An investigation of modelling parameters for surge phenomenon in axial compressors

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Abstract. Axial compressors have been used in areas such as propulsion and power generation for many decades now. The development of compressors has been accompanied by the identification of gas dynamic instabilities during their operation, such as surge and stall, and the subsequent development of technologies to mitigate such problems. A widely employed lumped model for studying post stall phenomenon, usually referred to as Moore-Greitzer model, involves combining the geometric and operating parameters of the compression system into certain non-dimensional groups. In this paper, a numerical study of the different parameters affecting the surge phenomenon in axial compressors is performed. By unfurling the non-dimensional groups in the Moore-Greitzer model, the significance of the actual geometric and operational variables is identified.

1 Introduction

Compression system performance is an important factor in the design and operation of gas turbines in both aviation and land-based power generation. The safe and efficient operation of gas turbines is closely dependent on the understanding of various instabilities associated with the compressor operation, especially in large axial machines. Two of the most prominent instability phenomena identified in axial compressor systems are rotating stall and surge. Rotating stall results in the compressor operating with extremely low frequency instabilities, causing excessive high internal temperature that has an adverse effect on blade life. Surge phenomenon can cause severe problems such as excessive built-up of pressure at the inlet and cyclic loading on compressor mounting resulting in the failure of blades to produce required loading, and the engine suffering from catastrophic damage.

Over the years, theoretical and experimental studies have been conducted on compressor systems to understand the stall inception [1-9]. These studies show that the surge in axial compressors is initiated by rotating stall. The post stall behaviour is studied theoretically by simplifying the compressor system. Some of the widely recognized works in this field come from Greitzer and Moore [10-13], which uses a lumped volume approach to model the compressor system. Among the different non-dimensional parameters introduced in this model, the Greitzer stability parameter B is used in these works to characterize the post stall behaviour. According to this model, lower values of B indicate small fluctuations in mass

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flow (rotating stall or mild surge), while higher values favour large fluctuations including reverse flow (deep surge). This approach also predicts that similar values of B shall give similar time-dependent flow behaviour.

Although Greitzer and Moore validated their model through experiments, a question arises on the impact of other geometric and operational parameters on the post stall behaviour. Studies have been conducted in the past to understand if the remaining non-dimensional parameters in this model are as insignificant as claimed by this model [14-16]. But these studies were limited to a small range of non-dimensional parameters. In order to understand the effect of all the variables in the Moore - Greitzer model, a detailed numerical analysis is carried out in this paper. The effect of variation of the parameters in the shape, frequency and amplitude of the post stall behaviour is studied and is reported here.

2 Modelling

The compression system model considered for this analysis was developed in MATLAB and based on the works of Greitzer and Moore [10,13]. The compressor is modelled as an actuator disk and an equivalent duct. The system also includes a plenum and a throttle valve, as shown in Fig. 1. The model is based on the non-dimensional governing equations (1) - (4).

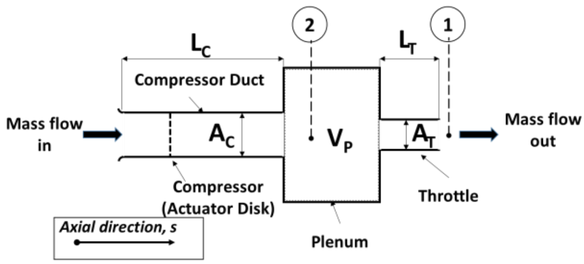

Fig. 1. Compression system (simplified model).

$$\frac{d\tilde{\dot{m}}_C}{d\tilde{t}} = B(\tilde{C} - \Delta\tilde{P}) \quad (1) \qquad \frac{d\tilde{\dot{m}}_T}{d\tilde{t}} = \frac{B}{G}(\Delta\tilde{P} - \tilde{F}) \quad (2)$$

$$\frac{d\Delta\tilde{P}}{d\tilde{t}} = \frac{1}{B}(\tilde{\dot{m}}_C - \tilde{\dot{m}}_T) \quad (3) \qquad \frac{d\tilde{C}}{d\tilde{t}} = \frac{1}{\tilde{\tau}}(\tilde{C}_{SS} - \tilde{C}) \quad (4)$$

where,

$$B = \frac{U_m}{2a}\sqrt{\frac{V_P}{A_C L_C}} \quad (5) \qquad G = \frac{L_T A_C}{L_C A_T} \quad (6)$$

$$K = \frac{{A_C}^2}{{A_T}^2} \quad (7) \qquad \Delta\tilde{P} = \frac{(P_{s2} - P_{s1})}{1/2\,\rho_1 {U_m}^2} \quad (8)$$

$$\tilde{F} = \frac{1/2\,\rho_1 {C_{x,T}}^2}{1/2\,\rho_1 {U_m}^2} = K{\tilde{\dot{m}}_T}^2 \quad (9) \qquad \tilde{\tau} = \frac{\pi R_m N_R}{L_C\,B} \quad (10)$$

The mean rotor radius (R_m), compressor inlet area (A_C), compressor duct equivalent length (L_C), throttle area (A_T), throttle length (L_T) and plenum volume (V_P) are used to describe the geometry of the compression system. Mean rotor speed (U_m), plenum pressure (P_{s2}), ambient pressure (P_{s1}), ambient density (ρ_1) and instantaneous compressor pressure rise ($\tilde{C}$) depend on the operating condition of the system. The variables are non-dimensionalised using reference conditions, i.e., using $1/2$ for pressure terms ($\Delta\tilde{P}$,$\tilde{F}$,$\tilde{C}_{ss}$,$\tilde{C}$) and using $\rho_1 A_C U_m$ for mass flow rates ($\tilde{\dot{m}}_C, \tilde{\dot{m}}_T$). The steady state characteristic curve ($\tilde{C}_{SS}$) for the system is taken from the compressor rig used by Greitzer during his experimental studies [11]. The time delay ($\tilde{\tau}$) is set to the value used by Greitzer in his work [11]. The governing equations are solved using 4th order Runge-Kutta method. The initial conditions are taken at the point on the steady state curve where the compressor just enters into stall condition, and $\tilde{C}_{SS}$ is set to rotating stall condition to initiate the post-stall condition.

$$\Delta\widetilde{P} = \tilde{C} = \tilde{C}_{stall} \quad (11) \qquad \tilde{\dot{m}}_C = \tilde{\dot{m}}_T = \tilde{\dot{m}}_{stall} \quad (12) \qquad \frac{L}{A} = \int_{s_i}^{s_f} \frac{ds}{A(s)} \quad (13)$$

As described by the equations (1) - (4), for low values of B, the plenum pressure decreases quickly compared to the time it takes for flow reversal to occur in the compressor ducting. This causes the compressor to operate around the rotating stall portion of the characteristics, i.e. very close to the conventional "surge line", as there is insufficient pressure energy to cause significant flow fluctuations. For high values of B, the plenum pressure remains high while the compressor moves into complete stall regime. Such stored pressure energy decelerates the compressor flow rapidly, resulting in surge, generally termed as mild surge, where large fluctuation of mass flow become apparent. For sufficiently high value of B, the compressor may enter into reverse flow region of the characteristics, and this is usually termed as deep surge.

These equations imply that surge will occur when the energy released from the high-pressure fluid in plenum overcomes the momentum inertia of the fluid in the compressor ducting, including intake, compressor channel, outlet. The non-dimensional parameter B can be viewed as a ratio of stored energy in the plenum to the work required to overcome the inertia. Parameter G is a representation of the volume ratio of compressor ducting and throttle ducting. Parameter K is the through flow area ratio of compressor ducting and throttle ducting.

3 Results & Discussion

The model is first validated against the results reported in [11]. The surge responses are shown in Fig. 2. As it is evident from the figure, the responses are in agreement with the experimental result.

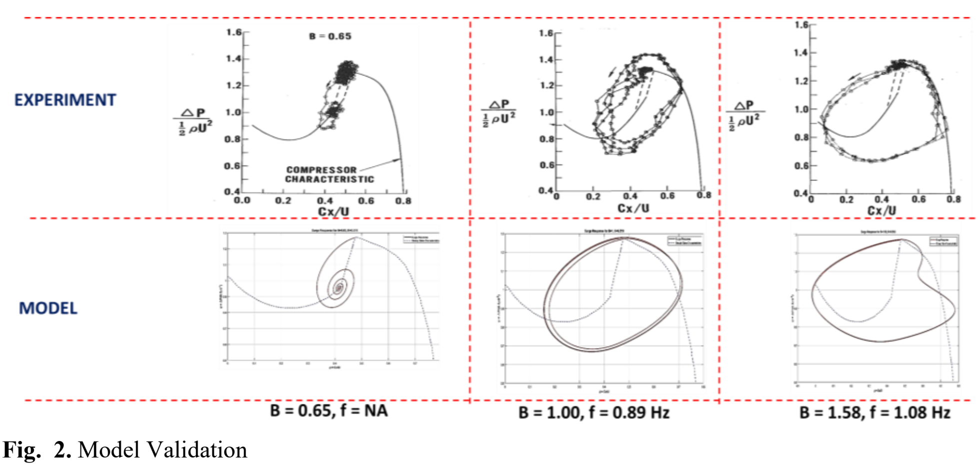

Fig. 2. Model Validation

The simulations are run for a wide range of B, G, K and L_C and the results are discussed in this section. The frequencies and amplitude of the surge response for different input conditions obtained from FFT analysis, for the compression system under investigation, are shown in Table 1.

Table 1. Summary of simulations.

Varied Parameter	B	G	K	L_C	A_C	L_T	A_T	R_m	U_m	a	V_P	τ	Frequency	Amplitude
	[-]	[-]	[-]	m	m²	m	m²	m	m/s	m/s	m³	[-]	Hz	[-]
B	***0.45***	0.36	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	7.19	0.055	N/A	N/A
	0.68	0.36	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	16.18	0.055	0.798	0.084
	0.90	0.36	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.145
	1.13	0.36	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	44.93	0.055	0.798	0.116
	1.35	0.36	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	64.70	0.055	0.997	0.162
G	0.90	***0.18***	5.52	1.46	0.15	0.11	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.147
	0.90	***0.27***	5.52	1.46	0.15	0.17	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.146
	0.90	***0.36***	5.52	1.46	0.15	0.22	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.145
	0.90	***0.45***	5.52	1.46	0.15	0.28	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.144
	0.90	***0.54***	5.52	1.46	0.15	0.34	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.143
K	0.90	0.36	***2.76***	1.46	0.15	0.32	0.09	0.26	59.00	377.05	28.76	0.055	N/A	N/A
	0.90	0.36	***4.14***	1.46	0.15	0.26	0.07	0.26	59.00	377.05	28.76	0.055	N/A	N/A
	0.90	0.36	***5.52***	1.46	0.15	0.22	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.145
	0.90	0.36	***6.90***	1.46	0.15	0.20	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.174
	0.90	0.36	***8.28***	1.46	0.15	0.18	0.05	0.26	59.00	377.05	28.76	0.055	0.798	0.194
Lc	0.90	0.36	5.52	***0.73***	0.15	0.11	0.06	0.26	59.00	377.05	14.38	0.055	N/A	N/A
	0.90	0.36	5.52	***1.10***	0.15	0.17	0.06	0.26	59.00	377.05	21.57	0.055	0.798	0.127
	0.90	0.36	5.52	***1.46***	0.15	0.22	0.06	0.26	59.00	377.05	28.76	0.055	0.798	0.145
	0.90	0.36	5.52	***1.83***	0.15	0.28	0.06	0.26	59.00	377.05	35.95	0.055	0.798	0.150
	0.90	0.36	5.52	***2.19***	0.15	0.34	0.06	0.26	59.00	377.05	43.14	0.055	0.798	0.146

3.1 Variation of B

Since B is mentioned as the most significant parameter by many in previous studies, its effect was the first one to be analysed here. As mentioned in literature, for the test rig used by Greitzer, the system enters surge when B exceeds 0.65. Therefore, B is varied by changing V_P while maintaining the other parameters constant. The surge responses and Fast Fourier transforms (FFTs) are shown in Fig. 3. The results in Fig. 3 (a), for steps of ±25% of B from 0.90, show the significant impact B has on the surge response. As B increases, the system moves from rotating stall to deep surge, as indicated by the post-stall behaviour plot. The change in the frequency is also visible from the FFT.

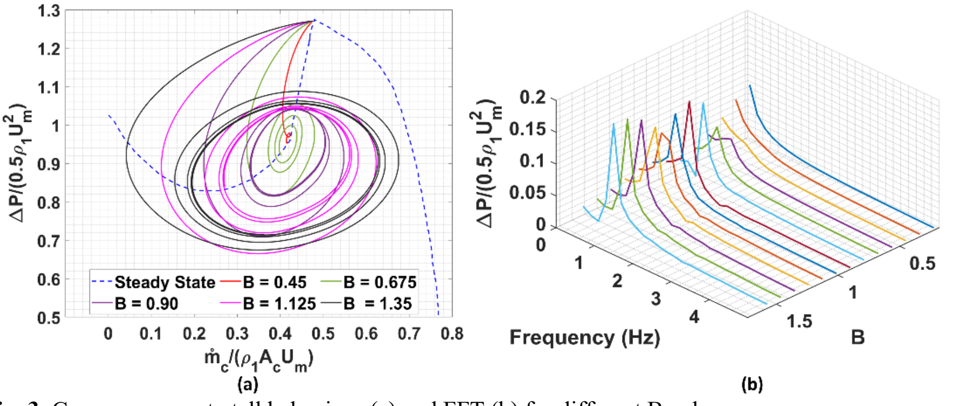

Fig. 3. Compressor post-stall behaviour (a) and FFT (b) for different B values.

3.2 Variation of G

The parameter G can be seen as an indirect representation of the ability of the system to maintain flow continuity and is a geometrical representation of the compression system. In the simulations, G is varied by changing L_T while maintaining other parameters constant.

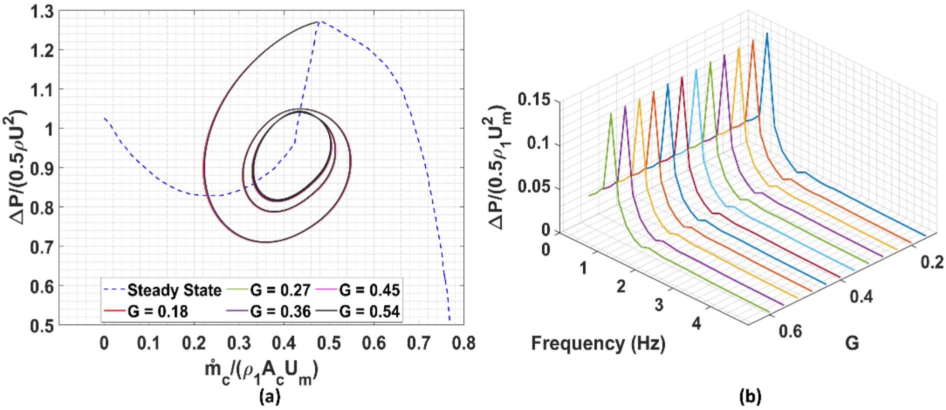

Fig. 4. Compressor post-stall behaviour (a) and FFT (b) for different G values.

It can be seen that G has negligible impact on the post-stall behaviour of this compressor system, represented in Fig. 4 (a) with steps of ±25% from G = 0.36, as inferred by Greitzer [10]. The impact on the FFT is also insignificant as shown in Fig. 4 (b) & Table 1.

3.3 Variation of K

The non-dimensional parameter K represents the throttling capacity of the compression system with high K values implying small throttling capacity. The variation of K is obtained by adjusting the values of A_T and L_T simultaneously, in order to maintain the other parameters constant. The results of the study of K are shown below. Variation of K, with steps of ±25% from K = 5.52 is in Fig. 5 (a). Fig. 5 (b) represents the effect of K variation on the FFT spectrum.

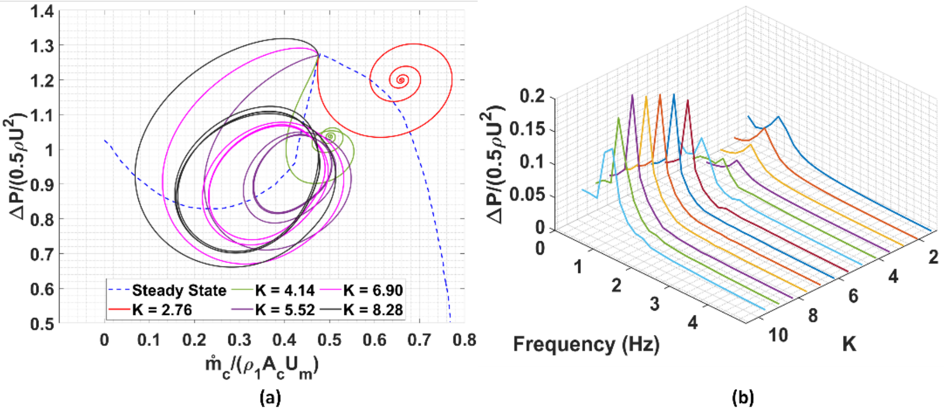

Fig. 5. Compressor post-stall behaviour (a) and FFT (b) for different K values.

It is clear from the plots that K has significant influence on the post stall behaviour of the compression system. The results are consistent with the experimental results of Greitzer [11]. It was observed that surge was present near K = 5.52 and the lower values of K represent either rotating stall or re-stabilization of the compressor operating point in the steady state side of the characteristics curve.

3.4 Variation of L_C

The geometric parameter L_C is the equivalent length of the compressor duct, and it is defined as the length of a duct, in which a given rate of change of mass flow produces the same unsteady pressure difference as in the actual duct (including a correction for end effects) and having a constant area equal to the compressor inlet area.

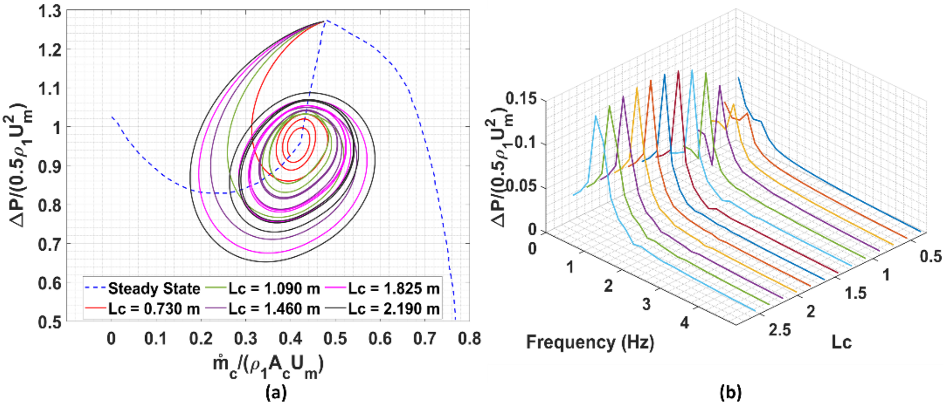

Fig. 6. Compressor post-stall behaviour (a) and FFT (b) for different L_C values.

The integration in Eq. (13) is carried out over all regions of the actual ducting in which the flow has significant kinetic energy. In order to study the impact of L_C, L_T and V_P are varied simultaneously to maintain the other non-dimensional parameters constant, with steps of ±25% from Lc = 1.46 m.

It is evident from the results, that the parameter L_C has an impact on the frequency and amplitude of the post-stall behaviour of the system. As L_C increases, the response extends towards the deep surge region (Fig. 6). It is to be noted that L_C is related to the Helmholtz resonance frequency of the system and the definition of L_C in the lumped volume approach could be responsible for this phenomenon. So, it is clear that the non-dimensional parameters used in the Moore-Greitzer model are not sufficient to fully characterise the unstable behaviour of compressor systems.

4 Conclusions

A parametric study of the widely used lumped volume approach to post-stall behaviour of a compression system is carried out in this work. The impact of different parameters in the Moore-Greitzer model is investigated by varying the non-dimensional terms & L_C. The results show that although the impact of B is the most significant, as claimed by Greitzer and others, the effect of K and L_C cannot be overlooked. The impact of the G parameter is minor, as inferred by Greitzer. The studies hint at the possibility of another variable that could capture the geometry of the compression system, which could be related to the definition of L_C. Further studies on this area using 1-D model for axial compressors and extension to radial compressors may be pursued at a later stage.

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References

1) J. Day, *Stall Inception in Axial Flow Compressors*, ASME. *J.* Turbomach.**115(1)**: 1–9 (1993). https://doi.org/10.1115/1.2929209
2) N. M. McDougall, N. A. Cumpsty, T. P. Hynes, *Stall Inception in Axial Compressors*, ASME. J. Turbomach. **112(1)**:116–123 (1990). https://doi.org/10.1115/1.2927406
3) G. Wilson, C. Freeman, *Stall Inception and Development in an Axial Flow Aeroengine*, ASME. J. Turbomach. **116(2)**: 216–225 (1994). https://doi.org/10.1115/1.2928356
4) J. Day, *Review of stall, surge and active control in axial compressors*, ISABE**93-7011** (1993).
5) G. L. Arnulfi, F. L. Ghiglino, A. F. Massardo, *Comparison Between Complete Hilbert Transform and Simplified Solutions of the Moore Rotating Stall Model*, ASME. J. Turbomach. **120(3)**: 446–453 (1998). https://doi.org/10.1115/1.2841737
6) G. L. Arnulfi, P. Giannattasio, C. Giusto, A. F. Massardo, D. Micheli, P. Pinamonti, *Multistage Centrifugal Compressor Surge Analysis: Part I—Experimental Investigation*, ASME. J. Turbomach., **121(2)**: 305–311 (1999). https://doi.org/10.1115/1.2841315
7) G. L. Arnulfi, P. Giannattasio, C. Giusto, A. F. Massardo, D. Micheli, P. Pinamonti, *Multistage Centrifugal Compressor Surge Analysis: Part II—Numerical Simulation and Dynamic Control Parameters Evaluation*, ASME. J. Turbomach., **121(2)**: 312–320 (1999). https://doi.org/10.1115/1.2841316
8) Abrassi, A. Traverso, L. Ferrari, *Turbocharger-Based Hybrid Systems: Modeling and Validation of a Free Spool Subject to Compressor Surge*, Proceedings of the ASME Turbo Expo 2018: Turbomachinery Technical Conference and Exposition. Volume 3: Coal, Biomass, and Alternative Fuels; Cycle Innovations; Electric Power; Industrial and Cogeneration; Organic Rankine Cycle Power Systems, **V003T06A019**, (2018). https://doi.org/10.1115/GT2018-76754
9) L. Mantelli, M.L. Ferrari, A. Traverso, *Dynamics and control of a turbocharged solid oxide fuel cell system*, Applied Thermal Engineering, Volume **191**:116862 (2021). https://doi.org/10.1016/j.applthermaleng.2021.116862
10) E. M. Greitzer, *Surge and Rotating Stall in Axial Flow Compressors - Part I: Theoretical Compression System Model*, ASME. J. Eng. Power, **98(2)**: 190–198 (1976) https://doi.org/10.1115/1.3446138
11) E. M. Greitzer, *Surge and Rotating Stall in Axial Flow Compressors - Part II: Experimental Results and Comparison With Theory*, ASME. J. Eng. Power, **98(2)**: 199–211 (1976). https://doi.org/10.1115/1.3446139
12) E. M. Greitzer, F. K. Moore, *A Theory of Post-Stall Transients in Axial Compression Systems: Part II—Application*. ASME. J. Eng. Gas Turbines Power, **108(2)**: 231–239 (1986). https://doi.org/10.1115/1.3239893
13) F. K. Moore, E. M. Greitzer, *A Theory of Post-Stall Transients in Axial Compression Systems: Part I—Development of Equations*, ASME. J. Eng. Gas Turbines Power, **108(1)**: 68–76 (1986). https://doi.org/10.1115/1.3239887

14) N. Yamaguchi, *Surge Phenomena Analytically Predicted in a Multi-stage Axial Flow Compressor System in the Reduced-Speed Zone*, International Journal of Fluid Machinery and Systems (2014). https://doi.org/10.5293/ijfms.2014.7.3.110
15) Y. Eddie Ng, N. Liu, *Parametric Study of Greitzer's Instability Flow Model Through Compressor System Using Taguchi Method*, Compressor Instability with Integral Methods, Springer, Berlin, Heidelberg (2007). https://doi.org/10.1007/978-3-540-72412-4_5
16) F. Grapow, G. Liśkiewicz, *Study of the Greitzer Model for Centrifugal Compressors: Variable L_c Parameter and Two Types of Surge*, *Energies* **13(22)**:6072 (2020). https://doi.org/10.3390/en13226072