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Diverse mathematical knowledge among indigenous Amazonians

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We investigate number and arithmetic learning among a Bolivian indigenous people, the Tsimane', for whom formal schooling is comparatively recent in history and variable in both extent and consistency. We first present a large-scale meta-analysis on child number development involving over 800 Tsimane' children. The results emphasize the impact of formal schooling: Children are only found to be full counters when they have attended school, suggesting the importance of cultural support for early mathematics. We then test especially remote Tsimane' communities and document the development of specialized arithmetical knowledge in the absence of direct formal education. Specifically, we describe individuals who succeed on arithmetic problems involving the number five—which has a distinct role in the local economy—even though they do not succeed on some lower numbers. Some of these participants can perform multiplication with fives at greater accuracy than addition by one. These results highlight the importance of cultural factors in early mathematics and suggest that psychological theories of number where quantities are derived from lower numbers via repeated addition (e.g., a successor function) are unlikely to explain the diversity of human mathematical ability.

mathematical cognition | education | arithmetic development

Significance

This work contributes to debates around the nature of human cognition and specifically the origins of mathematics in humans. In studying the correlates of numerical ability among indigenous Bolivian people, we find that cultural factors play a strong role—particularly schooling and economic activity. We also find evidence of remote adults without formal schooling who develop specialized arithmetical competence around the number 5 (due to local economic demands), even when they may not do arithmetic effectively with smaller numbers. Psychological theorizing about the cognitive nature of number and the innate components that support it should be tuned to accommodate the existence of such diverse trajectories.

It has been tempting for many developmental theorists to tie accounts of children's number learning to formalizations of number in mathematical logic. For example, the Peano axioms (1) define the natural numbers in logic through a simple axiomatic system where 0 is a number and every number x has a distinct successor $S(x)$. This successor function S inductively defines numbers in terms of the preceding number, with "one" defined as $S(0)$, "two" defined as $S(S(0))$, "three" defined as $S(S(S(0)))$, etc. Other axiomatizations of number rely on different foundations—for instance, in von Neumann's ordinals based on sets (2) or Church encoding of numbers into lambda calculus (3)—but still use analogous underlying logic of inductively defining each number in terms of the preceding. This recursive unfolding of natural numbers is compelling in mathematics because it allows an infinite system to be defined using a small collection of built-in rules.

These formalizations have motivated analogous theories in cognitive accounts of number. For example, it has been proposed that people are born endowed with "a little piece of algebra" equivalent to Peano's $S(x)$, which defines psychological representations of natural numbers starting from an innate concept of "one" (4, 5). Such accounts match theories where the principles of counting and number are innate and guide acquisition (6, 7). Even for accounts in which number is constructed, higher numbers are often defined in relation to what is essentially repeated addition from "one" (through the induction of a successor function), perhaps drawing on program learning (8) or an analogy or structural mapping that defines the higher numbers after the small numerosities (i.e., 1 to 4) have been memorized (9–11). Thus, diverse theories of numerical development share the general idea from mathematical logic that large numbers are defined eventually by some form of repeated addition.

However, there are reasons to suspect that number learning may be substantially more complex. First, some work has argued that children do not know principles concerning counting and cardinality until after learning counting itself (9, 12–19). Second, numerical cognition work has overwhelmingly focused on Western, Educated, Industrialized, Rich, and Democratic (WEIRD) populations (20) which have substantial cultural support for early number learning—for example, highly educated parents, educational toys and media, and even preschool education that emphasizes numeracy. Children's home math environment is an important predictor of children's mathematical knowledge (21–23), and there is considerable evidence that exposure to symbolic

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number, through both language and numerical activities, is crucial in the early development of numerical concepts (24–28). This makes WEIRD children, in some sense, the most difficult populations in which to study the foundations of number because of the influence of such cultural factors.

Although there is evidence of a nonsymbolic and approximate arithmetic ability likely based in innate numerical perception (7, 29–35), cross-cultural work has also argued for a key role of culture in counting and arithmetic development with symbolic number (35–42). In addition to some universals in how quantity perception works (34, 43), cross-cultural work has documented variation in the timing of acquisition (28, 44, 45), the role of language and the importance of the count list (42, 45, 46), and the coupling of arithmetic to a culture's social context (47–49) with novel forms developing outside of formal education contexts (50, 51).

Here, we focus on quantifying acquisition in a non-WEIRD culture with the goal of evaluating whether the psychological foundations of number are likely to rest on simple principles of logic. We first present a large meta-analysis of indigenous Bolivian children, the Tsimane', who have variable amounts of formal schooling and live a traditional (e.g., nonindustrialized) lifestyle. Our results provide strong evidence that Tsimane' children only induce a general counting procedure when they receive formal schooling. This fact is difficult to explain under theories in which the key parts of number are available innately. We then sought out individuals among the Tsimane' who were older and lived in a particularly remote area and who therefore had comparatively less cultural support for number learning. These people's performance on number tasks is especially informative regarding principles of number because their knowledge has been less affected by a pervasive culture of numeracy, specifically formal education and widespread monetary exchange. We show that among these individuals, number knowledge appears not to be based on a simple $S(x)$ logic: Participants often show better performance on some high numbers (multiples of five) compared to low numbers. For example, some people are better at adding $5 + 5$ than they are at adding $9 + 1$ or even $4 + 1$. This suggests that their route to success on $5 + 5$ is not through successive additions of 1 (or applications of $S(x)$). Instead, they succeed on fives likely due to directly constructing algorithms and a number system around the $+5$ operation itself, which is used in trade in that community as most items are priced in multiples of five Bolivianos ("pesos"). Theories concerning the psychological underpinning of number should seek mechanisms compatible with such diverse acquisition trajectories.

The Tsimane' Context

The studies we present here document in detail the numerical knowledge found among the Tsimane' of Bolivia. The Tsimane' are an indigenous farmer–foraging people living in a nonindustrialized society where both formal education and the value placed on numeracy can vary. Over a period of decades, Tsimane' society has been engaged in a process of market and national integration (52–56), and part of that complex history has involved outsiders exploiting both the Tsimane' people and their environmental resources (52, 57). More recently, there has been some protection afforded via integration with the national political system through the Gran Consejo Tsimane' (the representative political body based in the market town of San Borja) as well as through increasing rates of education and numeracy. As of today, many families in Tsimane' communities have become agriculturally active for commercial purposes, selling portions of their produce to visiting merchants.

The Tsimane' integration into broader Bolivian society has increased the need for numeracy and formal education, which has been the responsibility of the Bolivian government in Tsimane' territory since 2006. Schooling prior to this (from the 1950s to 2006) was conducted by missionaries, and the shift to government-administered education brought an increase in the number of non-Tsimane' teachers, instruction in Spanish, and exposure to a national curriculum including mathematics, language, and the natural and social sciences (58). Nearly every Tsimane' community now has a school, including the introduction of secondary schools in some areas (59), although teaching resources and school attendance vary considerably as remoteness increases. For these reasons, there is still an uneven distribution of numerical knowledge and ability even within communities, and this variability is especially pronounced in the more isolated areas. These contextual differences (e.g., from US society) have already generated research insights, for instance, that numeracy can affect children's notions of fairness (60), that words play a key representational role for number (42), and that understanding the logic of natural number and mastering counting itself may be partially separable conceptual achievements (18).

We have worked with the Tsimane' in a research capacity since 2012, in conjunction with Centro Boliviano de Investigacion y Desarrollo Socio Integral (CBIDSI), a Bolivian nonprofit organization that provides cultural expertise, consultation, and resources. Importantly, CBIDSI has enabled us to work with indigenous bilingual translators as well as negotiating formal approval and permission to enter Tsimane' lands through the governing political body. The following studies were approved by the Gran Consejo Tsimane' as well as the Committee for Protection of Human Subjects at the University of California, Berkeley.

Experimental Studies

Here, we report on four studies of Tsimane' number knowledge, using data collected across various field trips made between 2012 and 2019. In study 1, we present a meta-analysis of basic number knowledge at the population level, using data from 25 different Tsimane' communities to evaluate the importance of education in developing early numerical knowledge. For the remaining studies, we focused on a remote upriver community where there are a higher number of older adults who have had little to no exposure to the formal Bolivian schooling system. Working with this subpopulation of nonschooled older adults, we conducted two experimental studies of numerical cognition. In the first (study 2), we probed arithmetical abilities related to daily life, showing that the arithmetic knowledge that people develop is closely matched to what is practically relevant (i.e., via an arithmetic arising from local market practices where multiples of 5 are central). In study 3, we contrasted $+5$ with $+1$ logic for adults without formal education, finding evidence that arithmetical ability was not tied only to concrete market situations and also suggestion that the formalization of the successor function—learning via $+1$—likely does not accurately reflect number representation for some participants. Finally, to further highlight the possibility of differential development, we selected three participants and present brief cases studies in study 4—with one case suggesting a highly idiosyncratic (but context dependent) departure from the expectations of the successor function.

Table 1. Study 1: Meta-analysis data sources

Source	Give-N Method	N*	Ages
Piantadosi et al. (44)	Ordered	83†	3 to 12
Jara-Ettinger et al. (60)	Random	67‡	3 to 12
Jara-Ettinger et al. (18)	Random	63	4 to 11
Boni et al. (28)	Random	91§	4 to 11
Jara-Ettinger see ref. 61	Random	268	3 to 13
O'Shaughnessy et al. (61)	Ordered & random	240¶	4 to 13
O'Shaughnessy et al. (61)	Ordered & random	332	14 to 95

*$N = 1{,}144$ (812 children and 332 adults).

†We dropped 9 participants from the original dataset ($N = 92$) due to missing village data.

‡We dropped 3 participants from the original dataset ($N = 70$) who had too few Give-N trials for modeling purposes (i.e., those with 1 or 2 trials only).

§We dropped 9 participants from the original dataset ($N = 100$) due to unreliable education data.

¶We dropped 1 participant from the original dataset ($N = 241$) due to unreliable education data.

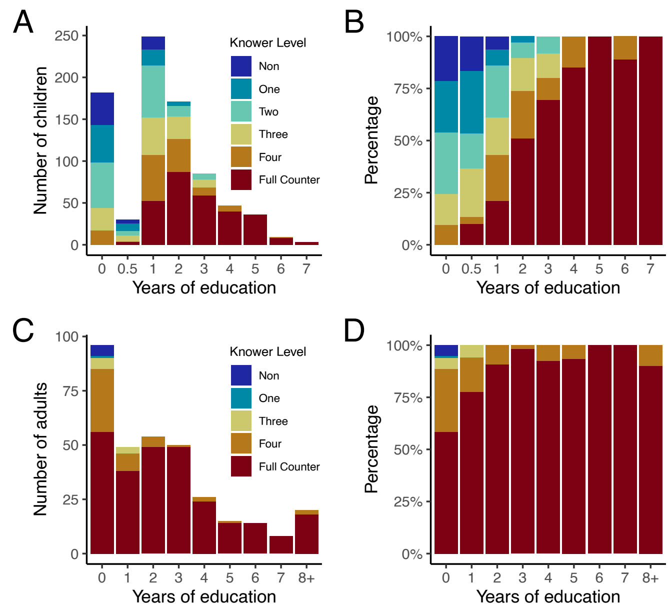

Fig. 1. (*A*) Stacked histogram of child knower levels per year of education (*N* = 812). (*B*) Percentages of child knower levels per year of education. (*C*) Stacked histogram of adult knower levels (*N* = 332) by year of education. (*D*) Percentages of adult knower levels per year of education.

Study 1: Tsimane' Number Knowledge Meta-Analysis. We pooled data from various studies (Table 1) to conduct a meta-analysis of the distribution of number knowledge across Tsimane' communities, with the aim of examining the demographic predictors of basic counting knowledge, as measured by success on a version of Wynn's Give-a-Number task (Give-N; 62). Give-N is a simple task which, in our version (*Materials and Methods*), required children to move some number of counters from one piece of paper to another (e.g., "Can you put four here?"). By asking for different numerosities, this task reveals which word meanings participants can reliably use. Work has shown that children in this task—including Tsimane' children (44)—tend to learn via a progression of stages called "knower levels" (10, 62–64), where they successively learn the numerical meanings of the first few words, in stages known as "subset-knowers." After the first few words, children have an insight into the meaning of higher words on their list. Children at this stage are called "full counters," although their knowledge is likely to be imperfect and takes longer to encompass all of the principles of cardinality (12, 18, 65, 66). In our meta-analysis studies, the highest number requested in Give-N sessions was 8 (*Materials and Methods*) largely due to constraints in fieldwork and also the fact that children classified as full counters will generally give correct responses from 1 to 8.

We analyzed Give-N results for our large sample ($N = 1{,}144$) of Tsimane' children ($n = 812$) and adults ($n = 332$), by classifying knower levels according to the Bayesian model described in Lee and Sarnecka (67, 68) (*Materials and Methods*). Below we provide results which describe the relationship between basic number knowledge (as captured by Give-N) and the relevant demographic variables of education, age, gender, and community remoteness. It is important to keep in mind that these demographic variables are imperfect. In particular, years of education and age can be based on participants' estimates, and community remoteness is a simplistic measure of market engagement. In addition, although reported ages in the child sample ranged from 3 to 14 y, 70.4% of the ages were from 5 to 8 y, primarily as a consequence of convenience sampling.

Results. Fig. 1*A* shows the number of children at each knower level, grouped by years of education, and Fig. 1*B* shows the same data expressed as percentages. Most children reported 0 to 2 y of education (632 children; 78%), with some reporting as high as 7. Strikingly, among the 182 children (22%) who had zero years of education (and a median age of 5 y; $SD = 1$; range: 3 to 8), none were classified as full counters. This gives a 99% binomial CI on the proportion of Tsimane' children who are full counters without schooling as $[0, 0.039]$. Thus, children in this population do not appear to learn productive counting without some level of formal education. However, children without formal education did transition through the subset-knower stages (from 0 to 4), with the majority classified as full counters by around 2 to 3 y of education.

Because age and education are correlated, we also compared the trends by education across children's age groupings in *SI Appendix*, Figs. S1 and S2, suggesting that education plays a role across all age brackets. We expand upon this analysis in *SI Appendix*, Figs. S3 and S4, respectively, showing children's overall average knower levels by education and the predicted knower levels from a linear regression taking both age and education into account (and see also the regression analysis controlling for age further ahead in Table 2). Given the importance of the count list in numerical development (28), we further analyzed the relationship between the highest verbal count and full counter status, using a subset of the data where this information was available ($n = 194$ children; $n = 329$ adults). *SI Appendix*, Table S1 suggests that experience with the count list may be one of the primary factors mediating the relationship between education and early numerical knowledge.

Results in the adult sample were markedly different. Fig. 1 *C* and *D* show the same histogram and percentages for participants aged 14 y and older. More than half of the 96 adults with no formal education were full counters (58%), with only a small percentage of nonknowers (5%), and the rest being subset-knowers. Full counter prevalence increased with a single year of education, and from 2 to 13 y of education, most adults were full counters, with the rest being four-knowers. The ages for adults with no formal education ($M = 55$ y; $SD = 19$; $n = 96$) were significantly higher than the rest of the adult sample ($M = 34$; $SD = 13$; $n = 236$) by t test: $t(133) = 9.73$, $P < 0.001$, Cohen's $d = 1.37$, 95% CI for d [1.11, 1.63]. The higher age for adults without formal education is consistent with the history of schooling in Tsimane' territories, which was not connected to the national education system until 2006 (58).

Table 2. Study 1: Logistic regression for full counting

	Full Counter	
	Children Estimate (*SE*)	Adults Estimate (*SE*)
Education (years)	0.95***	0.32**
	(0.12)	(0.12)
Gender (male)	0.76***	1.86***
	(0.20)	(0.45)
Age (years)	0.43***	−0.03**
	(0.08)	(0.01)
Distance from San Borja	−0.01	−0.03*
	(0.01)	(0.01)
Intercept	−5.68***	2.41***
	(0.56)	(0.65)
Observations	812	332
Log Likelihood	−323.93	−124.63
Akaike Inf. Crit.	657.85	259.25

Note: $^{*}P < 0.05$; $^{**}P < 0.01$; $^{***}P < 0.001$.

Given that some adults became full counters without formal schooling, we sought other origins for understanding their numerical knowledge. One likely candidate is the need to use number in economic transactions—with typical Tsimane' people mastering social and market skills (i.e., visiting San Borja, working outside the villages, selling products, etc.) between 15 and 20 y of age (69). If market activity is one such factor, then both community remoteness (i.e., proximity to a market town) and gender should be predictive, given the traditionally gendered division of labor in Tsimane' society where it is men who predominately engage in market labor and trading.

Table 2 shows a logistic regression predicting the likelihood of being a full counter, taking into account not only education but also age, gender, and community remoteness. For adults, education, age, and gender were also predictive of full counting. For adults with a mean age (40 for women and 42 for men) and living in a community the mean distance from San Borja, the predicted probabilities of an adult without education being a full counter were substantially higher than for children, which was true for both women (0.65) and men (0.92). The strong association with full counting for males suggests cultural effects on number development which involve the aforementioned gendered division of labor in Tsimane' society (69), where men are far more likely to engage in commercial activity than women (70). Moreover, increasing age and community remoteness were both negatively predictive of full counting for adults. This was likely related to the ongoing and relatively recent market integration for many Tsimane' communities, where the very oldest cohorts probably engage in less numerical market activity overall as well as having grown up prior to the introduction of extensive schooling. As with Tsimane' children, the relationship between the count list and full counting status was found for adults on the subset of participants for which these data were available ($n = 328$; see *SI Appendix*, Table S1), again pointing to the importance of the count list in the development of numerical concepts (and see ref. 42).

For children, education, age, and gender significantly predicted the probability of being a full counter. Girls with the mean age for all girls (7 y old) who had never gone to school had a 0.06 probability of being full counters, whereas the probability increased to 0.14 with a year of schooling and then increased to 0.52 after three years of education. For boys with the mean age of all boys (also 7 y old), the probability of being a full counter without education was similar (0.12) to that for girls, and the probability increased to 0.47 after 3 y of education. There was no relationship found for community remoteness.

Discussion. Meta-analyses for a large sample of Tsimane' participants showed diverging patterns of number knowledge between children and adults: On the one hand, we did not find even a single full-counting child who had reported 0 y of formal schooling, despite the fact that the ages of children without education in the sample ranged from 3 to 8 y old. This suggests that exposure to instruction and numerical content drives learning and the construction of integer concepts. It is also notable that there is cross-cultural evidence for educational contexts affecting the approximate system (26, 71, 72) indicating that the effects of formal schooling on numerical cognition may be far reaching.

On the other hand, while education also appeared to positively affect full counting for adults, more than half of the unschooled Tsimane' adults whom we surveyed were full counters. This shows that formal education is not necessary for learning number but that the timing of its acquisition will vary and be more protracted relative to those who receive formal education. Moreover, adults learning number without formal schooling is not guaranteed—just under half of our adult sample without formal education were subset-knowers. Regression analyses were consistent with an effect of market activity on number knowledge, which was largely divided along gendered lines.

Study 2: Everyday Arithmetic without School. Results from study 1 showed that while some Tsimane' adults without formal education become full counters, others remain with subset knowledge. Higher percentages of adults without formal education are found in the more geographically remote Tsimane' communities, and in our prior surveys of arithmetical ability in one especially remote community (see *Materials and Methods* for a brief description), we noted that adults without formal education often struggled with what would be considered "easy" problems if presented in a scholastic setting (e.g., $2 + 3$, or 2×6). This suggests that such arithmetic problems might be unnatural or difficult outside of a formal educational context. However, in the remote community, we noted a) the existence of a local economy where nearly all products are priced in multiples of 5, with 5 generally being the base unit of purchase and sale (see Fig. 2 for examples) and b) that many nonschooled adults are active participants in this economy. Based on field observations and in accordance with study 1, we hypothesized that in the absence of formal education, engagement in selling produce would shape the form of arithmetical knowledge available, similar to the documented development of a specialized mathematics in the case of young street vendors in Brazil (51). This out-of-school specialized mathematics was notable for its use of currency as the system of numerical representation and also the increasingly complex uses of ratios which exceeded the abilities of similarly aged nonselling peers.

For study 2, we focused on a single village which was particularly remote and tested adult participants who had little to no formal education. This remote community allowed for a unique cultural contrast but also made data collection more difficult and thus placed a limit on sample sizes. Nevertheless, we were able to work with a range of adults without formal education, with whom we presented arithmetic problems of multiplication and addition. The multiplication problems were relevant to the local market (i.e., all being multiples of 5), whereas the addition problems were not market-related (e.g., $1+2$ or $7+7$). Each problem was asked

Fig. 2. (*Left*) Jatata sheets sell for 5 Bolivianos (Bs) each in the remote community. They are used to roof houses but also as currency (units of 5) to pay off outstanding accounts with river traders. (*Right*) Plantains are also priced in multiples of 5, depending on the size of the bunch.

twice: once in a formal mathematical manner (scholastic) and once in a practical, or everyday manner (e.g., as a word problem about prices; see *Materials and Methods*, Table 5). Finally, there was a supplementary addition question (10 + 5) which was market related and was intended as a check for simple price memorization. Besides these arithmetic tasks, we also obtained a verbal count (to a maximum of 30) and tested numerical knowledge using the Give-N task (62), as used in study 1.

Results. Due to the focus on market activity, we analyzed the participants in terms of sellers and nonsellers, where a seller was defined as someone who performs actual price calculations when selling produce (*Materials and Methods*). Nonsellers, often a seller's spouse, might assist with sales activity in general, but they do not calculate prices. These two groups differed in their reported cultural experiences, but there was no strong statistical evidence for overall differences in verbal ability. With an upper limit of 30, the average verbal count was 25 (Mdn = 30) for the sellers (SD = 9) and 16 (Mdn = 12) for the nonsellers (SD = 11). This difference was not statistically reliable by the two-tailed Wilcoxon test: $W = 56$, $P = 0.090$, $r = 0.411$ (although it would be using a one-tailed test). Although counting is often done using a mix of Tsimane' and Spanish numerals (*Materials and Methods*), only one participant used a mixed approach, with the rest ($n = 17$) counting in Tsimane' only. Based on Give-N performance, we classified 9 of the 11 sellers (82%) as full counters, while we only classified 3 of the 7 nonsellers (43%) as such.

Fig. 3*A* shows the mean percentage of problems correct for nonsellers compared with sellers, collapsed across formal and practical phrasings. Nonsellers struggled in general and averaged only 1 correct question in multiplication and effectively zero questions correct in addition. The sellers outperformed nonsellers in both addition and multiplication but were more effective with multiplication than addition. We note, however, that we do not know how multiplication problems were solved by participants and they could have been completed using some form of repeated addition, especially given the diversity of strategies found in other samples (73). Fig. 3*B* shows that the formal phrasing was difficult for the sellers, whereas the practically phrased questions were easier. The sellers performed particularly well with the practically phrased multiplication questions, which were problems that had market relevance in both numerical content and phrasing and

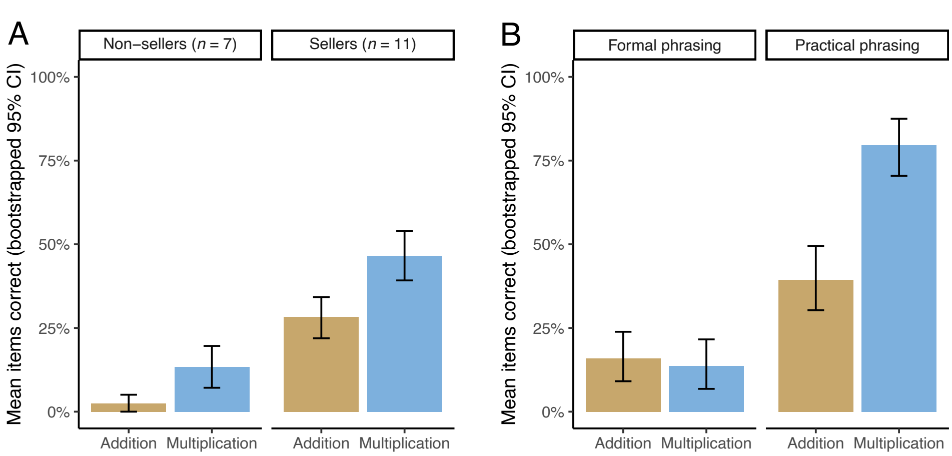

Fig. 3. Study 2: (*A*) Sellers outperformed nonsellers in both addition and multiplication (both phrasings); (*B*) Among themselves, sellers performed better in both addition and multiplication when questions were phrased as practical words problems but were notably stronger in multiplication.

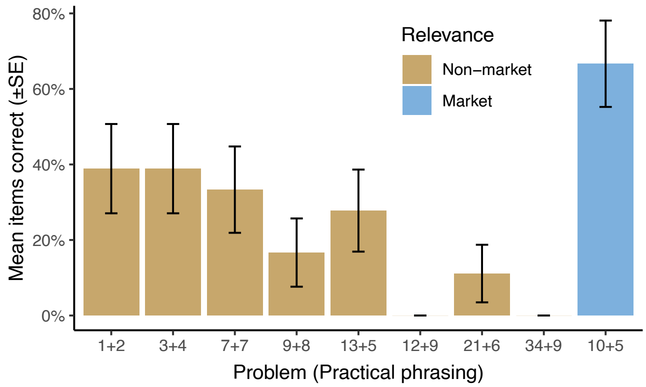

Fig. 4. Study 2: Market vs. nonmarket addition problems (whole-sample performance by item).

thus were closest to what they might encounter in their actual commercial activity.

Of the 11 sellers, 10 (91%) answered the market addition problem correctly (10 + 5). On the other hand, only 2 of the 7 nonsellers (29%) responded correctly, which was a reliable group difference (Fisher's Exact Test; $P = 0.013$). Fig. 4 compares responses to the market addition problem with the nonmarket addition problems for the whole sample, using the group's best overall performance condition (i.e., the practical/word problem phrasing). This figure shows that within this subpopulation of adults without formal education, 10 + 5 (as a price calculation) could be considered easier than 1 + 2 (as a mental addition of objects). Given that the majority of the sellers were full counters (i.e., they understood the numerical meanings of their count list), this is an especially surprising finding since difficulty in early arithmetic generally increases along with augend and addend magnitudes or problem size (74–76), and thus, an understanding of 1 + 2 would be considered to precede an understanding of 10 + 5 [but see Baroody (77) for a critique of the Wheeler (74) difficulty index].

Discussion. The results of study 2 show that many remote adults without formal schooling can succeed on numerical tasks when the problems are provided in a way that is relevant to the local economy, following past results in the tradition of "street" or "everyday" mathematics (50, 51, 78–80). Given that in the remote community, there is a) not large amounts of circulating state currency (and thus lack of access to different denominations and the everyday numerical activities associated with currency) and b) that the base unit of sales and purchasing is most often 5, sellers without formal schooling develop a mathematical practice that centers on what is directly useful in their day-to-day lives—multiples of 5. While this study did not measure how participants solved the multiplication problems, prior studies of adults without formal education have shown evidence of the use of one-to-many correspondences among Brazilian fishermen (81) and a variety of strategies among African adults without formal schooling (82).

These results suggest that remote Tsimane' people without formal education develop an arithmetic that suits their particular needs, rather than being driven by a logical derivation from repeated addition by one. The higher performance for multiplication over addition (even for nonsellers) is also notable given that addition is considered to precede multiplication in a school setting (74, 83). Indeed, in studies of early schooling, solving multiplication problems as repeated addition is one of the first intuitive multiplication strategies used (83, 84), although there is evidence that the conceptual basis for multiplication is found in one-to-many correspondence, and not in repeated addition (85).

Study 3: Fundamental Addition without School. The performance discrepancy between market and nonmarket problems led us to investigate arithmetical knowledge more exhaustively, specifically comparing performance on a range of +1 and +5 types of arithmetic problems. We selected +1 due to the centrality of recursively adding one in formalized notions of number (such as the Peano Axioms) and +5 due to the economic importance of multiples of five in the community. We further aimed here to test arithmetic without making reference to pricing, in order to probe performance and arithmetical transfer to nonmarket problems.

We posed a new set of problems to the same remote participants from study 2. In order to minimize effects of presentation format, we combined the previous study's formal and practical phrasings simultaneously for each item: For a single test item, we posed a verbal word problem along with its formal equivalent, including also its visual presentation, as in study 2. All of the arithmetic problems were presented verbally as additions of objects and not as market pricing problems (e.g., see the final item in Table 5). For the test items, we used a set of 12 augends (i.e., the *Left*-side number to add) to create 24 addition problems with the addends 1 and 5 (i.e., $x + 1$ and $x + 5$). The augend set was 1, 2, 3, 4, 5, 6, 9, 10, 14, 15, 19, and 20. These 12 augends and 2 addends made up 24 problems in total for each participant, with 12 for the +1 set and 12 for the +5 set. Items were presented in a pseudorandomized order.

Results. Fig. 5 shows the percentage correct at the item level for the two problem types ($x + 1$ and $x + 5$), with darker shading for problems with an augend that was divisible by 5 (e.g., **5** + 1, **10** + 1, **15** + 1, etc). Interestingly, 1 + 1 was not universally answered correctly (being correctly answered slightly less often than 5 + 1, or even 5 + 5), although +1 sums were overall more likely to be correct. However, performance spikes were evident across the set on sums with an augend that was divisible by 5.

Logistic regression analysis showed that being a seller was a strong sample-wide predictor of success, which was more predictive even than full counter classification. We therefore once again split the sample into sellers and nonsellers, and Table 3 shows the results of a logistic regression for each subgroup. This shows that the augend's divisibility by 5 was a large predictor, as was the seller's full counter status and, to a lesser degree, the resultant magnitude.

Indeed, when the augend was divisible by 5, sellers were correct with $x + 1$ type questions 75% of the time (i.e., for 5 + 1, 10 + 1, 15 + 1, and 20 + 1), yet they were correct 62% of the time for the other $x + 1$ questions. This difference was more pronounced for $x + 5$ type problems, with 73% correct when the augend was divisible by 5 and 53% for the rest of the $x + 5$ problems.

Discussion. Study 3 showed that remote Tsimane' participants without formal schooling were often able to transfer numerical knowledge to problems that did not involve pricing. However, this ability was not uniformly applied, and in addition to full counting and smaller result magnitude as predictors of success, mental additions using an augend of 5 were more likely to be correct. This effect was present both when adding 1 and 5, suggesting the construction of a numerical system that is at least partially selective of culturally salient features. It is notable also that most sellers were also full counters; therefore, the results do not suggest a numerical representation where only multiples of 5 exist, but instead, flexible arithmetical systems where multiples of 5 are particularly important, and indeed where arithmetical

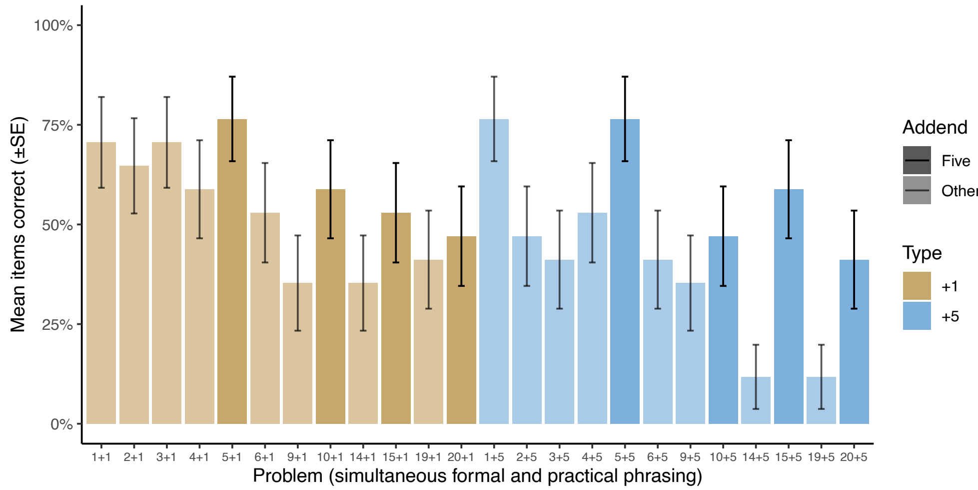

Fig. 5. Study 3: Nonprice addition performance on +1 and +5 (whole sample).

knowledge of +1 is not necessarily a required feature. In terms of learning mechanisms, this selective construction finds support in results at the behavioral and neural levels which highlight the importance of exposure frequency in the processing of symbolic number (86, 87).

Although we hypothesize a particular set of numerical relationships which differ from the documented WEIRD developmental trajectory, another consideration in explaining the results in Fig. 5 is that calculating with multiples of 5 may offer a simplification within the structure of a decimal count system. That is, such problems may be psychologically easier or at least offer a simplification of the rules or procedures which need to be established for calculation (77). However, it is difficult to compare measures of apparent objective difficulty across, for instance, WEIRD educational settings (74) and one where some people have not attended formal school and thus we expect are unlikely to mentally use a written place-value system.

Finally, while the mathematics of these adults is clearly determined in relation to the cultural context (and see refs. 49, 51, 88, and 89), we did not find their knowledge to be concrete in the sense of being bound only to its application of calculating prices but rather that the numerical knowledge could be generalized beyond pricing (79, 81), suggesting that routes to success in study 2 were not limited to price memorization.

Table 3. Study 3: Logistic regression for arithmetic performance

	Correct answer	
	Nonsellers Estimate (*SE*)	Sellers Estimate (*SE*)
Full Counter (Yes)	0.07	1.83***
	(0.46)	(0.37)
Augend (5 divisible)	1.45*	1.34***
	(0.57)	(0.35)
Magnitude (of result)	−0.25***	−0.09***
	(0.06)	(0.02)
Type ($x + 1$)	0.08	−0.02
	(0.47)	(0.30)
Intercept	0.96	−0.20
	(0.63)	(0.46)
Observations	144	264
Log likelihood	−65.00	−149.01
Akaike Inf. Crit.	140.00	308.03

Note: * $P < 0.05$; ** $P < 0.01$; *** $P < 0.001$.

Study 4: Brief Case Studies. So far, we have shown that without any formal schooling, some Tsimane' adults develop an understanding of number and mathematics that centers on a mathematics of fives which is useful in their particular social context. However, we also observed a diversity of numerical understanding in the remote community, and here, we present three case studies ranging from: a) one seller (ML) with nearly complete dependence on market-based arithmetic, despite being able to pass Give-N; b) another (PL) with an ability to add 5 without a complete understanding of adding 1 (including 1 + 1); and c) a third seller (GC) who had stronger arithmetic across both +1 and +5. Two of these case study participants were also asked an impromptu price calculation (equivalent to $(3 \times 5) + 10$), which provided some additional evidence against a simple memorization approach for the sellers.

Data for the case studies were drawn from studies 2 and 3 as well as a mini-interview conducted after the experimental studies (*Materials and Methods*). Finally, one participant (ML) had a follow-up session where he repeated the arithmetic problems from study 2 and a final follow-up session to confirm his understanding of the nonmarket additions (particularly 1 + 1).

ML. ML was a seller and widower of approximately 60 y of age. He had received no formal education but said he learned numbers around age 6 from his cousin. His method of counting to 30 was to count from 1 to 10 three times over (see *Materials and Methods* for a description of Tsimane' counting). He performed Give-N with perfect accuracy and placed the objects slowly and carefully. For study 2, he scored 50% on the multiplication questions (practical phrasing), but 0% on all others. He correctly answered the supplementary addition problem (10 + 5 = 15). For study 3, he scored 0% for all questions (i.e., +1 and +5; see Table 4).

In a follow-up testing session some days later, he scored 100% on the multiplication questions (practical phrasing), correctly answered the supplementary question again, and correctly answered a more complicated impromptu price calculation of "3 items for 5 Bolivianos [Bs], plus 1 item for 10 Bs": $(3 \times 5) + 10 = 25$. His answers for additions were incorrect but at times reflected attempts at pricing, for example, $1 + 1 = 20$, $1 + 2 = 30$, and $2 + 1 = 40$.

Table 4. Case study results (study 3 data)

Addend	1	2	3	4	5	6	9	10	14	15	19	20	% Correct
+1													
ML	10	4	20	6	20	60	70	70	30	30	260	100	0
PL	3	✓	✓	6	✓	8	11	✓	20	✓	11	11	42
GC	✓	✓	✓	✓	✓	✓	✓	✓	16	✓	✓	✓	92
+5													
ML	10	15	30	20	100	200	70	100	15	70	20	60	0
PL	✓	8	✓	✓	✓	✓	20	20	20	✓	20	20	50
GC	✓	✓	7	✓	✓	✓	✓	✓	20	✓	14	✓	75

In a final follow-up session, we removed the possibility of confusion about price and instead questioned about items without value (stones). When asked to say how many stones there would be if there was one stone, and then another one, he considered the question and answered "ĉanam' " (five). For two stones with one more, he answered "q̂uen'ĉan" (eight). Thus, ML is unusual in that he understands counting as repeated blocks of 10, understands the cardinalities of numbers for at least 1 to 8, but appears able to perform mental arithmetic in an exclusively market-centric way (i.e., working with prices that are multiples of 5). This market-centric arithmetic excludes even $1 + 1$, despite the fact that he was a full counter according to Give-N and could easily give two objects if asked for them.

PL. PL was a seller of around 55 y of age. Although having never been to school as a child, he had completed 2 mo of adult education through the Yo, Sí Puedo program some years back which he credited with imparting some numerical ability. His counting and Give-N performance were without errors, and in study 2, he scored 100% on practical multiplication (14% formal) and 38% practical addition (25% formal). He correctly answered the supplementary $10 + 5$ addition problem. However, like ML, he stumbled on $1 + 1$ as a mental addition. His $+1$ results were inconsistent overall, being roughly as good as his $+5$ results (Table 4). Thus, in a manner similar to ML, he was an active user of mathematics for sales, was a full counter, but did not seem to have developed a clear symbolic understanding of $+1$ logic as it applies to the count list. However, unlike ML, he was able to transfer some mathematical knowledge from his success at market arithmetic (study 2), to more general arithmetic (study 3).

GC. GC was another seller around 60 y of age, who was living with his family further out from the center of the community in a more isolated area. Like ML, he had no formal education and reported that he learned numbers on his own by matching his fingers to objects. Although his spouse and children were not sellers and had comparatively weak numerical abilities, his level of numerical skill was impressive, especially given that he had never been exposed to any formal schooling from childhood onward. Give-N was simple for GC, and he counted to 30 with ease. For the study 2 multiplication questions, he scored 100% in either phrasing, and for the addition questions, he scored 38% (practical) and 13% (formal). He correctly answered the supplementary addition question $(10 + 5 = 15)$ and also correctly answered the price for $(3 \times 5) + 10 = 25$. His results for study 3 were 92% for $+1$ and 75% for $+5$ (Table 4).

For comparative purposes, a 40-y old participant from a nearby community with two years of formal education scored 100% on every category except for the formal multiplication problems (scoring 75%). Thus GC's results are comparable to those of a participant with some formal schooling, with the exception of weaker arithmetic when the sums are not related to his livelihood.

Discussion. These case studies among Tsimane' sellers without formal education illustrate that numerical knowledge can be developed in diverse ways. For our sample, being classified as a full counter was not a guarantee that an understanding of $+1$ could be robustly applied to the count list, a result which finds support in recent developmental work (90).

General Discussion

Work with non-WEIRD populations has the potential to uncover universals of human nature (43, 91–93) but also to function as a check against overextended claims of universality (94–97). Perhaps surprisingly, the symbolic use of number appears to be one cognitive domain for which there is considerable diversity across human groups (41, 98). Although all humans, regardless of culture—and many other species—have an innate capacity to discern small numerosities exactly and larger ones approximately (34, 43, 99, 100), the development of exact, symbolic numerical systems (like verbal counting) which transcend these innate abilities appear to depend strongly on cultural factors. Our meta-analytic study of basic number knowledge among the Tsimane' shows that full counting and exposure to formal schooling are strongly coupled for these children. Without schooling and cultural supports, children could take a long time to learn number beyond the first few terms, if at all (and see refs. 34 and 101). The fact that gender is a strong predictor of number among the Tsimane' points to alternative cultural pathways to numerical knowledge, namely that market activity and wage labor have traditionally been almost exclusively men's work among the Tsimane' (70)—both domains in which number is highly useful, if not essential.

The existence of these alternative pathways led to our experimental field studies of arithmetic among remote Tsimane' adults, which are not the first to demonstrate novel forms of mathematics developing outside of schooling (51, 80, 89, 102), nor are they the first to document alterations in the form and function of indigenous mathematics resulting from the introduction of state-issued currency and market engagement (49). However, the often restricted nature of the arithmetic developed among adults without formal education in the remote community (through working primarily with multiples of 5) is difficult to explain under theories in which people derive their numerical knowledge via formal mathematical notions about successive addition by one. The pattern of number knowledge that we found among sellers did not fit an expected universal trajectory but rather seemed to be characterized by a range of conceptual constructions which are developed piecemeal, according to need. Computations which are believed to be fundamental, but are in a given context not relevant to daily life, can remain unlearned—at the extreme including even addition by one.

More broadly, when number is examined historically and cross-culturally, it appears that systems which symbolically represent large numerosities exactly are both difficult to invent and challenging for children to learn when compared to other word learning—facts which are hard to explain if the foundation of number itself is both simple and innate (10, 41). Cultures have also exhibited a striking diversity of systems for numerical representation, including some which are largely based in approximation (34), those that depend on the external body (103), potentially modulus body-part counting systems (104), number systems that allow the use of "vague"/indefinite number words (105), the incorporation of gesture to perform calculations (106), or the complete absence of exact symbolic numeration without even an unambiguous term for "one" (35, 101). Such diversity supports the theory that number's emergence was tied to more concrete, practical uses in specific cultural circumstances, rather than being predetermined by innate logic. As with models suggesting that there are many cognitive factors supporting the acquisition of mathematical knowledge (107), the psychological foundations of number may not be as simple or clean as mathematical formalizations would suggest.

Materials and Methods

Consent Procedure. Formal consent to enter Tsimane' lands and conduct research was granted by the Tsimane' representative political body, the Gran Consejo Tsimane'. Prior to any fieldwork or experimental data collection occurring in communities, we held a public meeting to explain the project. These meetings involved community members and leaders, experienced bilingual Tsimane'/Spanish translators, CBIDSI staff, and the research team. Interested individuals would then sign up for participation, with individual and parental (for children) consent being obtained immediately prior to experimental studies.

Data and Analyses. Datasets for the meta-analysis (study 1) and the experimental fieldwork (studies 2 to 3) are available online (61), along with the analysis code in R (108).

A. Study 1.

A.1. Participants. We conducted a meta-analysis on child and adult Tsimane' data ($N = 1,144$) from 25 Tsimane' communities, with the overall sample consisting of a) data collected in Tsimane' villages between 2012 and 2015 (18, 44, 60); b) data collected in 2018 (28); and c) data collected by the authors in 2019 (Table 1). For the child data, we analyzed Give-N scores for 812 participants (45% female) between 3 and 13 y old ($M = 7$; $SD = 2$). Children's formal schooling ranged from 0 to 7 y ($M = 1.6$; $SD = 1.5$). Researchers collected child data from 23 villages with distance to the closest major town (San Borja) ranging from 4 to 40 km ($M = 19$; $SD = 12$). All villages had their own school.

For the adult data, we analyzed scores for 332 participants (61% female) from data collected in 2019. Adult ages ranged from 14 to 95 y ($M = 40$; $SD = 18$), and formal schooling ranged from 0 to 13 y ($M = 2.5$; $SD = 2.8$). Data were collected in 19 villages ranging from 4 to 44 km ($M = 22$; $SD = 13$) away from San Borja.

A.2. Measures.

Give-N. Participants completed a version of the Give-a-Number (Give-N) task (62), in a presentation that was ordered, random, or both (Table 1). During the tasks, participants were presented with two half sheets of white paper and a set of 10 objects (such as coins or buttons). They were asked to move *N* objects (i.e., a quantity from 1 to 8) from one sheet to the other, with no feedback provided regarding accuracy.

A.3. Analyses. Knower levels were inferred using the Bayesian Data Analysis model described by Lee and Sarnecka (67, 68) implemented in Stan (109) and R (108). The model assumes that participants have a prior distribution over the amount of objects they will return regardless of instruction. When a participant receives a prompt for a given number, they will update their response distribution according to the prompt and their knower level. Using Bayes' rule, the model infers the prior distribution over responses, a likelihood temperature specifying how strongly children update their distribution given a prompt and a distribution over knower levels for each participant. The model was run with 2,000 Hamiltonian Monte Carlo steps for each of 4 chains. The knower level was extracted as the posterior mode (which was highly correlated with the posterior mean). Analysis code is available online (61).

B. Study 2. The data for study 2 were collected in a remote Tsimane' community along the Maniqui river, located about a day's travel from San Borja, first by car (to reach the port) and then canoe. The community does not have access to services such as the power grid or telecommunications, although merchants arrive by canoe from time to time to trade goods from the market towns. Since there is not a large amount of circulating currency in the community, purchases are often made on a trade basis with sheets of jatata (i.e., with each sheet priced at 5 Bolivianos), and purchases may be made on an account with expectation of future debt cancelation using jatata.

B.1. Participants. A total of 18 Tsimane' adults (10 men and 8 women) participated in the study. Within the sample, 11 were sellers, and the remaining 7 were nonsellers. The groups did not differ significantly in years of formal education or age. No participants had ever completed a full year of schooling.

Sellers. The sellers had a mean age of 53 ($SD = 7$) and modal years education of 0 (range: 0 to 0.25). Only three sellers had received some form of systematized education, although none had been through the Bolivian school system as children. Two of these participants had attended an adult missionary course in the past (1 of 4 mo duration, the other of 2 mo), and a third participant had completed a 2-wk adult learning course some years back.

Nonsellers. The nonsellers had a mean age of 54 ($SD = 13$) and modal years education of 0 (range: 0 to 0.42). Only one nonseller had received systematized education, having completed 4 mo of schooling when young (i.e., without finishing first year), in addition to a 1-mo adult "Yo, Sí Puedo" course.

B.2. Measures. All participants first answered demographic questions (age and years of education) and then were asked whether they sold produce or not. When participants said that they did sell produce, we then asked whether they performed calculations for the sales or not. We only considered those who answered positively to both questions to be sellers. Participants then in order completed a) verbal count, b) Give-N, and finally c) the arithmetic tasks from study 2 and then study 3. All questions were asked by the experimenter (the first author) in Spanish and were subsequently translated into Tsimane' by an experienced translator.

Verbal count. Participants were asked to audibly count as high as they could but were stopped if they reached 30. Counting could be either in Tsimane' (a base-10 system) or Spanish, although mixed counting was also accepted.

Ordered Give-N. We adapted the Give-N task from Wynn (62), and participants were asked to give an exact number of plastic buttons over 8 trials (with no accuracy feedback), moving the buttons from one sheet of white paper (containing all the buttons) to another sheet next to it (blank). The numbers 1 to 8 were used in sequence (ordered), irrespective of answer accuracy.

Arithmetic tasks. Two sets of arithmetic problems were each posed in two different ways, giving four conditions in total for each participant. The two sets of problems were multiplication (market-based) and addition (non-market-based). The multiplication problems were locally relevant to selling, being all based on some multiple of five, whereas the addition problems had limited to no local relevance to selling and thus were more scholastic in nature. Each set consisted of eight questions, and each set was presented twice: once in a formal mathematical phrasing and once in a practical phrasing (i.e., as a word problem). Thus, each participant was posed 32 problems in total (i.e., 4 sets of 8 questions). The specific questions in each set were as follows:

- Multiplication: Multiples of 5 (from 2 to 5), and multiples of 10 (also from 2 to 5).
- Addition: $1 + 2$;[*] $3 + 4$; $7 + 7$; $12 + 9$; $9 + 8$; $13 + 5$; $21 + 6$; $34 + 9$.

[*]The sum $1 + 2$ may appear to be fundamental to selling, but it is not strictly necessary if prices are always multiples of five, since the number of products for sale or purchase can be determined by counting (and then multiplied by 5).

Table 5. Example questions across conditions and styles

Item	Phrasing	Example question	Visual aid
2×5	Formal	What is two times five?	Yes
2×5	Practical	What is the value of two [jatata]?	No
$3 + 4$	Formal	What is three plus four?	Yes
$3 + 4$	Practical	If you had three [jatata] and your spouse had four [jatata], how many [jatata] would there be in total?	No

Examples of how these questions were asked are shown in Table 5. The actual objects referred to in the practical phrasings differed by participant and depended on their prior knowledge of item values (obtained via a brief mini-interview before the task). For instance, for the practical questions in Table 5, "jatata" would only be used if the participant knew beforehand that jatata had a value of 5 Bolivianos (Bs) in the community. Other possibilities included plantains, fish, chickens, and so on.

In the formal phrasing, participants were asked the problem verbally but were also shown the problem written on paper as a visual aid. In contrast, the practical phrasing was verbal only. The presentation order was pseudorandomized within sets, always avoiding an obvious linear order (e.g., 2×5, 3×5, 4×5, and 5×5).

Supplementary market addition. There was a supplementary addition problem asked at the close of the arithmetic tasks, equivalent to $10 + 5$. For example, a participant could be asked: "What is the total value of one sábalo" [a type of fish] and one jatata [a piece of roofing]?, where sábalo was known by the participant to be valued at 10 Bs, and jatata was known to be valued at 5 Bs. As above, other objects could be substituted depending on the participant's prior knowledge of local pricing conventions.

C. Study 3.

C.1. Participants. All participants from study 2 were included, except for one nonseller who was unable to correctly answer any arithmetic problems, making 17 participants in total.

C.2. Measures.

Arithmetic tasks. Two sets of arithmetic problems were presented in a pseudorandomized order, being drawn from a set of 12 addends: 1, 2, 3, 4, 5, 6, 9, 10, 14, 15, 19, and 20; with each addend used in both a $+1$ and $+5$ problem. Problems were presented in both the formal and practical phrasing of study 2. That is, for a given item, participants first heard the formal phrasing (e.g., "What is $3 + 5$?") along with written presentation of the sum (on a card), followed by the practical phrasing (e.g., "If you had 3 [objects] and your spouse had 5 [objects], how many [objects] would there be in total?").

D. Study 4. After completing the tasks for study 2 and study 3, all participants took part in a mini-interview where they were asked: a) how they learned numbers and b) whether their spouse could count and do calculations.

E. Notes on Tsimane' Counting. In terms of number and counting, contemporary Tsimane' culture engages with two base-10 systems: Tsimane' and Spanish. While we are not aware of any published analyses of the Tsimane' number system specifically, the Tsimane' language is considered part of the Mosetenan language family, and the analyses of Mosetén numerals provided by Sakel (110) show similarity with Tsimane' numerals. Historically, the count system may have been quinary (base-5; 110, p. 168), although it is presently a well-structured decimal system without irregular number words (such as "eleven" or "twelve" in English or similarly "once" and "doce" in Spanish). Although it may be tempting to speculate on a connection between the potentially base-5 history of the number system and the importance of fives that we report on here, the Tsimane' counting system does not privilege fives in any way.

In Tsimane' counting beyond ten, a common pattern is to give the decimal amount first, followed then by the remainder (as with numbers above 20 in English). For example, 11 would be roughly "ten one" (yiri'tac yiris), with 12 being "ten two" (yiri'tac pärä'), and so on. The words for 21 are roughly "twenty one" (pärä'qui'tac yiris), followed by "twenty two" (pärä'qui'tac pärä'), et cetera. However, in practice, these higher number words can be somewhat unwieldy, and it is common to hear speakers use Tsimane' for smaller numbers, while preferring Spanish for higher numbers. There is also variation in the manner of counting; for instance, older Tsimane' people tend to omit the decade component when counting up, so 1 to 20 would be counted out as "one" to "ten," then "one" to "nine," followed by "twenty," and so on (e.g., see ML's counting method in study 4).

Data, Materials, and Software Availability. Anonymized CSV data have been deposited in OSF (https://doi.org/10.17605/OSF.IO/75N6T) (111). Previously published data were used for this work. There are several, but the complete list of sources for our meta-analysis can be found in Table 1 of the manuscript (p. 3).

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1. G. Peano, *Arithmetices principia: Nova methodo exposita* (Fratres Bocca, 1889).
2. J. Von Neumann, Zur einführung der transfiniten zahlen. *Acta Litterarum ac Sci. Regiae Univ. Hung. Francisco-Josephinae, Sectio Sci. Math.* **1**, 199–208 (1923).
3. B. C. Pierce, *Types and programming languages* (MIT Press, 2002).
4. A. M. Leslie, R. Gelman, C. R. Gallistel, The generative basis of natural number concepts. *Trends Cognit. Sci.* **12**, 213–218 (2008).
5. A. M. Leslie, C. R. Gallistel, R. Gelman, "Where integers come from" in *The Innate Mind, Volume 3*, P. Carruthers, S. Laurence, S. Stich, Eds. (Oxford University Press, 2008).
6. R. Gelman, C. R. Gallistel, *The Child's Understanding of Number* (Harvard University Press, 1986).
7. C. R. Gallistel, R. Gelman, Preverbal and verbal counting and computation. *Cognition* **44**, 43–74 (1992).
8. S. T. Piantadosi, J. B. Tenenbaum, N. D. Goodman, Bootstrapping in a language of thought: A formal model of numerical concept learning. *Cognition* **123**, 199–217 (2012).
9. S. Carey, Bootstrapping & the origin of concepts. *Daedalus* **133**, 59–68 (2004).
10. S. Carey, D. Barner, Ontogenetic origins of human integer representations. *Trends Cognit. Sci.* **23**, 823–835 (2019).
11. J. Sullivan, D. Barner, How are number words mapped to approximate magnitudes? *Q. J. Exp. Psychol.* **66**, 389–402 (2013).
12. P. Cheung, M. Rubenson, D. Barner, To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognit. Psychol.* **92**, 22–36 (2017).
13. B. W. Sarnecka, S. Carey, How counting represents number: What children must learn and when they learn it. *Cognition* **108**, 662–674 (2008).
14. K. Davidson, K. Eng, D. Barner, Does learning to count involve a semantic induction? *Cognition* **123**, 162–173 (2012).
15. B. W. Sarnecka, M. C. Goldman, E. B. Slusser, "How counting leads to children's first representations of exact, large numbers" in *Oxford Handbook of Numerical Cognition*, R. C. Kadosh, A. Dowker, Eds. (Oxford University Press, 2015), pp. 291–309.
16. R. M. Schneider, J. Sullivan, K. Guo, D. Barner, What counts? Sources of knowledge in children's acquisition of the successor function *Child Dev.* **92**, e476–e492 (2021).
17. R. M. Schneider, A. Pankonin, A. Schachner, D. Barner, Starting small: Exploring the origins of successor function knowledge. *Dev. Sci.* **24**, e13091 (2021).
18. J. Jara-Ettinger, S. T. Piantadosi, E. S. Spelke, R. Levy, E. Gibson, Mastery of the logic of natural numbers is not the result of mastery of counting: Evidence from late counters. *Dev. Sci.* **20**, e12459 (2017).
19. R. M. Schneider, E. Brockbank, R. Feiman, D. Barner, Counting and the ontogenetic origins of exact equality. *Cognition* **218**, 104952 (2022).
20. J. Henrich, S. J. Heine, A. Norenzayan, The weirdest people in the world? *Behav. Brain Sci.* **33**, 61–83 (2010).
21. S. A. Hart, C. M. Ganley, D. J. Purpura, Understanding the home math environment and its role in predicting parent report of children's math skills. *PloS One* **11**, e0168227 (2016).
22. M. C. Daucourt, A. R. Napoli, J. M. Quinn, S. G. Wood, S. A. Hart, The home math environment and math achievement: A meta-analysis. *Psychol. Bull.* **147**, 565 (2021).
23. D. Leyva, M. E. Libertus, R. McGregor, Relations between subdomains of home math activities and corresponding math skills in 4-year-old children. *Ed. Sci.* **11**, 594 (2021).

24. S. C. Levine, L. W. Suriyakham, M. L. Rowe, J. Huttenlocher, E. A. Gunderson, What counts in the development of young children's number knowledge? *Dev. Psychol.* **46**, 1309 (2010).
25. E. A. Gunderson, S. C. Levine, Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Dev. Sci.* **14**, 1021–1032 (2011).
26. I. M. Lyons, S. Bugden, S. Zheng, S. D. Jesus, D. Ansari, Symbolic number skills predict growth in nonsymbolic number skills in kindergarteners. *Dev. Psychol.* **54**, 440–457 (2018-03).
27. K. S. Mix, The construction of number concepts. *Cognit. Dev.* **17**, 1345–1363 (2002).
28. I. Boni, J. Jara-Ettinger, S. Sackstein, S. T. Piantadosi, Verbal counting and the timing of number acquisition in an indigenous Amazonian group. *PLoS One* **17**, e0270739 (2022).
29. H. Barth, K. L. Mont, J. Lipton, E. S. Spelke, Abstract number and arithmetic in preschool children. *Proc. Natl. Acad. Sci. U.S.A.* **102**, 14116–14121 (2005).
30. H. Barth, L. Beckmann, E. S. Spelke, Nonsymbolic, approximate arithmetic in children: Abstract addition prior to instruction. *Dev. Psychol.* **44**, 1466–1477 (2008).
31. S. Dehaene, *The Number Sense: How the Mind Creates Mathematics* (OUP, USA, 2011).
32. J. F. Cantlon, Math, monkeys, and the developing brain. *Proc. Natl. Acad. Sci. U.S.A.* **109**, 10725–10732 (2012).
33. J. Whalen, C. R. Gallistel, R. Gelman, Nonverbal counting in humans: The psychophysics of number representation. *Psychol. Sci.* **10**, 130–137 (1999).
34. P. Pica, C. Lemer, V. Izard, S. Dehaene, Exact and approximate arithmetic in an Amazonian indigene group. *Science* **306**, 499–503 (2004).
35. M. C. Frank, D. L. Everett, E. Fedorenko, E. Gibson, Number as a cognitive technology: Evidence from Pirahã language and cognition. *Cognition* **108**, 819–824 (2008).
36. P. Gordon, Numerical cognition without words: Evidence from Amazonia. *Science* **306**, 496–499 (2004).
37. R. E. Núñez, Is there really an evolved capacity for number? *Trends Cognit. Sci.* **21**, 409–424 (2017).
38. S. Beller *et al*., The cultural challenge in mathematical cognition. *J. Numerical Cognit.* **4**, 448–463 (2018).
39. H. D. Cruz, "Testimony and children's acquisition of number concepts" in *Naturalizing Logico-Mathematical Knowledge* (Routledge, 2018), pp. 164–178.
40. M. Pantsar, The enculturated move from proto-arithmetic to arithmetic. *Front. Psychol.*, https://doi.org/10.3389/fpsyg.2019.01454 (2019).
41. D. M. O'Shaughnessy, E. Gibson, S. T. Piantadosi, The cultural origins of symbolic number. *Psychol. Rev.* **129**, 1442–1456 (2021).
42. B. Pitt, E. Gibson, S. T. Piantadosi, Exact number concepts are limited to the verbal count range. *Psychol. Sci.* **33**, 371–381 (2022).
43. S. Ferrigno, J. Jara-Ettinger, S. T. Piantadosi, J. F. Cantlon, Universal and uniquely human factors in spontaneous number perception. *Nat. Commun.* **8**, 13968 (2017).
44. S. T. Piantadosi, J. Jara-Ettinger, E. Gibson, Children's learning of number words in an indigenous farming-foraging group. *Dev. Sci.* **17**, 553–563 (2014).
45. A. Almoammer *et al*., Grammatical morphology as a source of early number word meanings. *Proc. Natl. Acad. Sci. U.S.A.* **110**, 18448–18453 (2013).
46. D. Barner, K. Chow, S. J. Yang, Finding one's meaning: A test of the relation between quantifiers and integers in language development. *Cognit. Psychol.* **58**, 195–219 (2009).
47. H. J. Reed, J. Lave, Arithmetic as a tool for investigating relations between culture and cognition. *Am. Ethnol.* **6**, 568–582 (1979).
48. M. C. Frank, D. Barner, Representing exact number visually using mental abacus. *J. Exp. Psychol.: General* **141**, 134 (2012).
49. G. B. Saxe, *Cultural Development of Mathematical Ideas: Papua New Guinea Studies* (Cambridge University Press, 2014).
50. T. N. Carraher, D. W. Carraher, A. D. Schliemann, Mathematics in the streets and in schools. *Br. J. Dev. Psychol.* **3**, 21–29 (1985).
51. G. B. Saxe, Candy selling and math learning. *Ed. Res.* **17**, 14–21 (1988).
52. T. L. Huanca, Tsimane' Oral Tradition, Landscape, and Identity in Tropical Forest (2008).
53. V. Reyes-García *et al*., Secular trends on traditional ecological knowledge: An analysis of changes in different domains of knowledge among Tsimane' men. *Learn. Individual Diff.* **27**, 206–212 (2013).
54. V. Reyes-García *et al*., Indigenous land reconfiguration and fragmented institutions: A historical political ecology of Tsimané lands (Bolivian Amazon). *J. Rural Stud.* **34**, 282–291 (2014).
55. M. Guèze *et al*., Shifts in indigenous culture relate to forest tree diversity: A case study from the Tsimane' Bolivian Amazon. *Biol. Conserv.* **186**, 251–259 (2015).
56. V. Vadez *et al*., Brief communication: Does integration to the market threaten agricultural diversity? Panel and cross-sectional data from a horticultural-foraging society in the Bolivian Amazon. *Hum. Ecol.* **32**, 635–646 (2004).
57. A. Chicchón, "Chimane Resource Use and Market Involvement in the Beni Biosphere Reserve, Bolivia," PhD thesis, University of Florida, FL (1992).
58. V. Reyes-García *et al*., Schooling and local environmental knowledge: Do they complement or substitute each other? *Int. J. Ed. Dev.* **30**, 305–313 (2010).
59. M. Gurven *et al*., Cognitive performance across the life course of Bolivian forager-farmers with limited schooling. *Dev. Psychol.* **53**, 160–176 (2017).
60. J. Jara-Ettinger, E. Gibson, C. Kidd, S. T. Piantadosi, Native Amazonian children forego egalitarianism in merit-based tasks when they learn to count. *Dev. Sci.* **19**, 1104–1110 (2015).
61. D. M. O'Shaughnessy *et al*., Diverse mathematical knowledge among indigenous Amazonians [Data set]. OSF (2023). https://doi.org/10.17605/OSF.IO/75N6T.
62. K. Wynn, Children's acquisition of the number words and the counting system. *Cognit. Psychol.* **24**, 220–251 (1992).
63. B. W. Sarnecka, M. D. Lee, Levels of number knowledge during early childhood. *J. Exp. Child Psychol.* **103**, 325–337 (2009).
64. S. Carey, *The Origin of Concepts* (Oxford University Press, 2009).
65. C. D. O'Rear, N. M. McNeil, P. K. Kirkland, Partial knowledge in the development of number word understanding. *Dev. Sci.* **23**, e12944 (2020).
66. P. Cheung, M. Toomey, Y. H. Jiang, T. B. Stoop, A. Shusterman, Acquisition of the counting principles during the subset-knower stages: Insights from children's errors. *Dev. Sci.* **25**, e13219 (2022).
67. M. D. Lee, B. W. Sarnecka, A model of knower-level behavior in number concept development. *Cognit. Sci.* **34**, 51–67 (2010).
68. M. D. Lee, B. W. Sarnecka, Number-knower levels in young children: Insights from Bayesian modeling. *Cognition* **120**, 391–402 (2011).
69. E. Schniter, M. Gurven, H. S. Kaplan, N. T. Wilcox, P. L. Hooper, Skill ontogeny among Tsimane forager-horticulturalists. *Am. J. Phys. Anthropol.* **158**, 3–18 (2015).
70. M. Gurven, J. Winking, H. Kaplan, C. von Rueden, L. McAllister, A bioeconomic approach to marriage and the sexual division of labor. *Hum. Nat.* **20**, 151–183 (2009).
71. M. Piazza, P. Pica, V. Izard, E. S. Spelke, S. Dehaene, Education enhances the acuity of the nonverbal approximate number system. *Psychol. Sci.* **24**, 1037–1043 (2013).
72. J. Nys *et al*., Does math education modify the approximate number system? A comparison of schooled and unschooled adults. *Trends Neurosci. Ed.* **2**, 13–22 (2013).
73. J. T. Mulligan, M. C. Mitchelmore, Young children's intuitive models of multiplication and division. *J. Res. Math. Ed.* **28**, 309–330 (1997).
74. L. R. Wheeler, A comparative study of the difficulty of the 100 addition combinations. *Pedagog. Seminary J. Genet. Psychol.* **54**, 295–312 (1939).
75. M. H. Ashcraft, B. A. Fierman, R. Bartolotta, The production and verification tasks in mental addition: An empirical comparison. *Dev. Rev.* **4**, 157–170 (1984).
76. N. J. Zbrodoff, G. D. Logan, "What everyone finds" in *Handbook of Mathematical Cognition*, J. I. D. Campbell, Ed. (Routledge, Abingdon, 2004), chap. 19.
77. A. J. Baroody, Mastery of basic number combinations: Internalization of relationships or facts? *J. Res. Math. Ed.* **16**, 83 (1985-03).
78. S. R. Guberman, The development of everyday mathematics in Brazilian children with limited formal education. *Child Dev.* **67**, 1609 (1996).
79. J. Lave, Cognitive consequences of traditional apprenticeship training in West Africa. *Anthropol. Ed. Q.* **8**, 177–180 (1977).
80. T. Nunes, A. Schliemann, D. Carraher, *Street Mathematics and School Mathematics* (Cambridge University Press, 1993).
81. T. Nunes, A. Schliemann, D. Carraher, *Reversibility and Transfer in the Schema of Proportionality* (Cambridge University Press, 1993), pp. 103–126.
82. A. L. Petitto, H. P. Ginsburg, Mental arithmetic in Africa and America: Strategies, principles, and explanations. *Int. J. Psychol.* **17**, 81–102 (1982).
83. F. B. Clark, C. Kamii, Identification of multiplicative thinking in children in grades 1–5. *J. Res. Math. Ed.* **27**, 41–51 (1996).
84. J. T. Mulligan, M. C. Mitchelmore, Young children's intuitive models of multiplication and division. *J. Res. Math. Ed.* **28**, 309 (1997).
85. J. H. Park, T. Nunes, The development of the concept of multiplication. *Cognit. Dev.* **16**, 763–773 (2001).
86. A. Krajcsi, G. Lengyel, P. Kojouharova, The source of the symbolic numerical distance and size effects. *Front. Psychol.*, https://doi.org/10.3389/fpsyg.2016.01795 (2016).
87. I. M. Lyons, S. L. Beilock, Characterizing the neural coding of symbolic quantities. *NeuroImage* **178**, 503–518 (2018).
88. J. Lave, Math lessons from Liberia. *Anthropol. Theory* **10**, 186–191 (2010).
89. R. T. Rosin, Gold medallions: The arithmetic calculations of an illiterate. *Anthropol. Ed. Q.* **15**, 38–50 (1984).
90. R. M. Schneider, A. Pankonin, A. Schachner, D. Barner, Starting small: Exploring the origins of successor function knowledge. *Dev. Sci.* **24**, e13091 (2021).
91. M. Sablé-Meyer *et al*., Sensitivity to geometric shape regularity in humans and baboons: A putative signature of human singularity. *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2023123118 (2021).
92. S. E. Koopman, A. M. Arre, S. T. Piantadosi, J. F. Cantlon, One-to-one correspondence without language. *R. Soc. Open Sci.* **6**, 190495 (2019).
93. Z. Witkower, J. L. Tracy, We don't make WEIRD faces: A brief history of emotion expression research in small-scale societies. *Evol. Hum. Behav.* **41**, 454–455 (2020).
94. M. Gurven, C. von Rueden, M. Massenkoff, H. Kaplan, M. L. Vie, How universal is the Big Five? Testing the five-factor model of personality variation among forager-farmers in the Bolivian Amazon. *J. Person. Soc. Psychol.* **104**, 354–370 (2013).
95. R. Sear, P. Sheppard, D. A. Coall, Cross-cultural evidence does not support universal acceleration of puberty in father-absent households. *Philos. Trans. R. Soc. B: Biol. Sci.* **374**, 20180124 (2019).
96. K. Cooperrider, J. Slotta, R. Núñez, The preference for pointing with the hand is not universal. *Cognit. Sci.* **42**, 1375–1390 (2018).
97. R. Núñez, K. Cooperrider, J. Wassmann, Number concepts without number lines in an indigenous group of Papua New Guinea. *PLoS One* **7**, e35662 (2012).
98. C. F. dos Santos, Enculturation and the historical origins of number words and concepts. *Synthese* **199**, 9257–9287 (2021).
99. A. Nieder, S. Dehaene, Representation of number in the brain. *Annu. Rev. Neurosci.* **32**, 185–208 (2009).
100. L. Feigenson, S. Dehaene, E. Spelke, Core systems of number. *Trends Cognit. Sci.* **8**, 307–314 (2004).
101. D. L. Everett, Cultural constraints on grammar and cognition in Pirahã. *Curr. Anthropol.* **46**, 621–646 (2005).
102. A. D. Schliemann, T. Nunes, A situated schema of proportionality. *Br. J. Dev. Psychol.* **8**, 259–268 (1990).
103. A. Bender, S. Beller, Nature and culture of finger counting: Diversity and representational effects of an embodied cognitive tool. *Cognition* **124**, 156–182 (2012).
104. E. P. Wolfers, The original counting systems of Papua and New Guinea. *Arith. Teacher* **18**, 77–83 (1971).
105. C. Bowern, J. Zentz, Diversity in the numeral systems of Australian languages. *Anthropol. Linguist.* **54**, 133–160 (2012).
106. N. B. Brooks, D. Barner, M. Frank, S. Goldin-Meadow, The role of gesture in supporting mental representations: The case of mental abacus arithmetic. *Cognit. Sci.* **42**, 554–575 (2017).
107. J. A. LeFevre *et al*., Pathways to mathematics: Longitudinal predictors of performance. *Child Dev.* **81**, 1753–1767 (2010).
108. R Core Team, *R: A Language and Environment for Statistical Computing* (R Foundation for Statistical Computing, Vienna, Austria, 2023).
109. Stan Development Team, *Stan Modeling Language User's Guide and Reference Manual* (2021).
110. J. Sakel, *A Grammar of Mosetén* (De Gruyter Mouton, 2011).
111. D. M. O'Shaughnessy *et al*., Diverse mathematical knowledge among indigenous Amazonians [Data set]. OSF. https://doi.org/10.17605/OSF.IO/75N6T. Deposited 27 July 2023.