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How Long is a Coast?

A Seemingly Trivial Question Leads to Fascinating Results

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Illustrated by Victoria Fisher & Emma Larson

Say on one lazy summer day, you pull up Google Earth and decided to measure the coastline of the United States. After multiple measurements, you come up with a number close to 45,000 km. How close were you compared to official figures? To your surprise, you're nowhere close. The World Factbook puts the number at 19,924 km. Skeptically, you search for another source and fall into another surprise. The World Resources Institute estimates the U.S. coastline to be nearly seven times as much, at 133,312 km. What is the reason for this discrepancy? It certainly couldn't be a rounding error. Soon enough, you see why. The Alexander Archipelago in southeast Alaska reveals kilometers upon kilometers of mountains, like the jagged edges of a piece of foil. The Puget Sound near Seattle forces itself into Washington State, engraving friezes into the rock. With these intricate details, it is absolutely crucial to take into account the length of the ruler you use to measure with.

The conflict of ruler size with coastline length is nothing new. In 1951, Lewis Fry Richardson, a mathematician investigating the causes of war, believed that factors leading to disputes could be modelled mathematically. As a result,

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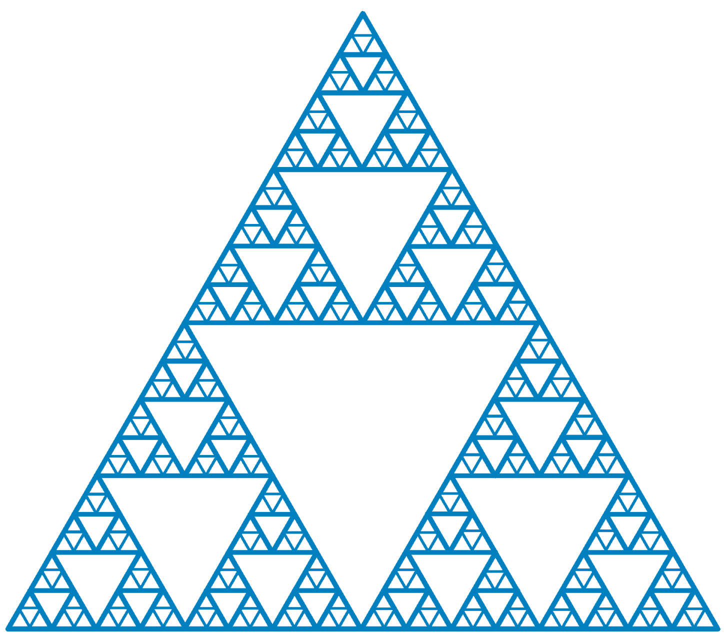
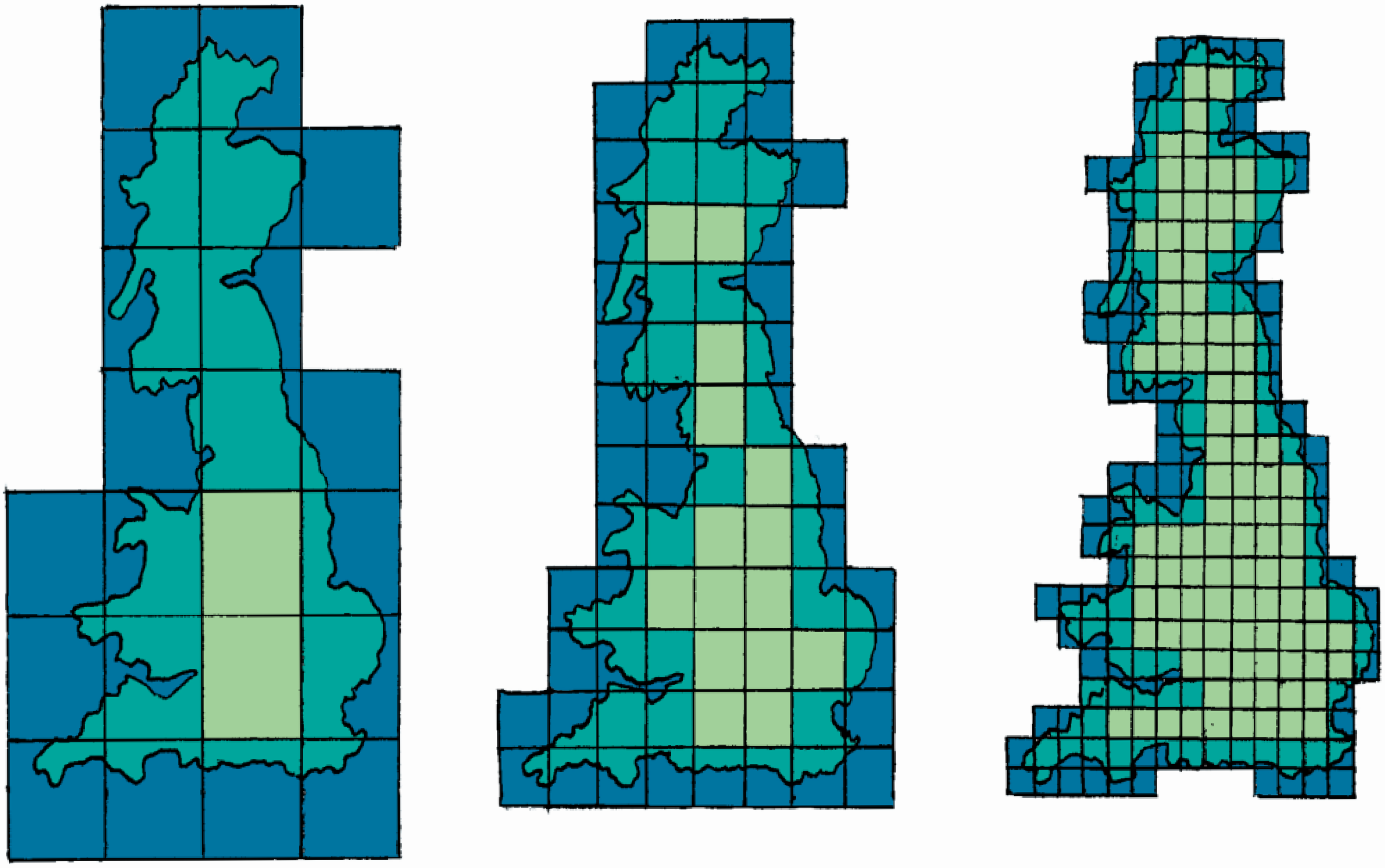
the chances of war could be predicted based on quantitative factors, much like the weather. One question he explored is how much more likely neighboring countries are to end up fighting one another, and it was here he engaged in a fascinating digression. It began with strange numbers: for the length of the Spain-Portugal border, Spain gave a figure of 987 km, while Portugal gave a figure of 1,214 km. To measure the length himself, Richardson studied from maps of various scales and counted off the number of "rulers" (of a fixed length) it took to cover the border. He discovered that as the size of the ruler decreases, the resulting border length increases like we have discussed before, because tinier rulers account for extra nooks and crannies hiding within the border. So, we should expect that as the ruler size gets smaller, the resulting border length should approach a specific value, right? Not quite. For reasons we shall see in a bit, as the ruler size decreases to infinitesimal lengths, the total length increases without bound. In other words, it's meaningless to ascribe an exact length to a border or coastline.

At first, you must think that there must be some kind of mistake. After all, the entire branch of calculus was developed on the idea that anything could be represented as a sum of infinitesimal building blocks.There must be a way to rectify the situation, literally and figuratively. Answers came nearly twenty years later, when Benoit B. Mandelbrot happened to stumble upon Richardson's results. In his seminal paper, "How Long is the Coastline of Britain? Statistical Self-Similarity and Fractional Dimension,", Mandelbrot sought to find a value that existed independently of the ruler size. He began by introducing the concept of a fractal, a shape with an infinite perimeter but a finite area. He proposed that the degree of roughness could be described by a number D, which possesses properties similar to a dimension. In our intuitive perspective of a dimension, we usually say that one-dimensional entities are lines, two-dimensional objects are squares, and three-dimensional objects are cubes. However, unlike the idea of whole number dimensions with which we usually associate objects, Mandelbrot said that the dimension can be fractional. To understand how objects could possess non-integer dimensions, we must define what a dimension means in fractal geometry.





Take a line, for example, characterized by only one "spatial descriptor" (corresponding to a dimension of one): length. Let's scale it by a factor of one-half. Notice how two of these scaled lines can fit inside the original line, a property known as self-similarity. Next, let's look at a square, characterized by two spatial descriptors, and also scale it by one-half. We are now able to fit four scaled pieces into the original, i.e. we have four self-similar pieces. When generalized to an object with D many spatial descriptors, the number of self-similar pieces will be

$$N = \frac{1}{x^D}$$

where x is the scaling factor. Now that we have a general definition, we can then attain a value for the dimension of a fractal. Take, for example, the Sierpinski triangle, a nested set of equilateral triangles in the shape of another equilateral triangle.a

Using the general rule we established, we end up with a number of about 1.58, a little more than halfway between a line and a plane. It seems strange to think of a shape that way, but it's a good example to start thinking about what a dimension really means.

Now that we know how to calculate the dimension of a self-similar object, you must be thinking: how are we supposed to find the dimension of something as complicated as a coastline? Fortunately, there is a way, and the method is simpler than you may think. To put it simply, we can place a country like Great Britain over a grid and count the number of squares the coastline touches, scaling the squares on the grid, and counting the boxes again. A square on the grid possesses a side length 1/x, where x is the factor we scale the original object with. After that, we can scale the squares down some more and recount the touched boxes. The rate of change between the touched boxes and side lengths is the dimension. Richardson and Mandelbrot estimated the dimension of Great Britain to be around 1.25.

The dimension here also represents the degree of roughness, or how much an object varies in a given space. If you look at Norway, a country riddled with fjords and islands, it's no surprise knowing that it possesses the greatest dimension of all, at 1.52. Though this number serves no apparent practical purpose, it eliminates the nonsensical result of infinite coastline length. This makes it possible to reasonably compare the nature of coastlines independent of ruler size.

There is no perfectly accurate method to measure the true length of a coastline. In fact, it's meaningless to even embark on such a task. However, one can hardly call the attempts to do so a failure. Mandelbrot's invention of fractal geometry transformed this apparent impossibility into a powerful mathematical concept, and its applications arise across disciplines, anywhere from chaos theory to seismology. The creation of such a branch of mathematics refines our intricate language of the universe, a place inhabited by uncertainty and complication, and brings us a little closer to understanding reality. • • •