THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Control and Communication-Schedule Co-Design for Networked Control Systems

Based on Robust Invariance and Model Predictive Control

Masoud Bahraini

Department of Electrical Engineering
Chalmers University of Technology
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MASOUD BAHRAINI
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Department of Electrical Engineering
Chalmers University of Technology
SE-412 96 Gothenburg, Sweden
Phone: +46 (0)31 772 1000
Email: masoudb@chalmers.se; masoud8841@gmail.com

Cover:
An illustration by the author of multi-agent networked control systems with a shared multi-channel wireless communication medium

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I dedicate this thesis to Nika Shakarami and brave women of Iran

این رساله دکتری را تقدیم میکنم به نیکا شاکرمی و زنان دلیر ایران

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Abstract

In a networked control system (NCS), the control loop is closed through a communication medium. This means that sensor measurements and/or control signals can be exchanged through a communication link. NCSs have many benefits, such as wiring reduction (elimination in the case of wireless communication), installation cost reduction, and simplification of upgrades and restructuring. However, network congestion, impairments of the wireless links (such as bandwidth limitations, packet losses, delays, and noises) may degrade system performance and even cause instability. These issues have motivated a great deal of research over the past 20 years and have given rise to a number of approaches to prevent congestion and compensate for delays and/or packet losses.

An interesting class of NCSs that has not received enough attention is an NCS whose systems are uncertain and subject to state and inputs hard constraints. These hard constraints may stem from the system itself, its environment, or be proposed by the designer in order to guarantee safety or a certain performance.

The contribution of this thesis is introducing a design framework that guarantees robust constraint satisfaction for a class of multi-agent NCSs with a shared communication medium that is subject to bandwidth limitation and prone to packet losses.

The proposed framework is built upon reachability analysis to determine the communication demand for each system such that local constraints are satisfied and scheduling techniques to guarantee satisfaction of the communication demands. The thesis explores offline and online scheduling designs under various communication topologies, optimal control designs under state and output feedback, and scheduling and control co-design for NCSs with hard constraints.

Keywords: Networked Control Systems, Robust Invariance, Model Predictive Control, Communication Scheduling.

List of Publications

This thesis is based on the following publications:

[A] Alessandro Colombo, **Masoud Bahraini**, Paolo Falcone, "Measurement scheduling for control invariance in networked control systems". CDC 2018.

[B] **Masoud Bahraini**, Mario Zanon, Alessandro Colombo, Paolo Falcone, "Receding-horizon robust online communication scheduling for constrained networked control systems". ECC 2019.

[C] **Masoud Bahraini**, Mario Zanon, Paolo Falcone, Alessandro Colombo, "Scheduling and Robust Invariance in Networked Control Systems". TAC 2021.

[D] **Masoud Bahraini**, Mario Zanon, Alessandro Colombo, Paolo Falcone, "Optimal Control Design for Perturbed Constrained Networked Control Systems". LCSS 2021.

[E] **Masoud Bahraini**, Mario Zanon, Alessandro Colombo, Paolo Falcone, "Optimal scheduling and control for constrained multi-agent networked control systems". OCAM 2022.

[F] **Masoud Bahraini**, Alessandro Colombo, Mario Zanon, Paolo Falcone, "Robust Control Invariance for Networked Control Systems with Output Feedback". CDC 2022.

[G] **Masoud Bahraini**, Mario Zanon, Alessandro Colombo, Paolo Falcone, "Communication Demand Minimization for Perturbed Networked Control Systems with Coupled Constraints". CDC 2023.

Other publications by the author, not included in this thesis, are:

[H] Alessandro Colombo, **Masoud Bahraini**, Mario Zanon, Paolo Falcone, "Simultaneously stabilizing networked systems with minimal communication". Submitted to IEEE Transactions on Automatic Control.

Acknowledgments

As I approach the end of this expedition, a profound realization dawns upon me - the essence of this journey lies in the cherished memories and remarkable experiences that I have encountered alongside my dear friends and colleagues. Thus, I find it imperative to pause and extend my heartfelt gratitude to all those who, in diverse ways, have contributed to making this expedition extraordinary.

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Masoud Bahraini,
Göteborg, August, 2023.

Acronyms

NCS:	Networked Control System
MAC:	Medium Access Control
MPC:	Model Predictive Control
RCI:	Robust Control Invariant
PP:	Pinwheel Problem
WSP:	Windows Scheduling Problem
RHC:	Receding Horizon Control
LTI:	Linear Time Invariant
RPI:	Robust Positive Invariant
MRPI:	Maximal Robust Positive Invariant
mRPI:	Minimal Robust Positive Invariant
RCI:	Robust Control Invariant
MRCI:	Maximal Robust Control Invariance
BPP:	Bin Packing Problem

Contents

II Papers 61

Part I

Overview

CHAPTER 1

Introduction

Networked Control Systems (NCSs) consist of a set of spatially distributed systems in which sensors, actuators, and controllers communicate through a shared band-limited digital network. The use of a shared network to connect spatially distributed elements enables a flexible structure with reduced maintenance and installation costs in general [1]. As a result, NCSs are used in a broad range of areas such as wireless sensor networks [2]–[5], autonomous driving [6], [7], remote surgery [8], [9], and industrial automation [10], [11], to name a few. However, imperfections of the communication link, such as bandwidth limitation and packet loss, may degrade the performance or cause instability [12]. Therefore, it is necessary to consider in control design the communication link imperfections and the Medium Access Control (MAC) mechanism employed. There are typically two types of MAC mechanisms: (a) random access (event-triggered), in which systems get access to the network randomly; and (b) scheduling (time-triggered), which determines which system gets access to the network at each time slot [13]. While event-triggered schemes may reduce the communication frequency in some cases, assessing their performance is generally challenging. Furthermore, scheduling schemes may perform better when the effect of the shared communication network is considered explicitly [14]. Moreover, constraint satisfaction is hard to guarantee, if possible at all, under random access schemes.

NCSs composed of systems with hard state and input constraints constitute a class that has been largely overlooked. State and input constraints may arise from physical limitations or control design to guarantee safety or performance requirements. This class of NCSs has applications in safety-critical systems, such as autonomous driving. Control and communication schedule co-design for constrained NCSs is the main topic of this thesis.

1.1 Research Question

The work presented in this thesis has specifically focused on addressing the following research question:

Research Question: How to co-design communication schedule and control for perturbed multi-agent NCSs with a lossy shared communication medium to ensure satisfaction of state and input constraints for all systems?

This primary research question can be further broken down into several sub-questions, considering a multi-agent NCS with a shared communication medium and constrained perturbed systems:

- **Q1:** How to determine the schedulability of the network when stabilizing feedback policies are applied to each system?
- **Q2:** If the network is found to be schedulable (as answered in **Q1**), how can a feasible schedule be obtained?
- **Q3:** How can **Q1** and **Q2** be addressed in the presence of a lossy communication medium?
- **Q4:** How does control design influence schedulability?
- **Q5:** What are the methodologies to design optimal control policies for the systems to enable schedulability?
- **Q6:** How can optimal scheduling and control be co-designed effectively?

To answer **Q1-Q6**, we employ reachability analysis, Model Predictive Control (MPC), and various scheduling techniques and findings from existing literature, which are briefly introduced in the following subsection.

Constrained Networked Control Systems

MPC, initially discovered in the 1960s and later rediscovered in the 1970s [15], stands as the primary systematic control design technique explicitly accounting for state and input constraints. A more recent development is the robust version of MPC, known as tube-based MPC, which ensures constraint satisfaction in the presence of bounded disturbances [16], [17]. Further advancements have been made in [18], where control design incorporates perturbed output measurements. However, these outcomes cannot be directly applied to NCSs due to potential feedback loop disruptions caused by network imperfections, such as delays and packet losses.

In the context of unconstrained linear systems, the co-design of control and communication schedules has been explored in [19] using the rollout approach. This method finds optimal control inputs and transmission decisions over a horizon, assuming that the optimal control policy is periodically employed after the horizon. The approach is extended in [20] to address state and input constraints. While investigations in [19], [20] consider a static network, where a fixed communication budget is used over a finite time, the rollout approach has been applied in [21], [22] under a traffic shaping scheme for communication, also known as a token-bucket network. In this dynamic setup, communication resources can be accumulated by not transmitting. The proposed rollout scheme in [22] is subsequently extended to handle constrained systems with bounded disturbances [23] through the use of the so-called multi-step, or H, robust control Invariant (RCI) sets. The multi-step RCI set ensures constraint satisfaction during intervals when no transmission is triggered. Building upon these principles, the Rollout scheme and H-RCI set concept are more recently utilized in [24] for output feedback design, an extension of robust output feedback MPC in [18]. While packet loss has been considered in [25], the focus was limited to undisturbed constrained linear systems.

The preceding research studies explore various aspects of perturbed constrained NCSs. However, they are limited to examining communication rates and lack proposals for a medium access scheme in multi-agent scenarios. Additionally, these studies have not addressed the issue of robust constraint satisfaction in the presence of packet loss.

Scope

This thesis centers around the co-design of control and communication schedules for multi-agent NCSs utilizing a shared and lossy communication medium. The systems are generally regarded as discrete linear time-invariant (LTI) with bounded process and measurement disturbances. Notably, the thesis does not delve into stochastic scenarios; rather, its primary objective is to guarantee robust satisfaction of state and input constraints.

Contributions

The main contributions of this thesis are as follows:

- Necessary and sufficient conditions are established for the existence of a feasible communication schedule for perturbed constrained multi-agent NCSs and heuristic approaches are proposed to finding feasible schedules under various communication topology scenarios, including those with packet loss (Paper A, Paper B, Paper C);
- The thesis presents optimal robust control designs, based on accurate state or perturbed output measurements, for perturbed constrained multi-agent NCSs. These designs enable schedulability by minimizing the communication demand for each system (Paper D, Paper F);
- Online joint communication schedule and control design method are introduced for constrained multi-agent NCSs (Paper E);
- The thesis proposes an optimal constraint decoupling technique for coupled multi-agent NCSs to enable schedulability (Paper G).

1.2 Thesis Outline

This thesis is divided into two parts. Part I comprises six chapters that lay the groundwork for the research. In Chapter 1, the research question is introduced, along with an overview of relevant studies, the scope of the research, and the thesis outline. Chapter 2 revisits technical definitions and results on MPC, reachability, invariance, and scheduling, providing the essential technical background for the subsequent chapters. Chapter 3 presents relevant background and results on scheduling techniques utilized in the thesis, while

Chapter 4 delves into the control techniques and results used. The summary of the included publications in Part II is presented in Chapter 5, and Chapter 6 offers concluding remarks, along with several suggestions for future research directions.

Part II comprises seven pertinent publications that contribute to addressing the research question posed in this thesis.

CHAPTER 2

Technical Background

This chapter serves as an introduction to MPC and two scheduling techniques used in the thesis. The definitions and results presented here lay the foundation of the rest of the thesis. In the first section, standard MPC, recursive feasibility, robust invariance, robust MPC, and robust output feedback MPC are reviewed. The second section focuses on the Pinwheel Problem (PP) and the Windows Scheduling Problem (WSP) from the scheduling literature, along with various available results on the schedulability of these problems.

2.1 Model Predictive Control

This section introduces MPC, recursive feasibility, robust invariance, robust MPC, and robust output feedback MPC. Interested reader is referred to [26] for a comprehensive reading.

Standard Model Predictive Control

Dynamic optimization is a widely used tool for decision-making across a diverse range of applications. These applications encompass tasks like determining the most fuel-efficient flight path for an airplane between two cities or identifying the most cost-effective approach to operate a chemical plant. Such problems can be formulated and solved as dynamic optimization problems. Dynamic optimization is often solved based on a dynamic model, i.e.,

$$x(k+1) = f(x(k), u(k)), \quad x(0) = x_0 \tag{2.1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, x_0 is the initial state, and $k \in \mathbb{Z}^+$ represents time in the discrete domain. The cost function for the optimization can be expressed as

$$\min_{u(0),\ldots,u(N-1)} \sum_{i=0}^{N-1} q(x(i), u(i)) + p(x(N)) \tag{2.2}$$

where $q(x, u)$ and $p(x)$ are the stage and terminal costs, respectively. Note that the cost function is defined over the finite horizon N. In order to find the optimal decisions, one can solve the problem

$$\min_{u(0),\ldots,u(N-1)} \sum_{i=0}^{N-1} q(x(i), u(i)) + p(x(N)) \tag{2.3a}$$

$$x(k+1) = f(x(k), u(k)), \quad \forall k \in \{0, 1, \ldots, N-1\} \tag{2.3b}$$

$$h(x(k), u(k)) \leq 0, \quad \forall k \in \{0, 1, \ldots, N-1\} \tag{2.3c}$$

$$x(0) = x_0 \tag{2.3d}$$

where inequities (2.3c) represents state and input constraints. Solving the optimization problem (2.3) yields the optimal decisions, i.e., $u^\star(0), \ldots, u^\star(N-1)$. However, usually not all optimal decisions are applied to the actual system. In order to deal with model inaccuracies, Receding Horizon Control (RHC) can be used as follows:

1. solve (2.3) and find optimal decisions $u^\star(0), \ldots, u^\star(N-1)$,
2. apply $u^\star(0)$ to the system and discard $u^\star(1), \ldots, u^\star(N-1)$,
3. set x_0 as the measured state value at the following time instant,
4. go to step 1.

The RHC scheme incorporates a closed-loop feedback mechanism that enhances the robustness of the calculated optimal decisions and mitigates model inaccuracies to a certain degree. Instead of solving the optimization problem (2.3) repeatedly, one might be able to solve it explicitly, i.e., $\mathbf{u}^\star = g(x_0)$, under some simplifying assumptions, which is beyond the scope of this thesis.

While the RHC scheme addresses model mismatches through repeated optimizations, closed-loop stability of the system is not guaranteed (stability is not discussed in this thesis). Furthermore, while the optimization problem (2.3) may be feasible for the initial state $x(0)$, there is no guarantee, in general, that admissible optimizers exist along the closed-loop state trajectory. The latter issue is known as recursive feasibility, which is discussed in the following subsection.

Invariance and Recursive Feasibility

In the RHC scheme, a constrained optimization problem is solved recursively. The optimization problem should be initially feasible, otherwise constraint softening may be applied when appropriate, see [27]. Given initial feasibility, recursive feasibility can be guaranteed using invariant sets, as described next.

Consider an autonomous discrete linear time invariant (LTI) system, described by

$$x(t+1) = Ax(t) \tag{2.4a}$$

$$x(t) \in \mathcal{X}, \quad \forall t \geq 0 \tag{2.4b}$$

where $\mathcal{X}$ is a polyhedron that represents the state's admissible set.

Definition 1 (Positive Invariant Set)**:** *Set $\mathcal{S} \subseteq \mathcal{X}$ is a positively invariant set for system* (2.4) *if*

$$x(0) \in \mathcal{S} \implies x(t) \in \mathcal{S},\ \forall t \geq 0. \tag{2.5}$$

Definition 2 (Maximal Positive Invariant Set)**:** *Consider $\{\mathcal{S}\}$ as the set of all invariant sets for system* (2.4)*. Then, $\mathcal{S}_\infty$ is the maximal positively invariant set for the system if $\mathcal{S}_\infty \in \{\mathcal{S}\}$ and*

$$\mathcal{S} \subseteq \mathcal{S}_\infty, \quad \forall \mathcal{S} \in \{\mathcal{S}\}. \tag{2.6}$$

Set $\mathcal{S}_\infty$ can be determined using Algorithm 3.1 in [28] (or similarly Algorithm 11.1 in [26]). While set $\mathcal{S}_\infty$ is not finitely determined in general, it becomes finitely determined under some conditions.

Theorem 1 (slightly rephrased Theorem 4.1 in [28])**:** *Suppose the following assumptions hold: i)* $\max|\lambda_i(A)| < 1$*, ii) set* $\mathcal{X}$ *is fully dimensional and bounded, iii) set* $\mathcal{X}$ *includes the origin in its interior. Then, set* $\mathcal{S}_\infty$ *is finitely determined.*

Invariant sets can also be defined for non-autonomous systems. Consider the following LTI system, described by

$$x(t+1) = Ax(t) + Bu(t) \tag{2.7a}$$

$$x(t) \in \mathcal{X}, \quad \forall t \geq 0 \tag{2.7b}$$

$$u(t) \in \mathcal{U}, \quad \forall t \geq 0, \tag{2.7c}$$

where sets $\mathcal{X}$ and $\mathcal{U}$ are polyhedrons that represent the state and input admissible sets.

Definition 3 (Control Invariant Set)**:** *Set* $\mathcal{C} \subseteq \mathcal{X}$ *is a control invariant set for system* (2.7) *if*

$$x \in \mathcal{C} \implies \exists u \in \mathcal{U} \;\; s.t. \; (Ax + Bu) \in \mathcal{C}. \tag{2.8}$$

Definition 4: *[Maximal Control Invariant Set] Consider* $\{\mathcal{C}\}$ *as the set of all control invariant sets for system* (2.7)*. Then,* $\mathcal{C}_\infty$ *is the maximal control invariant set for the system if* $\mathcal{C}_\infty \in \{\mathcal{C}\}$ *and*

$$\mathcal{C} \subseteq \mathcal{C}_\infty, \quad \forall \mathcal{C} \in \{\mathcal{C}\}. \tag{2.9}$$

One can use Algorithm 11.2 in [26] for computation of $\mathcal{C}_\infty$. In general, this algorithm may not be finitely determined and the approximated set computed iteratively in the algorithm may not even converge to $\mathcal{C}_\infty$. Convergence and finite determinability of $\mathcal{C}_\infty$ is guaranteed in [29] under several restrictive conditions. For example, in cases where control input $u(t)$ is unbounded. However, as conjectured by the study, set $\mathcal{C}_\infty$ may be finitely determined under a much more general setting. In cases where $\mathcal{C}_\infty$ is not finitely determined, one can use an invariant approximation of the set instead, which is finitely determined under certain conditions [30].

Given a control invariant set $\mathcal{C}$ for system (2.7), one can modify optimization problem (2.3) used in the RHC scheme to guarantee recursive feasibility as follows:

$$\min_{u(0),\ldots,u(N-1)} \sum_{i=0}^{N-1} q(x(i),u(i)) + p(x(N)) \tag{2.10a}$$
$$x(k+1) = Ax(k) + Bu(k), \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.10b}$$
$$x(k) \in \mathcal{X}, \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.10c}$$
$$u(k) \in \mathcal{U}, \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.10d}$$
$$x(N) \in \mathcal{C} \tag{2.10e}$$
$$x(0) = x_0. \tag{2.10f}$$

Optimization problem (2.10) is recursively feasible due to the terminal constraint (2.10e). Assume that (2.10) is initially feasible and consider the following sequence as its optimizer, $u^\star(0),\ldots,u^\star(N-1)$. After applying $u^\star(0)$ to the system, one feasible solution for the next iteration is $u^\star(1),\ldots,u^\star(N-1),\bar{u}$, where $\bar{u} \in \mathcal{U}$ is defined such that $(Ax(N)+B\bar{u}) \in \mathcal{C}$. Note that the control input $\bar{u}$ exists since $\mathcal{C}$ is control invariant. Interested reader is referred to [31], [32] for more details on recursive feasibility and stability.

While recursive feasibility can be guaranteed using control-invariant sets, as described above, the dynamical model of the system may be inaccurate, and recursive feasibility may not hold in the presence of unknown perturbations. This issue is addressed next.

Robust MPC

In this subsection, a robust MPC scheme is recalled, which guarantees stability and recursive feasibility in presence of bounded perturbations.

Consider the following system

$$x(t+1) = Ax(t) + Bu(t) + w(t), \quad \forall t \geq 0 \tag{2.11a}$$
$$x(t) \in \mathcal{X},\ u(t) \in \mathcal{U},\ w(t) \in \mathcal{W}, \quad \forall t \geq 0, \tag{2.11b}$$

where $w(t)$ is an unknown disturbance and sets $\mathcal{X}$, $\mathcal{U}$, and $\mathcal{W}$ are bounded polyhedrons which include the origin within their interiors.

In order to tackle recursive feasibility, robust invariance is recalled next.

Definition 5 (Robust Positive Invariant Set)**:** *Set $\mathcal{S} \subseteq \mathcal{X}$ is a robust positively invariant (RPI) set for system* (2.11) *with $B = \mathbf{0}$, if*

$$x(0) \in \mathcal{S} \implies x(t) \in \mathcal{S},\ \forall w(t) \in \mathcal{W},\ \forall t \geq 0. \tag{2.12}$$

Definition 6 (Maximal Robust Positive Invariant Set)**:** *Consider* $\{\mathcal{S}\}$ *as the set of all invariant sets for system* (2.11) *with* $B = \mathbf{0}$. *Then,* $\mathcal{S}_\infty$ *is the maximal robust positively invariant (MRPI) set for the system if* $\mathcal{S}_\infty \in \{\mathcal{S}\}$ *and*

$$\mathcal{S} \subseteq \mathcal{S}_\infty, \quad \forall \mathcal{S} \in \{\mathcal{S}\}. \tag{2.13}$$

Set $\mathcal{S}_\infty$ can be determined using Algorithm 11.4 in [26]. While set $\mathcal{S}_\infty$ is not finitely determined in general, it becomes finitely determined under some conditions. For instance, when eigenvalues of A are within the unitary circle. Also note that the algorithm might return an empty set, which implies that no RPI set for the system exists.

Definition 7 (minimal Robust Positive Invariant Set)**:** *Consider* $\{\mathcal{S}\}$ *as the set of all positively invariant sets for system* (2.11) *with* $B = \mathbf{0}$. *Then,* $\mathcal{S}_0$ *is the minimal robust positively invariant (mRPI) set for the system if* $\mathcal{S}_0 \in \{\mathcal{S}\}$ *and*

$$\mathcal{S}_0 \subseteq \mathcal{S}, \quad \forall \mathcal{S} \in \{\mathcal{S}\}. \tag{2.14}$$

The mRPI set, described by

$$\mathcal{S}_0 = \bigoplus_{i=0}^{\infty} A^i \mathcal{W}, \tag{2.15}$$

is generally impossible to explicitly characterize, see [33]. While the explicit characterization of $\mathcal{S}_0$ is only possible when A is nilpotent [34], one can use invariant outer-approximations of the mRPI set, see [33], [35].

Definition 8 (Robust Control Invariant Set)**:** *Set* $\mathcal{C} \subseteq \mathcal{X}$ *is a robust control invariant (RCI) set for system* (2.11) *if*

$$x \in \mathcal{C} \implies \exists u \in \mathcal{U} \quad s.t.\ (Ax + Bu + w) \in \mathcal{C},\ \forall w \in \mathcal{W}. \tag{2.16}$$

Definition 9 (Maximal Robust Control Invariant Set)**:** *Consider* $\{\mathcal{C}\}$ *as the set of all control invariant sets for system* (2.11). *Then,* $\mathcal{C}_\infty$ *is the maximal robust control invariant (MRCI) set for the system if* $\mathcal{C}_\infty \in \{\mathcal{C}\}$ *and*

$$\mathcal{C} \subseteq \mathcal{C}_\infty, \quad \forall \mathcal{C} \in \{\mathcal{C}\}. \tag{2.17}$$

Set $\mathcal{C}_\infty$ can be determined using Algorithm 11.5 in [26]. In general, MRCI set may not be finitely determined, as described following Definition 4.

An important problem that arises in robust MPC is that common definition of stability is not applicable to the states due to existence of a persistent unknown disturbance [17]. One alternative way of defining stability in this case is exponential convergence of the state to an RPI set $\mathcal{Z}$. This set may be regarded as the origin of the uncertain system and stability can be proved in this case using a Lyapunov function whose value is zero for all states in $\mathcal{Z}$. This notion of stability is used in a robust MPC framework that is described next based on the results in [17].

Consider system (2.11) and a constant feedback gain K such that $A_K = A + BK$ is stable. Also consider RPI set $\mathcal{Z}$ for the system under feedback policy $u(t) = Kx(t)$. The robust invariance of $\mathcal{Z}$ implies that

$$A_K\mathcal{Z} \oplus \mathcal{W} \subseteq \mathcal{Z}. \tag{2.18}$$

Invariance of set $\mathcal{Z}$ can be used to bound the error between the known and unknown parts of the dynamic as described next. Consider an unperturbed pair of the system (2.11), defined as

$$\bar{x}(t+1) = A\bar{x}(t) + B\bar{u}(t). \tag{2.19}$$

Considering a control input defined by

$$u(t) = \bar{u}(t) + K(x(t) - \bar{x}(t)), \tag{2.20}$$

and an initial condition that satisfies $x(0) \in \bar{x}(0) \oplus \mathcal{Z}$, one can prove that $x(t) \in \bar{x}(t) \oplus \mathcal{Z}$ for all $t \geq 0$ using (2.18), see Proposition 1 in [17]. As a result, a suboptimal MPC problem for the system (2.11) can be formulated as

$$\min_{\bar{x}(0),\bar{u}(0),\ldots,\bar{u}(N-1)} \sum_{i=0}^{N-1} q(\bar{x}(i), \bar{u}(i)) + p(\bar{x}(N)) \tag{2.21a}$$

$$\bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k), \quad \forall k \in \{0, 1, \ldots, N-1\} \tag{2.21b}$$

$$\bar{x}(k) \in \mathcal{X} \ominus \mathcal{Z}, \quad \forall k \in \{0, 1, \ldots, N-1\} \tag{2.21c}$$

$$\bar{u}(k) \in \mathcal{U} \ominus K\mathcal{Z}, \quad \forall k \in \{0, 1, \ldots, N-1\} \tag{2.21d}$$

$$\bar{x}(N) \in \mathcal{X}_f, \tag{2.21e}$$

$$x(0) \in \bar{x}(0) \oplus \mathcal{Z}, \tag{2.21f}$$

where the terminal set $\mathcal{X}_f$ satisfies

$$A_K\mathcal{X}_f \subset \mathcal{X}_f, \quad \mathcal{X}_f \subset \mathcal{X} \ominus \mathcal{Z}, \; K\mathcal{X}_f \subset \mathcal{U} \ominus K\mathcal{Z}. \tag{2.22}$$

Inclusions in (2.22) are used to guarantee recursive feasibility of the optimization problem (2.21). All points in $\mathcal{X}_f$ should also satisfy an additional technical condition to ensure stability, see [17].

In the optimization problem (2.21), nominal state and input constraints are tightened, i.e., $\mathcal{X} \ominus \mathcal{Z}$ and $\mathcal{U} \ominus K\mathcal{Z}$ instead of $\mathcal{X}$ and $\mathcal{U}$. However, the nominal system is not perturbed and its recursive feasibility is guaranteed through the choice of the terminal set $\mathcal{X}_f$. This results in robust satisfaction of $x(t) \in \mathcal{X}$ based on the following argument. Control input (2.20), coupled with the initial condition (2.21f), implies that $x(t) \in \bar{x}(t) \oplus \mathcal{Z}$ for all $t \geq 0$. This in turn implies that $x(t) \in \mathcal{X}$ for all $t \geq$, since $\bar{x}(t) \in \mathcal{X} \ominus \mathcal{Z}$ and $(\mathcal{X} \ominus \mathcal{Z}) \oplus \mathcal{Z} \subseteq \mathcal{X}$.

An innovating idea used in the optimization problem (2.21) is that the initial nominal state $\bar{x}(0)$ is a free variable, which enables the proof of stability and also eliminates some restrictions on the terminal set. Also note that state $x(t)$ is restricted to $\bar{x}(t) \oplus \mathcal{Z}$, a tube of trajectories whose center, i.e., $\bar{x}(t)$, can be controlled to a certain extent through the choice of $\bar{x}(0)$ at each iteration.

While the optimization problem (2.21) coupled with control input (2.20) can be used to design a robust MPC that is recursively feasible, exact measurement of the current state is assumed to be available at each time instant. To relax this assumption, a robust MPC scheme is presented next, which is designed based on perturbed output measurements.

Robust Output Feedback MPC

In this subsection, a robust output feedback MPC scheme is recalled, which guarantees stability and recursive feasibility given perturbed output measurements.

Consider the following system

$$
\begin{aligned}
x(t+1) &= Ax(t) + Bu(t) + w(t), \quad \forall t \geq 0 && (2.23a)\\
y(t) &= Cx(t) + v(t) && (2.23b)\\
x(t) &\in \mathcal{X},\ u(t) \in \mathcal{U},\ w(t) \in \mathcal{W},\ v(t) \in \mathcal{V}, \quad \forall t \geq 0, && (2.23c)
\end{aligned}
$$

where $v(t)$ and $w(t)$ are unknown disturbances and sets $\mathcal{X}$, $\mathcal{U}$, $\mathcal{W}$, and $\mathcal{V}$ are bounded polyhedrons which include the origin within their interiors. Pairs (A, B) and (A, C) are assumed to be controllable and observable, respectively.

A simple Luenberger observer can be used to estimate the state as follows:

$$\hat{x}(t+1) = A\hat{x} + Bu(t) + L(y(t) - \hat{y}(t)), \quad \hat{y}(t) = C\hat{x}(t). \tag{2.24}$$

Consider the state estimation error $\tilde{x}(t)$ defined by $\tilde{x}(t) := x(t) - \hat{x}(t)$. Then, the error dynamics can be described by

$$\tilde{x}(t+1) = (A - LC)\tilde{x}(t) + (w(t) - Lv(t)). \tag{2.25}$$

Assuming that all eigenvalues of $(A-LC)$ are within the unitary circle, an RPI set $\tilde{\mathcal{S}}$ for system (2.25) exists that can be finitely determined. Consequently, $\tilde{x}(0) \in \tilde{\mathcal{S}}$ implies that $\tilde{x}(t) \in \tilde{\mathcal{S}}$ for any $v(t) \in \mathcal{V}$ and $w(t) \in \mathcal{W}$, for all $t \geq 0$. Stated differently, $\hat{x}(t) \oplus \tilde{\mathcal{S}}$ represents a tube of trajectories which encapsulates the state trajectory $x(t)$. Consequently, one can guarantee $x(t) \in \mathcal{X}$ indirectly by restricting $\hat{x}(t)$ to $\mathcal{X} \ominus \tilde{\mathcal{S}}$, since

$$x(t) \in \hat{x}(t) \oplus \tilde{\mathcal{S}} \subseteq (\mathcal{X} \ominus \tilde{\mathcal{S}}) \oplus \tilde{\mathcal{S}} \subseteq \mathcal{X}. \tag{2.26}$$

Using the results from the previous subsection, one can formulate a robust MPC such that $\hat{x}(t) \in \mathcal{X} \ominus \tilde{\mathcal{S}}$ and $u(t) \in \mathcal{U}$ in order to design a robust output feedback MPC. Consider the nominal system (2.19) and nominal state error $e(t) = \hat{x}(t) - \bar{x}(t)$, i.e., the difference between the estimated state and the nominal state. Then, one can design the control input as in (2.20), i.e.,

$$u(t) = \bar{u}(t) + Ke(t), \tag{2.27}$$

where eigenvalues of $(A + BK)$ are inside the unitary circle. The state estimator dynamics can be represented by

$$\hat{x}(t+1) = A\hat{x}(t) + B\bar{u}(t) + BKe(t) + LC\tilde{x}(t) + Lv(t), \tag{2.28}$$

with the nominal state error dynamics described by

$$e(t+1) = (A + BK)e(t) + LC\tilde{x}(t) + Lv(t). \tag{2.29}$$

Note that term $LC\tilde{x}(t) + Lv(t)$ represents a bounded disturbance to the nominal state error, since it is within the set $LC\tilde{\mathcal{S}} \oplus L\mathcal{V}$. Since $(A+BK)$ is stable, an RPI set $\bar{\mathcal{S}}$ for system (2.29) exists that can be finitely determined. As a result, $\hat{x}(0) \in \bar{x}(0) \oplus \bar{\mathcal{S}}$ implies that $\hat{x}(t) \in \bar{x}(t) \oplus \bar{\mathcal{S}}$ for all $t \geq 0$. Therefore, a suboptimal output feedback MPC for the system (2.23) can be formulated by solving

$$\min_{\bar{x}(0),\bar{u}(0),\ldots,\bar{u}(N-1)} \sum_{i=0}^{N-1} q(\bar{x}(i),\bar{u}(i)) + p(\bar{x}(N)) \tag{2.30a}$$

$$\bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k), \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.30b}$$

$$\bar{x}(k) \in \mathcal{X} \ominus (\bar{\mathcal{S}} \oplus \tilde{\mathcal{S}}), \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.30c}$$

$$\bar{u}(k) \in \mathcal{U} \ominus K\bar{\mathcal{S}}, \quad \forall k \in \{0,1,\ldots,N-1\} \tag{2.30d}$$

$$\bar{x}(N) \in \mathcal{X}_f, \tag{2.30e}$$

$$\hat{x}(0) \in \bar{x}(0) \oplus \bar{\mathcal{S}}, \tag{2.30f}$$

where $x(0) \in \tilde{x}(0) \oplus \tilde{\mathcal{S}}$ and $\mathcal{X}_f$ is a positive invariant set for the nominal system which satisfies

$$A_K\mathcal{X}_f \subset \mathcal{X}_f, \quad \mathcal{X}_f \subset \mathcal{X} \ominus (\bar{\mathcal{S}} \oplus \tilde{\mathcal{S}}), \;\; K\mathcal{X}_f \subset \mathcal{U} \ominus K\bar{\mathcal{S}}. \tag{2.31}$$

States in $\mathcal{X}_f$ should also satisfy an additional condition for stability of the recalled scheme, see Theorem 1 in [18].

Note that the optimization problem (2.30), coupled with the state estimator (2.24), is recursively feasible and satisfaction of the state and input constraints is guaranteed, i.e., $x(t) \in \mathcal{X}$ and $u(t) \in \mathcal{U}$ for all $t \geq 0$.

Multi-step Robust Invariance

In NCSs, a system's feedback-loop may not be closed at all time instants. In such cases, one can still guarantee invariance, and therefore satisfaction of the state and input constraints, through H-step invariant sets [36], defined as follows.

Definition 10: *Set $\mathcal{S}$ is an H-step RPI set for the system* (2.11) *if $\mathcal{S} \subseteq \mathcal{X}$, $K\mathcal{S} \subseteq \mathcal{U}$, and*

$$(A+BK)^i\mathcal{S} \oplus \bigoplus_{j=0}^{i-1} A^j\mathcal{W} \subseteq \mathcal{S}, \quad \forall i \in \{1,\ldots,H\}. \tag{2.32}$$

Existence of an H-step RPI set for the system (2.11) implies that the feedback policy $u(t) = K\bar{x}(t)$, with

$$\bar{x}(t+1) = \begin{cases} (A+BK)\bar{x}(t), & \text{open-loop} \\ x(t+1), & \text{closed-loop} \end{cases}, \tag{2.33}$$

guarantees invariance of $\mathcal{S}$, and therefore satisfaction of the state and input constraints, assuming that $\bar{x}(0) = x(0) \in \mathcal{S}$ and the feedback loop is closed at least once during each H consecutive time instants for all $t \geq 0$.

2.2 Scheduling

This section introduces two scheduling problems from the literature: the Pinwheel Problem (PP), and the Windows Scheduling Problem (WSP). These specific scheduling problems serve as the foundation for communication schedule design in this thesis.

Pinwheel Problem

The PP emerged from communication of satellites with a ground station [37]. The problem is formulated as follows.

Problem 1 (PP)**.** Consider a set of positive integers $I = \{\alpha_1, \ldots, \alpha_q\}$. Infinite sequence (schedule) $\mathbf{S}$ of labels $\{1, \ldots, q\}$ is a feasible schedule for instance I if

$$i \in \{S(j), S(j+1), \ldots, S(j+\alpha_i-1)\}, \quad \forall j \geq 1, \ \forall i \in \{1, 2, \ldots, q\}. \tag{2.34}$$

Investigation of decidability of the PP in general appears to be intractable due to the infinite length of the schedule. Fortunately, the decision can be restricted to periodic schedules with a limited length due to the following result.

Theorem 2 (Theorem 2.1 in [37])**:** *If instance $I = \{\alpha_1, \ldots, \alpha_q\}$ is schedulable, as specified by* (2.34)*, then instance I has a cyclic schedule whose period is no greater than $\prod_{i=1}^{q} \alpha_i$.*

The above theorem suggests that the PP is decidable since exhaustive search over periodic schedules can be performed to find a feasible schedule if it exists. In order to categorize different instances of the PP, consider density function

$$\rho(I) = \sum_{i=1}^{q} \frac{1}{\alpha_i}. \tag{2.35}$$

Since each label i should appear at least once during each α_i consecutive elements of the schedule, $\frac{1}{\alpha_i}$ represents the minimum portion of the schedule's elements which has to be equal to i. As a result, any instance I with $\rho(I) > 1$ is not schedulable since more than 100% of the schedule should be allocated to the labels $\{1, \ldots, q\}$, see Theorem 2.3 in [37].

It is shown that all instances with $\rho(I) \leq 0.75$ are schedulable and the upper bound for schedulability is conjectured to be $\frac{5}{6}$ [38]–[40]. There indeed exists instances with $\frac{5}{6} \leq \rho(I) \leq 1$ that are schedulable and instances that are not schedule, for example instance $I_1 = \{2, 3, 6\}$ is not schedulable while instance $I_2 = \{3, 3, 3\}$ is schedulable.

Windows Scheduling Problem

The WSP is a generalized version of the PP, where m_c number of communication channels are available instead of only one [41], [42]. One can formulate the WSP as follows.

Problem 2. Consider a set of positive integers $I = \{\alpha_1, \ldots, \alpha_q\}$ and m_c communication channels. Infinite sequence (schedule) of ordered tuples $\mathbf{C}$ is a feasible schedule for the problem instance $\{m_\mathrm{c}, I\}$ if $C(t) = (c_{1,t}, c_{2,t}, \ldots, c_{m_\mathrm{c},t})$ is such that

$$\exists j \in \{1, \ldots, m_\mathrm{c}\},\ \exists k \in \{t, \ldots, t + \alpha_i - 1\} \text{ s.t. } \ c_{j,k} = i, \tag{2.36}$$

for all $i \in \{1, \ldots, q\}$ and for all $t \geq 1$.

Notice that the WSP reduces to the PP when one communication channel is available, i.e., $m_\mathrm{c} = 1$. Some of the results regarding schedulability of the PP can be extended to the WSP. For example, the necessary condition on the density function for schedulability of an instance of the PP, i.e., $\rho(I) \leq 1$, can be extended to $\rho(I) \leq m_\mathrm{c}$, which is the necessary condition for schedulability of an instance of the WSP. A heuristic for finding a feasible schedule for an instance of the WSP is to break instance I into m_c instances $I_1, \ldots, I_{m_\mathrm{c}}$ such that

$$\cup_{i=1}^{m_\mathrm{c}} I_i = I, \quad \rho(I_i) \leq 0.75, \quad \forall i \in \{1, \ldots, m_\mathrm{c}\}. \tag{2.37}$$

Note that (2.37) describes a Bin Packing Problem (BPP), see [42], and if a feasible solution for (2.37) exists, then one can find a feasible schedule for the WSP as explained in the following. Consider I_i as an instance of the PP and find a corresponding feasible schedule $\mathcal{S}_i$ for that instance; then schedule $\mathbf{C}$ with

$$C(t) = (S_1(t), \ldots, S_{m_\mathrm{c}}(t)), \tag{2.38}$$

is a feasible schedule for the instance $\{m_c, I\}$ of the WSP.

Note that finding a feasible schedule for the WSP through the BPP results in a so called *perfect* schedules. A perfect schedule, also known as a schedule with no migration, is a schedule where each system i is only scheduled through one of the communication channels. For example, in (2.37), all systems in instance I_j are scheduled through channel j. One of the open problems stated in [42] is whether migration is helpful? i.e., is there an instance of the WSP for which a feasible schedule exists but a perfect schedule does not?

2.3 Discussion

In this chapter, various results from control theory for constrained perturbed systems and a specific class of scheduling problems have been recalled. The first section presents the available control design techniques for closed-loop systems. In NCSs, the feedback loop is not always closed at all time instants, necessitating scheduling techniques to ensure frequent execution of each system's feedback loop. Thus, the second section introduces two scheduling problems to guarantee the timely closure of each system's feedback loop and avoid violations of state and input constraints. The subsequent two chapters address the utilization of these control and scheduling techniques in an NCS setup.

CHAPTER 3

Scheduling Design for Networked Control Systems

This chapter encompasses the scheduling-related contributions of the thesis, as proposed in Paper A, Paper B, Paper C, and Paper E. We introduce the concept of the *safe time interval*, which specifies the communication demand of a given system. The set of safe time intervals defines an instance of a scheduling problem, dependent on the network's topology, which we discuss throughout this chapter. In Section 3.1, we present offline scheduling approaches for different network topologies and provide necessary and sufficient conditions for schedulability. Furthermore, we demonstrate the design of online schedules that enhance performance based on current state measurements in Section 3.2.

3.1 Offline Scheduling

In this section we specify the communication demand for each system in an NCS and discuss our offline scheduling results for various communication topologies.

Safe Time Interval

Consider a multi-agent NCS, whose systems are described by

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t) + w_i(t), \quad \forall t \geq 0 \tag{3.1a}$$

$$x_i(t) \in \mathcal{X}_i,\ u_i(t) \in \mathcal{U}_i,\ w_i(t) \in \mathcal{W}_i, \quad \forall t \geq 0, \tag{3.1b}$$

where pair (A_i, B_i) are controllable, sets $\mathcal{X}_i$, $\mathcal{U}_i$, $\mathcal{W}_i$ are compact polytopes which include zero in their interiors, and $w_i(t)$ is an unknown disturbance.

Suppose that these systems have to share a communication medium, and only a subset of systems can transmit their state measurements to their corresponding controllers at each time instant. This implies that each system may evolve open-loop, and ensuring the satisfaction of its state and input constraints during the open-loop time intervals is not trivial.

Consider the set $\mathcal{S}_{i,\infty}$ as the MRPI set for the system (3.1) with $u_i(t) = K_i x_i(t)$. Furthermore, consider the following feedback policy

$$u_i(t) = K_i \hat{x}_i(t), \quad \hat{x}_i(t+1) = \begin{cases} (A_i + B_i K_i)\,\hat{x}_i(t), & \text{if disconnected} \\ x_i(t+1), & \text{if connected} \end{cases}, \tag{3.2}$$

where $\hat{x}_i(t)$ is the current state estimate, calculated in the controller, and $\hat{x}_i(0) = x_i(0)$. The state estimate is updated with the real value of the state at time instant t if system i gets access to the communication medium at this time instant. Consequently, the system may evolve open-loop during some time intervals. We define the safe time interval α_i for the system as

$$\alpha_i = \max\{t : \quad (A_i + B_i K_i)^t \mathcal{S}_{i,\infty} \oplus \bigoplus_{j=0}^{t-1} A_i^j \mathcal{W}_i \subseteq \mathcal{S}_{i,\infty}\}, \tag{3.3}$$

which is the longest time interval when system i can evolve in open-loop while its state remains within its MRPI set $\mathcal{S}_{i,\infty}$, assuming that the system's state belongs to this set initially. In another words, one can guarantee robust invariance, and hence satisfaction of the state and input constraints, of system i by guaranteeing that the system receives at least one state measurement during each α_i consecutive time instants.

Remark 1. Safe time interval α_i defined in (3.3), also similarly defined in papers A-C, is essentially the same as H, if maximized over the MRPI set $\mathcal{S}_{i,\infty}$, in (2.32). In Paper C through Paper G, the definition of α_i may differ, but the underlying concept remains consistent: it represents the longest time interval for which system i can evolve in open-loop while its state remains within a specified invariant set.

Single Channel

Consider a multi-agent networked control system, with q systems each of which described by (3.1). Furthermore, consider a shared communication medium with a single communication channel, i.e., only one system can communicate at each time instant.

Theorem 3: *[Theorem 1 in Paper A, rephrased] Consider $I = \{\alpha_1, \ldots, \alpha_q\}$ as the set of safe time intervals for the systems. The communication scheduling problem of the described NCS is equivalent to the PP with instance I.*

Theorem 3 connects the scheduling for the described NCS to the PP. A feasible schedule for instance I of the PP is such that each label i appears at least once during any time interval with length α_i. The same schedule is feasible for the communication scheduling problem, since each system i receives a state measurement at least once during each α_i consecutive time instants, i.e., the feedback loop for each system is closed frequently enough and robust invariance is preserved.

Since we connected the scheduling problem for the described NCS to the PP, we can use the available results [37]–[40], as recalled in the previous chapter, to find a feasible communication schedule.

Multi-Channels

Consider the described multi-agent NCS, with a shared communication medium which includes $m_\mathrm{c} \geq 1$ communication channels. In this case, m_c systems can receive their state measurements at each time instant.

Theorem 4: *[Theorem 3 in Paper C, rephrased] Consider $I_1 = \{\alpha_1, \ldots, \alpha_q\}$ as the set of safe time intervals for the systems. Communication scheduling for the described NCS is equivalent to the WSP with instance $I_2 = \{m_\mathrm{c}, I_1\}$.*

Theorem 4 connects the scheduling for the described NCS with the WSP and one can use available results on the WSP to find a communication schedule for the described NCS.

Next, we provide additional results on the WSP, which are proposed in Paper C. Consider $I_1 = \{m_\mathrm{c}, \{\alpha_1, \ldots, \alpha_q\}\}$ as an instance of the WSP.

Theorem 5: *[Theorem 4 in Paper C, rephrased] One can find a feasible schedule for instance I_1 of the WSP by finding a feasible schedule for instance $I_2 = \{m_c\alpha_1, \ldots, m_c\alpha_q\}$ of the PP.*

Given feasible schedule $\mathbf{C}_P$ for instance I_2 of the PP, a feasible schedule for instance I_1 of the WSP can be constructed as follows:

$$C(t) = (c_P(m_c t - m_c + 1), \ldots, c_P(m_c t)),\ t \geq 1. \tag{3.4}$$

Note that Theorem 5 provides a sufficient condition, i.e., if no feasible schedule for instance I_2 of the PP exists, a feasible schedule for instance I_1 of the WSP may still exist.

Lemma 1: *[Lemma 3 in Paper C, rephrased] A feasible perfect schedule for instance I_1 of the WSP exists only if a feasible schedule for instance $I_2 = \{m_c\alpha_1, \ldots, m_c\alpha_q\}$ of the PP exists.*

Lemma 1 suggests that existence of an admissible solution for instance I_2 of the PP is a necessary condition for existence of a perfect schedule for instance I_1 of the WSP.

Remark 2. In order to find a feasible schedule for an instance of the WSP, one can restrict the search space to perfect schedules and solve a BPP to break down the WSP into m_c number of PPs, see [42]. However, we advocate for solving the WSP without limiting the search space to perfect schedules. To achieve this, one can first find a feasible schedule for instance I_2 of the PP and then obtain an admissible schedule for the WSP using relation (3.4). Notably, this approach requires solving only one instance of the PP.

Our proposed heuristic for finding a feasible schedule for the WSP performs better than the one presented in [42] because our method will find a solution if a perfect schedule exists for the WSP, as demonstrated in Lemma 1. A material question, also mentioned as one of the open problems in [42], is whether restricting the scheduling search to only perfect schedules is limiting. If the answer to this question is affirmative, then does our proposed approach identify feasible schedules for the WSP even when no perfect schedule exists? These questions are addressed through the subsequent examples.

Example 1 (Example 4 from Paper C, rephrased)**.** Consider an instance of the WSP specified by

$$\{m_c, \{\alpha_i\}\} = \{2, \{2, 3, 4, 5, 5, 5, 7, 14\}\}. \tag{3.5}$$

In order to find a perfect schedule, one can first compute all possible allocations of the labels into two channels and verify that this instance of the WSP admits no perfect schedule. However, a schedule with the following cyclic part is feasible for instance $\{m_c\alpha_i\}$ of the PP:

$$2, 3, 4, 1, 7, 6, 2, 1, 5, 3, 2, 1, 4, 3, 6, 1, 2, 5, 7, 1, 4, 3, 2, 1, 6, 8, 5, 1.$$

The above schedule can be used to construct an admissible schedule for the WSP, as instructed in (3.4), with the following cyclic part:

$$\begin{aligned}\mathbf{C}_\mathrm{r} =&(2, 3), (4, 1), (7, 6), (2, 1), (5, 3), (2, 1), (4, 3),\\ &(6, 1), (2, 5), (7, 1), (4, 3), (2, 1), (6, 8), (5, 1). \end{aligned} \tag{3.6}$$

Example 1 illustrates that perfect scheduling is, indeed, restrictive. Secondly, it showcases that the proposed heuristic can identify a feasible schedule, even if it is non-perfect, when such a schedule exists. Notice that labels 5 and 6 appear in both channels in (3.6). However, it is important to note that the existence of a solution for the proposed heuristic is not a necessary condition for the existence of a feasible schedule for an instance of the WSP, as illustrated by the following example.

Example 2 (Example 3 from Paper C, rephrased)**.** Consider an instance of the WSP specified by

$$\{m_\mathrm{c}, \{\alpha_i\}\} = \{2, \{2, 3, 3, 4, 5, 5, 10\}\}. \tag{3.7}$$

While there is no feasible schedule for instance $\{m_\mathrm{c}\alpha_i\}$ of the PP, a schedule with the cyclic part

$$\begin{aligned}\mathbf{C}_\mathrm{r} =&(1, 2), (3, 4), (1, 6), (2, 5), (1, 3), (4, 7), (1, 2), (3, 6), (1, 5), (2, 4),\\ &(1, 3), (2, 4), (1, 6), (3, 5), (1, 2), (4, 7), (1, 3), (2, 6), (1, 5), (3, 4)\ , \end{aligned} \tag{3.8}$$

is feasible for instance $\{2, \{2, 3, 3, 4, 5, 5, 10\}\}$ of the WSP. Notice that (3.8) is not a perfect schedule, see labels 2, 3, and 4.

Proposition 1 (Proposition 4 from Paper C, rephrased)**:** *Given an instance $I = \{m_\mathrm{c}, \{\alpha_i\}\}$ of the WSP, $\rho(I) \leq 0.75m_\mathrm{c}$ is a sufficient condition for schedulability.*

The aforementioned proposition relies on the observation that all instances of the PP with $\rho(I) \leq 0.75$ are schedulable. The proposed heuristic, in turn, constructs a feasible schedule for the WSP based on a feasible schedule for instance $\{m_\mathrm{c}\alpha_i\}$ of the PP.

Connection Patterns

Consider the described multi-agent NCS, wherein a shared communication medium allows only a specific set of systems to communicate at each time instant. These sets are termed *connection patterns*, and the set of all such connection patterns is denoted by $\boldsymbol{\mathcal{C}}$. At each time t, the scheduler selects a connection pattern $C(t) \in \boldsymbol{\mathcal{C}}$, implying that system i receives communication through the network at time instant t if and only if i is part of $C(t)$. A feasible schedule for this NCS consists of an infinite sequence of connection patterns such that each label i appears at least once in a connection pattern during each consecutive set of α_i connection patterns for all i.

Remark 3. Finding a feasible schedule for the described NCS is equivalent to the PP when $\boldsymbol{\mathcal{C}} = \{\{1\}, \dots, \{q\}\}$. Likewise, finding a feasible schedule for the described NCS is equivalent to the WSP when $\boldsymbol{\mathcal{C}}$ is the set of all subsets of $\{1, 2, \dots, q\}$ with cardinality m_c.

While finding a feasible infinite schedule is not tractable in general, one can find a feasible cyclic schedule for the described NCS when a feasible schedule exists, see Corollary 1 in Paper C. In order to find a feasible schedule for the described NCS, one can solve the following optimization problem, which searches for a feasible periodic schedule among all schedules of period T_r.

$$\min_{C(1),\dots,C(T_\mathrm{r}),T_\mathrm{r}} \quad T_\mathrm{r} \tag{3.9a}$$

$$\text{s.t.} \quad C(1), \dots, C(T_\mathrm{r}) \in \boldsymbol{\mathcal{C}}, \tag{3.9b}$$

$$T_\mathrm{r} \leq \prod_{i=1}^{q} \alpha_i, \; T_\mathrm{r} \in \mathbb{N}, \tag{3.9c}$$

$$\sum_{k=t}^{t+\alpha_i-1} \eta_i(k) \geq 1, \qquad \forall i \in \{1, \dots, q\}, \; \forall t \in \{1, \dots, T_\mathrm{r}\}, \tag{3.9d}$$

$$\eta_i(k) = \begin{cases} 1 & \text{if } i \in C(k \bmod T_\mathrm{r}), \\ 0 & \text{otherwise.} \end{cases} \tag{3.9e}$$

Equation (3.9b) constrains the schedule elements to the given set of connection patterns, (3.9c) limits the length of the schedule's period, (3.9d) ensures that each system i is included in any α_i consecutive elements the schedule, and (3.9e) indicates which of the selected connection patterns $C(k)$ include label

i. Any feasible solution to the optimization problem (3.9) provides the periodic part of a feasible schedule for the described NCS, and the optimizer yields the periodic part of a feasible schedule with the shortest period. It is important to note that if (3.9) has no feasible solutions, there exists no feasible schedule for the NCS.

Regrettably, employing (3.9) to find a feasible schedule is generally impractical, given the combinatorial nature of the problem in relation to the number of systems and connection patterns. To tackle this issue, we propose a heuristic to find a feasible schedule based on the following optimization problem:

$$\min_{\hat{\rho}_j,\ \eta_{i,j}} \quad \sum_{j=1}^{l} \hat{\rho}_j \tag{3.10a}$$

$$\text{s.t.} \quad \hat{\rho}_j \geq \frac{1}{\alpha_i}\eta_{i,j}, \qquad \forall j \in \{1,\dots,l\},\ \forall i \in C_j, \tag{3.10b}$$

$$\sum_{j:i\in C_j} \eta_{i,j} \geq 1, \qquad \forall i \in \{1,\dots,q\}, \tag{3.10c}$$

$$\eta_{i,j} \in \{0,1\}, \qquad \forall i \in \{1,\dots,q\},\ \forall j \in \{1,\dots,l\}, \tag{3.10d}$$

where l is the number of connection patterns. Optimization problem (3.10) associates density $\hat{\rho}_j$ with each connection pattern C_j, effectively transforming the scheduling problem into the Pinwheel Problem (PP), as explained next. Consider

$$\hat{\alpha}_j := \frac{1}{\hat{\rho}_j^\star}, \quad \forall j \in \{1,\dots,l\},\ \hat{\rho}_j^\star > 0, \tag{3.11}$$

as safe time intervals of the connection patterns where $\hat{\rho}_j^\star$ is the optimizer of (3.10).

Theorem 6: *[Theorem 2 in Paper C, rephrased] If instance $I = \{\hat{\alpha}_j\}$ of the PP admits a feasible schedule, then the optimization problem* (3.9) *has a feasible solution.*

In order to find a feasible solution for the optimization problem (3.9), one can solve the optimization problem (3.10), find a feasible periodic schedule for instance $I = \{\hat{\alpha}_j\}$ of the PP, and reconstruct the solution as follows:

$$C(t) = C_{S(t)}, \quad \forall t \in \{1,\dots,T_r\}, \tag{3.12}$$

where $\mathbf{S} = S(1), S(2), \dots$ is the feasible schedule for the described PP and T_r is the period length of $\mathbf{S}$.

Remark 4. Note that the proposed heuristic is a sufficient condition for feasibility of the optimization problem (3.9), as outlined in Theorem 6. Therefore, the optimization problem (3.9) may still be feasible when the described PP does not admit a feasible schedule.

In summary, the proposed heuristic involves the following steps: first, select a subset of connection patterns with density $\hat{\rho}_j^\star > 0$; next, assign a safe time interval to each of the selected connection patterns, denoted as $\hat{\alpha}_j$; and finally, determine which of the selected connection patterns should be utilized at each time instant by finding a feasible schedule for the PP, using the set of assigned safe time intervals as its instance.

3.2 Online Scheduling

In the previous section, we utilized the safe time intervals of the systems and the communication topology to determine feasible communication schedules for a given NCS. The safe time interval for each system indicates the longest period during which the system's feedback loop must close at least once to ensure invariance and, consequently, satisfaction of the state and input constraints. In other words, there exists an initial condition and a sequence of disturbances for each system i such that x_i exits the invariant set if the feedback loop remains open for the following α_i time instants. However, when the initial condition or disturbance sequence is arbitrarily chosen, the state may still remain within the invariant set for longer periods with an open-loop feedback. Therefore, it is possible to compute a less conservative communication deadline for each system based on the current state measurements. This leads us to propose several online scheduling schemes, which are explained in this section.

Optimal Scheduling

In this subsection, we introduce online scheduling that relies on a set of feasible offline schedules. To achieve this, let's consider system (3.1), where the feedback loop was closed at time t_0, and its state is measured at time t. The scheduler has knowledge of t but the controller does not. Define *communication update deadline*

$$\gamma_i^x(t) := \max\{\tau : \; x_i(\tau) \in \mathcal{S}_{i,\infty}, \quad \forall w_i(j) \in \mathcal{W}_i, \; \forall j\}, \tag{3.13}$$

where $x_i(\tau, x_i(t))$ is the state at time $t+\tau$ and defined as

$$x_i(\tau, x_i(t)) := A_i^\tau x_i(t) + \sum_{j=0}^{\tau-1} A_i^j \left(B_i u_i(t+\tau-j-1, x_i(t_0)) + w_i(j)\right). \quad (3.14)$$

Note that u_i is a function of $x_i(t_0)$ and not $x_i(t)$, since the latter has not been transmitted to the controller. The communication update deadline $\gamma_i^x(t)$ is greater than, or equal to, $(t_0 + \alpha_i - t)$ by construction, i.e., the online deadline is lower bounded by the offline one. Given a feasible offline schedule $\mathbf{C}_o$, we define the *safety residual*

$$r_i(t, \mathbf{C}_o) := \gamma_i^x(t) - \gamma_i^{\mathbf{C}_o}(t), \quad (3.15)$$

where $\gamma_i^{\mathbf{C}_o}(t)$ is the waiting time for system i based on the offline schedule $\mathbf{C}_o$, specified by

$$\gamma_i^{\mathbf{C}_o}(t) := \min\{\tau : \; i \in C_o(t+\tau), \;\; \forall \tau \geq 0\}. \quad (3.16)$$

By construction, any feasible schedule ensures that $r_i(t, \mathbf{C}_o) \geq 0$ for all $i \in \{1, \ldots, q\}$ and $t \geq 1$. In other words, if $r_i(t, \mathbf{C}_o) < 0$ for some i and t, it implies that system i does not receive its subsequent measurement update on time, which may cause its state to leave its corresponding invariant set and, subsequently, violate a state constraint. Additionally, non-negative safety residuals at time instant t imply that the schedule $\mathbf{C}_o$ is a feasible schedule for the NCS from time t onwards, see Proposition 5 in Paper C. We use the safety residuals next to find feasible online schedules for the described NCS.

Consider $\mathbf{C}_o$ as a periodic feasible offline schedule whose periodic part is defined as

$$\mathbf{C}_r := C(1), \ldots, C(T_r), \quad (3.17)$$

where T_r is the period length of $\mathbf{C}_r$. Let us define rotations of $\mathbf{C}_o$ as follows

$$\mathbf{R}(\mathbf{C}_r, j) := C(j), \ldots, C(T_r), \mathbf{C}_r, \mathbf{C}_r, \ldots, \quad (3.18)$$

for $j \in \{1, \ldots, T_r\}$. Note that each schedule $\mathbf{R}(\mathbf{C}_r, j)$ is a feasible offline schedule for the NCS since each α_i time interval in $\mathbf{R}(\mathbf{C}_r, j)$ corresponds to an equivalent time interval in $\mathbf{C}_o$ for all i. This enables us to formulate an online optimal scheduling as

$$\begin{aligned} \min_{j(t)} \quad & J\left(t, \mathbf{R}(\mathbf{C}_r, j(t))\right) && (3.19a) \\ \text{s.t.} \quad & r_i\left(t, \mathbf{R}(\mathbf{C}_r, j(t))\right) \geq 0, \quad \forall i \in \{1, \dots, q\}, && (3.19b) \\ & j(t) \in \{1, 2, \dots, T_r\}, && (3.19c) \end{aligned}$$

where $J\left(t, \mathbf{R}(\mathbf{C}_r, j(t))\right)$ is a selected cost function. Our proposal for online scheduling involves solving the optimization problem (3.19) at each time instant t and selecting the first connection pattern in schedule $\mathbf{R}(\mathbf{C}_r, j^\star(t))$ as the optimal choice at time t, where $j^\star(t)$ represents the optimizer of (3.19).

Remark 5. The optimization problem (3.19) is recursively feasible as $\mathbf{R}(\mathbf{C}_r, j^\star(t))$ represents a feasible offline schedule for the NCS starting from the current time and moving forward.

Remark 6. In the optimization problem (3.19), it is important to observe that we have constrained the set of feasible schedules to $\mathbf{R}(\mathbf{C}_r, j)$, as defined in (3.18). However, the proposed online scheduling method is applicable to any finite set of feasible offline schedules.

The cost function $J(t, \mathbf{R}(\mathbf{C}_r, j(t))$ is defined as

$$J\left(t, \mathbf{R}(\mathbf{C}_r, j(t))\right) := \max_i \{-r_i(t, \mathbf{R}(\mathbf{C}_r, j(t))\}, \tag{3.20}$$

in Paper B and Paper C, which maximizes the minimum safety residual at each time instant. Moreover, in Paper E, $J(t, \mathbf{R}(\mathbf{C}_r, j(t)))$ is chosen as a quadratic cost of nominal states, enabling an optimal online communication schedule and control co-design.

Packet Loss Compensation

In this subsection, we use the proposed online scheduling framework to guarantee robust invariance in presence of a lossy communication channel. We provide necessary and sufficient conditions for existence of a feasible schedule in presence of packet loss under certain assumptions.

Let us consider a stochastic binary variable $\nu(t) \in \{0, 1\}$, where $\nu(t) = 1$ indicates that the packet transmitted at time t is lost. We assume that network's protocol is acknowledge-based and all transmissions are lost at time

t in case of multi-channel scenarios. Relaxation of the latter assumption is straightforward and omitted for simplicity. In addition, we make an assumption that only a limited number of packets may be lost during a given time interval, as this is necessary to ensure robust invariance.

Assumption 1. No more than $n_{l,i}$ packets are lost in any α_i consecutive time instants, i.e.,

$$\sum_{j=t}^{t+\alpha_i-1} \nu(j) \leq n_{\mathrm{l,i}}, \qquad \forall\, i \in \{1,\ldots,q\},\ \forall t \geq 0. \tag{3.21}$$

We define a feasible schedule as a sequence of connection patterns $C(t)$ such that every node i is connected at least once every α_i time instants in the presence of packet losses described by Assumption 1. Given a feasible baseline schedule $\mathbf{C}$ for a set of safe time intervals, we define the *shifted schedule* $\bar{\mathbf{C}}$ by

$$\bar{C}(t) := C\Big(t - \sum_{j=0}^{t-1} \nu(j)\Big), \tag{3.22}$$

which is used to compensate the packet losses. The shifted schedule selects the previous connection pattern when a packet is lost. We define the maximum time between two successive connections of node i, under baseline schedule $\mathbf{C}$ as

$$T_i := \max\left\{t_2 - t_1 :\ i \in C(t_2),\ i \in C(t_1),\ i \notin C(t),\ \forall t \in (t_1, t_2)\right\}. \tag{3.23}$$

Note that inequality $T_i \leq \alpha_i$ holds for all i due to feasibility of the baseline schedule $\mathbf{C}$.

Theorem 1 (Theorem 2 in Paper B, rephrased)**.** Shifted schedule $\bar{\mathbf{C}}$ defined by (3.22) is admissible under Assumption 1 if and only if

$$\alpha_i - T_i \geq n_{l,i}, \forall i. \tag{3.24}$$

The proof of the aforementioned theorem is grounded on the observation that each system i might encounter disconnection during its scheduled waiting time T_i, along with any retransmissions required due to packet losses within that time interval. Note that Theorem 1 solely confirms the feasibility of the shifted version of a given baseline schedule in the presence of packet losses.

However, it does not tackle the question of the existence of a feasible schedule. This matter will be addressed in the following.

Next we provide necessary and sufficient conditions for the existence of a baseline schedule which is robust against packet losses. To that end, consider a new set of safe time intervals specified by

$$\beta_i := \alpha_i - n_{l,i}, \tag{3.25}$$

and instance $I = \{\boldsymbol{\mathcal{C}}, \{\alpha_i\}, \{n_{\mathrm{l,i}}\}\}$ which describes the communication scheduling problem in presence of packet losses.

Theorem 7 (Theorem 5 in Paper C, rephrased)**:** *A feasible schedule for instance $I_1 = \{\boldsymbol{\mathcal{C}}, \{\alpha_i\}, \{n_{\mathrm{l,i}}\}\}$ exists if and only if a feasible schedule for instance $I_2 = \{\boldsymbol{\mathcal{C}}, \{\beta_i\}\}$ exists.*

The transformation introduced in Theorem 7 converts the scheduling design for a lossy network into a scheduling design for a non-lossy network. In essence, one can discover a feasible offline schedule for instance I_2, which can then serve as a baseline schedule for instance I_1 of the lossy network. Note that the existence of a feasible schedule for instance I_2 is both necessary and sufficient for the existence of a feasible schedule for instance I_1.

The feasible schedules explored in this subsection are essentially shifted versions of offline schedules. However, these shifted schedules can serve as a baseline schedule for designing optimal online schedules, as discussed in the previous subsection. Interested reader is referred to Proposition 7 and Algorithm 3 in Paper C.

CHAPTER 4

Control Design for Networked Control Systems

In the preceding chapter, various techniques and findings related to communication scheduling were introduced. The scheduling design was established based on a set of safe time intervals, where systems with higher safe time intervals require fewer communication resources. Thus, in this chapter, we delve into optimal control design schemes aimed at maximizing the safe time interval for each system under different scenarios. The results presented in this chapter are derived from Paper D, Paper E, Paper F, and Paper G. The chapter is organized into four sections, which explore the safe time interval, state feedback design, output feedback design, and control design in the presence of coupled constraints.

4.1 Safe Time Interval

In this section we study definition of the safe time interval and its relation with the selected invariant set.

Consider the following system

$$x(t+1) = Ax(t) + Bu(t) + Ev(t), \tag{4.1a}$$

$$x \in \mathcal{X}, \quad u \in \mathcal{U}, \quad v \in \mathcal{V}, \tag{4.1b}$$

with

$$\mathcal{X} := \{x \in \mathbb{R}^n : A_x x \leq b_x\}, \tag{4.2a}$$

$$\mathcal{U} := \{u \in \mathbb{R}^m : A_u u \leq b_u\}, \tag{4.2b}$$

$$\mathcal{V} := \{v \in \mathbb{R}^p : A_v v \leq b_v\}, \tag{4.2c}$$

where x, u, and v are the system's state, input, and disturbance. The safe time interval for system 4.1 can be defined as follows.

Definition 11 (Safe Time Interval)**:** *The safe time interval α is defined as*

$$\alpha := \max_t \{t : \forall x(0) \in \mathcal{O},\ \exists \mathbf{u} \in \mathcal{U} \text{ s.t. } F(t, x(0), \mathbf{u}, \mathbf{v}) \in \mathcal{O},\ \forall \mathbf{v} \in \mathcal{V}\}, \tag{4.3}$$

where

$$F(t, x(0), \mathbf{u}, \mathbf{v}) := A^t x(0) + \sum_{i=0}^{t-1} A^{t-i-1} \left(Bu(i) + Ev(i)\right) \tag{4.4}$$

and $\mathcal{O}$ is an RPI or RCI set for system 4.1.

Next, we provide a conservative upper bound for the safe time interval α as defined in (4.3).

Lemma 2 (Lemma 4 in Paper D, rephrased)**:** *Consider the safe time interval α as defined in* (4.3) *and assume that the admissible sets $\mathcal{X}$, $\mathcal{U}$, $\mathcal{V}$, defined in* (4.2)*, are symmetric w.r.t. the origin. Then, $\bar{\alpha} \geq \alpha$ holds where*

$$\bar{\alpha} := \max_t \left\{ \alpha : \bigoplus_{i=0}^{t-1} A^{t-1-i} E \mathcal{V} \subseteq \mathcal{X} \right\}. \tag{4.5}$$

The proof of Lemma 2 relies on the observation that any invariant set for the system is a subset of $\mathcal{X}$. Additionally, when the disturbance causes the state to leave the admissible set, the state also exits the invariant set.

We conjecture that

$$\mathcal{O}_1 \subseteq \mathcal{O}_2 \implies \alpha(\mathcal{O}_1) \leq \alpha(\mathcal{O}_2), \tag{4.6}$$

where $\mathcal{O}_1$ and $\mathcal{O}_2$ are arbitrary robust invariant sets and $\alpha(\mathcal{O}_i)$ is the safe time interval α when $\mathcal{O} = \mathcal{O}_i$. While relation (4.6) may not hold in general, we demonstrate its validity under specific conditions.

Lemma 3 (Lemma 5 in Paper D, rephrased)**:** *Inequality* $\alpha(\mathcal{O}_1) \leq \alpha(\mathcal{O}_2)$ *holds if*

$$\mathcal{O}_2 = \gamma \mathcal{O}_1,\ \mathcal{U}_2 = \gamma \mathcal{U}_1,\ \gamma \geq 1, \tag{4.7}$$

where $\mathcal{O}_i$ *is a robust invariant set for the system and* $\mathcal{U}_i$ *is the admissible set for the input used in* $\alpha(\mathcal{O}_i)$.

Lemma 4 (Lemma 6 in Paper D, rephrased)**:** *Assume that* $\Delta\mathcal{O}$ *and* $\Delta\mathcal{U}$ *are compact sets which contain the origin in their interiors and*

$$x \in \Delta\mathcal{O} \implies \exists u \in \Delta\mathcal{U} \text{ s.t. } Ax + Bu \in \Delta\mathcal{O}. \tag{4.8}$$

Then, inequality $\alpha(\mathcal{O}_1) \leq \alpha(\mathcal{O}_2)$ *holds when*

$$\mathcal{O}_2 = \mathcal{O}_1 \oplus \Delta\mathcal{O},\ \mathcal{U}_2 = \mathcal{U}_1 \oplus \Delta\mathcal{U} \tag{4.9}$$

where $\mathcal{O}_i$ *is a robust invariant set for the system and* $\mathcal{U}_i$ *is the admissible set for the input used in* $\alpha(\mathcal{O}_i)$.

Due to the above results, we use $\mathcal{O} := \mathcal{S}_\infty$, i.e., the MRPI set, when the feedback policy is given and $\mathcal{O} := \mathcal{C}_\infty$, i.e., the MRCI set, when no feedback policy is specified. Note that $\mathcal{S}_\infty \subseteq \mathcal{C}_\infty$ and we expect that $\alpha(\mathcal{S}_\infty) \leq \alpha(\mathcal{C}_\infty)$, which can be observed in the numerical examples in Paper D.

4.2 State Feedback

In this section, we delve into the optimal control design for constrained NCSs, assuming accurate state measurements. Initially, we examine control design with a constant feedback gain and subsequently explore MPC techniques to maximize the safe time interval for a given system.

Constant Feedback Gain

In this subsection, we seek a constant feedback gain that maximizes the safe time interval.

Consider the predicted state

$$\hat{x}(t+1) = \begin{cases} A\hat{x}(t) + Bu(t), & i \notin C(t) \\ x(t+1), & i \in C(t) \end{cases}, \tag{4.10}$$

with $\hat{x}(0) = x(0)$ as the initial condition and $u(t) = -K\hat{x}(t)$ with a constant feedback gain K. Next we study how to design K such that $\alpha(K)$ for the system (4.1) is maximized.

Given the control policy is already defined, we utilize $\mathcal{O} = \mathcal{S}_\infty(K)$ as the invariant set in (4.5). As α depends on set $\mathcal{S}_\infty(K)$, we proceed to specify this set. Given the feedback gain K, the admissible set for the state can be written as

$$\mathcal{A} := \{x \in \mathbb{R}^n : Hx \leq g\}, \; H := \begin{bmatrix} A_x \\ -A_u K \end{bmatrix}, \; g := \begin{bmatrix} b_x \\ b_u \end{bmatrix}, \tag{4.11}$$

and the MRPI set can be described by

$$\mathcal{S}_\infty(K) = \left\{x : HA_\text{c}^k x \leq g_k, \; 0 \leq k \leq n^*\right\}, \tag{4.12}$$

where $A_\text{c} := A - BK$, $g_0 := g$ and

$$g_k := g - \max_v \left(H \sum_{j=1}^{k} A_\text{c}^{j-1} E v(j) \right) \text{ s.t. } v(j) \in \mathcal{V}, \tag{4.13}$$

for $k > 0$, where the maximization is done component-wise and n^* is a positive integer such that

$$\mathcal{S}_\infty(K) \subseteq \{x : \; HA_\text{c}^n x \leq g_n\}, \; \forall n \geq n^*. \tag{4.14}$$

Since n^* is not known *a priori*, one can use a large positive number instead to make sure (4.14) holds for each n. Also note that (4.13) is a parametric optimization problem since K is unknown.

Next is the formulation to maximize α with respect to K, see (D.17),

$$\max_K \;\; \alpha \tag{4.15a}$$

$$\text{s.t. } \mathcal{S}_\infty(K) \subseteq \bigcap_{j=1}^{\alpha} \mathcal{S}_j(K), \tag{4.15b}$$

where $\alpha \in \mathbb{N}$, $K \in \mathbb{R}^{m \times n}$, and

$$\mathcal{S}_j(K) := \{x : \; HA_\text{c}^{k+j} x \leq g_k - \tilde{g}_{k,j}, \; 0 \leq k \leq n^*\}, \tag{4.16}$$

$$\tilde{g}_{k,j} = \max_v \left(HA_\text{c}^k \sum_{i=0}^{j-1} A^i E v(i) \right) \text{ s.t. } v(i) \in \mathcal{V}. \tag{4.17}$$

Similar to (4.13), (4.17) consists of elementwise parametric maximizations since K is unknown.

Remark 7. Maximizing α with respect to K poses a challenging problem due to its mixed-integer nature, nonlinear inequalities, and parametric optimizations. However, employing an evolutionary optimization scheme proves advantageous for solving this problem. The benefit of using evolutionary schemes lies in evaluating $\alpha(K)$ at different points of K, which avoids the need for parametric optimization. Nevertheless, a drawback is that evaluating $\alpha(K)$ is computationally intensive, and evolutionary schemes rely on assessing $\alpha(K)$ at multiple points.

Model Predictive Control

In this subsection we use MPC for maximization of the safe time interval.

Since the control policy is not given in this case, we use $\mathcal{O} = \mathcal{C}_\infty$ as the invariant set in (4.3). Note that unlike set $\mathcal{S}_\infty(K)$ described in the previous subsection, set $\mathcal{C}_\infty$ does not depend on the control policy. Hence, the maximum achievable α can be computed before designing the control policy, see Algorithm 1 from Paper D for more details. An MPC formulation can be used for designing a control policy which corresponds to the maximum α as follows

$$\min_{\bar{x},u} \sum_{k=0}^{\alpha-1} \left(\bar{x}_k^\top Q \bar{x}_k + u_k^\top R u_k\right) + \bar{x}_\alpha^\top P_f \bar{x}_\alpha \tag{4.18a}$$

$$\text{s.t. } \bar{x}_0 = x_0 \in \mathcal{C}_\infty, \tag{4.18b}$$

$$\bar{x}_{k+1} = A\bar{x}_k + Bu_k, \tag{4.18c}$$

$$u_k \in \mathcal{U}, \tag{4.18d}$$

$$\bar{x}_\alpha \in \bar{\mathcal{X}}_f, \tag{4.18e}$$

where Q, R, and P_f are positive definite matrices with appropriate sizes and terminal set $\bar{\mathcal{X}}_f := \mathcal{C}_\infty \ominus \left(\bigoplus_{i=0}^{\alpha-1} A^i E \mathcal{V}\right)$. The optimization problem (4.18) is solved every time the system receives a new measurement update, and the resulting optimal control inputs are successively applied to the system until the next measurement update is received. The proposed MPC scheme is recursively feasible, given a feasible communication schedule, and it guarantees the robust invariance, see Algorithm 2 and Lemma 3 from Paper D.

Unlike using a constant feedback gain, maximizing the safe time interval in the MPC case is straightforward to solve, and it leads to a greater, or equal, safe time interval, as conjectured and demonstrated through numerical examples in Paper D. It is important to note that formulation (4.18) exhibits significant differences from the common robust MPC scheme, as depicted in (2.21). For example, our objective is to design MPC in a manner that preserves invariance for the longest time horizon, whereas in (2.21), the aim is to design the controller such that the state converges exponentially to the mRPI set or an invariant outer approximation of it.

4.3 Output Feedback

In this subsection, we devise an output feedback scheme for a given system to maximize the safe time interval. We employ a Luenberger observer to estimate the state based on perturbed output measurements. The estimated states are then utilized to design a constant feedback gain, followed by designing an MPC scheme to maximize the safe time interval.

Observer Design

In this subsection, we design a Luenberger observer and establish a bound for the state-estimation error, which is used in the following subsections.

Consider the following system

$$x(t+1) = Ax(t) + Bu(t) + Fw(t), \quad \forall t \geq 0 \tag{4.19a}$$

$$y(t) = Cx(t) + Ev(t) \tag{4.19b}$$

$$x(t) \in \mathcal{X},\ u(t) \in \mathcal{U},\ w(t) \in \mathcal{W},\ v(t) \in \mathcal{V}, \quad \forall t \geq 0, \tag{4.19c}$$

as described in (2.23), an stabilizing observer gain L, and estimated state $\bar{x}$ described by

$$\bar{x}(t+1) = A\bar{x}(t) + Bu(t) + L(y(t) - \bar{y}(t)), \tag{4.20a}$$

$$\bar{y}(t) = C\bar{x}(t). \tag{4.20b}$$

The state-estimation error $e(t) := x(t) - \bar{x}(t)$ can be described by

$$e(t+1) = (A - LC)e(t) + Ev(t) - LFw(t). \tag{4.21}$$

Since $(A - LC)$ is assumed to be stable and perturbation $(Ev(t) - LFw(t))$ is within a compact polytope which includes the origin, the mRPI set $\mathcal{E}_0$ for system 4.21 exists. This in turn implies that if $e(0) \in \mathcal{E}_0$ then $e(t) \in \mathcal{E}_0$ for all $t \geq 0$, see Lemma 1 in Paper F. The invariance of the estimation error can then be used to guarantee satisfaction of the state constraints as described next.

The actual state is not accessible and satisfaction of the state constraints can only be achieved indirectly, i.e., through the estimated state in this case. In order to guarantee satisfaction of the state constraints, i.e., $x(t) \in \mathcal{X}$ for all $t > 0$, we use the sufficient condition $\bar{x}(t) \in \mathcal{X} \ominus \mathcal{E}_0$ for all $t > 0$. This indirect approach guarantees satisfaction of the state constraints since

$$x(t) = \bar{x}(t) + e(t),\ e(t) \in \mathcal{E}_0 \implies x(t) \in (\mathcal{X} \ominus \mathcal{E}_0) \oplus \mathcal{E}_0 \subseteq \mathcal{X}, \tag{4.22}$$

assuming that $\mathcal{X} \ominus \mathcal{E}_0$ is not empty.

Note that while in some cases it may not be possible to specify the mRPI set explicitly, one can use outer approximations of the set that are invariant, see (2.15) and the following descriptions.

Constant Feedback Gain

In this subsection, we design a feedback policy to maximize the safe time interval using constant feedback gain.

Consider the following predicted state

$$\hat{x}(t) := \begin{cases} \bar{x}(t), & \text{connected} \\ A\hat{x}(t-1) + Bu(t-1), & \text{not connected} \end{cases}, \tag{4.23}$$

with initial condition $\hat{x}(0) = \bar{x}(0)$, where the predicted state is updated by the estimated state whenever the corresponding state estimate is transmitted. Furthermore, consider a linear feedback policy with a constant feedback gain as $u(t) = -K\hat{x}(t)$. Then, the t-step reachable set for the estimated state can be described by

$$\bar{\mathcal{X}}^t := \{(A - BK)^t \bar{x}(0)\} \oplus \bigoplus_{j=0}^{t-1} A^j LC\mathcal{E}_0, \tag{4.24}$$

Given the reachable set for the estimated state $\bar{x}(t)$, the safe time interval can be defined as

$$\alpha := \max_{\tau} \left\{ \tau : \bar{\mathcal{X}}^\tau \subseteq \mathcal{S}_\infty \right\}, \tag{4.25}$$

where $\mathcal{S}_\infty$ is the MRPI set for the estimated stat, described by

$$\bar{x}(t+1) = A\bar{x}(t) + Bu(t) + LCe(t), \tag{4.26a}$$

$$\bar{x}(t) \in \mathcal{X} \ominus \mathcal{E}_0,\ u(t) \in \mathcal{U},\ e(t) \in \mathcal{E}_0,\ t \geq 0. \tag{4.26b}$$

The safe time interval α defined in (4.25) is the longest time interval during which one can guarantee satisfaction of the state constraints, given the feedback and observer gains K and L.

Next we discuss maximization of the safe time interval. In the described framework, the safe time interval is affected by

- the observer gain L,
- the RPI set $\mathcal{E}_0$ for the state estimation error,
- the feedback gain K,
- the RPI set $\mathcal{S}_\infty$ for the estimated state.

Given the observer gain L and the RPI set $\mathcal{E}_0$, the maximization of the safe time interval can be specified similarly to the previous section, i.e., the MRPI set $\mathcal{S}_\infty$ is used as the RPI set for the estimated state and K can be specified by solving an optimization problem similar to (4.15). Furthermore, set $\mathcal{E}_0$ represents bounds of the state estimation error $e(t)$ as specified in (4.26) and hence, the use of the mRPI set $\mathcal{E}_0$ yields the largest safe time interval. Since providing an explicit expression for $\alpha(K, L)$ is highly challenging, one can employ evolutionary schemes to discover optimal gains K and L, albeit at the cost of increased computational burden, see Algorithm 3 in Paper F.

Model Predictive Control

In this subsection we maximize the safe time interval of the system using MPC and a Luenberger observer.

Given an observer gain L, the reachable set for the estimated state $\bar{x}(t)$, specified in (4.26), is

$$\bar{\mathcal{X}}^t = \{A^t\bar{x}(0) + \sum_{j=0}^{t-1} A^{t-1-j}Bu(j)\} \oplus \bigoplus_{j=0}^{t-1} A^j LC\mathcal{E}_0. \tag{4.27}$$

The t-step open-loop reachable set $\bar{\mathcal{X}}^t$ can be used to define the safe time interval α as

$$\alpha := \max_t \left\{ t : \exists u(0), \ldots, u(t-1) \in \mathcal{U} \mid \tilde{\mathcal{X}}^t \subseteq \mathcal{C}_\infty \right\}, \tag{4.28}$$

where set $\mathcal{C}_\infty$ is the MRCI set for system 4.26.

Given the safe time interval α, one can use an MPC formulation to find a corresponding feedback policy as follows

$$\min_{\hat{\mathbf{x}},\mathbf{u}} \sum_{k=0}^{\alpha-1} \left(\hat{x}(k)^\top Q \hat{x}(k) + u(k)^\top R u(k) \right) + \hat{x}(\alpha)^\top P_f \hat{x}(\alpha) \tag{4.29a}$$

$$\text{s.t. } \hat{x}(0) = \bar{x}(0), \tag{4.29b}$$

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k), \tag{4.29c}$$

$$u(k) \in \mathcal{U}, \tag{4.29d}$$

$$\hat{x}(\alpha) \in \mathcal{X}_f, \tag{4.29e}$$

where $\bar{x}(0) \in \mathcal{C}_\infty$ is the latest received state estimate, $\hat{x}(k)$ is the predicted state at time k, Q, R, and P_f are positive definite matrices with appropriate sizes, and $\mathcal{X}_f = \mathcal{C}_\infty \ominus \left(\bigoplus_{j=0}^{t-1} A^j LC\mathcal{E}_0 \right)$ is the terminal set. We note that the described control policy is recursively feasible and it guarantees satisfaction of the state and input constraints, given the Luenberger observer (4.26) and a feasible communication schedule, see Lemma 2.

In the described framework, the safe time interval is affected by

- the observer gain L,
- the RPI set $\mathcal{E}_0$ for the state estimation error,
- the RCI set $\mathcal{C}_\infty$.

As discussed in the previous subsections, mRPI and MRCI sets are the best choices for the state estimation error and the estimated state invariant sets, respectively. Note that in this case the control design does not affect the safe time interval. The only remaining factor to consider for maximization of α is the observer gain L. Given the difficulty of explicitly specifying function $\alpha(L)$, we propose to determine the optimal observer gain using evolutionary optimization schemes, see Algorithm 3 in Paper F.

4.4 Coupled Constraints

In this section we consider maximization of the safe time intervals in presence of coupled state constraints. To that end, we formulate an optimal constraint decoupling problem and design a decentralized control policy for each system using MPC.

Consider a set of q systems described by

$$x_i(t+1) = A_i x_i(t) + B_i u_i(t) + F_i w_i(t) \tag{4.30a}$$

$$x_i(t) \in \mathcal{X}_i \subset \mathbb{R}^{n_i},\ u_i(t) \in \mathcal{U}_i,\ w_i(t) \in \mathcal{W}_i \tag{4.30b}$$

where $\mathcal{X}_i$ and $\mathcal{U}_i$ are admissible sets for state and input, and $\mathcal{W}_i$ is an admissible set for the unknown disturbance. Pair (A_i, B_i) is assumed to be controllable and admissible sets $\mathcal{X}_i$, $\mathcal{U}_i$ and $\mathcal{W}_i$ are convex polytopes which include zero in their interiors. Additionally, (4.30) is subject to

$$\sum_{i=1}^{q} G_i x_i \leq h, \tag{4.31}$$

where $G_i \in \mathbb{R}^{N \times n_i}$ and $h \in \mathbb{R}^N_{>0}$.

We have introduced a framework for control and communication scheduling design for systems (4.30) in absence of coupled constraints (4.31) in Section 4.2 and Chapter 3. Since we consider a decentralized control framework, i.e., no information exchange between the controllers, the coupled constraints should be decoupled. Consider a set of decoupled constraints as

$$G_i x_i \leq h_i,\ \sum_{i=1}^{q} h_i \leq h,\ h_i \geq \mathbf{0}, \tag{4.32}$$

where h_i is a vector and the inequalities are element-wise. Given vectors h_i for all i, one can describe the admissible set for the state of each system i by

$$\mathcal{X}_i^{\mathrm{u}}(h_i) := \{x_i \in \mathcal{X}_i :\ G_i x_i \leq h_i\}. \tag{4.33}$$

Using (4.33), the constraints are decoupled and control and scheduling design can be performed as discussed before. The remaining question is how to select vectors h_i such that the safe time intervals are optimal.

The primary aim of maximizing the safe time intervals is to enhance schedulability. So far, the control policy has been designed for each system, and the maximization of the safe time interval for each system has been pursued individually. However, in the presence of coupled constraints, maximizing the safe time interval for one system through the selection of h_i may lead to a lower safe time interval for another system. As discussed in previous sections, the safe time interval has a positive relation with the size of the corresponding invariant set. Moreover, the size of the maximal invariant set depends on the size of the state admissible set. Thus, a greater value of h_i can potentially increase the size of $\mathcal{X}_i^{\mathrm{u}}(h_i)$, resulting in a larger invariant set and safe time interval.

In order to choose h_i in a way that ensures schedulability for the resulting set of safe time intervals, we formulate an optimal constraint decoupling that explicitly considers schedulability as follows

$$\min_{C(1),\ldots,C(T_r),T_r,h_1,\ldots,h_q} \quad T_r \tag{4.34a}$$

$$\text{s.t.} \quad C(t) \in \boldsymbol{\mathcal{C}},\ T_r \in \mathbb{N}, \tag{4.34b}$$

$$T_r \leq \prod_{i=1}^{q} \alpha_i(h_i), \tag{4.34c}$$

$$\sum_{k=t}^{t+\alpha_i(h_i)-1} \eta_i(k) \geq 1,\ \forall i \in \{1,\ldots,q\},\ \forall t \in \{1,\ldots,T_r\}, \tag{4.34d}$$

$$\eta_i(k) = \begin{cases} 1 & \text{if } i \in C(k \bmod T_r) \\ 0 & \text{if } i \notin C(k \bmod T_r) \end{cases}, \tag{4.34e}$$

$$\sum_{i=1}^{q} h_i \leq h,, \tag{4.34f}$$

$$h_i \geq \mathbf{0}, \tag{4.34g}$$

where $C(0) := C(T_r)$. The solution to the optimization problem (4.34) yields an optimal constraint decoupling through h_i and also the periodic part of a feasible schedule through $C(t)$. We have selected the period length as the cost in order to find a feasible schedule with the shorted period; however, one can consider other costs such as the density function. Note that this optimization problem is mixed integer and it may be hard to explicitly describe the relation between α_i and h_i.

Next we substitute the optimization problem (4.34) with a more tractable optimization problem in order to enable schedulability. The new formulation maximizes the minimum safe time intervals lexicographically as follows

$$\operatorname{lex} \max_{h_1,\ldots,h_q} \left(\min_{i\in\{1,\ldots,q\}} \alpha_i(h_i) \right) \tag{4.35a}$$

$$\text{s.t.} \sum_{i=1}^{q} h_i \leq h, \ h_i \geq \mathbf{0}. \tag{4.35b}$$

The above formulation does not include any integer variables; however, it does not explicitly incorporate schedulability as a constraint. Instead, it indirectly enables schedulability by focusing on the maximization of the safe time intervals.

As the optimization problem (4.35) becomes challenging to solve due to the intricate relationship between α_i and h_i, we have presented a heuristic approach in G to find a suboptimal solution. Refer to Algorithm 1 in Paper G for more details. This algorithm relies on the conjecture that $\alpha_i(h_i)$ is an increasing function and can be outlined by the following steps:

- start from an initial guess h_i and compute $\alpha_i(h_i)$,
- set α to the minimum α_i,
- find the minimal h_i such that $\alpha_i(h_i) = \alpha$ for all systems,
- increase α by one step and find h_i such that $\alpha_i(h_i) = \alpha$,
- find minimal h_i such that $\alpha_i(h_i) = \alpha$,
- if a feasible set of h_i for achieving $\alpha_i(h_i) = \alpha$ does not exist, select one or several of the systems whose h_i is fixed such that $\alpha_i(h_i) = \alpha - 1$,
- repeat increasing α for the remaining systems.

See the numerical example in Paper G for illustration of the above steps.

CHAPTER 5

Summary of Included Papers

This chapter provides a summary of the included papers.

5.1 Paper A

Alessandro Colombo, **Masoud Bahraini**, Paolo Falcone
Measurement scheduling for control invariance in networked control systems
Published in 2018 IEEE 57st Conference on Decision and Control (CDC), pp. 3361–3366, Dec. 2018.
 DOI: 10.1109/CDC.2018.8619008 .

This paper examines the measurement schedulability of a specific class of multi-agent networked control systems sharing a communication medium. Reachability analysis is employed to determine a positive integer for each system, representing a time period required for the system to receive a measurement update via the communication link. This ensures robust satisfaction of the state and input constraints.
The paper assumes that each system is discrete, linear time-invariant, affected

by an unknown but bounded disturbance, and subject to state and input constraints. Additionally, stabilizing feedback gain is provided for each system, and all states are accurately measured. Furthermore, the systems share a communication medium with a single channel, implying only one system's state measurements can be transmitted through the medium at each time instant. The scheduling problem is transformed into the Pinwheel Problem from the scheduling literature. The paper provides necessary and sufficient conditions for the existence of a feasible offline schedule and recalls available techniques to find a schedule when it exists.
The author of the thesis contributed to the problem formulation, simulation results, and writing of the paper.

5.2 Paper B

Masoud Bahraini, Mario Zanon, Alessandro Colombo, Paolo Falcone
Receding-horizon robust online communication scheduling for constrained networked control systems
Published in 2019 18th European Control Conference (ECC),
pp. 2969-2974, Jun. 2019.
©2019 IEEE DOI: 10.23919/ECC.2019.8795822 .

This paper investigates online scheduling for a specific class of multi-agent networked control systems sharing a lossy communication medium. The approach utilizes available state measurements to update the communication deadline for each system at each time instant. These communication deadlines, along with feasible offline schedules, are then employed to devise an online schedule that is recursively feasible.
The paper assumes that each system is discrete, linear time-invariant, impacted by an unknown but bounded disturbance, and subject to state and input constraints. Moreover, it assumes that a stabilizing feedback gain is given for each system, and all states are accurately measured. Additionally, the systems share a lossy communication medium with a single channel, allowing only one system's state measurements to be transmitted through the medium at each time instant. It is assumed that the number of lost packets may not exceed a given threshold during a specific time interval.
The paper leverages the periodic nature of offline schedules to propose a recursively feasible online schedule that enhances performance and addresses

packet loss.
The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

5.3 Paper C

Masoud Bahraini, Mario Zanon, Paolo Falcone, Alessandro Colombo
Scheduling and Robust Invariance in Networked Control Systems
Published in 2021 IEEE Transactions on Automatic Control,
vol. 67, no. 6, pp. 3361–3366, Jul. 2021.
©2021 IEEE DOI: 10.1109/TAC.2021.3096917 .

This paper investigates both offline and online scheduling for a specific class of multi-agent networked control systems sharing a potentially lossy communication medium.
The paper assumes that each system is discrete, linear time-invariant, subject to an unknown but bounded disturbance, and adheres to state and input constraints. Additionally, it is assumed that a stabilizing feedback gain is given for each system, and all states are accurately measured. Moreover, the systems share a lossy communication medium with a generalized topology, allowing state measurements of a set of systems to be transmitted through the medium at each time instant.
To address offline scheduling in networks with multiple communication channels, the paper utilizes the Windows Scheduling Problem from the literature. Furthermore, it proposes an optimization problem for offline scheduling in networks where a set of states can be transmitted through the medium at any given time instant. Additionally, the paper provides a necessary and sufficient condition for the existence of online schedules in the presence of packet losses.
The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

5.4 Paper D

Masoud Bahraini, Mario Zanon, Alessandro Colombo, Paolo Falcone
Optimal Control Design for Perturbed Constrained Networked Control Systems

Published in 2021 IEEE Control Systems Letters,
vol. 5, no. 2, pp. 553 - 558, Apr. 2021.
 DOI: 10.1109/LCSYS.2020.3004204 .

This paper examines the control design for a specific class of multi-agent networked control systems sharing a common communication medium.
The paper assumes that each system is discrete, linear time-invariant, and influenced by an unknown but bounded disturbance while adhering to state and input constraints. Additionally, it assumes accurate state measurements. The paper investigates the impact of selected invariant sets and control policies on the communication demand of each system. To minimize the communication demand, the paper proposes optimal control designs within the scenarios of constant feedback gain and model predictive control.
The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

5.5 Paper E

Masoud Bahraini, Mario Zanon, Alessandro Colombo, Paolo Falcone
Optimal scheduling and control for constrained multi-agent networked control systems
Published in 2022 Wiley Optimal Control Applications and Methods,
vol. 43, no. 1, pp. 23–43, Jan. 2022.
 DOI: 10.1002/oca.2777 .

This paper explores the co-design of optimal control and scheduling for a specific class of multi-agent networked control systems that share a common communication medium.
The paper assumes that each system is discrete, linear time-invariant, and influenced by an unknown but bounded disturbance while adhering to state and input constraints. Additionally, it assumes accurate state measurements. The paper delves into the optimal joint design of offline/online communication schedules and tube-based model predictive control with the objective of minimizing an infinite horizon quadratic cost. It presents a heuristic approach for the optimal joint design based on a given set of feasible schedules.
The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

5.6 Paper F

Masoud Bahraini, Alessandro Colombo, Mario Zanon, Paolo Falcone
Robust Control Invariance for Networked Control Systems with Output Feedback
Published in 2022 IEEE 61st Conference on Decision and Control (CDC), pp. 7676-7681, Dec. 2022.
©2022 IEEE DOI: 10.1109/CDC51059.2022.9992732 .

This paper investigates optimal control design for a specific class of multi-agent networked control systems that share a common communication medium. The paper assumes that each system is discrete, linear time-invariant, and influenced by an unknown but bounded disturbance while adhering to state and input constraints. Additionally, it assumes that only perturbed outputs are available through measurements.
The paper delves into optimal output feedback design for each system with the goal of minimizing the corresponding communication demand. The design incorporates a Luenberger observer to estimate the states and is combined with a feedback policy based on either a constant feedback gain or tube-based model predictive control.
The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

5.7 Paper G

Masoud Bahraini, Mario Zanon, Alessandro Colombo, Paolo Falcone
Communication Demand Minimization for Perturbed Networked Control Systems with Coupled Constraints
Accepted for publication in 2023 IEEE 62st Conference on Decision and Control (CDC),
Dec. 2023.
©2023 IEEE DOI: .

This paper examines optimal control design for a specific class of multi-agent networked control systems sharing a common communication medium. The paper assumes that each system is discrete, linear time-invariant, and affected by an unknown but bounded disturbance while adhering to coupled state and input constraints. Additionally, it assumes accurate state measurements.

The paper focuses on optimal constraint decoupling to minimize communication demands and facilitate schedulability. It formulates an optimal constraint decoupling problem and proposes a heuristic method to address it.

The author of the thesis contributed to the problem formulation, theoretical and simulation results, and the writing of the paper.

CHAPTER 6

Concluding Remarks and Future Works

In this chapter, we will first provide a summary of how the thesis addresses the research question proposed in Chapter 1. Subsequently, we will suggest potential directions for future research based on the findings presented in this thesis.

6.1 Conclusion Remarks

The central research question of this thesis revolves around the co-design of communication schedules and control strategies for perturbed multi-agent NCSs with a shared, and potentially lossy, communication medium. The main question is further divided into six subquestions, each addressing specific aspects of the co-design problem.

The first three subquestions (**Q1**, **Q2**, and **Q3**) pertain to the scheduling aspects of the design, assuming given control policies. These questions are thoroughly investigated in Paper A, Paper B, and Paper C. The papers leverage existing scheduling techniques such as the Pinwheel Problem (PP) and the Windows Scheduling Problem (WSP), while also proposing a generalized framework and introducing novel results related to the scheduling techniques.

The next two subquestions (**Q4** and **Q5**) focus on the control design aspects of the problem and their impact on schedulability. These questions are addressed in Paper D, Paper E, Paper F, and Paper G. The papers introduce optimal control policies for various scenarios, including state feedback, output feedback, and the existence of coupled state constraints. The control strategies are designed to minimize the communication demand for each system, thereby facilitating schedulability.

Finally, the last subquestion (**Q6**) inquires about jointly designing the communication schedule and control strategies. This question is answered in Paper E, where we utilize offline scheduling and control designs within a recursive optimization framework.

6.2 Future Works

Although the proposed framework is mainly based on linear time-invariant systems, a potential research direction is to consider linear time-varying or linear parameter-varying systems. The implications of such changes could be explored to recreate the existing results and potentially unveil new findings.

In Paper F, state estimation is achieved using a Luenberger observer. To extend the results, one could consider receding horizon and set-based observers and explore their implications on the communication demand for each system.

The framework presented in Paper G addresses coupling between the states' constraints. An interesting extension would be to investigate systems with coupling in both states' constraints and dynamics, potentially leading to new insights and control strategies.

Finally, the application and implementation of the proposed framework in real-world scenarios are crucial tasks. While challenging, the potential applications could lead to new research topics that are more relevant to society, making the effort rewarding and impactful.

In conclusion, the thesis lays the groundwork for tackling important challenges in co-designing communication schedules and control strategies for multi-agent NCSs. Future research in the suggested directions can further enhance our understanding and contribute to the advancement of this interdisciplinary field.

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