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NEXT-TO-SOFT VIRTUAL RESUMMATION FOR QCD OBSERVABLES*

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We present a framework that resums threshold-enhanced logarithms, originating from soft-virtual and next-to-soft virtual (NSV) contributions in colour-singlet productions, to all orders in perturbation theory. The numerical impacts for these resummed predictions are discussed for the inclusive Drell–Yan di-lepton process up to next-to-next-to-leading logarithmic accuracy, restricting to only diagonal partonic channels.

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1. Introduction

Performing higher-order predictions in perturbative QCD involves complex Feynman loop integrals and many-body phase-space integrals. Due to the complexity in the computations, it is a general practise to look for alternative approaches by taking certain approximations. One good alternative is expanding the perturbative series around the production threshold, defined in terms of partonic scaling variable $z = \frac{Q^2}{\hat{s}} \approx 1$, where Q denotes the invariant mass of the final-state system produced in the partonic reactions with their centre-of-mass energy $\hat{s}$. Such an expansion also helps to understand the logarithmic structure in higher-order perturbative results. The leading term in the expansion, often called soft-virtual (SV) corrections, involves contributions from soft gluon emissions along with the Feynman loop corrections. At the production threshold, these soft gluon emissions result in large logarithms of the form of $\left(\frac{\ln^j(1-z)}{1-z}\right)_+, j \geq 0$, which needs to be resummed in order to get reliable predictions. The resummation framework for the SV logarithms is well-known [1–6] to the third order in logarithmic accuracy, thanks to the numerous efforts along this direction.

Recently, there have been numerous efforts to study the structure of next-to-leading term in the threshold expansion, with the form of $\ln^j(1-z), j \geq 0$ and their resummation to all orders in perturbation theory [7–14]. These contributions are called next-to-SV (NSV) logarithms. In [15, 16], we propose for the first time a framework to study the resummation of NSV logarithms beyond leading logarithmic (LL) accuracy for the colour singlet productions, restricting to only diagonal partonic channels. In the present article, we report on the numerical impacts of the NSV logarithms to third order in logarithmic accuracy for the Drell–Yan di-lepton process at the LHC.

2. Next-to-soft virtual framework

In the QCD-improved parton model, the differential invariant-mass distributions for a heavy colourless final states produced in hadron collisions take the form of a convoluted integral:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q}\left(q^2,\tau\right) = \sigma^{(0)} \int_{\tau}^{1} \frac{\mathrm{d}z}{z} \tilde{\Phi}_{ab}\left(\frac{\tau}{z},\mu_{\mathrm{F}}^2\right) \Delta_{ab}\left(q^2,\mu_{\mathrm{F}}^2,z\right) , \tag{2.1}$$

where $\sigma^{(0)}$ is the Born cross section. The partonic flux $\tilde{\Phi}_{ab}$ is defined to be

$$\tilde{\Phi}_{ab}\left(\frac{\tau}{z},\mu_{\mathrm{F}}^2\right) = \int_{\frac{\tau}{z}}^{1} \frac{\mathrm{d}y}{y} f_a\left(y,\mu_{\mathrm{F}}^2\right) f_b\left(\frac{\tau}{zy},\mu_{\mathrm{F}}^2\right) , \tag{2.2}$$

with the factorisation scale $\mu_{\rm F}$ and the incoming parton distribution function f_c. Also, $\tau = q^2/S$ is the hadronic scaling variable with hadronic center-of-mass energy S and $a, b = q, \overline{q}, g$ refer to incoming partonic states. The Δ_{ab} is the perturbatively calculable coefficient functions, which can be decomposed according to their singular behaviour

$$\Delta_{ab}(q^2, \mu_{\rm F}^2, z) = \delta_{ab}\Delta^{\rm SV}_{a\overline{a}}\left(q^2, \mu_{\rm F}^2, z\right) + \Delta^{\rm NSV}_{ab}\left(q^2, \mu_{\rm F}^2, z\right) + \Delta^{\rm reg}_{ab}\left(q^2, \mu_{\rm F}^2, z\right) . \tag{2.3}$$

Each of these terms is perturbatively expanded in terms of the renormalised strong coupling constant $a_{\rm s} = g_{\rm s}^2/16\pi^2$. For J = SV, NSV, reg, we have $\Delta^{(J)}_{ab}(q^2, \mu_{\rm F}^2, z) = \sum_{i=0}^{\infty} a_{\rm s}^i(\mu_{\rm R}^2)\Delta^{J,(i)}_{ab}(q^2, \mu_{\rm R}^2, \mu_{\rm F}^2, z)$, where $\mu_{\rm R}$ refers to renormalisation scale. The first term in (2.3) is the SV correction, which gets contributions from distributions of the form of $\left\{\delta(1-z), \left(\frac{\ln^k(1-z)}{1-z}\right)_+\right\}$ with $k \geq 0$. The second term comprises $\ln^k(1-z)$, $k \geq 0$,

$$\Delta^{{\rm NSV},(i)}_{ab}(z) = \sum_{k=0}^{2i-1} \Delta^{{\rm reg},(i)}_{ab,k} \ln^k(1-z) \,. \tag{2.4}$$

The rest are regular terms of the form of $(1-z)^k, k > 0$, when $z \to 1$.

The z-space coefficients in the above SV and NSV contributions involve convolutions, which are convenient to perform in the Mellin N-space. The soft limit $z \to 1$ in z-space corresponds to the large N limit in the Mellin space. These large logarithms with $a_{\rm s}$ produce $\mathcal{O}(1)$ terms at every order in $a_{\rm s}$, spoiling the truncation of perturbative series. Performing resummation resolves this by reorganising the series in terms of $\omega = 2a_{\rm s}(\mu_{\rm R}^2)\beta_0 \ln N$ at every order. The well-known formula for the SV resummation reads [1, 2]:

$$\lim_{N\to\infty} \ln \Delta^{\rm SV}_{c\overline{c},N} = \ln \tilde{g}^c_0(a_{\rm s}\left(\mu_{\rm R}^2\right)) + \ln N g^c_1(\omega) + \sum_{i=0}^{\infty} a_{\rm s}^i\left(\mu_{\rm R}^2\right) g^c_{i+2}(\omega) \,, \tag{2.5}$$

where $\Delta_{c\overline{c},N} = \int_0^1 {\rm d}z z^{N-1}\Delta_{c\overline{c}}(z)$. In (2.5) the resum coefficients $g^c_i(\omega)$ are universal and $\tilde{g}^c_0(a_{\rm s}(\mu_{\rm R}^2))$ collect N independent terms. Inclusion of successive terms in the expansion predicts the leading-logarithms (LL), next-to-LL (NLL), next-to-NLL (NNLL), and so on to all orders in $a_{\rm s}$. Including these higher logarithmic corrections improves the fixed-order results.

Following the formalism described in [15–18], we systematically resum NSV logarithms for the inclusive Drell–Yan di-lepton process, restricting to only the diagonal channels. In addition to threshold $\log N$, we include the $\mathcal{O}(1/N)$ terms in the large-N limit, in order to resum SV and NSV logarithms. Similar to the SV case in (2.5), the NSV resum formula reads

$$\lim_{N\to\infty} \ln \Delta^{\mathrm{NSV}}_{c\bar{c},N} = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i\left(\mu_R^2\right)\left(\bar{g}^c_{i+1}(\omega) + \sum_{k=0}^{i} h^c_{ik}(\omega)\ \ln^k N\right), \tag{2.6}$$

with NSV resum coefficients $\bar{g}^q_i(\omega)$ and $h^q_{ik}(\omega)$. These coefficients for the Drell–Yan process to NNLL are presented in the appendices of [17]. In order to avoid double counting threshold logarithms, we finally match the resummed results in the N-space to the fixed order corrections

$$\begin{aligned}\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} &= \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)}\sum_{ab}\int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}N}{2\pi i}(\tau)^{-N}\delta_{a\bar{b}} f_{a,N}\left(\mu_F^2\right) f_{b,N}\left(\mu_F^2\right)\\ &\quad\times\left(\Delta_{c\bar{c},N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{c\bar{c},N}\Big|_{\mathrm{tr\ N^nLO}}\right),\end{aligned} \tag{2.7}$$

where $\sigma_N^{\mathrm{N^nLO}}$ is the Mellin moment of $\mathrm{d}\sigma/\mathrm{d}Q$ to the n^{th} order in a_s. Also, $\overline{\mathrm{N^nLL}}$ denotes the SV+NSV resummation, while $\mathrm{N^nLL}$ refers to the resummation of only SV logarithms.

3. Phenomenology

The numerical setup we use for our study is detailed in [17]. In brief, we choose the center-of-mass energy of 13 TeV at LHC with MMHT2014 parton densities, the a_s is evolved to μ_R in $\overline{\mathrm{MS}}$-scheme and the electroweak parameters are chose to be: Z-boson mass = 91.1876 GeV and width = 2.4952 GeV, $\sin^2\theta_W = 0.22343$ and the fine structure constant $\alpha = 1/128$.

We begin with comparing fixed order corrections to the NSV resummed predictions, using the "K-factor" defined by $K(Q) = \frac{\mathrm{d}\sigma}{\mathrm{d}Q}\Big/\frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}Q}$ at $\mu_R = \mu_F = Q$. It is clear from Fig. 1 (left panel) that the resummed predictions improve the fixed order results. Quantitatively, for example at $Q = 2000$ GeV, the inclusion of $\overline{\mathrm{NLL}}$ enhances the NLO by 5.2% and $\overline{\mathrm{NNLL}}$ modifies NNLO by 1.2%. Further, the NLO + $\overline{\mathrm{NLL}}$ curve is closer to NNLO, indicating that the inclusion of higher logarithms mimics the entire second-order contributions.

To further see the effects of NSV logarithms in particular, we compare them against SV resummed results in the right panel of Fig. 1. In higher orders, both SV and NSV resum results are found to be closer, accounting for the better perturbative convergence upon including NSV effects.

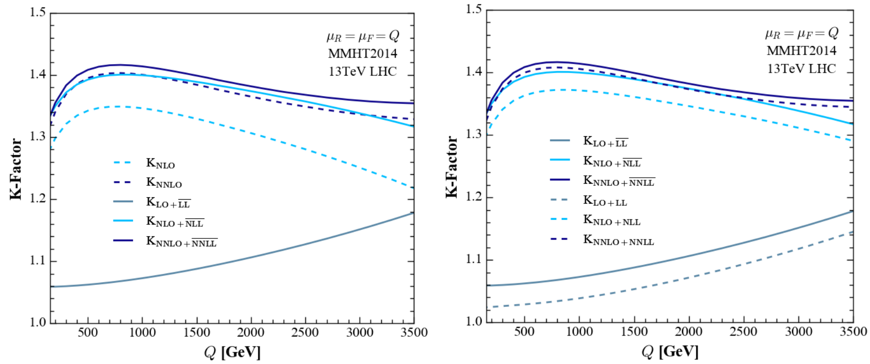

Fig. 1. K-factors (left) up to the NNLO+$\overline{\text{NNLL}}$ level at the central scale $Q = \mu_{\text{R}} = \mu_{\text{F}}$ and (right) for SV and NSV comparison.

To assess the impact of renormalisation and factorisation scales, we estimate the error using the canonical 7-point variation, with $\frac{1}{2} \leq \left(\frac{\mu_{\text{R}}}{Q}, \frac{\mu_{\text{F}}}{Q}\right) \leq 2$, excluding the extreme points (0.5,2) and (2,0.5). This is depicted in Fig. 2, where the resummed results are found to be not much improved. The reason could be due to the lack of an off-diagonal counter part, which will be evident in subsequent analysis.

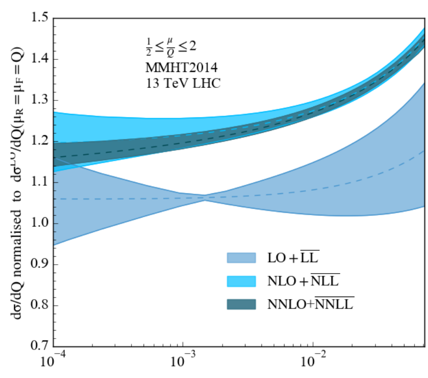

Fig. 2. 7-point scale variation of the resummed results for the central scale choice $(\mu_{\text{R}}, \mu_{\text{F}}) = (1, 1)Q$ for 13 TeV LHC.

In order to have a better understanding, we study the μ_{F} and μ_{R} scale variations separately as a function of τ in Fig. 3. The error band due to the μ_{F} variation has close resemblance to those of 7-point scales, suggesting that the uncertainty is largely due to μ_{F} variations. This is sensible, since the μ_{F} scales compensate between different partonic channels, which is not possible in this case due to the lack of off-diagonal resummed results. This is further emphasised by the $\mu_{\rm R}$-variation plot, where the partonic channels do not mix. We see the predictions are less sensitive to the $\mu_{\rm R}$ scale as expected. This essentially hints towards the importance of off-diagonal resummation, which requires further study.

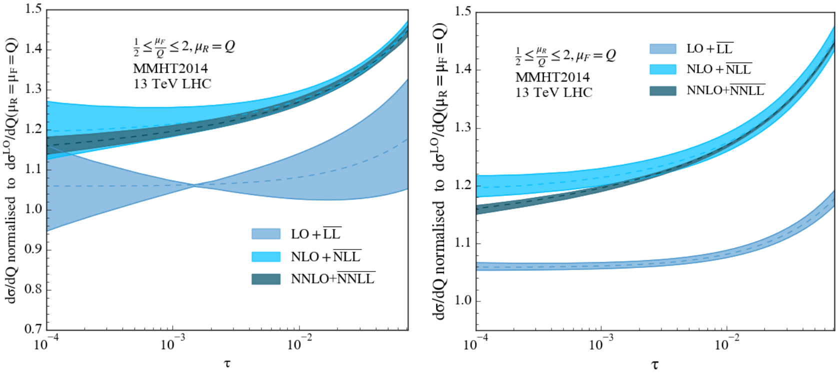

Fig. 3. $\mu_{\rm F}$ scale variation (left) of the resummed results with $\mu_{\rm R}$ held fixed and $\mu_{\rm R}$ scale variation (right) with $\mu_{\rm F}$ held fixed at Q for 13 TeV LHC.

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