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Published in:
European Journal of Operational Research

DOI:
10.1016/j.ejor.2023.01.032

Publication date:
2023

European Journal of Operational Research 309 (2023) 133–151

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing, Transportation and Logistics

Optimal trade-in and refurbishment strategies for durable goods

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ARTICLE INFO

Article history:
Received 14 August 2021
Accepted 17 January 2023
Available online 21 January 2023

Keywords:
Supply chain management
Trade-ins
Product refurbishment
Durable goods
Strategic consumers

ABSTRACT

Many manufacturers in the automobile industry accommodate the huge number of used cars by offering trade-in programs. In addition, some manufacturers have begun considering product refurbishment, a policy that is widely adopted in the electronics industry. Therefore, we are motivated to explore the reasons behind different practices in the automobile industry. We propose an analytical framework to identify when a manufacturer facing strategic consumers should offer trade-in (and refurbishment) programs. For that purpose, we analyze and compare the results of three models: no program, trade-in program only, and trade-in and refurbishment programs. This study establishes that the manufacturer can always increase his profit by improving the quality of new products and reducing the quality depreciation rate. Yet when the manufacturer does not (resp., does) offer a refurbishment program, his profit must (resp., need not) decrease with any increase in the production cost of new products. Finally, the manufacturer prefers to offer (a) trade-in programs only when the production cost of new products is low, (b) both trade-in and refurbishment programs when that cost is moderate, and (c) neither program when the cost to produce new products is high. Our numerical study reveals more management implications for the manufacturer's preferred decision.

1. Introduction

In the automobile industry, trade-ins are a convenient way for consumers to dispose of used cars because they involve relatively little paperwork and overall hassle. There are an estimated 41 million used cars in the United States, and changes triggered by the pandemic will add to that number (Rosenbaum, 2020). In response to this boom of used cars, many manufacturers are pro-actively offering trade-in programs. For example, Subaru created the Subaru Guaranteed Trade-In Program, tailored specifically so that owners are always given the highest possible trade-in value for their vehicle.[1] Tesla accepts passenger cars, trucks, vans, and SUVs for trade-in toward the purchase of a new Tesla.[2] BMW offers their customers a plethora of upgrade options, so current BMW owners can trade up to one of the newest BMW models at the same (or lower) monthly payment.[3] Toyota allows trade-ins as part or all of a down payment when buying a brand-new Toyota.[4]

In addition to offering trade-in programs, Toyota is exploring the possibility of incorporating one of Apple's best ideas by starting a refurbishment program at the brand's UK production facility (Duffy, 2022). In essence, Toyota would take back used cars, restore them to like-new condition, and thereby extend a vehicle's life cycle; this new process will form the backbone of a new mobility sub-brand called Kinto (Kalmowitz, 2022). The quality of a car can be partially restored by refurbishment, which is not the case when it is simply recycled. Toyota may perform refurbishment three times on a vehicle before turning its attention to recycling the car responsibly, which may include rebuilding batteries and reusing parts from the vehicle that are still in good condition. A brand-new Toyota could last for more than ten years with multiple refurbishments (Seto, 2022). The refurbished car could be sold again, becoming a new source of revenue for Toyota. The program is now being implemented only in Toyota's Burnaston plant, but if all goes well then similar schemes will pop up in numerous factories worldwide (Szymkowski, 2022).

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[1] https://www.suburbansubaru.com/guaranteed-trade-in-program-site.htm.
[2] https://www.tesla.com/support/trade-ins.
[3] https://www.plazabmw.com/bmw-trade-up-program.htm.
[4] https://www.buyatoyota.com/gst/trade-in-value/.

https://doi.org/10.1016/j.ejor.2023.01.032
0377-2217/

Therefore, we are motivated by the different practices in the automobile industry mentioned above, and the reasons behind them are explored in this paper. It is worth noting that, we focus on the automobile industry rather than the electronics industry because we have observed that in the automobile industry, some firms (Subaru, Tesla, and BMW) only implement trade-in programs, while others (Toyota) offer both trade-in and refurbishment programs. But in the electronics industry, almost all major manufacturers (Apple, Huawei, Xiaomi, et al.) offer both trade-in and refurbishment programs.

Although most consumers would like to purchase a new car, a large portion (43%) of them end up looking for a used car (AdColony, 2019). Consumers are heterogeneous not only in their preferences for old and new products but also regionally. For example, Gravy Analytics (2021) reports that the three US states with the greatest increase in auto buyers in 2021 were Alabama (32%), Arkansas (32%), and Mississippi (30%); an individual in any of these states might well be looking to trade in their current vehicle. Typically, consumers' product-purchasing and trade-in behaviors are strategic. For instance, consumers can learn important tips on buying a car, and decide whether to buy a new or an used one, by referring to Martucci (2022). Consumers can determine how best to dispose of old cars by consulting Agustinus (2022), which analyzes the pros and cons of consumers trading in or selling their old vehicles. Furthermore, we consider that under our two-period model setting, strategic consumers will make their first-period decisions by taking account of their utility across both periods (Kremer et al., 2017; Van Ackere & Reyniers, 1995). Understanding the strategic behavior in which heterogeneous consumers engage during the purchase process is critical for determining the manufacturer's optimal decisions, since choosing the right program helps the manufacturer increase his profits.

Given this set-up, we address the following three questions.

(i) How do heterogeneous consumers make purchase and trade-in decisions?
(ii) How do manufacturers make optimal decisons – with regard to selling prices and trade-in rebates – so as to maximize their profits, and what metrics can the manufacturer adjust to improve his profit margin?
(iii) Under what conditions will a manufacturer implement trade-in (and refurbishment) programs?

We address these questions by considering a manufacturer who sells new products to heterogeneous consumers for two periods and who can also offer a trade-in program to collect old products at the end of the first period, which can then be refurbished and resold to consumers in the second period. The new product's quality depreciates over the two periods, and the quality of old products is partially restored by the refurbishment program. At the beginning of the first period, strategic consumers make their purchase decisions while accounting for their expected utilities across both periods. Three models are proposed: no program, trade-in program only, and both trade-in and refurbishment programs. For each model, we start by analyzing the decisions of heterogeneous consumers to determine how many of them choose each option; we then solve for the manufacturer's optimal pricing and trade-in rebate decisions. Our analysis shows that the manufacturer's optimal pricing strategy is to set the same selling price for the new product in both periods. Finally, we identify the manufacturer's optimal strategy after comparing his profits in the three models. Our main findings are summarized as follows.

First, in order to ensure that manufacturers are willing to produce and sell new products and to offer trade-in and refurbishment programs, the production costs for new products must be limited. In particular: manufacturers with the best control over their new products' production costs can profitably offer both trade-in and refurbishment programs; manufacturers with intermediate control can profit only by offering just trade-in programs; and manufacturers with the least control must sell new products without offering any such programs at all.

Second, manufacturers can always increase profits by improving the quality of new products and reducing the rate of quality depreciation, regardless of whether or not they offer trade-in (and refurbishment) programs. Even if the manufacturer can get some revenue from the product depreciation by offering a trade-in program, he cannot change the trend of his profit decreasing with an increase in the depreciation rate. And despite the ability of manufacturers to exploit new sources of profit by offering refurbishment programs on a trade-in basis, they still cannot alter that monotonically decreasing trend. This outcome, which follows from the strategic behavior of consumers, is interesting because it runs counter to the "planned obsolescence" recommended to manufacturers by much of the literature.

Third, when the manufacturer does (resp., does not) offer a refurbishment program, his profit must (resp., need not) decrease with an increase in the production costs of new products. This finding suggests that manufacturers can cope with the loss of profits caused by higher production costs of new products by implementing a refurbishment program characterized by low costs and high levels of recovered quality. Moreover, the manufacturer's profit decreases with an increase in the refurbishment cost but increases with the quality recovery of refurbished products.

Finally, manufacturers prefer to offer only trade-in programs when the production cost of new products is low, to offer both trade-in and refurbishment programs when that production cost is moderate, and to offer neither program when the production cost is high. Also, our numerical study reveals that improving the quality recovery rates of refurbished products does not encourage manufacturers to offer trade-in programs – but it does incentivize the adoption of refurbishment programs by manufacturers that are already offering trade-in programs. An increase in the depreciation rate of new products not only encourage manufacturers to adopt trade-in programs, but also further encourage refurbishment adoption by those already offering trade-in programs.

Our research makes three main contributions. First, we observe the practice of existing trade-in activities and the emergence of product refurbishment activities in the automobile industry, which form the basis of our practical suggestions for manufacturers. Second, we adopt a two-period model to analyze the strategic behavior of consumers and give their number (endogenously) under different options; this approach supplements the method, used in the extant literature on trade-in and product refurbishment, of treating the consumer group as exogenously given. Third, we identify a condition – not discussed in previous studies – under which manufacturers should implement trade-in and refurbishment programs.

The rest of our paper proceeds as follows. Section 2 briefly reviews the relevant literature, and our model is introduced in Section 3. In Section 4, we analyze the respective equilibrium solutions of three models: no program, trade-in program only, trade-in and refurbishment programs. The three models are compared in Section 5, and we conclude in Section 6 with a brief review of our study's results and limitations. Mathematical proofs are given in the Appendix.

2. Literature review

This study focuses on the manufacturer's decision, given that consumers behave strategically, about whether to implement trade-in (and refurbishment) programs for old durable goods. Hence our work is most closely related to three streams of literature: research on (i) the durable goods monopoly problem with strategic consumers, (ii) trade-ins by strategic consumers, and

(iii) mechanisms for dealing with old products (i.e., second-hand old products, remanufactured products, leased products, and refurbished products).

First, our research is related to the classic durable goods monopoly problem, which has been extensively studied in microeconomics (for a detailed review of this field, interested readers are referred to Waldman, 2003). Given the strategic behaviors exhibited by consumers when purchasing products, the existing literature studies a variety of issues faced by the monopolist; examples include devising an optimal price policy (Board, 2008; Conlisk et al., 1984), the impact of rationing (Denicolo & Garella, 1999), the optimal sales strategy (Levinthal & Purohit, 1989), production and pricing decisions (Tilson & Zheng, 2014), the optimal dynamic price-quality strategy (Kumar, 2002), and whether selling is preferable to leasing (Chien & Chu, 2008). Along these lines, we study a manufacturer's pricing and trade-in decisions based on analyzing strategic consumer behavior. Some of the literature on durable goods suggests that monopolies tend to reduce product durability, causing products to break down more quickly and thus inducing repeat purchases from consumers; this phenomenon is known as "planned obsolescence" (Bulow, 1982; Fishman & Rob, 2000; Waldman, 1996). However, scholars (e.g., Agrawal et al., 2016b) have also suggested that firms can benefit from designing conspicuously higher durability into consumer durable goods. Our research confirms that the manufacturer's profit decreases as the product's depreciation rate increases, but we give a different reason: because a strategic consumer is less likely to buy new products if she anticipates their inefficient use in the future.

Second, our study is clearly related also to trade-in programs, which have been studied from multiple perspectives. Most relevant to our work are those studies that consider a two-period framework, treating products in period 1 as new products and products in period 2 as old products (Li et al., 2019; Miao et al., 2017). Strategic consumers usually make an optimal decision by comparing the total surplus they expect to derive from the product across two periods under different options (Liu et al., 2019; Van Ackere & Reyniers, 1995). While viewing the number of consumers under different options as being endogenously given, studies typically seek to identify the optimal price and rebate decisions for manufacturers (Hu et al., 2019; Ray et al., 2005; Yin et al., 2015). In addition, some scholars explore various issues in the context of trade-ins; topics include the optimal choice of trade-in provider (Tang et al., 2021), when and how an OEM should offer trade-ins (Agrawal et al., 2016a; Bian et al., 2019), the impact of implementing trade-ins (Dou & Choi, 2021; Rao et al., 2009; Zhu et al., 2016), the optimal trade-in rebate mechanism (Genc & De Giovanni, 2018; Guo et al., 2022), optimal subsidy schemes and budget allocations (Bai et al., 2021), and optimal scrappage subsidy levels (Zaman & Zaccour, 2021). However, none of these cited works addresses product refurbishment, which is now receiving concerted attention in the industry.

Consumers who buy a new product in the first period are usually called product owners (or holders); in the second period, they can choose whether or not to trade in their old product and receive the corresponding rebate. Consumers who do *not* purchase new products in the first period are often called acquirers (or nonholders) and may purchase old products in various forms in the second period. So with respect to the third related research stream, dealing with old products, the most intuitive resolution is for the second-hand (old) products to be sold directly to consumers without any processing. In general, old products connect product owners and acquirers by second-hand dealers or online peer-to-peer platforms (Crosno & Cui, 2018; Fernando et al., 2018; Fudenberg & Tirole, 1998; Guiot & Roux, 2010; Purohit, 1992; Rao et al., 2009; Vedantam et al., 2021; Yin et al., 2010). For example, Yin et al. (2010) examine how the emergence of retailer- and P2P-used product markets affected the manufacturer's product upgrading strategy and the retailer's pricing strategy in the context of textbooks. The three main differences between our study and theirs are as follows. First, they consider that consumers need textbooks only for a certain period of time and will not need them after that period. Unlike them, we consider consumers may need automobiles for multiple periods. Second, due to differences in research backgrounds, they consider that consumers have no opportunity to wait, while in our model, consumers may choose to wait strategically. Finally, when consumers face different choices (choosing which channel to return the old product through), the proportion of different types of consumers is exogenously given, while we endogenously compute the number of consumers of each type based on the utility of consumers. The above cited studies suggest that the second-hand old-product market will compete not only with the sales of new products but also with the recycling of old products involved in the trade-in program (if the trade-in program is provided). We differ by considering that the manufacturer can generate additional profits from refurbishing and selling the old products recycled. Note that the refurbished products considered here can, just like the second-hand old products, compete with the trade-in program.

The most widely discussed form of old-product disposal is that of the old products' "cores", recovered through trade-in, being remanufactured and sold to consumers. Against this background, scholars have addressed the following topics: choosing the appropriate reverse channel structure (Savaskan et al., 2004), analyzing the impact of trade-ins on economic and environmental values (Zhang & Zhang, 2018), the OEM's remanufacturing entry decision (Li et al., 2019), and the interactions between delegating collection and the channel structure (Fan et al., 2022). Here we must emphasize a key difference between the *remanufactured* products discussed in the literature and the *refurbished* products we study. In the model setting of most previous work, the quality of old products with a certain product category can be entirely restored by remanufacturing, which is sometimes even assumed to yield quality on the level of new products (e.g., Fan et al., 2022; Xiao et al., 2020a; Zhang & Zhang, 2018). However, real-world observations indicate that the quality of old products is never fully recovered by refurbishment – from which it follows that the manufacturer's profit will be affected by its product's depreciation rate. In fact, our research establishes that the higher the rate of product depreciation, the lower the manufacturer's profit.

Another mechanism that companies commonly use for old products is leasing programs. Agrawal et al. (2012) examine whether the leasing model is better for the environment than the selling model, and both Dou et al. (2017) and Jia et al. (2018) study whether the leasing model is more profitable than the selling model. In addition, Li & Xu (2015) derive the optimal pricing strategy under trade-in versus product leasing models in their study of manufacturers' preference for these two strategies. In the leasing model, consumers typically use the product at a lower price than the new product's sales price; but since the company still owns the product after use, it has an incentive to design a more durable product (Agrawal et al., 2012). In contrast, we examine a refurbishment program in which consumers still use the old (refurbished) product at a lower price but companies do not own the product after use. Nevertheless, one conclusion of interest is that, owing to the strategic behavior of heterogeneous consumers, the manufacturer can still benefit from making products more durable.

Most relevant to our research is work concerned with product refurbishment. Based on the two common disposal options of refurbishing to remarket and to fulfill warranty claims, Pinçe et al. (2016) examine how OEM should dynamically allocate consumer returns between fulfilling warranty claims and remarketing refurbished products. Borenich et al. (2020) investigate whether

Table 1
Comparison of this study with the most related literature.

Literature	Trade-in program	Product refurbishment	Strategic consumers
Kumar (2002)	×	×	√
Board (2008)	×	×	√
Chien & Chu (2008)	×	×	√
Tilson & Zheng (2014)	×	×	√
Ray et al. (2005)	√	×	√
Rao et al. (2009)	√	×	√
Li & Xu (2015)	√	×	√
Yin et al. (2015)	√	×	√
Zhang & Zhang (2018)	√	×	√
Hu et al. (2019)	√	×	√
Li et al. (2019)	√	×	√
Liu et al. (2019)	√	×	√
Vedantam et al. (2021)	√	×	√
Borenich et al. (2020)	√	√	×
Xiao & Zhou (2020)	√	√	×
Xiao et al. (2020b)	√	√	×
Li et al. (2022)	√	√	×
This study	√	√	√

manufacturers could sell products returned by consumers as refurbished items and thus motivate retailers to reduce consumer returns. Slama et al. (2022) address the capacitated disassembly lot-sizing problems under uncertain refurbishing durations, to minimize the expected total cost. However, these researches have nothing to say about trade-ins. Xiao & Zhou (2020) derive the optimal trade-in prices of old products and resale prices of refurbished products – for a firm that offers both "trade in for upgrade" and "trade in for cash" programs – to acquire old products, refurbish them, and then resell them together with new products. Xiao et al. (2020b) study the joint and dynamic decisions on the selling price of new products and the trade-in price of old products while assuming that returned old products can be sold on a secondary market after being refurbished or remanufactured. Li et al. (2022) derive and compare the optimal decisions of a monopolistic recycling platform under three trade-in modes (trade in for cash, trade in for refurbishment, and trade in for new). Our study differs from the extant literature in the following three key aspects. First, before proposing decision criteria for manufacturers, we analyze the strategic behavior of consumers in two periods and consider consumer groups with different preferences to be endogenous (rather than exogenous). Second, we assume that the quality of refurbished products depends on the quality of recycled old products – rather than a dismantled or weak relationship between refurbished and returned old products in prior literature. Third, we identify conditions under which manufacturers should implement trade-in and refurbishment programs.

In sum, we not only distinguish our study from each research stream in terms of research topic, but also discuss some aspects related to model setting, analysis, results and insights. Specifically, we have pointed out that we adopt a two-period model and introduced multiple consumer types under different behaviors. We have also made some comparisons among refurbished, remanufactured, second-hand, and leased products. Further, we have discussed some interesting results and insights of our study differ from the existing literature. In Table 1, we summarize the differences between our study and the closely related literature.

3. Modeling framework

We focus on automobile industry, and construct a two-period model to capture the crucial market characteristic that (a) products depreciate with time and usage and (b) the intertemporal substitution effect due to product durability, in the same spirit as most of the literature (Agrawal et al., 2012; Jiang & Tian, 2018; Li et al., 2019). We presuppose that the service life of each new product is designed to be only two periods, after which the product is fully depreciated functionally (Li & Xu, 2015) and is fit only for disposal (via recycling, incineration, or landfilling; Agrawal et al., 2012). That is, the product's salvage value is normalized to zero. At the beginning of the first period, the manufacturer sells the new durable product to consumers at price p_{n1}; this product will be sold at price p_{n2} in the second period. The new product has a quality of q and is produced at unit cost c. The product's quality becomes $q(1-\delta)$ at the end of the first period, where δ is the product's depreciation rate and $0 \le \delta \le 1$ (Guo et al., 2022; Li et al., 2019).

In addition to selling new products directly, the manufacturer also offers trade-in programs that give consumers a rebate r for each unit traded in (this set-up has been widely adopted in the literature since Van Ackere & Reyniers, 1993). We assume that the manufacturer also incurs a fixed cost F_t for implementing the trade-in program. After the manufacturer recovers the old product through trade-ins, he can refurbish it at unit cost γc; we use γ to denote the *cost rate* and $0 \le \gamma \le 1$, which is the cost of refurbishing an old product relative to the cost of manufacturing a new product. According to Chen et al. (2020), the purpose of refurbishment is to recycle old products so that they reach a certain quality standard and then continue to be put into the market, which is lower than the quality standard of new products. Thus, we assume that the quality of old products will be restored at the rate of β after refurbishment and $0 \le \beta \le 1$. This set-up can also reflect our observations from automobile industry. Since the refurbishment process of the automakers (e.g., Toyota) is essentially to replace some (but not all) used parts with new ones, the quality of the used parts that are not replaced will affect the quality of the refurbished products (Kalmowitz, 2022). Therefore, we assume that the quality of a refurbished product is $q(1-\delta+\beta\delta)$. The manufacturer also incurs a fixed cost F_r for implementing the refurbishment program. Finally, the refurbished products will be sold to consumers at price p_r in the second period.

Without loss of generality, we normalize the total number of consumers to 1. We assume that each consumer needs at most one unit of product in each period (Agrawal et al., 2016a; Vedantam et al., 2021) and that each consumer makes only one decision during each period (Van Ackere & Reyniers, 1995). Following the literature (see, e.g., Agrawal et al., 2016a; Biyalogorsky & Koenigsberg, 2014; Rao et al., 2009), we assume that heterogeneous consumers' taste θ for a product is uniformly distributed over [0,1]. Consumers can obtain *usage values* throughout both periods. Thus a consumer

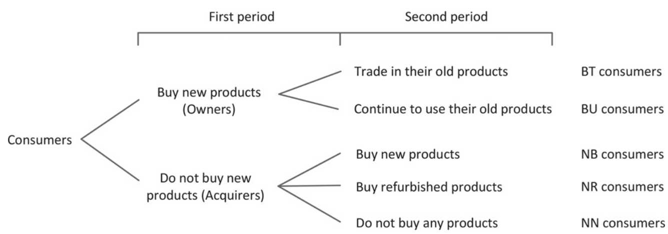


Fig. 1. Consumer behavior in the presence of trade-in and refurbishment programs.

with taste θ can gain usage value θq during the first period if she purchases a new product; her usage value is $\theta q(1-\delta)$ if she continues to use her old product during the second period, and it is θq if she trades in her old product. Also, a taste-θ consumer who has not bought a new product in the first period but who does buy a new (resp. refurbished) product in the second period gains usage value θq (resp. $\theta q(1-\delta+\beta\delta)$). For ease of exposition, we refer to consumers who purchase new products at the beginning of the first period as *owners* and to those who are unwilling to buy new products in the first period as *acquirers*.

Suppose that both the trade-in program and the refurbishment program have been implemented. At the beginning of the first period, consumers either buy or do not buy the new products. When the first period ends, consumers who have bought new products (i.e., the owners) either participate in the trade-in program (these are "BT" consumers) or continue to use the current old products (the "BU" consumers). At the same time, consumers who have not bought any new products (i.e., the acquirers) can either buy new products (thus becoming "NB" consumers), or buy refurbished products (thus becoming "NR" consumers) or exit the market without buying anything (the "NN" consumers). We normalize to zero any consumer's utility from an outside option. Consumer behavior is summarized in Fig. 1.

When purchasing automobiles, people are more likely to make strategic decisions. As the pandemic makes cars more expensive due to shortages of raw materials, George Hoffer suggests people might be better off waiting (see Tupponce, 2022). It follows that consumers are more cautious and "rational" or strategic about buying cars, not as impulsive and eager as they are about buying electronics (see Anchanto, 2022; Sha, 2022). Therefore, it is necessary to analyze the strategic behavior of heterogeneous consumers before solving the optimal decisions and formulating trade-in (and refurbishment) strategies for manufacturers. In other words, we consider each strategic consumer will take account of her utility across both periods at her first-period decision, and choose the option that gives her the largest surplus (Liu et al., 2019; Van Ackere & Reyniers, 1995).[5] It follows that the number of consumers under each option can be given endogenously. We use Q_B to denote the expected number of consumers who buy new products in the first period, Q_T the expected number of owners who participate in the trade-in program (if one is implemented), and Q_R the expected number of acquirers who buy refurbished products (if a refurbishment program is launched). In particular, we must have $Q_B \ge Q_T \ge Q_R$. These inequalities imply that the volume of refurbished products in the second period is limited by the trade-in amount, which in turn is limited by the volume of first-period sales. Moreover, we use Q_{NB} to denote the expected number of consumers who buy new products only in the second period.

The aim of our research is to discover under what circumstances the manufacturer should implement trade-in (and refurbishment) programs. Toward that end, we consider three models: Model N, in which neither the trade-in program nor the refurbishment program is implemented; Model T, where only the trade-in program is launched; and Model TR, for when both programs are implemented. We use the superscripts N, T, and TR to denote the three respective models. For each model, we consider a two-stage decision-making process for the manufacturer, and then followed by the consumers' strategic behavior. We now describe the sequence of events.

In Model N, in the first stage, the manufacturer decides on the new product's selling price p_{n1} for the first period; and in the second stage, he sets the new product's selling price p_{n2} for the second period. Then, strategic consumers make their purchase decisions while anticipating their utilities across the two periods.

In Model T, in the first stage, the manufacturer determines both the new product's selling price p_{n1} for the first period and the used product's trade-in rebate r; and in the second stage, the manufacturer determines the new product's selling price p_{n2} for the second period. Then, these choices are followed by the strategic consumers' purchase and trade-in decisions.

In Model TR, in the first stage, the manufacturer sets the optimal selling price p_{n1} for new products as well as the optimal trade-in rebate r for old products, and in the second stage, he sets the selling price p_{n2} for new products and the selling price p_r for refurbished products, to maximize his profit. Consumers make strategic purchase, trade-in, or exit decisions that maximize their utility across both periods and in accordance with the options illustrated in Fig. 1.

Our notation is summarized in Table 2.

4. Equilibrium analysis

The goal of this section is to discuss some measures that the manufacturer can adopt to increase his profit. More specifically, for each model we explore the manufacturer's optimal decisions by anticipating consumers' strategic behavior; these results are then used to show how the parameters affect manufacturer profits.

4.1. No program

Under the scenario that neither the trade-in program nor the refurbishment program is implemented (Model N). Because no trade-in progarm is offered, a consumer who has bought a new product in the first period has no choice but to continue using that product in the second period (thus becoming a BU consumer). Hence her utility across both periods is $U^{\mathrm{N}}_{\mathrm{BU}} = \theta q - p_{n1} + \theta q(1-\delta)$. However, consumers who have not bought new products in the first period cannot buy any refurbished products because this program is also not offered; but these consumers can either buy new products and obtain utility $U^{\mathrm{N}}_{\mathrm{NB}} = \theta q - p_{n2}$ or buy nothing thus obtain zero utility, i.e., $U^{\mathrm{N}}_{\mathrm{NN}} = 0$. When making her decision, a consumer compares the utility of the three options. Thus consumers will buy new products in the first period only if $U^{\mathrm{N}}_{\mathrm{BU}} \ge U^{\mathrm{N}}_{\mathrm{NB}}$ and $U^{\mathrm{N}}_{\mathrm{BU}} \ge U^{\mathrm{N}}_{\mathrm{NN}}$; consumers will buy new products in the second period only if $U^{\mathrm{N}}_{\mathrm{NB}} \ge U^{\mathrm{N}}_{\mathrm{BU}}$ and $U^{\mathrm{N}}_{\mathrm{NB}} \ge U^{\mathrm{N}}_{\mathrm{NN}}$; otherwise, she will exit the market.

Therefore, on the one hand, if the manufacturer adopts the markdown stategy, we focus on the nontrivial case that all options co-exist. So we obtain that the market demand for new products in both periods are $Q^{\mathrm{N}}_B = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$ and $Q^{\mathrm{N}}_{NB} = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}}{q}$. On the other hand, if the manufacturer does not adopt the markdown stategy, it is easy to verify that $U^{\mathrm{N}}_{\mathrm{BU}} \ge U^{\mathrm{N}}_{\mathrm{NB}}$. So we can ob-

[5] As defined in Bernstein & Martínez-de Albéniz (2017), "strategic consumers are forward looking and time their purchasing decision by anticipating changes in price or product value." Consumers' strategic behavior may be driven by different forces, for example, the product's price and stocking level (Kremer et al., 2017; Liu & Van Ryzin, 2008), product's value and technology level (Bernstein & Martínez-de Albéniz, 2017; Liu et al., 2019), consumers' strategic consideration when joining service systems (Guo & Zhang, 2013; Hu et al., 2022), etc.

Table 2
Notation.

Symbol	Description
c	Unit cost of manufacturing new products
q	Quality of new products
p_{n1}	Unit selling price of new products in the first period (decision variable)
p_{n2}	Unit selling price of new products in the second period (decision variable)
p_r	Unit selling price of refurbished products (decision variable)
r	Trade-in rebate (decision variable)
θ	Consumers' taste for quality ($0 \le \theta \le 1$)
δ	Depreciation rate of new products ($0 \le \delta \le 1$)
γ	Cost rate of refurbished products ($0 \le \gamma \le 1$)
β	Quality recovery rate of refurbished products ($0 \le \beta \le 1$)
Q_B	Number of consumers who buy new products during the first period
Q_T	Number of owners who trade in their old products during the second period
Q_R	Number of consumers who buy refurbished products during the second period
Q_{NB}	Number of consumers who buy new products only in the second period
U	Consumers' utility
π_m	Profit of the manufacturer

tain that the market demand for new products in both periods are $Q_B^{\mathrm{N}} = \frac{p_{n1}}{q(2-\delta)}$ and $Q_{NB}^{\mathrm{N}} = 0$.

Based on anticipating consumers' stategic behavior, the manufacturer sets the new product's selling prices for both periods, p_{n1} and p_{n2}, to maximize his profit π_m^{N}; formally,

$$\max_{p_{n1}, p_{n2}} \pi_m^{\mathrm{N}} = (p_{n1} - c)Q_B^{\mathrm{N}} + (p_{n2} - c)Q_{NB}^{\mathrm{N}}. \quad (1)$$

Lemma 1. *Let $\tilde{c}^{\mathrm{N}} = (2-\delta)q$. When $c \le \tilde{c}^{\mathrm{N}}$, the manufacturer sets his optimal selling price $p_{n1}^{\mathrm{N}*} = p_{n2}^{\mathrm{N}*} = \frac{c+(2-\delta)q}{2}$. Hence the market demand for new products in both periods are $Q_B^{\mathrm{N}*} = \frac{(2-\delta)q-c}{2(2-\delta)q}$ and $Q_{NB}^{\mathrm{N}*} = 0$. Accordingly, the manufacturer's profit is $\pi_m^{\mathrm{N}*} = \frac{((2-\delta)q-c)^2}{4q(2-\delta)}$.*

Lemma 1 gives a condition, $c \le \tilde{c}^{\mathrm{N}}$, that guarantees $Q_B^{\mathrm{N}*} \ge 0$. That is to say: in order to ensure non-negative market demand for a manufacturer's new products, their production costs cannot be too high. This condition ensures that the manufacturer is willing to produce and sell the new product; it makes sense because a manufacturer will always raise his selling prices to compensate for increased production costs, resulting in lower demand. Furthermore, our analysis shows that the manufacturer's optimal pricing strategy is to set the same selling price for the new product in both periods, i.e., $p_{n1}^{\mathrm{N}*} = p_{n2}^{\mathrm{N}*}$. That is, the manufacturer has no incentive to set different prices for each period. The reason is as follows. First, the manufacturer will not charge a higher price in period 2 (i.e., $p_{n2}^{\mathrm{N}*} > p_{n1}^{\mathrm{N}*}$), for otherwise no consumers will choose to wait. Second, if the manufacturer adopts a markdown strategy (i.e., $p_{n2}^{\mathrm{N}*} < p_{n1}^{\mathrm{N}*}$), some consumers may delay their purchases to period 2 for a lower price. However, such a waiting behavior will hurt the manufacturer's profit. Paradoxically, *the manufacturer actually does not lower the product price in the second period due to consumers' strategic waiting for a markdown*. As a consequence, the manufacturer will set the same selling price for the two periods to avoid the strategic waiting of consumers. This is similar to the price commitment policies that have been extensively studied in the existing literature (see, e.g., Lee et al., 2000; Liu et al., 2012; Taylor, 2001). It is worth noting that in reality, consumer strategy waiting may be caused by many factors (such as consumer irrationality, competition between firms, etc.), which is not within the scope of our current study.

Lemma 1 leads directly to our first proposition.

Proposition 1. *The effects of parameters c, q, and δ on the manufacturer's profit are as follows: $\frac{\partial \pi_m^{\mathrm{N}*}}{\partial c} \le 0, \frac{\partial \pi_m^{\mathrm{N}*}}{\partial q} \ge 0$, and $\frac{\partial \pi_m^{\mathrm{N}*}}{\partial \delta} \le 0$.*

If all else is held equal, then Proposition 1 has three implications. First, the increase in production costs c will lead to lower profits for manufacturers. Because manufacturers must raise their selling prices to cover increased production costs, demand for new products will fall and so manufacturer profits will suffer. Second, raising product quality q will increase manufacturer profits; this follows because the improvement of product quality can increase demand for the new product even at a higher selling price. Finally, the higher the depreciation rate δ, the lower the manufacturer's profit. We explain this result as follows. Under a high depreciation rate, strategic consumers are less willing to buy a new product because they expect it will deliver lower utility in the second period. Anticipating this response, the manufacturer must reduce the selling price so that his product will be more attractive to consumers. Yet our research establishes that reductions in the selling price cannot halt falling market demand for (rapidly depreciating) new products. As a result, the manufacturer's profit will inevitably decline. It is interesting that, as in Agrawal et al. (2016b), this conclusion runs counter to those who argue that monopolists should adopt a "planned obsolescence" strategy (see e.g., Bulow, 1982; Fishman & Rob, 2000; Waldman, 1996).

4.2. Trade-in program

Under the scenario that only the trade-in program is implemented (Model T), each consumer has four options, and she makes decisions while anticipating her utility across both periods. These options, and their respective utilities, are described as follows.

(i) If a consumer buys a new product in the first period and also participates in a trade-in program during the second period (a BT consumer), then her utility is $U_{\mathrm{BT}}^{\mathrm{T}} = \theta q - p_{n1} + \theta q - p_{n2} + r$.
(ii) If a consumer buys a new product in the first period and continues to use that product in the second period (a BU consumer), then she obtains utility $U_{\mathrm{BU}}^{\mathrm{T}} = \theta q - p_{n1} + \theta q(1-\delta)$.
(iii) If a consumer does not buy a new product in the first period but wait to buy a new product in the second period (a NB consumer), then she obtains utility $U_{\mathrm{NB}}^{\mathrm{T}} = \theta q - p_{n2}$.
(iv) If a consumer does not buy a new product in both periods and must therefore exit the market (an NN consumer), then her utility is $U_{\mathrm{NN}}^{\mathrm{T}} = 0$.

When making her final decision, the consumer once again compares the utility that she can derive from each option. Thus she selects option (i) only if $U_{\mathrm{BT}}^{\mathrm{T}} \ge U_{\mathrm{BU}}^{\mathrm{T}}$, $U_{\mathrm{BT}}^{\mathrm{T}} \ge U_{\mathrm{NB}}^{\mathrm{T}}$ and $U_{\mathrm{BT}}^{\mathrm{T}} \ge U_{\mathrm{NN}}^{\mathrm{T}}$; she selects option (ii) only if $U^{\mathrm{T}}_{\mathrm{BU}} \geq U^{\mathrm{T}}_{\mathrm{BT}}$, $U^{\mathrm{T}}_{\mathrm{BU}} \geq U^{\mathrm{T}}_{\mathrm{NB}}$ and $U^{\mathrm{T}}_{\mathrm{BU}} \geq U^{\mathrm{T}}_{\mathrm{NN}}$; she selects option (iii) only if $U^{\mathrm{T}}_{\mathrm{NB}} \geq U^{\mathrm{T}}_{\mathrm{BT}}$, $U^{\mathrm{T}}_{\mathrm{NB}} \geq U^{\mathrm{T}}_{\mathrm{BU}}$ and $U^{\mathrm{T}}_{\mathrm{NB}} \geq U^{\mathrm{T}}_{\mathrm{NN}}$. Otherwise, she selects option (iv).

Therefore, on the one hand, if the manufacturer adopts the markdown stategy, we focus on the nontrivial case where all options co-exist. Then, the expected number of consumers engaging in the trade-in program can be derived as $Q^{\mathrm{T}}_{T} = 1 - \frac{p_{n2}-r}{\delta q}$, the expected number of consumers who buy new products in the first period is $Q^{\mathrm{T}}_{B} = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$, and the expected number of consumers who buy new products in the second period is $Q^{\mathrm{T}}_{NB} = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}}{q}$. On the other hand, if the manufacturer does not adopt the markdown stategy, it is easy to verify that $U^{\mathrm{T}}_{\mathrm{BU}} \geq U^{\mathrm{T}}_{\mathrm{NB}}$. So we can obtain that the expected number of consumers engaging in the trade-in program $Q^{\mathrm{T}}_{T} = 1 - \frac{p_{n1}-r}{\delta q}$, as well as the expected number of consumers who buy new products in both periods are $Q^{\mathrm{T}}_{B} = 1 - \frac{p_{n1}}{q(2-\delta)}$ and $Q^{\mathrm{T}}_{NB} = 0$.

By anticipating consumers' stategic behavior, the manufacturer sets the new product's selling price for both periods, p_{n1} and p_{n2}, and the used product's trade-in rebate r to maximize his profit π^{T}_{m}:

$$\max_{p_{n1},p_{n2},r} \pi^{\mathrm{T}}_{m} = (p_{n1}-c)Q^{\mathrm{T}}_{B} + (p_{n2}-c-r)Q^{\mathrm{T}}_{T} + (p_{n2}-c)Q^{\mathrm{T}}_{NB} - F_t. \tag{2}$$

In this expression, the first term on the right-hand side corresponds to the manufacturer's profit from selling new products in the first period while the second term is the profit obtained from the second-period customers who trade in their old products. The third term refers to the manufacturer's profit from selling new products in the second period. Also, the manufacturer incurs a fixed cost F_t for implementing the trade-in program.

Lemma 2. *Let $\tilde{c}^{\mathrm{T}} = \delta q$. When $c \leq \tilde{c}^{\mathrm{T}}$, the manufacturer sets his optimal selling price $p^{\mathrm{T}*}_{n1} = p^{\mathrm{T}*}_{n2} = \frac{c+(2-\delta)q}{2}$ and trade-in rebate $r^{\mathrm{T}*} = q - \delta q$. We accordingly have the trade-in amount $Q^{\mathrm{T}*}_{T} = \frac{1}{2} - \frac{c}{2\delta q}$, as well as the market demand for new products in both periods are $Q^{\mathrm{T}*}_{B} = \frac{(2-\delta)q-c}{2(2-\delta)q}$ and $Q^{\mathrm{T}*}_{NB} = 0$. Accordingly, the manufacturer's profit is given by $\pi^{\mathrm{T}*}_{m} = \frac{c^2}{2\delta(2-\delta)q} + \frac{q-2c}{2} - F_t$.*

Lemma 2 gives a precondition, $c \leq \tilde{c}^{\mathrm{T}}$, that guarantees the inequalities $Q^{\mathrm{T}*}_{B} \geq Q^{\mathrm{T}*}_{T} \geq 0$. In other words, ensuring that a nonnegative number of consumers participate in the manufacturer's implemented trade-in program requires that the production cost of the manufacturer's new product should not be too high. The reason is that manufacturers will raise their new products' selling prices to compensate for increased production costs, which reduces consumers' willingness to engage with a trade-in program. Hence that program's successful implementation can be guaranteed only if $c \leq \tilde{c}^{\mathrm{T}}$. Here, we should point out that $\tilde{c}^{\mathrm{T}} \leq \tilde{c}^{\mathrm{N}}$; this inequality indicates that, if the manufacturer wants to implement a trade-in program, he must reduce production costs. For instance, Subaru, Tesla, and BWM who can improve their cost efficiency are qualified to offer trade-in programs. Whereas manufacturers whose production costs are below the threshold ($\tilde{c}^{\mathrm{T}}$) might not be willing to offer a trade-in program, those for whom production costs are above that threshold will definitely not offer any trade-in programs. Again, Lemma 2 shows that the manufacturer will set the same selling price for the two periods to avoid the strategic waiting of consumers.

Lemma 2 leads to our next proposition, as follows.

Proposition 2. *The effects of parameters c, q, and δ on the manufacturer's profit are: $\frac{\partial \pi^{\mathrm{T}*}_{m}}{\partial c} \leq 0, \frac{\partial \pi^{\mathrm{T}*}_{m}}{\partial q} \geq 0$, and $\frac{\partial \pi^{\mathrm{T}*}_{m}}{\partial \delta} \leq 0$.*

According to this proposition, the manufacturer's profit increases with improvements in product quality q but decreases with any increase in the production cost c or the depreciation rate δ – outcomes that are consistent with Proposition 1. In essence, the manufacturer's profit after implementing a trade-in program are still realized through the sales of new products. So whether or not the manufacturer does adopt a trade-in strategy, the influence of production costs and product quality on his profit remains unchanged.

An implication of perhaps greater interest concerns the relation between the manufacturer's profit and the depreciation rate δ. As the depreciation rate increases, products yield lower usage values in the second period. On the one hand, consumers who recognize this dynamic will forgo buying new products in the first period and thus reduce the manufacturer's profit from selling new products (a *negative* effect). On the other hand, more owners will choose to participate in trade-in programs rather than continue to use their old products, which results in more profits from the implementation of the trade-in program (a *positive* effect). One would therefore expect the manufacturer's profit to be non-monotonically related to the depreciation rate. Yet we establish the surprising result that the former (negative) effect always dominates the latter (positive) effect, so that the manufacturer's profit is monotonically decreasing in the depreciation rate. We stress that this result reflects the strategic behavior of consumers, who – so as not to be exploited by the manufacturer – make decisions at the beginning of the first period that account for their total utility across both periods. The management implication here is that although automobile manufacturers (e.g., Subaru, Tesla, BWM) can gain profit from offering trade-in programs, selling low-durability products will result in a profit loss for them since more consumers forgo buying new products. This suggests that manufacturers would be better off improving the durability of their products.

4.3. Trade-in and refurbishment programs

We now address the scenario in which both the trade-in program and the refurbishment program are implemented (Model TR). Consumers continue to make decisions while anticipating their utility across both periods. The consumers' five options and corresponding utilities are summarized as follows.

(i) If a consumer buys a new product in the first period and also participates in a trade-in program during the second period (a BT consumer), then her utility is $U^{\mathrm{TR}}_{\mathrm{BT}} = \theta q - p_{n1} + \theta q - p_{n2} + r$.
(ii) If a consumer buys a new product in the first period and continues to use that product in the second period (a BU consumer), then she obtains utility $U^{\mathrm{TR}}_{\mathrm{BU}} = \theta q - p_{n1} + \theta q(1-\delta)$.
(iii) If a consumer does not buy a new product in the first period but wait to buy a new product in the second period (a NB consumer), then she obtains utility $U^{\mathrm{TR}}_{\mathrm{NB}} = \theta q - p_{n2}$.
(iv) If a consumer does not buy a new product in the first period but does buy a refurbished product in the second period (an NR consumer), then her utility is $U^{\mathrm{TR}}_{\mathrm{NR}} = \theta q(1-\delta+\beta\delta) - p_r$.
(v) If a consumer does not buy a new product in both periods and exits the market (an NN consumer), then her utility is $U^{\mathrm{TR}}_{\mathrm{NN}} = 0$.

Again, each consumer compares the utility that she can derive from each option to make her final decision. Therefore, on the one hand, if the manufacturer adopts the markdown stategy, we focus on the nontrivial case where all options co-exist. Then, we have the number of consumers who participate in the trade-in program is $Q^{\mathrm{TR}}_{T} = 1 - \frac{p_{n2}-r}{\delta q}$; the expected number of consumers who buy new products in both periods are $Q^{\mathrm{TR}}_{B} = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$ and $Q^{\mathrm{TR}}_{NB} = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}-p_r}{\delta q(1-\beta)}$; also, the number of consumers who buy refurbished products in the second period is $Q_R^{\mathrm{TR}} = \frac{p_{n2}-p_r}{\delta q(1-\beta)} - \frac{p_r}{q-(1-\beta)\delta q}$. The remaining consumers exit the market. On the other hand, if the manufacturer does not adopt the markdown stategy, it is easy to verify that $U_{\mathrm{BU}}^{\mathrm{TR}} \geq U_{\mathrm{NB}}^{\mathrm{TR}}$. So we can obtain that the number of consumers who participate in the trade-in program is $Q_T^{\mathrm{TR}} = 1 - \frac{p_{n1}-r}{\delta q}$; the expected number of consumers who buy new products in both periods are $Q_B^{\mathrm{TR}} = 1 - \frac{p_{n1}-p_r}{q-\beta\delta q}$ and $Q_{NB}^{\mathrm{TR}} = 0$; also, the number of consumers who buy refurbished products in the second period is $Q_R^{\mathrm{TR}} = \frac{p_{n1}-p_r}{q-\beta\delta q} - \frac{p_r}{q-(1-\beta)\delta q}$. The remaining consumers exit the market.

In Model TR, the manufacturer's profit function is given by

$$\pi_m^{\mathrm{TR}} = (p_{n1}-c)Q_B^{\mathrm{TR}} + (p_{n2}-c-r)Q_T^{\mathrm{TR}} + (p_{n2}-c)Q_{NB}^{\mathrm{TR}} + (p_r-\gamma c)Q_R^{\mathrm{TR}} - F_t - F_r. \tag{3}$$

As before, the first term on the RHS refers to the manufacturer's profit from selling new products in the first period while second term is his profit from the second period's trade-in consumers. The third term is the profit from selling new products in the second period and the forth term is the profit from selling refurbished products. The manufacturer also incurs the fixed costs F_t and F_r of implementing, respectively, the trade-in and refurbishment programs. The manufacturer determines the prices p_{n1}, p_{n2} and p_r and trade-in rebate r, to maximize his profit.

Lemma 3. *Put* $\tilde{c}^{\mathrm{TR}} = \frac{\delta q(1-(1-\beta)\delta)(1-\beta\delta)}{1-(1-\beta)^2\delta^2-\gamma(2-\delta)\delta}$, $\bar{\gamma} = \frac{\beta\delta+\delta-1}{\delta}$, *and* $\bar{\bar{\gamma}} = \frac{1+\beta\delta-\delta}{2-\delta}$. *When* $c \leq \tilde{c}^{\mathrm{TR}}$ *and* $\bar{\gamma} \leq \gamma \leq \bar{\bar{\gamma}}$, *the manufacturer sets his optimal selling price* $p_{n1}^{\mathrm{TR}*} = p_{n2}^{\mathrm{TR}*} = \frac{c+(2-\delta)q}{2}$, *trade-in rebate* $r^{\mathrm{TR}*} = q-\delta q$, *and the refurbished product's optimal selling price* $p_r^{\mathrm{TR}*} = \frac{c\gamma-(1-\beta)\delta q+q}{2}$. *Then, we have the trade-in amount is* $Q_T^{\mathrm{TR}*} = \frac{1}{2} - \frac{c}{2\delta q}$, *the market demand for new products in both periods are* $Q_B^{\mathrm{TR}*} = \frac{1}{2} - \frac{c-c\gamma}{2(q-\beta\delta q)}$ *and* $Q_{NB}^{\mathrm{TR}*} = 0$, *as well as the market demand for refurbished products is* $Q_R^{\mathrm{TR}*} = \frac{c(\delta(\beta+\gamma-1)-2\gamma+1)}{2q(1-(1-\beta)\delta)(1-\beta\delta)}$. *Accordingly, the manufacturer's profit is given by* $\pi_m^{\mathrm{TR}*} = \frac{q-2c}{2} + \frac{c^2\left(1-(1-\beta-\gamma)^2\delta^2-2(1-\gamma)\gamma\delta\right)}{4\delta q(1-(1-\beta)\delta)(1-\beta\delta)} - F_t - F_r$.

To guarantee that $Q_B^{\mathrm{TR}*} \geq Q_T^{\mathrm{TR}*} \geq Q_R^{\mathrm{TR}*} \geq 0$, Lemma 3 stipulates conditions related to the production cost of a new product ($c \leq \tilde{c}^{\mathrm{TR}}$) and the cost rate of refurbishing a old product ($\bar{\gamma} \leq \gamma \leq \bar{\bar{\gamma}}$). Since we can easily verify that the refurbished product's selling price increases with the refurbishment cost rate γ, it follows that the market demand for refurbished products decreases. So in order to ensure a non-negative market demand for refurbished products (i.e., $Q_R^{\mathrm{TR}*} \geq 0$), the refurbishment cost rate should not be too high (i.e., $\gamma \leq \bar{\bar{\gamma}}$). At the same time, the refurbishment cost rate should not be too low ($\gamma \geq \bar{\gamma}$) in order to preclude the case where demand for new products is less than the number of consumers willing to trade in their old products ($Q_B^{\mathrm{TR}*} \geq Q_T^{\mathrm{TR}*}$) – a situation that arises when more consumers choose to buy refurbished rather than new products. And since the market demand for refurbished products should not exceed the trade-in amount ($Q_T^{\mathrm{TR}*} \geq Q_R^{\mathrm{TR}*}$), we conclude that the production cost for new products cannot be too high ($c \leq \tilde{c}^{\mathrm{TR}}$). The reason is that the new products' increased selling price, which is due to its increased production cost, will induce more consumers to buy refurbished products instead of new ones.

It is worth noting that $\tilde{c}^{\mathrm{TR}} \leq \tilde{c}^{\mathrm{T}}$ ($\leq \tilde{c}^{\mathrm{N}}$). These inequalities suggest that a manufacturer who (has already offered a trade-in program) wishes to continue offering refurbishment programs must has the ability to further reduce the production costs of new products. The management implication here is that the manufacturer with the best control over production costs is uniquely positioned to continue offering refurbishment programs in addition to the trade-in programs already provided. In short: firms whose production costs are below the threshold ($\tilde{c}^{\mathrm{TR}}$) may or may not be willing to offer both trade-in and refurbishment programs; but firms whose production costs are above that threshold should definitely not offer both programs.

Furthermore, Lemma 3 once again shows that the manufacturer has no incentive to set different prices for each period. Lemma 3 allows us to derive directly the effects of parameters c, q, δ, γ, and β on the manufacturer's profit, as shown in our next proposition.

Proposition 3. *If* $\beta \leq \beta_1$ *and* $\gamma \leq \gamma_1$, *then* $\pi_m^{\mathrm{TR}*}$ *first decreases and then increases with c. Otherwise,* $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} \leq 0$. *Moreover,* $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q} \geq 0$, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta} \leq 0$, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \gamma} \leq 0$, *and* $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \beta} \geq 0$.

It is intuitive to suppose that, as the cost c of producing a new product increases, the manufacturer will sell fewer new products at a lower unit sales revenue (in each of the two periods), so the manufacturer's profit always decreases with an increase in production cost. Our research confirms that this is indeed the case when the manufacturer does not offer a refurbishment program (see Propositions 1 and 2). However, Proposition 3 shows that this commonly held supposition may not hold when the manufacturer does provide refurbishment programs. It is interesting that, when both the refurbishment cost rate and quality recovery rate of refurbished products are low, the manufacturer's profit first decreases but then increases with increasing production costs.

We interpret this result as follows. As the production cost of new products increases, more consumers will become unwilling to buy new products and so will turn to their refurbished counterparts. Although manufacturers will respond by raising their price, consumers will continue being strongly inclined to buy refurbished products – which, in turn, eventually increases manufacturer profits from the sale of refurbished products. This positive incentive explains why the manufacturer's profit does not vary (decrease) monotonically with the production cost. It should be emphasized that these results are facilitated by our condition that both the refurbishment cost rate and quality recovery rate of refurbished products are not too high, so that manufacturers will neither bear a high production cost for refurbished products nor lose too much market demand for its new products because of competition from refurbished products. For management, these considerations imply that firms (e.g., Toyota) can cope with the loss of profits caused by higher production costs of new products by implementing refurbishment programs and successfully aiming for a lower refurbishment cost rate and quality recovery rate of refurbished products.

Once again, the manufacturer's profit decreases as the quality depreciation rate δ increases. This relation suggests that, even if the manufacturer (e.g., Toyota) also adopts refurbishment programs (on a trade-in basis) to increase his revenue sources, that does *not* change how the quality depreciation rate affects his profit. We reiterate that this outcome is due to consumers' strategic behavior: because their decisions at the start of the first period are made with future product quality in mind, refurbished products of low quality are unattractive to them. Finally, our results also show that the manufacturer's profit decreases with an increase in the refurbishment cost rate γ of refurbished products and increases with improvements in the quality q of new products and in the quality recovery rate β of refurbished products.

Our results suggest several ways for automobile firms (e.g., Toyota) to improve profits when implementing both trade-in and refurbishment programs. First, produce new products of higher quality and greater durability. Second, improve the quality recovery rate of refurbished products and reduce the corresponding cost rate. Finally, firms should always seek to reduce the cost of new product production – *unless* (a) the quality recovery rate of refurbished products and the corresponding cost rate are both low and (b) the production cost of new products is already high enough for the firm.

5. Comparison

We now compare the three models to investigate how the trade-in program and refurbishment program affect not only the manufacturer's decisions and profit but also consumer demand.

5.1. The manufacturer's preference

Proposition 4. *Comparing Model T and Model N yields the following statements.*

(i) $p_{n1}^{T*} = p_{n1}^{N*}, r^{T*} \geq r^{N*}, p_r^{T*} = p_r^{N*}, Q_B^{T*} = Q_B^{N*}, Q_T^{T*} \geq Q_T^{N*}$, *and* $Q_R^{T*} = Q_R^{N*}$.

(ii) *When* $F_t \geq \delta q/4$, *we have* $\pi_m^{T*} \leq \pi_m^{N*}$. *Yet when* $F_t \leq \delta q/4$, *we have* $\pi_m^{T*} \geq \pi_m^{N*}$ *if* $c \leq c_2$; *else, if* $c \geq c_2$ *then* $\pi_m^{T*} \leq \pi_m^{N*}$.

Part (i) of this proposition shows that, although manufacturers can promote the sale of new products in the second period and remain profitable if they offer trade-in programs, the cost of this approach is the need to pay trade-in rebates to consumers who recycle old products (i.e., $r^{T*} \geq r^{N*}$ and $Q_T^{T*} \geq Q_T^{N*}$). It follows that the manufacturer does not always profit from a trade-in program, especially when one considers that he also incurs a fixed cost F_t for implementing it. So when that fixed cost is high (i.e., $F_t \geq \delta q/4$), the manufacturer will have no interest in implementing a trade-in program (i.e., $\pi_m^{T*} \leq \pi_m^{N*}$). Yet when the fixed cost is low (i.e., $F_t \leq \delta q/4$), Proposition 4(ii) establishes that manufacturers will (resp., will not) implement trade-in programs if the production cost of new products is relatively low (resp., high). From a management perspective, manufacturers with lower production costs for new products are more likely to offer trade-in programs. The reason is that lower production costs help manufacturers preserve market demand by making it unnecessary to raise prices to cover production costs. Reducing production costs ultimately allows the manufacturer's profits to outweigh the fixed and variable costs associated with implementing a trade-in program.

Proposition 5. *Comparing Model TR and Model T yields the following statements.*

(i) $p_{n1}^{TR*} = p_{n1}^{T*}, r^{TR*} = r^{T*}, p_r^{TR*} \geq p_r^{T*}, Q_B^{TR*} \leq Q_B^{T*}, Q_T^{TR*} = Q_T^{T*}$, *and* $Q_R^{TR*} \geq Q_R^{T*}$.

(ii) *When* $F_r \geq F_{r,1}$, *we have* $\pi_m^{TR*} \leq \pi_m^{T*}$. *Yet when* $F_r \leq F_{r,1}$, *we have* $\pi_m^{TR*} \leq \pi_m^{T*}$ *if* $c \leq c_3$; *else, if* $c \geq c_3$ *then* $\pi_m^{TR*} \geq \pi_m^{T*}$.

Manufacturers who implement refurbishment programs on a trade-in basis encounter two opposite effects. On the one hand, manufacturers can increase their revenue streams by implementing refurbishment programs (since $p_r^{TR*} \geq p_r^{T*}$ and $Q_R^{TR*} \geq Q_R^{T*}$). On the other hand, the encroachment of refurbished products on new products will cause manufacturers to lose some new product sales revenue at the start of the first period (since $p_{n1}^{TR*} = p_{n1}^{T*}$ and $Q_B^{TR*} \leq Q_B^{T*}$). As shown by Proposition 5(ii), the trade-off between these two resulting revenue streams makes it less necessary for manufacturers to carry out refurbishment programs.

We must bear in mind that there is also a fixed cost F_r for implementing the refurbishment program. So when this fixed cost is high (i.e., $F_r \geq F_{r,1}$), the manufacturer will certainly *not* implement a refurbishment program (i.e., $\pi_m^{TR*} \leq \pi_m^{T*}$). But when the fixed cost is low (i.e., $F_r \leq F_{r,1}$), the manufacturer will (resp., will not) carry out refurbishment programs when the production cost of the new product is relatively high (resp., low). These outcomes reflect that refurbishment programs allow manufacturers to rely less on the sales of new products for their profits and thus to be less adversely affected by increases in their production costs. The management implication here is that a manufacturer who has already offered trade-in programs can improve his situation by implementing refurbishment programs if his production costs for new products is high.

Proposition 6. *Comparing Model TR and Model N yields the following statements.*

(i) $p_{n1}^{TR*} = p_{n1}^{N*},\ r^{TR*} \geq r^{N*},\ p_r^{TR*} \geq p_r^{N*},\ Q_B^{TR*} \leq Q_B^{N*},\ Q_T^{TR*} \geq Q_T^{N*}$, *and* $Q_R^{TR*} \geq Q_R^{N*}$.

(ii) *When* $F_t + F_r \geq \delta q/4$, *we have* $\pi_m^{TR*} \leq \pi_m^{N*}$, *and when* $F_t + F_r \leq F_{t,r}$, *we have* $\pi_m^{TR*} \geq \pi_m^{N*}$. *Yet when* $F_{t,r} \leq F_t + F_r \leq \delta q/4$, *we have* $\pi_m^{TR*} \geq \pi_m^{N*}$ *if* $c \leq c_4$; *else, if* $c \geq c_4$ *then* $\pi_m^{TR*} \leq \pi_m^{N*}$.

Proposition 6 (i) illustrates the impact of both trade-in and refurbishment programs on manufacturer profits. First, implementing a trade-in program benefits the manufacturer by boosting sales of new products in the second period – albeit at the expense of paying consumers a trade-in rebate to recycle old products (i.e., $r^{TR*} \geq r^{N*}$ and $Q_T^{TR*} \geq Q_T^{N*}$). Second, implementing refurbishment programs enables manufacturers to profit from the sale of refurbished products (since $p_r^{TR*} \geq p_r^{N*}$ and $Q_R^{TR*} \geq Q_R^{N*}$). Third, implementing a refurbishment program could cannibalize the manufacturer's new product sales at the start of the first period (since $p_{n1}^{TR*} = p_{n1}^{N*}$ and $Q_B^{TR*} \leq Q_B^{N*}$). The combination of these three factors makes it inadvisable for the manufacturer to implement both programs.

By Proposition 6(ii), if the sum of the fixed costs, $F_t + F_r$, of providing the two programs is relatively low (resp., high) then the manufacturer will (resp., will not) offer both programs. Yet if the sum of the fixed costs is moderate (i.e., $F_{t,r} \leq F_t + F_r \leq \delta q/4$), then the manufacturer will (resp., will not) implement both programs when the production cost of new products is low (resp., high). This follows because lower production costs for new products render manufacturers more willing to sell new and refurbished products at a lower price. Thus the manufacturer can ensure that the profit when implementing the two programs will offset his costs. In this case, the management implication is that only manufacturers who can reduce the production cost of new products should offer both trade-in and refurbishment programs.

Now suppose $F_r \leq F_{r,1}$ and $F_{t,r} \leq F_t + F_r \leq \delta q/4$, so that none of our three models is completely dominated by either of the others. In this case, we can summarize the results proposed in Propositions 4–6 as follows.

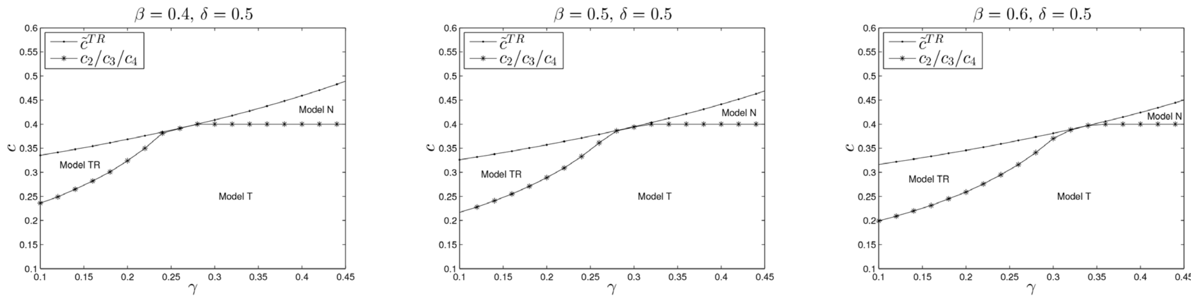
Proposition 7.

(i) *If* $c \leq \min\{c_2, c_3\}$, *then the manufacturer prefers Model T.*

(ii) *If* $c_3 \leq c \leq c_4$, *then the manufacturer prefers Model TR.*

(iii) *If* $c \geq \max\{c_2, c_4\}$, *then the manufacturer prefers Model N.*

Proposition 7 gives manufacturers' preferences for trade-in (and refurbishment) programs in different situations. In particular: manufacturers prefer to offer only trade-in programs when the production cost of new products is low, to offer both trade-in and refurbishment programs when that production cost is moderate, and to offer neither program when the production cost is high. This conclusion can be used to explain the different practices we have observed from the automobile industry. In recent years, due to the improvement of the core technology of automobile production (such as battery or fuel cell technology, lightweight structure), the production cost of automobile can be reduced (Peters et al., 2014). That is why many automobile manufacturers (e.g., Subaru, Tesla, BWM, Toyoto) can start to offer trade-in programs. However, the pandemic has resulted in a shortage of automobile semiconductor chips, increasing production costs for many automakers, such as Toyota (see Bellwood, 2022). That may be one reason why Toyota starts offering refurbishment programs on a trade-in basis. However, due to the high investment cost of providing refurbish-



(a) (b) (c)

Fig. 2. Effects of quality recovery rate β on manufacturer's preference.

Table 3
Parameter values.

Parameter	Description	Value
q	The quality of a new automobile	1
c	The production cost of new automobiles	$c \in [0.1, 0.6]$
γ	The cost rate of refurbishing automobiles	$\gamma \in [0.1, 0.45]$
δ	Product depreciation rate	0.5
β	Quality recovery rate	0.5
F_t	The fixed costs of implementing trade-in programs for automobiles	0.005
F_r	The fixed costs of implementing refurbishment programs for automobiles	0.005

ment programs, not all manufacturers can afford to offer such a programs.

The implication for management is that manufacturers should carefully choose whether to implement trade-in and refurbishment programs – that is, while accounting for the production cost of new products and other relevant factors, since the threshold value of that production cost can be influenced by a host of parameter values. Therefore, we conduct a numerical study to show more directly the various preferences of manufacturers.

5.2. Numerical study

Before performing the numerical study, we have the following considerations. First, the quality of a new automobile should be normalized to one for simplicity. Second, in order to ensure that manufacturers are willing to produce new automobiles (as well as offer trade-ins and refurbishment programs), the production cost of new automobiles should not be too high, which is the so-called low cost manufacturing (see, e.g., Tisza & Czinege, 2018; Tisza et al., 2017). Third, to ensure that the automobile manufacturer is willing to offer the refurbishment program, we need to have the cost rate of refurbishing automobiles neither too high nor too low. Forth, we consider a new automobile can be used for two periods and will be worthless at the end of the second period (Agrawal et al., 2012; Li & Xu, 2015). Fifth, we consider the quality of a refurbished automobile can reach a certain quality standard and will be lower than the quality standard of new products (Chen et al., 2020). Finally, in order to make any one of the three models has a chance to be the most preferred, we need to have the fixed costs of implementing trade-in and refurbishment programs for automobiles in the middle. Therefore, we set the parameter values accordingly and summarized them in Table 3.

Next, we will examine how the quality recovery rate β and the depreciation rate δ affect the automobile manufacturer's preference. Specifically, we vary β within the range [0.4,0.6] when analyzing the impact of the quality recovery rate β; to check for effects of the depreciation rate δ, we vary δ within the range [0.4,0.6].

Figs. 2 and 3 illustrate how the different preferences of manufacturers vary with the quality recovery rate β of refurbished products and the quality depreciation rate δ of new products. These figures verify the theoretical results proposed in Propositions 4–7 and also convey several more implications. First, when the production cost of new products and the cost rate of refurbished products are both high, manufacturers prefer not to provide either a trade-in program or a refurbishment program (Model N). Second, when the production cost of new products is low or when the production cost of new products is moderate and the cost rate of refurbished products is high, manufacturers prefer to offer only trade-in programs (Model T). Third, when the production cost of new products is moderate and the cost rate of refurbished product is low, manufacturers prefer to offer both trade-in and refurbishment programs (Model TR). This result suggests that although many automakers have began to offer trade-in programs (such as Subaru, Tesla, BWM, Toyoto), only those manufacturers with relatively high production costs but relatively low refurbishment costs (such as Toyota) have more incentive to offer additional refurbishment programs. Here, the refurbishment cost is another reason for the variation in automotive industry practices.

We can make several other observations with reference to Fig. 2. As the quality recovery rate β of refurbished products increases, the region where the manufacturer has the ability to offer both trade-in and refurbishment programs becomes smaller while the region where he prefers to offer both trade-in and refurbishment programs becomes larger. In addition, the regions associated with manufacturers' preferences for trade-in programs only and for declining to offer either program will be smaller. The practical implication here is that improved quality recovery rates of refurbished products do *not* encourage manufacturers to offer trade-in programs but *do* encourage those who already have a trade-in program to offer a refurbishment program as well. In the automobile industry, as refurbishment technology improves the quality recovery rate of refurbished products, more firms (which already offer trade-in programs) will be willing to offer another refurbishment programs like Toyota's.

Finally, a close observation of Fig. 3 reveals that, with an increase in the quality depreciation rate δ of new products, the region in which manufacturers are likely to provide both trade-in and refurbishment programs will expand, as will the region in

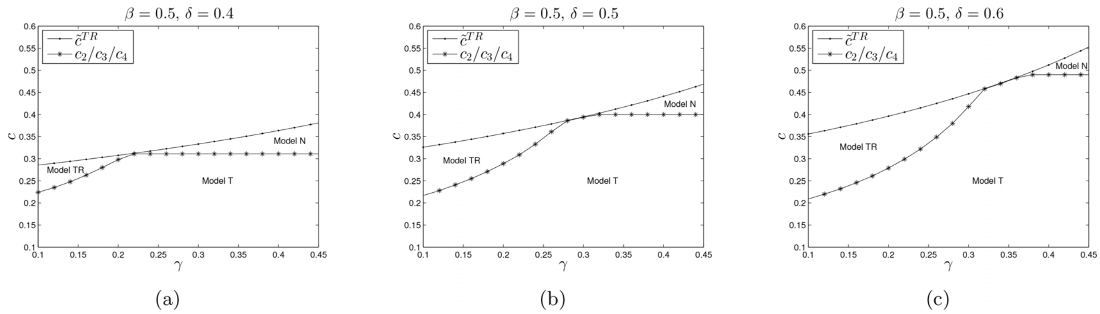


Fig. 3. Effects of depreciation rate δ on manufacturer's preference.

which manufacturers prefer to offer both programs. In addition, the region where manufacturers prefer to offer only trade-in programs becomes larger whereas the region where manufacturers prefer to offer neither program becomes smaller. The practical implication under these circumstances is that an increase in the depreciation rate of new products will not only induce manufacturers to implement trade-in programs; it will also encourage the adoption of refurbishment programs by those who are already offering trade-in programs. Therefore, as people use their cars more often, causing the depreciation rate to increase, it will also encourage more firms that have already implemented trade-in programs to offer refurbishment programs.

6. Conclusion

The goals of this research are (i) to determine when a manufacturer with strategic consumers should implement trade-in (and refurbishment) programs for durable goods and (ii) to show how this manufacturer can do so to improve his profit. We develop an analytical model in which a manufacturer sells new products to heterogeneous consumers in two periods and can also offer a trade-in program to collect old products at the end of the first period. The collected old products can be refurbished and resold to consumers in the second period. We address our study's goals by way of three models (no program, trade-in program only, trade-in and refurbishment programs both) that we describe, analyze, and compare.

Our principal results can be summarized as follows. First, manufacturers can always increase their profits by improving the quality of new products and thereby reducing the rate of quality depreciation, regardless of whether or not they offer trade-in (and refurbishment) programs. As described in the main text, this conclusion contradicts the apologists for planned obsolescence. Second, when the manufacturer does (resp., does not) offer a refurbishment program, his profit need not (resp., must) decrease with any increase in the production cost of new products. Besides, if the manufacturer implements a refurbishment program then his profit decreases with an increase in the refurbishment cost rate but increases with the quality recovery rate of refurbished products. Third, previous studies suggest that the firm should offer a trade-in program if it faces a competition from a third-party remanufacturer (Agrawal et al., 2016a), if it can take full advantage of forward-looking customer behavior (Zhang & Zhang, 2018), or if it has a high market share of old customers (Xiao et al., 2020a). We complement this research by showing that the manufacturer prefers to offer only trade-in programs when the production cost of new products is low, to offer both trade-in and refurbishment programs when that production cost is moderate, and to provide neither program when the production cost is high. Finally, our numerical study shows that improving the quality recovery rates of refurbished products does not encourage manufacturers to implement trade-in programs – though it does encourage the adoption of refurbishment programs by manufacturers who are already offering trade-in programs. However, an increase in the depreciation rate of new products will motivate manufacturers to offer trade-in programs while also encouraging the adoption of refurbishment programs by those who already offer trade-in programs.

The features we consider in our model, such as the decline of product quality over time and trade-in and refurbishment programs, also apply to the electronics industry. Therefore, although our model is based mainly on the automobile industry, under certain conditions (e.g., when the production cost of new products is moderate and the refurbishment cost of old products is low), manufacturers prefer to choose the TR model – providing trade-in and refurbishment services, which concurs with the practice of electronics industry.

This paper has the following main limitations. First, our model only describes the situation where the old products are at the same quality level. In reality, the old products may have multiple quality levels. Thus, it is important to establish a model that can capture different quality levels of old products. Second, we do not address how the manufacturer's optimal operating strategy is affected by (say) leasing new cars to consumers and then refurbishing them after a certain period of time. Third, in recent years, the pandemic has caused a shortage of raw materials to produce automobiles. Thus, it is valuable to consider not only the uncertainty of component supply in assembly problem, but also the imbalance between supply and demand of products. Forth, the shortage of raw materials will lead to different levels of product availability and different waiting times for consumers. We conjecture that due to supply uncertainty, consumers might be more or less willing to wait strategically. This depends on how the supply uncertainty affects the production of the two periods differently. For example, if the level of uncertainty is higher in the second period, it will discourage consumers from waiting strategically. This would reinforce the waiting behavior in the current model and results. Moreover, it is worth studying how the manufacturer allocates limited raw materials among different types of products. Competition between manufacturers would further intensify and complicate supply uncertainty.

Acknowledgement

The authors would like to thank the Editor Michael Yearworth and the review team for their constructive comments. This research was supported in part by National Natural Science Foundation of China (No. 71901197, 72172167, 71721001).

Appendix A. Proofs

Proof of Lemma 1. Under the scenario that neither the trade-in program nor the refurbishment program is implemented (Model N), we will discuss two cases as follows.

Case 1. The manufacturer adopts the markdown stategy. In the first stage, the manufacturer sets the new product's selling price in the first period p_{n1}; and in the second stage, he decides the new product's selling price in the second period p_{n2}, to maximize his profit $\pi_m^N = (p_{n1}-c)Q_B^N + (p_{n2}-c)Q_{NB}^N$ subject to $Q_{NB}^N \geq 0$, in which $Q_B^N = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$, and $Q_{NB}^N = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}}{q}$. We solve this model by backward induction.

In the second stage, to solve the constrained maximum problem for the manufacturer, we construct the following Lagrangian function

$$L^N(p_{n2},\lambda) = -(p_{n1}-c)Q_B^N - (p_{n2}-c)Q_{NB}^N - \lambda Q_{NB}^N, \tag{EC.1}$$

in which λ is the Lagrange multiplier. So the KKT conditions are

$$\begin{cases} \frac{\partial L^N}{\partial p_{n2}} = \frac{-c\delta+c+(\delta-2)\lambda+2p_{n1}+2(\delta-2)p_{n2}}{(\delta-1)q} = 0, \\ -Q_{NB}^N \leq 0, \\ \lambda \geq 0, \\ \lambda Q_{NB}^N = 0. \end{cases} \tag{EC.2}$$

Then, we indentify the manufacturer's selling prices for both new products and refurbished products, by discussing two subcases as follows.

Case 1a. If $\lambda = 0$, then from the KKT condition, Eq. (EC.9), we have

$$p_{n2}^N(p_{n1}) = \frac{2p_{n1} - c\delta + c}{4-2\delta}. \tag{EC.3}$$

In the first stage, by substituting Eq. (EC.3) into the profit function π_m^N, the manufacturer sets p_{n1} to maximize his profit. Since $\frac{\partial^2\pi_m^N}{\partial p_{n1}^2} = -\frac{2}{(2-\delta)q} \leq 0$, so the profit function π_m^N is concave in p_{n1}. Hence the unique optimal selling price p_{n1}^{N1*} can be obtained by solving the first-order condition $\frac{\partial\pi_m^N}{\partial p_{n1}} = 0$. Thus we obtain $p_{n1}^{N1*} = \frac{c+(2-\delta)q}{2}$. We accordingly have the market demand for new products in the second period is $Q_{NB}^{N1*} = -\frac{c}{2q} \leq 0$. As a result, this case is eliminated.

Case 1b. If $\lambda \neq 0$, then from the KKT condition, Eq. (EC.2), we have

$$\begin{cases} p_{n2}^N(p_{n1}) = \frac{p_{n1}}{2-\delta}, \\ \lambda = c - \frac{c}{2-\delta}. \end{cases} \tag{EC.4}$$

In the first stage, by substituting $p_{n2}^N(p_{n1})$ into the profit function π_m^N, the manufacturer sets p_{n1} to maximize his profit. Since $\frac{\partial^2\pi_m^N}{\partial p_{n1}^2} = -\frac{2}{(2-\delta)q} \leq 0$, so the profit function π_m^N is concave in p_{n1}. Hence the unique optimal selling price p_{n1}^{N1*} can be obtained by solving the first-order condition $\frac{\partial\pi_m^N}{\partial p_{n1}} = 0$. Thus we obtain $p_{n1}^{N1*} = \frac{c+(2-\delta)q}{2}$. We accordingly have the manufacturer's profit $\pi_m^{N1*} = \frac{((2-\delta)q-c)^2}{4q(2-\delta)}$.

Case 2. The manufacturer does not adopt the markdown stategy.

In this case, the manufacturer sets the new product's selling price in the first period p_{n1} to maximize his profit $\pi_m^N = (p_{n1}-c)Q_B^N$, in which $Q_B^N = \frac{p_{n1}}{q(2-\delta)}$ and $Q_{NB}^N = 0$. It follows from $\frac{\partial^2\pi_m^N}{\partial p_{n1}^2} = -\frac{2}{(2-\delta)q} \leq 0$ that the manufacturer's profit is concave in p_{n1}. Thus, the first-order condition $\frac{\partial\pi_m^N}{\partial p_{n1}} = 0$ yields the optimal selling price $p_{n1}^{N2*} = \frac{c+(2-\delta)q}{2}$. We accordingly have the market demand for new products $Q_B^{N2*} = \frac{(2-\delta)q-c}{2(2-\delta)q}$, and the manufacturer's profit $\pi_m^{N2*} = \frac{((2-\delta)q-c)^2}{4q(2-\delta)}$.

By comparing the manufacturer's profit in the above two cases, we have $\pi_m^{N1*} - \pi_m^{N2*} = 0$, which implies that the manufacturer has no incentive to mark down price, i.e., $p_{n2}^{N*} = p_{n1}^{N*} = p_{n1}^{N2*}$. Also, $Q_B^{N*} = Q_B^{N2*}$, $Q_{NB}^{N*} = 0$, and $\pi_m^{N*} = \pi_m^{N2*}$.

Moreover, to guarantee the market demand for new products is non-negative, i.e., $Q_B^{N*} \geq 0$, we need to have the precondition that $c \leq \tilde{c}^N$, in which $\tilde{c}^N = (2-\delta)q$. □

Proof of Proposition 1. In Model N, we have the precondition that $c \leq \tilde{c}^N$. Then, the effects of parameters (i.e., c, q and δ) on the manufacturer's profit are as follows:

$$\frac{\partial\pi_m^{N*}}{\partial c} = \frac{c}{4q-2\delta q} - \frac{1}{2}; \tag{EC.5}$$

$$\frac{\partial\pi_m^{N*}}{\partial q} = \frac{2-\delta}{4} - \frac{c^2}{4(2-\delta)q^2}; \tag{EC.6}$$

$$\frac{\partial\pi_m^{N*}}{\partial \delta} = \frac{c^2}{4(2-\delta)^2 q} - \frac{q}{4}. \tag{EC.7}$$

(i) Since we have

$$\frac{\partial \frac{\partial\pi_m^{N*}}{\partial c}}{\partial c} = \frac{1}{4q-2\delta q} \geq 0,$$

that is, $\frac{\partial\pi_m^{N*}}{\partial c}$ increases in c. Together with $\frac{\partial\pi_m^{N*}}{\partial c}|_{c\to\tilde{c}^N} = 0$, we can deduce that $\frac{\partial\pi_m^{N*}}{\partial c} \leq 0$.

(ii) Since we have

$$\frac{\partial \frac{\partial\pi_m^{N*}}{\partial q}}{\partial c} = -\frac{c}{2(2-\delta)q^2} \leq 0,$$

that is, $\frac{\partial\pi_m^{N*}}{\partial q}$ decreases in c. Together with $\frac{\partial\pi_m^{N*}}{\partial q}|_{c\to\tilde{c}^N} = 0$, we can deduce that $\frac{\partial\pi_m^{N*}}{\partial q} \geq 0$.

(iii) Since we have

$$\frac{\partial \frac{\partial\pi_m^{N*}}{\partial \delta}}{\partial c} = \frac{c}{2(2-\delta)^2 q} \geq 0,$$

that is, $\frac{\partial\pi_m^{N*}}{\partial \delta}$ increases in c. Together with $\frac{\partial\pi_m^{N*}}{\partial \delta}|_{c\to\tilde{c}^N} = 0$, we can deduce that $\frac{\partial\pi_m^{N*}}{\partial \delta} \leq 0$.

In conclude, we have $\frac{\partial\pi_m^{N*}}{\partial c} \leq 0$, $\frac{\partial\pi_m^{N*}}{\partial q} \geq 0$ and $\frac{\partial\pi_m^{N*}}{\partial \delta} \leq 0$. □

Proof of Lemma 2. Under the scenario that only the trade-in program is implemented (Model T), we will discuss two cases as follows.

Case 1. The manufacturer adopts the markdown stategy. In the first stage, the manufacturer sets the new product's selling price in the first period p_{n1} and the used product's trade-in rebate r; and in the second stage, he decides the new product's selling price in the second period p_{n2}, to maximize his profit $\pi_m^T = (p_{n1}-c)Q_B^T + (p_{n2}-c-r)Q_T^T + (p_{n2}-c)Q_{NB}^T - F_t$ subject to $Q_{NB}^T \geq 0$, in which $Q_T^T = 1 - \frac{p_{n2}-r}{\delta q}$, $Q_B^T = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$, and $Q_{NB}^T = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}}{q}$. We solve this model by backward induction.

In the second stage, to solve the constrained maximum problem for the manufacturer, we construct the following Lagrangian function

$$\begin{aligned} L^T(p_{n2},\lambda) = &-(p_{n1}-c)Q_B^T - (p_{n2}-c-r)Q_T^T - (p_{n2}-c)Q_{NB}^T \\ &+ F_t - \lambda Q_{NB}^T, \end{aligned} \tag{EC.8}$$

in which λ is the Lagrange multiplier. So the KKT conditions are

$$\begin{cases} \frac{\partial L^T}{\partial p_{n2}} = \frac{-(\delta-1)(\delta(c+q)+c+2r)+(\delta-2)\delta\lambda+2\delta p_{n1}+2((\delta-1)\delta-1)p_{n2}}{(\delta-1)\delta q} = 0, \\ -Q_{NB}^T \leq 0, \\ \lambda \geq 0, \\ \lambda Q_{NB}^T = 0. \end{cases} \tag{EC.9}$$

Then, we indentify the manufacturer's selling prices for both new products and refurbished products, by discussing two subcases as follows.

Case 1a. If $\lambda = 0$, then from the KKT condition, Eq. (EC.9), we have

$$p_{n2}^T(p_{n1}, r) = \frac{(\delta-1)(\delta(c+q)+c+2r)-2\delta p_{n1}}{2(\delta-1)\delta-2}. \tag{EC.10}$$

In the first stage, by substituting Eq. (EC.10) into the profit function π_m^{T}, the manufacturer sets p_{n1} and r to maximize his profit. Since $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2} = -\frac{2(\delta+1)}{(1+(1-\delta)\delta)q} \le 0$, $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2} = -\frac{4-2\delta}{(1+(1-\delta)\delta)q} \le 0$, and $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r} = \frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}} = \frac{2}{(1+(1-\delta)\delta)q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}}\right]=\frac{4}{(1+(1-\delta)\delta)q^2} \ge 0. \tag{EC.11}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{T} is jointly concave in (p_{n1}, r). Hence the unique optimal selling price $p_{n1}^{\mathrm{T1}*}$ and trade-in rebate $r^{\mathrm{T1}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{T}}}{\partial p_{n1}} = 0$ and $\frac{\partial\pi_m^{\mathrm{T}}}{\partial r} = 0$. Thus we obtain $p_{n1}^{\mathrm{T1}*} = \frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{T1}*} = \frac{q(1-\delta)}{2}$. We accordingly have the market demand for new products in the second period is $Q_{NB}^{\mathrm{T1}*} = -\frac{c}{2q} \le 0$. As a result, this case is eliminated.

Case 1b. If $\lambda \neq 0$, then from the KKT condition, Eq. (EC.9), we have

$$\begin{cases} p_{n2}^T(p_{n1}, r) = \frac{p_{n1}}{2-\delta}, \\ \lambda = \frac{(\delta-1)((\delta-2)(\delta(c+q)+c+2r)+2p_{n1})}{(\delta-2)^2\delta}. \end{cases} \tag{EC.12}$$

In the first stage, by substituting $p_{n2}^T(p_{n1}, r)$ into the profit function π_m^{T}, the manufacturer sets p_{n1} and r to maximize his profit. Since $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2} = -\frac{2+2(2-\delta)\delta}{(2-\delta)^2\delta q} \le 0$, $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2} = -\frac{2}{\delta q} \le 0$, and $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r} = \frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}} = \frac{2}{2\delta q-\delta^2 q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}}\right]=\frac{4}{(2-\delta)\delta q^2} \ge 0. \tag{EC.13}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{T} is jointly concave in (p_{n1}, r). Hence the unique optimal selling price $p_{n1}^{\mathrm{T1}*}$ and trade-in rebate $r^{\mathrm{T1}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{T}}}{\partial p_{n1}} = 0$ and $\frac{\partial\pi_m^{\mathrm{T}}}{\partial r} = 0$. Thus we obtain $p_{n1}^{\mathrm{T1}*} = \frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{T1}*} = \frac{(1-\delta)((2-\delta)q-c)}{2(2-\delta)}$. We accordingly have the manufacturer's profit $\pi_m^{\mathrm{T1}*} = \frac{c^2}{2\delta(2-\delta)q} + \frac{q-2c}{2} - F_t$.

Case 2. The manufacturer does not adopt the markdown stategy. In this case, the manufacturer sets the new product's selling price in the first period p_{n1} and the used product's trade-in rebate r, to maximize his profit $\pi_m^{\mathrm{T}} = (p_{n1}-c)Q_B^{\mathrm{T}} + (p_{n1}-c-r)Q_T^{\mathrm{T}} - F_t$; here $Q_T^{\mathrm{T}} = 1 - \frac{p_{n1}-r}{\delta q}$ and $Q_B^{\mathrm{T}} = 1 - \frac{p_{n1}}{(2-\delta)q}$.

Then, it follows that $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2} = -\frac{4}{(2-\delta)\delta q} \le 0$, $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2} = -\frac{2}{\delta q} \le 0$, and $\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r} = \frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}} = \frac{2}{\delta q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{T}}}{\partial r\partial p_{n1}}\right]=\frac{4}{(2-\delta)\delta q^2} \ge 0. \tag{EC.14}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{T} is jointly concave in (p_{n1}, r). Hence the unique optimal selling price $p_{n1}^{\mathrm{T2}*}$ and trade-in rebate $r^{\mathrm{T2}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{T}}}{\partial p_{n1}} = 0$ and $\frac{\partial\pi_m^{\mathrm{T}}}{\partial r} = 0$. Thus we obtain $p_{n1}^{\mathrm{T2}*} = \frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{T2}*} = q(1-\delta)$. We accordingly have the market demand for new products in the first period $Q_B^{\mathrm{T2}*} = \frac{(2-\delta)q-c}{2(2-\delta)q}$ and the trade-in amount $Q_T^{\mathrm{T2}*} = \frac{1}{2} - \frac{c}{2\delta q}$. Also, the manufacturer's profit is given by $\pi_m^{\mathrm{T2}*} = \frac{c^2}{2\delta(2-\delta)q} + \frac{q-2c}{2} - F_t$.

By comparing the manufacturer's profit in the above two cases, we have $\pi_m^{\mathrm{T1}*} - \pi_m^{\mathrm{T2}*} = 0$, which implies that the manufacturer has no incentive to mark down price, i.e., $p_{n2}^{\mathrm{T}*} = p_{n1}^{\mathrm{T}*} = p_{n1}^{\mathrm{T2}*}$. Also, $r^{\mathrm{T}*} = r^{\mathrm{T2}*}$, $Q_T^{\mathrm{T}*} = Q_T^{\mathrm{T2}*}$, $Q_B^{\mathrm{T}*} = Q_B^{\mathrm{T2}*}$, $Q_{NB}^{\mathrm{T}*} = 0$, and $\pi_m^{\mathrm{T}*} = \pi_m^{\mathrm{T2}*}$.

Moreover, we can directly show that the market demand for new products is larger than the trade-in amount, since $Q_B^{\mathrm{T}*} - Q_T^{\mathrm{T}*} = \frac{c(1-\delta)}{(2-\delta)\delta q} \ge 0$. Further, to guarantee the trade-in amount is non-negative, i.e., $Q_T^{\mathrm{T}*} \ge 0$, we need to have the precondition that $c \le \tilde{c}^{\mathrm{T}}$ in which $\tilde{c}^{\mathrm{T}} = \delta q$. □

Proof of Proposition 2. In Model T, we have the precondition that $c \le \tilde{c}^{\mathrm{T}}$. Then, the effects of parameters (i.e., c, q and δ) on the manufacturer's profit are as follows:

$$\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c} = \frac{c}{\delta q(2-\delta)} - 1; \tag{EC.15}$$

$$\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q} = \frac{1}{2} - \frac{c^2}{2(2-\delta)\delta q^2}; \tag{EC.16}$$

$$\frac{\partial\pi_m^{\mathrm{T}*}}{\partial\delta} = -\frac{c^2(1-\delta)}{(2-\delta)^2\delta^2 q} \le 0. \tag{EC.17}$$

(i) Since we have

$$\frac{\partial\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c}}{\partial c} = \frac{1}{\delta q(2-\delta)} \ge 0,$$

that is, $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c}$ increases in c. Together with $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{T}}} = \frac{1}{2-\delta} - 1 \le 0$, we can deduce that $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c} \le 0$.

(ii) Since we have

$$\frac{\partial\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q}}{\partial c} = -\frac{c}{(2-\delta)\delta q^2} \le 0,$$

that is, $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q}$ decreases in c. Together with $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q}|_{c\to\tilde{c}^{\mathrm{T}}} = 1 - \frac{1}{2-\delta} \ge 0$, we can deduce that $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q} \ge 0$.

In conclude, we have $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial c} \le 0$, $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial q} \ge 0$ and $\frac{\partial\pi_m^{\mathrm{T}*}}{\partial\delta} \le 0$. □

Proof of Lemma 3. Under the scenario that both the trade-in program and the refurbishment program are implemented (Model TR), we will discuss two cases as follows.

Case 1. The manufacturer adopts the markdown stategy. In the first stage, the manufacturer sets the new product's selling price in the first period p_{n1} and the used product's trade-in rebate r; and in the second stage, he decides the new product's selling price in the second period p_{n2} and the refurbished product's selling price p_r, to maximize his profit $\pi_m^{\mathrm{TR}} = (p_{n1}-c)Q_B^{\mathrm{TR}} + (p_{n2}-c-r)Q_T^{\mathrm{TR}} + (p_{n2}-c)Q_{NB}^{\mathrm{TR}} + (p_r-\gamma c)Q_R^{\mathrm{TR}} - F_t - F_r$ subject to $Q_{NB}^{\mathrm{TR}} \ge 0$, in which $Q_T^{\mathrm{TR}} = 1 - \frac{p_{n2}-r}{\delta q}$, $Q_B^{\mathrm{TR}} = 1 - \frac{p_{n1}-p_{n2}}{q(1-\delta)}$, $Q_{NB}^{\mathrm{TR}} = \frac{p_{n1}-p_{n2}}{q(1-\delta)} - \frac{p_{n2}-p_r}{\delta q(1-\beta)}$ and $Q_R^{\mathrm{TR}} = \frac{p_{n2}-p_r}{\delta q(1-\beta)} - \frac{p_r}{q-(1-\beta)\delta q}$. We solve this model by backward induction.

In the second stage, to solve the constrained maximum problem for the manufacturer, we construct the following Lagrangian function

$$L^{\mathrm{TR}}(p_{n2},p_r,\lambda)=-(p_{n1}-c)Q_B^{\mathrm{TR}}-(p_{n2}-c-r)Q_T^{\mathrm{TR}}-(p_{n2}-c)Q_{NB}^{\mathrm{TR}}-(p_r-\gamma c)Q_R^{\mathrm{TR}}+F_t+F_r-\lambda Q_{NB}^{\mathrm{TR}}, \tag{EC.18}$$

in which λ is the Lagrange multiplier. So the KKT conditions are

$$\begin{cases}\frac{\partial L^{\mathrm{TR}}}{\partial p_{n2}}=\frac{-\beta\delta\lambda-(\delta-1)(c(\beta+\gamma-2)+(\beta-1)(\delta q+2R))+\lambda+2(\beta-1)\delta p_{n1}-2(\beta+\delta-2)p_{n2}+2(\delta-1)p_r}{(\beta-1)(\delta-1)\delta q}=0,\\ \frac{\partial L^{\mathrm{TR}}}{\partial p_r}=\frac{(\beta-1)\delta\lambda+c(-\beta\delta+\gamma+\delta-1)+\lambda+2((\beta-1)\delta+1)p_{n2}-2p_r}{(\beta-1)\delta q((\beta-1)\delta+1)}=0,\\ -Q_{NB}^{\mathrm{TR}}\le 0,\\ \lambda\ge 0,\\ \lambda Q_{NB}^{\mathrm{TR}}=0.\end{cases} \tag{EC.19}$$

Then, we indentify the manufacturer's selling prices for both new products and refurbished products, by discussing two subcases as follows.

Case 1a. If $\lambda=0$, then from the KKT condition, Eq. (EC.19), we have

$$\begin{cases}p_{n2}^{TR}(p_{n1},r)=\frac{(\delta-1)(\delta(c+q)+c+2r)-2\delta p_{n1}}{2(\delta-1)\delta-2},\\ p_r^{TR}(p_{n1},r)=\frac{c\left(\delta^2(\beta+\gamma-1)-\gamma\delta-\gamma+\delta\right)+2\delta(-\beta\delta+\delta-1)p_{n1}+(\delta-1)\delta q((\beta-1)\delta+1)+2(\delta-1)r((\beta-1)\delta+1)}{2(\delta-1)\delta-2}.\end{cases} \tag{EC.20}$$

In the first stage, by substituting Eq. (EC.20) into the profit function π_m^{TR}, the manufacturer sets p_{n1} and r to maximize his profit. Since $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}=-\frac{2(\delta+1)}{(1+(1-\delta)\delta)q}\le 0$, $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}=-\frac{4-2\delta}{(1+(1-\delta)\delta)q}\le 0$, and $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}=\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}=\frac{2}{(1+(1-\delta)\delta)q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}\right]=\frac{4}{(1+(1-\delta)\delta)q^2}\ge 0. \tag{EC.21}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{TR} is jointly concave in (p_{n1},r). Hence the unique optimal selling price $p_{n1}^{\mathrm{TR1}*}$ and trade-in rebate $r^{\mathrm{TR1}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial p_{n1}}=0$ and $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial r}=0$. Thus we obtain $p_{n1}^{\mathrm{TR1}*}=\frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{TR1}*}=\frac{q(1-\delta)}{2}$. We accordingly have the market demand for new products in the second period is $Q_{NB}^{\mathrm{TR1}*}=-\frac{c-c\gamma}{2\delta q-2\beta\delta q}\le 0$. As a result, this case is eliminated.

Case 1b. If $\lambda\neq 0$, then from the KKT condition, Eq. (EC.19), we have

$$\begin{cases}p_{n2}^{TR}(p_{n1},r)=\frac{(\delta-1)(c(\delta(\delta(2\beta+\gamma-2)-\beta-2\gamma+3)-1)+(\delta-1)((\beta-1)\delta+1)(\delta q+2r))-2\delta(\beta\delta-1)p_{n1}}{2\delta\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+2},\\ p_r^{TR}(p_{n1},r)=\frac{(\beta\delta-1)(c(\delta(\delta(2\beta+\gamma-2)-\beta-2\gamma+3)-1)+(\delta-1)((\beta-1)\delta+1)(\delta q+2r))-2\delta((\beta-1)\delta+1)(\beta(2\delta-1)-\delta)p_{n1}}{2\delta\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+2},\\ \lambda=-\frac{(\delta-1)(c(-\delta)(\beta+\delta-2)+\gamma c((\delta-1)\delta-1)+c+2(\beta-1)\delta p_{n1}+(\beta-1)(\delta-2)\delta(\delta q+2r))}{\delta\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+1}.\end{cases} \tag{EC.22}$$

In the first stage, by substituting $p_{n2}^{TR}(p_{n1},r)$ and $p_r^{TR}(p_{n1},r)$ into the profit function π_m^{TR}, the manufacturer sets p_{n1} and r to maximize his profit. Since $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}=-\frac{2+(2-4\beta)\delta^2}{\delta q\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+q}\le 0$, $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}=-\frac{2(2-\delta)(1-\beta\delta)}{\delta q\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+q}\le 0$, and $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}=\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}=\frac{2-2\beta\delta}{\delta q\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}\right]=\frac{4-4\beta\delta}{q^2\left(\delta\left(2\beta(\delta-2)\delta+\beta-(\delta-1)^2\right)+1\right)}\ge 0. \tag{EC.23}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{TR} is jointly concave in (p_{n1},r). Hence the unique optimal selling price $p_{n1}^{\mathrm{TR1}*}$ and trade-in rebate $r^{\mathrm{TR1}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial p_{n1}}=0$ and $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial r}=0$. Thus we obtain $p_{n1}^{\mathrm{TR1}*}=\frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{TR1}*}=\frac{(1-\delta)(c(\gamma-1)-\beta\delta q+q)}{2-2\beta\delta}$. We accordingly have the manufacturer's profit $\pi_m^{\mathrm{TR1}*}=\frac{q-2c}{2}+\frac{c^2(1-(1-\beta-\gamma)^2\delta^2-2(1-\gamma)\gamma\delta)}{4\delta q(1-(1-\beta)\delta)(1-\beta\delta)}-F_t-F_r$.

Case 2. The manufacturer does not adopt the markdown stategy. In the first stage, the manufacturer sets the new product's selling price in the first period p_{n1} and the used product's trade-in rebate r; and in the second stage, he decides the refurbished product's selling price p_r, to maximize his profit $\pi_m^{\mathrm{TR}}=(p_{n1}-c)Q_B^{\mathrm{TR}}+(p_{n1}-c-r)Q_T^{\mathrm{TR}}+(p_r-\gamma c)Q_R^{\mathrm{TR}}-F_t-F_r$; here $Q_T^{\mathrm{TR}}=1-\frac{p_{n1}-r}{\delta q}$, $Q_B^{\mathrm{TR}}=1-\frac{p_{n1}-p_r}{q-\beta\delta q}$, and $Q_R^{\mathrm{TR}}=\frac{p_{n1}-p_r}{q-\beta\delta q}-\frac{p_r}{q-(1-\beta)\delta q}$. We solve this model by backward induction.

In the second stage, it follows from the manufacturer's profit function that π_m^{TR} is concave in p_r because $\frac{\partial(\pi_m^{\mathrm{TR}})^2}{\partial^2 p_r}=-\frac{2}{q-\beta\delta q}-\frac{2}{q-(1-\beta)\delta q}\le 0$. Thus, the first-order condition $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial p_r}=0$ yields the manufacturer's optimal refurbished product's selling price response, which is given by

$$p_r^{\mathrm{TR}}(p_{n1},r)=\frac{c(\delta(1-\beta-\gamma)+2\gamma-1)+2p_{n1}(1-(1-\beta)\delta)}{2(2-\delta)}. \tag{EC.24}$$

In the first stage, by substituting Eq. (EC.24) into the profit function π_m^{TR}, the manufacturer sets p_{n1} and r to maximize his profit. Since $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}=-\frac{4}{(2-\delta)\delta q}\le 0$, $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}=-\frac{2}{\delta q}\le 0$, and $\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}=\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}=\frac{2}{\delta q}$. We can now calculate the determinant of the Hessian matrix as

$$\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}^2}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r^2}\right]-\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial p_{n1}\partial r}\right]\left[\frac{\partial^2\pi_m^{\mathrm{TR}}}{\partial r\partial p_{n1}}\right]=\frac{4}{(2-\delta)\delta q^2}\ge 0. \tag{EC.25}$$

Therefore, the Hessian matrix is negative definite and so the profit function π_m^{TR} is jointly concave in (p_{n1},r). Hence the unique optimal selling price $p_{n1}^{\mathrm{TR2}*}$ and trade-in rebate $r^{\mathrm{TR2}*}$ can be obtained by simultaneously solving the two first-order conditions $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial p_{n1}}=0$ and $\frac{\partial\pi_m^{\mathrm{TR}}}{\partial r}=0$. Thus we obtain $p_{n1}^{\mathrm{TR2}*}=\frac{c+(2-\delta)q}{2}$ and $r^{\mathrm{TR2}*}=q(1-\delta)$. We accordingly have the optimal refurbished product's selling price $p_r^{\mathrm{TR2}*}=\frac{c\gamma-(1-\beta)\delta q+q}{2}$. Then, we have the trade-in amount is $Q_T^{\mathrm{TR2}*}=\frac{1}{2}-\frac{c}{2\delta q}$, the market demand for new products in the first period is $Q_B^{\mathrm{TR2}*}=\frac{1}{2}-\frac{c-c\gamma}{2(q-\beta\delta q)}$, and the market demand for refurbished products is $Q_R^{\mathrm{TR2}*}=\frac{c(\delta(\beta+\gamma-1)-2\gamma+1)}{2q(1-(1-\beta)\delta)(1-\beta\delta)}$. Also, the manufacturer's profit is given by $\pi_m^{\mathrm{TR2}*}=\frac{q-2c}{2}+\frac{c^2(1-(1-\beta-\gamma)^2\delta^2-2(1-\gamma)\gamma\delta)}{4\delta q(1-(1-\beta)\delta)(1-\beta\delta)}-F_t-F_r$.

By comparing the manufacturer's profit in the above two cases, we have $\pi_m^{\mathrm{TR1}*}-\pi_m^{\mathrm{TR2}*}=0$, which implies that the manufacturer has no incentive to mark down prices. As a result, we have $p_{n2}^{\mathrm{TR}*}=p_{n1}^{\mathrm{TR}*}=p_{n1}^{\mathrm{TR2}*}$. Also, $r^{\mathrm{TR}*}=r^{\mathrm{TR2}*}$, $p_r^{\mathrm{TR}*}=p_r^{\mathrm{TR2}*}$, $Q_T^{\mathrm{TR}*}=Q_T^{\mathrm{TR2}*}$, $Q_B^{\mathrm{TR}*}=Q_B^{\mathrm{TR2}*}$, $Q_{NB}^{\mathrm{TR}*}=0$, $Q_R^{\mathrm{TR}*}=Q_R^{\mathrm{TR2}*}$, and $\pi_m^{\mathrm{TR}*}=\pi_m^{\mathrm{TR2}*}$.

Moreover, first, to guarantee the market demand for new products is larger than the trade-in amount, i.e., $Q_B^{\mathrm{TR}*}\geq Q_T^{\mathrm{TR}*}$, we need to have the precondition that $\gamma\geq\bar{\gamma}$, in which $\bar{\gamma}=\frac{\beta\delta+\delta-1}{\delta}$. Second, to ensure that the market demand for refurbished products is non-negative, i.e., $Q_R^{\mathrm{TR}*}\geq 0$, we need to have $\gamma\leq\bar{\gamma}$ in which $\bar{\gamma}=\frac{1+\beta\delta-\delta}{2-\delta}$. Third, the trade-in amount should be larger than the market demand for refurbished products, i.e., $Q_T^{\mathrm{TR}*}\geq Q_R^{\mathrm{TR}*}$, we need to have $c\leq\tilde{c}^{\mathrm{TR}}$, in which $\tilde{c}^{\mathrm{TR}}=\frac{\delta q(1-(1-\beta)\delta)(1-\beta\delta)}{1-(1-\beta)^2\delta^2-\gamma(2-\delta)\delta}$. Also, we can verify that $\tilde{c}^{\mathrm{TR}}\leq\delta q(=\tilde{c}^{\mathrm{T}})$. □

Proof of Proposition 3. In Model TR, we have the precondition that $\bar{\gamma}\leq\gamma\leq\bar{\gamma}$ and $c\leq\tilde{c}^{\mathrm{TR}}(\leq\tilde{c}^{\mathrm{T}})$. Then, the effects of parameters (i.e., c, q, δ, γ and β) on the manufacturer's profit are as follows:

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}=\frac{c(\delta^2(\beta+\gamma-1)^2-2(\gamma-1)\gamma\delta-1)}{2\delta q((\beta-1)\delta+1)(\beta\delta-1)}-1;\tag{EC.26}$$

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial q}=\frac{c^2(-\delta^2(\beta+\gamma-1)^2+2(\gamma-1)\gamma\delta+1)}{4\delta q^2((\beta-1)\delta+1)(\beta\delta-1)}+\frac{1}{2};\tag{EC.27}$$

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial\delta}=\frac{c^2}{4q}\left(\frac{(1-\beta)\gamma^2}{((\beta-1)\delta+1)^2}+\frac{\beta(\gamma-1)^2}{(\beta\delta-1)^2}-\frac{1}{\delta^2}\right);\tag{EC.28}$$

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial\gamma}=-\frac{c^2(\delta(\beta+\gamma-1)-2\gamma+1)}{2q((\beta-1)\delta+1)(1-\beta\delta)};\tag{EC.29}$$

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial\beta}=\frac{c^2\delta(\delta(\beta+\gamma-1)-2\gamma+1)(1-\beta(2\gamma-1)\delta+\gamma\delta-\delta)}{4q((\beta-1)\beta\delta^2+\delta-1)^2}.\tag{EC.30}$$

(i) Since

$$\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c}=\frac{\delta^2(\beta+\gamma-1)^2-2(\gamma-1)\gamma\delta-1}{2\delta q((\beta-1)\delta+1)(\beta\delta-1)},$$

thus, we have

$$\frac{\partial(\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c})}{\partial\gamma}=-\frac{\delta(\beta+\gamma-1)-2\gamma+1}{q((\beta-1)\delta+1)(1-\beta\delta)}.$$

Let $\Phi_1=\delta(\beta+\gamma-1)-2\gamma+1$. As we have

$$\frac{\partial\Phi_1}{\partial\gamma}=\delta-2\leq 0,$$

that is, Φ_1 decreases in γ. Together with $\Phi_1|_{\gamma\to\bar{\gamma}}=0$, we can deduce that $\Phi_1\geq 0$. Thus, $\frac{\partial(\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c})}{\partial\gamma}\leq 0$. In other words, $\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c}$ decreases in γ. Together with

$$\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c}|_{\gamma\to\bar{\gamma}}=\frac{1}{2\delta q-\delta^2 q}\geq 0,$$

we can obtain that $\frac{\partial(\pi_m^{\mathrm{TR}*})^2}{\partial^2 c}\geq 0$. That is, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c. Next, we will show that whether $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ is negative or positive at the boundary point.

On the one hand, we have $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ at the lower bound is

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to 0}=-1\leq 0.$$

On the other hand, we have $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ at the upper bound is

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}=\frac{-(\beta+\gamma-1)^2\delta^2+2(\gamma-1)\gamma\delta+1}{-2(\beta-1)^2\delta^2+2\gamma(\delta-2)\delta+2}-1.$$

Thus, we have

$$\frac{\partial(\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}})^2}{\partial^2\gamma}=\frac{\delta(1-\delta)(1+(1-\beta)\delta)(1-(1-\beta)\delta)(1-\beta\delta)(\beta\delta+1)}{(1-(\beta-1)^2\delta^2+\gamma(\delta-2)\delta)^3}.$$

Let $\Phi_2=1-(\beta-1)^2\delta^2+\gamma(\delta-2)\delta$. Because

$$\frac{\partial\Phi_2}{\partial\gamma}=-(2-\delta)\delta\leq 0,$$

that is, Φ_2 decreases in γ. Together with $\Phi_2|_{\gamma\to\bar{\gamma}}=(1-\beta)\beta\delta^2-\delta+1\geq 0$, we can deduce that $\Phi_2\geq 0$. Thus, $\frac{\partial(\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}})^2}{\partial^2\gamma}\geq 0$, which implies that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}$ is convex in γ. So we have $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}$ at the boundary point are as follows

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}=\frac{(1-\delta)(1-(\beta+3)\delta)}{\delta((\beta-3)\delta+3)};\tag{EC.31}$$

$$\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}=\frac{1}{2-\delta}-1\leq 0.\tag{EC.32}$$

Let $\Phi_3=1-(\beta+3)\delta$, we have

$$\frac{\partial\Phi_3}{\partial\beta}=-\delta\leq 0,$$

that is, Φ_3 decreases in β. Thus, we have $\Phi_3\leq 0$ if $\beta\geq\beta_1$, else if $\beta\leq\beta_1$ then $\Phi_3\geq 0$, in which β_1 is the unique solution of $\Phi_3=0$, which is given by

$$\beta_1=\frac{1}{\delta}-3.\tag{EC.33}$$

Therefore, if $\beta\geq\beta_1$, we have $\Phi_3\leq 0$. Thus, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}\leq 0$. As a result, together with $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}$ is convex in γ and $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}\leq 0$, we can deduce that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}\leq 0$. Again, recall that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c and $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to 0}\leq 0$. Therefore, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c from negative to negative, that is, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}\leq 0$.

However, if $\beta\leq\beta_1$, we have $\Phi_3\geq 0$. Thus, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}\geq 0$. As a result, together with $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}$ is convex in γ and $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}},\gamma\to\bar{\gamma}}\leq 0$, we can deduce that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}$ either decreases from positive to negative or first decreases from positive and then increases to negative. In either case, we can deduce that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}\geq 0$ if $\gamma\leq\gamma_1$; else if $\gamma\geq\gamma_1$ then $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}\leq 0$. Here, γ_1 is the unique solution of $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}=0$ within the range of $[\bar{\gamma},\bar{\gamma}]$, which is given by

$$\gamma_1=\frac{\beta\delta^2-\delta+\sqrt{(2(\beta-1)\beta+1)\delta^4-2((\beta-1)\beta+1)\delta^3+2\delta}}{(2-\delta)\delta}.\tag{EC.34}$$

Based on the above analysis, we can conclude two results as follows.

On the one hand, if $\beta\leq\beta_1$ and $\gamma\geq\gamma_1$, then $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}\leq 0$. Again, recall that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c. Thus, we have $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}\leq 0$.

On the other hand, if $\beta\leq\beta_1$ and $\gamma\leq\gamma_1$, then $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to\tilde{c}^{\mathrm{TR}}}\geq 0$. Recall that $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c and $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}|_{c\to 0}\leq 0$. Therefore, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}$ increases in c from negative to positive, that is, $\pi_m^{\mathrm{TR}*}$ first decreases and then increases with the increase of c. In particular, $\frac{\partial\pi_m^{\mathrm{TR}*}}{\partial c}\leq 0$

if $c \le c_1$; else if $c \ge c_1$ then $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} \ge 0$, in which c_1 is the unique solution of $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} = 0$ within the range of $[0, \tilde{c}^{\mathrm{TR}}]$, which is given by

$$c_1 = \frac{2\delta q((\beta-1)\delta+1)(\beta\delta-1)}{\delta^2(\beta+\gamma-1)^2-2(\gamma-1)\gamma\delta-1}. \tag{EC.35}$$

In sum, if $\beta \le \beta_1$ and $\gamma \le \gamma_1$, we have $\pi_m^{\mathrm{TR}*}$ first decreases and then increases with c. Otherwise, we have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} \le 0$.

(ii) Since

$$\frac{\partial (\pi_m^{\mathrm{TR}*})^2}{\partial^2 q} = -\frac{c^2(\delta^2(\beta+\gamma-1)^2-2(\gamma-1)\gamma\delta-1)}{2\delta q^3((\beta-1)\delta+1)(1-\beta\delta)}.$$

Let $\Phi_4 = \delta^2(\beta+\gamma-1)^2-2(\gamma-1)\gamma\delta-1$, as we have $\Phi_1 \ge 0$ from (i), thus

$$\frac{\partial \Phi_4}{\partial \gamma} = 2\delta(\delta(\beta+\gamma-1)-2\gamma+1) = 2\delta\Phi_1 \ge 0,$$

that is, Φ_4 increases in γ. Together with

$$\Phi_4|_{\gamma\to\bar{\gamma}} = -\frac{2(1-\beta)\beta\delta^2+2(1-\delta)}{2-\delta} \le 0,$$

we can deduce that $\Phi_4 \le 0$. Thus, $\frac{\partial (\pi_m^{\mathrm{TR}*})^2}{\partial^2 q} \ge 0$. In other words, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}$ increases in q.

From the precondition that $c \le \tilde{c}^{\mathrm{TR}} = \frac{\delta q(1-(1-\beta)\delta)(1-\beta\delta)}{1-(1-\beta)^2\delta^2-\gamma(2-\delta)\delta}$ we can obtain the lower bound of q, that is,

$$q \ge \frac{c(1-(1-\beta)^2\delta^2-\gamma(2-\delta)\delta)}{\delta(1-(1-\beta)\delta)(1-\beta\delta)} (= q^{\mathrm{TR}}),$$

so we have

$$\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}|_{q\to q^{\mathrm{TR}}} = \frac{\delta((\beta-1)\delta+1)(\beta\delta-1)(-\delta^2(\beta+\gamma-1)^2+2(\gamma-1)\gamma\delta+1)}{4(1-(\beta-1)^2\delta^2+\gamma(\delta-2)\delta)^2} + \frac{1}{2}.$$

Since we have $\Phi_2 \ge 0$ from (i), thus

$$\frac{\partial (\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}|_{q\to q^{\mathrm{TR}}})}{\partial \gamma} = -\frac{\delta^2((\beta-1)\delta+1)^2(1-\beta\delta)^2(\delta(1-\beta-\gamma)+2\gamma+1)}{2\Phi_2^3} \le 0,$$

that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}|_{q\to q^{\mathrm{TR}}}$ decreases in γ. Together with $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}|_{q\to q^{\mathrm{TR}},\gamma\to\bar{\gamma}} = \frac{1}{\delta-2}+1 \ge 0$, we can deduce that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}|_{q\to q^{\mathrm{TR}}} \ge 0$. Together with the fact that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}$ increases in q, we know that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q}$ increases from positive, that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q} \ge 0$.

(iii) Since

$$\frac{\partial (\pi_m^{\mathrm{TR}*})^2}{\partial^2 \delta} = \frac{c^2}{2q}\left(\frac{\beta^2(1-\gamma)^2}{(1-\beta\delta)^3} + \frac{(1-\beta)^2\gamma^2}{((\beta-1)\delta+1)^3} + \frac{1}{\delta^3}\right) \ge 0,$$

that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ increases in δ. Next, we will show that whether $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ is negative or positive at the boundary point.

On the one hand, from $\gamma \ge \bar{\gamma}(= \frac{\beta\delta+\delta-1}{\delta})$, we have $\delta \le \frac{1}{\beta-\gamma+1}(= \bar{\delta}_1)$. So we obtain $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ at the upper bound is

$$\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}|_{\delta\to\bar{\delta}_1} = \frac{\beta c^2(1-\beta)(\gamma-\beta)(\beta-\gamma+1)^2}{q(\gamma-2\beta)^2}.$$

Thus, we have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}|_{\delta\to\bar{\delta}} \ge 0$ if $\gamma \ge \beta$; else if $\gamma \le \beta$, then $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}|_{\delta\to\bar{\delta}_1} \le 0$. Therefore, if $\gamma \le \beta$, we know that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ increases in δ to negative, that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta} \le 0$.

However, if $\gamma \ge \beta$, we know that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ may increases in δ to positive, so we will discuss the value of $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ on the other hand, i.e., when $\gamma \le \bar{\gamma}(= \frac{1+\beta\delta-\delta}{2-\delta})$. In this scenario, it follows that $\delta(1-\beta-\gamma) \le 1-2\gamma$. So we have the following two cases:

1. If $\gamma \le \frac{1}{2}$, we have $1-\beta-\gamma \ge 0$, then we obtain the other upper bound of δ, i.e., $\delta \le (\bar{\delta}_2 =)\frac{1-2\gamma}{1-\beta-\gamma} (\le 1)$. Thus, we have

$$\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}|_{\delta\to\bar{\delta}_2} = -\frac{c^2(\gamma-\beta)(1-\beta-\gamma)^3}{q(1-2\beta)^2(1-2\gamma)^2} \le 0,$$

which implies that $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ also increases in δ to negative, that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta} \le 0$.
2. If $\gamma \ge \frac{1}{2}$, to guarantee $\gamma \le \bar{\gamma}(= \frac{1+\beta\delta-\delta}{2-\delta})$, we must have $1-\beta-\gamma \le 0$ and $\delta \ge \frac{2\gamma-1}{\beta+\gamma-1} (\ge 1)$, which is conflict with $\delta \le 1$. In other words, this case is not exists.

In conclude, in the feasible domain, we always have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta}$ increases in δ to negative, that is, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta} \le 0$.

(iv) Recall from (i) that $\Phi_1 \ge 0$. So we can directly have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \gamma} = -\frac{c^2\Phi_1}{2q((\beta-1)\delta+1)(1-\beta\delta)} \le 0$.

(v) Let $\Phi_5 = 1-\beta(2\gamma-1)\delta+\gamma\delta-\delta$. Since we have

$$\frac{\partial \Phi_5}{\partial \gamma} = (1-2\beta)\delta,$$

which implies that Φ_5 is either increases or decreases in γ. Together with the fact that $\Phi_5|_{\gamma\to\bar{\gamma}} = 2\beta(1-\beta\delta) \ge 0$ and $\Phi_5|_{\gamma\to\bar{\gamma}} = \frac{2(1-\beta)\beta\delta^2+2(1-\delta)}{2-\delta} \ge 0$, we can deduce that $\Phi_5 \ge 0$. Recall from (i) that $\Phi_1 \ge 0$, so we have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \beta} = \frac{c^2\delta\Phi_1\Phi_5}{4q((1-\beta)\beta\delta^2+1-\delta)^2} \ge 0$.

In conclude, we have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} \ge 0$ if $\beta \le \beta_1$, $\gamma \le \gamma_1$, and $c \ge c_1$; otherwise, we have $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial c} \le 0$. Moreover, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial q} \ge 0$, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \delta} \le 0$, $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \gamma} \le 0$ and $\frac{\partial \pi_m^{\mathrm{TR}*}}{\partial \beta} \ge 0$. □

Proof of Proposition 4. (i) Using Lemmas 2 and 1, we compare the results for Models T and N and deduce that

$$p_{n1}^{\mathrm{T}*} - p_{n1}^{\mathrm{N}*} = 0; \tag{EC.36}$$

$$r^{\mathrm{T}*} - r^{\mathrm{N}*} = r^{\mathrm{T}*} \ge 0; \tag{EC.37}$$

$$p_r^{\mathrm{T}*} - p_r^{\mathrm{N}*} = 0; \tag{EC.38}$$

$$Q_B^{\mathrm{T}*} - Q_B^{\mathrm{N}*} = 0; \tag{EC.39}$$

$$Q_T^{\mathrm{T}*} - Q_T^{\mathrm{N}*} = Q_T^{\mathrm{T}*} \ge 0; \tag{EC.40}$$

$$Q_R^{\mathrm{T}*} - Q_R^{\mathrm{N}*} = 0; \tag{EC.41}$$

$$\pi_m^{\mathrm{T}*} - \pi_m^{\mathrm{N}*} = \frac{(c-\delta q)^2}{4\delta q} - F_t. \tag{EC.42}$$

Recall from Lemma 2 that $c \le \tilde{c}^{\mathrm{T}}(= \delta q)$. It follows from

$$\frac{\partial\left(\pi_m^{\mathrm{T}*} - \pi_m^{\mathrm{N}*}\right)}{\partial c} = \frac{1}{2}\left(\frac{c}{\delta q} - 1\right) \le 0,$$

that $\pi_m^{T*}-\pi_m^{N*}$ decreases in c.

Since we have

$$\left(\pi_m^{T*}-\pi_m^{N*}\right)|_{c\to 0}=\frac{\delta q}{4}-F_t,$$

and

$$\left(\pi_m^{T*}-\pi_m^{N*}\right)|_{c\to\tilde{c}^{T}}=-F_t\le 0.$$

Therefore, if $F_t\ge\frac{\delta q}{4}$, then $(\pi_m^{T*}-\pi_m^{N*})|_{c\to 0}\le 0$, that is $\pi_m^{T*}-\pi_m^{N*}$ decreases in c from negative to negative, i.e., $\pi_m^{T*}\le\pi_m^{N*}$. However, if $F_t\le\frac{\delta q}{4}$, then $(\pi_m^{T*}-\pi_m^{N*})|_{c\to 0}\ge 0$, that is, $\pi_m^{T*}-\pi_m^{N*}$ decreases from positive to negative. Thus, $\pi_m^{T*}\ge\pi_m^{N*}$ if $c\le c_2$; else if $c\ge c_2$, then $\pi_m^{T*}\le\pi_m^{N*}$, in which c_2 is the unique solution of $\pi_m^{T*}-\pi_m^{N*}=0$ within the range of $[0,\tilde{c}^{T}]$, that is

$$c_2=\delta q-2\sqrt{\delta qF_t}. \tag{EC.43}$$

In conclude, when $F_t\ge\frac{\delta q}{4}$, we have $\pi_m^{T*}\le\pi_m^{N*}$. However, when $F_t\le\frac{\delta q}{4}$, we have $\pi_m^{T*}\ge\pi_m^{N*}$ if $c\le c_2$; else if $c\ge c_2$, we have $\pi_m^{T*}\le\pi_m^{N*}$. □

Proof of Proposition 5. (i) Using Lemmas 3 and 2, we compare the results for Models TR and T and deduce that

$$p_{n1}^{TR*}-p_{n1}^{T*}=0; \tag{EC.44}$$

$$r^{TR*}-r^{T*}=0; \tag{EC.45}$$

$$p_r^{TR*}-p_r^{T*}=p_r^{TR*}\ge 0; \tag{EC.46}$$

$$Q_B^{TR*}-Q_B^{T*}=-\frac{c(\delta(\beta+\gamma-1)-2\gamma+1)}{2(2-\delta)q(1-\beta\delta)}; \tag{EC.47}$$

$$Q_T^{TR*}-Q_T^{T*}=0; \tag{EC.48}$$

$$Q_R^{TR*}-Q_R^{T*}=Q_R^{TR*}\ge 0; \tag{EC.49}$$

$$\pi_m^{TR*}-\pi_m^{T*}=\frac{c^2(\delta(\beta+\gamma-1)-2\gamma+1)^2}{4q(2-\delta)((\beta-1)\delta+1)(1-\beta\delta)}-F_r. \tag{EC.50}$$

(i) Recall from the proof in Proposition 3 that $\Phi_1=\delta(\beta+\gamma-1)-2\gamma+1\ge 0$, so we can directly have $Q_B^{TR*}-Q_B^{T*}=-\frac{c\Phi_1}{2(2-\delta)q(1-\beta\delta)}\le 0$.

(ii) It follows from

$$\frac{\partial(\pi_m^{TR*}-\pi_m^{T*})}{\partial c}=\frac{c(\delta(\beta+\gamma-1)-2\gamma+1)^2}{2q(2-\delta)(1-(1-\beta)\delta)(1-\beta\delta)}\ge 0,$$

that $\pi_m^{TR*}-\pi_m^{T*}$ increases in c.

Recall from Lemma 3 that we have $c\le\tilde{c}^{TR}(=c^{T})$. So we have

$$\left(\pi_m^{TR*}-\pi_m^{T*}\right)|_{c\to 0}=-F_r\le 0,$$

and

$$\left(\pi_m^{TR*}-\pi_m^{T*}\right)|_{c\to\tilde{c}^{TR}}=\frac{\delta^2q((1-\beta)\beta\delta^2+1-\delta)(\delta(\beta+\gamma-1)-2\gamma+1)^2}{4(2-\delta)(1-(\beta-1)^2\delta^2+\gamma(\delta-2)\delta)^2}-F_r.$$

Let

$$F_{r,1}=\frac{\delta^2q((1-\beta)\beta\delta^2+1-\delta)(\delta(\beta+\gamma-1)-2\gamma+1)^2}{4(2-\delta)(1-(\beta-1)^2\delta^2+\gamma(\delta-2)\delta)^2}. \tag{EC.51}$$

Therefore, if $F_r\ge F_{r,1}$, then $(\pi_m^{TR*}-\pi_m^{T*})|_{c\to\tilde{c}^{TR}}\le 0$, that is, $\pi_m^{TR*}-\pi_m^{T*}$ increases in c from negative to negative, i.e., $\pi_m^{TR*}\le\pi_m^{T*}$. However, if $F_r\ge F_{r,1}$, then $(\pi_m^{TR*}-\pi_m^{T*})|_{c\to\tilde{c}^{TR}}\ge 0$, that is, $\pi_m^{TR*}-\pi_m^{T*}$ increases from negative to positive. Thus, $\pi_m^{TR*}\le\pi_m^{T*}$ if $c\le c_3$; else if $c\ge c_3$, then $\pi_m^{TR*}\ge\pi_m^{T*}$, in which c_3 is the unique solution of $\pi_m^{TR*}-\pi_m^{T*}=0$ within the range of $[0,\tilde{c}^{TR}]$, that is

$$c_3=\frac{2\sqrt{q(2-\delta)((1-\beta)\beta\delta^2-\delta+1)F_r}}{\delta(\beta+\gamma-1)-2\gamma+1}. \tag{EC.52}$$

In conclude, when $F_r\ge F_{r,1}$, we have $\pi_m^{TR*}\le\pi_m^{T*}$. However, when $F_r\le F_{r,1}$, we have $\pi_m^{TR*}\le\pi_m^{T*}$ if $c\le c_3$; else if $c\ge c_3$, we have $\pi_m^{TR*}\ge\pi_m^{T*}$. □

Proof of Proposition 6. (i) Using Lemmas 3 and 1, we compare the results for Models TR and N and deduce that

$$p_{n1}^{TR*}-p_{n1}^{N*}=0; \tag{EC.53}$$

$$r^{TR*}-r^{N*}=r^{TR*}\ge 0; \tag{EC.54}$$

$$p_r^{TR*}-p_r^{N*}=p_r^{TR*}\ge 0; \tag{EC.55}$$

$$Q_B^{TR*}-Q_B^{N*}=-\frac{c(\delta(\beta+\gamma-1)-2\gamma+1)}{2(2-\delta)q(1-\beta\delta)}; \tag{EC.56}$$

$$Q_T^{TR*}-Q_T^{N*}=Q_T^{TR*}\ge 0; \tag{EC.57}$$

$$Q_R^{TR*}-Q_T^{N*}=Q_R^{TR*}\ge 0; \tag{EC.58}$$

$$\pi_m^{TR*}-\pi_m^{N*}=\frac{q-2c}{2}+\frac{c^2(1-(1-\beta-\gamma)^2\delta^2-2(1-\gamma)\gamma\delta)}{4\delta q(1-(1-\beta)\delta)(1-\beta\delta)}-\frac{((2-\delta)q-c)^2}{4q(2-\delta)}-F_t-F_r. \tag{EC.59}$$

(i) Recall from the proof in Proposition 3 that $\Phi_1=\delta(\beta+\gamma-1)-2\gamma+1\ge 0$, so we can directly have $Q_B^{TR*}-Q_B^{N*}=-\frac{c\Phi_1}{2(2-\delta)q(1-\beta\delta)}\le 0$.

(ii) Let $\Phi_6=\frac{\gamma^2}{1-(1-\beta)\delta}+\frac{(1-\gamma)^2}{1-\beta\delta}-\frac{1}{2-\delta}+\frac{1}{\delta}$. It follows that

$$\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})}{\partial c}=\frac{c\Phi_6}{2q}-\frac{1}{2},$$

and

$$\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})^2}{\partial^2 c}=\frac{\Phi_6}{2q}.$$

Recall from Lemma 3 that $\bar{\gamma}\le\gamma\le\bar{\gamma}$. Since we have

$$\frac{\partial\Phi_6^2}{\partial^2\gamma}=\frac{2-\delta}{q((1-\beta)\beta\delta^2+1-\delta)}\ge 0,$$

that is, $\frac{\partial\Phi_6}{\partial\gamma}$ increases in γ. Together with $\frac{\partial\Phi_6}{\partial\gamma}|_{\gamma\to\bar{\gamma}}=0$, we can deduce that $\frac{\partial\Phi_6}{\partial\gamma}\le 0$, that is, Φ_6 decreases in γ. As we have $\Phi_6|_{\gamma\to\bar{\gamma}}=\frac{1}{2\delta q}\ge 0$, we can deduce that $\Phi_6\ge 0$. Thus, $\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})^2}{\partial^2 c}\ge 0$, which implies that $\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})}{\partial c}$ increases in c.

Further, recall from Lemma 3 that $c\le\tilde{c}^{TR}(=\frac{\delta q(1-(1-\beta)\delta)(1-\beta\delta)}{1-(1-\beta)^2\delta^2-\gamma(2-\delta)\delta})$, and recall from the proof in Proposition 3 that $\Phi_1\ge 0$ and $\Phi_2\ge 0$. So we have

$$\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})}{\partial c}|_{c\to c^{TR}}=-\frac{\delta(1-\beta\delta+\gamma(2-\delta))\Phi_1}{2(2-\delta)\Phi_2}\le 0,$$

that is, $\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})}{\partial c}$ increases to negative, i.e., $\frac{\partial(\pi_m^{TR*}-\pi_m^{N*})}{\partial c}\le 0$. Thus, $\pi_m^{TR*}-\pi_m^{N*}$ decreases in c.

Since we have

$$(\pi_m^{TR*}-\pi_m^{N*})|_{c\to 0}=\frac{\delta q}{4}-F_t-F_r,$$

and

$$\left(\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*}\right)|_{c\to\tilde{c}^{\mathrm{TR}}} = \frac{\delta^2 q(\delta((1-\beta)\beta\delta+1-\delta)+1)(\delta(\beta+\gamma-1)-2\gamma+1)^2}{4(2-\delta)(\delta(\gamma(\delta-2)-(\beta-1)^2\delta)+1)^2} - F_t - F_r.$$

Let

$$F_{t,r} = \frac{\delta^2 q(\delta((1-\beta)\beta\delta+1-\delta)+1)(\delta(\beta+\gamma-1)-2\gamma+1)^2}{4(2-\delta)(\delta(\gamma(\delta-2)-(\beta-1)^2\delta)+1)^2}. \tag{EC.60}$$

Therefore, if $F_t + F_r \geq \frac{\delta q}{4}$, then $(\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*})|_{c\to 0} \leq 0$, that is, $\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*}$ decreases in c from negative, i.e., $\pi_m^{\mathrm{TR}*} \leq \pi_m^{\mathrm{N}*}$. If $F_t + F_r \leq F_{t,r}$, then $(\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*})|_{c\to\tilde{c}^{\mathrm{TR}}} \geq 0$, that is, $\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*}$ decreases in c to positive, i.e., $\pi_m^{\mathrm{TR}*} \geq \pi_m^{\mathrm{N}*}$. However, if $F_{t,r} \leq F_t + F_r \leq \frac{\delta q}{4}$, then $\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*}$ decreases in c from positive to negative. Thus, $\pi_m^{\mathrm{TR}*} \geq \pi_m^{\mathrm{N}*}$ if $c \leq c_4$; else if $c \geq c_4$, then $\pi_m^{\mathrm{TR}*} \leq \pi_m^{\mathrm{N}*}$, in which c_4 is the unique solution of $\pi_m^{\mathrm{TR}*} - \pi_m^{\mathrm{N}*} = 0$ within the range of $[0, \tilde{c}^{\mathrm{TR}}]$, that is

$$c_4 = \frac{\delta q - 4(F_t + F_r)}{\sqrt{\Phi_7} + 1}, \tag{EC.61}$$

in which $\Phi_7 = \frac{4(F_t+F_r)(\delta(2\gamma(\delta-2)((\beta-1)\delta+1)+\delta(\beta(2\beta(\delta-1)-3\delta+4)+\delta-1)+\gamma^2(\delta-2)^2-2)+2)-\delta^2 q(\delta(\beta+\gamma-1)-2\gamma+1)^2}{(\delta-2)\delta q((\beta-1)\beta\delta^2+\delta-1)}$.

In conclude, when $F_t + F_r \geq \frac{\delta q}{4}$, we have $\pi_m^{\mathrm{TR}*} \leq \pi_m^{\mathrm{N}*}$. When $F_t + F_r \leq F_{t,r}$, we have $\pi_m^{\mathrm{TR}*} \geq \pi_m^{\mathrm{N}*}$. However, when $F_{t,r} \leq F_t + F_r \leq \frac{\delta q}{4}$, we have $\pi_m^{\mathrm{TR}*} \geq \pi_m^{\mathrm{N}*}$ if $c \leq c_4$; else if $c \geq c_4$, we have $\pi_m^{\mathrm{TR}*} \leq \pi_m^{\mathrm{N}*}$. □

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