

MATHEMATICAL ANALYSIS OF 2-D STEADY SEEPAGE FLOW AND THE BEHAVIOR OF CONTAMINATES TRANSPORTATION THROUGH HOMOGENEOUS EARTH DAM USING COMSOL SOFTWARE

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Abstract: Seepage is a flow that happens through / under hydraulic structures or through the porous media such as in the case of earth dams from upstream to downstream due to difference in the hydraulic head. Study of seepage flow is important for hydraulic engineering. Before the structures being built, the behaviors of the seepage flows must be predicted because the seepage flow causes harmful to hydraulic structures. In this paper, a homogenous earth dam with sand material is proposed and constructed with horizontal bed filter at the Toe of the dam is taken as a case study. Through this Comsol is used to interpret the seepage of flow and contaminants transportation through the dam. The general objective of this paper is thus to present a prediction model aimed at quantifying a selected concentration of contaminants in the reservoir, and then predicted in the body of the dam, and at the exit from the dam.

Computation Fluid Dynamics (CFD) Module (Comsol version 4.2) with its sub-programs named (Free and Porous Media Flow and Species Transport through Porous Media) was used in the analysis and study the seepage flow and of the contaminants transportation. This software is based on the finite element techniques that were used for solving the governing equations of flow and transportation of contaminant through porous media. Also, it was used to determine the phreatic line, amount of seepage within the dam, the pressure head, the total head, and the amount of contaminates transported through the dam body.

From the comsol software, it is deduced that when the water level is at the maximum height (20m), it needs 12 days and (18) hr, at normal height (15m) it needs 29 days, while at a minimum height (8 m) it needs 81 days to reach the drain zone.

Keywords: *Steady seepage flow, transportation of contaminates, Comsol version 4.2*

1. Introduction

Earth dams are structures constructed from natural material obtained from pits located in the vicinity site. Dams are used for water storage, tourism and recreation, energy production, flood control and for irrigation purposes. Seepage is a very important parameter that should be studied carefully in the design of earth dams, since it is one of the main causes of the dam failure. [1]

Water seepage phenomena through porous media depend on many factors such as soil material, type of the dam, properties of the liquid and hydraulic gradient. Many methods have been developed to study seepage flow. These methods can be categorized as analytical, experimental and approximate (numerical) methods [2].

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Mishra and Singh (2005) defined the area that lies above the phreatic line in a homogeneous dam as a dry zone. The study concluded that the dry zone depends on the length of the drain and the capillary saturation. The study has shown that the location of the drain affect the capillary rise above the seepage line and on the downstream stability.

Cheng and Zhang (2009) studied physical and mechanical prosperities of material filling dam, dam operation and management status based on seepage of homogeneous earth dam of Longtougiao in China. According to state criterion of earth dam design, the program of seepage control was made, and they adopted technique of blocking in foreside and draining on back. They conclude that the effect of reconstructed is renovate to analyze the seepage control of the dam such that the measure will be significant.

Changchun (2012) presented a two-dimensional seepage flow model by using Auto-BANK software with finite element method. The seepage quantity and seepage gradient of earth dam were estimated depends on the steady seepage flow equation in porous media and definite conditions. This model can effectively determine and analyze multi-media seepage flow field with complex boundaries.

The rate of movement of a chemical constituent through porous media is determined by some transport mechanisms. These mechanisms often work simultaneously on the chemical and may include such processes as advection, dispersion, diffusion, linear equilibrium adsorption, and first-order production and decay. Since there are many mechanisms that affect solute transport, a full range (complete set) of analytical solutions must be provided, not only to predict the transfer of the actual solute in this field, but also to analyze the transport mechanisms themselves.

Analytical solutions to the advection-dispersion solute equation for different boundary conditions and solute-source forms using one, two and three dimensional systems of uniform groundwater flow was presented by Wexler (1992). Solutions in a simple format, with important assumptions in derivation and limitations to their use were reported.

Three analytical solutions for solute movement in saturated zone of homogeneous type of porous media was presented by Sim and Chrysikopoulos (1999). Accordingly, three dimensional model was developed for dispersion in a uniform flow field with first-order decay of aqueous phase and sorbed solutes with different decay rates and non-equilibrium solute sorption onto the solid matrix of the porous.

Advective transport of a soluble contaminant in a saturated homogeneous soil with non-linear sorption of the contaminant on to a porous media had been studied by SHENG et al. (1999). The non-linear sorption isotherms in the transport analysis were the Langmuir and Freundlich sorption isotherms. The linear sorption isotherm was a special case of the Freundlich sorption isotherm. Analytic solutions to the non-linear first-order hyperbolic equations were developed for a number of contaminant transport problems. In the Langmuir sorption isotherms case shock fronts developed at the leading edge of the concentration profile while for the Freundlich sorption isotherm shock fronts may develop at either the leading or trailing edge of the concentration profile.

Application of the advection – dispersion equation was presented by Zairi and Rouis (2000) in the numerical model. Validation tests were carried out by comparing with the analytical and semi-analytical solutions. A two dimensional finite element model was proposed for contaminant transportation. Experimental

laboratory tests on cadmium and chloride transportation through liner samples were also discussed then the results were compared with that resulted from the numerical model.

Rao et al. (2006) introduced the application of a multiple domain algorithm which integrated with a finite difference formulation to solve the two dimensional advection- dispersion equations. The results showed the primary purpose of using a multiple domain approach was met without any loss in the accuracy of the solution.

Patil and Chore (2015) tested the behavior of solute transport through porous media by using laboratory experiments. Sodium chloride was used as a conservative chemical in the experiment. Pulse boundary condition and continuous boundary conditions were used during the experiment. Experimental results have been presented for conservative solute transport in the sand. The pattern of the break through curve in all the cases of varying flow rate and initial concentration of conservative chemical remains almost same.

1.1. Contaminates Transportation

The contaminants in porous media move and spread as a result of:

1. Advection at which the solute contaminant transports by the solvent movement at the same rate as solvent (water). It moves with the flow in porous media according to Darcy's law (Fig. 1A). [13]
2. Dispersion which is the result of two processes- molecular diffusion and mechanical mixing. The process where ionic or molecular components move from regions of higher concentrations to regions of lower concentrations is called molecular diffusion. Mechanical mixing is dilution of the contaminant due to the mixing of contaminated groundwater with non-

contaminated groundwater [13]. Mechanism of the dispersion is illustrated in figure (1B).

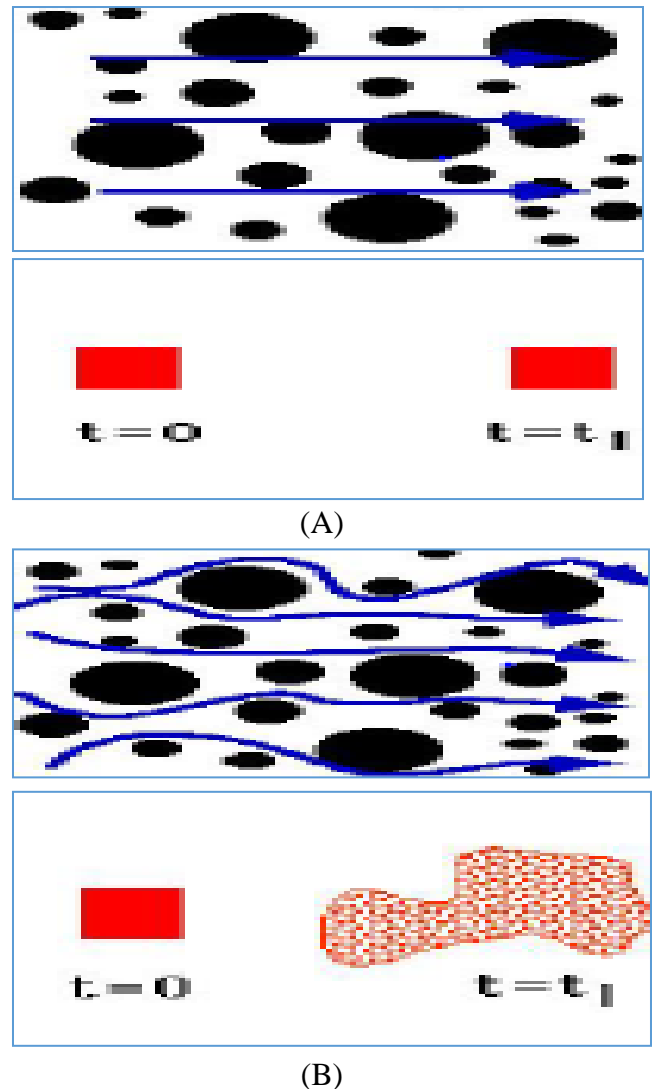


Figure 1. Transport of contaminants through groundwater by : (A) Advection, (B) Dispersion

3. Retardation due to sorption process. Sorption refers to the exchange of ions and molecules between two phases (solid phase and liquid phase). It includes adsorption and desorption. Adsorption is the attachment of ions and molecules from the solute to solid phase from which it will produce a decrease of the solute concentration. This process is also called Retardation (figure 2). The retardation depends on the distribution coefficients of the

contaminant and the characteristics of the porous medium. Desorption is the release of ions and molecules from the solid phase to the solute [13].

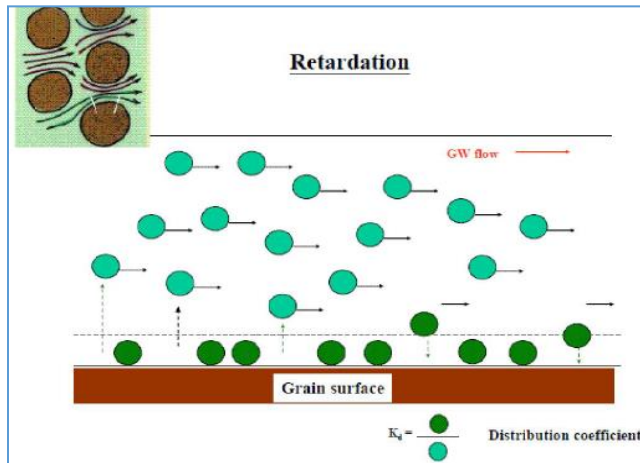


Figure 2. Contaminants retardation by adsorption on the soil surface

2. Homogeneous Embankment Dam

A homogeneous embankment type earth dam is chosen. The dam composed of a sandy soil. A suitable drainage system, of the form of toe drain is also provided. The height of the dam was (22.5) m and its length, $L = 107.25$ m, while the upstream side slope was 2.5H: 1V, and the downstream slope 2H: 1V (Figure 3). Table (1) illustrated the technical specifications of the dam.

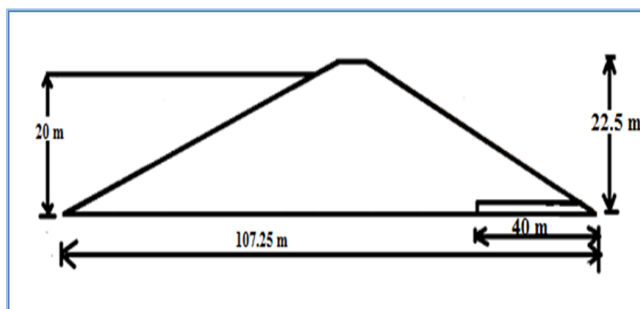


Figure 3. Homogeneous earth dam

Table below indicate the technical specifications of the embankment:

Table 1. The technical specifications of the embankment

Type of Dam	Homogeneous Earth dam
Dam	Height: 22.5 ; Length: 107.25 m
Freeboard	2.5 m
U/S Water level (maximum, normal, minimum level)	20 m, 15m, 8 m
Type of Material	Sandy Soil
bulk density of soil	1600 kg/m ³
Porosity	0.437
Hydraulic Conductivity,	2.827 m/ day
Intrinsic permeability	3.34 e-12 m ²
Slope of the dam	Upstream:2.5:1; Downstream: 2:1
Thickness of Filter	1 m

3. Theoretical Analysis

3.1. Comsol Version 4.2

Comsol was used for studying two states: the first state involves studies and interpretation of the flow net through porous media (the stream lines and equipotential lines through an earth dam). It depends on Continuity, Navier – Stokes equations, and the modified Navier–Stokes equation for a fixed bed porous medium; in addition to Forchheimer correction equation. These equations are as follows:

$$\rho \nabla \cdot u = 0 \tag{1}$$

$$\rho(u \cdot \nabla)u = \nabla \cdot [-pI + \mu(\nabla u + (\nabla u)^T)] + F \tag{2}$$

$$\frac{\rho}{\varepsilon_p} \left(\frac{u \cdot \nabla u}{\varepsilon_p} \right) = \nabla \cdot \left[-pI + \frac{\mu}{\varepsilon_p} (\nabla u + (\nabla u)^T) - \frac{2\mu}{3\varepsilon_p} (\nabla \cdot u)I - \left(\frac{\mu}{K} + \beta_F |u| + \frac{Q_{br}}{\varepsilon_p^2} \right) u \right] + F \tag{3}$$

$$C_F = \frac{1.75}{\sqrt{150\varepsilon_p^3}} \quad (4)$$

Where:

μ = the dynamic viscosity (Pa·s)

u = velocity of water (m/s)

ρ = the fluid's density (kg/m³)

p = the pressure (Pa.).

k = permeability of the porous medium (m²)

ε_p = the porosity (dimensionless)

C_f = the dimensionless friction coefficient.

The second state studies the species flow through porous media. It depends on Advection – Dispersion equation, and the Retardation equation:

$$R \frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} - \frac{\partial}{\partial x} (uc) - \frac{\partial}{\partial y} (vc) \quad (5)$$

$$R = 1 + \frac{\rho_b}{n} k_d \quad (6)$$

Where:

c = concentration of contaminants (mole/m³),

u = Velocity field in the x- direction (m/s),

v = Velocity field in the y- direction (m/s),

D_x = coefficient of dispersion in the x- direction (m²/s),

D_y = coefficient of dispersion in the y- direction (m²/s),

R = retardation which is caused by adsorption,

t = time,

e = porosity,

K_d = adsorption coefficient (m³/kg)

ρ_b = the bulk density of the soil (kg/m³)

The second state studies the species flow through porous media. It depends on Advection – Dispersion equation:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x} (uc) + \frac{\partial}{\partial y} (vc) = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} \quad (7)$$

3.1. Boundary Conditions

The boundary conditions are satisfied by the following points:

1- At inlet

$$P = \gamma H; P = 196200 \text{ Pa.},$$

$$C = C_0 = 100 \text{ mol/m}^3$$

2- At outlet

$$P = 9810 \text{ Pa.},$$

3- At ($t = 0$)

$$C(x, y) = 0$$

4- $R = 1$, there is no reaction between the soil material and the contaminant.

4. Results

A schematic diagram of homogeneous earth dam by using Comsol software is indicated in Figure (4). The finite element mesh which used for the analysis is shown in figure (5). The mesh type is Triangular which covers the whole body of the dam. The numbers of elements for all boundaries are 365 elements, minimum element size is 0.0322 m, and maximum element size is 7.19 m

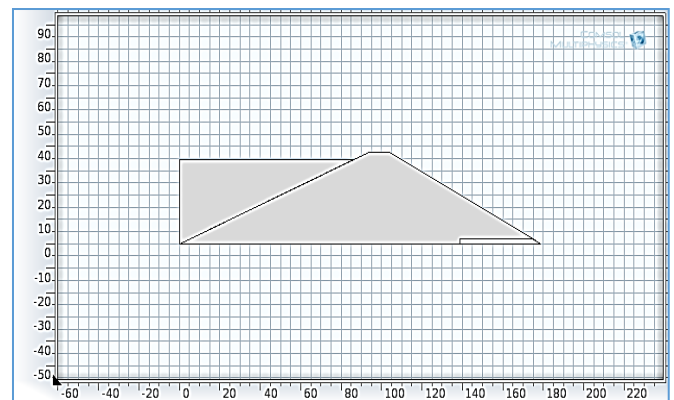


Figure 4. Cross section of homogeneous earth dam

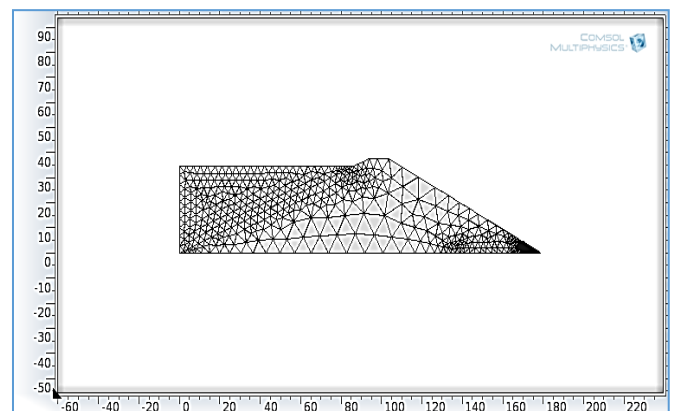
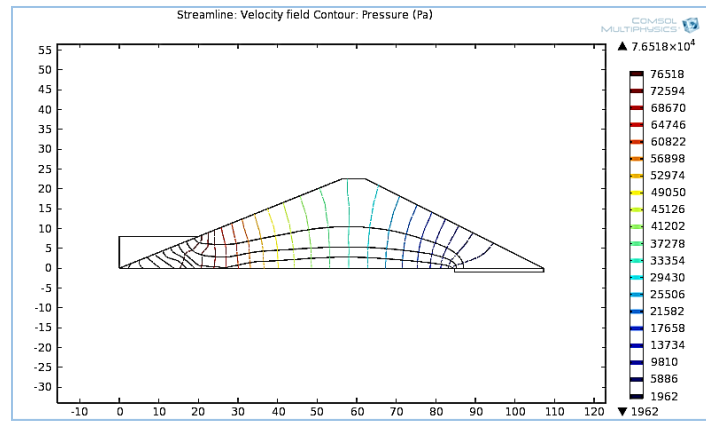


Figure 5. Finite Element Mesh

In homogeneous earth dam, the phreatic line starts from the water surface level and drops throughout the dam until it passes through the horizontal drainage filter and exit at the dam toe, throughout the filter. The object of the horizontal drainage filter is to pull the phreatic line down into the filter such that it does not intersect with the downstream side of the dam.

The flow net using comsol software is drawn and shown in figure (6). The colored bar line on the side represented a drop in the water pressure from (196200 Pa) to (9810 Pa) figure (6.A). The location of seepage line at steady state seepage flow through earth dam at (20m, 15m and 8m) upstream water level using Comsol software is indicated in figure (6.A, B, C) respectively.

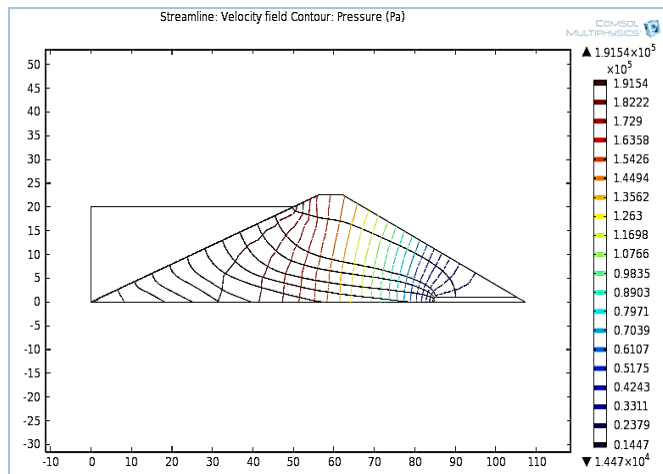


(C)

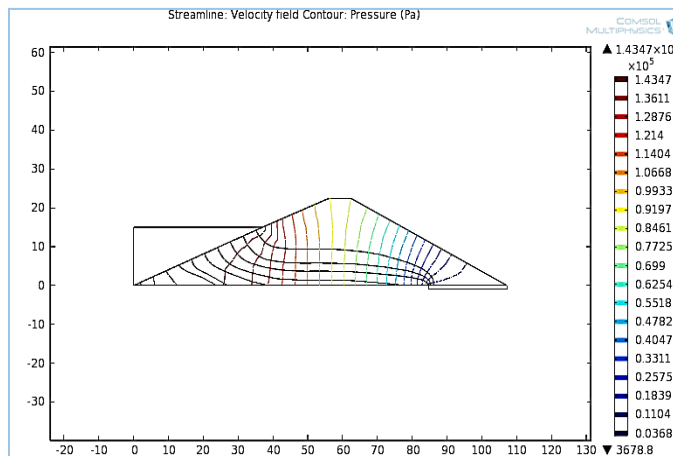
Figure 6.A,B,C. Flow net of homogeneous earth dam

The contaminants transportation model using Comsol software indicates the following results:

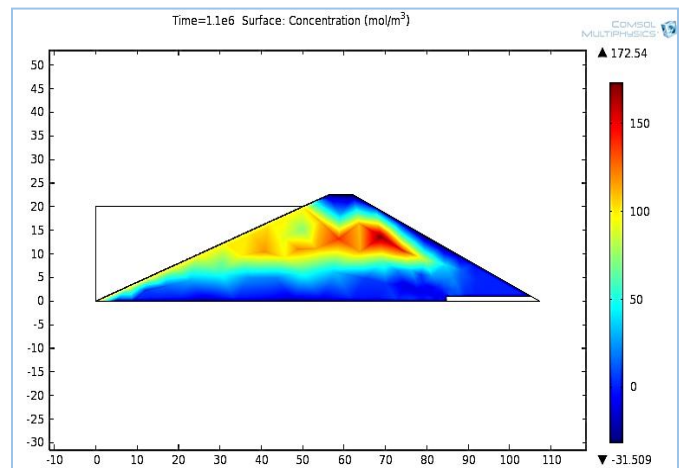
Unsteady state (time dependent) contaminates transportation through the dam is indicated in figure (7.A, B, C). The colored area represents the transmission of contaminates from the upstream or the source of the contaminants down through the earth dam. It is shown that the contaminate needs (1.1 e6 sec) about 12 days and 18 hr, (2.5 e6 sec) about 29 days, and (7e6 sec) about 81 days until reach the drain zone at (20m, 15m and 8m) upstream water level respectively as shown in figure (7.A, B, C).



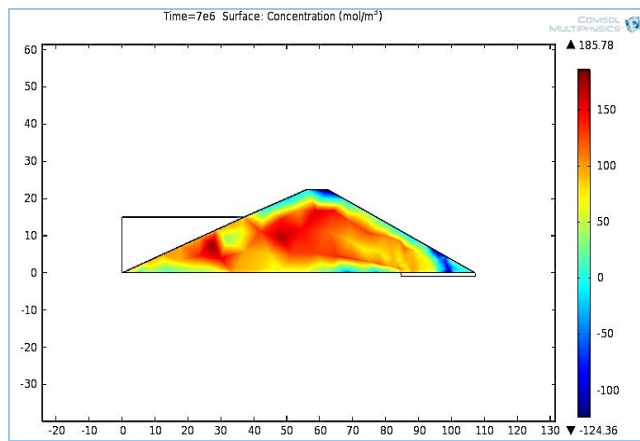
(A)



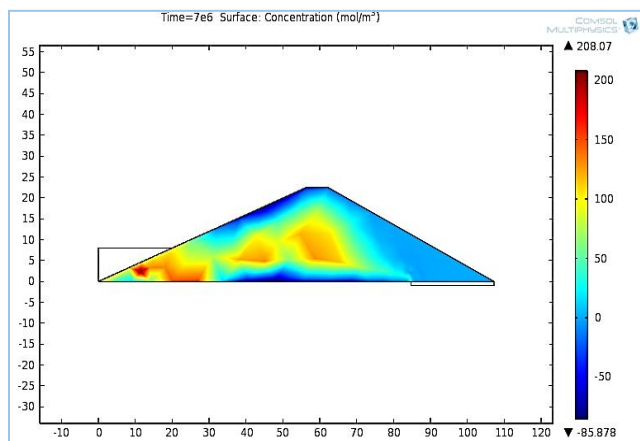
(B)



(A)



(B)



(C)

Figure 7.A,B,C. Computed transportation of contaminates until reach drain

Table (2) and figure (8) illustrate the effect of water level on the time rate of change of contaminates transportation through the earth dam till reaching the drain zone.

Table 2. Effect of reservoir water level on the time rate of contaminates transportation through the dam

Head (m)	Time (day)
8	100
15	30
20	12

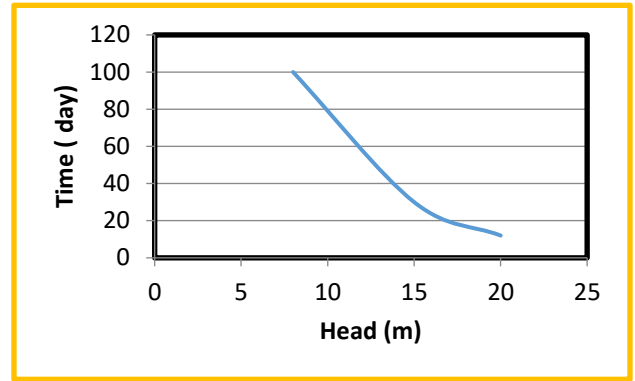


Figure 8. Variation of water level in the U/S with the contaminant transportation

5. Conclusions

The following conclusions can be drawn from this research:

1. The phreatic line and flow through a homogeneous earth dam is well presented by Computational Fluid Dynamics (CFD) Module (Comsol version 4.2). The phreatic line starts from the water surface level of the reservoir and drops throughout the dam until it passes through the horizontal drainage filter and exit at the dam toe, throughout the filter. The object of the horizontal drainage filter is to pull the phreatic line down into the filter such that it does not intersect with the downstream side of the dam.
2. Flow through porous media and transportation of the contaminants are simulated by a two-dimensional finite element seepage flow model based on the basic equations of steady seepage flow of homogeneous earth dam and definite conditions by using the comsol software. The program is applied on Homogeneous earth dam to specify the location of the free surface seepage line, through the dam, and the total head measurements.

3. Comsol software indicates that the time of transportation of the contaminants varies inversely with the water level such that the contaminants need (12) days and (18) hours until reach the drain zone when the water level head in the reservoir is at 20m. While the contaminants need (29) days and (81) days at 15m and 8m height of water level respectively.

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Abbreviations

A list of symbols should be inserted before the references if such a list is needed

μ	the dynamic viscosity
u	Velocity field in the x- direction
ρ	the fluid's density
p	the pressure
k	permeability of the porous medium
\mathcal{E}_p	the porosity
C_f	the friction coefficient
ε_v	volumetric strain
c	concentration of contaminants
v	Velocity field in the y- direction
D_x	coefficient of dispersion in the x- direction
D_y	coefficient of dispersion in the y- direction
R	retardation which is caused by adsorption
t	time
e	porosity
K_d	adsorption coefficient
ρ_b	the bulk density of the soil

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