

**Turbulent heat flux balance for natural convection
in air and sodium analysed by direct numerical
simulations**

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Introduction

- **Motivation**

turbulent heat transfer in buoyant flows

k- ϵ turbulence model

$$\overline{u_i' T'} = -\kappa_t \frac{\partial \overline{T}}{\partial x_i}, \quad \kappa_t = \frac{\nu_t}{Pr_t}$$

- inadequate for buoyant flows

full differential models for $\overline{u_i' T'}$

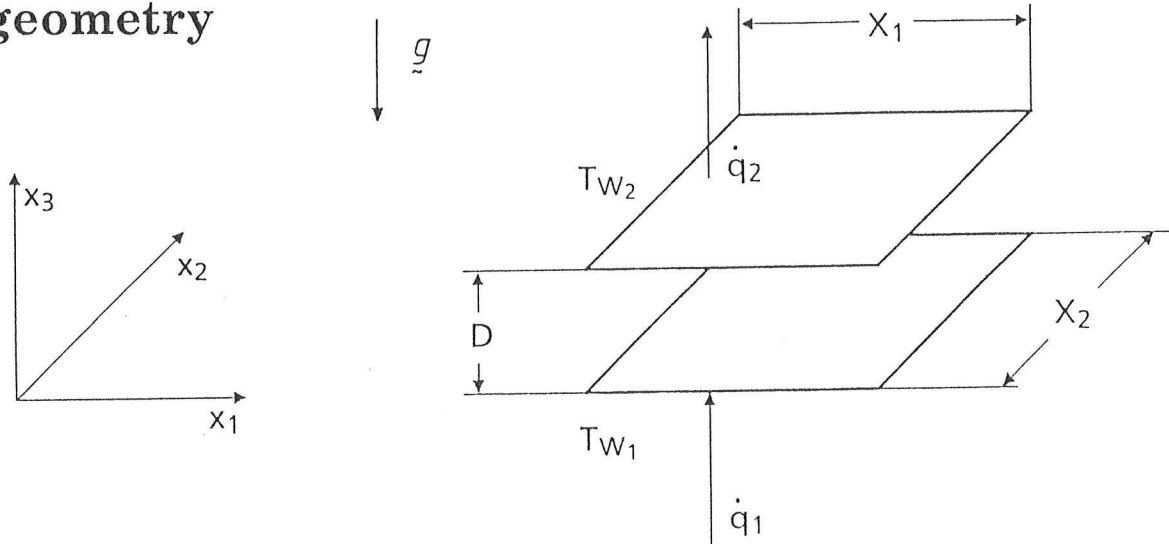
- closure of unknown correlations

- **Objective**

- detailed analysis of all terms in $\overline{u_i' T'}$ -equation
- turbulent natural convection in sodium and air
- evaluation of direct simulation results

Rayleigh-Bénard convection

- geometry



- dimensionless numbers

- Rayleigh-number:

$$Ra = \frac{g\beta(T_{W1} - T_{W2}) D^3}{\nu \kappa}$$

- Prandtl-number: $Pr = \nu/\kappa$

air: $Pr = 0.71$, sodium: $Pr = 0.006$

- Grashof number: $Gr = Ra/Pr$

Direct simulation method

- full conservation equations for
 - mass
 - momentum
 - energy
 - three-dimensional, time-dependent
 - resolve all scales
- no model assumptions
no parameters

Computer code TURBIT

- finite volume method
- spatial discretization
 - finite differences
 - staggered grid
- time integration
 - momentum equation
 - explicit Euler Leapfrog scheme
 - projection method of Chorin
 - thermal energy equation
 - semi-implicit Leapfrog-Crank-Nicholson scheme
- verified for natural and forced convection in various fluids

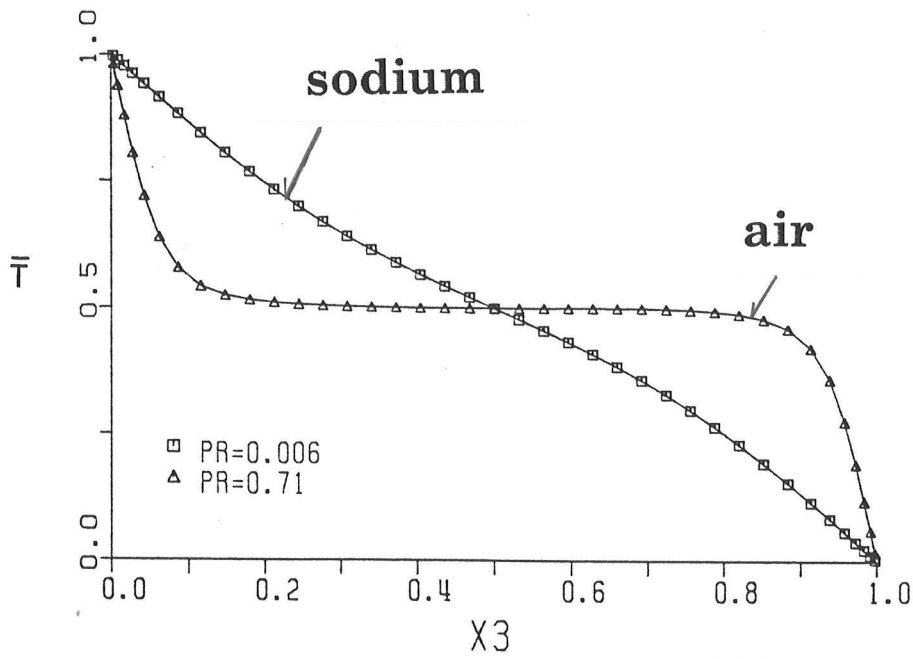
Case specifications

Fluid	Ra	Gr	grid
air	630,000	$0.9 \cdot 10^6$	200 · 200 · 39
sodium	24,000	$4 \cdot 10^6$	250 · 250 · 39

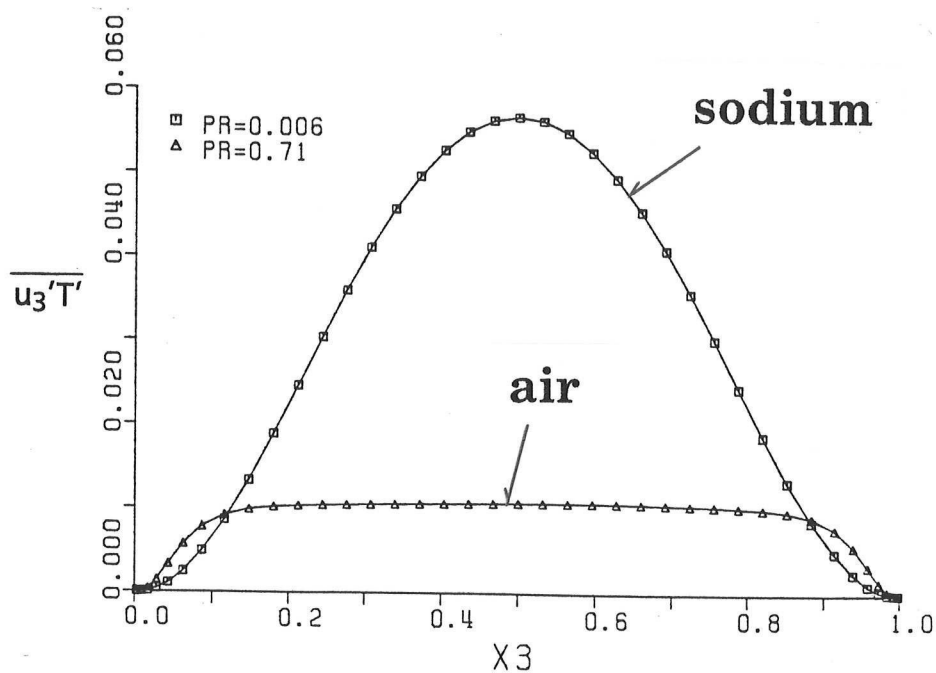
- boundary conditions:
 - periodic in horizontal direction ($X_{1,2} = 8$)
 - walls: no slip condition
constant wall temperatures
- initial conditions
 - final data of air / sodium simulations
with lower Ra

Evaluated results

- mean temperature



- vertical turbulent heat flux



Transport equation for $\overline{u_3 T}$

- turbulent Rayleigh-Bénard convection
 - no mean velocity $\overline{u_i} = 0$
 - no gradients in horizontal directions

$$0 = \underbrace{- \overline{u_3'^2} \frac{\partial \overline{T}}{\partial x_3} + \overline{T'^2}}_P$$

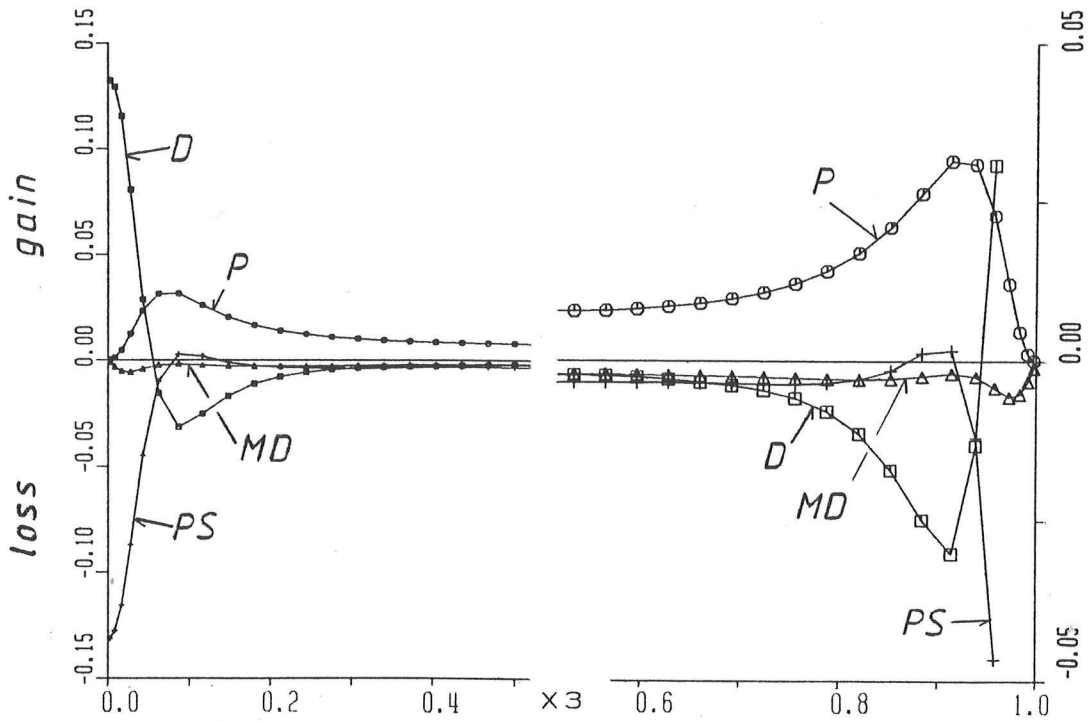
$$\underbrace{- \frac{\partial}{\partial x_3} \left(\overline{u_3'^2 T'} + \overline{p' T'} - \frac{1}{Pr \sqrt{Gr}} \overline{u_3' \frac{\partial T'}{\partial x_3}} - \frac{1}{\sqrt{Gr}} \overline{T' \frac{\partial u_3'}{\partial x_3}} \right)}_D$$

$$\underbrace{- \frac{1}{\sqrt{Gr}} \left(1 + \frac{1}{Pr} \right) \overline{\frac{\partial u_3'}{\partial x_i} \cdot \frac{\partial T'}{\partial x_i}}}_{MD}$$

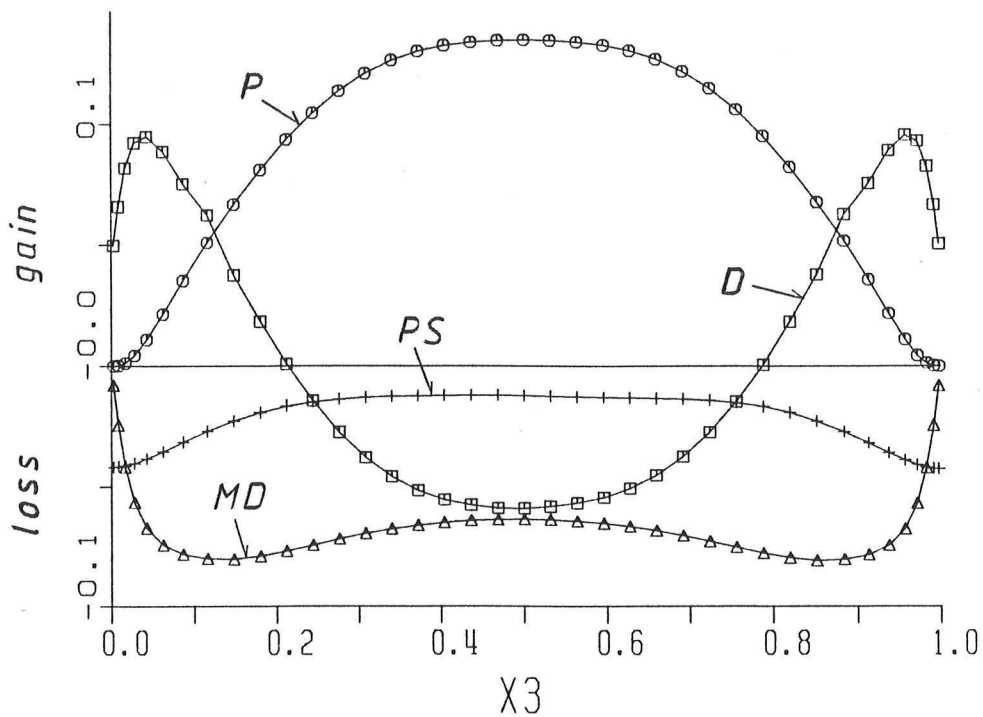
$$+ \underbrace{\overline{p' \frac{\partial T'}{\partial x_3}}}_{PS}$$

Turbulent heat flux budget

- air ($Pr = 0.71$)



- sodium ($Pr = 0.006$)

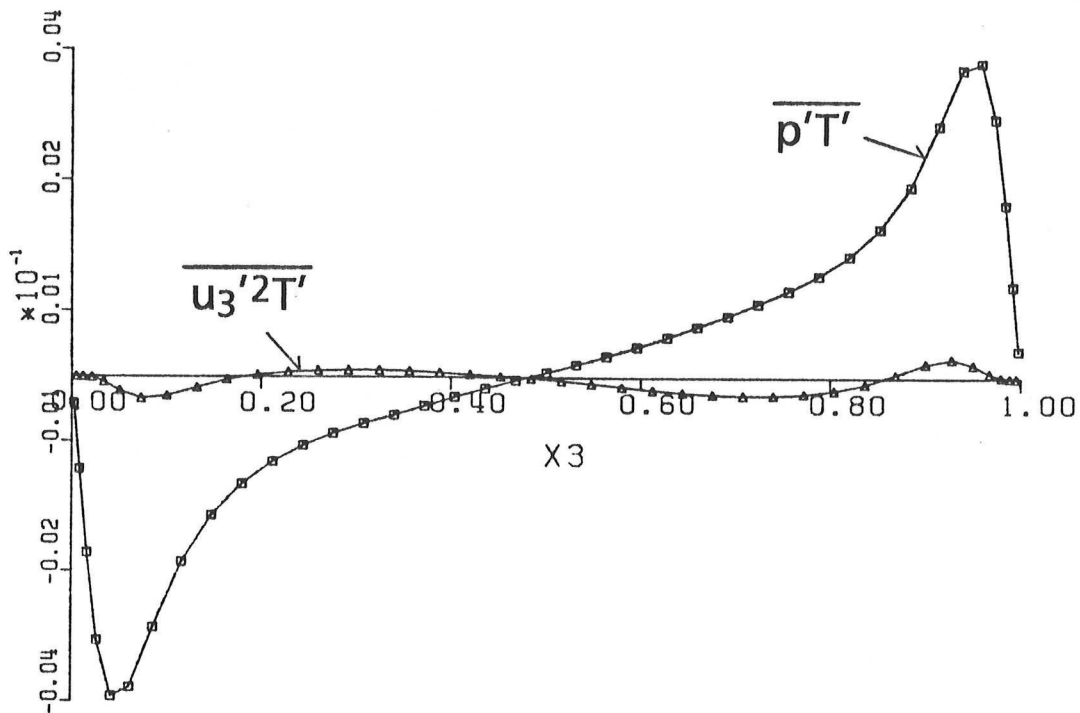


Analysis of diffusion of $\overline{u_3 T}$

$$-\frac{\partial}{\partial x_3} \left(\overline{u_3^2 T'} + \overline{p' T'} - \frac{1}{Pr \sqrt{Gr}} \overline{u_3 \frac{\partial T'}{\partial x_3}} - \frac{1}{\sqrt{Gr}} \overline{T' \frac{\partial u_3}{\partial x_3}} \right)$$

D

- air



Conclusions

- **Direct numerical simulation**
 - turbulent Rayleigh-Bénard convection
 - air and sodium
- **Balance of vertical turbulent heat flux**
 - no local equilibrium $P \neq PS$
 - molecular destruction is important sink
 - redistribution of $\overline{u_3'T'}$ by diffusion
 - turbulent diffusion mainly due to $\overline{p'T'}$
- **Standard turbulence models**
 - neglect pressure diffusion
 - neglect molecular destruction