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in der Helmholtz-Gemeinschaft
Institut für Reaktorsicherheit

Investigation of liquid phase turbulence based on direct numerical simulation of bubbly flows

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Content

- Introduction and motivation
- Direct numerical simulations
  - Verification for single bubbles
  - Results for bubble swarm flows for different values of the Morton number
- Analysis of k_L-equation
  - Profiles of balance terms (budget of k_L)
  - Assessment of closure assumptions
- Conclusions

Motivation

- Turbulence in bubbly flows
  - experiments show that bubbles may enhance turbulence or damp turbulence as compared to single phase pipe flow with same liquid flow rate
  - “pseudo turbulence” in bubble driven liquid flows (e.g. bubble columns)
  - phenomena are not fully understood
  - a reliable model to account for bubble induced turbulence (BIT) in Euler-Euler CFD codes is missing
- <u>Goal:</u> use DNS data to analyze turbulence kinetic energy equation for liquid phase and to test models

Motivation

- Liquid phase turbulence kinetic energy: $k_{\mathrm{L}} \equiv \overline{\overline{\mathbf{u}'_{\mathrm{L}} \cdot \mathbf{u}'_{\mathrm{L}}}}/2$
- Analytical transport equation* for k_{L}:

$$\frac{\partial}{\partial t}(\alpha_{\mathrm{L}} k_{\mathrm{L}}) + \nabla\cdot\left(\alpha_{\mathrm{L}} k_{\mathrm{L}} \overline{\overline{\mathbf{u}}}_{\mathrm{L}}\right) = \underbrace{\frac{1}{Re_{\mathrm{ref}}}\nabla\cdot\left(\alpha_{\mathrm{L}} \overline{\overline{\mathbb{T}'_{\mathrm{L}}\cdot\mathbf{u}'_{\mathrm{L}}}}\right) - \nabla\cdot\left[\alpha_{\mathrm{L}}\left(\overline{\overline{p'_{\mathrm{L}}\mathbf{u}'_{\mathrm{L}}}} + \frac{1}{2}\overline{\overline{(\mathbf{u}'_{\mathrm{L}}\cdot\mathbf{u}'_{\mathrm{L}})\mathbf{u}'_{\mathrm{L}}}}\right)\right]}_{DIFFUSION}$$

$$\underbrace{-\alpha_{\mathrm{L}} \overline{\overline{\mathbf{u}'_{\mathrm{L}}\mathbf{u}'_{\mathrm{L}}}} : \nabla\overline{\overline{\mathbf{u}}}_{\mathrm{L}}}_{PRODUCTION} \underbrace{-\frac{1}{Re_{\mathrm{ref}}}\alpha_{\mathrm{L}}\overline{\overline{\mathbb{T}'_{\mathrm{L}} : \nabla\mathbf{u}'_{\mathrm{L}}}}}_{DISSIPATION} + \underbrace{\overline{\overline{\left[\frac{1}{Re_{\mathrm{ref}}}\mathbb{T}'_{\mathrm{L;in}} - p'_{\mathrm{L;in}}\mathbb{I}\right]\cdot\mathbf{u}'_{\mathrm{L;in}}\cdot\mathbf{n}_{\mathrm{L;in}}a_{\mathrm{in}}}}}_{INTERFACIAL\ TERM}$$

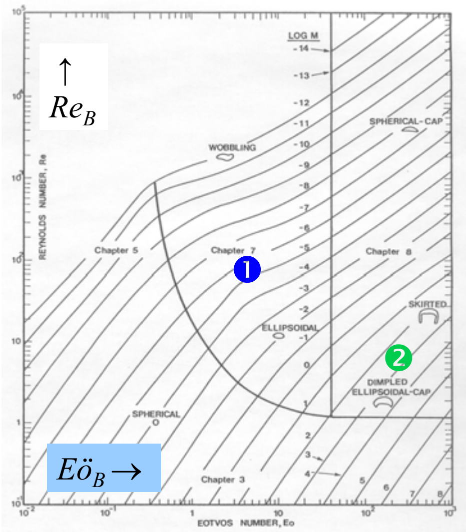
- Terms on right hand side of equation must be modeled
- Experimental data for individual closure terms is missing
  $\Rightarrow$ **<u>Direct numerical simulation</u> of bubble swarm flow**

* Kataoka & Serizawa, Int. J. Multiphase Flow, 15 (1989) 843

In-house code TURBIT-VOF

- Volume-of fluid method for interface tracking
  - Interface is locally approximated by plane (PLIC method)
- Governing equations for two incompressible fluids
  - Single field momentum equation with surface tension term
  - Zero divergence condition for center-of-mass velocity
  - Advection equation for liquid volumetric fraction *f*
- Solution strategy
  - Projection method resulting in pressure Poisson equation
  - Explicit third order Runge-Kutta time integration scheme
- Discretization in space
  - Finite volume formulation for regular staggered grid
  - Second order central difference approximations

Code verification



❶ Medium Morton number ellipsoidal bubble

DNS for $\Gamma_\mu = \mu_d / \mu_c = 1$,

$M = 3.09 \cdot 10^{-6}$, $Eö_B = 3.06$

$\Gamma_\rho = \rho_d / \rho_c = 0.5;\ 0.2;\ 0.1;\ 0.02$

❷ High Morton number ellipsoidal cap bubble

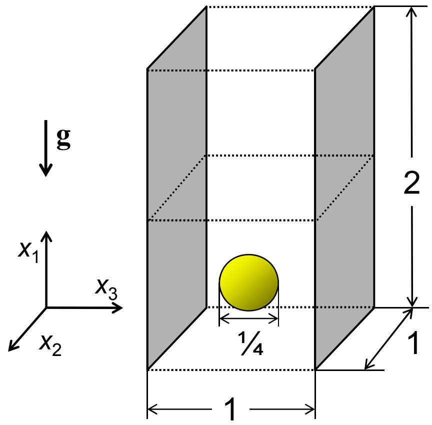
DNS for $\Gamma_\mu = \mu_d / \mu_c = 1$,

$M = 266$, $Eö_B = 243$

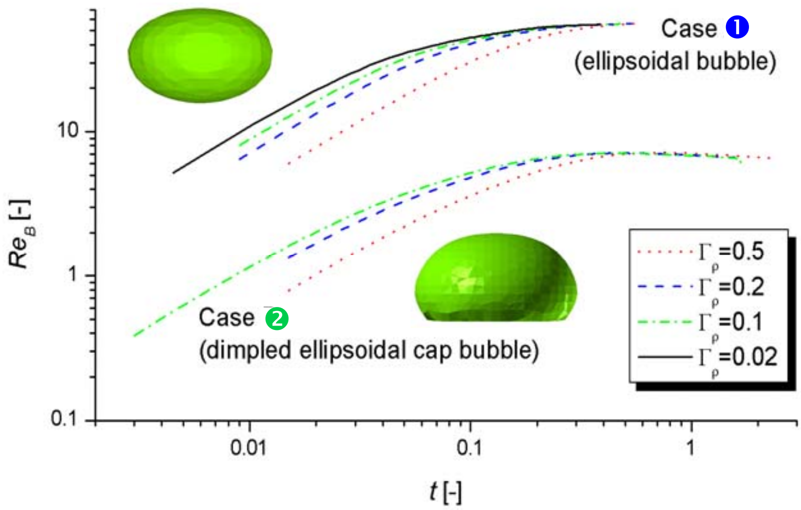
$\Gamma_\rho = \rho_d / \rho_c = 0.5;\ 0.2;\ 0.1$

Computational set-up

- Domain: 2 x 1 x 1
- Grid: 128 x 64 x 64
- Bubble diameter: 0.25
  (= 16 mesh cells)
- Gas holdup: ≈ 0.4%
- Boundary conditions
  - walls at x_3 = 0 and x_3 = 1
  - periodic in x_1 and x_2
- Liquid & gas initially at rest



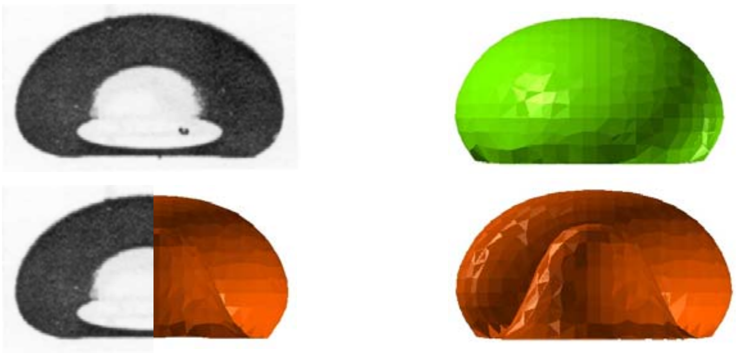
Bubble Reynolds number over time



Comparison of bubble shape (case ❷)

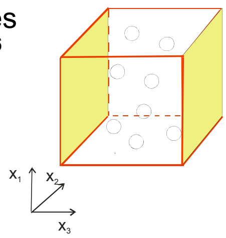
Experiment Bhaga & Weber
($\Gamma_\rho \approx 0.0008$; $\Gamma_\mu \approx 10^{-5}$)

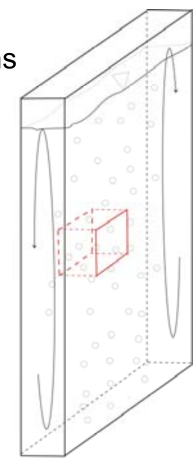
TURBIT-VOF
($\Gamma_\rho = 0.5$; $\Gamma_\mu = 1$)



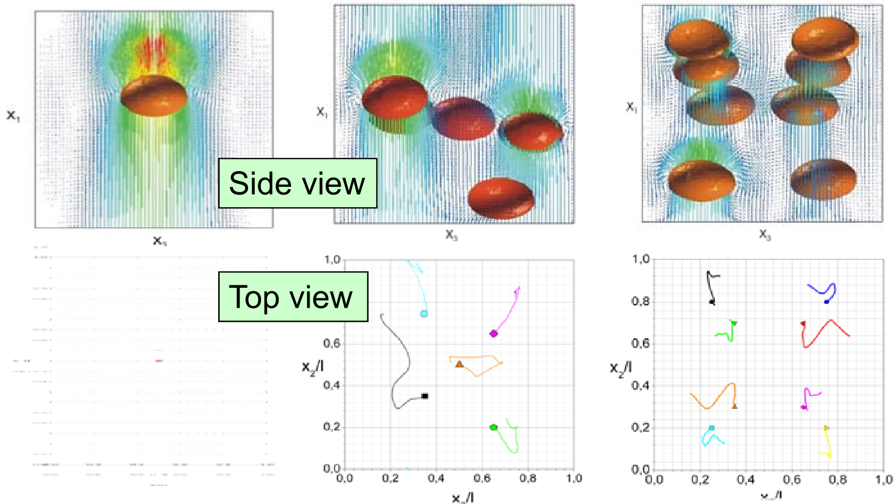
Bubble swarm simulations

- Simulations mimic part of a flat bubble column
  - two no-slip walls and periodic boundary conditions in vertical and lateral direction
- Parameters fixed: $Eö_B$ =3.06 ; $\Gamma_\rho = 0.5$; $\Gamma_\mu = 1$
- Parameters varied:
  - Gas content: 1, 5 and 8 bubbles (α = 0.8 – 6.4%) for $M = 3\times10^{-6}$
  - Morton number: $M = 3\times10^{-6}$, 3×10^{-4} , 3×10^{-2} (8 bubbles)
- Cubic computational domain
  - Grid: 64 × 64 × 64 cells
  - Resolution study: 100 × 100 × 100 cells



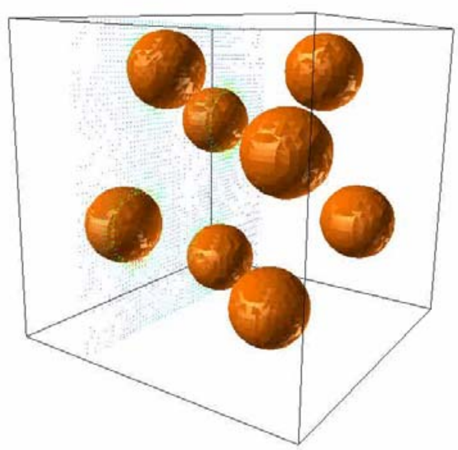


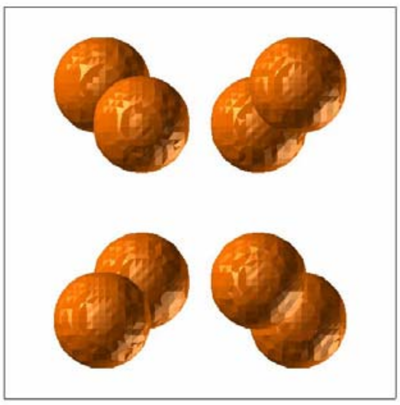
Bubble shape and path (case ❶)



1BM6: $M = 3\times10^{-6}$ | 5BM6: $M = 3\times10^{-6}$ | 8BM6: $M = 3\times10^{-6}$

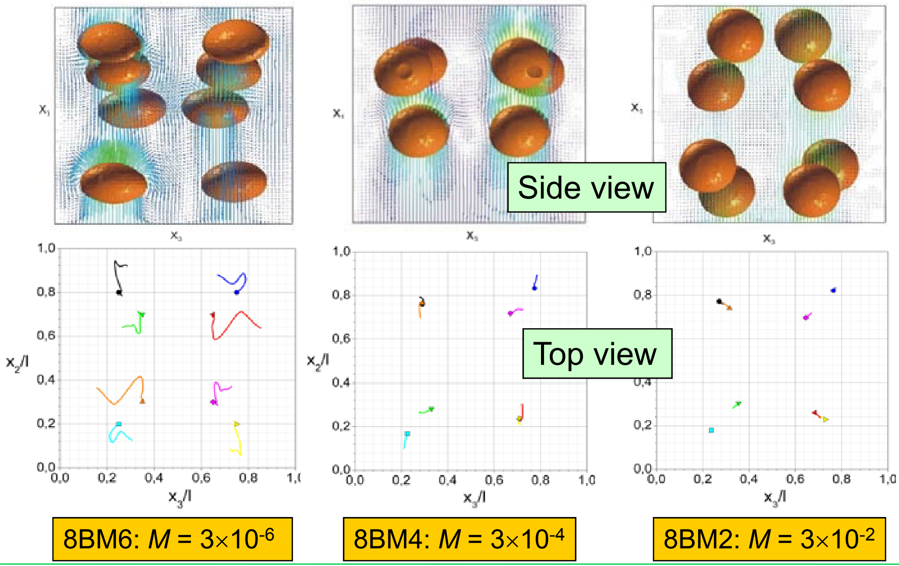
Visualization of bubble motion





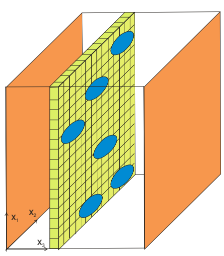
8BM6: $M = 3\times10^{-6}$

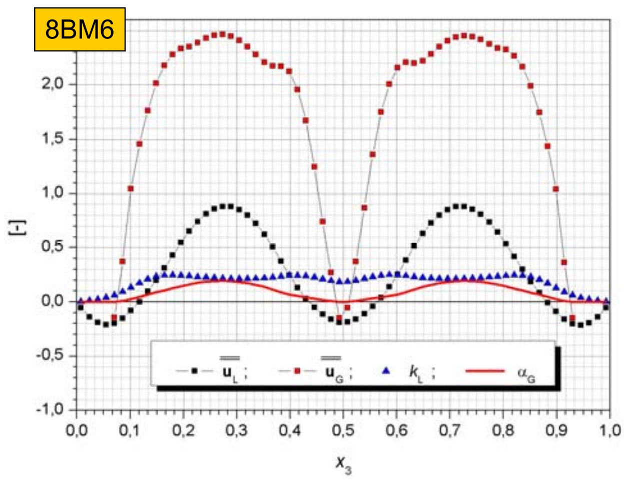
Bubble shape and path



Averaging of simulation results

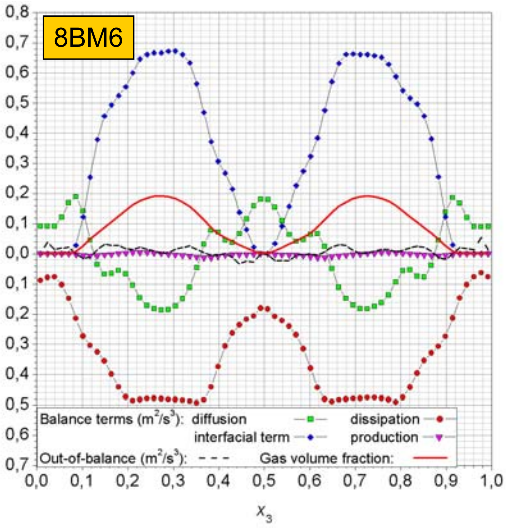
Averaging over planes parallel to the side walls



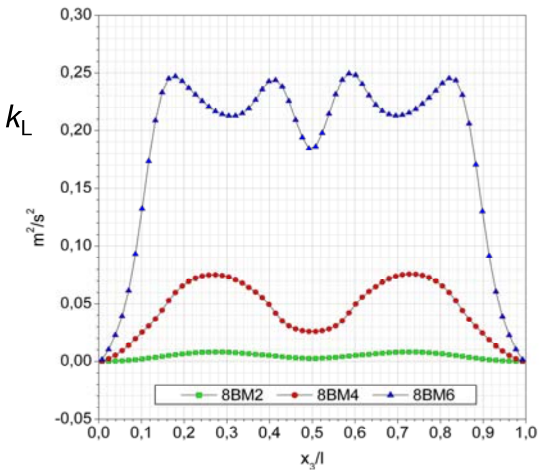


Terms in exact k_L-equation

$$\frac{\partial}{\partial t}(\alpha_L k_L) + \nabla\cdot\left(\alpha_L k_L \overline{\overline{\mathbf{u}_L}}\right) = \underbrace{\frac{1}{Re_{ref}}\nabla\cdot\left(\alpha_L \overline{\overline{\mathbb{T}'_L\cdot\mathbf{u}'_L}}\right) - \nabla\cdot\left[\alpha_L\left(\overline{\overline{p'_L\mathbf{u}'_L}} + \frac{1}{2}\overline{\overline{(\mathbf{u}'_L\cdot\mathbf{u}'_L)\mathbf{u}'_L}}\right)\right]}_{DIFFUSION} \underbrace{-\alpha_L \overline{\overline{\mathbf{u}'_L\mathbf{u}'_L}} : \nabla\overline{\overline{\mathbf{u}_L}}}_{PRODUCTION} \underbrace{-\frac{1}{Re_{ref}}\alpha_L\overline{\overline{\mathbb{T}'_L:\nabla\mathbf{u}'_L}}}_{DISSIPATION} + \underbrace{\overline{\overline{\left[\left(\frac{1}{Re_{ref}}\mathbb{T}'_{L;in} - p'_{L;in}\mathbb{I}\right)\cdot\mathbf{u}'_{L;in}\cdot\mathbf{n}_{L;in}a_{in}\right]}}}_{INTERFACIAL\ TERM}$$



Profiles of k_L for different Morton numbers

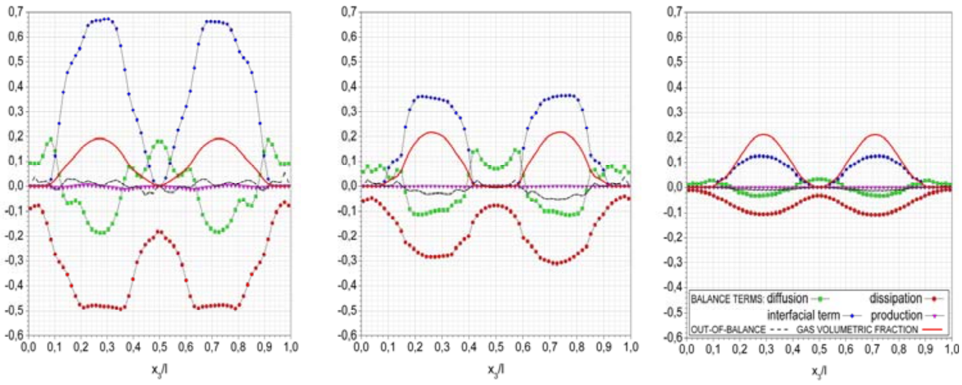


$Re_B \approx 60$

$Re_B \approx 10$

$Re_B \approx 1$

Budget of k_L for different Morton numbers



8BM6: $M = 3\times10^{-6}$ 8BM4: $M = 3\times10^{-4}$ 8BM2: $M = 3\times10^{-2}$

Models for production term

- Exact term: $\mathrm{Prod}(k_L) = -\alpha_L \overline{\overline{\mathbf{u}'_L \mathbf{u}'_L}} : \nabla \overline{\overline{\mathbf{u}_L}}$
- Common ansatz: $\mathrm{Prod}(k_L) \approx \alpha_L \nu_L^{\mathrm{eff}} \left[\nabla \overline{\overline{\mathbf{u}_L}} + \nabla \overline{\overline{\mathbf{u}_L}}^T \right] : \nabla \overline{\overline{\mathbf{u}_L}}$
- One-equation model:

$$\nu_L^{\mathrm{eff}} = \beta_1 l_{\mathrm{TP}} \sqrt{k_L} \quad \text{with} \quad \beta_1 = 0.56 \quad \text{and} \quad l_{\mathrm{TP}} = \alpha_G d_B / 3$$

- Two-equation model:

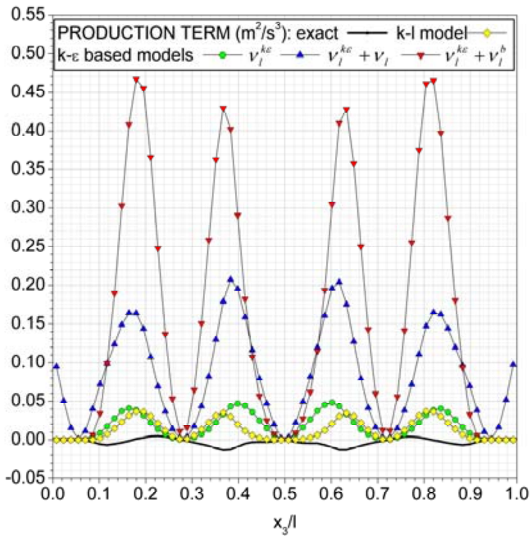
$$\nu_L^{\mathrm{eff}} = \nu_L^{k-\varepsilon} = C_\mu k_L^2 / \varepsilon_L$$

$$\nu_L^{\mathrm{eff}} = \nu_L^{k-\varepsilon} + \nu_L$$

$$\nu_L^{\mathrm{eff}} = \nu_L^{k-\varepsilon} + 0.6 \alpha_G d_B \left| \overline{\overline{\mathbf{u}}}_r \right| = \nu_L^{k-\varepsilon} + \nu_L^{\mathrm{B}}$$

Evaluation of production term models

8BM6



Models for diffusion term

- Exact term:

$$\mathrm{Diff}(k_{\mathrm{L}}) = \nu_{\mathrm{L}} \nabla \cdot \left(\alpha_{\mathrm{L}} \overline{\overline{\mathbb{T}_{\mathrm{L}}' \cdot \mathbf{u}_{\mathrm{L}}'}} \right) - \nabla \cdot \left[\alpha_{\mathrm{L}} \left(\overline{\overline{p_{\mathrm{L}}' \mathbf{u}_{\mathrm{L}}'}} + \frac{1}{2} \overline{\overline{(\mathbf{u}_{\mathrm{L}}' \cdot \mathbf{u}_{\mathrm{L}}') \mathbf{u}_{\mathrm{L}}'}} \right) \right]$$

- Common ansatz: $\mathrm{Diff}(k_{\mathrm{L}}) \approx \nabla \cdot \left(\alpha_{\mathrm{L}} \nu_{\mathrm{L}}^{\mathrm{Diff}} \nabla k_{\mathrm{L}} \right)$

- One-equation model:

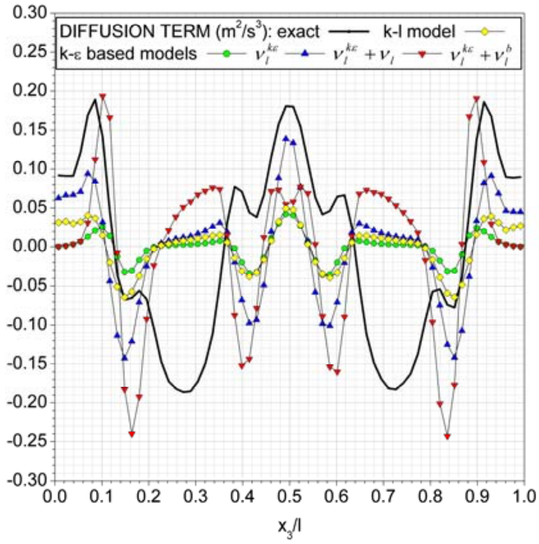
$$\nu_{\mathrm{L}}^{\mathrm{Diff}} = 0,5\nu_{\mathrm{L}} + \beta_2 l_{\mathrm{TP}} \sqrt{k_{\mathrm{L}}} \quad \text{with} \quad \beta_2 = 0,38 \quad \text{and} \quad l_{\mathrm{TP}} = \alpha_{\mathrm{G}} d_{\mathrm{B}} / 3$$

- Two-equation model:

$$\nu_{\mathrm{L}}^{\mathrm{Diff}} = \nu_{\mathrm{L}}^{k-\varepsilon} = C_\mu k_{\mathrm{L}}^2 / \varepsilon_{\mathrm{L}}$$
$$\nu_{\mathrm{L}}^{\mathrm{Diff}} = \nu_{\mathrm{L}}^{k-\varepsilon} + \nu_{\mathrm{L}}$$
$$\nu_{\mathrm{L}}^{\mathrm{Diff}} = \nu_{\mathrm{L}}^{k-\varepsilon} + 0,6\alpha_{\mathrm{G}} d_{\mathrm{B}} \left| \overline{\overline{\mathbf{u}}}_r \right| = \nu_{\mathrm{L}}^{k-\varepsilon} + \nu_{\mathrm{L}}^{\mathrm{B}}$$

Evaluation of diffusion term models

8BM6



Models for interfacial term

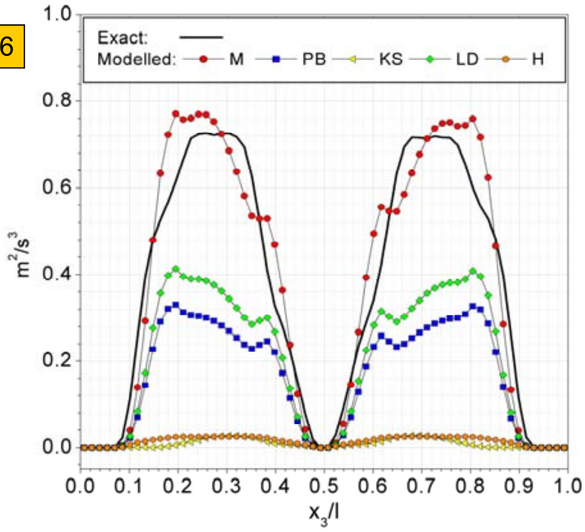
Exact term:

$$\mathrm{IFT}(k_{\mathrm{L}}) = \overline{\left[\frac{1}{Re_{\mathrm{ref}}}\mathbb{T}'_{\mathrm{L;in}} - p'_{\mathrm{L;in}}\mathbb{I}\right]\cdot \mathbf{u}'_{\mathrm{L;in}} \cdot \mathbf{n}_{\mathrm{L;in}} a_{\mathrm{in}}}$$

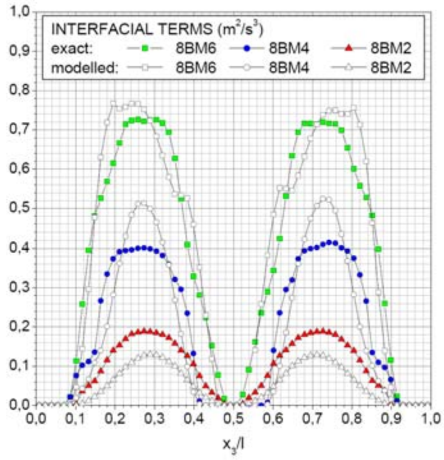
Reference	Work of drag force, W_{D}^{*}	Other contributions, W_{ND}^{*}
Kataoka & Serizawa (1997) Model 1, KS	$0.075 f_{\mathrm{w}}\left[\frac{3}{4}\alpha_{\mathrm{G}}\frac{C_{\mathrm{D}}}{d_{\mathrm{B}}^{*}}U_{\mathrm{T}}^{*3}\right]$	$-\alpha_{\mathrm{G}}\frac{k_{\mathrm{L}}^{*3/2}}{d_{\mathrm{B}}^{*}}$
Hill *et al.* (1995) Model 2, H	$\frac{3}{4}\frac{\alpha_{\mathrm{G}}C_{\mathrm{D}}}{d_{\mathrm{B}}^{*}}\left\|\overline{\mathbf{u}_{\mathrm{R}}^{*}}\right\|\left\{\frac{\mu_{\mathrm{L}}^{*}\overline{\overline{\mathbf{u}_{\mathrm{R}}^{*}}}\cdot\nabla^{*}\alpha_{\mathrm{G}}}{0.3\rho_{\mathrm{L}}^{*}\alpha_{\mathrm{L}}\alpha_{\mathrm{G}}}+2k_{\mathrm{L}}^{*}(C_{\mathrm{t}}-1)\right\}$	None
Lahey & Drew (2000) Model 3, LD	$\frac{1}{4}\alpha_{\mathrm{L}}\left(1+C_{\mathrm{D}}^{4/3}\right)\alpha_{\mathrm{G}}\frac{\left\|\overline{\mathbf{u}_{\mathrm{R}}^{*}}\right\|^{3}}{d_{\mathrm{B}}^{*}}$	None
Morel (1997) Model 4, M	$\frac{3}{4}\alpha_{\mathrm{G}}\frac{C_{\mathrm{D}}}{d_{\mathrm{B}}^{*}}\left\|\overline{\mathbf{u}_{\mathrm{R}}^{*}}\right\|^{3}$	$\frac{1+2\alpha_{\mathrm{G}}}{2\alpha_{\mathrm{L}}}\alpha_{\mathrm{G}}\left\{\frac{\mathrm{D}_{\mathrm{G}}\overline{\mathbf{u}_{\mathrm{G}}^{*}}}{\mathrm{D}t^{*}}-\frac{\mathrm{D}_{\mathrm{L}}\overline{\mathbf{u}_{\mathrm{L}}^{*}}}{\mathrm{D}t^{*}}\right\}\cdot\overline{\mathbf{u}_{\mathrm{R}}^{*}}$
Pfleger & Becker (2001) Model 5, PB	$1.44\alpha_{\mathrm{L}}\left[\frac{3}{4}\alpha_{\mathrm{G}}\frac{C_{\mathrm{D}}}{d_{\mathrm{B}}^{*}}\left\|\overline{\mathbf{u}_{\mathrm{R}}^{*}}\right\|^{3}\right]$	None

Evaluation of models for interfacial term

8BM6



Performance of Morel model for different values of the Morton number



$$IFT \approx \frac{3}{4} C_D \frac{\alpha_G \rho_L}{d_B} \left| \overline{\overline{\mathbf{u}}}_G - \overline{\overline{\mathbf{u}}}_L \right|^3 + C_{AM} \frac{1+2\alpha_G}{1-\alpha_G} \alpha_G \rho_L \left(\frac{D_G \overline{\overline{\mathbf{u}}}_G}{Dt} - \frac{D_L \overline{\overline{\mathbf{u}}}_L}{Dt} \right) \left(\overline{\overline{\mathbf{u}}}_G - \overline{\overline{\mathbf{u}}}_L \right)$$

The drag coefficient C_D is computed by the correlation of Tomiyama (1998):

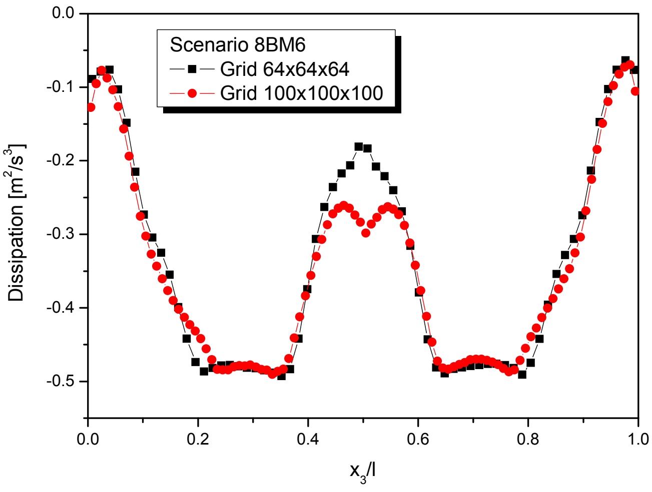
$$C_D = \max \left[\min \left\{ \frac{16}{Re_B} \left(1 + 0.15 Re_B^{0.687} \right), \frac{48}{Re_B} \right\}, \frac{8}{3} \frac{E\ddot{o}_B}{E\ddot{o}_B + 4} \right]$$

(Note that C_D formula in paper is not correct, Eq. (26))

Conclusions and outlook

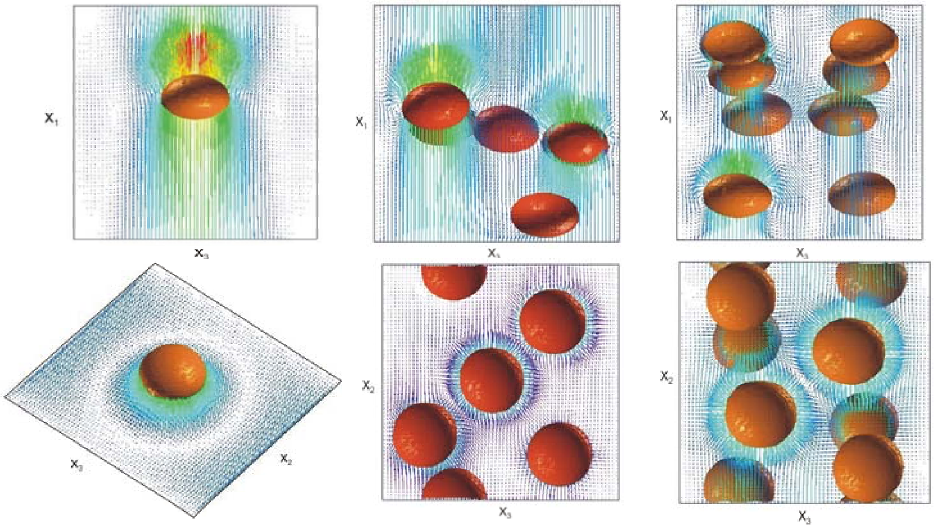
- Detailed analysis of transport eq. for liquid turbulence kinetic energy in bubble-driven flow for different Morton numbers
  - Production by shear stresses is negligible
  - Importance of interfacial term and diffusion term
- Evaluation of model assumptions
  - Production term and diffusion term: poor performance of standard single-phase type models
  - Interfacial term: Modeling as work of drag force together with Tomiyama correlation for C_D shows good performance
- Outlook
  - Development of improved models, implementation in CFD codes and recalculation of experiments for bubble columns

Grid refinement study for case 8BM6



Bubble-array flows with different number of bubbles

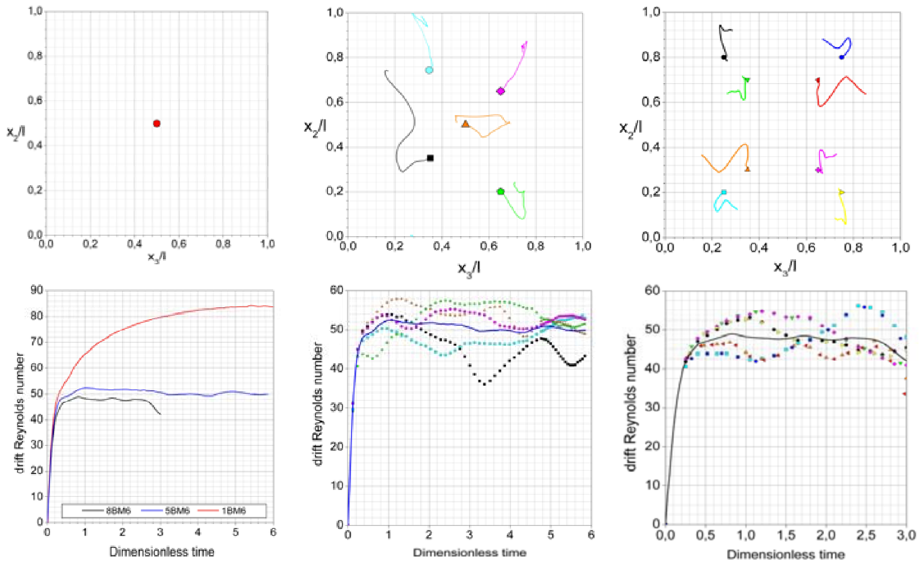
- bubble shape and bubble distribution -



Scenario 1BM6: κ = 1.555 | Scenario 5BM6: κ = 1.535 | Scenario 8BM6: κ = 1.526

Bubble-array flows with different number of bubbles

- lateral bubble movements and bubble rise velocities -



Turbulence kinetic energy equation for liquid phase (Kataoka & Serizawa, 1989)

$$k_{\mathrm{L}} \equiv \frac{1}{2}\overline{\overline{\mathbf{u}_{\mathrm{L}}' \cdot \mathbf{u}_{\mathrm{L}}'}} = \frac{1}{2}\frac{1}{U_{\mathrm{ref}}^{*\,2}}\overline{\overline{\mathbf{u}_{\mathrm{L}}^{*\prime} \cdot \mathbf{u}_{\mathrm{L}}^{*\prime}}}$$

$$\frac{\partial}{\partial t}(\alpha_{\mathrm{L}} k_{\mathrm{L}}) + \nabla\cdot\left(\alpha_{\mathrm{L}} k_{\mathrm{L}} \overline{\overline{\mathbf{u}_{\mathrm{L}}}}\right) = \underbrace{\frac{1}{Re_{\mathrm{ref}}}\nabla\cdot\left(\alpha_{\mathrm{L}}\overline{\overline{\mathbb{T}_{\mathrm{L}}' \cdot \mathbf{u}_{\mathrm{L}}'}}\right) - \nabla\cdot\left[\alpha_{\mathrm{L}}\left(\overline{\overline{p_{\mathrm{L}}'\mathbf{u}_{\mathrm{L}}'}} + \frac{1}{2}\overline{\overline{(\mathbf{u}_{\mathrm{L}}' \cdot \mathbf{u}_{\mathrm{L}}')\mathbf{u}_{\mathrm{L}}'}}\right)\right]}_{\mathit{DIFFUSION}}$$

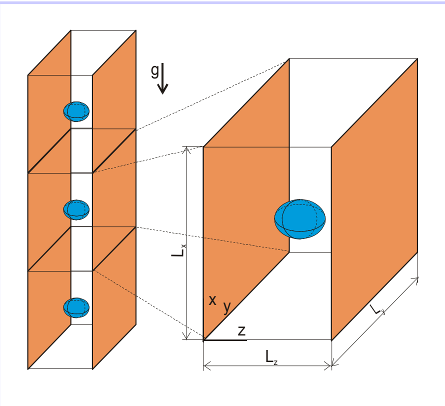
$$\underbrace{-\alpha_{\mathrm{L}}\overline{\overline{\mathbf{u}_{\mathrm{L}}'\mathbf{u}_{\mathrm{L}}'}} : \nabla\overline{\overline{\mathbf{u}_{\mathrm{L}}}}}_{\mathit{PRODUCTION}} \underbrace{- \frac{1}{Re_{\mathrm{ref}}}\alpha_{\mathrm{L}}\overline{\overline{\mathbb{T}_{\mathrm{L}}' : \nabla\mathbf{u}_{\mathrm{L}}'}}}_{\mathit{DISSIPATION}} + \underbrace{\overline{\left[\frac{1}{Re_{\mathrm{ref}}}\mathbb{T}_{\mathrm{L;in}}' - p_{\mathrm{L;in}}'\mathbb{I}\right]\cdot\mathbf{u}_{\mathrm{L;in}}' \cdot \mathbf{n}_{\mathrm{L;in}} a_{\mathrm{in}}}}_{\mathit{INTERFACIAL\ TERM}}$$

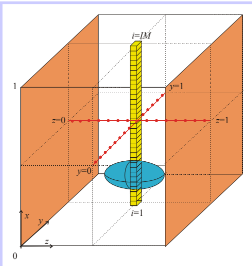
$$\mathbb{T}_{\mathrm{L}}' = \mu_{\mathrm{L}}\left[\nabla\mathbf{u}_{\mathrm{L}}' + (\nabla\mathbf{u}_{\mathrm{L}}')^{\mathrm{T}}\right]$$

$\overline{A_L}$ = averaging $\quad \overline{\overline{A_L}} = \overline{A_L X_L}/\overline{X_L} \quad \alpha_L = \overline{X_L}$

$A_L' = A_L - \overline{\overline{A_L}} \quad A_{Lin}' = A_{Lin} - \overline{\overline{A_L}}$

*k*L-equation for bubble train flow*





Line averaging:

$$\overline{A_L} = \overline{A_L}^{i}(j,k) = \frac{1}{IM}\sum_{i=1}^{IM} f_{i,j,k} A_{i,j,k}$$

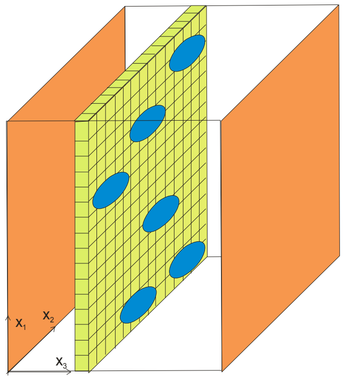
* see Ilić et al., *J. Nuclear Science & Technology* **41** (2004) 331-338

Averaging procedure

Plane averaging:

$$\overline{A_L} = \overline{A_L}^{i,j}(k) = \frac{1}{IM \cdot JM} \sum_{i=1}^{IM} \sum_{j=1}^{JM} f_{i,j,k} A_{i,j,k}$$

$$\overline{\overline{A_L}}(k) = \frac{\sum_{i=1}^{IM} \sum_{j=1}^{JM} f_{i,j,k} A_{i,j,k}}{\sum_{i=1}^{IM} \sum_{j=1}^{JM} f_{i,j,k}}$$



Bubble drag law of Tomiyama

– pure system

Schiller-Naumann for bubble (H-R correction)

$$C_D = \max\left[\min\left\{\frac{16}{Re_p}\left(1+0.15Re_p^{0.687}\right), \frac{48}{Re_p}\right\}, \frac{8}{3}\frac{Eö_B}{Eö_B+4}\right]$$

Potential flow around ridig sphere

Cap bubble

– slightly contaminated system

$$C_D = \max\left[\min\left\{\frac{24}{Re_p}\left(1+0.15Re_p^{0.687}\right), \frac{72}{Re_p}\right\}, \frac{8}{3}\frac{Eö_B}{Eö_B+4}\right]$$

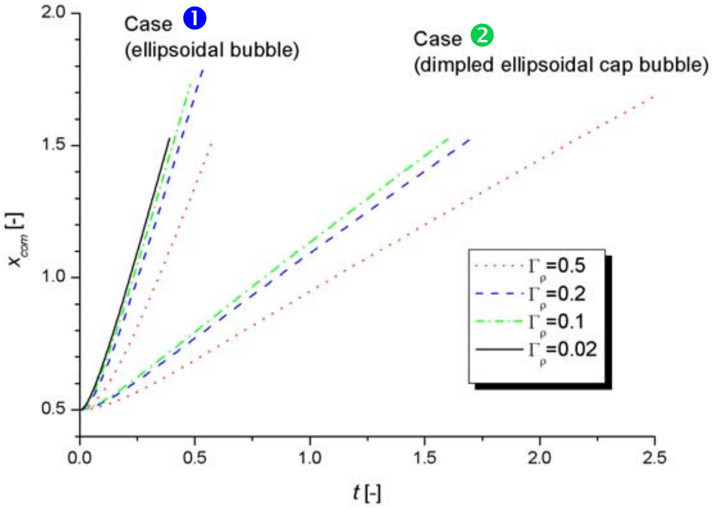
Schiller-Naumann for rigid sphere

Potential flow around bubble

– strongly contaminated system

$$C_D = \max\left[\frac{24}{Re_p}\left(1+0.15Re_p^{0.687}\right), \frac{8}{3}\frac{Eö_B}{Eö_B+4}\right]$$

Time history of vertical bubble position



Non-dimensional governing equations

$$\mathbf{x}=\frac{\mathbf{x}^*}{L^*_{ref}},\quad \mathbf{u}_k=\frac{\mathbf{u}^*_k}{U^*_{ref}},\quad t=\frac{t^*U^*_{ref}}{L^*_{ref}},\quad \rho_k=\frac{\rho^*_k}{\rho^*_c},\quad \mu_k=\frac{\mu^*_k}{\mu^*_c},\quad P_k=\frac{p^*_k+p^*_0-\rho^*_c\mathbf{g}^*\cdot\mathbf{x}^*}{\rho^*_c U^{*\,2}_{ref}}\quad (k\in c,d)$$

$$\frac{\partial}{\partial t}\rho_m\mathbf{u}_m+\nabla\cdot\rho_m\mathbf{u}_m\mathbf{u}_m=-\nabla P+\frac{1}{Re_{ref}}\nabla\cdot\left[\mu_m\left(\nabla\mathbf{u}_m+\nabla\mathbf{u}_m^{\mathrm{T}}\right)\right]-(1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*}+\frac{a_{int}\kappa\mathbf{n}_i}{We_{ref}}$$

$$\nabla\cdot\mathbf{u}_m=0 \qquad \frac{\partial f}{\partial t}+\nabla\cdot f\,\mathbf{u}_m=0 \qquad (f\equiv\alpha_c,\ 0\le f\le 1) \qquad \mathbf{u}_m\equiv\frac{1}{U^*_{ref}}\frac{f\rho^*_c\mathbf{u}^*_c+(1-f)\rho^*_d\mathbf{u}^*_d}{f\rho^*_c+(1-f)\rho^*_d}$$

$$\rho_m\equiv\frac{f\rho^*_c+(1-f)\rho^*_d}{\rho^*_c}=f+(1-f)\Gamma_\rho,\quad \mu_m\equiv\frac{f\mu^*_c+(1-f)\mu^*_d}{\mu^*_c}=f+(1-f)\Gamma_\mu$$

$$Re_{ref}\equiv\frac{\rho^*_c L^*_{ref}U^*_{ref}}{\mu^*_c},\quad E\ddot{o}_{ref}\equiv\frac{\left(\rho^*_c-\rho^*_d\right)g^*L^{*\,2}_{ref}}{\sigma^*},\quad We_{ref}\equiv\frac{\rho^*_c L^*_{ref}U^{*\,2}_{ref}}{\sigma^*}=\sqrt{\frac{M Re^4_{ref}}{E\ddot{o}_{ref}}}$$