THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Beyond the deterministic approach - on the feasibility of data assimilation methods in geotechnics

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ABSTRACT

The surge in economic and social development has resulted in significant challenges, especially for linear physical infrastructure. A substantial part of the ageing physical linear infrastructure has been built on problematic soils and often have poorly documented foundation solutions. A typical example is the case of embankments for infrastructure on soft ground conditions. Soft soils possess various important characteristics that contribute to their complex emerging soil response when subjected to hydro-mechanical loading. In recent times, numerous advanced constitutive models grounded in various theories and hypotheses have emerged to capture the behaviour of soft soils. These models differ from models commonly used in geotechnical engineering, as they encompass complex soft soil features, *e.g.* anisotropy, rate-dependency and degradation of bonding that enable reasonably accurate predictions for test data obtained under controlled laboratory conditions. However, their applicability for making informed decisions on large-scale field projects may be limited, as deterministic calculations alone may not adequately consider the variability in the behaviour of geomaterials encountered in real-world scenarios. Furthermore, not all model parameters have direct geotechnical significance as they are derived solely from mathematical expressions, posing challenges in their identification. With the growing utilisation of advanced constitutive models in engineering analysis, the input parameters for these models take on crucial roles as design parameters. This thesis provides a probabilistic methodology that enables the identification of parameters of constitutive models for geotechnics, through inverse analysis using Data Assimilation (DA). The primary objective of this thesis is to evaluate the applicability of existing Data Assimilation concepts in the field of geotechnical engineering. To achieve this, a modular framework that allows the implementation and use of multiple DA methods in conjunction with geotechnical numerical codes is created. A comprehensive and systematic comparison of contemporary state-of-the-art DA schemes specific to geotechnical engineering is performed along with examining the factors influencing their performance. Additionally, hybridisation of meta-heuristic algorithms with classical Data Assimilation methods has also been proposed to improve some of the observed drawbacks. In this thesis, the limitations of the deterministic approach has been demonstrated and the need for a robust probabilistic tool is shown to be paramount. It has also shown that it is time to start embracing the value of monitoring data which can be put to efficient use when a robust probabilistic framework like Data Assimilation is considered.

Keywords: Embankment, Soft soils, Uncertainty analysis, Data Assimilation

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First, I would like to express my gratitude to Prof. Jelke Dijkstra, my supervisor at Chalmers University of Technology, for his unwavering guidance and encouragement throughout the thesis. Additionally, I had the privilege of collaborating with Dr. Tara Wood from Ramboll Sverige AB, who provided valuable supervision and support. They not only provided me the opportunity to delve into a subject that genuinely motivated me but also proved to be exceptional mentors.

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A special note of appreciation to my family whose unconditional love and patience have provided me with a constant source of encouragement.

This thesis is composed of my original work, and any previously published or written material by others has been appropriately acknowledged in the text. The contributions of researchers from jointly-authored works has been clearly attributed.

NOMENCLATURE

Acronyms

CPT: Cone Penetration Test
CRS: Constant Rate of Strain
CSS: Current Stress Surface
GSA: Global Sensitivity Analysis
ICS: Intrinsic Compression Surface
IL: Incremental Loading
NCS: Normal Consolidation Surface
OCR: Over-consolidation ratio
PVD: Prefabricated Vertical Drains
SIR: Sequential Importance Resampling
SIS: Sequential Importance Sampling
SLS: Serviceability Limit State
ULS: Ultimate Limit State

Greek letters

α_0	initial inclination of NCS
χ_0	initial amount of bonding
$\dot{\Lambda}$	rate-dependent viscoplastic multiplier
ϵ_q^c	viscoplastic deviatoric strain
ϵ_q^e	elastic deviatoric strain
ϵ_v^c	viscoplastic volumetric strain
ϵ_v^e	elastic volumetric strain
η	Stress ratio
κ^*	modified swelling index
λ^*	modified compression index
λ_i^*	modified intrinsic compression index
μ_i^*	intrinsic modified creep index
ν	Poisson's ratio
ω	absolute effectiveness of rotational hardening
ω_d	relative effectiveness of rotational hardening
Φ	sample space
ψ	Particle Filter weight
σ'_{p_0}	initial preconsolidation pressure
τ	reference time
θ	model parameters
ξ	absolute rate of destructuration
ξ_d	relative rate of destructuration

Roman lower case letters

e_0	Initial void ratio
p'	mean effective stress
p'_{eq}	equivalent mean effective stress
p'_m	mean effective preconsolidation pressure
q	deviatoric stress

Miscellaneous

$\langle\bullet\rangle$	Macaulay brackets

Roman capital letters

$\mathcal{A}$	Event space
E	Event
G	Shear modulus
H	Observation operator
K	Bulk modulus
K_v	Vertical hydraulic conductivity
M_c	slope of critical state line in triaxial compression
M_e	slope of the critical state line in triaxial extension
N	Monte Carlo sample size, number of cycles
P	Probability measure
R	Observation error covariance

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A A. Amavasai, J.-P. Gras, N. Sivasithamparam, M. Karstunen, and J. Dijkstra (2017). "Towards consistent numerical analyses of embankments on soft soils". In: *European Journal of Environmental and Civil Engineering*, pp. 1–19

Paper B A. Amavasai, N. Sivasithamparam, J. Dijkstra, and M. Karstunen (2018). "Consistent Class A & C predictions of the Ballina test embankment". In: *Computers and Geotechnics* 93, pp. 75–86

Paper C A. Amavasai, T. Wood, and J. Dijkstra (2022). "Data assimilation for geotechnics - exploring the possibilities". In: *11th International Symposium on Field Monitoring in Geomechanics (ISFMG2022)*. UK

Paper D A. Amavasai, T. Wood, and J. Dijkstra (2023). "On the feasibility of data assimilation for uncertainty modelling in geotechnics". In: *10th European conference on Numerical Methods in Geotechnical Engineering*. UK

Paper E A. Amavasai, H. Tahershamsi, T. Wood, and J. Dijkstra (2023). "Data assimilation for Bayesian updating of predicted embankment response using monitoring data". Submitted to *Computers & Geotechnics*

Paper F A. Amavasai and J. Dijkstra (2023). "Particle Filter based on Jaya optimization for Bayesian updating of nonlinear models". Submitted to *Applied Soft computing* journal

The appended papers were prepared in collaboration with the co-authors. For Papers A and B, the author of this thesis was responsible for the implementation of the automated parameter derivation procedure, numerical simulation and preparation of the manuscript. For Paper C, the two co-authors collaboratively initiated and planned the study. The implementation and validation of various Data Assimilation algorithms were performed by the author of this thesis. The outcomes presented in Paper C subsequently led to further advancements, including validation using real case dataset and the development of Paper D. In Paper E, the author implemented the Data Assimilation framework in Plaxis Finite Element code for a synthetic case. The second author was responsible for the Global Sensitivity Analysis in this study. In Paper F, the author of this thesis was responsible for formulating the theory, developing the numerical implementations, leading the paper's planning, conducting the majority of the numerical simulations, and preparing the manuscript.

Other publications by the author

Karstunen, M. and Amavasai, A. (2017). *BEST SOIL : Soft soil modelling and parameter determination*. Technical report. Chalmers University of Technology.

CONTENTS

Part I
Extended summary

1 Introduction

1.1 Background

In recent decades, there has been a significant surge in demand for infrastructure around the globe. Natural geomaterials, such as soils and rocks, form a significant portion of the materials required for construction, as part of the engineered physical infrastructures, including roads and railways, as well as the subsoil built upon. As a result, understanding the characteristics and hydro-mechanical response of natural geomaterials plays a crucial role in engineering design and analysis. The progress in economic and social development has resulted in significant challenges, especially for linear physical infrastructure where the need to expand the capacity of existing roads and railways, and construction of new physical infrastructure have led to selection of sites with poor quality soils that have inferior engineering properties. A typical example is the case of constructing embankments on soft ground conditions that have a high compressibility.

Soft sensitive clays are present in several regions of Sweden, particularly in places with high urbanisation such as Stockholm and Gothenburg. Despite extensive and considerable experience contributing to the knowledge, embankment construction continues to present two major issues. The first issue is to ensure overall stability of the ageing infrastructure in a changing environment, *i.e.* to prevent drastic societal consequences. The other issue is to minimise deformations (within tolerable limits) for better serviceability of the structure, hence controlling the maintenance expenses. The former is termed as the ultimate limit state (ULS) and the latter the serviceability limit state (SLS). For linear infrastructure, especially for embankments constructed on soft soils, the design is primarily influenced by deformations *i.e.* SLS. Soft soils exhibit several key features, including sensitivity, anisotropy, and rate-dependency, which contribute to the complexity of their non-linear and time-dependent response to hydro-mechanical loading (Mitchell and Soga, 2005). Thus, it is crucial to employ suitable prediction models that incorporate most of these soft soil features, that generally are not part of traditional design methods used for SLS predictions. More reliable design techniques are needed, to improve accuracy, reduce uncertainties, reduce carbon footprint and ensure that foundations are optimised for their entire service life.

One of the most significant developments in geotechnical engineering is the introduction of the critical state soil mechanics framework based on the concepts of soil hardening (Drucker et al., 1957) and yielding (Roscoe et al., 1963). Constitutive models based on this framework are better equipped to capture the behaviour of the soil for any arbitrary stress path, given they account for the state of the soil. For example the Cam-Clay model (Roscoe and Burland, 1968) was one of the first models to capture the elastic-plastic response for soils by introducing hardening laws that evolve with loading history. The most common approach for solving boundary value problems using these models in computational geomechanics is by the Finite Element Method. Over the past several decades, the sophistication of constitutive models has increased (*e.g.* Al-Tabbaa, 1987; Al-Tabbaa and Muir Wood, 1989; Whittle, 1993; Koskinen et al., 2002; Karstunen et al., 2005;

Karstunen and Koskinen, 2008; Gras et al., 2018) leading to increasing accuracy in the prediction of the overall soft soil behaviour. Some of the advanced models have been developed with stress probing tests (Mašín, 2007) and constant stress path testing (Wheeler et al., 2003), and hence the hardening laws in those models are more thoroughly validated than for the simple models.

Before applying more advanced constitutive models for designing real-world linear infrastructure, a prior validation on well documented test embankments is necessary. One of the motivations is that the site investigation for test embankments are often more comprehensive than typically found in commercial projects, providing sufficient information to calibrate advanced models. Also, test embankments can be regarded as being equivalent to a controlled laboratory test at boundary value level, due to the absence of embedded elements in the subsoil, and providing a broad range of stress paths throughout the domain. Despite the well-instrumented sites, advanced constitutive models are still unable to fully capture the actual soil response. This is partly due to the disturbances from extracting samples from the soil, which in turn affect the soil properties evaluated from laboratory tests (Karlsson et al., 2016). However, in reality, even with high quality sampling, the deviation in parameters is still expected due to the unavoidably large variations of the soil properties in a large soil volume when compared to the number of samples retrieved for testing.

In geotechnics, uncertainties are prevalent, so most analysis of soil behaviour must take the uncertainties of the properties of the soils into account and provide methods for determining how they affect the final predictions. Even though the fidelity of computational models has improved, and their accuracy has been validated with data from controlled tests, their true predictive power at the field scale is still constrained by a deterministic framework. In the deterministic methods the uncertainties are addressed by using a global or partial safety factor, leading to over-design in some settings whilst being unsafe in other situations. Even in the simplest scenario where the geometry of the system is known with sufficient accuracy and a constitutive model which is well validated with laboratory test data, capturing the behaviour of the soil *in-situ*, is still a major challenge in geotechnical engineering.

The observational method (Peck, 1969) has been recommended to address this issue and given the rise in cost-effective and robust monitoring solutions for geotechnical structures in recent years (Klar et al., 2006; Bennett et al., 2010; Cheung et al., 2010; Schwamb et al., 2014), have enabled large-scale projects to be equipped with tools to monitor the system behaviour in real time. This leads to new possibilities for validating the ever-growing optimisation techniques for inverse analysis, and subsequent uncertainty reduction in the model prediction. The inverse problem consists of using observations to infer the (updated) values of the model parameters that characterise the system. Although there has been several significant contributions from the deterministic approach (Gioda and Sakurai, 1987; Ledesma et al., 1996; Calvello and Finno, 2004; Lecampion and Constantinescu, 2005; Rechea et al., 2008; Levasseur et al., 2009; Hashash et al., 2010; Levasseur et al., 2010), the inherent focus on the best possible fit between the model prediction and observations ignores the uncertainties stemming from measurement noise, or model uncertainties and their effect on the model predictions.

As opposed to the deterministic approach, the Bayesian statistical framework is most effective for updating of model parameters using monitoring data (Wu et al., 2007; Zhang et al., 2010; Juang et al., 2013). Here the *a priori* information on the model parameters is represented by a probability distribution over the 'model space'. This gives the practitioners the additional advantage to assess the range of possible behaviour of the geo-structure whilst using monitoring data to update the model predictions. The uncertainties from both observation and forward simulation can be considered here with a practical margin of error, but a systematic procedure is required.

Data Assimilation (DA) is a well-known and versatile mathematical discipline used predominantly in the field of atmospheric science for numerical weather prediction. DA follows the Bayesian approach more exactly in terms of representing uncertainty (Geer, 2021) acting as a bridge integrating theoretical models and observations to estimate the state of an evolving system enabling realistic estimations with a reduced overall variance in the prediction. The weather forecasts are generated from the output of a DA system (Fletcher, 2017). Modern DA builds on "state estimation" techniques developed in control systems engineering (Raanes, 2016) and is one of the most useful inverse analysis schemes. Geotechnics could greatly benefit from approaches used in DA, which have evolved to deal with real observations that are uncertain and sparse, holding potential to make better use of monitoring data and provide reliable estimates of uncertainty in model prediction. DA has been applied to some geotechnical problems since the 1980s (e.g Murakami and Hasegawa, 1985; A. Murakami, 1991) until recently (Hommels and Molenkamp, 2006; Shuku et al., 2013; Liu et al., 2018a; Tao et al., 2020; Mohsan et al., 2021). However, a rigorous and systematic comparison between recent state-of-the-art DA schemes for geotechnical engineering is still missing.

1.2 Aim of research

The aim is to validate an advanced constitutive model, and identify the limitations of the deterministic approach, enabling to further investigate the feasibility of using DA techniques for joint state and parameter estimation in geotechnical engineering based on monitoring data. The main goal is to assess the adoption of various state-of-the-art DA techniques and evaluate their relevance for geotechnical engineering. Specifically, the methods are employed on test embankments in order to evaluate the performance of each technique, compare their efficiency and identify the obstacles encountered in their application. As part of the overall aim, the following questions are solved in this thesis:

- Are advanced constitutive models always better in capturing the true state of the system?
- How do different DA methods apply to geotechnical numerical models of varying complexity?
- Can DA still help capture the response of the true system, even if the model formulation does not match the physics of the geotechnical system considered, be it simple or complex?
- Does the field monitoring setup *i.e.* the quantity, type and location of measurements has any effect on the performance of the DA?
- Is there a possibility to improve the efficiency of a DA technique?

1.3 Method

To answer the previously stated research questions, the following tasks were carried out:

- Perform deterministic analyses of test embankments using an advanced constitutive model that captures most of the soft soil features and quantify its limitations.
- Create a modular framework that allows the implementation and use of multiple DA methods in conjunction with geotechnical numerical codes that allow scripted inputs.
- Test the framework by assessing the performance of the implemented DA techniques on different constitutive models with varying levels of complexity.
- Identify the merits and limitations of employing DA for geotechnics and propose a novel method.

1.4 Limitations

- The effect of temperature on the hydro-mechanical behaviour of soils has not been considered, *i.e.* the evaluated model parameters are not corrected for differences in temperature between the lab and *in-situ* conditions.
- The boundary value level validation is limited to static loading conditions, such as those of test embankments constructed on soft ground.
- The non-linear behaviour at small magnitudes of strain amplitude is not considered.
- Since sufficiently high quality dataset were made available from test embankments, the screening for outliers was not necessary in this study, but it is prudent to include them as part of the developed module for future practical applications.
- The external applied load, soil layer thickness and geometry of the geostructure are considered deterministic *i.e.* fully certain during the inverse analysis.
- The spatial heterogeneity of the soil parameters is not considered and is reserved for future studies.

2 Deterministic analysis

2.1 Embankments on soft soil

Prior to the early 1900s, studies on the behaviour of embankments were basic and there was limited understanding of the mechanical behaviour of soils, especially soft clays. Frontard (1914) was one of the first to attribute the failure of embankments to the development of pore-water pressures based on observations from experiments. The understanding of the mechanical behaviour of soils developed gradually and the groundwork for modern soil mechanics was laid by a set of significant contributions in the mid-1930s, (Buisman, 1936; Casagrande, 1936; Hvorslev, 1936; Rendulic, 1936). Later, Terzaghi's contribution on the effective stress framework in soil mechanics, proposed in his seminal work (Terzaghi, 1943), revolutionised the field of geotechnical engineering and has been instrumental in rationalising the design and improving understanding of foundation behaviour and the influencing factors. Terzaghi's work on one-dimensional consolidation theory was groundbreaking, and formed the basis for settlement calculations which still are regarded as a point of reference by numerous geotechnical engineers today.

During this era and until early 1960s, a considerable amount of attention was given to the stability of embankments constructed on soft soils, such as natural clays. Skempton and Golder (1948) and Odenstad (1948) carried out practical studies to validate total stress calculations. Bishop and Bjerrum (1960) subsequently built upon this work. Bishop (1955) introduced a method for stability analysis using circular failure surfaces, which is still used in many projects today, and provides a indicator of the safety factor for modern-day projects. Janbu (1957) presented a similar technique for non-circular failure surfaces. As a result, models for embankment stability analysis in soils have become well-established since around 1960.

During this period, there was limited progress in the development of techniques for calculating the serviceability of embankments, which is a critical aspect of overall design. Unfortunately, this area was primarily limited to Terzaghi's initial proposal of settlement calculation based on one-dimensional consolidation theory. The phenomenon of secondary settlement was often ignored in most projects until the contributions of Bjerrum (1967), which were well-received by engineers at that time. However, empirical methods contained elements of conservatism, and practitioners encountered unacceptable outcomes when using them in real-world problems. Unfortunately, the industry at that time was hesitant to adopt Cam-Clay type models developed by Roscoe and Burland (1968) along with Finite Element methods, citing complexity as a primary reason.

In order to address the uncertainties associated with existing calculation methods for embankments constructed on soft soils, the industry turned to *in-situ* testing while still relying on empirical models. The economic consequences of inaccurate calculations for large projects were recognised, and the use of trial embankments became a more widely accepted approach after 1970 (Leroueil, 1990). These test sites were typically included in a project, ideally with geotechnical conditions similar to those of the entire project, to collect data such as surface settlements, settlements at different depths, pore-water pressure development, and horizontal displacement. The purpose of these trial embankments was to gather high-quality data to test the accuracy of existing calculation methods and modify them if needed to suit specific site conditions, particularly for projects in new or unknown territories. As a result, a significant amount of experience has been gained in terms of stability and long-term settlements of embankments on soft soil.

Trial embankments are considered to be highly valuable asset for large-scale projects, despite the fact that the interpretation of the results can take many years, particularly for long-term settlement data. As a result, it is critical to consider the use of trial embankments early on in the planning of major projects. To expedite consolidation, techniques have been developed, such as the use of prefabricated vertical drains (PVDs).

The empirical methods have continued to survive to this day and are still used in several major projects. While the Finite Element Method and accompanying constitutive models also existed in parallel, the latter has only recently begun to be more widely accepted for analysing geotechnical problems. Even with the advent of new models and sophisticated tools, the purpose of trial embankments has not changed *i.e.* to test the validity of the calculation models and, if necessary, adapt them to particular site conditions. However, the difference is that the knowledge of the complexities of the soft soil behaviour have now been significantly improved and the new advanced models are able to capture most of the soft soil features at the laboratory scale in controlled conditions. This is a significant improvement compared to the empirical models generally used in practice. The prominent features of the soft soil behaviour are discussed in the next section followed by the brief introduction of the advanced model considered in this thesis.

2.2 Facets of soft soil behaviour

The behaviour of soft soil can be observed as a collection of distinct features such as compressibility, primary and secondary consolidation, anisotropy and bond degradation. During construction of embankments on soils of this type, the effect of these features are inevitable (and inseparable) and a model with a unified approach is needed. Despite being commonly treated as continuum materials, soft soils are actually porous fine-grained materials, and their behavior is influenced by both the particles and their arrangement, as well as the presence of liquid and/or gas in the pores, resulting in coupled hydro-mechanical behaviour. It is essential to differentiate between stresses carried by the solid phase and those carried by the pore phase, since while the particle assembly can transmit both the normal and shear stresses, the pore-water can only transmit hydrostatic stress. The subsequent section will give a brief outline of the various aspects of the behaviour of soft soils that need to be incorporated into the constitutive model chosen for embankment analyses (which will be discussed in the following part of this section).

Deposition history

Soft soil behaviour is best described by the emerging strength and the deformation response under different initial conditions and loading paths. The former depend on the constituents (as reflected by the index properties) and the loading history. Therefore, the processes during geological formation help to understand the origin of the soil properties and subsequent response under loading. Soft soils are usually deposited over large areas of extent. During and after the deposition process the deformation experienced is generally one-dimensional (Muir Wood, 1991). The movement of soil particles has been predominantly vertical with lower tendency to move laterally. The particle orientation is primarily horizontal due to the consolidation process after deposition and this is experimentally corroborated in Birmpilis et al. (2019). Hence, the stress state and emerging fabric remains highly three dimensional and the emerging response of the soft soils in the deposit is different between the vertical and the horizontal directions. This leads to cross-anisotropic behaviour commonly observed in soft soils. In addition to the physical processes, several chemical and biological processes influence the initial structure of the soil during deposition (Mitchell and Soga, 2005). More complex time-dependent behaviour due to viscosity can be observed post deposition. These phenomena have a significant impact on the soft soil behaviour, in terms of stiffness and mobilised strength, and affect the hydro-mechanical response of the material.

Anisotropic yield surface

The fabric of a soft soil deposit refers to how the individual particles are oriented, distributed, and shaped, as well as the way they come into contact with one another. Soft soils, in general, exhibit yielding at different magnitudes of strain and effective stress for different loading paths, which in triaxial stress space represents a rotated yield surface (As shown in Figure 2.1 plotted in the triaxial stress space and normalised by the preconsolidation pressure). This is typical for (intact) natural soft soils found in the Nordic countries and Canada.

The effect of anisotropy has been studied by several researchers, and for different natural clays the anisotropic nature of the yield surface (Larsson, 1977; Larsson, 1981; Leroueil and Vaughan, 1990; Wheeler et al., 2003; Karstunen and Koskinen, 2008) has been reported. The *in-situ* anisotropic state (inherent anisotropy) evolves with subsequent loading (induced anistropy) and is governed by the particle scale mechanisms (Hicher et al., 2000). This has a major impact on the emerging (undrained) shear strength at continuum level which becomes anisotropic, as *e.g.* observed in the changes in strength between triaxial tests sheared under compression and extension.

Considering the significance of anisotropy, it is highly important to incorporate this mechanism in the constitutive model. The effect of anisotropy on the emerging embankment behaviour has been studied by several authors (Graham, 1979; Potts et al., 2002; Yildiz et al., 2009; Yildiz and Uysal, 2016). The main finding is that with an anisotropic model a reasonable prediction of the lateral displacements at the toe of the embankment is obtained

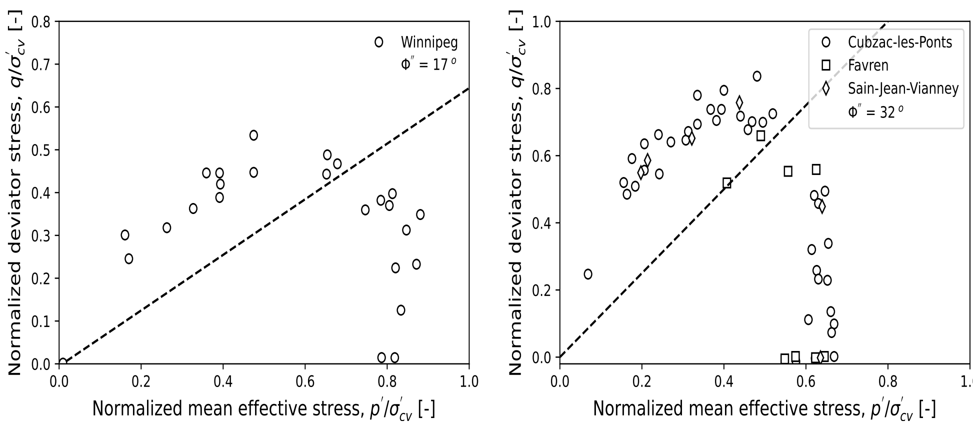


Figure 2.1: *Yield points of different natural soft soils (reproduced from (Länsivaara, 1999)).*

Apparent bonding

Soft soils are typically assumed to be normally consolidated, but the natural deposits in their undisturbed state often demonstrate an initial stiffness that is relatively high and comparable to the stiffness during unloading and reloading, which suggests a lightly overconsolidated state. This is due to the presence of creep and cementation effects that result in inter-particle bonding (Burland, 1990; Cudny, 2013). The reduction in void ratio due to secondary compression leads to an increased apparent preconsolidation pressure and emerging shear strength. The effect of structure, defined here as a combination of fabric and the (apparent) bonding between the particles (Leroueil and Vaughan, 1990), on the mechanical behaviour of the soils depends strongly on the stress history and void ratio. The mechanical behaviour of structured soft soils, consequently, differs significantly from their reconstituted counterparts.

Several authors have evaluated the effect of structure on soft soil behaviour (Bjerrum and Lo, 1964; Bishop, 1971; Leroueil et al., 1979; Burland, 1990; Leroueil and Vaughan, 1990; Gens and Nova, 1993; Cotecchia and Chandler, 1997; Koskinen et al., 2002; Karstunen et al., 2003). The state of a soil, which has had all its original form of structure re-arranged from continuous loading is referred to as the intrinsic state and is linked to the initial intact state via a model feature that represents the bonding.

Figure 2.2 shows clearly the difference in response between an intact and a reconstituted sample, where the slope of the normal compression line (λ) tends towards the intrinsic value (λ_i) at large effective stress levels. After reaching the yield point (σ'_{p0}) from its initial stress (slope denoted by the slope of the swelling line 'κ'), the bonds between the particles start to gradually collapse until, the intrinsic state is reached. The effect of structure on soft soil behaviour in triaxial shearing, compression and swelling is well described by Leroueil and Vaughan (1990). The relevance of considering degradation of structure in embankment problems is that it can occur as a result of irreversible (plastic) strains and lead to more realistic predictions of settlements *in-situ* (Yildiz et al., 2009; Cheang et al., 2016).

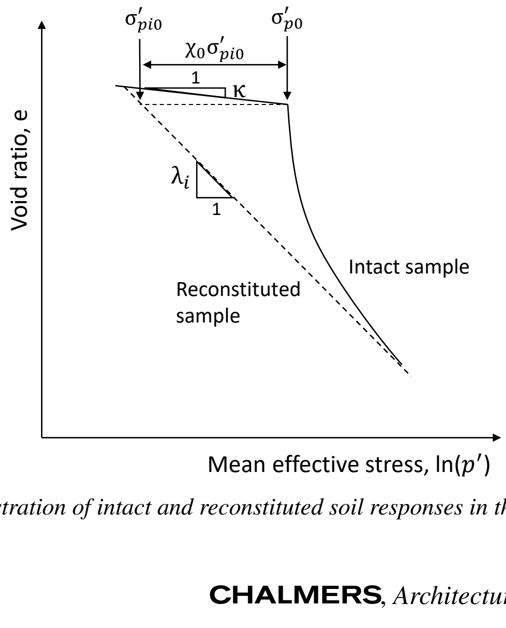


Figure 2.2: *Illustration of intact and reconstituted soil responses in the $e - ln\ (p')$ plot.*

Rate-dependent behaviour

The effect of rate-dependency on the emerging strength and compressibility of natural soft soils has been studied by several authors (Crawford, 1964; Bjerrum, 1973; Sällfors, 1975; Janbu et al., 1981; Graham et al., 1983a). Furthermore, the location of the yield envelope in effective stress space is also a function of loading-rate (Tavenas et al., 1978; Lew, 1981).

The importance of modelling rate-dependency on *e.g.* embankment construction has been demonstrated by several researchers (Kerry and Hinchberger, 1998; Rezania et al., 2016). An example on the viscous effect of soft soil is best described by comparing the mechanical response between a young and aged normally consolidated clay. The old deposit would have a reduced void ratio due to creep under self-weight leading to higher yield stress than expected for the same soil that is younger (Bjerrum, 1973). Hence due to creep, the yield surface has expanded over time. When the soil is subjected to additional stress, the new stress state passes the original yield point and only at a higher effective stress level the structure starts to evolve (degradation of bonds, re-arrangement of fabric), falling towards the normal consolidation line again. This effective stress level in oedometric conditions is referred to as the apparent preconsolidation pressure or quasi-preconsolidation pressure as shown in Figure 2.3a (Leroueil et al., 1996).

Although it is true that secondary compression is driven by pure creep, these are not entirely the same since creep also occurs outside the secondary compression regime (Tavenas et al., 1978; Degago et al., 2011). Due to the viscoplastic nature of the soft soils, the induced strain rate will influence the stress-strain response of the sample as shown in Figure 2.3b. An increase in strain rate leads to an increase of the yield stress. Furthermore, for constant-rate of strain (CRS) tests a system level interpretation is required, as the mobilisation of excess pore-water pressures (at large levels of strain) is non-uniform (Muir Wood, 2016). The increase in strain rate is more pronounced in soils with a higher plasticity index. However, the strength at critical state is largely independent of strain rate effects. The effect of high creep rates in soft soils below an embankment increases the risk of stability problems both during and after construction, in addition to affecting the prediction accuracy of long term settlements.

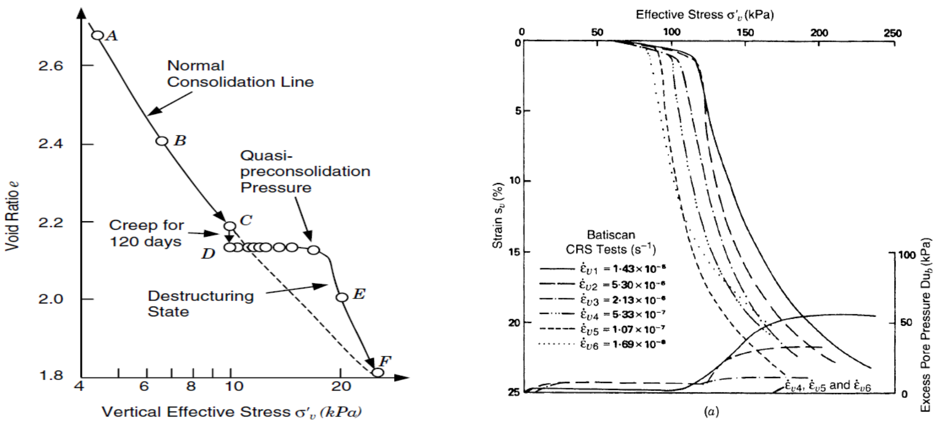


Figure 2.3: *(a) Illustration of creep and apparent preconsolidation pressure for Jonquiere clay (Leroueil et al., 1996) (b) Constant rate of strain tests with different strain rates for Batiscan clay (Leroueil et al., 1985).*

Critical state

Soil when subjected to deviatoric stresses eventually tend towards an ultimate condition where plastic (deviatoric) strains, *i.e.* shear, continue indefinitely under constant volume. This state is termed as the critical state (Muir Wood, 1991). Critical state is the ultimate condition reached irrespective of the original state (void ratio, anisotropy and bonding) prior to shearing. Hence, the critical state is used as a reference state to describe the strength properties and the effect of overconsolidation ratio and void ratios in different soils.

Figure 2.4 shows the critical state concept in the triaxial stress space. When the stress ratio reaches the critical state line ($\eta = M$) in compression or extension, the soil is said to have reached the ultimate state (Mitchell and Soga, 2005). The critical state of a soil is independent from strain rate effects, which makes it a useful reference criterion for design of structures where loads can go beyond the peak strength and progressive failure might occur due to the deformation in the structure. A distinct difference in behaviour can be observed between initially normally consolidated and overconsolidated states (Muir Wood, 1991). In general, normally or lightly overconsolidated soils whose stress state lies above and right to the critical state line (wet side) in the compression plane, exhibit higher drained strength with volumetric hardening or positive excess pore-water pressure generation. In contrast, for heavily overconsolidated clays, the stress state lies below and left to the critical state line (dry side) and exhibits dilatancy or negative excess pore pressure.

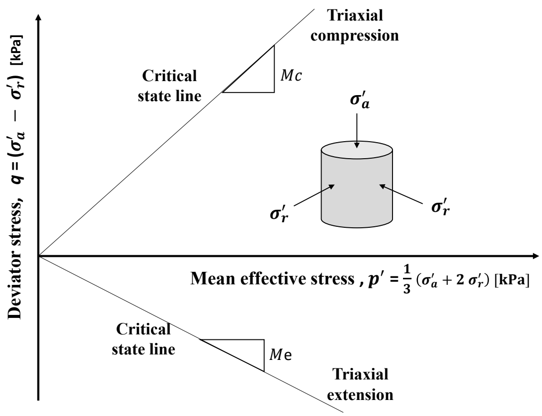


Figure 2.4: *Illustration of the critical state in the $p' - q$ space.*

2.3 Modelling based on Critical-State soil mechanics

The emergence of the framework of critical state soil mechanics by the research team at the University of Cambridge in the late 1950s (Roscoe et al., 1958) is considered one of the most significant developments in geotechnical engineering. Constitutive models that are based upon the critical state soil mechanics framework are most effective in representing, and understanding, the behaviour of soft soils under any given stress path. Roscoe and his team conducted experiments and developed mathematical formulations based on yielding and the critical state concepts, which ultimately resulted in the Cam-Clay models (Roscoe and Burland, 1968; Schofield and Wroth, 1968). Unfortunately, the relative complexity of their formulation delayed their use in practice until the mid 1970s. However, with the continuous efforts of the geotechnical community to develop better and more sophisticated constitutive models, the challenge of predicting and simulating complex features of soft soil behaviour has been improved. A version of a constitutive model for soft soils which can be traced back to the Cam-Clay type formulation is described briefly in the next section. The constitutive model used in this thesis has been refined in its predictive accuracy by hierarchically introducing additional model features.

An advanced constitutive model: Creep-SCLAY1S

Creep-SCLAY1S is an advanced rate-dependent constitutive model for normally consolidated or lightly overconsolidated soft soils formulated in the general stress space. One of the major advantages of Creep-SCLAY1S is that it is a hierarchical rate-dependent model, similar to SCLAY1S (Karstunen et al., 2005). Thus, features such as hardening laws that capture anisotropy and destructuration can be "switched off" by appropriate choice of input parameters (Koskinen et al., 2002; Karstunen et al., 2005; Gras et al., 2018). Therefore, the user can explore different features of soft soil behaviour in an intuitive and simple manner. The model has been tested and successfully validated at the element level (Yannie and Sivasithamparam, 2016; Gras et al., 2018) and against several diverse boundary value problems (Cheang et al., 2016; Sexton et al., 2016; Tornborg et al., 2021, 2023). The downside of the model is that it requires rather many (14) input parameters, of which five are similar to those used in the (isotropic) modified Cam–Clay model. Three of those parameters are state parameters, used for the initialisation of the model and updated during the analysis. Although the required parameter set is large, most of the parameters have a physical meaning and can thus be derived directly from conventional element level laboratory tests. There is no purely elastic domain in this model, similar to the Anisotropic Creep Model (Leoni et al., 2008); hence viscoplastic deformations are assumed to occur at all effective stress states. The total strain rate is decomposed into elastic and viscoplastic component:

$$\dot{\epsilon_v} = \dot{\epsilon_v^e} + \dot{\epsilon_v^c} \qquad \dot{\epsilon_q} = \dot{\epsilon_q^e} + \dot{\epsilon_q^c} \tag{2.1}$$

where the superscripts 'e' and 'c' refer to the elastic and viscoplastic strain components, respectively, and the subscripts 'v' and 'q' refer to the volumetric and deviatoric part.

The model assumes isotropic elastic behaviour similar to the Modified Cam-Clay model. The elastic volumetric strain rate ($\dot{\epsilon_v^e}$) and elastic deviatoric strain rate ($\dot{\epsilon_q^e}$) are defined in relation to the stress dependent bulk modulus, $K = p'/\kappa^*$ and shear modulus, $G = 3K(1-2\nu')/2(1+\nu')$:

$$\dot{\epsilon_v^e} = \frac{\dot{p'}}{K} \qquad \dot{\epsilon_q^e} = \frac{\dot{q}}{3G} \tag{2.2}$$

where the invariants p' and q represent, respectively, the mean effective stress and deviatoric stress in the triaxial stress space. Figure 2.5 shows the surfaces used in the Creep-SCLAY1S model in triaxial stress space. The Normal Consolidation Surface (NCS), whose size is defined by the isotropic preconsolidation pressure p'_m, acts as a bounding surface and delimits the small and large creep strain rates. The current effective stress state is defined by the current stress surface (CSS) with the size defined by p'_{eq}. An imaginary intrinsic compression surface (ICS) proposed by Gens

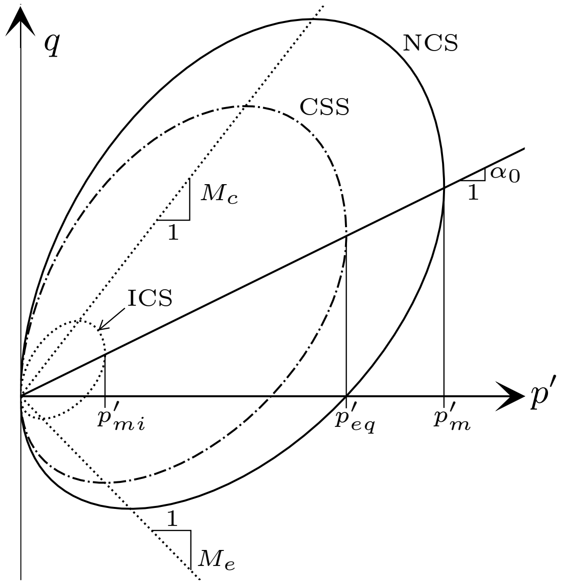


Figure 2.5: *Illustration of model surfaces used in Creep-SCLAY1S model (Tornborg et al., 2021).*

and Nova (1993) is introduced, with size (p'_{mi}) is related to p'_m through the bonding parameter 'χ' to capture the degradation of bonding observed in soft sensitive soils.

The size of ICS increases as a function of incremental volumetric creep strains relating to the isotropic hardening rule. The subscript 'i' in Equation 2.3 refers to the intrinsic value.

$$p'_m = p'_{mi}(1+\chi) \qquad \dot{p}'_{mi} = \frac{p'_{mi}}{\lambda^*_i - \kappa^*}\dot{\epsilon}^c_v \tag{2.3}$$

All three surfaces have similar shape and orientation, with the mathematical expression that defines the size for triaxial conditions:

$$p'_s = p' + \frac{(q - \alpha p')^2}{(M(\theta_\alpha)^2 - \alpha^2)\,p'} \tag{2.4}$$

where p'_s can be equal to p'_m, p'_{eq} or p'_{mi} to define the sizes of NCS, CSS or ICS, respectively.

The orientation of the surfaces is governed by the scalar variable α_0 representing the initial anisotropy of the soil. The inclination of the critical state line M has been made a function of Lode angle (Sheng et al., 2000), used to control the slope of the critical state line in triaxial extension (M_e) and triaxial compression (M_c). Due to the incorporation of a smooth failure surface, the model has a numerical advantage over the Mohr-Coulomb failure surface by avoiding sharp corners. To avoid repetition, the expressions related to Lode angle dependent critical state line are not elaborated here and the reader is directed to Gras et al. (2017) for more details.

An associated flow rule is assumed in the Creep-SCLAY1S model which is reasonable when evolution of anisotropy is included in the formulation (Wheeler et al., 2003). Therefore, the viscoplastic strain rate is considered similar to the viscous formulation inspired by the Anisotropic Creep Model (Leoni et al., 2008):

$$\dot{\epsilon^{c}_{ij}} = \dot{\Lambda}\frac{\partial p'_{eq}}{\partial \sigma'_{ij}} \tag{2.5}$$

In the above equation, the viscoplastic multiplier following the idea of Grimstad et al. (2010) is considered:

$$\dot{\Lambda} = \frac{\mu_i^*}{\tau}\left(\frac{p'_{eq}}{(1+\chi)p'_{mi}}\right)^{\frac{\lambda_i^*-\kappa^*}{\mu_i^*}} \frac{M_c^2-\alpha^2_{K_0^{NC}}}{M_c^2-\eta^2_{K_0^{NC}}} \tag{2.6}$$

where μ_i^* is the modified intrinsic creep index, measured in the volumetric strain - ln (time) plot. μ_i^* is an intrinsic value, and hence should be derived from a load step where all bonding in the material has been degraded (either at large effective stress levels, or alternatively from tests on reconstituted samples). The reference time τ is usually taken as 24 hours, which is the standard time for the load steps in incremental loading (IL) oedometer tests, but can be adjusted to match the actual rate of testing. λ_i^* is the modified intrinsic compression index and κ^* the modified swelling index that can be derived directly from the $\epsilon_v - ln\ p'$ plot. The right term in the Equation 2.6 has been added to ensure that under oedometric conditions the resulting creep strain corresponds to the measured volumetric creep strain rate. $\alpha_{K_0^{NC}}$ defines the inclination of the ellipses in the normally consolidated state (assuming normally consolidated K_0 consolidation history).

The scalar state variable α represents the amount of anisotropy, and the changes in anisotropy are calculated by tracking the evolution of the surfaces as a function of viscoplastic strain rates. In the general stress space a fabric tensor, analogous to the deviatoric stress tensor, is used to account for principal stress rotations, and the consequent fabric rotations, in three dimensions. For cross-anisotropic samples in the triaxial stress space, the rotational hardening law is simplified into:

$$\dot{\alpha_d} = \omega\left(\left[\frac{3\eta}{4} - \alpha_d\right]\langle\dot{\epsilon^c_v}\rangle + \omega_d\left[\frac{\eta}{3} - \alpha_d\right]\left|\dot{\epsilon^c_d}\right|\right) \tag{2.7}$$

where η is the stress ratio and ω & ω_d are model constants. Here ω controls the absolute effectiveness of rotational hardening and ω_d controls the relative effectiveness of rotational hardening due to deviatoric viscoplastic strain rate. The Macauley brackets $\langle\rangle$ are used here to maintain a sensible prediction on the dry side of the critical state line, and a modulus sign for the viscoplastic deviatoric strain rate due to sign convention in triaxial testing.

The initial inclination of the yield surface, α_0, can be determined with drained radial stress probing at different stress ratios. The yield points can subsequently be used to fit a yield surface with a unique α_0 value (Koskinen et al., 2002; Wheeler et al., 2003). However, this is not done in everyday applications due to complexity in testing and long duration of drained tests on fine-grained soils. Assuming an associated flow rule for the Creep-SCLAY1S model, 'α_0' can be calculated from a simplified relation recommended by Wheeler et al. (2003).

$$\alpha_0 = \frac{\eta_{K_0}^2 + 3\eta_{K_0} - M_c^2}{3} \qquad\qquad \eta_{K_0} = \frac{3(1 - K_0^{NC})}{(1 + 2K_0^{NC})} \tag{2.8}$$

The anisotropic hardening formulation assumes that the inclination of the Normal Consolidation Surface tends towards a target value for a given stress ratio, depending on the magnitude of the volumetric and deviatoric viscoplastic strains. The constants in equation, ω and ω_d control the absolute and relative effectiveness of the anisotropic change observed in the test.

In order to derive the value of ω for a given soil, indirect methods such as conducting experimental tests involving significant rotation of the yield surface are required. *e.g.* An undrained shearing in triaxial extension is most suitable to assess the value of ω. In the absence of suitable experimental data, a typical value of ω ranging between '$10/\lambda_i$' to '$15/\lambda_i$' suggested by Yin and Karstunen (2011) can be taken, which suggests that within this range the model predicts the complete degradation of anisotropy of the soil when subjected to stresses three times larger than the yield stress. Parameter bounds related to structure and anisotropy for Creep-SCLAY1S model are discussed in Gras et al. (2018). For ω_d, a simplified formulation is proposed:

$$\omega_d = \frac{3}{8} \frac{(4M_c^2 - 4\eta_{K0}^2 - 3\eta_{K0})}{(\eta_{K0}^2 - M_c^2 + 2\eta_{K0})} \tag{2.9}$$

It is evident from Equations 2.9 and 2.8 that the value of both ω_d and α_0 depend strongly on the slope of the critical state line in triaxial compression. The higher the value for the friction angle, the higher the anisotropy, and relative effectiveness of rotational hardening.

The rate of bond degradation is given as a function of viscoplastic volumetric and deviatoric strain rate. Again there are two model constants ξ and ξ_d which controls the absolute destructuration rate and the relative effectiveness due to viscoplastic deviatoric strain rate:

$$\dot{\chi} = \xi \left([0 - \chi] \, |\dot{\varepsilon_v^c}| + \xi_d \, [0 - \chi] \, \dot{\varepsilon_d^c} \right) = -\xi \chi \left(|\dot{\varepsilon_v^c}| + \xi_d \dot{\varepsilon_d^c} \right) \tag{2.10}$$

The rate of bond degradation due to volumetric and deviatoric viscoplastic strains is controlled by the parameters ξ and ξ_d respectively. To estimate ξ, the sample needs to be isotropically compressed to stress states where the initial bonding in the sample is erased and there would practically be volumetric strains only. In order to determine the parameter ξ_d, ξ is used along with high stress ratio loading in triaxial tests to obtain the bond degradation from shear. In practice, however, drained tests on such stress paths are rarely available, and usually data from IL or CRS stress paths are used to fit the values of these two parameters. In this case, an optimisation procedure can be employed to fit laboratory data for the parameters controlling the rate of destructuration, but uniqueness is not guaranteed (Gras et al., 2017). In most cases, the deviation of the values remains in a small range for a typical region of the soft soil profile.

Succinctly, the input parameters of the Creep-SCLAY1S model and their description are tabulated in Table 2.1.

Table 2.1: Description of Creep-SCLAY1S model parameters

Feature	Parameter	Unit	Description
Critical state	κ^*	–	Modified swelling index
	ν'	–	Poisson's ratio
	λ_i^*	–	Modified intrinsic compression index
	M_c	–	Slope of critical state line in triaxial compression
	M_e	–	Slope of the critical state line in triaxial extension
Anisotropy	ω	–	Absolute effectiveness of rotational hardening
	ω_d	–	Relative effectiveness of rotational hardening
	α_0 †	–	Initial inclination of the reference surface
Bonding	ξ	–	Absolute rate of destructuration
	ξ_d	–	Relative rate of destructuration
	χ_0 †	–	Initial amount of bonding
Viscous	τ	d	Reference time for creep and definition of OCR
	μ_i^*	–	Intrinsic modified creep index
Initial conditions	OCR †	–	Over-consolidation ratio (corresponding to reference time τ)
	e_0 †	–	Initial void ratio

† Initial state variables

2.4 Model sophistication: boon or bane ?

Advanced geotechnical models use a large number of parameters to simulate the soil response in geotechnical problems. Identification of some of these parameters can be challenging, as they are not directly related to measurable soil properties, but are instead derived from mathematical equations. Therefore, it is important to have a systematic procedure to first evaluate and fix the model parameters that can be directly derived from lab data, before fitting the remaining parameters for a suitable loading path.

Advanced models may not be appealing to practicing engineers because of their complex formulation and large number of input parameters. In addition, many of these models use parameters, such as the anisotropy and bonding parameters of the Creep-SCLAY1S model that require non-traditional loading paths in traditional laboratory tests. This kind of testing may not be practical in the industry due to the limited access to advanced laboratory facilities and the associated time for testing and the (perceived) costs. In most projects, there are also issues with poor sample quality resulting from extraction, transport and storage (*e.g.* Lunne et al., 1997; Karlsson et al., 2016).

Moreover, even the simplest of elastoplastic models can be significantly impacted in accuracy by a small number of samples used to derive their model parameters. Due to the typically large parameter set involved in more advanced models, the evaluated values, when done manually, are prone to error, and as discussed, sometimes the values are non-unique. This creates a rigid dichotomy between sophistication and robustness. On the one hand, sophistication helps to closely fit the physical system. The latter demands more information, such as the parameters that define the constitutive model, which is quite difficult to get, and comes with additional uncertainties.

2.5 The need for consistent parameter derivation

For cases where a comprehensive dataset is made available for the purpose of calibrating an advanced model, the accuracy in the derived parameter values play a key role in the performance of the model. This paves the way for optimisation techniques to adjust model parameters to suit laboratory and field data so that the model prediction matches the experimental/measured data.

Typically the parameter space is bounded to logical values (either known from experience or from prior sensitivity analyses) before optimisation. The mathematical framework involves defining an error function that quantifies the deviation between the measured and the computed response, and which is then iterated with perturbations in the parameter space until the error function converges to a tolerable value. A plethora of optimization techniques, both deterministic and stochastic, exist and each of them have their own advantages and drawbacks. However, the common limitations incurred include i) getting trapped in a local minimum, ii) sensitivity to initial conditions, iii) model-intrinsic control parameters, iv) inconsistency in the model parameters (*i.e.* the fitted values vary each time the algorithm is run for the very same input data).

A comparative analysis of some of the commonly used optimisation algorithms in geotechnical engineering is summarised in Yin (2017). The use of such optimisation procedures only further complicates the use of advanced constitutive models. Hence, it is obvious that a consistent methodology for deriving parameters from raw data (from both laboratory and field) in an automated environment is paramount to minimise the dependence on external optimisation techniques.

Several researchers have shown that optimisation techniques, although a useful tool to predict non-conventional parameters, have failed to show uniqueness in the fitted values for models even simpler than the Creep-SCLAY1S model, and hence cannot be relied upon entirely.

In this thesis, a module for automated parameter derivation has been created using a series of custom scripts that work with actual measurement data from both laboratory and field. The purpose of this automated approach is to enable the user to process numerous dataset simultaneously, and thus efficiently derive a precise and consistent set of parameters in a short period, while minimizing the risk of errors. To manage the level of required user intervention, which would depend on the project and quality of dataset, a toggle for adjusting different levels of automation (from semi-automated to fully automated) is provided in the module. This module has been implemented for analysing trial embankments in **Paper A** and **Paper B** and shall be detailed in Chapter 5.1.

2.6 Limitations of the deterministic approach

Although a highly advanced model with a systematic approach for deriving parameters can provide relatively accurate results at laboratory scale, it can still encounter inaccuracies in predicting the field-scale behaviour, due to the complexity of geological materials and the incomplete understanding of their formation history. Additionally, the differences between the design and as built, and the impact of construction practices that might be different among different contractors, further complicate the system. Hence, discrepancies between measured and predicted outcomes at system level are likely to occur, and it is challenging to fully rely on a single deterministic analysis. Moreover, due to the limited amount of data available, numerical models based on such information may also result in further uncertainty. Therefore, relying solely on the model without considering other factors can pose a risk in decision-making.

A common approach in industry to overcome uncertainties is to perform parametric analyses. In a parametric analysis various combinations of parameter values are tested, which can be both time-consuming and expensive, and may not always produce satisfactory outcomes. In view of this issue, practitioners for real-world projects resort towards ad-hoc strategies such as applying global factor of safety (or partial factors of safety). Alternative strategies to address the uncertainty involve choosing conservative input parameters, drawing insights from past experiences and historical data, revising computation models and construction methods based on testing and observations conducted on actual projects. Overall, the integral part in all these approaches is the engineering judgement.

The quality of predictions using the Finite Element Method in geotechnics is mainly dependent on the constitutive model used and the calibration of the model parameters. Several constitutive models based on different theories and hypotheses have been developed and used for embankments. However, most of these methods are deterministic and do not consider uncertainties in the parameters. Application of probabilistic analysis of embankments is relatively rare in the literature (Liu et al., 2018b; Zheng et al., 2018; Tian et al., 2022a).

In order to bridge the gap between the research and practical applications, it is important to shift the focus towards developing analyses that are practical and realistic. When considering uncertainty, incorporating probabilistic methods can aid in identifying model parameters. In probabilistic methods, model parameters are treated stochastically, using probability distributions for each parameter. As constitutive models for soils are increasingly utilised in engineering analysis, the input parameters for these models become critical design parameters that must also be managed probabilistically (Most, 2010; Jin et al., 2019).

3 Probabilistic analysis

3.1 Uncertainty in geotechnics

There are two primary forms of uncertainty, known in general as aleatory and epistemic uncertainty. Aleatory uncertainty, also known as irreducible uncertainty or inherent variability, pertains to the inherent randomness or variability in natural processes. In contrast, epistemic Uncertainty refers to those arising from a lack of knowledge or information about the mechanisms governing the behaviour of a system (Kiureghian and Ditlevsen, 2009).

Aleatory uncertainties, relate to the natural variability of the ground conditions caused by geological processes that constantly modify the spatio-temporal properties of the ground. It is not affected by the accumulation of knowledge and is considered independent (Baecher and Christian, 2003). Quantifying its impact on the overall uncertainty and determining its significance can be challenging since it is highly dependent on site-specific conditions.

There are two types of errors in measurement: systematic bias and random errors. They arise from errors in the test equipment or from procedural-operator errors during the measurement process. These errors are part of epistemic uncertainty, which can be reduced as knowledge improves. Improvements in measurement techniques and equipment can help to minimise the presence and magnitude of measurement errors (Phoon and Kulhawy, 1999).

The uncertainty that emerges from inferring soil properties and underground stratigraphy using a limited amount of information is called statistical uncertainty. This is a form of knowledge uncertainty and it cannot be eliminated unless the complete subsurface is investigated, which is not feasible. Therefore, statistical uncertainty is an unavoidable aspect of site investigation.

When dealing with real-life scenarios, it is usual to merge various forms of variabilities and uncertainties, and treat them as a single entity known as total variability. Among this collective entity, the uncertainties related to selecting the appropriate models, their parameters, and soil layer boundaries, among others, exert significant influence on the predictive results.

Uncertainty in the model and its parameters

The precise knowledge of the mechanical behaviour of the *in-situ* ground conditions requires a better understanding of the site, while the extent of knowledge and experience the user has on the model reflects the better understanding of the model. However, the quality of prediction is only optimal for a specific combination of model prediction and observed behaviour. This discrepancy can be attributed to: (1) the evaluation of design parameters for the model, either stiffness or strength depending on the design (2) the formulation of the model itself. Due to the influence of empiricism when formulating geotechnical prediction models, uncertainties associated with the model formulation can be substantial. As outlined in Steinberg et al. (2001) and illustrated in Figure 3.1, the overall uncertainty in the predictions made by a model arises from sources including the formulation defining the model complexity, the number of model parameters (depending on the model complexity), and other factors such as the initial and boundary conditions. The development of a model with ideal complexity that works perfectly for all situations is unlikely, and the effectiveness of a model typically is site-specific.

In practice, despite the complexity of soil behaviour, reasonable predictions for the serviceability

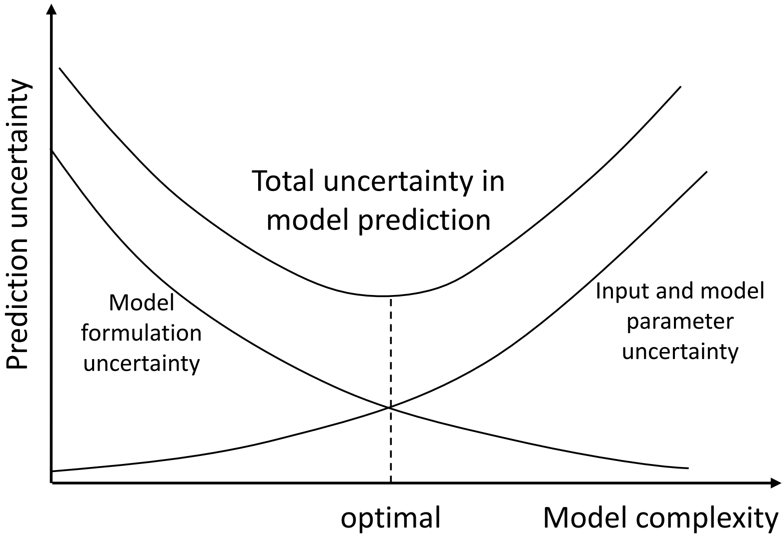


Figure 3.1: *Balancing the uncertainty from model formulation and model parameters.*

limit state, *e.g.* settlements below the centre line of an embankment, are still achieved through empirical calibrations for local conditions. However, the results are non-generic outside the bounds of the model capabilities and dataset used. Although the generality of complex models is higher, this approach can lead to a high number of model parameters required. The complex model fits data well, due to the higher degree of freedom, resulting in a less biased model. However, there may be several non-unique parameter choices that can produce a similar fit, leading to high parameter variance. As a result, the problem becomes ill-posed and the estimated parameters may not accurately reflect the system dynamics.

The simpler models, on the other hand, are stiff, *i.e.* only a moderate fit to the dataset is obtained by adjusting the model parameters making it relatively more biased, since there are typically only a few parameter choices that can provide the moderate fit.

One of the major difficulties in geotechnical modelling is to find or create a model with sufficient rigidity, with few parameters, while still being able to fit the data well. This is almost impossible, because to enhance the generic capabilities of the model, a large parameter set is inevitable and some of its sensitivity varies with different stress paths, stress levels, and boundary conditions, as explained in Tahershamsi and Dijkstra (2021). The efficacy of the chosen inverse analysis procedure can reduce the bias and variance of the estimates of the model parameters, regardless of the model complexity. However, the success of this approach can only hold value if the calibrated model parameters fall within the bounds of logical values (discussed in **Paper D**).

3.2 Probability theory

Probability theory is a commonly used mathematical framework for dealing with uncertainties. It is based on events and combinations of events. The mathematical theory of probability starts with the definition of the *probability space* which is represented by the three components: (Φ, $\mathcal{A}$, P). Probability theory is utilised to model situations in which outcomes happen randomly. These situations are commonly known as experiments, and the *sample space*, Φ, consists of all possible outcomes of the experiment. The *event space* $\mathcal{A}$ is the set of elements of Φ, also known as σ-algebra on Φ. The third component is the probability measure P.

The first axiom follows that the probability of an event is a non-negative real number and is always finite, $P : \mathcal{A} \rightarrow [0; 1]$. This gives the probability value for each event, $E_i \in \mathcal{A}$. The condition $P(\Phi) = 1$ needs to be satisfied which is the second axiom. The third axiom states that any countable sequence of disjoint events should satisfy $P(\bigcup_i E_i) = \sum_i P(E_i)$. These axioms fit well with the intuitive understanding of probability (Reichenbach, 1949; Billingsley, 1976; Jaynes, 2003). The following interesting properties are also noted:

- **Conditional probability**: The notion of conditional probability, along with its related concepts of independence and dependence, is among the most significant concepts in probability theory. The conditional probability of event A given B is written as $P(A|B)$.

- **Marginal probability**: This is defined as the probability of an event occurring in isolation, and may be thought of as an unconditional probability, P(A). If the event A is independent of B, then the conditional probability becomes equal to the marginal probability as shown below:

$$P(A|B) = P(A) \tag{3.1}$$

- **Joint probability**: $P(A \cap B)$. Joint probability is the intersection of two or more events. The probability of the intersection of A and B may be written as:

$$P(A \cap B) = P(A)\ P(B|A) = P(B)\ P(A|B). \tag{3.2}$$

  This can be arranged to give the relationship among the conditional probabilities known as *Bayes theorem*:

$$P(B|A) = \frac{P(A|B)\ P(B)}{P(A)} \tag{3.3}$$

  The *theorem of total probability* can be applied here for the denominator which is

$$P(B|A) = \frac{P(A|B)\ P(B)}{\sum_{i=1}^{n} P(A|B_i)\ P(B_i)} \tag{3.4}$$

Random variable

For a discrete univariate set X, the function to calculate the probability of event A in terms of probability mass function (pmf) over the sample space, Φ, is given below,

$$P(A) = \sum_{A} f_X(x)\ , \quad [x \in A]\ ; \quad \sum_{i=1}^{n} f_X(x_i) = 1 \tag{3.5}$$

If the random variable is continuous, then the probability is dealt in terms of distribution (pdf) which then requires integration over the sample space, Φ,

$$P(A) = \int_{A} f_X(x)\, dx\ , \quad [x \in A]\ ; \quad \int_{-\infty}^{\infty} f_X(x)\, dx = 1 \tag{3.6}$$

The cumulative mass function and cumulative distribution function of X is shown below.

$$F_X(x_j) = \sum_{i=1}^{j} f_X(x_i) \ \text{[discrete]} \quad ; \quad F_X(x_i) = \int_{-\infty}^{x_i} f_X(x)\, dx \ \text{[continuous]} \tag{3.7}$$

Moments of probability distribution

Descriptive statistics are necessary to express probability information. The mean is a widely used measure of central tendency and represents the centre of mass of the distribution. For both probability mass function (pmf) and probability density function (pdf), the first moment of the distribution around the origin provides the expectation or mean.

$$\mathrm{E}(X) = \frac{1}{N} \sum_{i=1}^{N} x_i \quad \text{[discrete]} \tag{3.8}$$

$$\mathrm{E}(X) = \int_{-\infty}^{+\infty} x\, f_X(x)\, dx \quad \text{[continuous]} \tag{3.9}$$

A way to convey the variability in a dataset is by using the range, which is the difference between the largest and smallest values. Nevertheless, the range is not a reliable statistical measure because it is heavily influenced by extreme values. Instead, the variance is a preferable measure in many statistical analyses.

$$\mathrm{Var}(X) = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mathrm{E}(X))^2 \quad \text{[discrete]} \tag{3.10}$$

$$\mathrm{Var}(X) = \mathrm{E}(x - \mathrm{E}(X))^2 = \int_{-\infty}^{\infty} (x - \mathrm{E}(X))^2\, f_X(x)\, dx \quad \text{[continuous]} \tag{3.11}$$

The *coefficient of variation* (C.O.V.) is commonly used to measure uncertainty in soil properties. The standard deviation is divided by the mean and represents a second-order assessment of data dispersion. C.O.V. of parameters accounts for measurement errors, model uncertainty, and inherent uncertainty. However, not all parameters are considered uncertain, *e.g.* unit weights are often deemed deterministic parameters with a C.O.V. of less than 10%. (Kulhawy et al., 2000). A comprehensive investigation into the uncertainty of soil properties used in design was conducted by Phoon and Kulhawy (1999).

For the case of a bivariate dataset, X and Y, the covariance is the extension of their corresponding variances.

$$\begin{aligned}\mathrm{Cov}(X,Y) &= \mathrm{E}((x-\mathrm{E}(X))(y-\mathrm{E}(Y))) \\ &= \sum_x \sum_y (x-\mathrm{E}(X))\,(y-\mathrm{E}(Y))\, f_{XY}(x,y) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mathrm{E}(X))\,(y-\mathrm{E}(Y))\, f_{XY}(x,y)\, dx\, dy\end{aligned} \tag{3.12}$$

The correlation between the two variables is assessed by the Pearson correlation coefficient, ρ_{XY}, defined as,

$$\rho_{XY} = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} \tag{3.13}$$

3.3 Uncertainty propagation

While deterministic design methods still play a crucial role, probabilistic methods provide a systematic and quantitative approach for geotechnical engineers to infuse uncertainties into their calculation. In order to evaluate the impact of uncertainty in soil properties on the behaviour of a system, it is logical to go beyond a deterministic approach and employ probabilistic analyses. These analyses consider the properties of homogenised material layers as random variables, enabling the treatment of uncertainty in material properties.

The model to predict the soil behaviour works by mapping the input space (which is controlled by the parameters denoted as θ) to the output space (also known as the prediction space denoted as $\mathcal{Y}$) using the forward dynamics function, denoted as F. If the model parameters are treated as random variables, the uncertainties associated with these parameters are carried forward to the output through the forward dynamics.

$$\mathcal{Y} = F(\theta) \tag{3.14}$$

Various methods exist to propagate uncertainties in forward calculations, but one of the most commonly employed approaches is Monte Carlo simulation (MCS) (Fishman, 1995). With advancements in computing power, MCS has become increasingly important. MCS involves establishing a probability distribution for each independent variable, and then running a simulation where, during each iteration, a random value from the distribution function for each parameter is selected and used in the calculation. The independent random samples are generated with size N_X as $\theta = [\ \theta_1, \theta_2, \theta_3, \ldots, \theta_{N_X}\ ]$. Once the simulation is executed repeatedly, it produces multiple results represented as $\mathcal{Y} = [\ F(\theta_1), F(\theta_2), F(\theta_3), \ldots, F(\theta_{N_X})\ ]$. The number of samples needed is determined by factors such as the number of input variables, the complexity of the model, and the accuracy required for the output. The result of the simulation is a probability distribution of the output parameter. The statistics of the output is calculated as shown below:

$$E(Y) \approx \frac{1}{N_X} \sum_{y \in \mathcal{Y}} y \tag{3.15}$$

$$\mathrm{Var}(Y) \approx \frac{1}{(N_X - 1)} \sum_{y \in Y} (y - E(Y))^2 \tag{3.16}$$

Theoretically, the rate at which the error in the estimate reduces is proportional to the square root of the number of samples (Baecher and Christian, 2003). The fundamental concept of Monte Carlo simulation is easy to understand and straightforward. Even though the importance of uncertainty analysis is acknowledged, the implementation in engineering practice is still lagging, due to the difficulties in quantifying all the uncertainties in a project *e.g.* loads, geologic site interpretations, geotechnical properties, computation models, *etc.* thereby increasing the dimension of the problem. Monte Carlo methods, in this regard, are advantageous for geotechnical problems, as they are generally applicable and can handle high-dimensional random variables and complex problems, making them ideal for geotechnical applications.

In this thesis, a method known as the Data Assimilation technique which incorporates observations into numerical forecasting models as part of an inverse analysis procedure is pursued. Among the different Data Assimilation tools, Monte-Carlo based techniques are considered and the details of these algorithms are mentioned in Chapter 4.

3.4 Inverse analysis

In geotechnics, inverse modelling techniques have been used for estimation of parameters as part of model calibration. During model calibration, the parameters are modified until the model response matches the measured response for a certain loading path. The physical meaning of the model parameters, however, are preserved only when the model accurately represents the system complexity (as demonstrated in **Paper C & D**). Usually, numerical models are calibrated by using *ad-hoc* trial and error methods. The main benefit of using an inverse analysis is its ability to automatically determine the best parameter values that match the observed and computed results.

The approach of inverse analysis with Bayesian statistical framework is the most effective for updating of model parameters (Wu et al., 2007; Zhang et al., 2010; Juang et al., 2013). In probabilistic methods the *a priori* information on the model parameters is represented by a probability distribution over the 'model space'. This gives the engineer the additional advantage to assess the range of possible behaviour of the geo-structure, whilst using monitoring data to update the model predictions.

Recent developments in other scientific domains have shown the efficacy of a powerful method, known as Data Assimilation (DA), which systematically incorporates observations into numerical forecasting models. There are several types of DA algorithms some of which follow the Bayesian approach more exactly, in terms of representing uncertainty (Geer, 2021), *i.e.* they rigorously integrate observations and numerical forecasting models, and account for their uncertainties to estimate the state of an evolving system. This enables realistic estimations with a reduced overall variance in the prediction. Chapter 4 is dedicated to this method.

3.5 Bayesian Inference

Benefits of the Bayesian approach in geotechnics

Despite following the same axioms of probability theory, the distinction between the frequentist and Bayesian approaches stems from their different interpretation of probability. Frequentists view probability as a measure of frequency, while Bayesians view it as a measure of belief. Frequentist methods do not require the specification of a prior probability distribution and focus on the probabilities of observed and unobserved data. The frequentist approach operates on the assumption that the probability of an event happening is equivalent to the frequency with which that event occurs over a long period of time.

Bayesian statistics has two qualities that make it suitable for geotechnical applications: the ability to assign probabilities to states of nature, such as site conditions or engineering parameters, and the allowance of subjective probabilities. In contrast, frequentist statistics does not permit either of these. (Baecher, 2021). The crucial difference between Bayesian and frequentist statistics is explained in Figure 3.2.

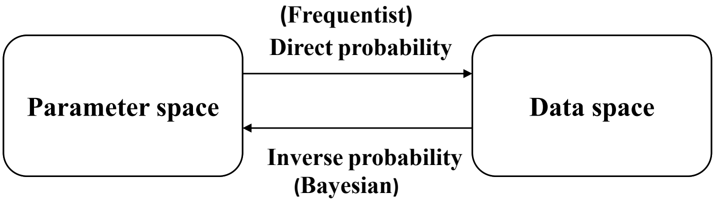


Figure 3.2: *Illustrative plot to show the difference between Bayesian and Frequentist approaches. Adapted from Baecher (2021)*

Bayesian methods can access the full potential of data in geotechnical practice. These methods offer a robust approach for estimating probabilities for field conditions using available geotechnical measurements, including *in situ* and laboratory tests, as well as field monitoring data, such as displacements and excess porewater pressure. In essence, the Bayesian approach is a systematic method for making logical deductions when faced with uncertainty. The advantages of using Bayesian approach are numerous, and include the capacity to model complex problems using robust algorithms (*e.g.* from Data Assimilation in Chapter 4) to analyse data while considering uncertainty.

Framework

Bayesian inference is a method of fitting a statistical model by utilising prior knowledge on the model parameters (Θ) and combining it with the observed monitoring data, using Bayes' theorem (Equation 3.3). In this context, all unknowns are considered as random variables, and the distribution of the model parameters, conditioned on the monitoring data, is expressed using Bayes' theorem (shown in Figure 3.3 assuming a Gaussian distribution for both the parameter and data space). Figure 3.4 shows the working of a typical Bayesian inverse analysis procedure.

$$P(\Theta \mid \text{data}) = \frac{P(\text{data} \mid \Theta)\ P(\Theta)}{P(\text{data})} \quad \propto \quad P(\text{data} \mid \Theta)\ P(\Theta) \tag{3.17}$$

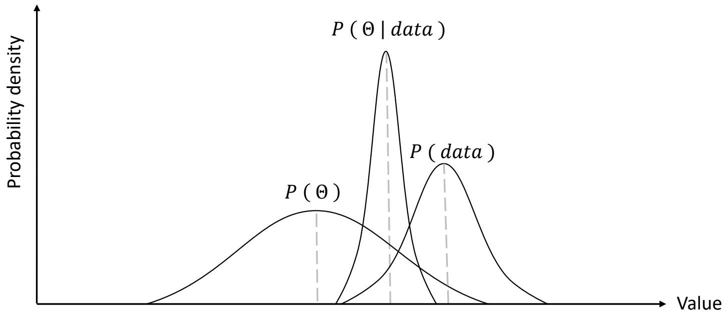


Figure 3.3: *Illustration of Bayesian analysis*

The following definitions apply:

- $P(\textbf{data} \mid \Theta)$, also represented as $\mathcal{L}(\Theta; \text{data})$, denotes the likelihood function which represents the degree to which the parameter distribution describes the data. Considering the case of a univariate Gaussian distribution, $\Theta \sim N(\mu_\Theta, \sigma_\Theta)$, and a single measurement, y, with noise σ, the likelihood function is then defined as,

$$P(\text{y} \mid \Theta) = \frac{1}{\sigma\sqrt{2\pi}}\exp\Big(-\frac{1}{2}\Big(\frac{y - \mu_\Theta}{\sigma_\Theta}\Big)^2\Big) \tag{3.18}$$

  For more than one measurement, $\mathcal{Y} = [y_1, y_2, \dots, y_N]$, and assuming independence between the measurements, the likelihood function is given as:

$$\mathcal{L}(\Theta; \mathcal{Y}) = \prod_{i=1}^{N} P(y_i \mid \Theta) \tag{3.19}$$

- $P(\Theta)$ denotes the *a priori* state of the parameters. The selection of this distribution should be based on the available information about the parameters before any monitoring is conducted. Further details are provided in the next section.

- $P(\textbf{data})$ is the evidence (also called the marginal distribution) and is often used for normalisation. Based on Equation 3.4, the *Theorem of total probability* applies: $P(\text{data}) = \int P(\text{data} \mid \Theta)\ P(\Theta) = \sum_i P(\text{data} \mid \Theta_i)\ P(\Theta_i)$.

- $P(\Theta \mid \textbf{data})$ is the posterior distribution which reflects the updated knowledge on the model parameters after considering the prior knowledge and site observation data.

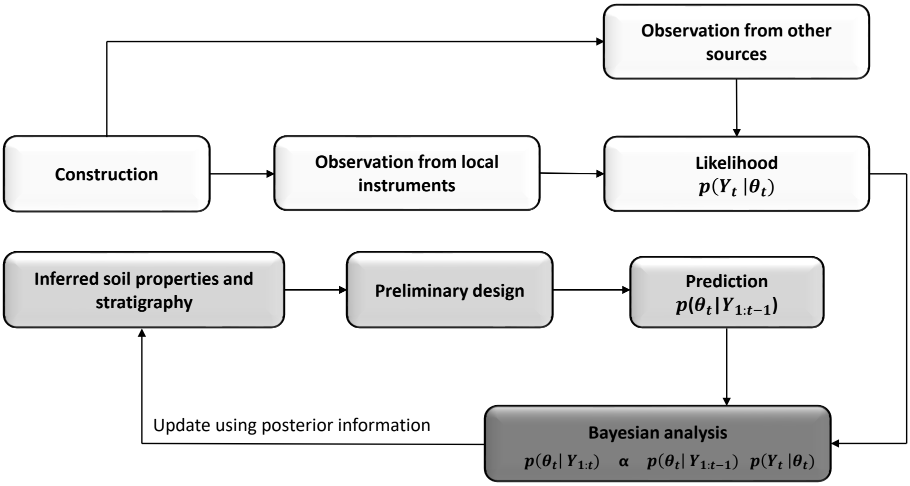


Figure 3.4: *Schematic diagram of the Bayesian inverse analysis approach.*

3.6 Prior knowledge

The source for the prior estimates from the preliminary design, *i.e.* in this context the prior distribution of the model parameters, is attributed to the combination of site-specific information and initial prior knowledge relevant to the site.

In cases where there is no prior knowledge of the parameters available regarding the site, a weakly informative prior distribution can be used, where all possible values of the model parameters are treated equally likely. This type of distribution is commonly incurred in practice. The knowledge of the physical meaning of soil properties ensures that their range is generally well-bounded. For model parameters that are mutually independent, the prior knowledge can be represented by a uniform distribution, $P(\theta)$, as shown below.

$$P(\theta) = \begin{cases} 1/(\theta_{max} - \theta_{min}) & \text{for } \theta \in [\theta_{min}, \theta_{max}] \\ 0 & \text{for otherwise} \end{cases} \tag{3.20}$$

When initial prior knowledge is improved in terms of quality and quantity, it becomes relatively more informative and, sometimes, sophisticated. The log-normal distribution is a commonly used informative prior distribution in geotechnical engineering due to its simple relationship with the normal distribution and effectiveness in handling negative and inconsistent model parameter values without requiring truncation. The choice of log-normal distribution in an inverse analysis is expected to yield similar results compared to other distributions such as gamma, Weibull and Rayleigh (Griffiths et al., 2013). If C.O.V. of a particular parameter decreases with the accumulation of information, then the parameter distribution is said to have become well-informed or highly confident. Hence, the level of confidence in the knowledge of the parameter depends on the C.O.V. of its distribution. In this thesis, both the aforementioned types of prior distribution have been used in the inverse analysis procedure (**Papers C, D, E & F**).

4 Data Assimilation for geotechnics

4.1 Introduction

What is Data Assimilation ?

Data Assimilation (DA) is a scientific approach that involves integrating numerical forecasting models with observations of a system, in order to create an updated forecast that is better than the model or the observations on their own. The process of DA is essential for creating accurate forecasts in various disciplines. Both the numerical models and observations of the system play important roles, but neither are perfect. By combining the numerical models with observations, a relatively more accurate forecast can be produced than relying solely on the numerical model without information on the current measurement.

Forecasting models simulating the real-world physics are used to predict the future response. In turn, the real-world observations can be used to update the model using DA which is an inverse problem and this cycle of verification and validation continues. Figure 4.1 shows the significance of DA in this deductive spiral where it acts as a bridge between model and observation.

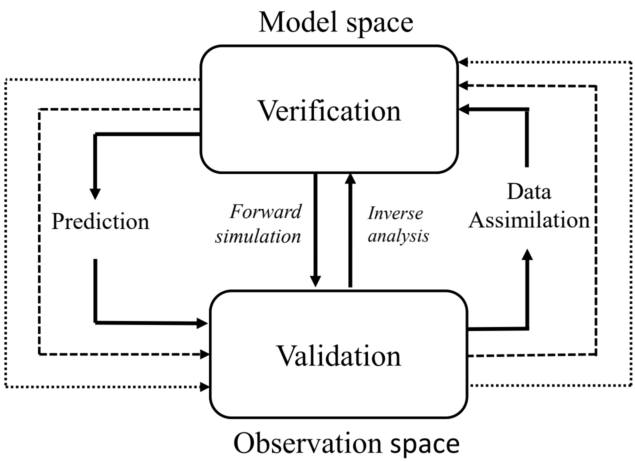


Figure 4.1: *Illustration of the deductive spiral (re-illustrated from Asch et al. (2016))*

The evolution of Data Assimilation (DA) is closely linked to meteorology and DA was predominantly used in this field for numerical weather prediction. Currently, DA is utilised in various domains and geotechnical engineering stands to gain considerably by adopting the techniques employed in DA. Developed to handle incomplete and uncertain real-world observations, these techniques can enable geotechnical engineers to make better use of monitoring data and generate more precise estimates of uncertainty in model predictions.

Types of Data Assimilation algorithms

There are two main approaches of Data Assimilation: variational and statistical. An optimal solution is the aim in both these approaches. The difference is that the statistical approach seeks a solution with minimum variance while the variational approach seeks the minimum of a cost function. The implementation of the variational technique, however, is rather impractical for general geotechical engineering by requiring the use of tangent linear and adjoint models (Hommels and Molenkamp, 2006). Hence, the variational technique suffers from limited applicability (Evensen, 2007).

Furthermore, when dealing with practical inverse problems, the data being analysed is imprecise and includes random noise. In order to effectively address this measurement error, only statistical models can provide the necessary rigour. The statistical approach is relatively easy to implement and observations are assimilated in the model prediction each time they become available (*i.e.* sequentially).

Although the observational method is a useful concept in geotechnics, using observed movements to control construction in a timely manner can be challenging in typical projects where contractors are constrained by time. Furthermore, quantitative assessments of the progress of the engineering work is difficult. Therefore, it is necessary to assimilate observations sequentially, as they become available.

Using sequential data assimilation techniques is preferable, particularly for highly critical projects in geotechnical engineering. The statistical approach explicitly solves a series of equations to find the posterior state of the system. One of the earliest and well known examples include the Kalman Filter (KF) which follows a Bayesian state-estimation algorithm formulated by Kalman for linear systems with Gaussian uncertainties (Kálmán, 1960). Later, the KF was modified into the Extended Kalman filter (EKF) to accommodate lightly nonlinear systems (Jazwinski, 1970). For many decades, the Extended Kalman Filter (EKF) has been the primary Bayesian algorithm used for state-estimation in nonlinear systems. Despite its widespread use, the EKF is only a reliable method for systems that are nearly linear within the updating time interval (Julier et al., 2000).

In Civil Engineering, systems can become highly nonlinear, which raises questions about the effectiveness of the Kalman Filter (KF) and its extended version. Despite being used by researchers for many years, the suitability of these techniques for highly nonlinear systems has received little attention (Beck, 1978; Hoshiya and Saito, 1984; Koh and See, 1994). This is because, for highly nonlinear problems, in most applications, the prior background error covariance matrix, cannot be specified explicitly and requires an approximation. To achieve this, the probabilistic formulation of the inverse problem requires a resolution in terms of 'samples' of the *a posteriori* probability distribution in the model space.

As a result, the ensemble extensions of the classical Kalman filter, namely the Ensemble Kalman Filter [EnKF] (Evensen, 1994), the Unscented Kalman Filter [UKF] (Julier and Uhlmann, 1997) and Particle Filter [PF] (Gordon et al., 1993; Kitagawa, 1996) were proposed. These methods provide strategies to approximate the background error covariance matrix using a statistically consistent ensemble of states for nonlinear problems. As a result, Monte-Carlo based data assimilation systems have started to garner widespread use in several applications. Each sample is forecast individually by the forward model, and the spread of these samples define the uncertainty of the estimate.

DA in geotechnical engineering

DA has been applied to some geotechnical problems since the 1980s (e.g Murakami and Hasegawa, 1985; A. Murakami, 1991). Recently, more research on the use of advanced DA techniques has been reported for geotechnical problems. DA by Particle filter (PF) has attracted attention for updating model parameters in geotechnical engineering (Shuku et al., 2012, 2013; Nguyen et al., 2014; Murakami et al., 2017; Shibata et al., 2019). However, the issue with degeneracy in PF is not reported in any of these works, and comparison with other DA techniques is yet to be conducted.

Tao et al. (2020) demonstrated the use of EnKF through synthetic and real-case data to predict the settlement of road embankments. The effect of sensitivity of model parameters, ensemble size and observation error were studied. The use of elastoplastic model combined with EnKF is studied by Mohsan et al. (2021) to update the factor of safety of a slope based on synthetic monitoring data. Mohsan et al. (2021) noted that the state variables of the numerical model such as effective stress, strain and pore-water pressures need to be updated with a recursive/restarting algorithm to achieve proper convergence. The effect of sensor location is, however not reported.

Tao et al. (2021) used the Modified Cam-Clay elastoplastic model combined with EnKF for back-calculating the spatial variability of the stiffness parameters, and showed the influence of the quantity of sensors. Literature on some comparative studies between DA methods for geotechnics do exist, *e.g.* Hommels and Molenkamp (2006) studied the effect of using Unscented Kalman Filter (UKF) and Ensemble Kalman Filter (EnKF) for an embankment problem, but using a simplified elastic-perfectly plastic Mohr-Coulomb model which is not suitable for general serviceability calculations in geotechnical engineering. Tao et al. (2022) compared EnKF with Markov Chain Monte Carlo (MCMC) algorithm for a consolidation settlement problem, but only for a simple analytical model.

A comparison between the EnKF, UKF and PF algorithms have not been performed in previous studies. This is crucial, since the purpose of the comparison is to assess the combination of forward model and DA in terms of the extent of complexity required from them. Also the comparison between the DA algorithms need to be performed for models with rate-dependency and bond degradation features since natural geomaterials, especially natural soft clays (as seen in Section 2.2) are highly complex in nature. Combined with multilayered soil profiles, the estimation can become a high dimensional Bayesian update problem.

Additionally, it is imperative to check whether DA can be used for models whose formulation does not match the physics of the system, which can be expected in general geotechnical practice. It should be noted that because of the constantly evolving nature of the DA discipline, an exhaustive validation of all the different types of DA methods is a monumental task, and not practical within the current scope. However, the commonly used recent DA methods for geotechnics, mentioned in this section, can serve as a good starting point to answer the aforementioned points as part of the objectives of this thesis.

4.2 Basic principles

Considering the Data Assimilation (DA) window is represented by $t \in [0, T]$, the state of the system evolves via the forward model $\mathbb{F} : f(t, x) \to f(t + \Delta t, x)$, $\forall (t, x) \in [0, T] \times \Omega$ with initial condition $f(t_0 = 0, x)$. Starting from the prior distribution, represented as $p(x_0)$ in the model space, the system state $x_k \in \mathbb{R}^m$ evolves via the relation shown in Equation 4.1. The true state of the system is observed via a set of instruments treated as the observation space modelled by $y_k \in \mathbb{R}^n$. The variables from the model space are mapped to the observation space through the operator $\mathbb{H} : f(t,x) \to g(t,x),\ \forall\ (t,x) \in [0,T] \times \Omega$.

$$\begin{aligned} x_k &= \begin{pmatrix} u_k \\ p_k \end{pmatrix} \in \mathbb{R}^m \\ x_{k+1} &= F(x_k) + q_k \\ y_k &= H(x_k) + v_k \end{aligned} \tag{4.1}$$

where, u_k is the displacement vector and p_k is the pore-water pressure vector at the nodes for time-step k of the discretised system. The process noise is given by $q_k \sim N(0, \sigma_q)$ due to modelling errors and the noise corrupting the measurement is given as $v_k \sim N(0, \sigma_v)$. The error covariance matrix for the process noise is given as $E[q_k\ q_k^T] \to Q_k$ and for the observation error covariance matrix is defined as $E[v_k\ v_k^T] \to R_k$.

The state of the system is updated using all the available noisy observations until time k to construct the posterior density. The previous posterior from time k-1 is projected forward using the transition density $p(x_k|x_{k-1})$, to generate the prior distribution at time k based on the Chapman-Kolmogorov equation (Chatzi and Smyth, 2009).

$$p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1})\ p(x_{k-1}|y_{1:k-1})\ dx_{k-1} \tag{4.2}$$

Subsequently the updated posterior at time k can be obtained, by incorporating the latest observation likelihood:

$$p(x_k|y_{1:k}) = \frac{p(y_k|x_k)\ p(x_k|y_{1:k-1})}{p(y_k|y_{1:k-1})} \tag{4.3}$$

where $p(x_k|y_{1:k})$ defines the *a posteriori* estimate. A solution for the above Bayesian integral equations are usually intractable, especially for geotechnical problems. Hence, algorithms that approximate the exact solution are necessary, which is described in the coming sections.

The process of sequential assimilation of observations is summarised in Figure 4.2. The typical assimilation scheme is made up of two major steps: (1) a prediction/forecast step and (2) a update/analysis step. At time k, the analysis, x_k^a, which is a result of the forecast x_k^f and set of observations, y_k, is projected to the next time step, $k+1$ using the forward model. The result of the forecast is denoted x_{k+1}^f and becomes the background, or initial guess, so that by using the observation y_{k+1} is then updated to x_{k+1}^a, and so forth, for the subsequent time steps.

4.3 Joint state and parameter estimation

In geotechnical engineering, the problem of estimating parameters for a forward model involves finding the joint probability density function of the parameters and the model state, given a set of measurements. In practice, however, a series of *ad-hoc* approaches is employed to "find" an estimate of the parameters that is either close to prior experience, and logical deduction or, just plainly, fit the model response in a random manner to be as close as possible to a set of measurements. The latter approach is ill-advised, and the parameter set so obtained may not contain sufficient

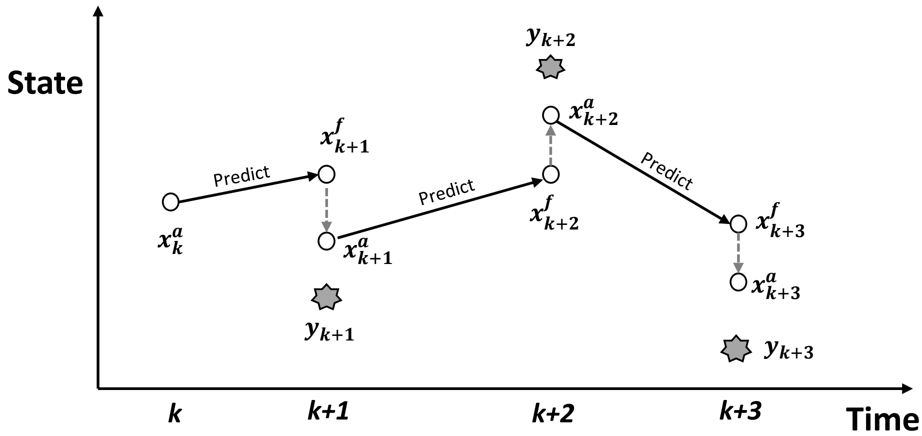


Figure 4.2: *Illustration of sequential data assimilation.*

information to predict the future response. The aim here is to achieve a set of parameters in the model which results in a model solution that is consistent with a set of measurements while also maintaining the physical meaning of that set such that it is more representative for the site.

The field of parameter estimation is well developed in Data Assimilation (DA), and is vastly different from the aforementioned *ad-hoc* approach. Traditionally DA has been used exclusively for state estimation, and the sentiment still exists that as long the state is accurately predicted, the parameters can be fixed. This is acceptable when the sensitivity of the results at the current timestep largely depend on the previous state of the system, rather than the model parameters.

In contrast, in geotechnical engineering the current state of the system depends on both the model parameters and the state history of the system. DA with state estimation alone is not expected to provide reasonable results, and also it does not provide any scientific knowledge. The approach for parameter estimation can differ depending on the purpose. However, in this thesis, success is measured mainly by how close the parameter estimates are to their true value, since it is believed that this approach would provide a more consistent way of describing the future behaviour of the system, and to better understand the underlying physics.

In order to estimate the model parameter set $\theta \in \mathbb{R}^p$ concurrently with the evolving model state, the state space $x \in \mathbb{R}^m$ needs to be augmented to create a joint state-parameter space, allowing to update the model parameters jointly with the state variables as part of the assimilation process (Bocquet and Sakov, 2013; Iglesias et al., 2013).

$$\tilde{x}_k = \begin{pmatrix} x_k \\ \theta_k \end{pmatrix} \in \mathbb{R}^{m+p} \tag{4.4}$$

This augmented system state can be combined with the governing equations of the model state evolution in the usual manner, with the exception that the observation operator needs to be augmented, *i.e.*

$$\tilde{H}_k = \begin{pmatrix} H_k & 0 \end{pmatrix} \in \mathbb{R}^{n\times(m+p)} \tag{4.5}$$

The augmented state vector allows for the calculation of the cross-covariance between the states and parameters. The inference about the unobserved parameter, and its uncertainty relies crucially on the cross covariance matrix (Equation 4.6). The off-diagonal elements of the (augmented) state error covariance matrix, *i.e.* $P_{x\theta}$ & $P_{\theta x}$, pass information from the data assimilated state to improve the estimate of the unobserved parameters. Since the model parameters are constant, the persistence model is applied to the parameters, meaning the parameter set remains constant during the model state evolution until the subsequent assimilation cycle.

$$\mathrm{E}\left[\,(\tilde{x} - \mathrm{E}[\tilde{x}])\,(\tilde{x} - \mathrm{E}[\tilde{x}])^T\,\right] = \begin{pmatrix} P_{xx} & P_{x\theta} \\ P_{\theta x} & P_{\theta\theta} \end{pmatrix} \tag{4.6}$$

In this study the model error is not considered explicitly, since due to the adopted persistence model, the evolution of parameters with time is perfect ($d\theta/dt = 0$). Therefore, any additional model error taken into account may weigh down the significance of the previous assimilation, which in turn affects the convergence (Trudinger et al., 2008).

4.4 Data Assimilation algorithms

Unscented Kalman Filter

The unscented Kalman filter employs the non-linear unscented transformation principle, which uses a set of predetermined sigma points with corresponding weights to parameterise the mean and covariance of the prior Gaussian distribution. This approach avoids propagating the full information and achieves results equivalent to the original Kalman Filter for linear systems, while elegantly generalised for nonlinear systems. The effectiveness of this method for nonlinear systems is due to the fact that approximating a probability distribution with a set of points and weights from the probability density function is simpler than approximating a nonlinear function.

The number of sigma points generated is $2L + I$, where $L \to m + p$ is the dimension of the augmented state variable reflecting the statistics of the system state. The first sigma point is chosen as the mean of the distribution, and the remaining points are computed by scaling the covariance matrix by $\lambda \to \alpha^2(L + \kappa) - L$, where α distributes the sigma points around the mean (value between $10^{-4} \le \alpha \le 1$) and κ is the secondary scalar parameter to guarantee positive semi-definiteness of the covariance matrix. κ is usually chosen ≥ 0. A good default choice is 0 (Merwe and Wan, 2003). These points are illustrated in Figure 4.3 and expressed as

$$\begin{aligned} \chi_0 &= \hat{\tilde{x}}_{k-1|k-1} \\ \chi_i &= \hat{\tilde{x}}_{k-1|k-1} + [\sqrt{(L+\lambda)\tilde{P}_{k-1|k-1}}]_i \qquad i = 1, ...L \\ \chi_i &= \hat{\tilde{x}}_{k-1|k-1} - [\sqrt{(L+\lambda)\tilde{P}_{k-1|k-1}}]_{i-L} \quad i = L+1, ...2L \end{aligned} \tag{4.7}$$

and the associated weights are given below as

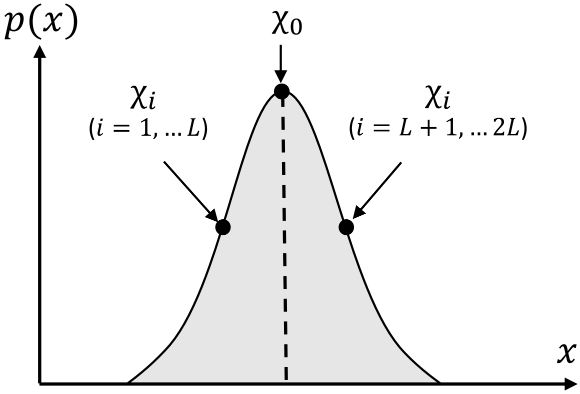


Figure 4.3: *Illustration selecting sigma points for the unscented Kalman filter.*

$$
\begin{aligned}
W_0^m &= \frac{\lambda}{L+\lambda} \\
W_0^c &= \frac{\lambda}{L+\lambda} + 1 - \alpha^2 + \beta \\
W_i^m &= W_i^c = \frac{1}{2(L+\lambda)} \quad i = 1 \ldots 2L
\end{aligned} \tag{4.8}
$$

Here β is a positive real number. An optimal choice for β would be $\beta = 2$, for true Gaussian priors. Each of the sigma points in Equation 4.7 are then propagated in the time domain (Equation 4.9) through the nonlinear function and the statistics are computed using the weights (Equation 4.8).

$$
\chi^i_{k|k-1} = F(\chi^i_{k-1}), \quad i = 0, \ldots, 2L \tag{4.9}
$$

Here, $L \to m+p$ is the dimension of the augmented state system and the statistics of the transformed sigma points are computed using the associated weights where the mean and covariance of the projected prior at time k is calculated using the unscented transform,

$$
\begin{aligned}
\hat{\tilde{x}}_{k|k-1} &= \sum_{i=0}^{2L} W_i^m \, \chi^i_{k|k-1} \\
\tilde{P}_{k|k-1} &= \sum_{i=0}^{2L} W_i^c \, (\chi^i_{k|k-1} - \hat{\tilde{x}}_{k|k-1})(\chi^i_{k|k-1} - \hat{\tilde{x}}_{k|k-1})^T + Q_{k-1}
\end{aligned} \tag{4.10}
$$

The sigma points are mapped to the measurement space and the mean is calculated.

$$
\begin{aligned}
Z &= \tilde{H}(\chi^i_{k|k-1}) \\
\hat{y}_{k|k-1} &= \sum_{i=0}^{2L} W_i^m \, Z_i
\end{aligned} \tag{4.11}
$$

The covariance of the sigma points for the measurement and the cross covariance between the state and the measurements are defined as:

$$
\begin{aligned}
P_k^{yy} &= \sum_{i=0}^{2L} W_i^c\ (Z - \hat{y}_{k|k-1})(Z - \hat{y}_{k|k-1})^T + R_k \\
P_k^{xy} &= \sum_{i=0}^{2L} W_i^c\ (\chi_{k|k-1}^i - \hat{\hat{x}}_{k|k-1})\ (Z - \hat{y}_{k|k-1})^T
\end{aligned}
\tag{4.12}
$$

The Kalman gain is then computed as the ratio between the belief in the state of the system to that in the measurement. Subsequently the Kalman gain is used to update the estimate of the state (mean and variance) using the measurement y_k

$$
\begin{aligned}
K_k &= P_k^{xy}(P_k^{yy})^{-1} \\
\hat{\hat{x}}_{k|k} &= \hat{\hat{x}}_{k|k-1} + K_k\ (y_k - \hat{y}_{k|k-1}) \\
\tilde{P}_{k|k} &= \tilde{P}_{k|k-1} - K_k\ P_k^{yy}\ K_k^T
\end{aligned}
\tag{4.13}
$$

Figure 4.4 shows the illustration of the working of the Unscented Kalman Filter (UKF). Compared to Monte-Carlo techniques that need numerous sampling points, the UKF is a relatively more straightforward method. Its implementation is simpler, with the generation of sigma points during each update being the only computationally intensive aspect. However, joint estimation of the state and parameter can increase the dimensional complexity, particularly with large parameter sets, leading to instability when combined with a highly nonlinear process model. Furthermore, large noise in the dataset can decrease the overall performance.

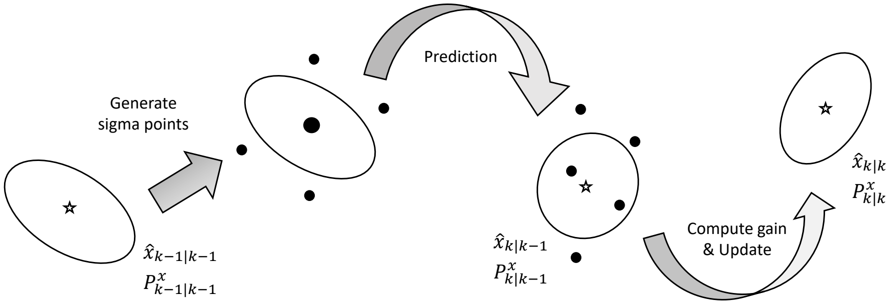


Figure 4.4: *Illustration of the working of the Unscented Kalman filter.*

Ensemble Kalman Filter

The Ensemble Kalman Filter (EnKF), which was proposed by Evensen (1994), is based on the Kalman filter formulation. In this method, the statistics of the state variable are represented by a set of ensemble members, which are propagated in time using the nonlinear dynamics of the system. The standard Kalman filter analysis scheme is then applied to this ensemble set to calculate the actual posterior mean and variance at each time step. Despite its widespread use in the geosciences community, EnKF is still a relatively new technique in geotechnics.

The Ensemble Kalman Filter is popular since it can effectively handle large dimensional problems using a small ensemble size (Schillings and Stuart, 2016), and it is straightforward to implement. It utilises a derivative-free optimisation technique, where the ensembles act as a substitute for derivative information and provide an approximation of the error covariance matrix. As the ensemble size increases, the sampling error decreases proportionally at a rate of $\sqrt{1/N}$. Over the years, EnKF has developed a large user group, and also has gained popularity in geotechnical engineering. Recent studies have explored its potential in this field (*e.g.* Hommels and Molenkamp, 2006; Vardon et al., 2016; Mavritsakis, 2017; Liu et al., 2018a).

The ensemble representation of the augmented state vector is represented as:

$$\tilde{x}_k^N = \begin{pmatrix} x_k^1 & x_k^2 & \dots & x_k^N \\ \theta_k^1 & \theta_k^2 & \dots & \theta_k^1 \end{pmatrix} \in \mathbb{R}^{N\times(m+p)} \tag{4.14}$$

The mean, anomaly matrix and the subsequent covariance matrix of the augmented ensemble forecast state vector is estimated as:

$$\begin{aligned} \tilde{x}_k^f &= \tilde{x}_k^{f,N} \cdot \mathbf{1}_N \\ \boldsymbol{X}_k^f &= \frac{1}{\sqrt{N-1}} (\tilde{x}_k^{f,N} - \tilde{x}_k^f) \\ \tilde{\boldsymbol{P}}_k^{e,f} &= (\boldsymbol{X}_k^f)(\boldsymbol{X}_k^f)^T \approx \tilde{\boldsymbol{P}}_k^f \end{aligned} \tag{4.15}$$

where superscript e represents the quantities estimated from ensembles with $\mathbf{1}_N$ representing the equal weight vector for calculating the mean. In order to maintain consistency in the error covariance matrix between EnKF and KF, an ensemble of perturbed observation with covariance $\boldsymbol{R}$ is defined (Burgers et al., 1998). Hence, the method can also be called the stochastic-EnKF (Hoteit et al., 2015).

$$\begin{aligned} y_k^j &= y_k^t + v_k^j \quad j = 1, 2, \dots, N \\ \boldsymbol{Y}_o' &= \frac{1}{\sqrt{N-1}} \left[v_k^1, v_k^2, \dots, v_k^N \right] \\ \boldsymbol{R}^e &= (\boldsymbol{Y}_o')(\boldsymbol{Y}_o')^T \approx \boldsymbol{R} \end{aligned} \tag{4.16}$$

The ensemble based Kalman gain matrix is obtained and each ensemble member is updated in the analysis step (see Equation 4.17):

$$
\begin{aligned}
\boldsymbol{K}^e &= \tilde{P}_k^{e,f}\, \tilde{H}^T \left[\tilde{H} \tilde{P}_k^{e,f}\, \tilde{H}^T + \boldsymbol{R}^e \right]^{-1} \\
\tilde{x}_{n,k}^a &= \tilde{x}_{n,k}^f + \boldsymbol{K}^e \left[y_{n,k} - \tilde{H} \tilde{x}_{n,k}^f \right], \;\; 1 \leq n \leq N \\
\tilde{P}_k^{e,a} &= [\boldsymbol{I} - \boldsymbol{K}^e \tilde{H}]\, \tilde{P}_k^{e,f}
\end{aligned} \tag{4.17}
$$

The EnKF analysis method is an approximation because it does not properly consider non-Gaussian contributions in the predicted ensemble. Unlike the particle filter, EnKF does not solve the Bayesian update of a non-Gaussian probability density function. Instead, the updated ensemble inherits most of the non-Gaussian properties from the forecast ensemble since only the updates defined by the right-hand side of Equation 4.17 for calculating $\tilde{x}_{n,k}^a$ are linear. According to Verlaan and Heemink (2001), EnKF can be used for strongly nonlinear problems. In geotechnical engineering, several studies have used prior variables with assumptions other than Gaussian, such as the uniform distribution (Tao et al., 2020) and the prior distribution of parameter fields sampled from the log-normal distribution (Liu et al., 2018a; Tao et al., 2021).

In order to avoid divergence of the filter, it is necessary to treat observations as random variables, as noted by Burgers et al. (1998), who explained that failure to do so could result in underestimation of the analysis covariance. To compare and assess the impact of perturbed observations, a deterministic version of EnKF, known as the Ensemble Square Root Filter (EnSRF), is implemented. Instead of introducing noise to the observations, as in Equation 4.16, the Kalman gain is adjusted so that Equation 4.17 is satisfied, and independent observations are assimilated sequentially, as demonstrated in Whitaker and Hamill (2002). This factor, shown in Equation 4.18, can be a scalar between 0 and 1 meaning that to obtain the desired analysis error covariance, a reduced form of the traditional Kalman gain is used. The calculation of $[\boldsymbol{I} - \tilde{K}^e \tilde{H}]$ involve the square root of the background error covariance and is hence called the Ensemble Square Root Filter (EnSRF) (Whitaker and Hamill, 2002).

$$
\tilde{K}^e = \left(1 + \sqrt{\frac{R}{\tilde{H} \tilde{P}_k^{e,f}\, \tilde{H}^T + R}} \right)^{-1} K^e \tag{4.18}
$$

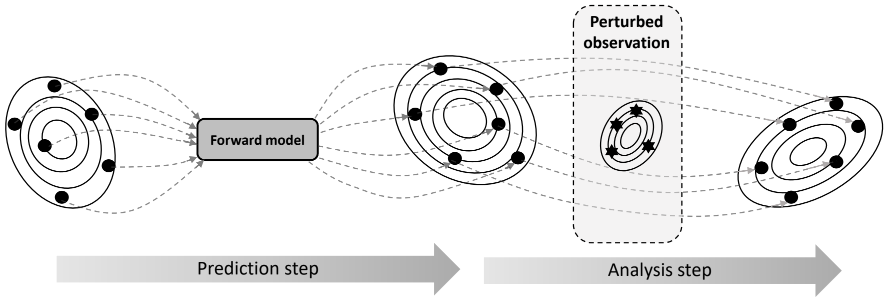


Figure 4.5: *Illustration of the principle of the ensemble Kalman filter*

Particle Filter

The Particle filter (PF) (Kitagawa, 1996; Arulampalam et al., 2002; Doucet et al., 2013) uses a set of independent particles, *e.g.* $\{x^{(1)}_{k-1|k-1}, x^{(2)}_{k-1|k-1}, \dots x^{(N)}_{k-1|k-1}\}$ with associated weights $\{\psi^{(1)}_{k-1}, \psi^{(2)}_{k-1}, \dots, \psi^{(N)}_{k-1}\}$ at time 'k-1'. The approximation using the particles represented with Dirac delta masses, 'δ', is shown below (Shuku et al., 2012).

$$p(x_{k-1}|y_{1:k-1}) \approx \sum_{i=1}^{N} \psi^{(i)}_{k-1}\, \delta(x_{k-1} - x^{(i)}_{k-1|k-1}) \tag{4.19}$$

The prediction step of Equation 4.2 is approximated as,

$$\begin{aligned} p(x_k|y_{1:k-1}) &\approx \sum_{i=1}^{N} \int \psi^{(i)}_{k-1}\, \delta(x_{k-1} - x^{(i)}_{k-1|k-1})\, p(x_k|x_{k-1})\, dx_{k-1} \\ &= \sum_{i=1}^{N} \psi^{(i)}_{k-1}\, \delta(x_k - (F(x^{(i)}_{k-1|k-1}) + q^{(i)}_k) \quad \dots (4.1) \\ &= \sum_{i=1}^{N} \psi^{(i)}_{k-1}\, \delta(x_k - x^{(i)}_{k|k-1}) \end{aligned} \tag{4.20}$$

and the Bayesian integral from Equation 4.3 can be approximated as,

$$p(x_k|y_{1:k}) \approx \sum_{i=1}^{N} \psi^{(i)}_{k} \delta(x_k - x^{(i)}_{k|k-1}) \tag{4.21}$$

As samples cannot be simulated directly from this distribution, the Importance Sampling technique is used, where the samples are generated from a proposal density, $q(x_k|y_{1:k})$ which approximates the true filtering distribution. The importance weights account for the deviation from this true distribution as shown below (Chatzi and Smyth, 2009):

$$\psi^{(i)}_k \;\propto\; \frac{p(x^{(i)}_k|y_{1:k})}{q(x^{(i)}_k|y_{1:k})} \tag{4.22}$$

As the state is evolved, during the assimilation period, the importance weights are estimated recursively using the relation below (Carrassi et al., 2017; Tamboli, 2021):

$$\psi^{(i)}_k \;\propto\; \psi^{(i)}_{k-1}\, \frac{p(y_k|x^{(i)}_k)\, p(x^{(i)}_k|x^{(i)}_{k-1})}{q(x^{(i)}_k|x^{(i)}_{k-1}, y_k)} \tag{4.23}$$

with weights normalised to 1. The choice of a relevant proposal density may not be straightforward and depends on the problem. The most commonly used approach is to use the transitional prior as the importance density function, $q(x^{(i)}_k|x^{(i)}_{k-1}, y_k) = p(x^{(i)}_k|x^{(i)}_{k-1})$. This process is called the Sequential Importance Sampling (SIS) procedure, representing the bootstrap version of the Particle Filter (Gordon et al., 1993), and is the most widely used due to its ease of implementation. The above equation then reduces to:

$$\psi_k^{(i)} = \psi_{k-1}^{(i)} \; p(y_k | x_{k|k-1}^{(i)}) \tag{4.24}$$

$$p(y_k | x_{k|k-1}^{(i)}) \;\propto\; exp\left[\frac{-(y_k - Hx_{k|k-1}^{(i)})^T \; R_k^{-1} \; (y_k - Hx_{k|k-1}^{(i)})}{2}\right] \tag{4.25}$$

The importance weights are, in this case, essentially dependent on the likelihood function. For geomaterials with elastic-viscoplastic law the behaviour at the current timestep is highly dependent on the information from the previous state of the stress, strain and pore-water pressure of the system. These are preserved when using the SIS algorithm by keeping the initially generated model trajectories constant during the entire filtering process, updating only the weights based on sequentially observed data. Although limiting to prediction and correction steps for updating the weights is reasonable for most geotechnical applications, the SIS suffers from sample degeneracy. Due to the curse of dimensionality, the maximum weight in the bootstrap PF converges to one unless the sample size grows exponentially with the problem dimension (Bengtsson et al., 2008; Bickel and Bengtsson, 2008; Fearnhead and Künsch, 2018). There are two general ways for dealing with the degeneracy of the particle filter, one is finding a good proposal density and the other is resampling.

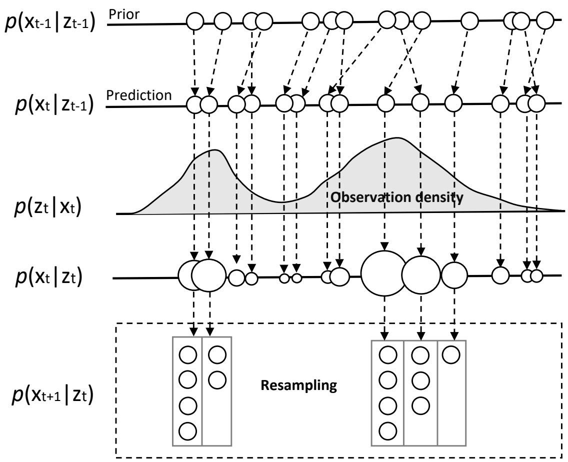


Figure 4.6: *Illustration of the working of Particle Filter with resampling procedure.*

The importance weights of the individual particle trajectories are subsequently updated, and used to assess whether resampling is necessary. If the effective sample size has dropped below a user-defined threshold, particles with lower weights are relinquished, and resampling is used to create more promising trajectories that adequately capture the evolving posterior state distribution. For geotechnical problems, the number of particles is severely constrained by the computational cost, which further penalises this approach. This can be resolved by reinforcing the filter with a sequential importance resampling (SIR) technique that uniformly resets the weights to N^{-1} using balanced sampling schemes (*i.e.* stratified or systematic (Carpenter et al., 2000)) but is, quite often, still not sufficient to counteract this issue as will be shown in this thesis.

4.5 Nature of data

There are several factors that can diminish the true value of a geotechnical dataset, and they all can be categorised into two components: quantity and quality. Disparity between theory and actual field conditions exist, and the magnitude of this depends on several factors. However, one of the major issues that amplifies this discrepancy is the shortage of data. The reason is due to the high costs associated with acquiring data. Even with a comprehensive data collection, the magnitude of geotechnical uncertainty remains considerable due to the fact that the volume of geomaterials investigated is much smaller than the spatial volume considered in the analysis. As a result, making decisions always involves risk in geotechnical engineering, and is only further amplified due to simplifications in modelling and testing errors. Measurement error arises from equipment, procedural–operator, and random testing effects. Equipment effects result from inaccuracies in the measuring devices and variations in equipment geometries and systems employed for routine testing (Phoon and Kulhawy, 1999). In general, tests that are highly operator dependent, and that have complicated test procedures, will have greater variability than otherwise. Based on the information provided in this section, the criteria for identifying the optimal Data Assimilation algorithm for geotechnical applications is mentioned in the next section.

4.6 Criteria

The appropriate data assimilation (DA) method for a geotechnical problem should be efficient against the following factors:

- *Problem dimension*: Practical problems in geotechnical engineering are usually associated with significant uncertainties. Hence the number of variables of interest that need to be estimated is usually high. Therefore, the selected DA method should be stable for high dimensional problems.

- *Limited information*: The data needed for utilisation of probabilistic assessment are not available to the sufficient extent. This is investigated in the context of geotechnical problems whether a meaningful level of accuracy and precision in state and parameter estimation are achieved with the selected DA method based on limited information.

- *Data quality*: Several different sources of uncertainties can affect the observation data retrieved during various phases of a geotechnical project. The low quality of data is a real issue in geotechnical engineering when considered for inverse analysis. Hence, the selected DA method needs to be consistent in the accuracy and precision of the results when faced with low quality datasets *i.e.* highly noisy dataset.

- *Computational efficiency*: Although some data assimilation methods may offer improved convergence, the high computational cost may make them less attractive for most geotechnical applications. Therefore, it is important to choose a DA method that strikes a balance between convergence and computational efficiency.

5 Summary of Appended Papers

Structure of the Chapter

In this Chapter, the results from the appended papers of this thesis are summarised. This Chapter is structured according to the research questions mentioned in Section 1.2 (see Figure 5.1). Since both **Papers A and B** deal with deterministic analyses of trial embankments with a similar methodology, the summary of these papers are combined in this Chapter. In **Paper C**, the performance of various Data Assimilation (DA) algorithms with different constitutive models have been studied for a synthetic case to identify the most robust algorithm. This method is further evaluated against monitoring data from a real test case in an extension to **Paper C**. In **Paper D**, the numerical model from **Paper C** is used but with the difference here to investigate whether DA can help capture the response of a system even when the model formulation does not match the physics of that geotechnical system. The effect of field monitoring setup on the performance of DA is studied in **Paper E**. In **Paper F**, an improved Particle Filter is proposed where its generic formulation is combined with an optimisation algorithm to mitigate its limitation for geotechnical application.

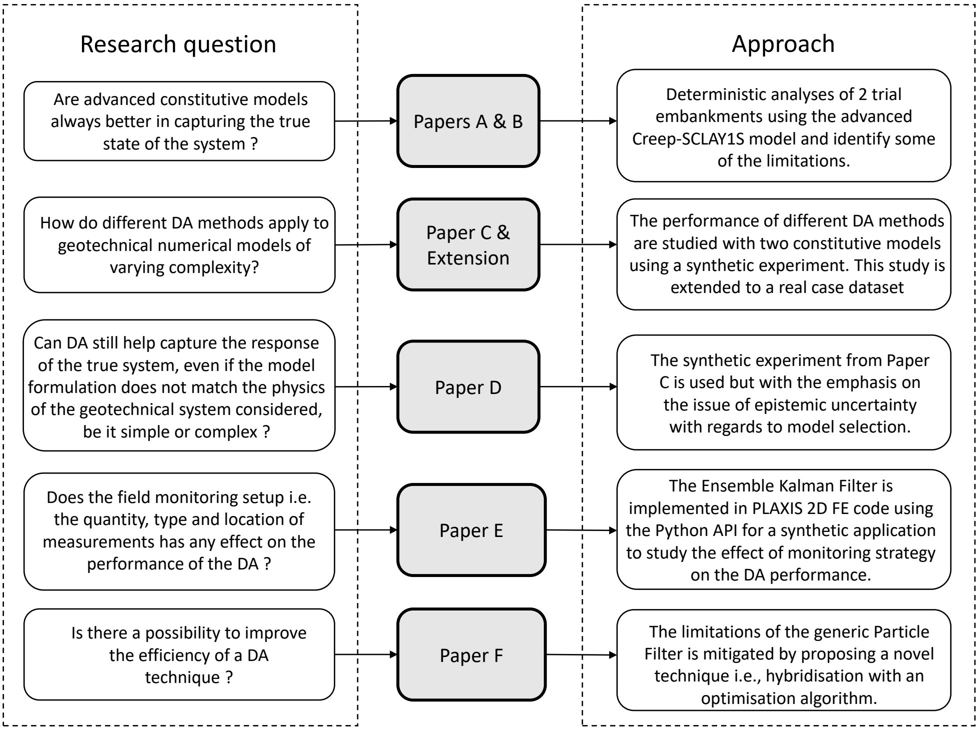


Figure 5.1: *Schematic diagram on the structure of this Chapter.*

5.1 Papers A & B

Paper A: "Towards consistent numerical analyses of embankments on soft soils" & Paper B: "Consistent Class A & C predictions of the Ballina test embankment"

Introduction

In **Paper A**, the analysis of a trial embankment constructed in Haarajoki, Finland by the Finnish National Road Administration between July and August 1997 was conducted. The embankment has a length of 100 meters, width of 8 meters, and a height of 2.9 meters. Half of the embankment is constructed without any ground improvement, while the other half is improved with prefabricated vertical drains.

Paper B deals with the prediction of the deformation behaviour of a trial embankment constructed in Ballina, Australia, which was part of a prediction Symposium in September 2016, inviting practitioners and academics world-wide. The trial embankment (80 m long and 16 m wide with a 3 m high crest) was constructed with drains to study the settlement behaviour of the soft ground. Before the embankment was built, high-quality soil samples were taken from the site and subjected to advanced laboratory testing at the University of Newcastle (Pineda et al., 2016), which was supplemented by sophisticated in-situ testing (Kelly et al., 2017).

Both these embankment were extensively instrumented to measure pore pressures, vertical and horizontal deformations, and total stresses at key locations. Based on the provided test data, it can be observed that the soil deposits in Haarajoki and Ballina display high sensitivity, rate dependency, and anisotropy. Therefore, the Creep-SCLAY1S model (Gras et al., 2017) is chosen to simulate this behaviour, as it takes all these features into account. The properties of the soil would change as a function of depth, based on the index properties, necessitating the use of separate layers with uniform properties, each with its own set of model parameters.

Automated parameter derivation

When employing complex models like Creep-SCLAY1S, determining accurate parameters becomes challenging due to the large number of parameters in the model. Manual derivation can introduce inconsistencies, which subsequently compromises the accuracy of the input parameters, and ultimately impact the performance of the model. Hence, the process of determining model parameters using techniques that work well with actual measurement data has been automated to a large extent to produce a reliable set of parameters. The advantage of this automated approach is that it allows the user to process multiple datasets at once, and to obtain a consistent set of parameters in a short time with minimal error. Moreover, a large number of laboratory tests can be easily integrated into the analysis.

Despite the requirement of a considerable number of parameters for the Creep-SCLAY1S model, a consistent approach is maintained during the entire procedure, and the user can assess the quality of the process at each stage. With minimal user intervention, the method operates seamlessly. The data for the Haarajoki and Ballina test site are imported into this framework and processed using the module as shown in Figure 5.2. The validation of the method was carried out on both the test sites, where laboratory and field measurements were used.

Figure 5.2 shows the workflow of this algorithm. A high level overview of the workings of this algorithm can be described as follows. Initially the data is imported into the repository and processed for noise. For this purpose, the Savitzky-Golay filter (Savitzky and Golay, 1964) is applied to smooth and correct the noisy data. Then the parameters are derived in an automated manner using a series of functions and criteria preset in the algorithm. Combined with the data from index test, the layers for the soil profile is defined to capture the variation of properties with depth. After finalising the layers for the soil profile, the optimal value from each layer is selected and a unique parameter set is obtained for each layer.

Using the parameter values from each layer, laboratory test simulations are conducted for respective samples from each of these layers. In general practice, this process is tedious but due to an automated environment, it is quite convenient for the user to execute this inspection in a time efficient manner. After validation with the laboratory data, boundary value simulations with the same parameter set are used to demonstrate the effectiveness of the procedure.

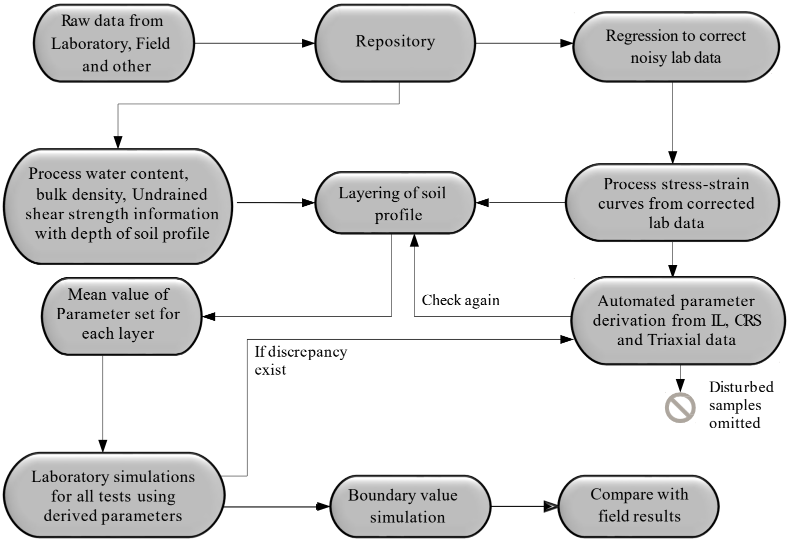


Figure 5.2: *Schematic diagram of workflow.*

Layering

Due to the large variation in the properties of the soft soils between different depths, the hydro-mechanical behaviour changes with depth. Therefore, the soil profile is divided into layers of similar characteristics to avoid oversimplification and perform a reliable validation. Among others, information on the index properties, undrained shear strength from CPT tests, unit weight of the soil with depth are some of the key criteria in the layering process.

Sometimes the quality of such data may be affected due to manual and/or instrument errors and other information such as shear wave velocity, organic content, mineralogy, sensitivity, data from dilatometer tests and other field observations, which although can also be subject to variation, can provide valuable complementary input to assess the existing deviations.

Stress-strain curves from laboratory tests are another important pre-requisite to assess the layering process. The choice of data from CRS or IL tests depends, however, on the application at boundary value level. In addition to the amount, quality and reliability of the provided data, the design of the experimental programme (strain rates, stress levels) further affects the usefulness of the data.

If the validation cases involved are test embankments, then IL tests are preferred, since data from CRS tests do not provide reliable data on creep, and are thus not representative for staged construction of an embankment at the field scale. Data from triaxial tests provides data on the strength of the soil at critical state, which is another key feature that is included in the layering process. All these data are imported into the algorithm and processed using conditional arguments and logical operators. It should be noted that information from two boreholes should not be mixed during the layering process but should be interpreted separately to assess the horizontal extent of the layering of the soil profile. Although this process is automated, user intervention may still be required to interpret the results before finalising the layering process.

Result

The results of the simulation are compared against field measurements. Using this methodology, a closer match between the predicted and measured behaviour is achieved without the use of any cumbersome optimisation methods or any tampering with the parameter set. The consistency of the parameter set is achieved from the laboratory scale to the boundary value simulations. The latter is successfully demonstrated by the predictions of the performance of Haarajoki and Ballina trial embankments. However, even with a consistent approach minor discrepancies do exist and they are mentioned as follows.

Haarajoki: The results obtained from the model were compared to the measured settlements in space and time for the normal cross-section without ground improvement. The predictions were found to be in good agreement with the measured values, without requiring any further parameter modifications. The settlement predictions are in good agreement with the measured values, but the horizontal displacement is overestimated, indicating that one of the possible reasons can be that the predicted K_0 value may not be a good representation of the site condition. Although there were some discrepancies with the measured pore pressure profiles, those fall within the accuracy of the field instrumentation. To analyse the PVD improved subsoil in areas with ground improvement, a method proposed by Chai et al. (2001) is used. This method involves using an equivalent vertical hydraulic conductivity (K_{ve}) that approximates the increase in the normal vertical hydraulic conductivity (K_v) after PVD installation. By using this methodology, the PVD improved subsoil can be analysed under the same 2D plane strain conditions as the area without PVD, with only the input hydraulic conductivity values changing.

The properties of the smear zone were found to have a significant impact on the predicted results, but their assessment is challenging due to uncertainties in the ground condition during and after PVD installation. Additionally, the presence of the desiccated crust layer has a large sensitivity on the results.

Ballina: The main reason for the difference between the predicted settlements and the field measurements in Class A predictions is due to (1) uncertainty in the preconsolidation pressure obtained from CRS data, which even after correcting for strain-rate effects, remained overly high, and (2) uncertainty in the characteristics of the smear zone adjacent to the drains (similar to that observed in Amavasai et al. (2017)). Since settlement predictions, and the model used, are highly sensitive to the magnitude of the preconsolidation pressure, obtaining accurate values is critical. For Class C predictions, the measurement data provided after the Class A prediction is subsequently used to re-evaluate the preconsolidation pressure from the high quality incremental loading oedometer tests with a reference time consistent with the model. The assumptions made regarding the hydraulic conductivity of the system were also re-evaluated, and as a result, the accuracy of the predictions was improved.

5.2 Paper C

Title: "Data assimilation for geotechnics - exploring the possibilities"

Introduction

Researchers and practitioners have started to realise the potential of using Bayesian methods as part of the inverse analysis for geotechnical problems. Data Assimilation (DA) is a powerful tool and can be used to integrate forecast and monitoring data for state estimation and parameter inference. Recent contributions have shown the efficacy of using DA in geotechnics, however, there are various methods within the DA framework, each with their own benefits and drawbacks, and a comprehensive evaluation of these algorithms is yet to be undertaken. There is currently a lack of systematic research on the stability and efficiency of different DA methods for geotechnical applications. In this paper, the potential of using different DA methods in geotechnics is studied. To demonstrate and compare the effectiveness of different DA algorithms in geotechnics, the geotechnical forward model benchmark case is simplified to a synthetic and classical one-dimensional settlement problem of an embankment on soft soil. This simplified approach serves as a proof-of-concept example and allows for a comparison of various DA techniques while maintaining computational efficiency. If needed, the forward model can be replaced with a more advanced numerical model within the implemented general framework. The study evaluates the performance of different DA algorithms, including the Unscented Kalman Filter, Ensemble Kalman Filter, Ensemble Square Root Filter, and Particle filter, as described in Chapter 4.

Methodology

In this paper, a set of synthetic examples are created to evaluate the performance of the aforementioned DA algorithms. This allows to evaluate the DA performance in a controlled environment where modifications can be systematically incorporated. The methodology has the following steps:

- Using a pre-defined set of parameters the forward model is run which serves as the synthetic truth. A set of measurements are derived from this synthetic true experiment with pre-defined virtual sensor locations. The retrieved measurements are perturbed to generate a realistic noisy observed dataset.

- Based on the initial prior knowledge of the parameters, the forward model is simulated. Whenever synthetic observations are made available at specific time intervals, the forecasted state in the model space is converted into the observation space. By employing data assimilation (DA), the posterior distribution of the parameter set is then estimated.

- The predicted state *i.e.* settlement and excess pore water pressure at a specific location and time interval, depend not only on the magnitude of the model parameters but also on the history of the state variables, such as stress, strain, and porewater pressure. Hence a recursive algorithm is employed where after each assimilation process, the forward simulation is restarted from the initial time period in order to update the aforementioned states along with the model parameters to achieve proper convergence, albeit, at an unavoidably increased computational cost (Mohsan et al., 2021).

- The performance of different DA algorithms is evaluated. For any modifications to enable parametric assessment, adequate changes to the observational network, error statistics or the model are employed and the methodology is repeated.

The current study examines two constitutive formulations: a basic elastoplastic formulation and an advanced elastoviscoplastic formulation with structural degradation, coupled with consolidation (Józefiak and Zbiciak, 2017), which are used to calculate settlements and excess pore water pressure under an embankment loading in a one-dimensional section. The study aims to investigate the impact of model complexity on the performance of various DA algorithms, including the updating of parameters and settlement predictions (see Figure 5.3).

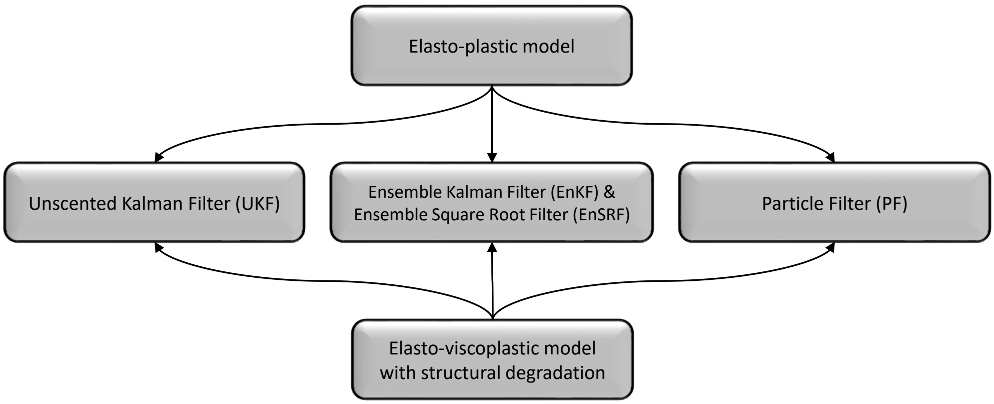


Figure 5.3: *Constitutive models and Data Assimilation procedures chosen in this study.*

A fully coupled hydro-mechanical finite difference simulation based on Rahman and Can Ulker (2018) has been implemented. The overall workflow is implemented and solved in a Python environment (Van Rossum and Drake Jr, 1995). This study shall aim to touch upon some of the typical questions on the effect of model complexity, and the convergence of the model parameters on the DA performance. The knowledge gained from this analysis is expected to be useful to identify a robust DA method for application for combined state and parameter estimation in geotechnical engineering. The performance of different DA algorithms are assessed based on accuracy and precision of the parameter estimation (see Figure 5.4).

Effect of sensor location on convergence of parameters

The performance of DA is influenced by the sensitivity of the parameters. The presence of insensitive parameters means that a unique solution cannot be guaranteed, and additional strategy is needed to restrict the parameter space, typically by conducting a preliminary sensitivity analysis,

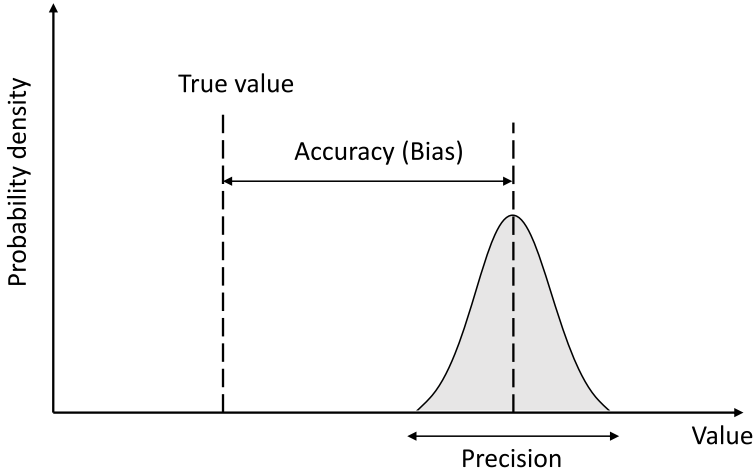


Figure 5.4: *Illustration of accuracy and precision.*

and/or utilising prior knowledge to determine the most significant parameters, and consequently constrain the less sensitive parameters to a reasonable value (Chen et al., 2013).

Nonetheless, in this study, no parameters were excluded as that allows to investigate how each DA procedure performs in the presence of these insensitive parameters. The sensitivity of the parameters is not constant in the spatio-temporal domain due to the change in the effective stress level in the system (Tahershamsi and Dijkstra, 2021, 2022). Consequently, the performance of the DA will also vary depending on the sensor location. In view of this, the time and location of our measurement, which we include in our DA scheme, dictates the convergence of the parameters.

To illustrate this point, let us consider the same scenario but with a change in the drainage boundaries at the bottom of the model. In this case, the drainage is closed at the bottom boundary and settlement measurements are taken at a depth of 4 m with high frequency (0.5 days) for the first 50 days, followed by the time intervals used previously. This was selected based on engineering judgement, however, a prior spatiotemporal sensitivity assessment of the model parameters can be an effective approach for optimal sensor placement (Hölter et al., 2015; Schoen et al., 2022).

Figure 5.5 depicts the performance of EnKF in estimating the parameters for the model. We observe that the modified swelling index (κ^*) converges faster than the other parameters in the first 50 d, which is likely due to the effective stress level being still in the overconsolidated region. This finding is consistent with the results reported in Tahershamsi and Dijkstra (2021) from a sensitivity point of view. Now continuing the simulation with the previously used regular time intervals (shown until 300 days in Figure 5.6), the other parameters reach convergence to their true value, as now the stress level has reached the elastoplastic region. In the synthetic truth, the time of stiffness transition from the elastic to elastoplastic region at 4 m depth is around 140 days and this corresponds well with Figure 5.6d where the assimilation of the modified compressibility index starts around this time.

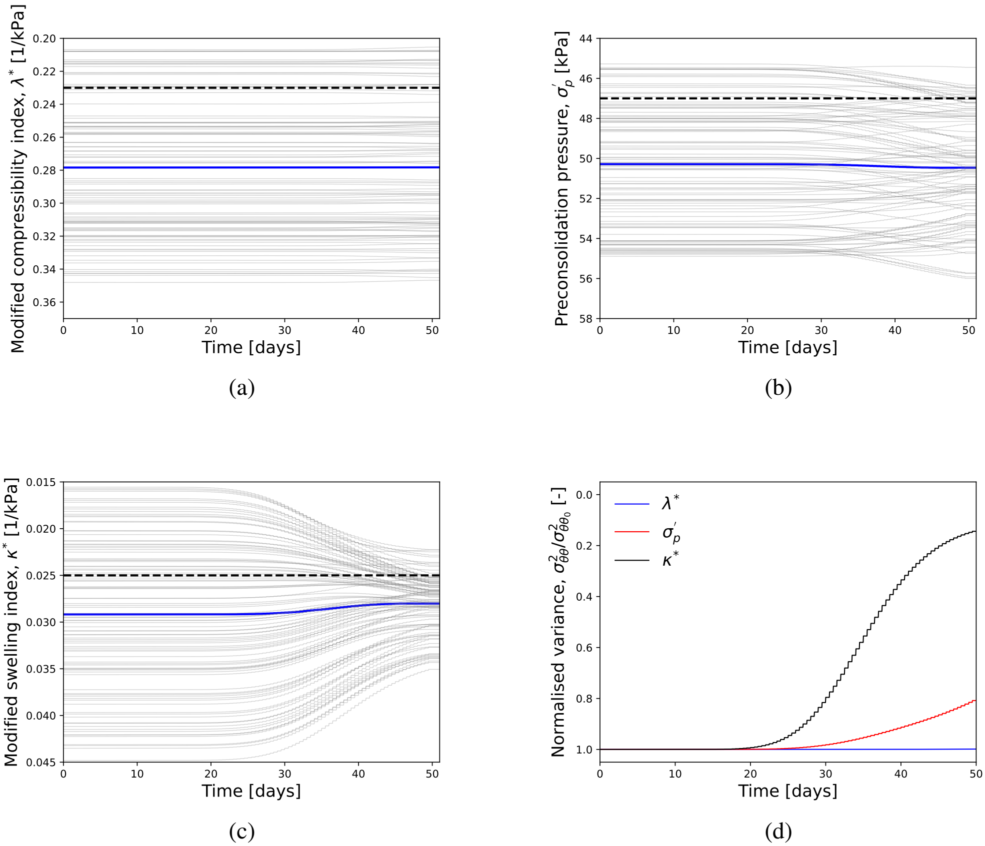
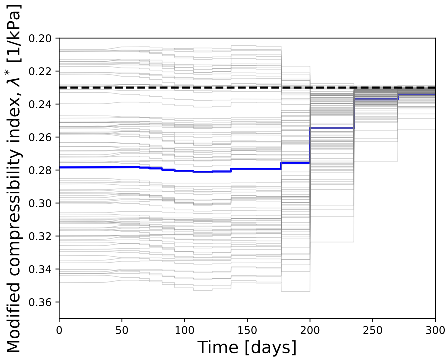
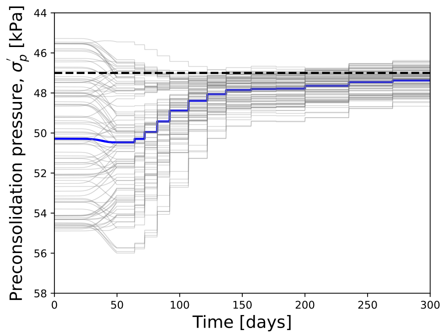


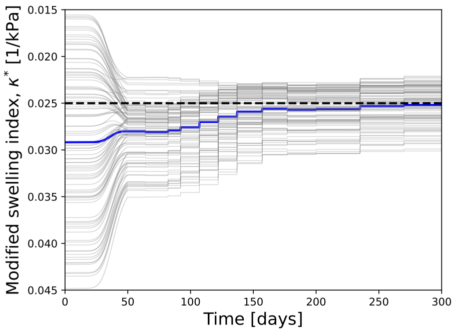
Figure 5.5: *Convergence of particles of EnKF (0–50* d*) for Elastoplastic model parameters in (a),(b),(c) [Color representation: Solid Gray: ensemble, Solid Blue: mean of the ensemble, Dashed black: Synthetic true value] and (d) normalised variance of all parameters.*



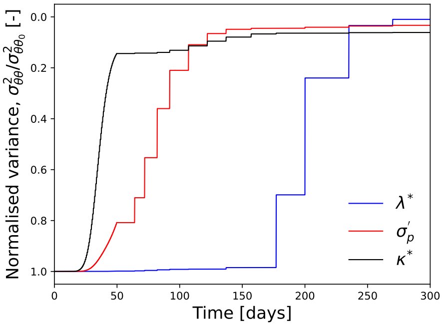
(a)



(b)



(c)



(d)

Figure 5.6: *Convergence of particles of EnKF (0–*300 d*) for Elastoplastic model parameters in (a),(b),(c) [Color representation: Solid Gray: ensemble, Solid Blue: mean of the ensemble, Dashed black: Synthetic true value] and (d) normalised variance of all parameters.*

Conclusions

The results indicate that the Ensemble Kalman Filter (EnKF) with perturbed observation performs substantially better compared to its square root variant (EnSRF) and all other filters, even for complex elastoviscoplastic models. Also EnKF can be relatively more stable in the presence of wild outliers in the data as demonstrated by Lei et al. (2010). The effect of number of ensembles of the EnKF for a geotechnical application has been studied by several researchers (Hommels and Molenkamp, 2006; Vardon et al., 2016; Mohsan et al., 2021) and not repeated here for the sake of brevity. Among the Data Assimilation techniques tested, the Unscented Kalman Filter (UKF) is the most computationally efficient and can be recommended for simpler geotechnical models. The UKF requires inherent parameters for choosing sigma points, due to its deterministic algorithm. With increasing values for α (from equation 4.7 and 4.8), the points move away from the mean. Meanwhile, the weight associated with the center particle increases while for the other particle decreases. Nevertheless, the effect of UKF performance based on the location of chosen sigma points ($0.80 \geq \alpha \geq 0.10$) is found to have negligible difference in the convergence of the parameters but this needs further evaluation with other applications.

The Particle Filter is shown to be less suitable for estimation of moderate to large sets of model parameters in geotechnics. One of the reason is due to the relatively high accuracy of the instruments considered in this exercise, leading to issues such as particle degeneracy *i.e.* losing the diversity of the particles. Although this can be circumvented by increasing the sample size or using an appropriate proposal distribution to sample the posterior, the issue can still persist. The traditional PF needs improvement in order to make it feasible for general uncertainty quantification in geotechnical engineering (see **Paper F**).

From a practical point of view, the convergence of modified swelling index may not be essential in real-life embankment situations, as long as the short and long term behaviour are estimated accurately. However, this exercise demonstrates that the stability of the parameter estimation depends on the relationship between the parameters and the observations. The available observations must contain adequate information to determine the unknown parameters of interest. Therefore, it is demonstrated that parameter sensitivity is dependent on the observation strategy. Regardless of the Data Assimilation method used, its effectiveness relies on an optimally designed monitoring network, which includes selecting an optimal sensor location and measurement interval. This finding highlights the impact of sensor placement on the quality of the implemented inverse analysis method.

5.3 Paper C Extension

Title: "Analysis of Ballina trial Embankment using Data Assimilation"

Introduction

Based on the synthetic tests, the Ensemble Kalman Filter (EnKF) is identified as the most robust among the considered Data Assimilation (DA) algorithms. However, the ideal synthetic test considered is a homogeneous soil profile, while real world soil investigation data significantly deviate from this state and usually have a layered profile (change of material properties with depth). Hence as a next step, the EnKF algorithm is evaluated against monitoring data from a real test case *i.e.* Ballina trial embankment. The background information on the Ballina test embankment, soil layering, in situ and laboratory tests, instrumentation and in-situ measurements are presented in Kelly et al. (2018). Some of the previous works dealing with Bayesian updating of the Ballina embankment (Tan et al., 2019; Tian et al., 2022b) involve the use of soft soil creep model (SSC) for the estuarine clay layer for the purpose of updating the model parameters. Research reported on the use of an advanced elastoviscoplastic model with structural degradation for Bayesian updating is limited. In contrast, the use of advanced models in deterministic analysis for the Ballina trial embankment is more prevalent (*e.g.* Amavasai et al., 2018; Rezania et al., 2018). Even when using such complex models, discrepancies still exist between the model prediction and the system behaviour, highlighting the difficulties in capturing accurately the *in-situ* behaviour. Both the Elastoplastic model (EP) and the Elastoviscoplastic model with structural degradation (EVP-S) are combined with EnKF algorithm to evaluate their performance with the Ballina trial embankment. The aim in this section is actually two-fold:

- To validate the efficiency of EnKF for high dimensional Bayesian updating of a real world embankment problem.
- To investigate whether high fidelity can be achieved for the simpler EP model by integrating monitoring data from a complex system with the help of Data Assimilation.

Methodology

Analyses are performed along the section with the magnetic extensometer (MEX1). The 1D hydro-mechanical coupled finite difference model proposed by Yang and Carter (2018) for the Ballina embankment is used. Similar to their Class-C prediction, only the estuarine clay layer (1.5 m to 11.1 m) was analysed with three sub-layers (Layer 1: 2 m thick, Layer 2: 5 m thick & Layer 3: 2.6 m thick) due to its high plasticity and the effect of prefabricated vertical drains are modelled using increased values for the vertical hydraulic conductivity along the soft soil profile. The monitoring data for the DA procedure is extracted from the magnets at depths of around 2 m (Magnet-1), 5 m (Magnet-2), 8 m (Magnet-3) and 10.9 m (Magnet-4). The prior statistics for the model parameters are considered the same for all layers and are represented by the log-normal distribution with the coefficient of variation (C.O.V.) values corresponding to Liu et al. (2018b) (see Table 5.1 and 5.2). A total of 200 ensemble members have been generated from this distribution for the analysis.

Table 5.1: Prior statistics of uncertain parameters of the EP constitutive model for all 3 layers.

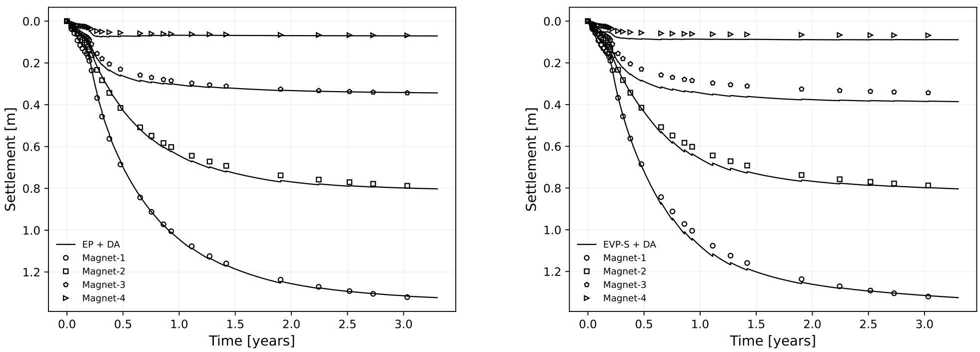
Parameter	Mean	COV
λ^* [-]	0.216	0.26
κ^* [-]	0.032	0.25
σ_p' [kPa]	55.0	0.23
k [m/day]	9e-4	0.30
C_k [-]	0.50	0.20

Table 5.2: Prior statistics of uncertain parameters of the EVP-S constitutive model for all 3 layers.

Parameter	Mean	COV
λ_i^* [-]	0.150	0.26
κ^* [-]	0.032	0.25
σ_p' [kPa]	55	0.23
χ_0 [-]	7.0	0.20
ρ [-]	10.0	0.20
μ_i^* [-]	0.0030	0.20
τ [days]	3	0.20
k [m/day]	9e-4	0.30
C_k [-]	0.50	0.20

Results

The ensemble mean of the assimilated settlements for both the EP and the EVP-S model using EnKF is shown in Figure 5.7. It should be noted that similar to **Paper C**, the restart algorithm is employed where after each assimilation, the forward model is restarted from the initial time period to update the state variables *i.e.* stress, strain and pore-water pressure along with the model parameters. Using EnKF, an accurate state prediction is achieved, regardless of model complexity.



(a) (b)

Figure 5.7: *Comparison of the assimilated settlement based on monitoring data using the EnKF Data Assimilation (DA) algorithm for (a) EP & (b) EVP-S model*

A preliminary simulation with only prior knowledge from Tables 5.1 and 5.2 without EnKF is conducted using the EP and EVP-S model. Figure 5.8 shows the difference between the simulated settlements using only prior knowledge of the random variables and that when combined with EnKF. So using only prior knowledge with no Data Assimilation, the EVP-S model would overestimate the settlements, and in the case of EP there would be an underestimation.

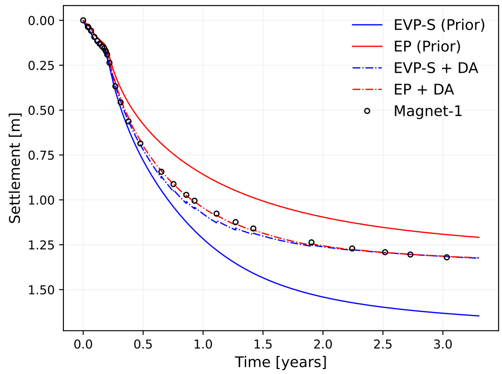
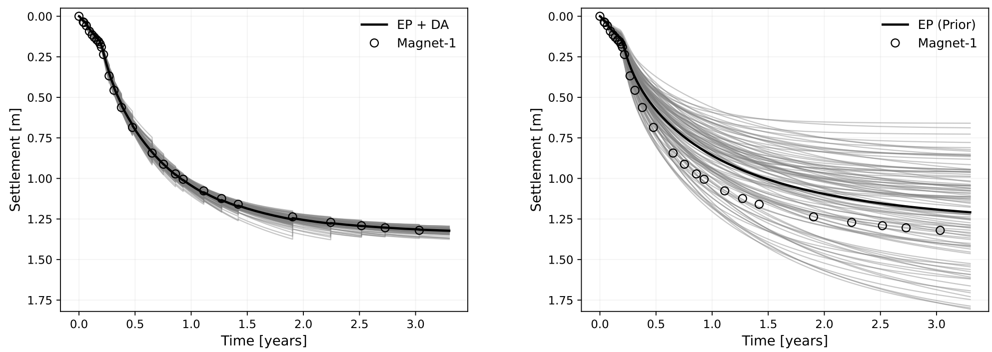


Figure 5.8: *Comparison of the data assimilated state predictions with that from prior knowledge for the EP and EVP-S models.*

The ensemble distribution is shown in Figure 5.9 for the EP model and in Figure 5.10 for the EVP-S model to show the difference in simulation between using Data Assimilation to update the ensemble prediction and that using only prior knowledge. The results, especially Figure 5.9a. and 5.10a., are similar to those observed in Tao et al. (2022).



(a) (b)

Figure 5.9: *Comparison of ensemble distribution for settlement prediction using EP model with (a) using EnKF and monitoring data (b) using only prior knowledge (Color representation: Solid Gray: ensemble, Solid black: Ensemble mean, marker: Measurement from field)*

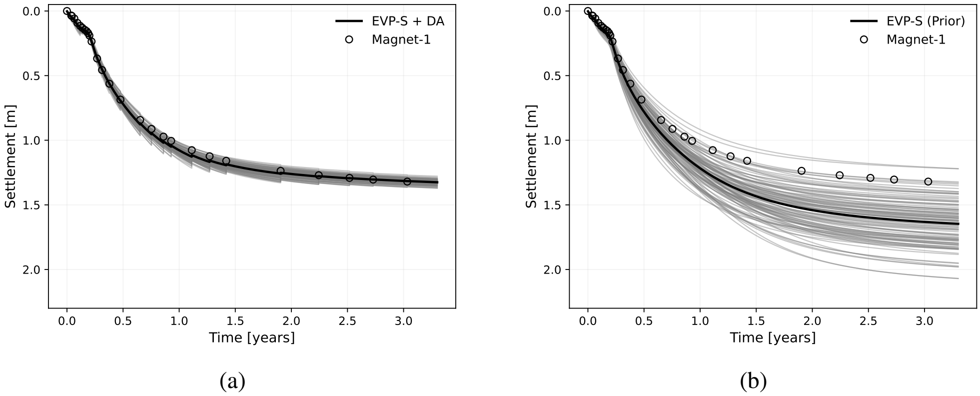


Figure 5.10: *Comparison of ensemble distribution for settlement prediction using EVP-S model with (a) using EnKF and monitoring data (b) using only prior knowledge (Color representation: Solid Gray: ensemble, Solid black: Ensemble mean, marker: Measurement from field)*

With regards to the precision of the state prediction using DA, the standard deviation of the settlement after 3 years at the location of Magnet-1, are recorded as 2.9 mm and 2.57 mm for the EP and EVP-S model respectively. A simple model still captures the system behaviour not just accurately but also with a precision that is on par with that of an advanced model, when using EnKF. When checking the assimilated parameters for the EP model (see Table 5.3), however, the modified compressibility index (λ^*) in the first layer is slightly elevated and the preconsolidation pressure for all the layers show lower values than reported in the laboratory data for samples from those depths (Pineda et al., 2016).

Table 5.3: Assimilated EP model parameters for the Ballina embankment.

Parameters	Layer 1		Layer 2		Layer 3	
	mean	COV	mean	COV	mean	COV
λ^*	0.250	0.14	0.185	0.02	0.150	0.01
σ_p'	46.26	0.10	42.45	0.08	49.87	0.15
κ^*	0.028	0.17	0.030	0.20	0.027	0.23
k	0.00132	0.09	0.00177	0.09	0.00148	0.06
C_k	0.532	0.14	0.498	0.17	0.440	0.14

Since the EP model does not consider the deformations arising from rate dependency (creep) and loss of bonding (destructuration), some parameters in the EP model deviate from the expected values from laboratory tests, to compensate for the effects that are not explicitly captured by the model. This shows that by using monitoring data to update model parameters, the system behaviour can be predicted with improved fidelity, regardless of the forecasting model. When a simpler model is used to capture more complex system behaviour, the model parameters may not necessarily represent the true soil parameters, but rather become fitting parameters to accommodate the additional features of the system. The final assimilated hydraulic parameters are reasonably

Table 5.4: Assimilated EVP-S model parameters for the Ballina embankment.

Parameters	Layer 1		Layer 2		Layer 3	
	mean	COV	mean	COV	mean	COV
λ_i^*	0.106	0.04	0.095	0.02	0.088	0.03
σ_p'	47.816	0.02	62.01	0.02	71.53	0.02
κ^*	0.0247	0.15	0.029	0.14	0.036	0.16
μ_i^*	0.0028	0.09	0.0030	0.06	0.0031	0.07
χ_0	8.334	0.18	10.56	0.10	7.197	0.16
ρ	8.713	0.10	10.08	0.07	6.338	0.17
τ	2.71	0.17	4.32	0.14	3.24	0.18
k	0.00110	0.06	0.00143	0.13	0.00165	0.04
C_k	0.426	0.26	0.611	0.16	0.599	0.18

close for both models, and most of the assimilated parameters for the EVP-S model (see Table 5.4) correspond to those adopted independently in Amavasai et al. (2018).

Conclusions

- This study demonstrates that even with a reliable estimation algorithm like EnKF, the parameter estimation outcomes may be unsatisfactory when a simpler model is used. A more effective approach is to utilise a forward model that accurately represents the system behavior. Therefore, a successful state and parameter estimation technique depends on a robust forward model that captures the fundamental processes in the behaviour of the system, along with a reliable DA estimation algorithm.

- Although the EVP-S model performs relatively satisfactory for this case study, it is not necessarily the best model to use. There may exist other variants of this formulation for viscous behaviour that are rigid in the sense defined earlier (Section 3.1), and yet able to fit the response well. A similar approach to consider the effect of PVDs need to be considered using different methods (Hansbo et al., 1981; Chai et al., 2001; Walker and Indraratna, 2006; Walker et al., 2009).

- It should be noted that the difference in using the informative log-normal distribution, and the weakly informative uniform distribution has already been discussed in Tian et al. (2022b) for the Ballina embankment, and hence not repeated here for the sake of brevity.

5.4 Paper D

Title: "On the feasibility of data assimilation for uncertainty modelling in geotechnics"

Introduction

Due to the inherent complexity of soil behaviour, geotechnical applications typically rely on advanced constitutive models that involve a large number of model parameters to achieve high fidelity solutions. However, a common issue in geotechnical problems is that the scarcity of relevant data makes it difficult to accurately characterize these models, leading to increased uncertainties in the predictions. To avoid this situation, simpler models are preferred, but they may not possess the sufficient features to represent the complexities of a geotechnical system. This study aims to investigate the problem of epistemic uncertainty in terms of model selection, to determine whether a simpler model can be used to capture a complex system when combined with a Data Assimilation (DA) algorithm. The Ensemble Kalman Filter (EnKF) is used as the DA procedure in this paper.

Methodology

The workflow undertaken in this paper is shown in Figure 5.11. The measurements are generated synthetically with noise using any model from the module which comprises of advanced elasto-viscoplastic model with structural degradation (EVP-S), Elastoplastic model (EP) and, the newly proposed, Elastoplastic model with structural degradation (EP-S). The uncertain variables are assumed to be log-normally distributed to avoid negative values. For each case, when a specific constitutive model is chosen to represent the synthetic truth of the system, a different model is then chosen to run forward in time, and then integrated with the observations using EnKF algorithm to jointly estimate the state and model parameters.

Results

The numerical model from **Paper C** is chosen in this study for the synthetic experiment but with the methodology mentioned in the previous section. The study has shown that using the EnKF to integrate observations allows the model to capture the behaviour of a system, regardless of its complexity, within the monitoring time window. The calibrated parameters of the model also enable accurate predictions of the long-term behavior of the system beyond the monitoring window, where observations are not available. The study suggests that the choice of a suitable model does not depend solely on capturing all the physical processes of the system, as long as it is coupled with a Data Assimilation procedure (or any other robust Bayesian inverse analysis technique).

However, the use of a simplified model can result in the calibrated parameters to become more fitting parameters rather than informative of the system behaviour. In some cases, this can becomes a major limitation, especially in the prediction window when there is no monitoring data beyond a certain time period for which the model has been calibrated. The future forecast of this calibrated simple model may diverge from the true behaviour of the complex system when there is a drastic change in the boundary condition in contrast to the simple static loading problem considered in this study.

Nevertheless, the synthetic cases used in this study has helped to understand better the reason behind the convergence of those fitted values by retracing to the difference in complexity between the model formulation and the synthetic system. For systems with time-dependent behaviour, the simpler non-viscous models can still capture the creep settlement beyond the monitoring window, albeit with some slight deviation. This minor discrepancy can be alleviated by monitoring over a more extended period, although this may not be cost-effective in most projects.

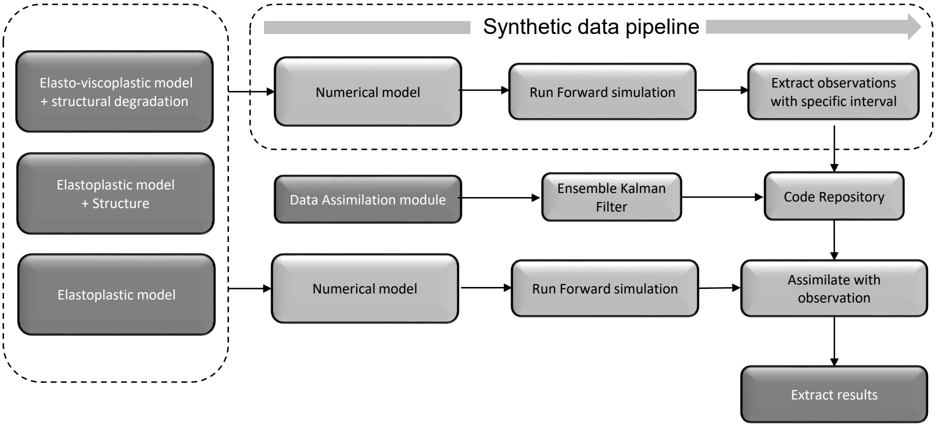


Figure 5.11: *Workflow for including Data Assimilation with numerical forward model to estimate state and parameters.*

5.5 Paper E

Title: "Data assimilation for Bayesian updating of predicted embankment response using monitoring data"

Introduction

The accurate and precise determination of model parameters is pivotal for the successful implementation of the Data Assimilation (DA) procedure, as it significantly contributes to comprehending the underlying physics of the soil behavior. The objective of this study is to estimate the parameters of the constitutive model by analyzing field measurement data from a synthetic trial embankment constructed in the Plaxis Finite Element code through the employment of a Data Assimilation (DA) procedure. The Ensemble Kalman Filter (EnKF) is used as the DA procedure in this study. Although using DA as an inverse analysis procedure is an efficient approach, the aspect of determining which measurement to include is often overlooked. This is because the accuracy and precision of the obtained parameter is directly linked to the selected experimental configuration, including factors like the quantity and type of sensors, as well as their location. An optimal sensor configuration can dramatically increase the quality of the DA analysis. However, the instrumentation set-up for most geotechnical applications, particularly for trial embankments, are standardised *e.g.* settlement gauge under the centre of embankment, inclinometer near the toe *etc.* It should be noted that even with this straightforward set-up, the type of observation included in the Data Assimilation (DA) procedure still has a large influence on the convergence of some constitutive model parameters. This is due to the sensitivity of those model parameters to certain measurements and in order to assess this, a Global Sensitivity Analysis (GSA) using factorial design (Tahershamsi and Dijkstra, 2022) is performed in this study. The effect of chosen prior distribution of the model parameters *i.e.* weakly-informed to well-informed distribution is considered.

Methodology

The integration of the Ensemble Kalman filter (EnKF) into a geotechnical application implemented within the PLAXIS Finite Element code is illustrated in Figure 5.12. The behavior of the system is simulated in a controlled setting by producing synthetic measurement data with noise, specifically vertical and horizontal displacements. The Data Assimilation (DA) procedure is then implemented for the considered time-dependent geotechnical system, which involves an embankment on soft soil undergoing consolidation. Following initialisation, the Soft Soil (SS) model utilizes the set of ensembles, each with unique set of parameters, to predict the geotechnical response in the time domain up to a specified time step. These ensembles represent the prior belief of the parameter values leading up to the time of available measurement. The predicted state in the model space is subsequently transformed into the observation space, and through the use of DA, the posterior distribution of the parameter set is estimated. The displacement at a specific time interval depend not only on the magnitude of the model parameters but also on the state variables, such as stress, strain, and porewater pressure distribution, at each time step. Hence, a recursive algorithm is necessary to update these states along with the model parameters at each assimilation cycle to achieve proper convergence, albeit, at an unavoidably increased computational cost (Mohsan et al., 2021). The converged parameters are assessed in terms of physical meaningfulness and subsequent model performance on future forecasts post-assimilation window. A hydro-mechanical Finite Element (FE) simulation serves as the forward model.

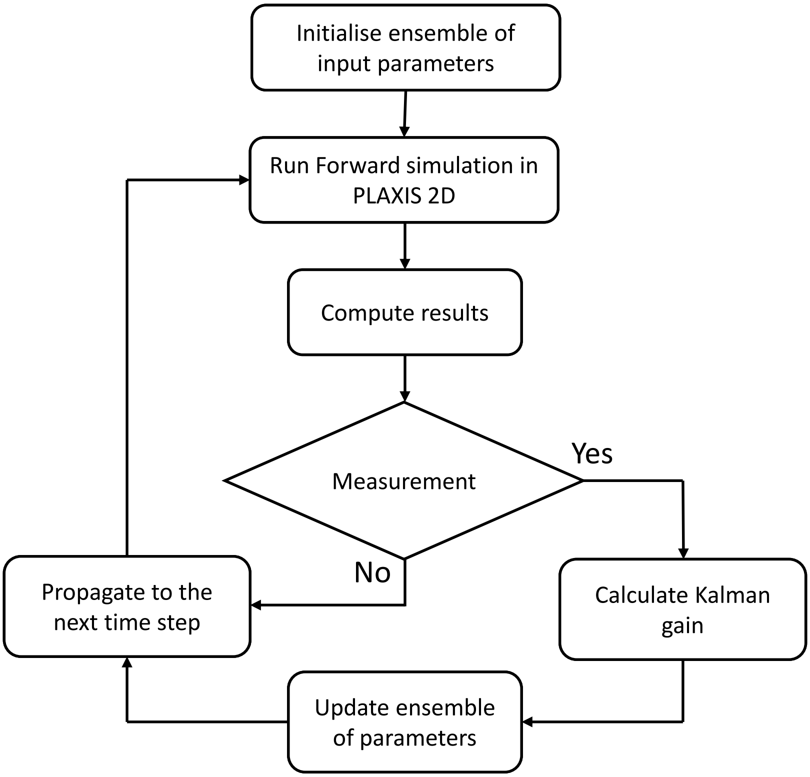


Figure 5.12: *Illustration of the integrated workflow of PLAXIS FE with Ensemble Kalman Filter*

The study investigates the impact of various factors such as sensor characteristics, prior distribution type, and parameter sensitivity on the Bayesian update of embankment behaviour. The entire workflow is implemented and solved in a Python environment (Van Rossum and Drake Jr, 1995), utilizing the PLAXIS Python interface to dictate the Finite Element calculations in PLAXIS and integrating it with the DA algorithm in the same code. A basic version of this script to made available as open source in the link: https://github.com/amaran1988/DA-PLAXIS2D.git. The dimensions of the numerical model along with the sensor locations are shown in Figure 5.13. For more details the reader is directed to Amavasai et al. (2023).

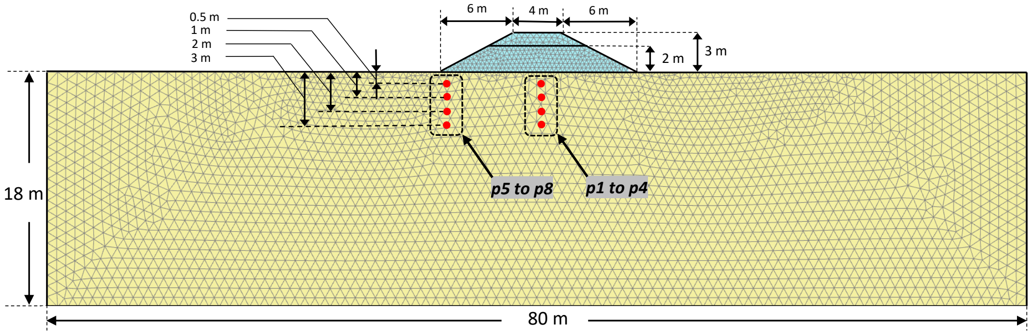


Figure 5.13: *Mesh discretisation and dimensions of the numerical model.*

Results

The results from this study show some important points regarding joint state and parameter estimation using DA for geotechnical analyses. For instance, among several factors that influence the performance of the DA algorithm such as the error statistics from measurements or from background covariance matrix, the sensor strategy *i.e.* the quantity and type of observation play a significant role. It is shown in this study that it is not just the quantity of sensors but also the type of sensors and its combination that dictates the performance of the DA procedure, with the latter having the most significant influence. The reason for this is due to the varying sensitivity of the parameters for different types of observation which correlates to the convergence of the parameters when DA is employed for inverse analysis. This strong correlation between the type of observation and convergence of parameters, although does not depend on the type of prior information (be it well or weakly-informed as shown in this study), strongly depends on the location of that observation since the sensitivity of the parameter varies in the spatiotemporal domain.

5.6 Paper F

Title: "Particle Filter based on Jaya optimization for Bayesian updating of nonlinear models"

Introduction

The Particle Filter (PF) is a method in the Data Assimilation (DA) framework that has gained popularity in geotechnical applications recently (Shuku et al., 2012; Murakami et al., 2013; Shibata et al., 2019). It is a Sequential Monte Carlo technique that uses observational data to update model predictions as they are received in a sequential manner. The probability density is represented by a finite number of particles with associated weights making it efficient for nonlinear state estimation. The traditional versions of PF, however, struggles with two major drawbacks when working with limited sample size: the problem with degeneracy and impoverishment (Li et al., 2014) affecting the diversity in the distribution of particles in the state space.

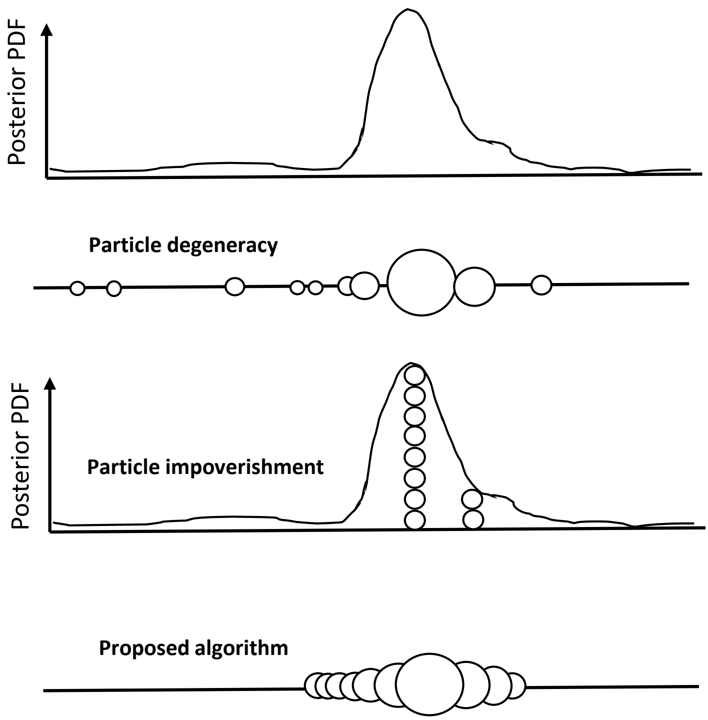


Figure 5.14: *Illustration of particle degeneracy and impoverishment and the solution for these problems using the proposed PF-JAYA algorithm.*

As shown in Figure 5.14, degeneracy occurs when the distribution of weights among the particles become extremely uneven such that few particles get large weights and the rest end up with negligible weights. This issue can be countered by employing an additional resampling step, but when faced with limited number of particles, suffers from impoverishment as shown in the same Figure. Introducing noise after the resampling step can help mitigate the problem mentioned earlier, but this approach is unlikely to enhance the estimation accuracy, because this step only improves particle diversity in the state space, and does not consider the recent observation.

Proposed method

In this paper, a novel hybrid Particle Filter based on Jaya Optimization (Rao, 2016) [PF-JAYA] is proposed. The algorithm works by taking into account both the current measurement information and the previous state of the particles to move them towards high likelihood regions in the state space as shown in Figure 5.15. This method effectively addresses the problems of particle degeneracy and impoverishment, while also achieving accurate estimates, making it a robust and efficient algorithm to improve the performance of PF, both in theory and in practice.

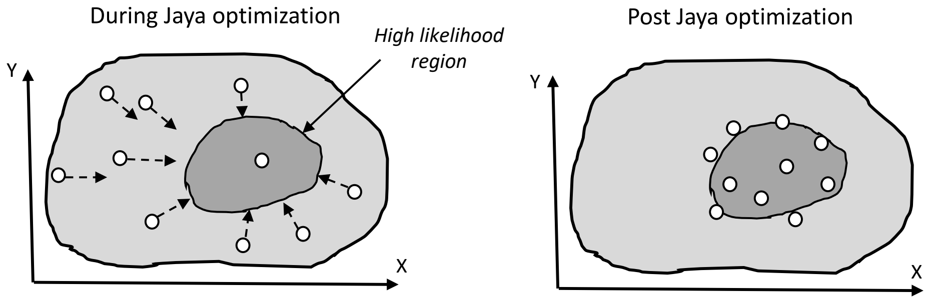


Figure 5.15: *Illustration of the working of PF-JAYA during and after the Jaya optimization procedure.*

Validation cases

This paper demonstrates the effectiveness of the proposed hybrid PF-JAYA algorithm through two geotechnical engineering-specific joint state and parameter estimation problems. These problems involve the prediction of settlement in soft ground under embankment loading, where the first problem uses an analytical linear elastic settlement model, while the second problem utilises a one-dimensional hydro-mechanical coupled finite difference numerical model with an elastoplastic constitutive model. PF based on the resampling procedure (PF-SIR) is used as a reference baseline for comparing the performance of the proposed hybrid PF-JAYA algorithm. Additionally, the Ensemble Kalman Filter (EnKF) is also used in the second validation case as a means to compare the performance of the PF-JAYA algorithm. To further demonstrate the effectiveness of PF-JAYA, an additional validation case is conducted using the Lorenz model (Lorenz, 1963).

Results

The first example case demonstrated that PF-JAYA outperforms PF-SIR in terms of accuracy and convergence in both state and parameter estimation, even with a limited number of particles, and the choice of prior distribution had little effect on the results. PF-JAYA was found to be less affected by increasing sparsity in the monitoring data.

In the second example, PF-JAYA outperformed all DA algorithms, including EnKF, in terms of accuracy and precision in parameter estimation. The sensitivity of model parameters, determined by sensor strategy, was found to dictate the convergence of the DA algorithm.

In the third example, using the Lorenz '63 model as a non-geotechnical validation case, PF-JAYA demonstrated superior performance over EnKF in terms of state estimation. Although the proposed approach has achieved remarkable improvement in the accuracy, robustness, and convergence compared to its predecessors, it has resulted in increased computational requirements. In order to make PF-JAYA practical for large scale problems, computationally efficient surrogate models are necessary.

Factors influencing Data Assimilation for geotechnics

This study suggests that combining Data Assimilation with a deterministic geotechnical forecasting model holds promise for addressing state and parameter estimation for time-dependent geotechnical problems. However, the performance of the Data Assimilation procedure is influenced by several factors. It is crucial to understand these factors before engaging their use in general geotechnical practice, some of which are mentioned as follows:

- *Sensor Strategy*: The choice of sensor strategy can significantly impact the performance of the assimilation procedure as shown in **Paper C** and **Paper E**. Sensor strategy refers to the selection of sensors, their locations, quantity and the frequency of measurements. However, this is not always straightforward, and the cost of sensors and their maintenance should also be considered as part of this factor.

- *Error statistics*: The effect of model uncertainty (as studied in **Paper D**) has a large effect on the physical meaning of the assimilated parameters. Also, the errors associated with measurements defines the accuracy of the estimation. It is crucial to identify and quantify these errors as they hold a significant influence on the DA performance.

- *Complexity*: The choice of the assimilation algorithm and the predictive forward model along with the sensitivity of its parameters can significantly impact the performance of DA as shown in **Paper C** and **Paper F**. The computational requirement of the DA algorithm and the dimension of the forward model defines the criteria for choosing in practical applications.

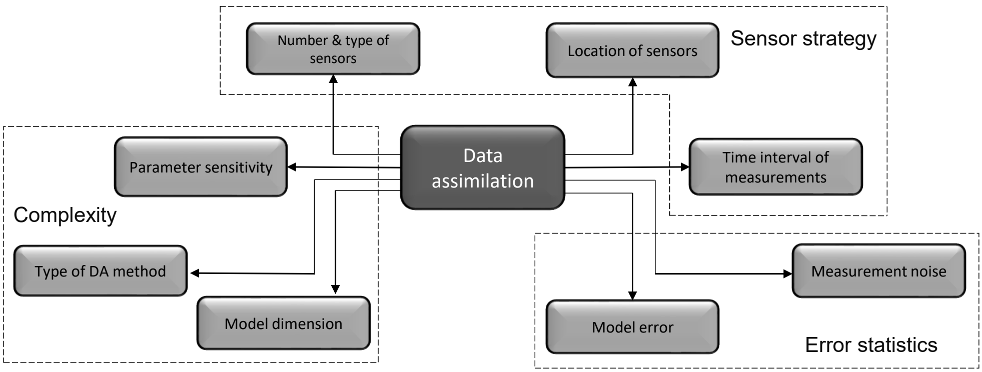


Figure 5.16: *Data Assimilation influence chart.*

6 Conclusions and recommendations

6.1 Conclusions

Data Assimilation (DA) methods are commonly presented in the literature as a complex mathematical framework constrained predominantly to numerical weather prediction. As a result, this precluded the use of DA in geotechnical engineering. In contrast, this thesis offers an intuitive description of the use of DA procedures in geotechnical engineering, for which a modular framework with various DA algorithms has been implemented. The algorithms have been applied to several synthetic examples and real case dataset. Based on the findings reported in the appended papers, the main conclusions are summarised as follows:

- Data from two trial embankments have been used to validate an advanced rate-dependent constitutive model in **Paper A** (Haarajoki embankment) and **Paper B** (Ballina Embankment). A consistent parameter set has been derived for the constitutive model using standard laboratory tests and and field data of both these embankments, using a custom built semi-automated parameter derivation module. One of the drawbacks of the advanced constitutive model used is the requirement to perform non-standard laboratory tests for the anisotropic and destructuration parameters, which even for the test embankments considered in this thesis were not available. Hence, values based on empirical relation were taken.

- For the Haarajoki embankment in **Paper A**, the settlements predicted by the model agreed well with that measured in the field for the section without ground improvement. For the improved area with vertical drains, however, the effect of the smear zone properties had a large impact on the predicted results. A simplified relation to account for the smear zone properties has been considered in the model to obtain the equivalent hydraulic conductivity without any further change in the numerical model. Despite not fully modelling the drains and their installation in full detail, a remarkably good agreement was achieved for the settlements under the embankment for the same set of model parameters, leading to the fact that a consistent parameter set is achieved. However, it should be noted that the model predictions are sensitive for the properties of the dessicated crust on top of the soft soil layers. Furthermore, the horizontal displacements at the toe of the embankment are overestimated, meaning that the predicted K_0 value is not representative of the site.

- Following a similar parameter derivation approach for the Ballina embankment in **Paper B**, the observed discrepancies between the Class A predictions and the field measurements were mainly due to the over-estimated apparent preconsolidation pressure from the constant rate of strain compression tests that were provided early in the prediction contest. This is a classic case of error stemming from engineering judgement, due to limited data and unavailable prior knowledge about the site. The Class C predictions used the preconsolidation values derived from the incrementally loaded oedometer tests instead, with a reference time that is consistent with the model, giving a substantial improvement in the model performance. A second uncertainty stemmed from the estimation of the hydraulic conductivities affected by the smear zone resulting from the installation of the drains.

- Both **Paper A** & **Paper B** have shown that an advanced constitutive model implemented in a hydro-mechanically coupled Finite Element code, even with its excellent predictive capabilities validated at the laboratory scale under controlled conditions, still is unable to fully capture the system behaviour in the field scale. The latter indicates that model sophistication on its own is not the practical way forward, as long as a deterministic approach is followed. Therefore, the need to consider uncertainties in a probabilistic framework, whilst maintaining a sufficiently accurate prediction model is paramount, especially for model parameter estimation in real-world applications.

- **Paper C** dealt with the implementation of different DA methods and assessed their performance, benefits and drawbacks when applied to different constitutive models. The criteria involved the assessment of the accuracy and precision of each DA method when applied to various constitutive models with differing complexity. This paper demonstrated that DA is a promising tool to address joint state and parameter estimation for time-dependent problems in geotechnical engineering. Caution is, however, required when choosing the appropriate method and observation data. The impact of different monitoring points on the assimilation of model parameters has been studied and it is found that the effectiveness of the DA process is determined by the location of sensors and the time interval of measurements, due to the variation in parameter sensitivity in the spatiotemporal domain. Among the DA methods that were evaluated, the Unscented Kalman Filter was found to be the most computationally efficient and reasonably accurate (and precise), and thus it is recommended for simpler geotechnical models. However, for more complex constitutive models that have a large model parameter set, the Ensemble Kalman Filter (EnKF) was found to perform better, in terms of accuracy and precision, than the other techniques. Particle Filter (PF) is shown to be less effective in high dimensions (even with resampling after the particle weight update step). Under the constraint of limited sample size, PF can fail to capture the true value of the model parameters while EnKF have a potential advantage.

- The EnKF is further evaluated in **Paper D** where its effectiveness in trying to capture the behaviour of a complex system using a simplified model is studied. The results demonstrate that by using DA, the model is still able to capture the behaviour of that system within the monitoring time window and by using the calibrated parameters, able to capture the long term behaviour of the system well beyond the monitoring window where observations are not (yet) available. This shows that for state estimation the choice of a suitable model may not necessarily be based on capturing all the physical processes of the system, as long as it is augmented with a DA procedure. However the calibrated parameters, in most situations, can be at risk of losing their physical meaning *i.e.*, values beyond the bounds of logical comprehension and prior experience.

- The difference between the effect of type and number of observations included in the DA procedure is studied in **Paper E**. The Ensemble Kalman filter (EnKF) is integrated into the PLAXIS Finite Element code to analyse a synthetic embankment case. The sensitivity of the parameters has a large influence on its convergence during DA . Depending on the type of observation included, the convergence of the model parameters during DA would change.

- A novel Particle Filter based on Jaya optimisation (PF-JAYA) has been proposed in **Paper F**. This proposal is an improvement of the original particle filter (PF) algorithm which reduces the effects of particle degeneracy and impoverishment, the two well known drawbacks of PF. The theoretical and practical effectiveness of this algorithm has been systematically studied and is shown to outperform the classical DA algorithms. The PF-JAYA has shown consistency in its superior performance under cases with various types of *a priori* information and model complexity.

- To summarise, the limitations of the deterministic approach are demonstrated and the need for a robust probabilistic tool is shown to be paramount. Considering DA has been around for several decades, it is still a new concept when implementing for geotechnics, so in that sense this study is preliminary, but has revealed several insights to the attractive concept of state and parameter estimation for geotechnical engineering.

6.2 Future scope

The work done so far on implementing DA algorithms for geotechnical engineering can be viewed as an initial step, and there are many avenues yet to be explored. These techniques have numerous advantages with many promising features and can be used for a wide range of problems. In this section we present some potential areas for future research:

- The effect of sensor strategy on the performance of DA, regardless of the technique used, motivates to perform a detailed study regarding optimal sensor placement. However, this is non-trivial and a general guideline to optimal sensor placement is difficult to establish until a large test bed of synthetic geotechnical scenarios are investigated.
- To account for spatial heterogeneity, the work is planned to be extended toward estimation of parameter fields based on monitoring data. This should be done in 3D since assuming a plane strain assumption for random fields can be erroneous, and not representative of the site, unless the variability in the out of plane dimension is considered.
- In recent years, the focus of research has shifted from theoretical proposals to models that reflect real-world data. Hence, the current developed module would be extended to include data driven techniques with online learning capabilities.

6.3 Recommendations

- The current norm includes practitioners to manually interpret data to match the input requirements of a physics-based model in an ad-hoc manner. This strategy needs to be reviewed and probabilistic methods that make sense of monitoring data are needed to offer insights into critical decisions for ongoing projects.
- Data assimilation, like Finite Element analysis, can be prone to misuse when there is lack of proper understanding. Therefore, it is important to have some prior knowledge of the factors that can influence the DA tool (as shown in Chapter 5.6) before exercising its use.
- Using 3D Finite Element models to perform forward calculation is computationally demanding. Adding a probabilistic inverse analysis wrapper for such models can be far from

practical. To make the most of monitoring data, computationally efficient surrogate models are recommended for engineering practice.

- The scope for optimal sensor placement in practical applications depend on the model used for predicting the system behaviour. In order to propose a sensor strategy for a site requires first for the model to capture the site response accurately. Paradoxically to achieve this, data from that site is required in order to calibrate the model preferably with a Data Assimilation tool. One solution is to first have a well instrumented site with a comprehensive monitoring scheme based on engineering judgement and past experience. Subsequently, one can use that data to first refine the model (*i.e.* to choose the right model, calibrate the parameters, test the accuracy of future state prediction *etc.*) and then optimise the quantity of sensors (and possibly reuse the rest of the sensors for another project). In this way a reliable Digital twin of the site is obtained which can help plan a proper maintenance program for efficient asset management.

- The value of historical datasets are not fully recognised, and is often stored away without utilizing it in any ongoing projects. A comprehensive analysis of such datasets can provide valuable prior information which is an important step before employing Data Assimilation in geotechnical engineering practice.

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