity [36]	MPC [36]
Body travel $(X_{body})(\text{m})$	0.00302	0.0039	0.0071	0.0041
Body acceleration (X_{body}) (m/s^2)	0.07545	0.1683	0.1588	0.1252
Suspension deflection $(X_{body} - X_{wheel})(\text{m})$	0.00280	0.0044	0.0072	0.0046
Tire deflection $(X_{wheel} - X_{road})(\text{m})$	0.00015	-	-	-
Actuator $(F)(\text{N})$	59.29	64	95.5	80.7

The best values for error performance are highlighted in bold in Table 7. According to the results of car body motion obtained utilizing the ITAE performance criteria in Table 7, the PSO-based fuzzy-LQR control method showed the lowest error performance, and its value was 0.00302 m. According to the performance criteria in the studies reported in the literature, the car body motion performance is determined by the harmonic, MPC, and H-infinity control methods, respectively, and their values are 0.0039, 0.0041, and 0.0071 m, respectively. According to the results of car acceleration in the table, the minimum error value is determined by the fuzzy-LQR control method, MPC control method, H-infinity control method, and harmonic control method, respectively, and their values are 0.07545, 0.1252, 0.1588, and 0.1683 m/s^2, respectively. According to the suspension deflection results of another parameter given in the table, the minimum error value is determined by the PSO-based fuzzy-LQR control method, the harmonic control method, the MPC control method, and the H-infinity control method, and their values are 0.00280, 0.0044, 0.0072, and 0.0046 m, respectively. Finally, according to the actuator force results shown in the table, the minimum error values were determined by the PSO-based fuzzy-LQR control method, the harmonic control method, the MPC control method, and the H-infinity control method, and their values are 59.29, 80.7, and 95.5 N, respectively. Considering the above results, the proposed PSO-based fuzzy-LQR control method showed superior performance in terms of all parameters compared with the studies in the literature when compared with the ITAE criteria.

5. Conclusions

In this paper, the model of the suspension system of a quarter-car is established, and an active control system is implemented with three different control methods. In developing the controller, the goal of improving the car handling performance and passenger comfort was achieved, as shown in the graphs. Active control was performed using the ASS, LQR, fuzzy logic control (FLC), and fuzzy-LQR control algorithms. To enhance the performance of the FLC and fuzzy-LQR control methods, the optimal values of the coefficients of the points where the feet of the membership functions touch were calculated using the particle swarm optimization (PSO) algorithm. Then, the designed controllers were simulated in the computer environment. The success of the control performance of the applied methods concerning the passive suspension system was compared in percentages. The methods were compared against one another and the research in the literature using the integral time-weighted absolute error (ITAE) criterion. As a result, it was found that the proposed control method (fuzzy-LQR) is about 84.2% more successful in body motion, 90% in car acceleration, 84.5% in suspension deflection, and 86.7% in tire deflection compared to the studies in the literature. All these results show that car ride comfort can be improved significantly. The proposed controller's (fuzzy-LQR) successful results have demonstrated that this sort of control may be expanded upon and applied to both linear and nonlinear systems. Future studies can aim to investigate the method's behavior with a real-time experimental application of the proposed method. Furthermore, the method can be applied to half- and full-car models in simulation and experimental environments.

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Nomenclature

m_b	Mass of a quarter-car body
m_w	Mass of the wheel assembly
k_s	Spring coefficient of the suspension system
b_s	Damper coefficient of the suspension system
k_t	Spring coefficient of the tire
b_t	Damper coefficient of the tire
x_b	Body travel
$\ddot{x}_b$	Body acceleration
x_w	Displacement of the wheel assembly
x_r	The road roughness (distortion)
$x_b - x_w$	Suspension deflection
$x_w - x_r$	Tire deflection
F	Actuator control force
ASS	Active suspension system
LQR	Linear quadratic regulator
PSO	Particle swarm optimization algorithm
FLC	Fuzzy logic control
$Fuzzy - LQR$	Fuzzy linear quadratic regulator

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