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Incentive based policies on climate change: taxes and tradable emissions permits

Políticas de cambio climático basadas en incentivos: impuestos y derechos de emisión

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INCENTIVE BASED POLICIES ON CLIMATE CHANGE: TAXES AND

TRADABLE EMISSION PERMITS

POLITICAS DE CAMBIO CLIMÁTICO BASADAS EN INCENTIVOS:

IMPUESTOS Y DERECHOS DE EMISION

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Abstract

The reduction of Greenhouse Gases (GHG) plays a central role in the environmental policies considered by countries for implementation not only at its own level but also at supranational levels. This thesis is dedicated to investigate some aspects of two of the most relevant climate change policies. The first part is dedicated to emission permit markets and the second part to optimal carbon taxes.

On emission permit markets we explore the strategic behavior of oligopolistic firms operating in polluting industrial sectors that are regulated by cap and trade systems. Our aim is to identify how market power influences the main results obtained under perfect competition assumptions and to understand how actions taken in one market affects the outcome of the other related market.

A partial equilibrium model is developed for this purpose with specific abatement cost functions. In Chapter 2 we use the model to explain some of the most relevant literature results. In Chapter 3 the model is used to analyze different oligopolistic structures in the product market under the assumption of competitive permits market. There are two significant findings. Firstly, under the assumption of a Stackelberg oligopoly, firms have no incentives for lobbying in order to manipulate permit prices up, as they have under Cournot competition. Secondly, incentives appear

if they receive free permits (grandfathering) and incentives grow as the number of free permits increase.

In Chapter 4 our analysis is based on the assumption of imperfect competition in both markets. The main consequences of our study are: A dominant firm in the emissions permit market is always making profit either as a net buyer or as a net seller of permits, whatever the oligopolistic structure in the output market. In the absence of grandfathering, the leader firm in the product market has an advantage in terms of output and profits but the introduction of grandfathering can partially or totally compensate that situation if the follower receives enough more free permits than the leader.

The second part of this thesis (Chapter 5) focuses on the role that fossil fuel extraction costs play on the optimal carbon taxes imposed to internalize the externality created for the accumulation of greenhouse gases in the atmosphere. A general equilibrium model with capital accumulation, a damage function, and the dynamics of a non-renewable resource, is considered. The extraction cost function takes into account the flow of extraction and also the scarcity effect in relation with the stock not yet extracted. This is the so-called stock effect.

The main finding is that the stock effect could create important distortions in the dynamics of the optimal tax, and mainly in relation with potential shocks in the proven reserves of fossil fuels. This statement is supported on analytical basis and confirmed by a related quantitative analysis. Moreover, it can be the case that the optimal carbon tax is not time consistent and a second policy instrument would be needed to attain a first best policy.

Resumen

La reducción de gases de efecto invernadero juega un papel fundamental en las políticas de los países, tanto a nivel nacional como internacional. Esta tesis se dedica a investigar algunos aspectos de las dos políticas más relevantes sobre el cambio climático. La primera parte se dedica a los mercados de derechos de emisión y la segunda a la imposición óptima sobre emisiones.

En la primera parte estudiamos el comportamiento estratégico de empresas que operan en sectores industriales contaminantes sometidos a limitación de emisiones a través de derechos negociables, con el propósito de identificar las diferencias entre los resultados obtenidos cuando hay poder de mercado o en competencia perfecta. Y además entender como las acciones tomadas en uno de esos mercados afectan al otro.

Para ello se desarrolla un modelo de equilibrio parcial con funciones específicas de costes de reducción de emisiones. En el Capítulo 2 este modelo se utiliza para explicar alguno de los resultados más relevantes de la literatura. En el Capítulo 3 se utiliza para analizar diferentes estructuras de oligopolio cuando el mercado de permisos es competitivo. Encontramos dos hechos significativos. En primer lugar y en un modelo de Stackelberg, se determina que las empresas no tienen incentivos para presionar al regulador con el objetivo de manipular al alza el precio de los permisos, a diferencia de

lo que ocurre cuando el oligopolio es de Cournot. Y en segundo lugar esos incentivos aparecen si las empresas reciben permisos negociables gratuitos, incentivos que aumentan a medida que aumenta el número de dichos permisos.

En el capítulo 4, y bajo la hipótesis de competencia imperfecta en ambos mercados, las principales consecuencias que se obtienen de nuestro estudio son: La firma dominante en el mercado de permisos obtiene siempre beneficios, tanto si actúa como comprador o como vendedor de permisos, cualquiera que sea el tipo de oligopolio que se considere en el mercado de producto. Si no hay reparto de títulos gratuitos, la firma líder en el mercado de producto obtiene ventas y beneficios superiores al seguidor, pero la ventaja del líder puede ser parcial o totalmente compensada a medida que el seguidor recibe más títulos que el líder.

La segunda parte de esta tesis (Capítulo 5) se centra en estudiar el papel que desempeñan los costes de extracción en el establecimiento de la imposición óptima a la externalidad creada por la acumulación de gases de efecto invernadero. Consideramos un modelo de equilibrio general con acumulación de capital, una función de daños y la dinámica de un recurso no renovable. La función de costes de extracción considera tanto el flujo de extracción como el efecto escasez asociado al recurso aún no extraído.

La contribución más significativa del capítulo es que el "stock effect" puede crear distorsiones importantes en la dinámica del impuesto, especialmente ante variaciones en el nivel de reservas de combustibles fósiles. Este hecho se soporta en bases analíticas y se confirma a través del análisis cuantitativo. El impuesto óptimo obtenido al considerar el "stock effect" no es consistente en el tiempo y por tanto un segundo instrumento sería necesario para alcanzar el llamado "first best".

Palabras Clave: Derechos de emisión, Poder de Mercado, Impuestos Óptimos, Costes de Extracción.

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List of Abbreviations

BGP   Balanced Growth Path

CAT   Cap and trade

CBD   Convention on Biological Diversity

CDM   Clean Development Mechanism

COP   Conference of the Parties

DAI   Dangerous anthropogenic interference

DICE   Dynamic Integrated Climate E-conomy Model

DHSS   Dasgupta-Heal-Solow-Stiglitz

ERC   Emission-Reduction Credit

EUETS   European Union Emission Trading Scheme

FOC   First Order Condition

GHG   Greenhouse Gases

IAM   Integrated Assessment Model

IPCC   United Nations Intergovernmental Panel on Climate Change

NO_x Oxides of Nitrogen

RCK   Ramsey-Cass-Koopmans Model

RICE   Regional Integrated Climate Economy Model

SCC	Social Cost of Carbon
TEP	Tradable Emissions Permit
UN	United Nations
UNCED	UN Conference on Environment and Development
UNCHE	UN Conference on the Human Environment
UNEP	UN Environment Program
UNFCCC	UN Framework Convention on Climate Change
WMO	World Meteorological Organization
WSSD	World Summit on Sustainable Development

Chapter 1

Introduction

1.1 Summary

Carbon taxes and emissions permit markets are the most important environmental policies that have been put in place in order to reduce the negative effects of climate change. This thesis is devoted to the study of some important issues regarding both policies.

From an economic point of view, climate change is a global massive externality, and therefore is subject to the free rider issue. This is the reason of the required international involvement and cooperation. In this introductory chapter we offer an overview of the main issues regarding climate change and the policies used to deal with it. Specifically we first review the most important advances in the acknowledgment and identification of the magnitude of the problem, presenting a historical overview from 1972 to 2015.

Then we introduce the different climate change policies, giving particular attention to those that incorporate economic incentives since they will be the object of our study. We show the different arguments that have been used to support carbon taxes and emission permits markets and also the main issues linked to these policies. We finally explain which circumstances can make both policies equivalent in economic terms, and we also introduce the new policy approach that suggests combining them.

It follows a brief review of the emissions permit markets subject, explaining its nature and most relevant characteristics. We mainly focus on the two most important elements of concern that has been covered by the literature. First, the price volatility issue and the different mechanism that have been proposed to overcome this problem. Secondly, the market power issue, which is the central object of analysis in the first part of this thesis. We identify and explore the impact of market power on the efficiency of this policy instrument, the incentives that firms face to manipulate permit prices and finally the interactions between the permit market and the polluting product markets were the firms are operating.

The last section in this chapter introduces the carbon tax policy. We start considering the theoretical aspect of the policy. The Pigouvian tax equals the marginal external value, known as the social cost of carbon. There are a big number of estimates due to the lack of accurate information and this is the main difficulty to put in practice this policy. Then we explain the main conflicting elements of the policy. First we focus on the target point of application since there is a trade-off between upstream versus downstream. And finally we describe the equity and efficiency issues. Optimal taxes are the object of the second part of this thesis.

1.2   Background and context of study

Nearly all economic activities produce carbon dioxide (CO_2) and other greenhouse gas (GHG) emissions that are responsible for a phenomenon of global warming in the atmosphere with serious consequences in the climate of the planet. This climate change is the most important environmental problem that scientists face in the twenty-first century and has become a major economic and political issue. In response to mounting scientific evidence that human activities are contributing significantly to global climate change, decision makers are devoting substantial attention to public policies to reduce GHG and thereby prevent or reduce such change.

The so-called greenhouse gases include not only CO_2, but also methane, nitrous oxide, fluorocarbons (including hydrofluorocarbons and perfluorocarbons), tropospheric ozone (precursors of which include nitrogen oxides, non-methane hydrocarbons, and carbon monoxide), and Sulphur hexafluoride. However, CO_2 accounts for the bulk of the aggregate warming potential and so we will refer to it most of the time.

It has taken a long time to acknowledge the magnitude of the problem and the need for corrective policies and to a certain extent it is still a controversial issue. The first significant step in this direction took place in the United Nations sponsored Conference on Humans and the Environment (UNCHE). At that time, the UN was looking to expand its role into managing global environmental problems. Through bringing together government representatives from 114 countries, it hoped to lay the groundwork for architecture of global environmental governance that would serve the planet for decades to come.

UNCHE, byname Stockholm Conference, was the first conference that focused on international environmental issues. The conference, held in Stockholm, Sweden, from June 5 to 16, 1972, reflected a growing interest in conservation issues worldwide

and laid the foundations for global environmental governance. The final declaration of the Stockholm Conference was an environmental manifesto that was a forceful statement of the finite nature of Earth's resources and the necessity for humanity to safeguard them. The Stockholm Conference also led to the creation of the United Nations Environment Program (UNEP) in December 1972 to coordinate global efforts to promote sustainability.

Since 1972, global environmental governance has been associated with the negotiation and implementation by nation states of international (multilateral) environmental treaties and agreements on an issue by issue basis. By that time it was clear that climate change is a global common externality that presents a classic free-rider problem. The benefits of any action taken at a country level are distributed globally. That is the reason why international cooperation is essential.

After 1972, the UN sponsored two major international summits on environment and development: the UN Conference on Environment and Development (UNCED), held in Rio de Janeiro in 1992, and the 2002 World Summit on Sustainable Development (WSSD), held in Johannesburg. At Rio, two major conventions were opened for signature: the UN Framework Convention on Climate Change (UNFCC) and the Convention on Biological Diversity (CBD).

The normative focus of the summits shifted from focusing primarily on environmental protection ("the human environment") to sustainable development, conventionally defined in the 1987 Brundtland Report as development that "meets the needs of the present without compromising the ability of future generations to meet their own needs". Such a vague definition can hardly be used as a working tool but it is

important to consider its call on intergenerational justice when dealing with exhaustible resources, environmental quality or ecosystem damages.[1]

In 1988, a semi-political conference held in Toronto recommended that, as a first step, CO_2 emissions should be reduced by 20 per cent from the 1988 level by 2005. Barrett (1998) argues that this so-called 'Toronto target' was arbitrary, but the idea that countries should commit to meeting a target for emission reduction had endured and it was the background of the Kyoto protocol.

In the same year that the Toronto conference was held, the Intergovernmental Panel on Climate Change (IPCC) was formed, at the request of the UN General Assembly. The IPCC is the international body for assessing the science related to climate change. The IPCC was set up in 1988 by the World Meteorological Organization (WMO) and the United Nations Environment Program (UNEP) to provide policymakers with regular assessments of the scientific basis of climate change, its impacts and future risks, and options for adaptation and mitigation.

The startup of the IPCC represented the international recognition of the dimension of the problem and the need for a serious and accurate evaluation of the effects of the global warming in human wellbeing. The IPCC was asked to report on what was known and not known about climate change, on the potential impacts of climate change, and on what could be done to forestall and adapt to climate change. The IPCC's first assessment report, published in 1990, concluded that 'emissions resulting from human activities are substantially increasing the atmospheric concentrations of the greenhouse gases . . . [and] will enhance the greenhouse effect, resulting on average in an additional warming of the Earth's surface' (IPCC, 1990, p. 1). The report calculated that 'the long-lived gases [including CO_2] would require immediate reductions in

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[1] UN General Assembly established the World Commission on Environment and Development chaired by Norwegian Prime Minister Gro Harlem Brundtland in 1983.

emissions from human activities of over 60 per cent to stabilize their concentrations at today's levels', and it predicted that, under the 'Business-as-Usual' scenario, global mean temperature would rise by between $0,2^0$C and $0,5^0$C, and average global sea level would rise by between 3 and 10cm, per decade during the 21st century.

Article 2 of the UNFCC (United Nations 1992) commits signatory nations to stabilize GHG concentrations in the atmosphere at a level that ''would prevent dangerous anthropogenic interference (DAI) with the climate system.''

In an effort to provide some insight into the impacts that might be considered DAI, authors of the Third Assessment Report of the IPCC identified 5 ''reasons for concern'' and the IPCC Fourth Assessment Report (AR4) states that ''the reasons for concern identified in the Third Assessment Report remain a viable framework for assessing key vulnerabilities''. The reasons for concern (RFC) are:

1. **Risk to Unique and Threatened Systems.** This RFC addresses the potential for increased damage to or irreversible loss of unique and threatened systems, such as coral reefs, tropical glaciers, endangered species, unique ecosystems, biodiversity hotspots, small island states, and indigenous communities.

2. **Risk of Extreme Weather Events.** This RFC tracks increases in extreme events with substantial consequences for societies and natural systems. Examples include increase in the frequency, intensity, or consequences of heat waves, floods, droughts, wildfires, or tropical cyclones.

3. **Distribution of Impacts.** This RFC concerns disparities of impacts. Some regions, countries, and populations face greater harm from climate change, whereas other regions, countries, or populations would be much less harmed— and some may even benefit. The magnitude of the harm can also vary within regions and across sectors and populations.

4. **Aggregate Damages.** This RFC covers comprehensive measures of impacts. Impacts distributed across the globe can be aggregated into a single metric, such as monetary damages, lives affected, or lives lost. Aggregation techniques vary in their treatment of equity of outcomes, as well as treatment of impacts that are not easily quantified. This RFC is based mainly on monetary aggregation techniques available in the literature.

5. **Risks of Large-Scale Discontinuities.** This RFC represents the likelihood that certain phenomena (sometimes called singularities or tipping points) would occur, any of which may be accompanied by very large impacts. These phenomena include the deglaciation (partial or complete) of the West Antarctic or Greenland ice sheets and major changes in some components of the Earth's climate system, such as a substantial reduction or collapse of the North Atlantic Meridional Overturning Circulation

The First Conference of the Parties (COP 1) of the UNFCC took place in Berlin 1995, and established that Annex 1 Countries (basically OECD countries) will commit to targets for emission reductions. The Kyoto Protocol, negotiated in December 1997, is the first international treaty to limit emissions of greenhouse gases. It is a climate change treaty with an important difference with respect to the previous ones. Unlike the UNFCC, the Kyoto Protocol incorporates targets and timetables—that is, ceilings on the emissions of greenhouse gases and dates by which these ceilings must be met. The Kyoto protocol came into force in February 2005 and began to restrict emissions for ratified countries in 2008. The Kyoto protocol expired at the end of 2012.

After Kyoto the nations of the world were continuing negotiating mainly through the UNFCCC. Attention to climate change reached unprecedented levels in 2006, largely due to the publication of the IPCC fourth Assessment Report (AR4) that warned

of the serious consequences of doing nothing to control the build-up of GHG. It stated:
"As a result of the buildup of GHG´s, it is expected that significant climate changes will
occur in the coming decades and beyond". It was released just before the Bali
conference. The 13th Cop of the UNFCCC and the Parties to the Kyoto Protocol met in
Bali (COP 13), where the participants settled on a road map for negotiating a new
climate agreement by the end of 2009

After Copenhagen 2009 (COP 15), the Durban Negotiations (COP 17) extended
Kyoto protocol for a second commitment period from 2013 to 2020. But in Doha
negotiations (COP 18) only the European Union and Australia participate in that second
period. That means about a 14% of global emissions. In addition, many of the world´s
largest emitting countries have held a series of meetings under the auspices of the Major
Economies Forum for Energy and Climate and the G20.

Only the European Union and Australia have policies which include formal
climate policy targets. Strand (2013) argues that these countries might at later stages be
joined by other high-income countries (including Canada, Japan and the U.S.), and
perhaps also by some major emerging economies (among which China and South
Africa have already signaled a willingness to impose GHG pricing in the relatively near
future). What seems not achievable, in the near future, is a set of comprehensive and
coordinated climate policies for all GHG emitters globally. Paris 2015 is the next
opportunity to progress although no substantial advances are expected.

As long as technical economic aspects are concerned and in spite of the great
effort that has been made in the last 25 years we still have to address issues in the
evaluation of potential damages and the action plan needed to deal with the different
impacts. Three different areas which require interdisciplinary action can be identified as
the most extensively considered: Mitigation, Adaptation and Negotiation. We focus in

this thesis on the first area, considering policies oriented to reduce GHG emissions. In short, our main concern is: what are the best options and how can we ensure they are selected.

The main problem we face is that climate mitigation policies require more detailed and accurate information than the one currently available. This is an unprecedented challenge for economic science. The evaluation of damages in money terms faces enormous difficulties mainly due to the huge uncertainty about future impacts and the very long term nature of them. While reducing the uncertainty is crucial to determining the right actions, the timing problem is also critical and makes valuations highly dependent on the discount rate.

An enormous amount of academic work has been done on the economics of climate change but the most influential contributions have been the Nordhaus integrated assessment model and the Stern report.

Nordhaus (1993) introduced the DICE model as an integrated model that incorporates the dynamics of emissions and climate-change impacts as well as the economic costs of policies to curb emissions. This model extends earlier studies by integrating the economic costs and benefits of GHG reductions with a simple dynamic representation of the scientific links among emissions, concentrations, and climate change. The model was updated during the following years and the fifth version was published in 2007. There also exists a regional version named RICE.

In November 2006 the U.K. government published *The Economics of Climate Change: The Stern Review*, written by a team led by Nicholas Stern. The Stern Review (2006) makes an economic case for prompt and significant action to reduce GHG emissions. It is generally understood that the climate system is a global public good and

the emission of GHG is a massive negative externality. The Stern Review refers to it as possibly the greatest market failure in history.

These are the most influential studies on the field and astoundingly they show huge differences in the damage evaluation and the cost of inaction. The Stern report estimated that the damages under a business-as-usual scenario are between 5 and 15% of global GDP. The cost of action to the global economy would be roughly 1% of GDP while the costs of inaction could be from 5 to 20% of GDP. But Nordhaus estimates the cost of inaction in the range of 2-5% of GDP. That big difference is mainly due to three factors:

- Stern includes intangible values not considered by Nordhaus

- Discount rates are close to zero in Stern´s review while Nordhaus use the estimated market return on capital as the discount rate.

- Stern weights more the damages in poor countries

As noted by Heal (2009) there is an amazing disjunction between economists and natural scientists on this issue: most natural scientists take it as completely self-evident that the consequences of climate change justify significant actions to mitigate the buildup of GHG, whereas there is a range of opinions on this matter among economists.

Heal (2009) also argues that with such a large un-internalized externality, the business as usual scenario with no action on climate change obviously cannot be Pareto efficient, so if we move to correct the externality it must in principle be possible to make a Pareto improving (or "win–win") change to the world economy. And this point would be valid whatever the numbers.

Apart from Nordhaus and Stern, there have been numerous academic papers on the subject. Tol (2009) analyses fourteen different estimates of the global economic impact of Climate Change. Some of them are based on the so called "enumerative method" while some others use the statistical approach. The main conclusions of Tol´s study are:

1. The welfare effect of a doubling of the atmospheric concentration of greenhouse gas emissions on the current economy is relatively small—a few percentage points of GDP

2. Some estimates point to initial *benefits* of a modest increase in temperature, followed by losses as temperatures increase further

3. Although greenhouse gas emissions per person are higher in high income countries, relative impacts of climate change are greater in low-income countries

1.3   Climate Change Policies

From an economic point of view, the need for environmental policy to control the climate change is due to the fact that the atmosphere is a common property resource and in the absence of regulation would be excessively polluted. A particular implication is due to CO_2 emissions that create the greenhouse effect and the consequent increase in temperature. The need of implementing an environmental policy is based on the belief that optimal abatement is ultimately cheaper than adapting to or suffering from the damages. Therefore it looks for putting in place policies that bring about meaningful reductions in the emissions of greenhouse gases.

The four standard approaches to environmental policy are property rights, binding quota restrictions, pigouvian taxes and subsidies, and markets for pollution permits (se for example, Van der Ploeg & Withagen 1991).

Property rights are based on the argument that the market can solve the problem if property rights become explicit and transferable (Coase 1960). He argued that the market could play a substantial role not only in valuing these rights but also in assuring that they gravitated to their best use.

The rather counterintuitive thesis of Coase is that as long as property rights are well defined, and under the assumption of zero transaction costs, the market and the corresponding exchanges of rights will naturally lead to the highest valued use of resources in total, no matter how the rights were initially allocated. Under these conditions, different initial allocations will lead to different wealth and transfers among actors, but they will all lead to the same optimal outcome for a same total quantity of rights.

Binding quota restrictions are implemented through command-and-control regulations, which consist of setting specific standards to emission sources, enforced by administrative controls and penalties. Command-and-control regulatory standards are either technology-based or performance-based. The first method specifies equipment, processes or procedures, like energy efficient motors, combustion processes or landfill-gas collection technologies. Performance-based standards are more flexible. They specify allowable levels of pollutant emissions, but leaving the specific methods of achieving the target to regulated entities. Aldy & Stavins (2011) argues that uniform technology and performance standards can – in principle – be effective in achieving some environmental purposes. But, given the ubiquitous nature of greenhouse gas emissions from diverse sources in an economy, it is unlikely that technology or ordinary performance standards could form the center-piece of a meaningful climate policy.

The economic literature opposes command-and-control policies, in which the public authority set up standards and rules to directly reduce environmental damages,

with policies based on "economic tools" that aim at changing the behavior of economic agents through the modification or the introduction of prices, which reflect the cost of environmental damages in a context where traditional markets fail to account for environmental externalities. As pointed out by Tietenberg (2010) economists and policy makers developed visions of how pollution-control policy should be conducted. But these two visions were worlds apart. Policy makers preferred controlling pollution through a series of legal regulations or quantity based policies while economists promoted price based mechanisms, that is to say policies based on "economic tools" that aim at changing the behavior of economic agents through the modification or the introduction of prices.

The support for systems of tradable emission permits and environmental taxes over systems of command and control is particularly strong in the literature of environmental economics. One of the main arguments is that command-and-control regimes generally are not cost-effective. As stated by Tietenberg (2010), theory proved that command-and-control regulation typically was not cost effective, but empirical work demonstrated that the degree of inefficiency was very large indeed.

A main theoretical attraction of emissions pricing is its potential to achieve emissions reductions at lower cost than what is possible under direct regulations such as mandated technologies or performance standards. Since competitive firms equalize their marginal abatement costs to the price of pollution, notably under an emission tax rate or a price for tradable permits, a socially optimal allocation can be decentralized. This is because (a) marginal abatement costs are leveled out among all the polluters, and (b) marginal abatement costs are equalized to marginal damage.

However, while the cost-efficiency of taxes and competitive emission permit markets is independent of the product market structure, the overall efficiency of these

systems depends on the assumption of perfectly competitive markets. In the case of imperfectly competitive markets, both tax and permits systems might not lead to the optimization of the resource allocation problem. (Sartzetakis 97)

Carbon pricing (either carbon taxes or emissions trading) is viewed as a critical instrument for limiting future climate change, not only because it fosters the transition to a low carbon economy but also because it is the more cost-effective way to achieve the transition. If the world is to succeed in reducing emissions in line with the two-degree target, an international price on carbon emissions must be established. And carbon pricing is considered our most important policy instrument in the fight against global climate change. Major options for carbon pricing are carbon taxes, cap and trade systems, emission reduction credits, clean energy standards and fossil fuel subsidy reductions. This thesis explores only some aspects of the two main tools: pigouvian carbon taxes and tradable permits. While the academic literature is in general more in favor of taxes, cap and trade programs have been used in USA and Europe as the unique tool with the only exception of Scandinavian Countries. We review the main contributions to clarify this field.

1.3.1 Tax or Cap

There is wide agreement among economists as to the potential advantages of emissions pricing, but there is much debate as to which particular form – carbon taxes or cap and trade – is the better climate policy option. In this subsection we summarize the most important arguments used in the literature.

Authors in favor of carbon taxes note that the overall administrative costs are higher under a cap-and-trade program. As argued by Goulder and Schein (2013) the reason is that cap and trade imposes an additional administrative responsibility: the

regulator must not only monitor emissions but also establish a registry for allowances and keep track of allowance trades and the associated changes in ownership of allowances. Historically in general emissions trading has been targeted at large sources (e.g. electrical generation and emission-intensive facilities), while taxes have been targeted on more diffuse sources such as household and transport emissions.

Price volatility is another issue for a cap-and-trade system. The supply of allowances is perfectly inelastic, hence shifts in demand can cause significant price changes – and irregular shifts in demand can produce price volatility. Nordhaus (2009) notes that demand for allowances is also likely to be highly inelastic in the short run, leading to even greater potential for high price volatility. He argues that allowance trading programs' price volatility represents a reason to favor carbon taxes over cap and trade.

Uncertainty has been used as an argument to favor both systems. The price versus quantity issue does not look, a priori, like a timing issue: the question is whether an environmental policy should aim at controlling the user price of the polluting good or its quantity. A carbon tax and a cap-and-trade program address uncertainty differently. The carbon tax stipulates the price of emissions, while leaving uncertain the aggregate emissions level. Cap and trade stipulates aggregate emissions, leaving the price uncertain.

The fact that a carbon tax does not guarantee that emissions will be kept within a given limit is considered by some groups a crucial liability. Under a carbon tax it remains possible that emissions will significantly exceed the levels considered safety. At the same time, some business groups highlight the fact that cap and trade leaves prices uncertain. They emphasize that uncertainty about emissions prices constrains the business community's ability to respond to climate policy: changing the input mix and

investing in research toward new technologies is more risky when future allowance prices are uncertain.

The question of whether it would be better to control certain forms of pollution by setting emission standards or by charging the appropriate pollution taxes was firstly addressed by Weitzman (1974 & 1978) who established that under complete knowledge and perfect certainty, both models are identical, while in a world where planners do not have the required information and outcomes are uncertain there is a relative advantage in regulating prices as the number of production units becomes larger or the level of substitution among products becomes greater. He also compared the expected efficiency gains under uncertainty of a price-based approach (as with carbon taxes) and a quantity-based approach (as with cap and trade). The relative advantage depends on the slopes of the functions that express marginal environmental damages and marginal costs as functions of emissions. The quantity-based approach emerges as superior when the marginal damage function is relatively steep; otherwise the price-based approach is more attractive.

Kaplow and Shavell (2002) emphasize the role of a dubious set of assumptions in their critique of Weitzman (1974). First they argue that taxes are constrained to be linear even though marginal harm is taken to be non-linear (rising in the quantity of emissions) which violates the basic Pigouvian prescription. And secondly taxes are taken to be fixed for all time.

Goulder and Schein (2013) examine the relative attractions of a carbon tax, a "pure" cap-and-trade system, and "hybrid" options (a cap-and-trade system with a price ceiling and/or price floor) and arrive to the following conclusions:

1) Policies that specify emissions prices exogenously have several attractions relative to policies that do not. Emissions prices are exogenous under the carbon tax: the

specified carbon tax rate is the emissions price. A hybrid system – that is, a cap-and-trade system that establishes a ceiling and/or floor price – also has exogenous prices when the floor or ceiling price is in effect. Exogenously specified prices confer several attractions. One is that they prevent emissions price volatility. Another is that they are likely to minimize expected policy errors in the face of uncertainties about benefits and costs. Two additional and important attractions – which have received relatively little attention – are that exogenous prices help avoid problematic interactions with other climate policies, and avoid large wealth transfers to oil exporting countries.

2) There are four dimensions along which, contrary to frequently made claims, the two approaches are equivalent.

a) The carbon tax and cap and trade offer qualitatively equivalent incentives to reduce emissions, regardless of whether the allowances are introduced through auction or free provision. Even when allowances are received for free, each additional unit of emissions carries an opportunity cost

b) In principle any distributional outcome under cap and trade can be matched via a carbon tax. The same redistribution brought about through free allocation of allowances can be produced through the granting of partial or full exemptions to the carbon tax.

c) The potential for downstream implementation is not exclusively enjoyed by a carbon tax. A cap and trade system can also be introduced downstream.

d) An offset is a credit for emissions reductions achieved by an entity in a sector that is not covered by a given CAT system. However it is possible to include or

exclude offsets as part of a carbon tax program. Thus, considerations relating to offsets have no bearing on the choice between cap and trade and the carbon tax.[2]

Some other authors have criticized potential deficiencies in the CAT programs. Pratlong (2005) argues that a high abatement cost industry (regulated through a tradable permits system) can increase its market shares compared to its rivals (regulated through pollution taxes system). The reverse applies for a low abatement cost industry. He concludes that a permits system is not always beneficial for each industry.

Fischer and Preonas (2010) have pointed out a potentially important advantage of a carbon tax over cap and trade. They have shown that, in the presence of a cap-and-trade program, introducing an additional GHG-reducing policy such as a performance standard might yield no further reductions in overall emissions. The reason is that overall emissions are determined by the overall cap or, equivalently, by the number of allowances in circulation. In contrast, introducing an additional GHG-reducing policy in the presence of a carbon tax can lead to a reduction in overall emissions.

This theoretical debate is completed and influenced by practical experiences. Both price-based and quantity-based instruments have effectively been used to control environmental externalities, for example carbon taxes in Northern Europe or SO_2 emission trading system in North America in the beginning of the 1990s. The European Union, which was initially planning to establish a tax on carbon emissions in the same period, eventually favored an emission trading system after the negotiation of the Kyoto Protocol, in order to help the implementation of the Kyoto Protocol among European Member States. This led to the creation of the European Union Emission Trading System (EU ETS), to date the largest system in the world which effectively puts a price on the greenhouse gas emissions of energy intensive industries.

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[2] The authors cited the example of Australia´s emissions pricing program that allows the use of some offsets during its "fixed price period" from 2012 to 2014. A fixed price is essentially a carbon tax.

Traditionally, the analysis on the choice of instruments has focused on identifying whether carbon taxes or emissions trading is the superior instrument but nowadays it tends to frame the issue in terms of how they can best be combined. In Europe, while the EUETS system is already in place, there are also energy and carbon taxation schemes in several EU member states that have been guided by the so-called environmental tax reform (ETR). This reform of national tax systems seeks to shift the tax burden from conventional sources, such as labor and capital, to alternative sources such as environmental pollution or natural resource use. Another case came recently from United Kingdom, which decided to impose a tax on CO_2 emissions caused by electric power generators in the country. For each unit of emissions, these generators had to pay this tax in addition to the price that they paid for EU ETS emissions allowances.

1.4 Emission Permits Market

In cap-and-trade programs (CAT), regulators impose a cap on the total quantity of emissions permitted and distribute a corresponding number of tradable emission permits. Emission permits markets are based on legal regulations for the limitation of certain economic activities (e.g. taxi licenses in large cities). When emission trading is used to set the level of allowable emissions, the regulator does not need to know the damage or the cost functions to achieve cost effectiveness, since the price mechanism and the transferability of rights will end up in accomplishing the required reduction at the lowest cost.

Coase's contribution laid the basis for the development of a new approach to economics and to environmental regulation in particular. If factors of production are thought of as rights, the right to do something which has a harmful effect can also be a

factor of production. The right to damage the environment up to a certain point, seen as a limited factor of production, can be materialized as a tradable right. The regulator has to define the total number of rights and ensure their legal force, but do not directly fix their price. The price of permits is determined by exchanges between entities on the market, which under perfect conditions, leads to the most efficient use of the permits. In this case, the environmental goal is obtained at the least possible cost, without the regulator having to evaluate *ex ante* the costs involved.

In an emission trading program, the volatility of allowance prices can undermine the climate policy, what explains the deep attention given to "cost containment" measures. These measures include offsets, banking and borrowing, safety valves and price collars.

An offset provision links a CAT system with an emission-reduction credit system. The most important version of this measure is the Clean Development Mechanism (CDM) where abatement of emissions can be purchased from countries that do not have a climate policy in order to comply with the requested cap. An objective of the CDM is to make it easier (and less costly) for emitters in the policy countries to reach their emissions caps.

Banking of permits occurs when regulated entities are allowed to hold unused permits for future compliance. The cap considers cumulative emissions over a period of years, rather than a cap on annual emissions. Banking thus diminishes the supply in the short term, but raises the supply in the future. Banking makes it difficult for the price of allowances to fall down to zero as long as the anticipation horizon is distant or highly uncertain

Borrowing is symmetrical to banking. In this case, permits from future compliance periods can be used in advance. Borrowing thus diminishes demand in the

short term, but raises demand in the future because the allowances used in advance have to be paid back. Nevertheless in the case of a price spike, borrowing can prove to be an efficient short term response.

A safety valve puts an upper bound on the costs of abatement by offering additional allowances at a predetermined fee (the safety valve "trigger price"). This is a hybrid approach to climate policy: a cap-and-trade system that transitions to a tax under high mitigation costs, although in this case, the aggregate emissions exceed the emission cap.

Murray et al (2009) proposed a mechanism that includes features of both price and quantity instruments called allowance reserve. While the safety valve stipulates that an unlimited number of allowances be made available at the specified safety valve price, the allowance reserve stipulates both a ceiling price at which cost relief is provided and a maximum number of allowances to be issued in exercising that relief. Much like a safety valve mechanism can mimic either a pure price or pure quantity control, depending on how the cap and safety-valve price are set, an allowance reserve can mimic a pure price, pure quantity, or safety-valve approach, depending on how the ceiling price and volume are set.

A price collar combines the ceiling of a safety valve with a price floor created by a minimum price in auction markets or a regulator commitment to purchase allowances at a specific price

Nowadays, emission permit markets are being used at an unprecedented scale to regulate externalities, and it is likely that they will play a key role in any future international agreement concerning GHG emissions. They have been used with success in other environmental domains as well as for pricing CO_2 emissions. The U.S. sulfur dioxide (SO_2) cap and trade program cut U.S. power plant SO_2 emissions more than

50% after 1990 and resulted in compliance costs one half of what they would have been under conventional regulatory mandates (Carlson et al, 2000). The success of the SO_2 allowance trading program motivated the design and implementation of the EU ETS, the world's largest cap and trade program, focused on cutting CO_2 emissions from power plants and large manufacturing facilities throughout Europe (Ellerman & Buchner, 2007).

1.5 Carbon Taxes

Environmental taxes are usually established according to the Pigouvian rule.[3] The appropriate policy to internalize the externality, according to Pigou, involved imposing a per unit tax on emissions. The tax rate should be set equal to the marginal external social damage caused by the last unit of pollution at the efficient allocation. While this policy can be identified and found out within theoretical models, it is very difficult to put in practice since it requires knowing the marginal damage at the equilibrium point.

The marginal damage cost of carbon dioxide, also known as the "social cost of carbon" (SCC), is defined as the net present value of the incremental damage due to a small increase in carbon dioxide emissions. For policy purposes, the marginal damage cost (if estimated along the optimal emission trajectory) should be equal to the Pigouvian tax that could be placed on carbon, thus internalizing the externality and restoring the market to the efficient solution.

SCC is nowadays a very important concept in global warming economics and has attracted a massive academic attention. Tol (2008) makes an analysis of over 200

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[3] The theory behind the integration of environmental damages in the economy dates back to the work of English economist Arthur C. Pigou (1920) which remains central to modern welfare economics and particularly to environmental economics.

estimates and conclude that the big differences found are driven to a large extent by the choice of the discount rate, reason why the estimation of the Stern report is the highest one representing an outlier in the sample. He also found a downward trend in the estimates, in clear contradiction with the 2007 IPCC report (AR 4). In 2013 the same author reported 75 studies with 588 estimates.

Nordhaus (2011) estimated social cost of carbon for the current year (2015) considering uncertainty, equity weighting, and risk aversion is $44 per ton of carbon (or $12 per ton CO_2) in 2005 US$ and international prices. Uncertainty increases the expected value of the SCC by approximately 8 percent. All these data are based on the RICE 2011 model. This value is in big contrast with the one given by Stern ($85 per ton CO_2)

Another central point in pricing carbon through a tax is the election of the target point of application. Carbon tax can be established at a rate per ton of CO_2 emissions or based on the carbon content of the three main fossil fuels (coal, petroleum and natural gas). The carbon tax could be applied at a variety of points in the product cycle of fossil fuels, from the suppliers like refineries or importers (upstream taxation) to final emitters at the point of energy generation (downstream taxation). In general terms emissions can be controlled indirectly (via "upstream" targeting), more directly (via "downstream" targeting), or via a hybrid involving some combination of the two. In an upstream system the taxes or allowance requirements would be targeted at the point of extraction, production, import, processing, or distribution of substances. A downstream point of regulation would focus control on the point of use, where emission into the atmosphere would occur.

Mansur (2010) examines the tradeoffs of regulating greenhouse gases (GHG) upstream versus downstream. He sets out some key issues in deciding what level of a

vertical chain of industries to target in designing regulation. He concluded first, that upstream regulation could substantially reduce transactions costs. He argues that incomplete regulation will affect the types of goods produced, traded, and consumed, if all nations do not harmonize carbon prices and concludes that the magnitude of the regulatory leakage depends on whether the policy regulates firms upstream or downstream.

According to Metcalf and Weisbach (2009), leaving aside international trade concerns, the best place to impose the tax would be at a point in the supply chain where carbon content could be easily measured and the number of taxpayers relatively small. For coal, this would be at the mine, for petroleum at the refinery, and for natural gas at processing facilities or, for those that bypass them, the wellhead. Marron and Toder (2014) noted that going upstream to oil wells and importers would expand administrative and compliance burdens without increasing the effectiveness of the tax, while going downstream from these points would weaken the link between the tax and actual carbon emissions.

Aldy and Stavins (2011) argues that focusing on the carbon content of fuels, or upstream taxation, would enable the policy to capture about 98% of US CO_2 emissions, with a relatively small number of covered firms. A crediting system for downstream sequestration could complement the emission tax system including potentially emission-reduction projects (offsets) in other countries.

The big issue that always arises when carbon taxes are considered is equity. It is argued that the tax will raise the price of energy and poor households will be seriously penalized. The cost burden of a carbon pricing program is estimated to be regressive (particularly for non-transport emissions in industrialized countries) in the absence of any redistribution of the revenues because lower income households use a larger

proportion of their earnings to purchase energy intensive products (gas and electricity being the most important).

Analysts generally assume that a carbon tax would be passed forward onto consumers both directly in higher prices for their energy purchases and indirectly in higher prices for other goods and services based on the carbon-intensity of production but analysts typically focus on the long run when most costs will be passed onto consumers. Like other taxes on consumers, a carbon tax would be regressive: its burden would be higher as a share of income for low-income households than for high-income ones because low-income households consume a greater share of their income and spend relatively more on carbon- intensive goods and services

Some authors have proposed to address this issue by using the proceeds in a way that offsets the regressivity of this kind of taxes. Nordhaus (2009) argues that the tax system raises substantial revenues that can be used to alleviate the economic hardships of low-income households through reducing other taxes or increasing benefits. Alternatively, some of the tax revenues could be used for research and development on low carbon energy systems

Mathur and Morris (2012) found that in the USA, a carbon tax averaging 1.7 percent of consumption imposes a burden of 2.1 percent of consumption in the bottom decile, but only 1.3 percent in the top decile.

Tax relief could offset the disproportionate effect of the tax on the poor and reduce the economic distortions of the existing tax system. Such a relief could take the form of lower income or payroll taxes or new tax credits.

Marron and Toder (2013) estimate that offsetting 50 percent of carbon tax revenues with a refundable tax credit and 50 percent with a cut in the corporate income tax rate would leave both low-income and upper-income households better off but raise

net taxes on middle-income households. Adding payroll tax cuts to the mix would redistribute some of these benefits to the middle class.

Jointly to the equity issue appears the efficiency issue. The analysis of the trade-offs between economic growth and climate policies fighting global warming is central to the research on optimal carbon taxes. To date, academic researchers have relied heavily on deterministic neoclassical growth models called IAM (Integrated Assessment Models), like DICE or the one by Golosov et al (2014) to address optimal climate policy.

The optimal tax results in a first best policy are heavily dependent upon some key assumptions like the level of fossil fuel proven reserves or the backstop technologies. Other key assumptions like discount rates, social cost of carbon and uncertainty levels have already been discussed in this chapter.

Second best policies have also been investigated, but subjects like the double dividend or the role of distortionary taxes in climate policies are well beyond our scope.

1.6 Outline of the thesis

This thesis performs an evaluation of some particular aspects of the main climate change policies based on economic incentives. Particularly we examine the role of market power in the behavior of firms participating in emission permit markets, and the impact of extraction costs and proven reserves in the optimal carbon taxes setting.

The remainder of the document is divided in two distinct parts: chapters 2, 3 and 4 are focused on tradable emission permits while chapter 5 deals with environmental taxes.

In the first part, our emission permits study covers three chapters: The first one (Chapter 2) consists of a comprehensive literature survey on theoretical models dealing

with the market power issue. Our aim in this chapter is to summarize the main findings in one simple and canonical model that will be the base for the next two chapters.

The second chapter of this part (Chapter 3) is dedicated to the analysis of the existence of scarcity rents and the possibility that such rents provide incentives for strategic price manipulation when the correspondent output market is oligopolistic but firms are price takers in the emission permits market. Our work is closely related to Sartzetakis (97 and 2004) and Erhard et al (2008) who used a Cournot model to analyse the impact of market power in efficiency and welfare. We use a Stackelberg model which is the main difference with them, since we are interested in studying the role of output asymmetries in the firm´s behavior. We particularly focus on the impact of scarcity rents and we compare the results of this analysis under different oligopolistic structures.

The third chapter of this part (Chapter 4) covers again Cournot and Stackelberg oligopolies in the product market but introducing a dominant firm in the emissions permit markets which make endogenous the permit price. We show the key role that grandfathering is playing at the level of output and profit to compensate the leadership advantage of one firm in the output market

Optimal climate policy is investigated in an IAM very close to Golosov et al (2014). These models are widely used to evaluate the size of the social cost of carbon and hence the optimal carbon tax. Our model is a neoclassical growth model in discrete time.

We firstly deal with theoretical aspects of the optimal tax when varying extraction costs are considered. Our main contribution is the analysis of the stock effect impact on the dynamics of the optimal carbon tax and their relationship with the green paradox. We examine conditions under which the optimal carbon tax increases or

decreases based on the level of reserves and assumptions for the stock-dependent oil extraction costs.

Secondly we perform a quantitative assessment, also in line with Golosov *et al* (2009) parameterization, to explore whether the role of extraction costs are significant in the optimal tax setting.

Chapter 2

Market Power and Emission Permit Markets

2.1 Introduction

As we have discussed in Chapter 1, there are two ways of introducing a price that incorporates environmental externalities in the markets: price-based and quantity-based regulation. Quantity-based policies usually consist of cap-and-trade (CAT) or baseline and credit programs, which create tradable emission rights that aim at reducing pollution emissions in the most efficient way, i.e. in the sense of minimizing the total cost of pollution abatement.

The CAT programs are attractive from an economic point of view for, as long as marginal abatement costs differ, incentives for trade exist and the market can play a positive role in achieving a pre-specified target at a minimum cost. In addition to this practical advantage, emission trading allows pollution emitters some flexibility to comply with the regulation, either reducing emissions or acquiring emissions reductions

from other firms. All with the aim of being economically efficient (cost effective).

Emissions trading change the nature of the regulatory process with respect to traditional command-and-control policies. The burden of identifying the appropriate control strategies is shifted from the control authority to the polluter. Tradable permits allow flexibility in the timing of control investments and different pollution reduction strategies can take place. Temporal flexibility in emissions trading is provided by banking, borrowing and advance auctions. Banking allows holding allowances beyond their designated year for later use. Borrowing allows an allowance to be used before its designated date. Advance Auctions sell allowances that can be used after some future date, commonly 6 or 7 years hence.

CAT programs represent nowadays a common tool used by authorities to regulate pollution emissions, although this policy instrument is not without its own limitations. To mention one of the most obvious, a policy relying on quantity restrictions, by definition requires an estimate of the optimal amount of emissions, which is not an easy task to accomplish. Tradable emissions permit systems[4] (TEP) are in place for several pollutants at national levels within Europe and the US. In the US there already exists a nation-wide TEP system for sulphur-dioxide (Verbon & Whitagen 2005). A TEP system was implemented for the entire EU for greenhouse gas emissions. Such a system, known as EUETS (European Union Emission Trading System) is nowadays the most important emissions market in the world.

One of the central results in the literature shows that a CAT system is effective in attaining the pollution reduction objective at the least cost under a set of assumptions. But this result is challenged by the violation of some of those assumptions. The literature has mainly focused on three important aspects of CAT systems: Different

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[4] TEP refers to CAT programs in a little more restricted way, only including local and global pollutants.

mechanisms to make the initial allocation of permits, banking and borrowing regulations and market imperfections. After a brief review of all three aspects, this work mainly focuses on the third one, imperfect competition.

DIFFERENT PERMITS ALLOCATION.

Two main methods are considered as the most important to distribute permits among polluters. The first one, which consists of free allocation of permits based on past emissions, is called grandfathering. The second one is auctioning the permits. We will focus our study along this and the next two chapters on the grandfathering case, due to its relevance, notably in the EUETS market implementation and its impact on effectiveness and welfare. Grandfathering involves a transfer of wealth, equal to the value of the allowances, to existing firms, whereas, with an auction, this same wealth is transferred to the government. To mitigate potentially adverse competitiveness impacts, and to engender political support for the program, it has become standard to allocate some percentage (or all) of these emissions permits for free to industrial stakeholders (Joskow and Schmalensee, 1998; Hahn and Stavins, 2011).

Under a grandfathering regime, permits are freely distributed to regulated sources based on a pre-determined criterion, such as historic emissions. At least two different issues regarding grandfathering have been investigated: The effect of sequential versus simultaneous allocation on the one hand and imperfect competition on the related output market on the other hand.

The sequential version was used in the early phases of the EUETS, when domestic permit allocations were often announced at separate times.[5] MacKenzie (2011) concludes that this option may result in strategic behavior from the different countries

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[5] This secuential allocation can happen in schemes with sovereign governments, if the regulator allows them the option to announce their permit allocations at different dates.

involved. Each government announces an allocation level in order to maximize social welfare in its region and social welfare can be reduced because aggregate emissions differ between sequential and simultaneous allocation announcements.

Imperfect competition allows firms to pass pollution costs on to consumers. If they receive permits for free, they essentially get reimbursed for costs they never had to incur. According to Hintermann (2011), existing firms favor freely allocated tradable permits not only because they convey rents (known in the literature as windfall profits), that represent a wealth transfer from consumers to firms, but also for the fact that it sets entry barriers, as long as the newcomers have to purchase permits.

BANKING AND BORROWING REGULATIONS

With banking of permits, one agent that has reduced emissions more than indicated by the permits it holds, can keep the excess permits for future use. Thus, if borrowing is allowed, an agent can increase its emissions in excess of the permits it holds against future emission reductions

Banking and borrowing, as opposed to the basic trading case, change the nature of standards since firms can emit above the standard at some points in time. When social damages from emissions are related not only to the cumulative emission level, but also to the emission level in each period, least-cost allocation of emissions for agents through time can be different from the socially optimal emission level in each period.. Hence, restrictions on banking and borrowing can be required, for instance, by only allowing a certain amount of permits to be banked and borrowed through time.

MARKET COMPETITION

Unfortunately, perfect market assumptions rarely hold in practice. Indeed, emissions permits markets can suffer from several impediments, such as uncertainties, transaction costs (see Cason & Gangadharan, 2003; Montero, 1997), market power (see Hahn, 1984; Liski & Montero, 2005; Misiolek & Elder, 1989; Hinterman 2011), and imperfect compliance behaviors (Keeler, 1991; van Egteren & Weber, 1996; Malik, 1990, 2002; Arguedas et al 2010).

These problems did raise questions regarding the cost effectiveness of CAP policies and cost bearing. In order to understand and explain the way in which the market works, a number of models were developed that can be static or dynamic, and consider different market structures, like Cournot, Stackelberg or models of conjectural variations.

Our investigation aims at a consolidation of this work. It brings together the different strands, and highlights the main results in one specific model. For this purpose in Section 2.3, we set up a two period model where two firms compete in a polluting product market and receive some tradable permits by means of grandfathering. We specifically focus on cross-market links when firms maximize profits in both markets.

2.2 Market Power Literature Review

The theory behind emissions trading was formalized by Baumol & Oates (71) for the case of uniformly mixed pollutants and Montgomery (1972) for the case of non-uniformly mixed pollutants. Baumol & Oates proposed to establish a set of arbitrary standards and then impose a set of charges on emissions sufficient to attain these standards. They admitted that the system do not generally produce a Pareto-efficient allocation of resources, but they also showed some important optimality properties as

achieving a specified reduction in pollution levels at a minimum cost.

Under the assumption of competitive markets, cost effectiveness is achieved regardless of the allocation rule chosen (Montgomery 1972). Under the assumption of competitive permits market, the initial allocation of emission permits does not affect the equilibrium, and the allocation scheme does not influence the results (Sartzetakis 1997).

Cost effectiveness holds when permits are bankable in a competitive permit market with perfect foresight, but only if all the firms are not subject to profit regulations (Cronshaw & Kruse 1996). Rubin (1996) considers banking and borrowing of quotas in an intertemporal, continuous time model. Within this framework he shows that in a competitive permit market an equilibrium solution exists and is cost effective, which means that marginal abatement costs are equalized across all agents (static optimum), present values of marginal abatement costs are equalized (dynamic optimum) and there are equal marginal abatement costs across agents and across periods (cost effectiveness)

But the majority of emissions regulated under existing and planned regulations come from industries that are highly concentrated like electricity, cement or refining. Therefore it is not realistic to assume perfect competition and accordingly a literature analyzing the relationship between imperfect competition and emission permits has been developed. Since the results are sensitive to the assumption of competitive market, either in emission permit markets or the polluting product market, this literature can be divided in three different lines where market power is introduced on the permits market, the good market or both simultaneously.

The first line considers market power just in the permit market. The ground-breaking paper is Hahn (1984), which, based on a static model a la Stackelberg, stated that the efficiency loss due to market power depends on the initial allocation of permits

and the permit price is an increasing function of the leader's allocation. The dominant firm will manipulate the price (upwards if it is a seller and downwards if it is a buyer) unless the initial allocation equals the cost-effective one, which requires a perfectly informed regulator. Hagen & Westskog (1998) extended the Hahn setting in a dynamic two-period model and found a non-optimal distribution of abatement in an imperfectly competitive market with banking and borrowing.

On the second line, authors have addressed the issue of imperfect competition in the goods market combined with perfect competition in the permit market. Some articles have shown that perfect competition in the permit market might not be sufficient to render a cost-effective outcome if the product market is not perfectly competitive. Within the framework of a Cournot duopoly, Sartzetakis (1997) compares the efficiency of a competitive emissions market to a command-and-control regulation. Emissions trading modifies the allocation of emissions among firms and hence their production choices. Sartzetakis (2004) shows that welfare can decrease when emission trading is allowed between asymmetric firms endowed with different abatement and production technologies. The permit price that clears the market is a weighted average of the value of emissions of firms under command and control and therefore the cost of the more inefficient firm is reduced while the cost of the more efficient one is increased when permit trading is introduced.

According to Liski & Montero (2005) borrowing of permits from future vintages could also be included, and may be efficient. For both to actually happen, permits allocations must decrease overtime and at least at a rate higher than the discount rate for some period of time. They also concluded that a dominant firm exhausts its stock of banked permits slower than a competitive firm, because the dominant firm will manipulate the permit price upwards.

Ehrhart, Hope and Löschel (2008) use game theoretical methods to show that, if the output market is not perfectly competitive, firms may have incentives to collude in the permit market even if there is no explicit market power on the latter. These authors conclude that, under certain circumstances an increase in the permit prices lead to higher profits due to a decrease in product quantities as a result on a higher cost and a decrease in the output.

Meunier (2011) analyzes the efficiency of emission permit trading between two imperfectly competitive product markets and conclude that even if firms are price takers in permit markets, the integration of permit markets can decrease welfare because of imperfect competition in product markets. Theoretically, if markets are perfectly competitive, a unique global permit market that covers all polluting activities would be efficient to allocate an aggregate emissions level. If markets are not perfect, but some firms enjoy market power, several permit markets may be more efficient than an integrated one.

Chapter 3 of this dissertation fits in this second line of research and extends the Ehrhart et al (2008) model to analyze an oligopoly ala Stackelberg, introducing cost asymmetry between agents.

A third line addresses the concurrent existence of market power in both permit and output markets. Misiolek & Elder (1989) extended Hahn's setting to the product market and concluded that a single dominant firm can manipulate the permit market to drive up the fringe firm's cost in the product market. Hinterman (2011) found that the threshold of free allocation above which a dominant firm will set the permit price above its marginal abatement costs is below its optimal emissions in a competitive market, and that overall efficiency cannot be achieved by means of permit allocation alone.

Tanaka & Chen (2012) consider a Cournot-fringe model with market power in

both product and permits market to simulate the California electricity market and they show that Cournot firms can significantly raise both power price and permit price, which results in a great loss in social surplus.

The market power in both markets lead to results that are based on the fact that the dominant firm may increase its profits by increasing industry costs. In the industrial organization literature this strategy is known as "raising rivals´ costs".

Chapter 4 of this dissertation fits in the third line. We will consider an Stackelberg model where one firm is dominant in both markets and the same model with two different leaders, one for the output market and another one for the permit market.

2.3 A Canonical Model

In this section, we develop a two-period partial equilibrium model with two markets. The permit market can integrate several polluting sectors as designated by the regulator and it allows full banking and borrowing. One of these sectors is composed by firms that produce a final good (energy, for instance) by emitting a global pollutant. The model is conceived for the analysis of different market structures in both markets.

Our aim is twofold. First we try to show in (slightly modified versions of) one single model some of the basic results of the literature under different market structures. Second, we set up a framework that will be used in the following chapters to explore some specific cases in order to answer some questions that have not yet been considered in academic literature, as far as we know. Our model is based in the conjectural variations structure as presented for example in Ehrhart et al (2008). Modified versions of this framework will be the base of the Stackelberg model that will be developed in the next chapters.

2.3.1 Basic Elements

We consider two firms labeled as i = 1, 2 and two periods j = 1, 2. We denote the variables with subscripts where the first subscript refers to the agent (i), and the second is related to the period (j), unless total variable per period, which only has a time subscript. For example, we denote output of firm 2 in period 1 as x_{21} and total output of period 1 as X_1. Individual firms will be considering in analyzing competitive output market, Cournot duopoly and Stackelberg model.[6]

Let $X_j := x_{1j} + x_{2j}$ be output quantity for period j. Firms face the inverse demand function $P(X)$, where $\frac{dP}{dX_j} < 0$. Production results in emissions of a pollutant. We denote as e_{ij} the emissions net of abatement of firm "i" in period "j". The cost function of firm I at time j, $C_{ij}(x_{ij}, e_{ij})$, depends on output and emissions and is continuous and twice differentiable in both arguments with the following properties:

$$\frac{\partial C_i}{\partial x_i} > 0 , \quad \frac{\partial C_i}{\partial e_i} < 0 , \quad \frac{\partial^2 C_i}{\partial e_i^2} > 0 , \quad \frac{\partial^2 C_i}{\partial x_i \partial e_i} < 0. \tag{1}$$

These cost assumptions are common in the emission trading literature. They mean that cost is increasing with respect to output and decreasing and convex with respect to emissions. These assumptions can also be thought as involving a positive and increasing marginal abatement cost. The fourth condition implies that marginal cost is decreasing in emissions.

Under a cap and trade scheme, emissions become a factor of production that has to be paid for. As highlighted by Hahn (84), the assumption of downward sloping demand curves for emission permits is equivalent to the assumption that marginal

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[6] In some imperfect competition models in the literature, there is one dominant firm and a fringe of competitive firms. To some extent, the Stackelberg version of our model can be linked to this type of model by considering the firm 1 as leader and there is a single price taking firm indexed by i = 2.

abatement costs are increasing. This assumption implies that the firm attains a regular minimum in solving the profit maximization problem.

The firms can receive an initial endowment of permits in each period and they can go to the market in order to purchase additional required permits or sell remaining ones. Let $S_j = S_{1j} + S_{2j}$ represent total and individual endowments of period j and let y_{ij} represent the purchases or sales by firm i to firm j A positive value of y_{ij} corresponds to a purchase and a negative value to a sale of permits by firm i. Unused permits may be banked (saved for next period use) or sold. Let B_{ij} be the permits banked in period j by agent i.

The stock of permits evolves overtime according to the following banking conditions:

$$B_{i1} = S_{i1} - e_{i1} + y_{i1}$$
$$B_{i2} = S_{i2} - e_{i2} + y_{i2}$$

(2)

The amount of permits banked by firm i in period 1 equals the difference between the total amount of permits hold by the firm (initial endowment plus purchases) and its total emissions net of abatement. At the end of the second period it is optimal for both firms not to bank any permit. $B_{i2} = 0, i = 1, 2$ We also assume, as it is frequently seen in the literature that the emissions generated by both firms can be perfectly monitored without cost by the regulatory authorities, which implies that firms cannot emit more than the holdings of permits allows them to.

The competition in the product market is modeled using the conjectural variations model of oligopoly.[7] In this approach instead of considering that each agent

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[7] Conjectural variation is typically viewed as a reduced form approximation to the repeated dynamic games. The conjectural variations model includes monopolistic and competitive behavior as special cases. This model is discussed by Bresnahan (1981) and Seade (1980) and is surveyed by Dixit (1986).

takes the action of the rival as given, agents form conjectures about the rival's response to their own strategy.

If we assume that both firms have constant and identical conjectural variations equal to δ, it follows that:

$$\delta = \left(\frac{\partial x_2}{\partial x_1}\right) = \left(\frac{\partial x_1}{\partial x_2}\right); \frac{dX}{dx_{11}} = \frac{dx_{11}}{dx_{11}} + \frac{dx_{21}}{dx_{11}} = 1 + \delta \tag{2a}$$

which means that when one firm increases its own production by one unit, it conjectures that total output in the market will increase by $1 + \delta$ units.

The attractive of this approach is that it can be seen as a general framework to include different well known market structures that can be seen as particular cases by taking particular values of δ. Cournot equilibrium is obtained when δ equals zero. The competitive equilibrium corresponds to the case in which δ equals minus one and a collusive cartel is obtained when δ equals one.

We can also assume firms have constant but different conjectural variations. In this case

$$\delta_1 = \left(\frac{\partial x_2}{\partial x_1}\right); \frac{dX}{dx_{11}} = \frac{dx_{11}}{dx_{11}} + \frac{dx_{21}}{dx_{11}} = 1 + \delta_1$$

$$\delta_2 = \left(\frac{\partial x_1}{\partial x_2}\right); \frac{dX}{dx_{21}} = \frac{dx_{21}}{dx_{21}} + \frac{dx_{11}}{dx_{21}} = 1 + \delta_2 \tag{2b}$$

The Stackelberg equilibrium in the case that firm 1 is the leader, corresponds to δ_2 equals zero and δ_1 equals minus one half. To have a full description of the model we need to determine whether the output and the permit markets are competitive or not. In what follows we study each case separately. Specifically we analyze a competitive output market in subsection 2.3.2 and imperfect competition in the output market in subsection 2.3.3

2.3.2 Competitive Output Market

In this subsection we study the case in which the output market is competitive, which in turn, gives raise to two different scenarios depending on whether the permit market is competitive or not.

Competitive permit market

We consider first that both markets (output and permit) are competitive. Let β be the discount factor. Firm i faces the following two-period profit maximization problem

$$Max_{\{x,e,y,B\}} P_1(X) x_{i1} - C(x_{i1}, e_{i1}) - p_1 y_{i1} +$$
$$+ \beta \left[P_2(X) x_{i2} - C(x_{i2}, e_{i2}) cx_{i2} - p_2 y_{i2} \right]$$

$$s.t. \quad B_{i1} = S_{i1} - e_{i1} + y_{i1}$$
$$B_{i2} = S_{i2} - e_{i2} + y_{i2} + B_{i1} \tag{3}$$
$$B_{i1} \geq 0$$
$$B_{i2} \geq 0$$

Let λ_j and μ_j be the multipliers on the equality and inequality constraints respectively. The Kuhn-Tucker conditions related to the variables that are linked to the emissions permit market (y,e,B) are the following:

$$FOC(y_{i1}) \Rightarrow \lambda_1 = p_1 \tag{4}$$

$$FOC(e_{i1}) \Rightarrow -\frac{\partial C}{\partial e_{i1}} = \lambda_1 \Rightarrow e_{i1} = f(p_1) \tag{5}$$

$$FOC(y_{i2}) \Rightarrow \lambda_2 = \beta p_2 \tag{6}$$

$$FOC(e_{i2}) \Rightarrow -\beta \frac{\partial C}{\partial e_{i2}} = \lambda_2 \Rightarrow e_{i2} = f(p_2) \tag{7}$$

$$FOC(B_{i1}) \Rightarrow -\lambda_1 + \lambda_2 + \mu_1 = 0 \tag{8}$$

$$FOC(B_{i2}) \Rightarrow -\lambda_2 + \mu_2 = 0 \tag{9}$$

$$B_{i1} \geq 0; \mu_1 \geq 0; B_{i1}\mu_1 = 0 \qquad\qquad (10)$$

$$B_{i2} = 0; \mu_2 > 0 \qquad\qquad (11)$$

We assume throughout the analysis that all agents' marginal abatement costs are strictly positive and therefore, it is not optimal for any firm to keep unused permits by the end of period 2. That is the reason why the conditions in Equation (11) differ from those in Equation (10).

As it is shown in the previous literature, under the assumption of competitive markets, cost effectiveness is achieved regardless of the allocation rule chosen (Montgomery 1972). In our case this result straightforwardly follows from equations (5) and (7) which are the standard conditions, according to which the marginal cost of abatement equals the permit price and therefore the marginal cost is equalized across firms. In our framework this equality holds every period regardless of the value of *S*.

Using equations (4) and (6) into equation (8) and taking into account that the multiplier μ is non negative, we conclude that if the problem has a unique solution, the present value permit prices are not decreasing over time. Formally

$$p_1 - \beta p_2 = \mu_1 \geq 0$$

In a similar way, it can be established by using equations (5) and (7) into equation (8) that the marginal abatement cost is not increasing over time in present value

$$\frac{\partial C}{\partial e_{i1}} - \beta \frac{\partial C}{\partial e_{i2}} = \mu_1 \geq 0$$

According to equations (10) and (11) the amount of banked permits in any period can be positive only if the multiplier μ for that period is zero. This is in line with one of the results in Cronshaw & Kruse (1996). They stated "Suppose that one of the

firms is not subject to profit regulation. Then that firm is only willing to bank permits if the futures price is the same in the two periods, or equivalently if the spot price rises with the rate of interest" (p. 185). It should be also noted that in this case, the present value of marginal abatement cost is also equated among periods. In our case, it is rational to bank permits only in the first period, not in the second, as the economic value of remaining permits at the end of the second period is zero.

Imperfect Competition in the Permit Market

Now we consider that there is market power in the permit market and specifically, that firm 1 is a dominant firm, and firm 2 is a price taker.[8] The dominant firm solves the following problem with regard to the emissions permit market:

$$
\begin{aligned}
Min_{\{e_{11}, y_{11}, e_{12}, y_{12}, p_1, p_2\}} \quad & C(x_{11}, e_{11}) + p_1 y_{11} + \beta \left[C(x_{12}, e_{12}) + p_2 y_{12} \right] \\
s.t. \quad & B_{11} = S_{11} + y_{11} - e_{11} \\
& B_{12} = S_{12} + y_{12} - e_{12} + B_{11} \\
& y_{11} = -y_{21} \\
& y_{12} = -y_{22}
\end{aligned}
\tag{12}
$$

while taking into account the follower´s demand for permits.

The static one-period model addressed by Hahn (1984) can be seen as a particular case of this framework by considering β = 0 and not allowing for banking. We start by analyzing this case and in this way, we adapt our model to represent the seminal Hahn (1984) static framework. The follower FOC´s are exactly the same as in equations (4) and (5). Now firm 1´s problem can be stated as:

$$
\begin{aligned}
Min_{\{p_1\}} \quad & C(x_{11}, e_{11}) + p y_{11} \\
s.t. \quad & y_{11} = e_{11} - S_{11} = S_{21} - e_{21}(p)
\end{aligned}
\tag{13}
$$

where $e_{21}(p)$ is the follower´s demand function for permits, resulting from the follower´s profit maximization problem.

---

[8] Qualitatively similar results that can be obtained if firm 2 is interpreted as a fringe of competitive firms

The leader´s FOC with respect to p yields

$$\frac{\partial C}{\partial e_{11}}\frac{\partial e_{11}}{\partial e_{21}}\frac{\partial e_{21}}{\partial p_1}+\frac{\partial y_{11}}{\partial e_{11}}\frac{\partial e_{11}}{\partial e_{21}}\frac{\partial e_{21}}{\partial p_1}p+y_{11}=0 \tag{14}$$

This is the first Hahn (1984) result. He stated "Suppose that there is one firm with market power. If it does not receive an amount of permits equal to the number that it holds in equilibrium, then the total expenditure on abatement will exceed the cost-minimizing solution." (Proposition 1 p. 756)

A simple manipulation in equation (14) makes Hahn´s statement clear in our model:

$$\frac{\partial e_{11}}{\partial e_{21}}=-1\Rightarrow\left(-\frac{\partial C}{\partial e_{11}}-p\right)\frac{\partial e_{21}}{\partial p}+y_{11}=0$$

The dominant firm equals its marginal abatement cost to the permit price only when there is no trade ($y_{11} = 0$)

The second Hahn´s result says that for the case where a regular interior minimum exists, a transfer of permits from any of the price takers to the firm with market power will result in an increase in the equilibrium price. This result can be immediately shown by differentiating (14).

$$\frac{\partial p}{\partial S_{11}}=\left[\left(-\frac{\partial C}{\partial e_{11}}-p_1\right)\left(\frac{\partial^2 e_{21}}{\partial p_1^2}\right)+\left(\frac{\partial^2 C}{\partial e_{11}^2}\right)\left(\frac{\partial^2 e_{21}}{\partial p_1^2}\right)-2\left(\frac{\partial e_{21}}{\partial p_1}\right)\right]^{-1}$$

Note that the expression in brackets is just the second order condition and must be positive for a minimum. An immediate corollary to this result is that the number of permits that the firm with market power uses will increase as its initial allocation of permits is increased.

We have already seen that when permit markets are competitive, as in Montgomery (1972), the distribution of permits is strictly an equity issue, but as soon as

we relax the perfect competition assumption, the distribution of permits matters, with regard not only to equity considerations but also to cost.

Let us now consider a two period framework with banking and borrowing. For convenience denote gross emissions (i.e. emissions in the absence of abatement activities) as Z and emission units abated as q. Then net emissions equal the difference between gross emissions (Z) and abatement (q).

$$e_{ij} = Z_{ij} - q_{ij} \tag{15}$$

Taking into account that it is optimal for both firms not to keep permits at the end of the second period, firm 2 faces the following constraint

$$q_{21} + q_{22} = Z_{21} + Z_{22} - S_{21} - S_{22} - (y_{21} + y_{22}) \tag{16}$$

Consider the situation where firm 2 is a net buyer of permits, which implies that $y_{21} + y_{22} > 0$. In this case firm 2 solves the following problem

$$Min_{\{q_{21}, y_{21}, q_{22}, y_{22}\}} \; C(x_{21}, q_{21}) + p_1 y_{21} + \beta \left[C(x_{22}, q_{22}) + p_2 y_{22} \right]$$
$$s.t. \qquad Eq.(16) \tag{17}$$

The FOC´s of this problem are:

$$FOC(y_{21}) \Rightarrow \lambda = p_1 \tag{18}$$

$$FOC(q_{21}) \Rightarrow \frac{\partial C}{\partial q_{21}} = \lambda \tag{19}$$

$$FOC(y_{22}) \Rightarrow \lambda = \beta p_2 \tag{20}$$

$$FOC(q_{22}) \Rightarrow \beta \frac{\partial C}{\partial q_{22}} = \lambda \tag{21}$$

Equations (18) and (20) imply that the present value price of permits must be constant over time in equilibrium. Equations (19) and (21) show that it is optimal for firm 2 in equilibrium to abate emissions until the present value of marginal abatement costs equals the present value price of permits. These are again the previously shown

results of Cronshaw and Kruse (1996). From equations (18) to (21) we can get the permit price as a function of the correspondent abated quantities.

The dominant firm faces a constraint similar to the follower's

$$q_{11} + q_{12} = Z_{11} + Z_{12} - S_{11} - S_{12} - (y_{11} + y_{12}) \tag{22}$$

The leader minimizes the cost of abatement in both periods minus the income from selling permits. From equations (18) to (21) we can get permit prices as a function of the correspondent traded permits

$$
\begin{aligned}
&Min_{\{q_{11},y_{11},q_{12},y_{12}\}} \quad C(x_{11},q_{11}) - p_1(y_{21})y_{11} + \beta\left[C(x_{12},q_{12}) + p_2(y_{22})y_{12}\right] \\
&s.t. \qquad\qquad Eq.(22) \\
&\qquad\qquad\qquad y_{1j} = -y_{2j}
\end{aligned} \tag{23}
$$

The first order conditions show the following results:

$$\frac{\partial C}{\partial q_{11}} = \frac{\partial p_1(y_{21})}{\partial y_{21}} \frac{\partial y_{21}}{\partial y_{11}} y_{11} + p_1(y_{21}) \tag{24}$$

$$\frac{\partial C}{\partial q_{12}} = \frac{\partial p_2(y_{22})}{\partial y_{22}} \frac{\partial y_{22}}{\partial y_{12}} y_{12} + p_2(y_{22}) \tag{25}$$

In words, it is optimal for the dominant firm in equilibrium to abate emissions until the present value of marginal abatement cost in each period equals the present value of the marginal revenue from selling permits.

The follower's marginal abatement costs will exceed the marginal abatement costs of the dominant firm in each period. The dominant firm sells too few quotas and hence abates too little compared to a cost-effective distribution of abatement across agents. This result is equivalent to the one in Hagem & Westskog (1998). According to them, "in the banking and borrowing system the monopolist extracts the full monopoly rent from the total sale of permits over both periods." (p. 95)

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2.3.3 Imperfect Competition in the Output Market

Consider now that the output market is imperfectly competitive. To analyze this case let us come back to problem (3).

Competitive permit market

We keep the assumption that the permit market is competitive and therefore equations (4) to (11) hold. With regard to the output market, we consider conjectural variations in line with equation (2a). This problem yields the following necessary conditions for an optimal solution:

$$FOC\left(x_{i1}\right) \Rightarrow P_1 + x_{i1}\frac{dP_1}{dX_1}\left(1+\delta\right) - \frac{\partial C}{\partial x_{i1}} = 0 \tag{26}$$

$$FOC\left(x_{i2}\right) \Rightarrow P_2 + x_{i2}\frac{dP_2}{dX_2}\left(1+\delta\right) - \frac{\partial C}{\partial x_{i2}} = 0 \tag{27}$$

Several authors have noted that the output market structure matters when considering the efficiency of CAP policies. Meunier (2011) based his analysis on the observation of the EUETS and concluded that even if the firms are price takers in the permit market, the integration of these permit markets can decrease welfare because of imperfect competition in the product markets. A qualitative equivalent result can be obtained in our framework. To this aim, let us consider that i = 1,2 represent polluting product markets instead of agents and there are n_i different firms in each of them. We denote by X_i the aggregate quantity of good i produced at market i and by P_i (X_i) the inverse demand function, which is assumed to satisfy the conditions required to ensure existence and uniqueness of Cournot equilibrium. Particularly the price function is not too convex and quantities are strategic substitutes. (See Meunier 2011 for details)

In each market a CAP is implemented with the local price of emissions denoted as p_i. Let us assume the static version of our canonical model with $\beta = 0$ and assume that there is no allocation of free permits, $(S_i = 0)$. Then, problem (3) becomes:

$$Max_{\{x_i,e_i\}} P_i(X_i)x_i - C(x_i,e_i) - p_i y_i$$
$$e_i = y_i \tag{28}$$

If we consider the same cost function for all agents, the equilibrium is symmetric in the sense that output and emissions are equally distributed among firms on each output market. Therefore individual quantities are given by X_i/n_i and individual emissions by E_i/n_i where E_i is the overall quantity of emissions in market $i = 1,2$.

Total quantity produced can be written as a function of total emissions $X_i{}^*(E_i)$ as the unique solution of the following equation

$$P_i + P_i' \frac{X_i}{n_i} = \frac{\partial C_i}{\partial x_i}\left(\frac{X_i}{n_i}, \frac{E_i}{n_i}\right) \tag{29}$$

The demand for permits of each firm in market i, as a function of the permit price $E_i{}^*(p_i)$ is implicitly determined by the following condition

$$p_i = -\frac{\partial C_i}{\partial e_i}\left(\frac{X_i^*\left(E_i^*(p_i)\right)}{n_i}, \frac{E_i^*(p_i)}{n_i}\right), i = 1,2 \tag{30}$$

The welfare implications of the interaction of a competitive market for emission permits with an oligopolistic product market can be analyzed with the introduction of the following welfare function:

$$W(X_1, X_2, E_1, E_2) = \sum_i Z(X_i) - n_i C_i\left(\frac{X_i}{n_i}, \frac{E_i}{n_i}\right) \tag{31}$$

Welfare is the sum of surpluses net of production cost. Gross surplus from consumption is $V(X)$ with $dV/dX = P(X)$. The optimal allocation of emissions denoted $(E_1{}^*, E_2{}^*)$ solves the following problem:

$$Max_{\{E_1,E_2\}} W(X_1(E_1), X_2(E_2), E_1, E_2) \ s.t. \ E_1 + E_2 = \bar{E} \tag{32}$$

On each market $i = 1, 2$ an additional unit of emissions increases the local net surplus by:

$$\frac{dW_i}{dE_i} = \left(P_i - \frac{\partial C_i}{\partial x_i} \right) \frac{\partial X_i^*}{\partial E_i} - \frac{\partial C_i}{\partial e_i} \tag{33}$$

The first term is the effect related to market power. An additional permit increases production and because of the existence of market power, this has a strictly positive effect on welfare.

If it is interior, the optimal allocation of emissions satisfies the first order condition

$$\left(P_1 - \frac{\partial C_1}{\partial x_1} \right) \frac{\partial X_1^*}{\partial E_1} - \frac{\partial C_1}{\partial e_1} = \left(P_2 - \frac{\partial C_2}{\partial x_2} \right) \frac{\partial X_2^*}{\partial E_2} - \frac{\partial C_2}{\partial e_2} \tag{34}$$

With an integrated permit market, local permit prices are equalized and the marginal costs of emissions for each firm are equalized across output markets. If the difference between the product price and the marginal cost is not the same in both markets, then the market allocation does not satisfy Eq. (34) and welfare is lower than in Eq. (32). This is the result of Meunier (2011), which says that the integration of markets does not increase welfare in general. The inefficiency of an integrated permit market arises from the divergence between price and marginal cost and the sensitivity of production to emissions.

Ehrhart et al (2008) investigate the effect of a permit price increase on firms´ profit and consumer surplus under the assumption of imperfect competition in the product market. The influence of an increase of the permit price on firm´s profit is ambiguous because there are two counteracting effects. A negative effect is due to more expensive permit purchasing costs and a positive effect is due to higher revenues in the output markets. This positive effect is directly related to the imperfect competition product market because a permit price increase will lead to a decreasing output level, and to an increase in the output price. That means a revenue increase in an imperfect

competition market. Under particular conditions a higher permit price will make firms´ profit increase and consumer´ surplus and social welfare decrease.

To derive these findings in our framework we solve problem (28) in two stages by backward induction. In the second stage, the cost minimization problem for firm "i" is given by:

$$Min_{e_i} C(x_i, e_i) + pe_i \qquad s.t. \quad e_i \geq 0 \tag{35}$$

Marginal cost of emissions equals permit price which is the FOC in a competitive market. Based on the conditions stated in Eq. (1), the cost function is convex in emissions and the second order condition is always fulfilled. Let us denote e_i* the emissions amount that minimizes abatement costs

In the first stage we solve the profit maximization problem given by:

$$Max_{(x_i, x_{-i}, p)} P(x_i + x_{-i}) x_i - TC(x_i, e_i^*(x_i, p)) \tag{36}$$

The FOC for maximization is:

$$P(x_i + x_{-i}) + (1+\delta) \frac{\partial P(x_i + x_{-i})}{\partial X} x_i - \frac{\partial TC(x_i, p)}{\partial x_i} = 0 \tag{37}$$

The equilibrium is symmetric and so we can write $x_i = x_{-i} = x^*$. Differentiating the profit function, and taking into account Eq. (37) yields

$$\frac{d}{dp}\left(P(2x^*)x^* - TC^*(x^*, p)\right) = (1-\delta)x^* \frac{\partial P(2x^*)}{\partial X} \frac{\partial x^*}{\partial p} - \frac{\partial TC^*}{\partial p} \tag{38}$$

The sign of (38) is ambiguous. The condition for being positive is:

$$\left| \frac{(1-\delta)x^* \frac{\partial P(2x^*)}{\partial X}}{(3+\delta) \frac{\partial P(2x^*)}{\partial X} - \frac{\partial TC^*(x^*, p)}{\partial x_i \partial x_i}} \right| \frac{\partial e_i^*}{\partial x_i} - e_i^* > 0 \tag{39}$$

which is Ehrhart et al (2008) result. They stated " that under certain parameter ranges, a higher permits price induces higher firm profits for all types of expected competition, with the exception of a monopoly scenario ($\delta = 1$)." (p. 352)

As it was defined in Eq (31), welfare is the sum of surpluses net of production cost.

$$W\left(x_{1},x_{2},p\right)=\int_{0}^{2x^{*}}P\left(z\right)dz-2TC^{*}\left(x^{*},p\right)$$

Independent of the effects on firms' profits, an increasing permit price never leads to an increase in social welfare, because the negative effect on the consumers' surplus always outweighs the possibly increasing profits.

$$\frac{\partial W\left(x_{1},x_{2},p\right)}{\partial p}=\left[2P\left(2x^{*}\right)-2\frac{\partial TC^{*}\left(x^{*},p\right)}{\partial x^{*}}\right]\frac{dx^{*}}{dp}-2\frac{\partial TC^{*}\left(x^{*},p\right)}{\partial p}\leq 0 \qquad (40)$$

This is the second Ehrhart et al (2008) result. We investigate in chapter 3, the same problem under a different market structure. Specifically we use a Stackelberg model which is asymmetrical in nature instead of the Cournot symmetrical model we have just shown.

2.4 A Particular Model with Endogenous Permit Prices

The model used in previous sections has allowed us to show some of the main results provided by the literature, but we need to introduce some specific production and abatement functions to study how different technologies influence the oligopolistic firm´s behavior and markets equilibrium. These particular functions will be used in this chapter to get a more precise view of the connections between the output market and the permit market. And they will also be used in chapters 3 and 4 to analyze in more detail some aspects within a duopoly Stackelberg model.

In the output market we keep a duopolistic framework, with linear demand. The inverse demand function is $P = a - bX$. On the production side we assume that the firms face a constant marginal cost of production c. Gross emissions are assumed to be proportional to the firms' output, (rx_i), where r is the pollution intensity, which is assumed to be common for both firms. The firms can reduce emissions by either reducing output or reducing emissions. A quadratic abatement function is used. Firm i's total abatement cost is $(d + tq_i)q_i$, where q_i is firm's total abatement.

The profit maximization problem considered in Equation (3) becomes

$$
\begin{aligned}
Max \quad & \left(a - bx_{i1} - bx_{-i1}\right)x_{i1} - cx_{i1} - \left(d + t_i q_{i1}\right)q_{i1} - p_1 y_{i1} + \\
& + \beta\left[P_2\left(x_{i2} + x_{-i2}\right)x_{i2} - cx_{i2} - \left(d + t_i q_{i2}\right)q_{i2} - p_2 y_{i2}\right] \\
s.t. \quad & B_{i1} = S_{i1} - rx_{i1} + q_{i1} + y_{i1} \\
& B_{i2} = S_{i2} - rx_{i2} + q_{i2} + y_{i2} + B_{i1} \\
& B_{i1} \geq 0 \\
& B_{i2} \geq 0
\end{aligned}
\tag{3a}
$$

The corresponding Kuhn Tucker conditions from equations (4) to (11) and (26) to (27) take the following form:

$$
FOC\left(y_{i1}\right) \Rightarrow \lambda_1 = p_1
\tag{4a}
$$

$$
FOC\left(q_{i1}\right) \Rightarrow d + 2tq_{i1} = p_1 \Rightarrow q_{i1} = \frac{p_1 - d}{2t}
\tag{5a}
$$

$$
FOC\left(x_{i1}\right) \Rightarrow P_1 - bx_{i1}\left(1 + \delta\right) = c + rp_1
\tag{26a}
$$

$$
FOC\left(y_{i2}\right) \Rightarrow \lambda_2 = \beta p_2
\tag{6a}
$$

$$
FOC\left(q_{i2}\right) \Rightarrow -\beta\left(d + 2tq_{i2}\right) = \lambda_2 \Rightarrow q_{i2} = \frac{p_2 - d}{2t}
\tag{7a}
$$

$$
FOC\left(x_{i2}\right) \Rightarrow P_2 - bx_{i2}\left(1 + \delta\right) = c + rp_2
\tag{27a}
$$

$$
FOC\left(B_{i1}\right) \Rightarrow -\lambda_1 + \lambda_2 + \mu_1 = 0
\tag{8a}
$$

$$
FOC\left(B_{i2}\right) \Rightarrow -\lambda_2 + \mu_2 = 0
\tag{9a}
$$

$$B_{i1} \geq 0; \mu_1 \geq 0; B_{i1}\mu_1 = 0 \tag{10a}$$

$$B_{i2} = 0; \mu_2 > 0 \tag{11a}$$

From equations (26a) and (27a) it is trivial to see that the product price is increasing in the permit price, as a simple algebraic manipulation yields the following values for the output market equilibrium:

$$X_1 = 2 \cdot \frac{P_1 - c - rp_1}{b(1+\delta)} \Rightarrow P_1 = \frac{(1+\delta)a + 2(c+rp_1)}{3+\delta} \tag{41}$$

Permit market equilibrium is obtained under the assumption that both firms are the only agents operating in the emissions permit market and they are price takers. Therefore the market is closed based on the following condition:

$$r(X_1 + X_2) - S_1 - S_2 - (q_{11} + q_{21}) - (q_{12} + q_{22}) = 0 \tag{42}$$

Total output in each period is given by:

$$X_j = \frac{2(a - c - rp_j)}{b(3+\delta)} \tag{43}$$

From the first order conditions, abatement is a function of the permit price, and using Eq. (5a) and Eq. (7a) into Eq. (42) and Eq. (43) lead us to the permit price.

$$p_2 = \frac{8rt^2(a-c) - 2b(3+\delta)\left[(S_1 + S_2)t^2 - 2dt\right]}{(1+\beta)\left[4r^2t^2 + 2bt(3+\delta)\right]} \tag{44}$$

From this equation we can analyze and compare the endogenous permit price in the different market structures, represented by the parameter δ. From Eq (43) we know that total output is decreasing in both the permit price and the conjectural variation parameter.

LEMMA 1

The quantities abated and the permit price decrease as the product market becomes less competitive. They are decreasing at a decreasing rate.

PROOF

We show in the Appendix at the end of this chapter that the sign of the derivative of p with respect to δ in (44) is unambiguously negative for both periods.

PROPOSITION 1

The price of permits is higher when the product market is controlled by a Stackelberg leader than in the case of a cartel. It is also higher than the case of a Cournot competition. But it is lower if the product market is competitive.

PROOF

We show in the Appendix that in the Stackelberg model the corresponding product output and permit price become:

$$X_j = \frac{3(a-c-rp_j)}{4b} \tag{45}$$

$$p_2 = \frac{6rt^2(a-c) - 4b\left[(S_1 + S_2)t^2 - 2dt\right]}{(1+\beta)\left[3r^2t^2 + 4bt\right]} \tag{46}$$

Then the comparison of Equation (46) with the different values of Equation (44) yields the result. Details of these comparisons can be found in the Appendix.

The permit price level under different market structures follows the same behavior as the total quantities produced in each type of market. As the equilibrium output quantities decrease so the permit prices do. It is worth noting that the permit price depends on the output technology (parameter c) and the abatement technology

(parameters d and t) which are assumed to be the same for both firms. Therefore only the total output quantity matters regardless the output of any single firm.

2.5 Concluding Remarks

This chapter has described the role played by market competition within CAT programs, paying special attention to the interaction between two related markets. One is the tradable permits market and the other one is the polluting product market associated to it. And the market power issue affects to both of them. Along this chapter and within the framework of a canonical model, we have reviewed some of the main results provided by the literature about the impact that imperfect competition has over the effectiveness of this type of environmental policy.

Another goal of this chapter was to set up a specific model with a double objective. First we have used this model as a unified framework to address some of the main literature results related to the effect of different market structures on the effectiveness of a CAP program. Secondly we want to use this model as the base for our investigation within the next two chapters. In this sense, the conjectural variations model provides a natural framework to make comparisons between different oligopolies structures.

We have also analyzed in a particular but rather standard model how market power in the polluting product market affects the price in the emissions market. Our analysis shows that the permit price level under different market structures follows the same behavior as the quantities produced. As the equilibrium output quantities decrease so the permit prices do. We have analyzed four different structures that we enumerate from higher to lower equilibrium output quantities (from higher to lower price of

permits): A competitive market, a Stackelberg oligopoly, a Cournot oligopoly and a cartel.

This result is based on the assumption that product and abatement cost functions are the very same for both firms. Therefore only the total quantity produced affects the price of permits regardless each firm output. In the following chapters we will address this issue considering different production and abatement technologies between agents.

APPENDIX

Proof of Lemma 1

Consider the following changes to simplify the expression of the emissions permits price in period 2.

$$A = 8r(a-c)t^2$$
$$B = 2b\left[(S_1 + S_2)t^2 - 2dt\right]$$
$$C = (1+\beta)4r^2t^2$$
$$D = (1+\beta)2bt$$

Now equation (44) and its partial derivative with respect to delta becomes

$$p_2 = \frac{8r(a-c)t^2 - 2b(3+\delta)\left[(S_1+S_2)t^2 + 2dt\right]}{(1+\beta)\left[4r^2t^2 + 2bt(3+\delta)\right]} \Rightarrow p_2 = \frac{A-(3+\delta)B}{C+(3+\delta)D}$$

$$\frac{\partial p_2}{\partial \delta} = \frac{-B\left[C+(3+\delta)D\right]-\left[A-(3+\delta)B\right]D}{\left[C+(3+\delta)D\right]^2} = \frac{-BC-AD}{\left[C+(3+\delta)D\right]^2}$$

(A1)

$A > 0$ Note that $(a - c - rp_2)$ must be positive if product quantities are positive. C and D are unambiguously positive. The sign of B is ambiguous.

Assume (A1) is positive. Therefore B must be negative and $-BC > AD$. If this is the case we have the following inequality

$$-2b\left[(S_1+S_2)t^2 - 2dt\right](1+\beta)4r^2t^2 > 8r(a-c)t^2(1+\beta)2bt$$
$$-\left[(S_1+S_2)t^2 - 2dt\right]r > (a-c)2t$$
$$2rdt - (a-c)2t > (S_1+S_2)rt^2 \Rightarrow rd - (a-c) > 0$$

But the last expression must be negative if product quantity is positive because

$$a-c-rp_2 > 0 \Rightarrow a-c-r(d+2tq_{i2}) > 0 \Rightarrow a-c-rd > 0 \Rightarrow rd-(a-c) < 0 \quad \text{(A2)}$$

We have used equation (7a) to substitute p_2 in (A2)

$$q_{12} = \frac{p_2 - d}{2t} \Rightarrow p_2 = d + 2tq_{12}$$

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We conclude that AD > -BC and (A1) is negative.

To see that is decreasing at a decreasing rate, we just take the second order derivative

$$\frac{\partial^2 p_2}{\partial \delta^2} = \frac{2D(BC + AD)}{\left[C + (3 + \delta)D\right]^3} > 0$$

Proof of Proposition 1

Let us consider firm 1 as the Stackelberg leader. The conjectural variations are $\delta_2 = 0$ and $\delta_1 = -1/2$. Based on Eq. (26a) the total output for the first period is:

$$X_1 = x_{11} + x_{21} = \frac{2(P_1 - c - rp_1)}{b} + \frac{(P_1 - c - rp_1)}{b} = \frac{3(P_1 - c - rp_1)}{b} \tag{A3}$$

And the corresponding product price is:

$$P_1 = a - bX_1 = \frac{a + 3(c + rp_1)}{4}$$

Leading to the standard Stackelberg result where the leader is producing a double quantity than the follower as long as the marginal product cost is the same.

$$X_1 = x_{11} + x_{21} = \frac{a - c - rp_1}{2b} + \frac{a - c - rp_1}{4b} = \frac{3(a - c - rp_1)}{4b} \tag{A4}$$

Now Eq. 36 becomes

$$r\left(\frac{3(a - c - rp_1)}{4b} + \frac{3(a - c - rp_2)}{4b}\right) - S_1 - S_2 - \frac{p_1 - d}{t} - \frac{p_2 - d}{t} = 0$$

And taking into account that $p_1 = \beta p_2$ the last expression lead us to

$$r\left(\frac{6(a - c)}{4b} - \frac{3rp_2(1 + \beta)}{4b}\right) - S_1 - S_2 - \frac{(p_2(1 + \beta) - 2d)}{t} = 0$$

Therefore second period permit price is

$$p_2 = \frac{6rt(a - c) - 4b\left[(S_1 + S_2)t - 2d\right]}{(1 + \beta)\left[3r^2t + 4b\right]} = \frac{6rt^2(a - c) - 4b\left[(S_1 + S_2)t^2 - 2dt\right]}{(1 + \beta)\left[3r^2t^2 + 4bt\right]} \tag{A5}$$

We make the second expression for comparison purposes.

Consider the following changes to simplify the expression of the emissions permit price in period 2.

$$A = rt^2(a-c)$$
$$B = b\left[(S_1 + S_2)t^2 - 2dt\right]$$
$$C = (1+\beta)r^2t^2$$
$$D = (1+\beta)2bt$$

A > 0 Note that (a – c - rp$_2$) must be positive if product quantities are positive. C and D are unambiguously positive. The sign of B is ambiguous.

The simplified expression for Equation (A5) is

$$p_2^S = \frac{6A - 4B}{(1+\beta)[3C + 2D]} \tag{A6}$$

Now we proceed to compare the Stackelberg permit price with other oligopolistic structures. The conjectural variation for a Cournot competition takes $\delta = 0$ in Equation 38 and the permit price simplified form is

$$p_2^C = \frac{8A - 6B}{(1+\beta)[4C + 3D]} \tag{A7}$$

The comparison shows that the Stackelberg permit price is higher

$$(6A - 4B)[4C + 3D] = 24AC + 18AD - 16BC - 12BD$$
$$(6A - 4B)[4C + 3D] = 24AC + 16AD - 18BC - 12BD$$
$$B > 0 \Rightarrow 18AD - 16BC > 16AD - 18BC$$

The result is also valid if B < 0, because we have shown in the proof of Lemma 1 the result (AD > –BC). That implies:

$$AD > -BC \Rightarrow 2AD > -2BC \Rightarrow 18AD - 16BC > 16AD - 18BC$$

We apply the same rational to prove that the emission permits price is also higher in the Stackelberg model than in the case of a cartel but is lower when the product market is competitive.

The permit price under a cartel structure is:

$$p_2^{Cartel} = \frac{8A - 8B}{(1+\beta)[4C + 4D]}$$ (A8)

The comparison with (A6) yields the proposed result

$$(6A - 4B)[4C + 4D] = 24AC + 24AD - 16BC - 16BD$$
$$(8A - 8B)[3C + 2D] = 24AC + 16AD - 24BC - 16BD$$
$$B > 0 \Rightarrow 24AD - 16BC > 16AD - 24BC$$
$$B < 0 \Rightarrow 24AD - 16AD > -24BC + 16BC \Rightarrow AD > -BC$$

The permit price under a competitive structure is

$$p_2^{comp} = \frac{8A - 4B}{(1+\beta)[4C + 2D]}$$ (A9)

This price is higher than in Stackelberg

$$(6A - 4B)[4C + 2D] = 24AC + 12AD - 16BC - 8BD$$
$$(8A - 4B)[3C + 2D] = 24AC + 16AD - 12BC - 8BD$$
$$B > 0 \Rightarrow 16AD - 12BC > 12AD - 16BC$$
$$B < 0 \Rightarrow 4AD > -4BC$$

Chapter 3

Imperfect Competition in the Product Market[9]

3.1 Introduction

In this chapter we examine the existence of scarcity rents and incentives for firms to collude in order to inflate the price of emission permits under the assumption of a leader-follower relationship in the output market. Both scarcity rents and price manipulation have been pointed out as relevant factors in the European Union Emission Trading System (EU ETS).

The main reason why cap-and-trade (CAT) programs are so attractive and popular among economists is that they theoretically allow emissions to be reduced in a cost-effective way by means of a price system. Regardless of the initial allocation rule chosen for the permits, the cost-effectiveness property is well documented in the

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[9] This chapter represents joint work together with Francisco J. André (Universidad Complutense). It is published in MPRA Paper nº 61770 under the title "Scarcity Rents and Incentives for Price Manipulation in Emissions Permit Markets with Stackelberg Competition"

literature under the assumption of perfect competition (see Montgomery 1972). Unfortunately, the perfect-market assumption rarely holds in practice and the cost-effectiveness property is in fact challenged if there is market power in either the permit market, in the associated product market or in both. As it has been discussed in Chapter 2 (Section 2) the literature analyzing the relationship between imperfect competition and emission permits can be divided in three different branches, depending on whether market power is introduced in the permit market, in the good market or in both simultaneously.

This paper fits within the branch of research that considers market power in the product market, but not in the permit market. The reason to choose this branch is twofold. First, as noted by Montero (2009) and Muller *et al.* (2002), whereas market power among firms is very common in output markets, the existence of market power in emissions permits is more likely to appear when the relevant players are countries rather than firms or facilities.[10] In the latter case, there are normally a very large number of them, which makes it very difficult for market power to arise. It can be argued that this is the case in the EU ETS, with more than 11,000 facilities involved. Moreover, the latest steps taken by the European Commission seem to be aimed at increasing the degree of competition even more (for example, by enlarging the number of involved sectors, centralizing the allocation of permits or moving from grandfathering to auctioning). On the other hand, among the economic sectors that are subject to the EU ETS, it is rather realistic to assume that at least in some of them there is some market power in the output market (see, e.g. Smale *et al.* 2006 or Hinterman 2011).

As a second reason for this line of research, the EU-ETS price shock in 2005

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[10] As an example regarding Annex 1 countries in the Kyoto Protocol, Russia initially received roughly a fifth of the permits and a third went to the USA. Countries with market power can easily manipulate prices up (down) through tariffs on permit exports (domestic subsidies to cleaner technologies) and also implement policies regarding the linkage between domestic and foreigner markets. See Montero (2009) or Barrett (1998) for a related discussion.

generated a great deal of interest in issues related to market power. Initially, the price of allowances was far in excess of expectations, but it suddenly fell in April 2006, reaching zero in mid-2007. Empirical studies have not been able to perfectly explain these excessively high price levels when the number of permits exceeded emissions in every year of the first phase (see, e.g. Ellerman *et al.* 2010). It is therefore natural to ask whether the reason for these variations in price might be linked to the output market rather than the permit market insofar as permits could somehow be used to obtain windfall profits in the output market.

The closest paper to ours is the one by Ehrhart *et al.* (2008), which claims that under some conditions a permit price increase leads to higher firms' profit due to a decrease in product quantities, which in turn increases the output price. This result can be seen as an important case of scarcity rents. As far as permits are a limited input, output price will reflect the scarcity value of the permits.[11] Due to the tradable nature of emission permits, some firms can take the opportunity to obtain additional revenues by selling permits. Empirical evidence suggests that this phenomenon has been rather important in the first phase of the European Union Emission Trading System (EU ETS). For example, Newell *et al.* (2013) point out that power generators extracted rents by receiving carbon allowances for free and then passing along the opportunity costs of these allowances to their customers. For an analysis of this phenomenon, see Ellerman and Joskow (2008) or Ellerman *et al.* (2010).

Ehrhart *et al.* (2008) show that under some conditions firms benefit from a higher price of permits even if they are net buyers rather than seller of permits. Although an increase in the permit price has the direct effect of increasing one's cost,

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[11] See, e.g. Fullerton and Metcalf (2001) for a discussion on scarcity rents. In a perfect competition framework, Mohr and Saha (2008) claim that, via the generation of scarcity rents, a stricter environmental regulation might have a distributional impact in the sense of increasing firm's profits and passing the cost onto consumers. André *et al.* (2009) make a similar discussion in a strategic setting with quality competition.

seeing as it also raises the rival's cost, it can generate scarcity rents for both firms by restricting the quantity and increasing the price of output. They conclude that, under these conditions, firms have incentives to collude in order to push the price of permits upwards.

Importantly, Ehrhart *et al.* (2008) also claim that, although there is apparently no explicit market power as such in the EU ETS, there are loopholes in the trading law that allow collusive behavior among firms to manipulate the price of permits. The most important of these mechanisms are first, the possibility to influence the initial allocation of permits (to make it more stringent); second, the 'opt-in' rule, which enables industries not committed to participating in the permits trading system to do so voluntarily; third, the possibility to implement project-based mechanisms and pay more for these credits than they would in the market and fourth, by paying additional emissions duties. It has also been argued that price manipulation practices might be responsible for the variations in price observed during the first phase of the EU ETS. For example, Hinterman (2011) concludes that the largest electricity producers in Germany, the UK and the Nordpool market might have found it profitable to manipulate the permit price upwards and he claims that this could explain the elevated allowance price level during the first 18 months of the EU ETS.[12]

This paper addresses the question as to whether, via the generation of scarcity rents, firms' interests could be aligned to push the price of permits up (and therefore if there are incentives to collude) under Stackelberg competition in the output market. We thus investigate whether the colluding incentives reported by Ehrhart *et al.* (2008) in a symmetric scenario might still arise in a setting that is asymmetric in nature in the sense that there is a leader and a follower. This seems a relevant case to consider seeing as, in

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[12] Note, however, that Hinterman's analysis is not fully comparable to ours as he assumes explicit market power in both output and permit markets.

the EU ETS, there are some big leading firms together with small firms that probably act as followers. As an additional contribution of this model, it fills a gap in the literature by considering Stackelberg competition in the third line of research, among the three reported above. In fact, leader-follower competition has been addressed in the first line by Hahn (1984) and in the second one by Hinterman (2011) but, as far as we know, it has not been studied in the third line, as we do.

As in Ehrhart *et al.* (2008), we take the permit price as given and thus we do not explicitly model the permit market. Consequently, we restrict ourselves to testing the existence of incentives for collusion to manipulate the price of permits rather than modeling price manipulation itself or determining if such manipulation has taken place in practice.

We first use a general model to show that a higher permit price increases the firms' cost of purchasing permits but also restricts output and increases the output price, generating some scarcity rents. Therefore, the final effect on both the leader's and the follower's profit is ambiguous. Hence, the possibility that firms benefit from a price increase still exists as in Ehrhart *et al.* (2008), but in our case the asymmetric role of the firms means that such a possibility arises under different conditions for the leader and the follower. This introduces the possibility of one firm being interested in raising and the other in decreasing the permit price.

We subsequently proceed to explore a particular case with a separable cost function to gain more accurate insights. We start with a basic case in which both firms have the same cost function and there is no grandfathering. As a first core finding, under the reasonable assumption that the solution is interior (both firms produce, pollute and abate to some extent), we conclude that both firms face a profit function that is convex in the permit price. Moreover, within the relevant range, when the price is sufficiently

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low, both firms will benefit from a further price reduction, whereas for sufficiently high prices, the follower will benefit from a price increase, while the leader will still prefer the price to decrease. Hence, in the absence of grandfathering, there is no room for collusion in the latter range. This is contrary to Ehrhart *et al.* (2008), which uses a symmetric model where both firms' interests are always aligned. The implication of this finding is that the existence of leadership in output markets reduces the room for collusion in the permit market. In fact, in our specific example with a separable function, we conclude that the collusive region shrinks to the extent that it disappears.

As a first extension, we consider the possibility that some permits are distributed for free (by means of grandfathering) and conclude that this possibility opens up the way for collusion. In fact, apart from the two regions identified in the simple case, there is a third region in which both firms are interested in pushing the price up and this region becomes wider the more permits are distributed for free. This result represents an important argument against grandfathering insofar as it could introduce incentives to foster collusive behavior for price manipulation.

As a second extension, we explore the effect of asymmetries and conclude that the likelihood of facing an environment that is conductive to collusion is sensitive to the cost parameters of both firms and the allocation of free permits received by the leader, but not by the follower. In short, those parameter changes that tend to undermine the leader's advantage in output production (i.e., an increase in the leader's cost, or a decrease in the follower's cost) have the effect of making the firms more symmetric in a certain sense and hence increase the likelihood of observing collusive behavior. The opposite occurs with abatement costs: the likelihood of collusive behavior tends to decrease with the leader's abatement cost and to increase with the follower's. The reason is that, seeing as the leader produces more output than the follower, its cost is

more sensitive to the permit price and thus it is more difficult for the former to benefit from such a price increase, and this is truer, the higher the leader's abatement cost. On the other hand, an increase in the follower's abatement cost reduces the possibility of its being optimal for it to pollute zero, which widens the interior solution range and hence also the scope for collusion.

Section 2 expounds the basic model. A particular abatement cost function is considered in Section 3, including the basic case and the two extensions. Concluding remarks are given in Section 4. All the mathematical proofs are gathered in the appendix.

3.2 The general model

Let us consider a simple duopoly Stackelberg model of a polluting industry that is subject to a tradable permit system. Firm 1 is a leader and firm 2 is a follower in the output market. Following Ehrhart *et al.* (2008), we assume no explicit market power in the permit market and thus the permit price is taken as an exogenous value. The game has three stages: in the two first stages firms sequentially decide on their output levels, x_1 and x_2, a la Stackelberg, facing the inverse demand function $P(X)$, where $X := x_1 + x_2$ and $\frac{dP}{dX} < 0$. In the third stage, they simultaneously choose their cost-minimizing emission levels, e_1 and e_2.

The cost function of firm $i \in (1, 2)$, $C_i(x_i, e_i)$, depends on output (x_i) and emissions (e_i) and is continuous and twice differentiable in both arguments with the following properties:

$$\frac{\partial C_i}{\partial x_i} > 0 , \quad \frac{\partial C_i}{\partial e_i} < 0 , \quad \frac{\partial^2 C_i}{\partial e_i^2} > 0 , \quad \frac{\partial^2 C_i}{\partial x_i \partial e_i} < 0. \tag{1}$$

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This function integrates production and abatement costs and reflects the fact that producing clean (with low emissions) is more costly than producing dirty. Each unit of emissions must be covered by an emission permit. Initially, each firm i is endowed with a given amount of permits S_i, and additionally required permits, $e_i - S_i$, can be obtained on the market at a given price, p. Considering the cost of permit purchasing, the total cost of firm i is given by

$$TC_i(x_i, e_i) := C_i(x_i, e_i) + p(e_i - S_i). \tag{2}$$

The model is solved by backward induction. In the third stage of the game, both firms decide on their emissions levels to minimize their total cost, $TC_i(x_i, e_i)$, while taking their output levels and the price of permits as given. If the solution is interior, we obtain the standard first-order condition (FOC),[13]

$$\frac{\partial C_i}{\partial e_i} + p = 0, \tag{3}$$

from which we obtain each firm's demand for permits, $e_i^*(x_i, p)$.[14] Total differentiation of the FOC shows that optimal emissions are increasing in output and decreasing in the permit price:

$$\frac{\partial^2 C_i}{\partial e_i^2} de_i + \frac{\partial^2 C_i}{\partial e_i \partial x_i} dx_i = 0 \Rightarrow \frac{\partial e_i}{\partial x_i} = \frac{-\dfrac{\partial^2 C_i}{\partial e_i x_i}}{\dfrac{\partial^2 C_i}{\partial e_i^2}} > 0, \tag{4}$$

$$\frac{\partial^2 C_i}{\partial e_i^2} de_i + dp = 0 \Rightarrow \frac{\partial e_i}{\partial p} = \frac{-1}{\dfrac{\partial^2 C_i}{\partial e_i^2}} < 0. \tag{5}$$

Using the Envelope Theorem, we conclude that the minimized total cost function defined as

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[13] The second order condition is always fulfilled due to the convexity of C_i in emissions.

[14] Throughout the paper we use asterisks to denote equilibrium values.

$$TC_i^*(x_i, p) := TC(x_i, e_i^*(x_i, p)) = C(x_i, e_i^*(x_i, p)) + p[e_i^*(x_i, p) - S_i] \qquad (6)$$

has the following properties:

$$\partial TC_i^* \Big/ \partial p = e_i^* - S_i, \qquad (7)$$

$$\frac{\partial TC_i^*}{\partial x_i} = \frac{\partial C_i}{\partial x_i} + \left(\frac{\partial C_i}{\partial e_i} + p\right)\frac{\partial e_i}{\partial x_i} = \frac{\partial C_i}{\partial x_i}, \qquad (8)$$

$$\frac{\partial^2 TC_i^*}{\partial x_i^2} = \frac{\partial^2 C_i}{\partial x_i^2} + \frac{\partial^2 C_i}{\partial x_i \partial e_i}\frac{\partial e_i}{\partial x_i} = \frac{\partial^2 C_i}{\partial x_i^2} - \frac{\left(\dfrac{\partial^2 C_i}{\partial x_i \partial e_i}\right)^2}{\dfrac{\partial^2 C_i}{\partial e_i^2}}, \qquad (9)$$

$$\frac{\partial^2 TC_i^*}{\partial x_i \partial p} = \frac{\partial^2 C_i}{\partial x_i \partial e_i}\frac{\partial e_i}{\partial p} = \frac{-\dfrac{\partial^2 C_i}{\partial x_i \partial e_i}}{\dfrac{\partial^2 C_i}{\partial e_i^2}} > 0. \qquad (10)$$

We now move on to the first and the second stages, in which the firms choose their output levels. The follower faces the following maximization problem:

$$\underset{x_2}{Max}\ \Pi_2\left(x_1, x_2, e_2^*(x_2, p), p\right) = P(x_1 + x_2)x_2 - TC_2^*(x_2, p). \qquad (11)$$

The FOC of this problem is

$$P + \frac{dP}{dX}x_2 - \frac{\partial TC_2^*}{\partial x_2} = 0, \qquad (12)$$

which, solving for x_2, gives the reaction function of the follower, $x_2^R(x_1, p)$. Differentiating the FOC and operating, we conclude that the optimal follower's output is decreasing in the leader's output and the price of permits:

$$\frac{\partial x_2^R}{\partial x_1} = \frac{-\dfrac{dP}{dX}}{2\dfrac{dP}{dX} - \dfrac{\partial^2 TC_2}{\partial x_2^2}} < 0, \qquad \frac{\partial x_2^R}{\partial p} = \frac{\dfrac{\partial^2 TC_2}{\partial x_2 \partial p}}{2\dfrac{dP}{dX} - \dfrac{\partial^2 TC_2}{\partial x_2^2}} < 0. \qquad (13)$$

Finally, in the first stage, the leader takes the follower's reaction function into account when maximizing its own profit. The FOC of the corresponding problem is

$$P(x_1 + x_2) + \frac{dP}{dX} x_1 \left(1 + \frac{\partial x_2^R}{\partial x_1} \right) - \frac{\partial TC_1}{\partial x_1} = 0, \qquad (14)$$

from which we obtain the leader's optimal output as a function of the permit price, $x_1^*(p)$. By differentiating (14), we conclude that the leader's output supply is also decreasing in the price of permits:

$$\frac{dx_1^*}{dp} = \frac{\dfrac{\partial^2 TC_1^*}{\partial x_1 \partial p}}{\dfrac{dP}{dX} \left(2 + \dfrac{\partial x_2^R}{\partial x_1} \right) - \dfrac{\partial^2 TC_1^*}{\partial x_1^2}} < 0. \qquad (15)$$

Equations (13) and (15) show how the leader and the follower react to a permit price increase. While the follower only takes into account the effect of its own output variation on the output price, the leader incorporates, not only its own, but also the follower's. This tends to make the denominator smaller in absolute value and, hence, the whole value of (15) greater in absolute value.

Using the equilibrium output values we can express the profit of both firms solely as a function of the permit price: $\Pi_1^*(p)$, $\Pi_2^*(p)$. We are now ready to address the main question of this paper, namely the effect of an increase in the price of permits on firm's profit. The question is: could both firms benefit simultaneously from a price increase as predicted by Ehrhart *et al.* (2008) in a symmetric setting? The motivation behind this question is that, if the answer happens to be positive, both firms might have incentives to collude in order to manipulate the price of permits upwards. For the sake of realism, it is relevant to ask this question in a setting in which the firms play different roles regarding their market power, as this situation is commonly observed in the real world and, specifically, in the EU ETS. As in Ehrhart *et al.* (2008), we do not model

explicitly price manipulation. We simply test for the existence of firms' incentives to do so.

By direct differentiation of the profit functions, and dropping the terms that cancel out due to the FOCs, we conclude that the marginal effect of the price of permits on both firms' profit has two effects: on the one hand, it drives cost up, which tends to reduce firm's profit. On the other hand, however, it also causes output to decrease and therefore the product price to increase, which is beneficial for both firms. Formally,

$$\frac{d\Pi_1^*}{dp} = \underbrace{\frac{dP}{dX}\frac{\partial x_2^R}{\partial p}x_1^*}_{SR_1} - \left(e_1^* - S_1\right),\tag{16}$$

$$\frac{d\Pi_2^*}{dp} = \underbrace{\frac{dP}{dX}\frac{dx_1^*}{dp}x_2^*}_{SR_2} - \left(e_2^* - S_2\right).\tag{17}$$

The first summand in equations (16) and (17) can be seen as the scarcity rents from the point of view of firms 1 and 2 respectively (SR_1 and SR_2), *i.e.*, the additional revenue that each firm will receive thanks to the reduction in output supply. It is interesting to note that a higher value of p causes the output of both firms to decrease but each firm can only benefit from the effect that is due to the rival's output reduction. The reason is that decreasing the own output has a positive effect (increasing the price and decreasing the cost) and a negative effect (decreasing the number of sold units) and in equilibrium both effects cancel out as both firms are at the profit maximizing output level. Note also that the effect of a price increase on the follower's output has two components: a direct one and an indirect one through the leader's output. Formally,

$\frac{dx_2^*}{dp} = \frac{\partial x_2^R}{\partial p} + \frac{\partial x_2^R}{\partial x_1}\frac{dx_1^*}{dp}$. Nevertheless, the latter effect is already accounted for in the

leader's optimizing process and hence only the former matter to determine the leader's scarcity rent.

Direct comparison of (16) and (17) shows that $SR_1 \geq SR_2 \Leftrightarrow \varepsilon_{x_2^R,p} \geq \varepsilon_{x_1^*,p}$, where

$\varepsilon_{A,B}$ denotes the elasticity of A with respect to B, i.e., a firm can enjoy more scarcity

rents than its rival if its rival's output is more sensitive to the permit price than its own

output.

The second term in (16) and (17) is the marginal increase in cost due to a higher

permit price, which simply equals each firm's purchase of permits. It determines which

part of the scarcity rents is captured by each firm. In the most favorable case, if all the

permits were distributed for free, $e_i^* = S_i$, then the marginal impact of p on the cost

would be zero and each firm would capture the whole available scarcity rent.

The sign of both (16) and (17) is ambiguous. If the first term (the scarcity rent)

dominates the first (the marginal cost) then profit will increase with the price of permits.

If this occurs simultaneously for both firms, there exist incentives to collude in order to

manipulate the price upward, as Ehrhart *et al.* (2008) noted in a symmetric setting.

Actually, their analysis is conducted in the absence of free permits ($S_i = 0$), which is the

less favorable case for the firms and they conclude that, even in this case, the net effect

might be positive. According to our interpretation, the question is to determine to what

extent a higher permit price generates enough scarcity rents for both firms to

compensate for the higher cost of purchasing permits.

Equations (16) and (17) also show that the conditions under which a higher price

is profit-enhancing are different for the leader and the follower. This is due, not only to

the fact that they may have different cost structures, but also to the fact that their

reactions to a price increase, given in (13) and (15), are different. This opens up the

possibility of disagreement between the firms insofar as one of them is interested in a

price increase and the other one in a price decrease. This is contrary to Ehrhart *et al.*

(2008), in which both firms are symmetric and therefore either both firms are better-off

or both are worse-off after a price increase. Hence, the introduction of asymmetry seems to reduce the scope for collusion. At this level of generality, it is not possible to gain more specific insights. For that reason, we explore a specific case in the next section.

3.3 A Separable Function

To gain some additional insight, we assume now a separable cost function. As is done in Ehrhart *et al.* (2008), we initially consider a basic case with no grandfathering ($S_1 = S_2 = 0$) and the same cost functions for both firms, so that the only difference between them is due to their roles as leader and follower. After studying this basic case, we first explore the effect of distributing free permits to the firms and second, the consequences of considering cost asymmetries.

3.3.1 Basic case

Let us assume that production and abatement costs are separable in the following way. The production cost of firm i is given by cx_i, so there is a constant marginal production cost equal to c. The (inverse) demand function for output has the linear form $P(X) = a - bX$. Each unit of output generates r units of pollution, where $r > 0$ is a constant coefficient of pollution intensity (the gross emissions of firm i are hence given by rx_i). By performing abatement activities, firms can reduce their flow of pollution. Let us denote as $q_i \geq 0$ the amount of emissions abated by firm i. Thus, net emissions are given by $e_i = rx_i - q_i$. Following Sarzetakis (1997), we assume the following quadratic abatement cost function, which is common to both firms:

$$AC(q_i) = q_i(d + tq_i),\qquad(18)$$

where d and t are positive parameters. Adding up all the costs we have the cost function

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$$TC_i\left(x_i, e_i\right) = cx_i + \left(rx_i - e_i\right)\left(d + t\left(rx_i - e_i\right)\right) + pe_i. \qquad (19)$$

To ensure an interior solution, we bound the relevant parameters by including the following technical assumption:

Assumption 1: $d < p < \bar{p}$, *where* $\bar{p} := p \, / \, e_2^*\left(x_2^*, p\right) = 0$. $\qquad (20)$

This assumption rules out uninteresting solutions in which either of the firms produces zero, pollutes zero or abates zero. The lower bound for p prevents abatement from being negative (see Equation (22) below). To understand this result, note that d is the marginal cost of abatement at $q_i = 0$. If the price of permits is even lower than the cost of the first unit of abatement it will never be profitable for the firms to abate, as buying permits is a cheaper option. The upper bound for p is defined as that value of the permit price such that, in equilibrium, it is optimal for the follower to pollute zero.[15] The reason to include this assumption is that, in our setting, the follower's emissions is the first variable to reach a zero value as p increases and hence this is a sufficient condition to ensure a nonnegative solution.[16]

Proceeding as in the general model, we first solve the third stage, in which both firms choose their emission levels. Endowed with our specific analytical expressions, we can compute the optimal amount of emissions of firm *i* as a function of output:

$$e_i^*\left(x_i, p\right) = rx_i - \frac{p - d}{2t}, \qquad (21)$$

and it is straightforward to conclude that firm *i*'s optimal abatement is

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[15] The specific expression for $\bar{p}$ can be found in the appendix. Specifically, it is given by (A3) in the basic model, but takes a different form in the subsequently developed extensions.

[16] If both the follower's abatement and the follower's net emissions are nonnegative, it is straightforward to conclude that the follower's gross emissions, rx_2, are nonnegative, which implies that the follower's output is nonnegative. As we subsequently show, in equilibrium the leader always produces more and pollutes more than the follower and thus Assumption 1 ensures that all the relevant variables of the model are nonnegative in equilibrium.

$$q_i^*(p) = \frac{p-d}{2t} > 0, \quad i = 1, 2, \tag{22}$$

which, due to separability, is independent of output and, due to cost symmetry, is common for both firms. Using (21) in (19), we obtain the expression for the minimized cost function, which reveals that the marginal product cost is constant in output and increasing in permit price:

$$TC_i^*(x_i, p) = x_i(c + pr) - \frac{(p-d)^2}{4t}. \tag{23}$$

We now move on to solve the two first stages, in which both firms decide on their output levels. By standard methods,[17] we obtain

$$x_1^* = \frac{a - c - rp}{2b}, \tag{24}$$

$$x_2^* = \frac{a - c - rp}{4b}. \tag{25}$$

From (24) and (25), we conclude that the leader's output is twice that of the follower's, as in the classical Stackelberg model with linear demand and constant marginal cost, both firms' output depend positively on the demand intercept, a, and negatively on the demand slope, b, and all the cost parameters c, r and p. The equilibrium profits can be now be written as a function of the price of permits, defined as:

$$\Pi_i^*(p) := \left[a - b(x_1^* + x_2^*) \right] x_i^* - TC_i^*(x_i^*, p). \tag{26}$$

Using our specific functions to substitute in (16) and (17) we obtain

$$\frac{\partial \Pi_i^*}{dp} = \underbrace{\frac{r}{2} x_i^*}_{SR_i} \underbrace{- r x_i^* + \frac{p-d}{2t}}_{-e_i^*}, \tag{27}$$

---

from which we conclude that the scarcity rent due to the rise in the permit price that accrue to firm i is given by $rx_i^*/2$. The second and the third terms in (27) determine the marginal increase in cost due to permit purchasing. The second term is gross emissions and measure how much cost would increase in the absence of abatement. Finally, the third term measures how much the firms are able to save by performing abatement activities.

There are some straightforward insights that we can get from equation (27). First, if the firms were not able to abate (and thus the third term would be absent), scarcity rents by themselves would never be able to compensate for the cost increase and thus firms would never benefit from a higher permit price. Second, the positive abatement effect is increasing in the price of permits, which means that the higher the permit price the more firms can gain by using abatement to adapt themselves to the market conditions.

As a third important insight, the simple form of equation (27) allows a straightforward comparison between the effects on the leader's and the follower's profit. Indeed, simple manipulation of (27), together with (24) and (25) gives

$$\frac{\partial \Pi_1^*}{dp} - \frac{\partial \Pi_2^*}{dp} = -\frac{r}{2}\left(x_1^* - x_2^*\right) = -\frac{r(a-c-rp)}{8b} < 0, \qquad (28)$$

where the inequality always holds under interior solution. Thus, we conclude that a rise in the permit price will always benefit the follower more than the leader or will harm the leader more than the follower. According to (28), the reason for this result lies in the output difference: since the leader produces more output than the follower it also pollutes more and, therefore, its cost is more sensitive to a higher permit price.

To study the effect on firms' incentives, we conduct now a more detailed study of the profit functions. For notational convenience, we denote as $\hat{p}_i$ the value of the permit price that minimizes firm i's profit. Formally,

$$\hat{p}_i := \arg\min_p \Pi_i^*(p) \quad i = 1, 2.\tag{29}$$

Lemma 1 and Proposition 1 show the main results of this part of the paper. Lemma 1 determines the shape of the equilibrium profit functions in terms of the permit price and Proposition 1 splits the relevant range in two regions with different consequences for the firms' interests regarding the evolution of p.

LEMMA 1

$\Pi_1^*(p)$ *and* $\Pi_2^*(p)$ are strictly convex functions of p with $d < \hat{p}_2 < \hat{p}_1 = \overline{p}$.

■

PROPOSITION 1

If $d < p < \hat{p}_2$*, a price decrease will make the profit of both firms increase. If* $\hat{p}_2 < p < \overline{p}$*, a price increase will decrease the leader's profit and increase the follower's profit.* ■
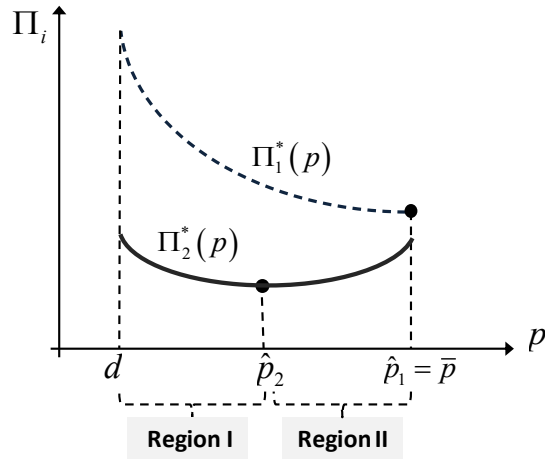
FIGURE 3.1: Equilibrium profits as a function of p (basic case)

The results in Lemma 1 and Proposition 1 are shown in Figure 1. There are two important facts worth highlighting in this figure. First, profits are strictly convex in p with a minimum at $\hat{p}_i$ (for $i=1, 2$). As it can be concluded from equation (27), the convex shape of the profit functions is a direct implication of the firm's reaction to a price increase by abating more and purchasing fewer permits.

The second insight from Figure 1 is that the minima of the profit functions are unambiguously ordered such that $\hat{p}_2 < \hat{p}_1$; i.e., the follower reaches a minimum for a lower price than the leader. Hence, we have that, if $p < \hat{p}_2$, both firms are situated in the decreasing part of their profit functions, which implies that their profit will increase if the permit price decreases. If, instead, $\hat{p}_2 < p < \hat{p}_1$, the follower is situated in the increasing part (and so will benefit from a price increase), whereas the leader is still in the decreasing part (and therefore will still prefer the price to decrease). As we can conclude form our previous discussion, the reason why firm 2's profit reaches a minimum before firm 1's is that, being a Stackelberg follower, it is optimal for firm 2 to produce less than firm 1 and therefore to pollute less. This implies that the direct effect of a price increase on its cost is less pronounced that it is for the leader.

Apparently, if $p > \hat{p}_1$, the leader enters the increasing part of its profit function and both firms will benefit from a higher price. Under our specification, however, we have that $\hat{p}_1 = \bar{p}$; i.e., the minimum of the leader's profit function is reached precisely at the highest value of the price that is compatible with an interior solution (specifically, (21) renders $e_2 < 0$ for any $p > \hat{p}_1$) and hence there is no feasible range under which both firms will benefit from a price increase.

Regarding the existence of incentives for collusion, the main consequence of Lemma 1 and Proposition 1 is that, in our example, there is a range within which both firms are interested in decreasing the price but, unlike the symmetric case developed by Ehrhart *el al.* (2008), it is never the case that both firms simultaneously profit from a price increase. Therefore, they never have incentives to collude in order to push the price up. Moreover, there is a range of disagreement within which the interests of both firms diverge, which can never occur in the symmetric case.

In this example, we have shown how asymmetry between firms (in the form of a leader-follower relationship) reduces the likelihood of collusive behavior to such an extent that they disappear. In the next subsections, we show two generalizations of this example in which the result is not so extreme in the sense that the likelihood of collusion, though smaller than in a purely symmetric setting, does not fully disappear.

3.3.2 Grandfathering

In the basic case, for the sake of comparability with Ehrhart *et al.* (2008), we have assumed that firms do not have any initial allocation of permits and therefore have to buy all the permits they need on the market. In reality, it is common for the participants in CAP systems to receive some permits for free by means of a grandfathering scheme. In fact, as is discussed for example in Alvarez and André

(2014), grandfathering has traditionally been the most widespread method used to distribute permits.

We now extend our setting to consider the possibility that some permits are initially distributed with no cost for the firms via a grandfathering scheme.[18] Hence, firms need only buy those permits that exceed their initial allocation and, moreover, they have the option to sell permits if they pollute less than their initial allocation.

Let us consider that both firms receive an equal allocation of free permits, S, and denote as y_i the amount of permits that firm i buys (if $y_i > 0$) or sells (if $y_i < 0$) on the market, which can be calculated as the difference between net emissions and the allocation of permits:

$$y_i = e_i - S = rx_i - q_i - S, \tag{30}$$

from which we have that $e_i = y_i + S$; i.e., the net emissions of a firm must be covered by permits that either come from its free allocation or are bought on the market. Therefore, firm i's total cost function is now given by the expression

$$TC_i(x_i, y_i) = cx_i + (rx_i - y_i - S)(d + t(rx_i - y_i - S)) + py_i, \tag{31}$$

which can be written in terms of output and net emissions as

$$TC_i(x_i, e_i) = cx_i + (rx_i - e_i)(d + t(rx_i - e_i)) + p(e_i - S). \tag{32}$$

Solving the third stage of the game, we conclude that the optimal levels of emissions and abatement for each firm are still given by (21) and (22), respectively, and it is straightforward to obtain the optimal traded permits and the corresponding minimized cost function:

$$y_i^*(x_i, p) = \frac{d - p}{2t} + rx_i - S, \tag{33}$$

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[18] Actually, the fact that permits are distributed for free is not crucial for our results. The only important assumption is that firms enjoy an exogenously given amount of permits.

$$TC_i^*\left(x_i, p\right) = x_i\left(c + pr\right) - \frac{\left(p-d\right)^2}{4t} - pS,\qquad(34)$$

where separability entails that the minimized cost function has the same structure as in the basic case, except for the fact that the initial permit endowment appears as a reduction in the cost. Regarding the sensitivity of profits to the permit price, as we know from equations (16) and (17), the inclusion of grandfathering does not affect the overall value of scarcity rents, but it modifies the cost term and thus has an impact on the part of the scarcity rents that each firm is able to capture in equilibrium.

Lemma 2 and Proposition 2 are the main results of this part of the paper. We still use the notation introduced in (29) to refer to the value of the permit price that minimizes each profit function. For notational convenience, we also define the following threshold value for S:

$$\tilde{S} := \frac{r\left(a - c - dr\right)}{8b}.\qquad(35)$$

LEMMA 2

When both firms are initially endowed with the same free allocation of permits, S, the equilibrium profit functions for both firms are strictly convex with a unique minimum each at $\hat{p}_i$ for $i = 1, 2$, with $\frac{\partial \hat{p}_i}{\partial S} < 0$. Moreover, we have the following ordering:

a) If $S < \tilde{S}$, then $d < \hat{p}_2 < \hat{p}_1 < \bar{p}$.

b) If $\tilde{S} < S < 2\tilde{S}$, then $\hat{p}_2 < d < \hat{p}_1 < \bar{p}$.

c) If $S > 2\tilde{S}$, then $\hat{p}_2 < \hat{p}_1 < d < \bar{p}$. ■

PROPOSITION 2

When both firms are initially endowed with a free allocation of permits, the following results hold:

a) If $S < \tilde{S}$, the relevant range of values for p has three regions: In region I, defined by $d < p < \hat{p}_2$, both firms become better off when p decreases. In region II, defined by $\hat{p}_2 < p < \hat{p}_1$, the leader becomes better off when p decreases and the follower becomes better off when p increases. In region III, defined by $\hat{p}_1 < p < \bar{p}$, both firms become better off when p increases.

b) If $\tilde{S} < S < 2\tilde{S}$, region I disappears and region II is delimited by $d < p < \hat{p}_1$.

c) If $S > 2\tilde{S}$, regions I and II disappear and region III is defined by the entire feasible range, $[d, \bar{p}]$. ∎
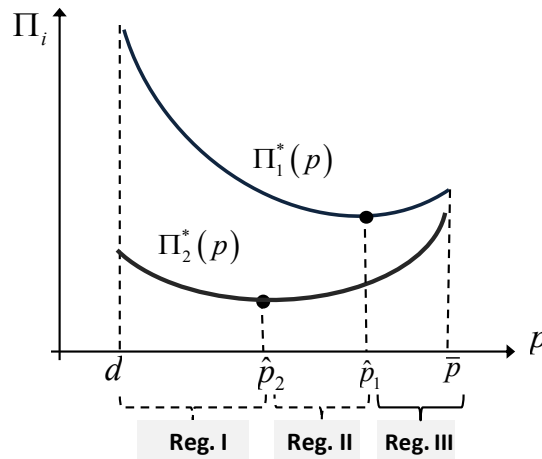
FIGURE 3.2: Equilibrium profits as a function of *p* (grandfathering)

The consequences of Lemma 2 and Proposition 2 are the following. The profit of both firms is still strictly convex in the price of permits, with a minimum at $\hat{p}_i$, $i = 1, 2$. When grandfathering is introduced, the values of the permit price at which the minima are reached, $\hat{p}_1$ *and* $\hat{p}_2$, shift to the left and do so to a greater extent the higher the value of S. This shift implies that, for each firm, there is a wider range of the permit price such that it becomes better-off when the price increases. The reason is that the

existence of free permits makes permit purchasing less costly for firms. Moreover, it opens the way for obtaining positive revenues by selling some permits.

More importantly, the inclusion of grandfathering opens up the possibility of collusion. Let us focus first on case a) (with $S < \tilde{S}$). We have now three regions instead of two, as shown in Figure 3.2. In region III, to the right of $\hat{p}_1$, both firms benefit from a price increase, while the solution is still interior ($e_1, e_2 > 0$). The technical reason why this new region arises is that the direct effect of a price increase on cost is now less pronounced, as the firms have to buy fewer permits and thus they can capture a higher part of the scarcity rents. Furthermore, if the price is high enough, it can also be the case that it is profitable for the firms to sell part of their free endowment instead of buying additional permits, which provides a new opportunity to increase profits. Nevertheless, it can be shown that at $\hat{p}_1$ we have $y_1 > 0$; i.e., at the point where the leader starts finding it profitable to increase the price, it is still a net buyer of permits and hence the profit-enhancing effect is not yet due to selling permits.

Moreover, if the initial allocation of permits is large enough, it could be the case that region I disappears, which implies that the follower is always interested in increasing the price of permits (case b in Lemma 2 and Proposition 2), or even that both regions I and II disappear, which implies that both the leader and the follower are always interested in manipulating the price upward. This is the most favorable case for collusion.

The focus of this paper is on region III, given that this is the only region within which firms can find it profitable to collude in order to push the price up. One natural question is how large this region is, or, in other words, how likely it is to fall within this region. To answer this question, we focus on case a) ($S < \tilde{S}$), which is perhaps the most realistic. The discussion of the other two cases is more straightforward. Region III is

thus delimited by two threshold values for p. First, $\hat{p}_1$, which is the price above which it is profitable, not only for the follower, but also for the leader to push the price up. The second threshold is the upper bound, $\bar{p}$, which is the highest value of the price compatible with an interior solution. The size of region III is thus given by the difference between these two thresholds:

$$\bar{p} - \hat{p}_1 = \frac{4btS}{2b + tr^2},$$ (36)

which depends positively on the number of free permits as well as the slope of the demand curve, b, and the abatement cost parameter t, whereas it depends negatively on the emissions intensity parameter, r.

3.3.3 Asymmetric cost

In the previous subsections we have assumed both firms to be fully symmetric in terms of cost functions and also, in the case of grandfathering, of free permit endowment. There are two reasons for making this assumption. The first is for the sake of simplicity. The second is to focus on the leader-follower relationship as the (only) source of asymmetry between firms. As a sensitivity analysis, in this subsection we consider the possibility that firms are asymmetric in terms of cost and/or initial permit endowment and explore the effect of these asymmetries on the likelihood of generating a propitious environment for collusive behavior. In other words, we explore the effect of different parameters on the size of region III as defined in the previous subsection.

To account for cost asymmetry, we denote the production cost of firm i as $c_i x_i$, where c_i is a firm-specific unit cost parameter. Seeing as we have postulated that firm 1 is a leader and firm 2 is a follower in the output market, it is natural to conjecture that $c_1 < c_2$; i.e., the position of the leader might well be due to the fact that it enjoys a cost

advantage. However, nothing prevents us from considering the opposite case as a theoretical possibility. Analogously, firm i's abatement cost function is given by:

$$AC_i(q_i) = q_i(d_i + t_i q_i), \qquad i = 1, 2. \tag{37}$$

Finally, each firm might receive an initial free endowment of permits, S_i, which is not necessarily constant across firms. Proceeding as in the basic case, we conclude that the optimal amounts of emissions, abatement and purchase of permits for each firm in the third stage are given, respectively, by[19]

$$e_i^*(x_i, p) = \frac{d_i - p}{2t_i} + rx_i, \tag{38}$$

$$q_i^*(p) = \frac{p - d_i}{2t_i}, \tag{39}$$

$$y_i^*(x_i, p) = rx_i - \frac{p - d_i}{2t} - S_i, \tag{40}$$

and, moving on to the first and second stages, we can compute the equilibrium levels of output:

$$x_1^* = \frac{a + c_2 - 2c_1 - rp}{2b}, \tag{41}$$

$$x_2^* = \frac{a + 2c_1 - 3c_2 - rp}{4b}. \tag{42}$$

To investigate the likelihood of observing collusive behavior, we proceed by analyzing the effect of different parameters on the size of region III. In the previous subsection we concluded that, simply by introducing a constant initial allocation of permits, three different cases arise. Now, due to the larger number of varying

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[19] Unlike the other parameters, we assume that the emissions intensity parameter, r, is common to both firms; i.e., $r_1 = r_2 = r$. There are two pragmatic reasons for this simplification. First, the sensitivity analysis results related to these parameters are unclear and so we do not gain any valuable insight by exploring them. Second, the sign of some equilibrium values for some of the key variables are affected by the terms $2r_1 - r_2$ and/or $3r_1 - 2r_2$ and this fact forces us to keep the asymmetry between these parameters bounded so as to avoid meaningless results.

parameters, by choosing the right combination of these parameters we could generate almost any imaginable case. Hence, we need to bound the range of possibilities in some way so as to avoid, on the one hand, meaningless results (such as negative output, negative abatement or negative emissions) and, on the other, a qualitative change in the nature of the solution. For this reason, we introduce the following assumptions in this subsection:

Assumption 1': $d_1, d_2 < p < \bar{p}$, *where* $\bar{p}$ *is defined in* (20).

Assumption 2: $e_1 > e_2$.

Assumption 3: $\hat{p}_2 < \hat{p}_1$.

The two first assumptions ensure nonnegative values for all the relevant variables. The idea is that the leader will still be the one who produces a larger amount of output and a larger amount of emissions. Hence, the follower will still be the one who finds it profitable to pollute zero for a lower value of p and such a value determines the upper bound for the range that is compatible with an interior solution, $\bar{p}$. If this is the case, it is natural to accept that Assumption 3 also holds; i.e., it is easier for the follower than it is for the leader to benefit from a price increase.

Under these assumptions, region III is still delimited by $\hat{p}_1$ and $\bar{p}$ and hence its size increases if $\bar{p}$ increases and/or $\hat{p}_1$ decreases. Proposition 3 summarizes how the size of this region depends on the parameters of the model. Table 1 presents a taxonomy of all the relevant effects.

PROPOSITION 3

The size of region III is increasing in the following cases:

a) *If the leader's marginal production cost, c_1, increases or the follower's marginal production cost, c_2, decreases.*

b) *If the parameter of the linear term in the abatement cost function decreases for the leader (d_1) or increases for the follower (d_2).*

c) *If the parameter of the quadratic term in the leader's abatement cost function, t_1, decreases (provided the number of free permits is moderate) or the equivalent follower's parameter, t_2, increases.*

d) *If the number of free permits received by the leader, S_1, is increasing regardless of the free permits received by the follower.* ■

Effects on thresholds	Changes in model parameters							
	c_1	c_2	d_1	d_2	t_1	t_2	S_1	S_2
$\Delta\bar{p}$	+	-	0	+	0	+	0	0
$\Delta\hat{p}_1$	-	+	+	0	+ (*)	0	-	0
$\Delta(\bar{p}-\hat{p}_1)$	+	-	-	+	- (*)	+	+	0

Table 3.1. Summary of sensitivity analysis results.

(*) For a moderate value of S_1.

Regarding point a) in Proposition 3, increasing the leader's production cost or reducing the follower's cost tends to erode the leader's advantage with respect to the follower, which has the effect of making the firms more symmetric in terms of their

position in the market. The more symmetric the firms are, the more aligned their interests will be and hence it is more likely for them to find it profitable to collude.

Table 3.1 shows that increasing c_1 has a twofold effect. On the one hand, $\bar{p}$ grows because the output of the follower increases, which makes it less likely for firm 2 to decide not to emit at all (in other words, the range of prices under which there is an interior solution widens). On the other hand, $\hat{p}_1$ decreases, as, due to the higher cost, firm 1 tends to produce less and to emit less and hence its total cost will be less sensitive to an increase in the price of permits. Both of these effects tend to enlarge the collusion region. Just the opposite occurs when c_2 increases. Firm 1 tends to produce more and pollute more and hence its cost becomes more sensitive to an increase in the price of permits (which increases the value of $\hat{p}_1$), whereas the follower tends to produce less and to reach the point where it finds it profitable to stop polluting ($\bar{p}$ decreases) sooner, which reduces the size of the collusion region.

As to the parameters of the abatement cost function (d_i and t_i), notice that, due to separability, each firm's parameters are only relevant for the own firm, but not for its rival. Both the linear and the quadratic term of firm 2 are irrelevant in determining the value of $\hat{p}_1$. However, increasing either of them makes the follower's abatement cost increase, which in turn makes it less likely to reach the point where it decides to pollute zero. In other words, it enlarges the relevant feasible range. The corresponding parameters for firm 1 are immaterial in determining the value of $\bar{p}$, their only relevant effect being on $\hat{p}_1$. Assuming a moderate value of the leader's initial endowment of permits, any increase in d_1 and t_1 makes the leader's abatement cost higher, which makes firm 1 become more sensitive to increases in the price of permits.

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Finally, the initial allocation of permits is irrelevant for the upper bound of p, as it represents simply a fixed term in the cost (and the profit) function and so the optimal decisions are not affected. The value of a firm's profits is affected by its own endowment (not the rival's) and hence only S_1 is relevant in determining the size of region III. When the leader's free endowment increases, its cost becomes less sensitive to an increase in the price of permits and it will hence be more receptive to the idea of pushing the price up, thereby increasing the likelihood of observing collusive behavior.

3.4 Conclusions and policy implications

We have explored the possibility that two firms that compete a la Stackelberg in the output market and are subject to a CAT system could have incentives to manipulate the price of permits upward and increase their profits. We do so within a framework similar to that proposed by Ehrhart *et al.* (2008), with the difference that these authors restrict their study to symmetric situations, whereas we explore a situation that is asymmetric in nature. We also include a reading of their results in terms of scarcity rents generation. The main research question is whether the incentives for this type of collusive behavior still exist in a situation in which some firm has a dominant position and the other or others act as followers.

We have shown in a general model that the effect of a permit price increase on the firms' profit has an ambiguous sign as it has two effects: on the one hand, it raises cost but, on the other hand, it creates scarcity rents, of which each firm can only benefit from that part that is due to the rival's output reduction. This ambiguity opens the way for firms to benefit from a price increase and the possibility of colluding in order to manipulate the price upward. However, the asymmetric role of each firm means that the conditions under which a price is profit-enhancing are different for each of them.

Under a separable cost function, we first show that the profit functions are strictly convex in the permit price and secondly that the minima of the profit functions are different for both firms, which creates a region of disagreement within which the leader prefers the price to go down, whereas the follower prefers it to go up. This situation is ruled out in Ehrhart *et al.* (2008) by construction, as the interests of fully symmetric firms are always aligned.

The main message is that a leader-follower relationship in the output market reduces the scope for collusion to manipulate the price of permits upward. Actually, in a standard separable case with symmetric costs functions, if no free permits are distributed among the firms, the region within which there is incentives to collude shrinks to the extent of disappearing. The main policy implication of this finding is that a situation of market power in the product market can preclude the existence of incentives for collusion in the permit market.

Another central policy implication of our research is that distributing some permits for free (e.g. by means of grandfathering) allows the firms to capture a larger share of the scarcity rents and thus opens up the possibility for collusive behavior. The greater the number of permits distributed by a non-market scheme, particularly to firms that enjoy market power, the more incentives there are for collusion. The European Union is reducing the use of grandfathering and increasing the use of auctioning to distribute emission permits. The 2008 revised European Emission Trading Directive established the mandate that auctioning of allowances is to be the default method for allocating allowances as a fundamental change for the third trading period, starting in 2013. The arguments put forward by the European Commission (EC) to support the introduction of auctions are that auctioning "best ensures the efficiency, transparency and simplicity of the system, creates the greatest incentives for investment in a low-

carbon economy and eliminates windfall profits".[20] Our results suggest an additional argument to reduce the use of grandfathering (and arguably to increase the use of auctioning), as it might introduce incentives for price manipulation.

Our final insight is that the likelihood of firms finding collusion profitable is very sensitive to the cost asymmetries between them. In general terms, the more asymmetric the firms are, the more difficult collusion becomes. Moreover, if a grandfathering scheme exists, the more permits are allocated to firms enjoying market power, the more likely collusion becomes.

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[20] See http://ec.europa.eu/clima/policies/ets/cap/auctioning/faq_en.htm, section "Why are allowances being auctioned?". Alvarez and André (2014) present a discussion on the efficiency argument.

APPENDIX 1

A.1 A Extension: Scarcity Rents in a Cournot Model[21]

A.1.1 Introduction

In this section we change the output market structure and introduce a Cournot model while keeping the same assumptions for the emission permits market. The objective is twofold. Firstly we make a comparison with the most relevant results shown in this chapter for the Stackelberg model particularly the ones related to scarcity rents. Secondly we revisit the paper by Erhard et al (2008) in terms of scarcity rents and extend it by introducing grandfathering with the purpose of evaluating the effects of these changes in the firm´s incentives to collude in order to manipulate the price of permits.

Now the game has two stages. In the first one both firms simultaneously decide their output levels (reaction curves) and then in the second stage they decide at the same time on their cost minimizing emission levels. We solve the model by backward induction. Due to the symmetry of the Cournot model we denote firms as (i, -i)

A.1.2 General Model

Within the general model considered in subsection 2, the firms behavior in the emission permits market is the very same as above and equations (1) to (10) apply in this model. In the output market both agents solve its profit maximization problem which yields an equal first order condition for them

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[21] This Appendix is not part of the joint work with Francisco J. André that has been the content of this chapter.

$$P\left(x_i + x_{-i}\right) + \frac{\partial P}{\partial X} x_i - \frac{\partial TC_i}{\partial x_i} = 0 \tag{12a}$$

From this equation we get the reaction function of the firms

$$x_i^* = x_i^*\left(x_{-i}, p\right) \tag{12b}$$

In a similar way as we did for equation (13) we get

$$\frac{\partial x_i^R}{\partial x_{-i}} = \frac{-dP/dX}{2\,dP/dX - \partial^2 TC_i/\partial x_i^2} < 0 \qquad \frac{\partial x_i^R}{\partial x_{-i}} = \frac{\partial^2 TC_i/\partial x \partial p}{2\,dP/dX - \partial^2 TC_i/\partial x_i^2} < 0 \tag{13a}$$

and so, one agent′s output is decreasing in the other agent′s output and the price of permits.

Using both reaction functions, we can get the final equilibrium as a function of the permit price and we can also write both agents′ profit as a function of permit prices.

$$\Pi_i\left(x_i^*, x_{-i}^*, e_i^*, p\right) = P\left(x_i^* + x_{-i}^*\right) x_i^* - TC\left(x_i^*, e_i^*\right) \tag{14a}$$

Differentiating this function with respect to permit prices and taking into account equations (7) and (12a) we conclude

$$\partial \Pi_i/\partial p = \left[\frac{\partial P}{\partial x_{-i}^*} \partial x_{-i}^*/\partial p\right] x_i^* - e_i^* \tag{16a}$$

The marginal effect of the permit price on the profit profit has two components. The first one is the scarcity rent which means a profit increase while the second one is the cost increasing effect due to the higher permit price. This is the same result as in the Stackelberg model in qualitative terms. The main difference is that both agents are in the same position because there is no leader or follower. Since the sign of the equation is ambiguous it means that both firms will benefit from a price increase at the same time, so there is no room for different strategies. This is basically Ehrhart et al (2008) result.

The comparison in quantitative terms with our Stackelberg model is not possible in this general case, so we will now consider the same separable cost function as in section 3 and the same linear demand function.

A.1.3 A Separable Function

Abatement and Total Cost functions do not change and equations (18) and (19) apply. We still consider assumption 1 to rule out meaningless solutions. Equation 20 becomes:

$$d < p_C < \bar{p}_C \quad \text{where} \quad \bar{p}_C := p / e_i^*\left(x_i^*, p\right) = 0 \tag{20a}$$

The optimal amount of emissions and optimal abatement are given by equations (21) and (22), and the minimized cost function is given by equation (23). The output level which is common for both firms is given by

$$x_i^* = x_{-i}^* = \frac{a - c - rp}{3b} \tag{24a}$$

The equilibrium profits can be stated as a function of the price of permits, and equation (26) applies. The marginal profit is common for both agents

$$\frac{\partial \Pi_i(p)}{\partial p} = \frac{1}{3}rx_i - rx_i + \frac{p - d}{2t} \tag{26a}$$

Once again the sign of this derivative is ambiguous. The first term is the scarcity rent, whiles the second and third relates to gross emissions and the abatement cost. The scarcity rents as a proportion of output are lower for Cournot firms (r/3) than for Stackelberg firms (r/2). But the amount that the firms are able to save by performing abatement activities is equal in both models. This means that the positive abatement effect is more important for Cournot firms as it is the incentive to abate when the permit price increases.

As it was proven in Section 3, the profit function is convex and a critical point can be established. This critical point shows a particular price that yields the minimum profit.

Lemma 1.A

There is an equilibrium permit price that is common for both agents where the marginal profit with respect to the permit price is zero. For higher prices the profit is increasing with price. For lower prices is decreasing.

We denote as $\hat{p}_C$ the value of the permit price that minimizes firm i's profit. In the Appendix we prove that

$$d < \hat{p}_C < \bar{p}_C \tag{29a}$$

For comparison purposes we also denote the critical prices for the Stackelberg leader and follower as

$$\hat{p}_L := Leader's\ price \qquad \hat{p}_F := Follower's\ price$$

Proposition 1.A

The critical price for Cournot firms is lower than the Stackelberg leader but higher than the follower´s. The upper bound for p, (defined as that value of the permit price such that, in equilibrium, it is optimal to pollute zero) is higher in the Cournot model.

$$\hat{p}_L > \hat{p}_C > \hat{p}_F \qquad\qquad \bar{p}_C > \bar{p}_F$$

A.1.4 Grandfathering in the Cournot model

We consider a grandfathering scheme as in subsection 3.2. The amount of traded permits and the total cost function are in accordance to equations (30) to (32) and the

optimal traded permits and the correspondence minimizes cost functions like equations (33) and (34).

In a similar way as happened in the Stackelberg model, the introduction of free permits shifts the minimum permit price to the left while the upper bound price remains the same, because equation (21) applies. It means that the range of prices for which a price increase implies a profit increase is wider now. Even more, the region where profit decreases as price increases just disappears for a certain value of S, as in the Stackelberg model.

Let us denote by Region I the range of permit prices where firm´s profit are decreasing and Region II the range of permit prices where profits are increasing. Let $\tilde{S} = \dfrac{2r(a-c-dr)}{9b}$ be a particular amount of permits. Now:

Proposition 2.A

The introduction of grandfathering wideness Region II, to the extent that Region I disappears when the amount of free permits exceed the quantity $\tilde{S}$.

We prove it in the Appendix 2

APPENDIX 2

Proof of Lemma 1

Using (24) and (25) in (21), we obtain the equilibrium values for emissions:

$$e_1^*\left(x_1^*, p\right) = \frac{db + rt(a-c) - p\left(b+tr^2\right)}{2bt}, \tag{A1}$$

$$e_2^*\left(x_2^*, p\right) = \frac{2bd + tr(a-c) - p\left(2b+tr^2\right)}{4bt}, \tag{A2}$$

and using the definition given in (20), we compute the value of $\bar{p}$ by equating (A2) to

zero:

$$e_2^*\left(x_2^*, p\right) = 0 \Rightarrow p = \bar{p} = \frac{2bd + rt(a-c)}{2b + tr^2}. \tag{A3}$$

Using (24) and (25) in the inverse demand expression $P(X) = a - bX$, we get

the equilibrium price of output: $P = \dfrac{a + 3(c + pr)}{4}$. Using the equilibrium expressions

for x_1, x_2, e_1, e_2 and P together with (19), we obtain the expressions for the

equilibrium profits of both firms:

$$\Pi_1^*(p) = \frac{t(a-c-pr)^2 + 2b(p-d)^2}{8bt},$$

$$\Pi_2^*(p) = \frac{t(a-c-pr)^2 + 4b(p-d)^2}{16bt}.$$

Differentiating twice with respect to p, we conclude that the second derivative

of both functions is positive and thus both of them are strictly convex in p. By

inspection of the first derivative and bearing in mind (A3), we conclude that Π_1 has a

minimum at $p = \hat{p}_1 = \bar{p}$, which implies that Π_1 is decreasing in p for all the feasible

values of p below $\bar{p}$. We similarly conclude that Π_2 has a minimum at

$\hat{p}_2 := \dfrac{tr(a-c) + 4bd}{4b + tr^2}$, which implies that Π_2 is decreasing in p for $p < \bar{p}_2$ and

increasing for $p > \bar{p}_2$.

The last step is to check that the thresholds are ordered in the right way. By

direct comparison, we conclude that

$$d < \hat{p}_2 < \bar{p} \Leftrightarrow a - c > dr.$$

To prove that the last inequality is true, using (22) and the definition of abatement ($q_i = rx_i - e_i$), we conclude that, within the relevant range, $x_2 > \dfrac{e_2}{r} > 0$.

Using the expression for x_2^* given in (25), we conclude that $x_2 > 0$ implies $a - c > rp$, and that this inequality, together with $d < p$ (Assumption 1), implies $a - c > dr$.

QED.

Proof of Proposition 1

The result follows straightforwardly from Lemma 1: the relevant range for p is delimited by d and $\bar{p}$. As Π_2^* is strictly convex and reaches a minimum at $\hat{p}_2$, we conclude that it is strictly decreasing between d and $\hat{p}_2$ and strictly increasing between $\hat{p}_2$ and $\bar{p}$. As Π_1^* is strictly convex and reaches a minimum at $\hat{p}_1 = \bar{p}$, it is strictly decreasing between d and $\bar{p}$. This completes the proof.   QED.

Proof of Lemma 2

As the expressions for e_i ($i=1,2$) are the same as in the basic case and the minimized cost function (34) is the same as (23), except for a constant term, it immediately follows that the expressions for x_i ($i=1,2$) are also the same as in the basic case. Using these values, we get the equilibrium profits of both firms

$$\Pi_1^*(p) = P\left(x_1^* + x_2^*\right)x_1^* - TC_1^*\left(x_1^*, p\right) = \frac{t(a - c - pr)^2 + 2b(p - d)^2 + 8btpS}{8bt},$$

$$\Pi_2^*(p) = P\left(x_1^* + x_2^*\right)x_2^* - TC_2^*\left(x_2^*, p\right) = \frac{t(a - c - pr)^2 + 4b(p - d)^2 + 16btpS}{16bt}.$$

The second derivative reveals that these functions are still strictly convex. Differentiating them with respect to p, we conclude that they have respective minima at

$$Arg\min_p \Pi_1^*(p) \equiv \hat{p}_1 = \frac{rt(a-c)+2bd-4btS}{2b+tr^2},$$

$$Arg\min_p \Pi_2^*(p) \equiv \hat{p}_2 = \frac{rt(a-c)+4bd-8btS}{4b+tr^2},$$

and it follows straightforwardly that both $\hat{p}_1$ and $\hat{p}_2$ depend negatively on S.

Regarding the order of the thresholds, by direct comparison we conclude that $\hat{p}_1 > \hat{p}_2 \Leftrightarrow 2brt(a-c-dr+2rSt) > 0$; however, in the proof of Proposition 1 we have proved $a-c-dr \geq 0$, which ensures that $\hat{p}_1 > \hat{p}_2$. Moreover, using (A3) we also conclude that $\bar{p} = \hat{p}_1 + \frac{4btS}{2b+tr^2} > \hat{p}_1$. Hence, we have that $\hat{p}_2 < \hat{p}_1 < \bar{p}$. To determine the relative position of d, let us first recall that, from Lemma 1, we know that $d < \bar{p}$ and hence we only have to check whether d is below $\hat{p}_2$, in the interval $(\hat{p}_2, \hat{p}_1)$ or in the interval $(\hat{p}_1, \bar{p})$. By direct comparison, we conclude the following:

$$\hat{p}_1 > d \Leftrightarrow S < \frac{r(a-c-dr)}{4b} = 2\tilde{S}, \qquad (A4)$$

$$\hat{p}_2 > d \Leftrightarrow S < \frac{r(a-c-rd)}{8b} = \tilde{S}. \qquad (A5)$$

This completes the proof. QED.

Proof of Proposition 2

Let us first consider statement a). The results in regions I and II follow from Lemma 2 according to a similar reasoning to that used in the proof of Proposition 1. In region III, between $\hat{p}_1$ and $\bar{p}$, it is straightforward to conclude that both $\Pi_1^*(p)$ and $\Pi_2^*(p)$ are strictly increasing in p. Statements b) and c) follow straightforwardly from (A4), (A5) and Assumption 1. QED.

Proof of Proposition 3

Using (41) and (42) in (38), we obtain the equilibrium values for emissions:

$$e_1^*\left(x_1^*, p\right) = \frac{b\left(d_1 - p\right) + t_1 r\left[a + c_2 - 2c_1 - rp\right]}{2bt_1},$$

$$e_2^*\left(x_2^*, p\right) = \frac{2b\left(d_2 - p\right) + rt_2\left(a + 2c_1 - 3c_2 - rp\right)}{4bt_2}.$$

By imposing the non-negativity conditions on the follower's emissions, we obtain the upper bound value for the permit price, $\bar{p}$ in this case:

$$e_2^* \geq 0 \Leftrightarrow p \leq \bar{p} = \frac{2bd_2 + rt_2\left(a - 3c_2 + 2c_1\right)}{2b + r^2 t_2}. \qquad (A6)$$

By substitution of the relevant variables in the profit function, we obtain the expression for the leader's profit function in terms of the model parameters:

$$\Pi_1^*(p) = \frac{\left[a + c_2 - 2c_1 - rp\right]^2}{8b} + \frac{\left(p - d_1\right)^2}{4t_1} + pS_1$$

Differentiating with respect to p, we obtain

$$\frac{\partial \Pi_1^*}{\partial p} = \frac{2b\left(p - d_1\right) + 4bt_1 S_1 - rt_1\left[a + c_2 - 2c_1 - rp\right]}{4bt_1}$$

and, by equating this derivative to zero, we get the minimum value of p such that the leader finds it profitable to push the price up, $\hat{p}_1$:

$$\frac{\partial \Pi_1^*}{\partial p} \geq 0 \Leftrightarrow p \geq \frac{rt_1\left(a + c_2 - 2c_1\right) + 2bd_1 - 4bt_1 S_1}{r^2 t_1 + 2b} = \hat{p}_1. \qquad (A7)$$

By direct differentiation of the values of $\bar{p}$ and $\hat{p}_1$, we obtain the results in the proposition:

$$\frac{\partial \bar{p}}{\partial c_1} = \frac{2rt_2}{2b + r^2 t_2} > 0; \qquad \frac{\partial \hat{p}_1}{\partial c_1} = \frac{-2rt_1}{2b + rt_1} < 0;$$

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$$\frac{\partial \overline{p}}{\partial c_2} = \frac{-3rt_2}{2b+r^2t_2} < 0; \qquad \frac{\partial \hat{p}_1}{\partial c_2} = \frac{rt_1}{2b+rt_1} > 0;$$

$$\frac{\partial \overline{p}}{\partial d_1} = 0; \qquad \frac{\partial \hat{p}_1}{\partial d_1} = \frac{2b}{2b+rt_1} > 0;$$

$$\frac{\partial \overline{p}}{\partial d_2} = \frac{2b}{2b+r^2t_2} > 0; \qquad \frac{\partial \hat{p}_1}{\partial d_2} = 0;$$

$$\frac{\partial \overline{p}}{\partial S_1} = \frac{\partial \overline{p}}{\partial S_2} = 0;$$

$$\frac{\partial \hat{p}_1}{\partial S_1} = \frac{-4bt_1}{2b+rt_1} < 0; \qquad \frac{\partial \hat{p}_1}{\partial S_2} = 0;$$

$$\frac{\partial \hat{p}_1}{\partial t_1} = \frac{2br(a+c_2-2c_1-d_1r)-8b^2S_1}{\left[r^2t_1+2b\right]} \le 0 \Leftrightarrow S_1 \le \frac{r(a+c_2-2c_1-d_1r)}{4b};$$

$$\frac{\partial \overline{p}}{\partial t_2} = \frac{2br\left[a-3c_2+2c_1-rd_2\right]}{\left(2b+r^2t_2\right)^2} > 0,$$

where, in an interior solution, the numerator of the last expression must be positive for the follower's output to be positive. QED.

PROOF OF LEMMA 1.A

The equilibrium values for emissions can be obtained from equations (21) and (24a).

Equating this value to zero we get the maximum permit price

$$e_i^* = \frac{2rt(a-c-rp)-3b(p-d)}{6bt} = 0 \Rightarrow \overline{p}_C = \frac{2rt(a-c)+3bd}{3b+2r^2t} \qquad \text{(A3a)}$$

Using equations (19) and (24a) together with the inverse demand function yields the following expression for the equilibrium profit

$$\Pi_i(p) = \frac{4t(a-c-rp)^2 + 9b(p-d)^2}{36bt}$$

Differentiating the function with respect to p we get the critical price

$$\hat{p}_C = \frac{4rt(a-c)+9bd}{9b+4r^2t}$$

By comparison of these two values and taking into account equation (20a) we conclude

$$\bar{p}_C - \hat{p}_C = 6brt(a-c-rd) \Rightarrow d < \hat{p}_C < \bar{p}_C$$

PROOF OF PROPOSITION 1.A

We just compare the corresponding expressions, compute the difference and check the sign to prove the first part of the proposition.

$$\hat{p}_L > \hat{p}_C \Leftrightarrow \frac{rt(a-c)+2bd}{2b+r^2t} > \frac{4rt(a-c)+9bd}{9b+4r^2t}$$
$$[rt(a-c)+2bd](9b+4r^2t) - [4rt(a-c)+9bd](2b+r^2t) =$$
$$= brt(a-c) - dbr^2t = brt(a-c-dr) > 0$$
$$\hat{p}_C > \hat{p}_F \Leftrightarrow \frac{4rt(a-c)+9bd}{9b+4r^2t} > \frac{rt(a-c)+4bd}{4b+r^2t}$$
$$[4rt(a-c)+9bd](4b+r^2t) - [rt(a-c)+4bd](9b+4r^2t) =$$
$$= 7brt(a-c) - 7dbr^2t = 7brt(a-c-dr) > 0$$

By direct comparison of equation (A3a) with the equation (A3) we prove the second part of the proposition

$$\frac{3bd+2rt(a-c)}{3b+2r^2t} - \frac{2bd+rt(a-c)}{2b+r^2t} = rbt(a-c-dr) > 0$$

PROOF OF PROPOSITION 2.A

The equilibrium profit is now

$$\Pi^*_i(p) = \frac{4t(a-c-rp)^2 + 9b(p-d)^2 + 36bptS}{36bt}$$

Differentiating the function we obtain the minimum price value

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$$\hat{p}_C^G = \frac{4rt(a-c)+9bd-18btS}{9b+4r^2t}$$

Now we compare this minimum profit price with and without calculation and check the sign to prove the first part of the proposition

$$\hat{p}_C - \hat{p}_C^G = \frac{18btS}{9b+4r^2t} > 0$$

To prove the second part of the proposition we note that Region I disappears when the lower bound of prices equals the minimum price. We can formally write this condition like $d = \hat{p}_C^G$. We calculate the limit value of free permits to meet this condition

$$\hat{p}_C^G > d \Leftrightarrow \frac{4rt(a-c)+9bd-18btS}{9b+4r^2t} > d \Leftrightarrow S < \frac{2r(a-c-dr)}{9b} = \tilde{S}$$

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Chapter 4

Imperfect Competition in Product and Permit Markets

4.1 Introduction

In this chapter we analyze the strategic behavior of polluting oligopolistic firms that interact strategically both in the output and the permit markets. The main difference with Chapter 3 is that we now consider the price of permits as endogenous instead of an exogenously given variable. We consider a model of two firms competing under different oligopolistic structures while there is a dominant firm in the permit market. In this framework, we study and compare the effect of market power under three different situations.

We first study the permit market assuming that there is a dominant firm and another one that behaves as a price taker. We show that the equilibrium of such a market is crucially determined by the equilibrium in the output market and the initial allocation of permits. Then, we move on to study the output market and its link with the permit market

Regarding the output market, we consider three alternative market structures. In the first version we consider a Cournot oligopoly whereas in the other two there are a leader and a follower a la Stackelberg. Specifically, in the second version we consider that the firm that acts as a leader in the output market (Firm 1) is the follower in the permits market. As far as we know, a double duopoly with different leaders has not been analyzed yet. In this case we look for conditions to ensure that being a leader in one of these markets implies a competitive advantage or, in other words, which is the best place to exercise market power. In the third version, we consider a dominant firm in the emissions permits market that is also a Stackelberg leader in the product market.

Some papers have used the Stackelberg model to analyze the impact of market power in the efficiency of cap-and-trade environmental policies. Hahn (84) considers the existence of a dominant firm in the emissions permit market while the output market is competitive. The third chapter of this thesis considers the Stackelberg model in the output market while the related permits market is competitive.

Hinterman (2011) considers a firm which is a dominant firm in both markets and found that such a firm will set the permit price above its marginal abatement costs and therefore efficiency cannot be achieved by means of the permit allocation alone.

Chevalier (2008) considers a permit market with both spatial and intertemporal trading. Market power is introduced by assuming a large dominant agent in a Stackelberg position and a large number of small firms who are nonstrategic but

forward looking. The equilibrium is characterized for the monopoly case and for intermediate cases.

The leader-fringe models are also very close to ours. Tanaka & Chen (2012) consider a Cournot-fringe model with market power in both product and permits market to simulate the California electricity market and they show that Cournot firms can significantly raise both power price and permit price, which results in a great loss in social surplus.

We try to address situations in which there is a strong interaction between the strategic behavior of firms or countries in both product and permit markets. The NOx permit market in California is one example. Nearly 25% of the NOx permits were allocated to facilities that sell power into the California electricity market, which has been recognized for its (unilateral) market power problems. In fact, Kolstad and Wolak (2003) argue that electric utilities used the NOx market to enhance their ability to exercise (unilateral) market power in the electricity market[22]

Hagem and Maestad (2005) analyses the optimal strategies for a country like Russia that could have market power in an international market for emission permits and at the same time participates in a non-competitive fuel export market. By means of numerical simulations, they concluded that Russia could benefit from coordinating its permit exports with its oil and gas exports during the commitment period.

In the same line, Montero (2009) argues that some of the large countries in an eventual global carbon market are also big players in energy markets. One paradigmatic example is the Russia´s role in the Kyoto Protocol. It has been argued[23] that USA rejection resulted from the fact that its least costly way to implement targets would have

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[22] See Fowlie (2010) for the analysis of the effects on permit market efficiency.
[23] See Russia's Role in the Kyoto Protocol *Alain Bernard, Sergey Paltsev, John M. Reilly Marc Vielle and Laurent Viguier* Report No. 98 *June 2003* The MIT Joint Program on the Science and Policy of Global Change

involved large purchases of emission credits from Russia, which is also a major fossil fuel exporter.

Some other key aspects have already been covered by the literature. Malueg and Yates (2009) develop a model that deals precisely with a permit market in which there are only strategic players due to the observation that for some permit markets, the oligopoly fringe structure may not apply.

Another concern is that under cap and trade, the economic rents can be lost to energy exporting countries. Berger et al (91) showed that when the supply side of fossil fuel markets is imperfectly competitive, cap and trade could lead to the transfer of policy-generated rents from the domestic economy to fossil-fuel-exporting countries.

The remainder of the chapter has the following structure: In the next section (4.2) we describe the basic elements of the model. Section 4.3 analyses the equilibrium in the emissions permit market. In the next three sections we explore the implications of grandfathering in the firms output and profit within three different oligopolistic structures. In section 4.4 we present a Cournot model. Section 4.5 describes a model where the price taker firm in the permit market is a Stackelberg leader in the related product market. In Section 6, the dominant firm in the permit market is also the Stackelberg leader in the product market. Section 7 states our conclusions.

4.2 The Model. Basic Elements

We consider a model with two firms labeled $i = 1, 2$. Both firms enjoy an initial firm-specific free allocation of permits, S_i ($i=1,2$). We set up the model with the same particular functions as in section 2.4. In the output market, the inverse demand function is $P=a–bX$. On the production side we assume that the firms face a constant marginal cost of production c. Gross emissions are assumed to be proportional to the firms´

output (rx_i) where r is the pollution intensity which is common for both firms. The firms can reduce emissions by either reducing output or making abatement. Abatement cost is a quadratic function $q_i (d + tq_i)$ where d and t are technological parameters. We assume the same abatement technology for both firms. Finally both firms trade permits. We denote permits purchased (sold) by firm i y_i, which in equilibrium has to be equal to

satisfy $y_i = -y_{-i}$, where $-i$ refers to the firm other than i.

We also assume, as it is common in the literature that the emissions generated by both firms can be perfectly monitored without cost by the regulatory authorities and firms cannot emit more than the number of permits they hold. Alternatively, we can interpret that a high enough penalty has to be paid to ensure that there is no room for moral hazard.

4.3 A dominant firm in the emission permits market

Regardless the oligopolistic structure considered in the product market, we will solve every model by backward induction. It means that Firm 2 solves its abatement cost problem in the last stage. This problem is:

$$Min_{\{q_2\}} q_2 (d + tq_2) + py_2$$
$$s.t. \quad S_2 + q_2 + y_2 = rx_2 \tag{1}$$

We can substitute the constraint into the objective function and arrive at the familiar first-order condition, which states that marginal abatement costs equal the permit price.

$$\underbrace{d + 2tq_2}_{MAC_2} = p \tag{2}$$

and combining this expression with the constraint we get optimal demand for permits of firm 2:

$$y_2 = rx_2 - S_2 - \frac{p-d}{2t}. \tag{3}$$

These conditions define firm 2´s optimal emissions and permit purchase decisions as a function of the permit price.

The dominant firm minimizes its own costs anticipating the reaction of the follower. It faces a permit market-clearing condition given by

$$y_1 = rx_1 - q_1 - S_1 = -y_2 = -rx_2 + q_2(p) + S_2. \tag{4}$$

Solving the problem yields the optimal abatement of the dominant firm (details can be found in the Appendix):

$$q_1 = \frac{r(2x_1 + x_2) - 2S_1 - S_2}{3}. \tag{5}$$

Equations (4) and (5) let us find firm 2´s optimal abatement and demand for permits in equilibrium as a function of output and the initial allocation of permits:

$$q_2 = \frac{r(2x_2 + x_1) - 2S_2 - S_1}{3}, \tag{6}$$

$$y_2 = -y_1 = \frac{(S_1 - S_2) - r(x_1 - x_2)}{3} = q_2 - q_1. \tag{7}$$

Equation (7) shows that the dominant firm will be a net buyer of permits ($y_1 > 0$) when its abatement exceed the follower´s. It means that, in such a case, the dominant firm buys permits at a price that is lower than its marginal cost of abatement, as can be easily proof considering equation (2)

$$p = \underbrace{d + 2tq_2}_{MAC_2} < \underbrace{d + 2tq_1}_{MAC_1}.$$

In the same fashion it can be stated that if the dominant firm is a net seller of permits the price will exceed its marginal cost of abatement. In both cases the dominant position is an instrument that firm 1 can use to reduce its cost and increase its profit.

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Equation (7) also reveals that the equilibrium in the permit market is driven by two main elements: first, the equilibrium in the output market, and more specifically, the difference in both firms' output, and second, the difference in the initial permit allocation. If we consider the particular case where both firms receive the same amount of free permits, Equation (7) shows that the net demand for permits is proportional to the difference in output which means that the firm that produces more (less) output acts as a net buyer (seller) of permits. If both firms produced the same, there would not be any trade of permits, and both firms will abate the same amount of emissions.

To have a full picture, in the following sections we investigate the equilibrium of the output market and how such equilibrium is influenced by the initial allocation. We consider two possibilities: first, both firms compete simultaneously in output a la Cournot, and second, there is a follower and a leader, a la Stackelberg. In the second case, in turn, we consider two possibilities depending on whether the leader in the output market is the same firm that has a similar position in the permit market or not.

4.4 A Cournot Model

In this section we assume a Cournot oligopoly in the product market. Therefore, both firms choose their output simultaneously, anticipating that in the emissions permit market firm 1 is a dominant firm while firm 2 acts as a price taker.

Then, our model results in a game with two stages, which we solve by backward induction.

In the first stage of the game the firms solve simultaneously its profit maximization problem anticipating the equilibrium of the second stage given by equations (2), (5), (6) and (7). As in the standard Cournot model, the FOC´s yield the reaction functions as we show below.

The profit maximization problem on output for firm 2 is:

$$Max\left(a - b\left(x_1 + x_2\right)\right)x_2 - cx_2 - q_2\left(d + tq_2\right) - p\left(rx_2 - q_2 - S_2\right)$$
$$s.t. \text{ to } Eq(2) \text{ and } (6) \tag{8}$$

And the resulting reaction curve is

$$x_2 = \frac{9\left(a - c - dr\right) + 2rt\left(8S_2 + S_1\right)}{18b + 16r^2t} - \frac{9b + 2r^2t}{18b + 16r^2t}x_1. \tag{9}$$

In a similar way we obtain firm 1's reaction curve:

$$x_1 = \frac{9\left(a - c - dr\right) + 6rt\left(2S_1 + S_2\right)}{18b + 12r^2t} - \frac{9b + 6r^2t}{18b + 12r^2t}x_2. \tag{10}$$

Solving the system given by equations (9) and (10) we obtain the optimal output of both agents in terms of the model parameters:

$$x_2 = \frac{\left(a - c - dr\right)}{\left(3b + 2r^2t\right)} + \frac{rtS_2\left(26b + 20r^2t\right) - 8brtS_1}{\left(9b + 10r^2t\right)\left(3b + 2r^2t\right)}, \tag{11}$$

$$x_1 = \frac{\left(a - c - dr\right)}{\left(3b + 2r^2t\right)} + \frac{rtS_1\left(22b + 20r^2t\right) - 4brtS_2}{\left(9b + 10r^2t\right)\left(3b + 2r^2t\right)}. \tag{12}$$

Note that output is not necessarily equal for both firms as it would be the case in the standard Cournot model with symmetric firms. Combining (11) and (12) we conclude that the difference in output is given by

$$x_1 - x_2 = \frac{10rt\left(S_1 - S_2\right)}{\left(9b + 10r^2t\right)}. \tag{13}$$

This result is somewhat surprising. If both firms initially receive the same amount of permits, firm 1´s dominant position in the emission permits market does not lead to any advantage in practical terms as both firms will produce the same amount of output $\left(x_1 = x_2\right)$ and, as a consequence, abatement will be the same for both firms, as can be seen from equations (5) and (6) and there will not be any permit trade.

Plugging equations (11) and (12) into equation (5) we obtain the optimal abatement of the dominant firm in terms of the parameters of the model.

$$q_1 = \frac{r(a-c-dr)-b(2S_1+S_2)}{(3b+2r^2t)} \ .$$
(14)

A similar procedure is employed to obtain the abatement of firm 2

$$q_2 = \frac{r(a-c-dr)}{(3b+2r^2t)} - \frac{2bS_2(9b+8r^2t)+bS_1(9b+14r^2t)}{(3b+2r^2t)(9b+10r^2t)} \ .$$
(15)

Combining both expressions it is immediate to compute the difference in the abatement made by both firms which, according to (7), provides the equilibrium of the permit market:

$$q_1 - q_2 = \frac{3b(S_2-S_1)}{9b+10r^2t} \ .$$
(16)

Summing up, although there is a dominant position in the permit market, if both firms receive the same amount of permits and compete a la Cournot in the output market, both firms will produce the same, and therefore, there will be no trade of permits. As a consequence, both firms will also make the same profit.

From equations (13) and (16) it can easily be proved that both firms increase output and decrease abatement by the same amount when they receive one more permit. But the marginal profit that Firm 1 obtains from one additional permit is higher than the marginal profit that Firm 2 would obtain in the same event, due to the fact that Firm 1 buys (sells) permits at a lower (higher) price than its marginal abatement cost.

We can summarize the main results regarding firms´ behavior when they receive a different amount of free permits in the following proposition.

PROPOSITION 1

a) When firm 1 *receives more (less) free permits than firm* 2, *it produces more (less) output and consequently its gross emissions are higher (lower). Although firm* 1 *increases (decreases) its abatement as increases (decreases) its output with respect to the case with symmetric allocation, it still abates a lower (higher) quantity of emissions than firm* 2 *and become a net seller (buyer) of permits.*

b) The firm that receives more free permits makes a higher profit

The first part of the proposition has a trivial demonstration. We prove the second part in the Appendix.

4.5 A Stackelberg Model with a Different Leader in Each Market

In this section we analyze another situation where one firm dominates the product market and the other dominates the permit market. The rest of the elements of the model are the same as in the previous section.

With such an approach we aim at determining the potential advantages of being a leader in each of these markets. For consistency with the previous sections, we denote as "Firm 2" the one that acts as a leader in the output market (and as a price taker in the permit market) while "Firm 1" is still the dominant firm in the permit market and act as a follower in the output market. Initially, we consider that both firms enjoy an equal initial free allocation of permits $S_1 = S_2 = S$.

The game has four stages that develop as follows:

1. Firm 2 sets its output acting as a Stackelberg leader

2. Firm 1 sets its output as a follower

3. Firm 1 decides its abatement level (and thus its demand for permits) and the price of permits anticipating the reaction of Firm 2.

4. Firm 2 decides its level of abatement and demand for permits acting as a price taker.

The model is solved by backward induction. Stages 3 and 4 have already been solved in Section 4.3. Due to the assumption $S_1 = S_2 = S$, equations (5) and (6) become

$$q_1 = \frac{r(2x_1 + x_2) - 3S}{3} \tag{17}$$

$$q_2 = \frac{r(2x_2 + x_1) - 3S}{3} \tag{18}$$

and the associated permit price is

$$p = d - 2tS + \frac{2tr(2x_1 + x_2)}{3}. \tag{19}$$

From (17) and (18) we get the number of traded permits as a proportion of the difference between the agent´s output quantities, and also as the difference between the abated quantities.

$$y_1 = \frac{r(x_1 - x_2)}{3} = q_1 - q_2 \tag{20}$$

which is a particular case of (7) when both firms receive the same amount of free permits.

In the first two stages of the game the agents solve its profit maximization problem sequentially as it is common in the Stackelberg models. As in a standard Stackelberg model, the product leader (firm 2) considers the follower´s reaction curve and closes the market. After some algebra we get optimal outputs in terms of the parameters (detailed calculations are given in the Appendix):

$$x_2 = \frac{\left(6b + 6r^2t\right)\left(a - c - dr + 2rtS\right)}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} > 0 \tag{21}$$

$$x_1 = \frac{\left(3b + 6r^2t\right)\left(a - c - dr + 2rtS\right)}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} > 0 \tag{22}$$

Throughout our analysis, we assume $(a\text{-}c\text{-}dr) > 0$, to make sure that output is always positive even in the case of null allocation of free permits ($S = 0$).

Using expressions (21) and (22), we can immediately compare both outputs:

$$x_2 - x_1 = \frac{3b\left(a - c - dr + 2rtS\right)}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} > 0 \tag{23}$$

Equation (23) shows that as long as both firms produce positive amounts, the leader´s output is always greater than the follower´s, which is a standard result in the Stackelberg model. Actually, in the standard Stackelberg model with linear demand and constant marginal cost, the leader produces exactly double as much as the follower. In our case, the ratio between the outputs of both firms equals

$$\frac{x_2}{x_1} = \frac{2b + 2r^2t}{b + 2r^2t} \in \left(1,2\right) \tag{24}$$

and so the leader produces less than double except if $r = 0$ or $t = 0$, which would lead us to the standard Stackelberg model. The higher the value of r and t, the more similar the outputs of both firms are. The interpretation of this result is that, the more important the environmental problem (as determined by the pollution intensity and the convexity of the abatement cost function) the more able firm 1 is to overcome the leadership position of firm 2 in the output market, although it will never be able to leapfrog firm 2 in terms of output.

To find out how firm 1 takes advantage of its leadership in the permit market, note the fact that $x_2 > x_1$, combined with (17) and (18) implies that $q_2 > q_1$, i.e., firm 2 makes more abatement than firm 1, and since the abatement cost function is strictly convex, we conclude that, in equilibrium, the marginal abatement cost of firm 2 is higher than that of firm 1 and this result, combined with equation (2), implies that firm 1 exerts its market power in the permit market by setting a price that is above its marginal abatement cost (and equal to that of firm 2). The main features of the equilibrium are summarized in the following proposition.

PROPOSITION 2

In equilibrium, the following results hold:

> *a) Firm 2 produces more and abates more than firm 1.*

> *b) Firm 2 is a net buyer and firm 1 is a net seller of permits.*

> *c) The permit price is above firm 1´s marginal cost of abatement.*

The proof of this proposition is straightforward. Equation (23) and equation (20) directly imply a) and b). If we take these equations combined with equation (2) we obtain the following inequality that proves c):

$$p = d + 2tq_1 > d + 2tq_2.$$

Plugging equations (21) and (22) into equations (17) and (18), abatement and the amount of traded permits can also be stated in terms of the parameters of the model as follows

$$q_2 = \frac{r\left(5b + 6r^2t\right)\left(a - c - dr + 2trS\right)}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} - S \tag{25}$$

$$q_1 = \frac{r(4b + 6r^2t)(a - c - dr + 2trS)}{(6b + 4r^2t)(2b + 3r^2t)} - S \qquad (26)$$

It is trivial to see that both firms output and abatement are increasing in the demand intercept (parameter *a*) and in the number of allocated free permits (parameter *S*) and decreasing in marginal output cost (parameter *c*) and the marginal cost of the first unit of abatement (parameter *d*). The difference of outputs and abatements follows the same rule, meaning that all changes have a greater impact on the product market leader´s variables.

$$q_2 - q_1 = \frac{r(x_2 - x_1)}{3} = \frac{br(a - c - dr + 2trS)}{(6b + 4r^2t)(2b + 3r^2t)} = y_2 \qquad (27)$$

Regarding profit, we come up with the following Proposition:

PROPOSITION 3

*a) The profit of the output market leader (*Firm 2*) is always greater than the profit of the emissions permit market leader (*Firm 1*).*

b) The difference between Firm 2's and Firm 1's profits is increasing in parameters "a" and "S" and decreasing in parameters "c" and "d".

We prove Proposition 3 in the Appendix. The economic interpretation of the results regarding parameters *S* and *c* is straightforward. As long as the number of free permits is increasing, less abatement is needed for a fixed production and the product market leader is taking a bigger advantage. On the other hand, if the marginal product cost is increasing firm 1 is suffering less impact in its profits since its production level is always lower than the production level of firm 2.

Asymmetric free allocation of permits

In this subsection we relax the assumption that both firms receive the same amount of initial permits by allowing for S_1 being different from S_2. For the sake of completeness, we consider, not only the case that firm 2 receives more permits than firm 1, which could be justified if the regulator allocates permits in proportion to production amounts, but also the opposite case, which could explain the dominant position of firm 1 in the permit market. For comparison purposes we also assume that the regulator sets the same total cap as in the symmetric case. That means $2S = S_1 + S_2$.

The abatement quantities and the demand for permits follow from equations (5), (6) and (7). Proceeding in a similar way as in the symmetric case we obtain the following corresponding expressions for the output quantities.

$$x_2 = \frac{\left(6b + 6r^2t\right)\left(a - c - dr\right) + 4rt\left[b\left(4S_2 - S_1\right) + 3r^2tS_2\right]}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} \tag{21.a}$$

$$x_1 = \frac{\left(3b + 6r^2t\right)\left(a - c - dr\right) + 2rt\left[b\left(5S_1 - 2S_2\right) + 6r^2tS_1\right]}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} \tag{22.a}$$

We compute the difference as:

$$x_2 - x_1 = \frac{3b(a - c - dr) + 2brt\left(10S_2 - 7S_1\right) + 12rt\left(r^2t\right)\left(S_2 - S_1\right)}{\left(6b + 4r^2t\right)\left(2b + 3r^2t\right)} \tag{23.a}$$

And now the sign is ambiguous. The free allocation of permits directly impact both firm's outputs and consequently firm's profits.

Consider first $S_2 > S_1$. By simple differentiation it can be immediately seen that firm 2 increases production and decreases abatement as the difference is increasing, while firm 1´s output decreases and abatement increases.

From Equation (23.a) we immediately obtain

$$\frac{\partial(x_2 - x_1)}{\partial S_2} = \frac{4rt(5b + 3r^2t)}{(6b + 4r^2t)(2b + 3r^2t)} > 0.$$
(24.a)

From Equations (7) and (23.a) we conclude

$$\frac{\partial(q_2 - q_1)}{\partial S_2} = \frac{-4b^2 - 2br^2t}{(6b + 4r^2t)(2b + 3r^2t)} < 0.$$
(25.a)

As we know from equation (7) the difference in abatement determines the amount of sold and bought permits. Therefore, (25.a) implies that the number of permits demanded by firm 2 is decreasing as a consequence of the reduction in the difference of abated quantities. And the permit price is also decreasing because Firm 2 lower abatement means a decrease in its marginal abatement cost and therefore in the permit price. The number of permits sold reach zero at a certain value of the difference as it is shown in the following proposition (which is proved in the Appendix).

PROPOSITION 4

a) There is a particular allocation of free permits, with $\hat{S}_2 > \hat{S}_1$; $2S = \hat{S}_2 + \hat{S}_1$, such that the market is closed without transactions. At this point both firms abate the same amount of emissions. Specifically,

$$\hat{S}_2 = \frac{r(a - c - dr) + 8S(b + r^2t)}{2(4b + 3r^2t)}$$
(28)

b) For any allocation between the symmetric one ($S_1 = S_2$) and the one given by (28), Firm 2 is a net buyer of permits and the permit price exceeds firm 1's marginal cost of abatement.

c) For any allocation above the threshold value $\hat{S}_2$ firm 2 becomes a net seller of permits. The permit price is below firm 1's marginal cost of abatement.

It trivially follows that whenever $S_2 \geq S_1$ firm 2 produces 2 more and makes a higher proft than firm 1.

Consider now the case $S_1 > S_2$. By simple differentiation, as in the previous case, it can be shown that, as the difference S_1-S_2 increases, firm 1 increases output and decreases abatement while firm 2 decreases output and increases abatement. The number of permits sold is increasing as it is the permit price because the marginal cost of abatement of firm 2 is also increasing. At a certain value of the difference, both firms produce exactly the same amount of output.

PROPOSITION 5

a) There is a particular allocation of free permits S_1^*, S_2^* (satisfying $S_1^* > S_2^*$) such that both firms produce the same amount of output. Specifically,

$$S_1^* = \frac{3b(a-c-dr)+4rtS(10b+6r^2t)}{2rt(17b+12r^2t)} = 2S - S_2^* \qquad (29)$$

b) For any allocation between the symmetric one ($S_1 = S_2$) and the one defined by (29), firm 1 produces less than firm 2. Beyond this threshold firm 1 produces more than firm 2.

c) At the threshold point and beyond, Firm 1´s profits are higher than firm´s 2 profit.

We prove the proposition in the Appendix. The importance of this proposition is to show that the dominant position that firm 2 enjoys in the output market can be offset if the cost advantage enjoyed by firm is reinforced by assigning it a larger number of free permits. In such a case, firm 1 can make higher profits because marginal product cost is the same for both firms and the dominant position in the emissions permit market makes that firm 1 is selling permits at a price exceeding its marginal cost of abatement.

4.6 A Stackelberg Model with the same Leader in Both Markets

In this section we assume that firm 1 is not only a dominant firm in the emission permits market but also in the product market. To some extent, our approach is similar to the followed by Hinterman (2011) but introducing several significant differences. First, Hinterman considers a dominant firm and a competitive fringe in the output while we are analyzing a duopoly. In our model the follower that acts strategically, by placing itself in its reaction curve, while the Hinterman's approach the firms in the fringe are price takers i.e.: price equals their marginal cost in the optimum. A second depart from Hinterman's paper is the fact that we are setting a separable cost function considering a particular abatement function and a constant ratio between output and emissions. Finally we consider a linear demand function.

Now the timing of the game is the following:

First, Firm 1 set its output acting as a Stackelberg leader and simultaneously decides its level of abatement and the price of permits anticipating the reaction of the price taker.

Second, Firm 2 set its output and abatement level as a follower.

Both firms solve a profit maximization problem to determine their optimal levels of abatement, taking into account the cost of buying permits. The follower´s profit maximization problem is

$$
\begin{aligned}
&Max \left[a - b(x_1 + x_2) \right] x_2 - cx_2 - q_2(d + tq_2) - py_2 \\
&s.t. \quad y_2 = rx_2 - q_2 - S_2
\end{aligned}
\tag{30}
$$

We can substitute the constraint into the objective function and arrive at the familiar first-order conditions that marginal abatement costs equal the permit price, and the reaction curve for the output.

$$FOC(x_2) \Rightarrow x_2 = \frac{1}{2b}(a-c-rp) - \frac{1}{2}x_1, \tag{31}$$

$$FOC(q_2) \Rightarrow q_2 = \frac{p-d}{2t}. \tag{32}$$

The dominant firm takes equations (31) and (32) into account when maximizing its own profits. It also incorporates the permit market-clearing condition, which is given by

$$y_1 = rx_1 - q_1 - S_1 = -y_2 = -rx_2(x_1) + q_2(p) + S_2. \tag{33}$$

Equation (33) lets us find the permit price as a function of the leader's output and abatement:

$$p = \frac{bt(rx_1 - 2q_1) + bd - 2bt(S_1 + S_2) + rt(a-c)}{b + r^2 t}. \tag{34}$$

The permit price is increasing in leader's output and decreasing in leader's abatement. If we combine Equation (34) with Equation (31), and applying the chain rule, we find the slope of the reaction curve of the follower:

$$\frac{\partial x_2}{\partial x_1} = \frac{-rt}{2(b + r^2 t)} - \frac{1}{2}. \tag{35}$$

Equation (35) captures the double effect of an increase in the leader's output: The standard effect due to the dominant position in the output market, and the indirect effect due to the permit price increase that further reduce the follower's output. This effect can be identified as the so called "raising rival's cost" that can be found in the related literature. In short, a dominant firm can improve its position in the product market indirectly via manipulation of input prices (in our case, the price of emission permits).

The leader's profit maximization problem on output and abatement is:

$$Max\left(a - b\left(x_1 + x_2(x_1, p(x_1, q_1))\right)\right)x_1 - cx_1 - q_1(d + tq_1) - p(x_1, q_1)y_1(x_1, q_1) \tag{36}$$

The resulting FOC's are:

$$a - 2bx_1 - x_2 - x_1\frac{\partial x_2}{\partial x_1} - c - p\frac{\partial y_1}{\partial x_1} - y_1\frac{\partial p}{\partial x_1} = 0 \qquad x_1 > 0 \tag{37}$$

$$(-bx_1)\frac{\partial x_2}{\partial p}\frac{\partial p}{\partial q_1} - d - 2tq_1 - y_1\frac{\partial p}{\partial q_1} - p\frac{\partial y_1}{\partial q_1} = 0 \qquad q_1 > 0 \tag{38}$$

Combining (37) and (34) the solution to the leader´s problem is

$$x_1 = \frac{3(a - c - dr) + 2rt(2S_1 + S_2)}{6b + 4r^2t} \tag{39}$$

$$q_1 = \frac{2r(a - c - dr) - 2b(2S_1 + S_2)}{6b + 4r^2t} \tag{40}$$

And the follower output comes from plugging equation (39) in (31)

$$x_2 = \frac{a - c - dr + 2rtS_2}{4(b + r^2t)} \tag{41}$$

Once we have solved the model we analyze whether a particular allocation of free permits can alter the Stackelberg model standard results, with the following finding:

PROPOSITION 6

In any interior solution, the leader´s output is always higher than the follower´s regardless the initial allocation of free permits.

We prove this proposition in the Appendix.

We have checked that, for some range of parameter values, although the leader always produces more than the follower, the follower can still make a higher profit than the leader if the follower's allocation of permits is large enough as compared to the leader's.

4.7 Concluding Remarks

In this chapter we have considered three different oligopolistic structures in the product market under the common assumption of imperfect competition in the related permit market. The main focus of our analysis has been to analyze the role of grandfathering in the outcome of firms in terms of output and profits.

For the Cournot model, we have shown that if both firms initially receive the same amount of permits, there will not be any permit trade. Both firms will produce the same quantities and make the same profit and therefore firm 1´s dominant position in the emission permits market does not lead to any competitive advantage. If the allocation is not symmetric, the firm who receives more free permits produces a higher quantity and makes higher profits.

For the Stackelberg model with two different leaders, one in each market, the Stackelberg leader produces more and make more profits than the dominant firm in the permit market under a symmetric allocation of free permits but as soon as we consider that the permit market leader is receiving more permits, both firms output tends to equalize first and it comes to a point where the Stackelberg leader in the product market produces less and makes less profits than the follower.

When there is only one leader for both markets, and regardless the allocation of free permits, the leader is always producing more than the follower, although the follower can still make a higher profit than the leader if the follower's allocation of permits is large enough as compared to the leader's.

APPENDIX

Permit Market Leader Optimal Solution

The leader solves the following problem

$$\min_{\{q_1, y_1\}} \quad q_1(d + tq_1) + py_1$$
$$s.t. \quad S_1 + q_1 + y_1 = rx_1$$
$$p = d + 2tq_2 = d + 2t(rx_2 - S_2 - y_2) \tag{A1}$$

Solving for p in the above equation leads to a single variable problem

$$p = d + 2t(rx_2 - S_2 + y_1) = d + 2t(rx_2 + rx_1 - q_1 - S_2 - S_1)$$
$$\min_{\{q_1\}} \quad q_1(d + tq_1) + (rx_1 - S_1 - q_1)[d + 2t(rx_2 + rx_1 - q_1 - S_2 - S_1)] \tag{A2}$$

The FOC of this problem is

$$d + 2tq_1 - 2t(rx_1 - q_1 - S_1) - d - 2t(rx_2 + rx_1 - q_1 - S_2 - S_1) = 0$$

Solving for q_1

$$6tq_1 = 2t(2rx_1 + rx_2 - 2S_1 - S_2) \Rightarrow q_1 = \frac{2rx_1 + rx_2 - 2S_1 - S_2}{3}$$

Proof of Proposition 1

Let us consider the function

$$\Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - dy_1 - t(q_1^2 - q_2^2) - 2py_1$$
$$\Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - dy_1 - t(q_1 + q_2)(q_1 - q_2) - 2py_1 \tag{A3}$$

This function shows de difference between profits. We have taken into account equation (7) and we denote the product price as P. By simple algebraic manipulation we obtained the following expression

$$\Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - y_1(d + 2p + t[r(x_1 + x_2) - (S_1 + S_2)]) \tag{A4}$$

We have considered equations (5) and (6) to arrive at the following equation

$$q_1 + q_2 = r(x_1 + x_2) - (S_1 + S_2) \tag{A5}$$

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The sum of both firm´s abatement must be non-negative as long as any firm abatement is non-negative. If firm 1 is a net buyer of permits ($y_1 > 0$), the second term of the function is positive while the first one is negative ($x_1 - x_2 < 0$). Therefore the profit made by firm 2 exceeds the profit made by firm 1. The opposite case can be proved in a similar way.

Model 2 Firm 1 Problem in the Output Market

Firm 1 solves the following problem

$$Max\left[a-b\left(x_1+x_2\right)\right]x_1-cx_1-\left(d+tq_1\right)q_1-py_1 \tag{A6}$$

FOC lead us to the reaction curve after some algebraic operations

$$a-bx_2-2bx_1-c-d\frac{\partial q_1}{\partial x_1}-2tq_1\frac{\partial q_1}{\partial x_1}-\frac{\partial p}{\partial x_1}y_1-p\frac{\partial y_1}{\partial x_1}=0$$

$$a-bx_2-2bx_1-c-\frac{2dr}{3}-\frac{4tr}{3}\left(\frac{2rx_1+rx_2-3S}{3}\right)-\frac{2tr}{3}\left(\frac{r\left(x_1-x_2\right)}{3}\right)-$$

$$-\left(\frac{3d-6tS+2tr\left(2x_2+x_1\right)}{3}\right)\frac{r}{3}=0$$

$$x_1=\frac{3\left(a-c-dr\right)+6trS}{6b+4tr^2}-\frac{1}{2}x_2 \tag{A7}$$

Model 2 Firm 2 Problem in the Output Market

The leader in the output market solves the following problem

$$Max\left[a-b\left(x_1+x_2\right)\right]x_2-cx_2-\left(d+tq_2\right)q_2-py_2 \tag{A8}$$

The FOC for optimal output is

$$a-2bx_2-bx_1-bx_2\frac{\partial x_1}{\partial x_2}-c-d\frac{\partial q_2}{\partial x_2}-2tq_2\frac{\partial q_2}{\partial x_2}-\frac{\partial p}{\partial x_2}y_2-p\frac{\partial y_2}{\partial x_2}=0$$

The partial derivatives take the following values

$$\frac{\partial y_2}{\partial x_2} = \frac{r}{3} + \frac{r}{6} = \frac{r}{2}; \frac{\partial q_2}{\partial x_2} = \frac{2r}{3} - \frac{r}{6} = \frac{r}{2}; \frac{\partial p}{\partial x_2} = \frac{4tr}{3} - \frac{tr}{3} = tr$$

Plugging these values on the FOC and taking into account the reaction curve of the follower, lead us to the following expression

$$a - 2bx_2 + \frac{1}{2}bx_2 - b\left[\frac{3(a-c-dr)+6trS}{6b+4tr^2} - \frac{1}{2}x_2\right] - c - \frac{dr}{2} - tr\left(\frac{2rx_2 + rx_1 - 3S}{3}\right) -$$

$$-\left(\frac{3d - 6tS + 2tr(2x_2 + x_1)}{3}\right)\frac{r}{2} - tr\left(\frac{rx_2 - rx_1}{3}\right) = 0$$

Now some algebraic operations yields the optimal output value of the leader as it is shown in Equation (21)

$$x_2 = \frac{(3b+3tr^2)(a-c-dr+2trS)}{(3b+2tr^2)(2b+3tr^2)} = \frac{(6b+6tr^2)(a-c-dr+2trS)}{(6b+4tr^2)(2b+3tr^2)} \tag{A9}$$

Plugging this value in the reaction curve of the follower yields Equation (22)

Proof of Proposition 3

We define the function of the difference between firms profit as:

$$\Pi = \Pi_2 - \Pi_1 = (P-c)(x_2 - x_1) - d(q_2 - q_1) - t(q_2^2 - q_1^2) - 2py_2 \tag{A10}$$

Considering Eq (20) we compute the function as

$$\Pi = (a - bx_1 - bx_2 - c)(x_2 - x_1) - dy_2 - ty_2(q_1 + q_2) - 2(d + 2tq_1)y_2 \tag{A11}$$

We plug equations (21), (22), (23), (25), (26), and (27) in (A11) and after some tedious calculations we obtain the following expression

$$\Pi = \frac{b^2}{4}\frac{(a-c-dr+2rtS)^2(9b+13r^2t)}{(2b+3r^2t)(3b+2r^2t)} \tag{A12}$$

The expression is obviously strictly positive and proves the first part of this Proposition.

To prove the second part, we take into account that a necessary condition for producing and selling positive quantities is:

$$a - c - dr > 0,$$

We differentiate (A12) and we find the following results:

$$\frac{\partial \Pi}{\partial S} = \frac{b^2 tr (a - c - dr + 2trS)(9b + 13tr^2)}{(2b + 3tr^2)^2 (3b + 2tr^2)^2}$$

$$(a - c - dr + 2trS) > 0 \Rightarrow \frac{\partial \Pi}{\partial S} > 0$$

$$\frac{\partial \Pi}{\partial d} = \frac{-1}{2} \frac{b^2 tr (a - c - dr + 2trS)(9b + 13tr^2)}{(2b + 3tr^2)^2 (3b + 2tr^2)^2}$$

$$(a - c - dr + 2trS) > 0 \Rightarrow \frac{\partial \Pi}{\partial d} < 0$$

$$\frac{\partial \Pi}{\partial c} = \frac{-1}{2} \frac{b^2 tr (a - c - dr + 2trS)(9b + 13tr^2)}{(2b + 3tr^2)^2 (3b + 2tr^2)^2}$$

$$(a - c - dr + 2trS) > 0 \Rightarrow \frac{\partial \Pi}{\partial c} < 0$$

$$\frac{\partial \Pi}{\partial a} = \frac{1}{4} \frac{b^2 tr (a - c - dr + 2trS)(9b + 13tr^2)}{(2b + 3tr^2)^2 (3b + 2tr^2)^2}$$

$$(a - c - dr + 2trS) > 0 \Rightarrow \frac{\partial \Pi}{\partial a} > 0$$

Proof of Proposition 4

Based on equation (7) the condition for a positive or null demand of permits is:

$$r(x_2 - x_1) - (S_2 - S_1) \geq 0 \tag{A13}$$

Plugging equation (23a) in (A13) yields

$$3br(a - c - dr) - (12b^2 + 6br^2 t) S_2 + (12b^2 + 12br^2 t) S_1 \geq 0 \tag{A14}$$

Solving (A14) in the equality case for S_2, and taking into account that $S = S_1 + S_2$, we obtain Equation (28), which proves the first part of the proposition.

Firm 2 is a net buyer of permits if

$$r(x_2 - x_1) - (S_2 - S_1) > 0 \tag{A15}$$

which is the case when both firms receive the same amount of free permits, because in this case firm 2´s output exceeds firm 1´s, as can be seen in Equation (23.a). When the value of S_2 is above the threshold in Equation (28), the expression (A14) is strictly positive and this fact proves the second part of the proposition.

The last part of the proposition follows immediately when we consider

$$r(x_2 - x_1) - (S_2 - S_1) < 0 \tag{A16}$$

which is the condition for Firm 2 being a net seller of permits. In that case equation (A14) becomes

$$3br(a - c - dr) - (12b^2 + 6br^2t)S_2 + (12b^2 + 12br^2t)S_1 < 0 \tag{A17}$$

and S_2 satisfies the required condition of being beyond the threshold.

Proof of Proposition 5

To prove the first part of the proposition we simply set equation (23.a) to zero and solve for S_1 to find the threshold value. Since the output functions are continuous and we have already proved that $x_2 > x_1$ at the symmetric allocation of permits, the second part of the proposition is trivially proved.

Now consider the function

$$\Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - d(q_1 - q_2) - t(q_1^2 - q_2^2) - 2p(y_1 - y_2) \tag{A18}$$

At the threshold point, both outputs are equal $x_1 = x_2$ and Firm 1 is a seller of permits ($y_1 < 0$). The above equation can be reduced to

$$\Pi = \Pi_1 - \Pi_2 = -dy_1 - 2ty_1(q_1 + q_2) - 2py_1 > 0 \tag{A19}$$

Beyond the threshold point the first term on the right side of (A18) is positive and the rest of the terms of that expression is (A19). Both equations are positive and the proposition is proved.

Proof of Proposition 6

Based on Equations (39) and (41) we see that the difference between outputs is increasing in S_1 and decreasing in S_2.

$$\frac{\partial(x_1 - x_2)}{\partial S_1} = \frac{4rt}{(6b + 4r^2t)} > 0; \frac{\partial(x_1 - x_2)}{\partial S_2} = \frac{2rt}{(6b + 4r^2t)} - \frac{2rt}{(4b + 4r^2t)} < 0$$

We consider that the maximum number of free permits that can be allocated is covering total gross emissions. The minimum difference is obtained when all the free permits go to the follower, therefore $S_2 = r(x_1 + x_2)$. Plugging this result into equations (39) and (41), both firms output are given by

$$x_1 = \frac{3(a - c - dr) + 2r^2t(x_1 + x_2)}{6b + 4r^2t}; x_2 = \frac{(a - c - dr) + 2r^2t(x_1 + x_2)}{4b + 4r^2t} \tag{A20}$$

The solution for the above system is given by

$$x_1 = \frac{3b(a - c - dr) + 2r^2t(a - c - dr)}{b(6b + 5r^2t)}; x_2 = \frac{3b(a - c - dr) + 4r^2t(a - c - dr)}{2b(6b + 5r^2t)} \tag{A21}$$

And the difference between both outputs (in this limit case) shows that the leader is always producing more than the follower

$$x_1 - x_2 = \frac{3(a - c - dr)}{2(6b + 5r^2t)} > 0 \tag{A23}$$

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Chapter 5[24]

Optimal Taxation on Fossil Fuels with Varying Extraction Costs

ABSTRACT

In this chapter we analyze the optimal taxation on fossil fuels in general equilibrium under alternative assumptions on extractions costs. The tax instruments include a profit tax and an ad valorem tax. Without extraction costs these tax instruments are equivalent, but these is no longer the case when extraction costs depend on two different factors, the flow of extraction and the stock not yet extracted. The quantitative importance of those alternative assumptions is illustrated over relevant specification of the damage function in line with Golosov et al. (2009). Finally, per unit of extraction taxes are also considered even though they are not typically an available policy tool.

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[24] This chapter represents a joint work with Luis A. Puch, and has been presented in a seminar in Madrid (2015) and a congress in Vigo (2012). Still incomplete, this chapter is the basis for a future publication as a working paper

5.1 Introduction

Carbon taxes are often proposed as a policy instrument to reduce emissions of greenhouse gases. Emissions of greenhouse gases are generated by fossil fuel burning and therefore the extraction path of such non- renewable resources becomes a key issue. Under non-renewable use, Hotelling (31) states the first optimality condition, which requires the growth rate of the resource price equalizing the market interest rate. This arbitrage condition shows that the firm is indifferent between extracting the resource now or delays it for one period. Alternatively, the basic condition for efficiency stated by Solow (1974), Stiglitz (1974) and Dasgupta and Heal (1974) implies that the marginal product of resource growth rate has to be equal to the marginal product of capital. Indeed, in the competitive equilibrium of the economy, this condition (DHSS, Dasgupta-Heal-Solow-Stiglitz) is the very same as Hotelling´s one.

None of these conditions takes into consideration the existence of extraction costs and externalities. Sinn (2007) generalized the DHSS efficiency condition, by taking into account both climate change as an externality and an extraction cost following the Herfindahl rule. That efficiency condition is Pareto optimal and implies a flatter extraction path than Hotelling's rule.[25] In this setting, the marginal product of capital equals the sum of three elements: a) the growth rate of the marginal product of the resource net of extraction cost, b) the rate of return of a better environmental quality (global warming effect) and c) the rate of return of the delayed use of resources in terms of its marginal cost.

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[25] Withagen (94) stated that current resource consumption should be lower if pollution is to be taken into account, and accordingly extraction has to be postponed.

Sinn (2008) argues that if suppliers feel threatened by a gradual greening of economic policies, they will extract their stocks more rapidly, thus accelerating global warming. This argument gave rise to the so called "Green Paradox"[26]

Optimal taxation in such a setting has been recently addressed by Sinn (2008) and Golosov et al (2009, 2011 and 2014) among other authors. While Sinn focus on the supply side of the non-renewable resources in partial equilibrium, the Golosov et al sequence of papers consider a General Equilibrium model in discrete time, to characterize optimal tax and subsidy policies.[27] We build upon these two approaches to focus on varying fossil fuel extraction costs. Some interesting results related to optimal taxes can be found in the existing literature based on the different treatment of extraction costs. For instance, if we consider no extraction costs (or insignificant), then a constant ad valorem tax will have no effect on the extraction path.[28] In such a circumstance what matters is the time path of the tax regardless its level. Further, without extraction costs a profit tax and an ad valorem tax are equivalent, but this is no longer the case when extraction costs depend on the general path of extraction.

Several authors have considered the existence of extraction costs, like Cremer (79), Long and Sinn (85) and Ulph and Ulph (94), but it is Sinn (2008) that noticed that Hotelling´s rule differs whether we consider marginal extraction costs depending on the

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[26] Sinn (2008) also stated that subsidizing a carbon-free backstop can have adverse climate effects as the anticipation of oil being made obsolete more quickly by such renewable encourages oil extraction, but van der Ploeg and Withagen (2010) argue that the Green Paradox occurs for relatively expensive but clean backstops (such as solar or wind), but does not occur if the backstop is sufficiently cheap relative to marginal global warming damages (e.g., nuclear energy) as then it is attractive to leave fossil fuels unexploited and thus limit CO2 emissions.

[27] The literature does not provide a uniform view about the optimal environmental policy, mainly a tax on the polluting resource. While Sinclair (92,94) shows that an optimal ad valorem tax on the use of non renewable resources is decreasing, Ulph & Ulph (94) analyzed a special case when carbon tax would be rising when the initial stock of pollutant is small and Hoel & Kverndokk (96) stated that optimal tax increases and then decreases. Moreover Grimaud and Rouge (2005) show that the optimal ad valorem tax is either increasing or decreasing according to assumptions taken on pollution´s marginal disutility and the discount rate. And Dasgupta(82) and Ploeg & Withagen (91) show that if the stock of CO_2 is below its steady state level, then a carbon tax should rise overtime.

[28] Dasgupta and Heal (79)

current flow of extraction or depending on the stock not yet extracted. We follow this line of research but assuming that the way in which extraction costs depend on the general path of extraction depends on any or both the flow and the stock effects. The quantitative importance of those alternative assumptions is illustrated over relevant specification of the damage function in line with Golosov et al. (2009).

It is worth noting that the negative climate change externality raises several issues related to optimal policies and their impact on the economy. These issues have been recently studied in general equilibrium by comparing the decentralized equilibrium and the social planner´s optimum. It is widely accepted that it is the stock of CO_2 rather than the flow of emissions what should be taken into account when considering the damage costs caused by climate change. Then the carbon tax required to maximize some measure of social welfare should be calculated according to this assumption. We analyze the impact of the use of non-renewable resources in the production process and the optimal taxation needed to cancel out the externalities of climate change.

The aggregate production function is neoclassical with positive and decreasing marginal products in capital and energy. The climate variable is also part of the production function with positive and increasing marginal damage. Within this framework we consider an extraction cost function depending upon resource extracted and the existing stock.

The chapter is organized in five parts of which the introduction is the first. The second states the social optimum whereas Section 5.3 presents the decentralized equilibrium. Section 5.4 discusses the role of the different tax instruments in connection to alternative assumptions on extraction costs. A quantitative assessment of these alternative assumptions on extraction costs is illustrated in Section 5.5 Concluding remarks are given in the last section.

5.2 The general setting and the planner´s problem

The household´s period utility function depends only on consumption and we write the intertemporal utility function as:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

As usual the marginal utility of consumption is positive and decreasing and β is the constant utility discount rate. The economy is subjected to the following budget constraints:

$$K_{t+1} = F(K_t, N_t, E_t, S_t) - c_t + (1-\delta)K_t - Q(E_t, R_t) \tag{2}$$

$$S_t = L(E^t) = \sum_{s=0}^{t+T} (1-d_s)E_{t-s}; E^t = \{E_{-T}, E_{-T+1}.....E_t\} \tag{3}$$

$$R_{t+1} = R_t - E_t; \sum_{t=0}^{\infty} E_t \leq R_0 \tag{4}$$

The capital accumulation is described by Equation (2).[29] The final good production function uses physical capital (*K*), labour (*N*), energy input (*E*) and the climate variable (*S*) as production factors. It means that polluting emissions are a consequence of the use of non-renewable resources, and the pollution flow is modeled as an identity function of the extracted exhaustible resources. Not only the flow of emissions enters the production function, but the climate stock does. It is represented by one variable, which is taken to be the global concentration of carbon in the atmosphere denoted by S_t.

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[29] We could also consider a two sectors economy as in Golosov et al (2011). In this case equation (2) is replaced by three new equations:

$$K_{t+1} = F_{1,t}(K_{1,t}, N_{1,t}, E_{1,t}, S_t) - c_t + (1-\delta)K_t \tag{2.1}$$

$$E_t = F_{2,t}(K_{2t}, N_{2t}, E_{2t}, R_t, R_{t+1}) \tag{2.2}$$

$$K_1 + K_2 = K; N_1 + N_2 = N; E_1 + E_2 = E \tag{2.3}$$

Resources are extracted and transformed according to the production function described in Equation (2.2). The transformation activity uses capital, labor and energy while the stock effect is represented by R.

The production function presents positive and decreasing marginal products in K,N and E, while $F_S < 0$ and $F_{SS} > 0$, that is to say positive and increasing marginal damage due to the stock of carbon in the atmosphere. Physical capital depreciates at a δ constant rate.

Total extraction costs are represented by $Q = Q(E, R)$ a function of resources extracted (E) and the existing stock (R). In line with Sinn (2007), the sequence of extraction is in inverse order of the site specific extraction costs. We assume marginal cost is an increasing function of the extraction rate and will likely increase as more of the resource is extracted. That means $Q_E > 0$ and $Q_R < 0$.

Equation (3) represents the dynamics of the environmental quality. We follow Golosov et al (2011) in this general form of the carbon cycle, as a function of the fossil fuel extracted in the past. Equation (4) describes the use of the non renewable resource. The initial stock is denoted by R_0.

The social planner maximizes (1) subject to (2) – (4). The Lagrangian is:

$$L = \sum_0^\infty \beta^t u(c_t) + \beta^t \lambda_t^K \left[F - c + (1-\delta)K - Q_t(E_t, R_t) - K_{t+1} \right] -$$
$$- \beta^t \lambda_t^S \left(S_t - \sum_{s=0}^{t+T} (1 - d_s) E_{t-s} \right) + \beta^t \lambda^R (R_t - E_t - R_{t+1}) \tag{5}$$

We use the minus in front of λ^S to ensure that the co-state variable can be interpreted as a positive shadow price. And also for simplicity we abstract from the use of labor (N = 1).

The first order conditions and the characteristic Ramsey and Hotelling rules are shown in the Appendix. We have omitted time subscripts for ease of notation.

$$\frac{U'(c_t)}{\beta U'(c_{t+1})} = F_{K,t+1} + (1-\delta) \equiv \rho_{t+1} \tag{6}$$

$$\rho_{t+1} = \frac{NP_{E,t+1} - Q_{R,t+1} - \Lambda^S_{t+1}}{NP_{E,t} - \Lambda^S_t} \tag{7}$$

We have defined:

$$\Lambda_t^s \equiv \frac{-\sum_{j=0}^{\infty} \beta^j \lambda_{t+j}^K F_{S,t+j} \frac{\partial S_{t+j}}{\partial E_t}}{u'(c_t)} > 0 \tag{8}$$

$$NP_{E,t} \equiv F_{E,t} - Q_{E,t} \tag{9}$$

Equation (8) measures the climate change externality. It is the marginal cost of a unit of carbon in the atmosphere. Note that $F_{S,t+j} < 0$. It represents the present value of damage cost in terms of period-t consumption.

Equation (9) is the marginal product of energy net of extraction cost. Equation (7) is a modified Hotelling rule and deserves further examination.[30] It shows the marginal condition for optimal resource extraction under stock and flow dependent extraction costs and climate change externalities. This efficiency condition requires that the social rate of discount should equal the rate of return from holding the resource, which is the sum of the rate of capital appreciation plus the effects due to stock dependent extraction costs and marginal damages due to climate change. This is the Hotelling rule in terms of the efficiency condition established in the DHSS model when extraction costs and climate change externalities are considered.

It is very useful to analyze this expression as a portfolio choice between extracting one more unit of the non-renewable resource this period or delay the extraction to the next one. An alternative expression for equation (7) is as follows:

$$(\rho_{t+1})(NP_{E,t} - \Lambda_t^s) = NP_{E,t+1} - \Lambda_{t+1}^s - Q_{R,t+1} \tag{10}$$

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[30] The corresponding equation in the two sectors model is:

$$\rho_{t+1} = \frac{\dfrac{\partial F_{1,t+1}}{\partial E_{t+1}} - \left(1 - \dfrac{\partial F_{2,t+1}}{\partial R_{t+1}}\right)\dfrac{\lambda_{t+1}^2}{\lambda_{t+1}^1} - \Lambda_{t+1}^S}{\dfrac{\partial F_{1,t}}{\partial E_t} - \left(1 + \dfrac{\partial F_{2,t}}{\partial R_{t+1}}\right)\dfrac{\lambda_t^2}{\lambda_t^1} - \Lambda_t^S} \tag{7.1}$$

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The efficiency condition requires that the marginal cost of not extracting an additional unit of the stock (left side) should equal the marginal benefit (right side).

The left side of the equation is what can be obtained in period t+1 if one additional unit of the resource is extracted in period t and its net marginal product is invested at the net marginal product of capital. It is also called the holding cost of the resource stock because is the marginal cost of not extracting an additional unit of the stock.

The right side of the equation is what can be obtained for the same unit if extracted in period t+1. The first term is again the net marginal product of the resource. The second term is the additional return due to the delay of the marginal damage. And the third term is the present value of the extraction costs that is avoided by a delayed use of a unit of resource. It should be noted that a period ahead marginal extraction cost will be lower as more resources remain available.

In the absence of the stock effect ($Q_R = 0$) the optimal rate of extraction reduces to:

$$\rho_{t+1} = \frac{NP_{E,t+1} - \Lambda_{t+1}^{s}}{NP_{E,t} - \Lambda_{t}^{s}} \tag{11}$$

The interest rate equals a rate of return of the resource which is lower than in Eq (10). The marginal benefit of not extracting one unit is lower than before and the optimal rate of extraction accelerates.

The comparison between (10) and (11) also shows that the level of the net product of the resource is only affected by the cost related to the extraction rate. But the rate of increase of the net product of the resource has to be lower when considering the stock effect. Therefore the stock effect will slow down the rate of extraction. It means that the growth rate of prices is lower when considering the stock effect, known also as the scarcity rent.

5.3. Decentralized equilibrium

When we consider the decentralized equilibrium, the effect of emissions on climate damages is assumed to be a pure externality, not taken into account by any private agent.

We define Π^F as profits from final goods production and Π^E as profits from resource extraction. Price of physical capital is r and the price of the resource is P^E.

The problem of the goods production firm under perfect competition takes prices as a data. Final good is the "numeraire".

$$Max \quad \sum_{s=t}^{\infty} \Gamma_t^s \left\{ F\left(K_s, E_s\right) - \delta K_s - r_s K_s - P_s^E E_s \right\}$$

We have denoted

$$\Gamma_t^s \equiv \prod_{j=0}^{s} \frac{1}{\rho_{t+j}}; 1 + r_s = \rho_{t+s}$$

First order conditions read that factors are paid its marginal product.

$$FOC\left(K_{t+1}\right) \Rightarrow F_{K,t+1} - \delta = r_{t+1} \tag{12}$$

$$FOC\left(E_t\right) \Rightarrow F_{E,t} = P_t^E \tag{13}$$

A representative individual solves:

$$Max \quad \sum_0^{\infty} \beta^t U\left(c_t\right)$$

$$s.t. \quad C_t + K_{t+1} = K_t + r_t K_t + \Pi_t^F + \Pi_t^E + T_t$$

Profits from final goods production equal zero in equilibrium while profits from resource extraction are strictly positive representing the stock value of fossil fuels in the ground.

Hamiltonian and First order condition are shown in Appendix 2. It yields the well-known Ramsey rule

$$\frac{U'\left(C_t\right)}{\beta U'\left(C_{t+1}\right)} = 1 + r_{t+1} = \rho_{t+1} \tag{14}$$

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And it is the same as (6) when we consider (12).

The problem of the extraction firm will be solved considering the existence of a profit tax (τ_t) and a per unit tax (θ_t).

$$Max \quad \sum_{s=t}^{\infty} \Gamma_t^s \left[\left(P_s^E - \theta_s \right) E_s - Q(E_s, R_s) \right] \left(1 - \tau_s^E \right); \quad \Gamma_t^s = \prod_{j=0}^{s} \frac{1}{\rho_{t+j}}$$

$$s.t. \quad R_{t+1} = R_t - E_t; \sum_0^{\infty} E_t \leq R_0$$

Lagrangian and First Order Conditions are shown in the Appendix where we also obtained the optimal extraction rate in the different cases considered. They yield:

$$F_{E,t} = P_t^E \Rightarrow NP_{E,t} = P_t^E - Q_{E,t}$$

A) Without taxes $\quad \rho_{t+1} = \dfrac{NP_{E,t+1} - Q_{R,t+1}}{NP_{E,t}}$ (15)

B) Fixed taxes $\quad \rho_{t+1} = \dfrac{NP_{E,t+1} - \theta - Q_{R,t+1}}{NP_{E,t} - \theta}$ (16)

C) Var taxes $\quad \rho_{t+1} = \dfrac{\left(NP_{E,t+1} - \theta_{t+1} - Q_{R,t+1} \right)\left(1 - \tau_{t+1}^E \right)}{\left(NP_{E,t} - \theta_t \right)\left(1 - \tau_t^E \right)}$ (17)

Based on (13) the price of the resource equals its marginal product and therefore these are Hotelling formulas updated with extraction cost and taxes. The price net of the marginal extraction cost rises at a rate equal to the market rate of interest.

It is worth noting that Q_E affect the price level (static efficiency condition) but the price growth rate is only affected by Q_R (dynamic efficiency condition).[31]

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[31] The corresponding equations when two sectors are considered, follows the same process. In this case the price is the cost for the final good sector and η is the relative shadow price of energy in terms of the shadow price of the final good:

A) Without taxes $\quad 1 + r_{t+1} = \dfrac{P_{t+1}^E - \eta_{t+1}^E \left(1 - \dfrac{\partial F_{2,t+1}}{\partial R_{t+1}} \right)}{P_t^E - \eta_t^E \left(1 + \dfrac{\partial F_{2,t}}{\partial R_{t+1}} \right)}$ (15.1)

If we do not consider taxes then, the decentralized equilibrium is not considering the climate change externality and differs from the central planner optimum in (10). The firm will extract the resource at a quicker rate than the central planner, because the social cost decelerates extraction. When we consider Pigouvian taxes, the rate of extraction has to be the same as in the central planner equilibrium. The issue we explore in the next section is whether the optimum tax (increasing or decreasing) will accelerate extraction, leading to the green paradox.

5.4 Policy instruments and extraction costs

From the previous equations the following results can be derived:

PROFIT TAX

1. A constant profit tax rate has no impact on the allocation of resources. Equation (17) under this assumption and no ad valorem tax reads as equation (15). This is a standard result in fiscal policy as explained by Sinn (2008). This result is valid regardless the stock effect extraction cost and considering a model with one or two sectors.

2. A changing profit tax rate gives rice to substantial intertemporal distortions. An increasing profit tax rate leads to more extraction. As pointed out by Sinn 2008, a

B) Fixed taxes $1 + r_{t+1} = \dfrac{P_{t+1}^E - \theta - \eta_{t+1}^E\left(1 - \dfrac{\partial F_{2,t+1}}{\partial R_{t+1}}\right)}{P_t^E - \theta - \eta_t^E\left(1 + \dfrac{\partial F_{2,t}}{\partial R_{t+1}}\right)}$ (16.1)

C) Var taxes $1 + r_{t+1} = \dfrac{\left\{\left(P_{t+1}^E - \theta_{t+1}\right) - \eta_{t+1}^E\left(1 - \dfrac{\partial F_{2,t+1}}{\partial R_{t+1}}\right)\right\}\left(1 - \tau_{t+1}^E\right)}{\left\{\left(P_t^E - \theta_t\right) - \eta_t^E\left(1 + \dfrac{\partial F_{2,t}}{\partial R_{t+1}}\right)\right\}\left(1 - \tau_t^E\right)}$ (17.1)

firm facing increasing profit tax rates is the same as a firm that maximizes the present value of profit net of taxes facing a higher interest rate or a lower growth in the resource price leading to an acceleration of the extraction. Considering equation (17) without ad valorem taxes the firm finds the following situation

$$
\begin{aligned}
&1+r_{t+1} = \left[\frac{\left(NP_{E,t+1} - Q_{R,t+1} \right)\left(1-\tau^E_{t+1} \right)}{NP_{E,t}\left(1-\tau^E_t \right)} \right] \Rightarrow \\
&\Rightarrow \left(1+r_{t+1} \right) NP_{E,t}\left(1-\tau^E_t \right) = \left(NP_{E,t+1} - Q_{R,t+1} \right)\left(1-\tau^E_t - \Delta\tau^E_t \right) \\
&\Delta\tau^E_t > 0 \Leftrightarrow \left(1+r_{t+1} \right) NP_{E,t}\left(1-\tau^E_t \right) > \left(NP_{E,t+1} - Q_{R,t+1} \right)\left(1-\tau^E_t - \Delta\tau^E_t \right)
\end{aligned}
\tag{18}
$$

The result is the same regardless the model has one or two sectors.

3. Optimal profit tax rate is a single rule when the stock effect is not considered. From equations (10) and (17) and taking into account that there is no ad valorem tax:

$$
\frac{NP_{E,t+1}\left(1-\tau^E_{t+1} \right)}{NP_{E,t}\left(1-\tau^E_t \right)} = \frac{NP_{E,t+1} - \Lambda^S_{t+1}}{NP_{E,t} - \Lambda^S_t} \Leftrightarrow \tau^E_t = \frac{\Lambda^S_t}{NP_{E,t}} \quad \forall t
\tag{19}
$$

The negative relationship between profit tax rate and energy net prices is consistent with the objective of internalize the marginal damage. Higher net prices mean a higher amount of profits. Therefore, a lower profit tax rate cover the amount needed to cope with the marginal damage.

4. The stock effect introduces a substantial impact in the optimal profit tax rate. From equations (11) and (17) and no ad valorem tax:

$$
\frac{\left(NP_{E,t+1} - Q_{R,t+1} \right)\left(1-\tau^E_{t+1} \right)}{NP_{E,t}\left(1-\tau^E_t \right)} = \frac{\left(NP_{E,t+1} - Q_{R,t+1} \right) - \Lambda^S_{t+1}}{NP_{E,t} - \Lambda^S_t}
\tag{20}
$$

$$
\tau^E_t = \frac{\Lambda^S_t}{NP_{E,t}} \quad ; \quad \tau^E_{t+1} = \frac{\Lambda^S_{t+1}}{NP_{E,t+1} - Q_{R,t+1}}
\tag{21}
$$

Equation (20) implies that the profit tax rate is lower when the stock effect is taken into account. Equation (21) should be understood as either the tax at t+1 is a

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function of the tax at t, or we need two instruments to correct for the externality in order to overcome a time inconsistency problem. The second case is less interesting for our study in terms of dynamics, so I try to characterize a g function as

$$\tau_{t+1}^{E} = g\left(\tau_{t}^{E}\right) \tag{21a}$$

In so doing, we also need at the steady state a consistent general relation. In Section 5.5.3 we explore that function on particular cases. Provided such a g function exists, then equation (21) can also give us a policy rule to deal with the evolution of the proven reserves. By differentiation:

$$\frac{\partial \tau_{t+1}^{E}}{\partial R_{t}} = \frac{\partial \tau_{t+1}^{E}}{\partial NP_{E,t+1}} \frac{\partial NP_{E,t+1}}{\partial R_{t}} + \frac{\partial \tau_{t+1}^{E}}{\partial Q_{R,t+1}} \frac{\partial Q_{R,t+1}}{\partial R_{t}} \tag{22a}$$

The first term on the right side of equation (22a) is measuring the impact of reserves on the tax rate due to the profit change.

$$\frac{\partial \tau_{t+1}^{E}}{\partial NP_{E,t+1}} \frac{\partial NP_{E,t+1}}{\partial R_{t}} = -\Lambda_{t+1}^{S}\left(NP_{E,t+1} - Q_{R,t+1}\right)^{-2} \frac{\partial NP_{E,t+1}}{\partial R_{t}} > 0 \tag{22b}$$

The positive sign of equation (22b) depends on the way that an increase in proven reserves influences the profit level. Both revenue and cost are decreasing in R_{t} but we guess that marginal profit is null at the optimum under the assumption of competitive markets.

The sign of the second term on the right side of equation (22a) is:

$$\frac{\partial \tau_{t+1}^{E}}{\partial Q_{R,t+1}} \frac{\partial Q_{R,t+1}}{\partial R_{t}} = \Lambda_{t+1}^{S}\left(NP_{E,t+1} - Q_{R,t+1}\right)^{-2} \frac{\partial Q_{R,t+1}}{\partial R_{t}} > 0 \tag{22c}$$

Equation (22c) is positive under the assumption of convex cost ($Q_{RR} > 0$), and therefore an increase in the amount of oil reserves implies an increase in the profit tax rate for the next period.

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PER UNIT TAX

5. Optimal tax per unit of oil is equal to the marginal externality cost. If profit taxes are constant, optimal tax per unit of oil must be set equalizing equations (10) and (17). Therefore:

$$\frac{NP_{E,t+1} - \theta_{t+1}}{NP_{E,t} - \theta_t} = \frac{NP_{E,t+1} - \Lambda_{t+1}^S}{NP_{E,t} - \Lambda_t^S} \Rightarrow \theta_t = \Lambda_t^S, \forall t \tag{23}$$

The tax equals the marginal social damage generated by externalities. This result does not change when the stock effect is introduced.

AD VALOREM TAX

An ad valorem tax is close to a per unit tax under the following relation

$$P_t - \theta = P_t(1-v)$$

Therefore we will update the Equations (15) (16) and (17) accordingly

6. A constant ad valorem tax makes the extraction path flatter and is neutral without extraction costs. This result is in line with Sinn (2008)

The firm equilibrium before tax implementation is:

$$\left(P_t^E - Q_{E,t}\right)\rho_{t+1} = P_{t+1}^E - Q_{E,t+1} - Q_{R,t+1}$$

After the tax the firm faces the following inequality

$$\left(P_t^E(1-v) - Q_{E,t}\right)\rho_{t+1} < \left(P_{t+1}^E(1-v) - Q_{E,t+1} - Q_{R,t+1}\right)$$

And the firm postpones extraction. Note that in the absent of any kind of costs the price growth at the interest rate, but the stock effect increases the return of extracting later and therefore the price is growing at a lower rate than the interest rate. Therefore the loss coming from the tax reduces de difference and the firm postpones the extraction

$$\rho_{t+1}P_t^E - P_{t+1}^E = \Delta \Rightarrow (1-v)\left(\rho_{t+1}P_t^E - P_{t+1}^E\right) = (1-v)\Delta < \Delta$$

7. Optimal ad valorem tax rate equals the ratio between marginal damage cost and the price of the resource. As long as the extraction cost is positive, the optimal ad valorem tax rate is lower than the optimal profit tax rate.

$$\frac{P_{t+1}^E\left(1-v_{t+1}\right)-Q_{E,t+1}-Q_{R,t+1}}{P_t^E\left(1-v_t\right)-Q_{E,t}}=\frac{P_{t+1}^E-Q_{E,t+1}-Q_{R,t+1}-\Lambda_{t+1}^S}{P_t^E-Q_{E,t}-\Lambda_t^S}$$

$$P_t^E v_t=\Lambda_t^S\forall t\Rightarrow v_t=\frac{\Lambda_t^S}{P_t^E}\forall t$$

(24)

The result is the same in all the cases considered. The optimal ad valorem tax rate could be increasing constant or decreasing, based on the ratio between the growth of the marginal damage cost and the price increase of the resource.

8. An increasing ad valorem tax produces different impacts based on the extraction costs:[32]

8.1 With no extraction cost accelerates extraction.

8.2 With significant extraction costs the acceleration is mitigated

8.3 With sufficiently strong extraction cost it even produces a deceleration.

8.4 A borderline case exists. An absolute tax wedge that increases at the rate of discount

Sinn (2008) shows that with an increasing tax rate, the growth rate of prices would have to be higher. That means steeper rather than flatter extraction rate. It is the same result regardless extraction costs. With sufficiently strong extraction costs, current extraction may even move in the right direction as it was shown by Long and Sinn (85). Taxation is neutral for the extraction path if the tax wedge increases at the rate of discount since the discounted revenue loss per unit of the extracted resource is constant.

---

[32] The market reactions to a changing ad-valorem tax rate were studied by Sinn (82) Ulph and Ulph (94) and Sinclair (94).

Faster increase implies the resource firms anticipate extraction and a smaller increase implies they will postpone.

Before implementing the tax, the equilibrium is given by

$$\left(P_t^E - Q_{E,t}\right)\rho_{t+1} = P_{t+1}^E - Q_{E,t+1} - Q_{R,t+1} \tag{25}$$

Once the tax is implemented, the condition for indifference extraction is:

$$\left(P_t^E\left(1-v_t\right)-Q_{E,t}\right)\rho_{t+1} = \left(P_{t+1}^E\left(1-v_{t+1}\right)-Q_{E,t+1}-Q_{R,t+1}\right) \tag{26}$$

The difference between (26) and (25) yields the condition for tax neutrality

$$\rho_{t+1}P_t^E v_t = P_{t+1}^E v_{t+1} \Rightarrow \left(1+r_{t+1}\right)P_t^E = \left(1+v\right)P_{t+1}^E$$

This is the same result as Sinn (2008).

5.4.1 Extraction Costs Discussion

Several assumptions on extraction costs have been used when studying the role of carbon taxes in reducing GHG emissions. Herfindahl rule says that reserves with lower cost to extract are used first, implying that stock dependent extraction costs are rising as reserves diminish. Constant or even decreasing extraction cost are justified on the basis of technological advances and also based on the assumption that variables costs are insignificant and only fixed cost should be taken into account.

How the extracting firm will react to a constant ad valorem tax depends on the extraction cost path. If it is constant, then the extraction path becomes flatter and this effect is even greater if the firm is facing decreasing costs. Under the assumption of increasing costs, a constant ad valorem tax can have different impacts on the extraction path that could even accelerate in some cases. A borderline case exists when the rate of growth of extraction costs equals the interest rate.

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As far as profit tax is concerned, only a changing trend gives rise to distortions. An increasing rate accelerates extraction regardless its optimality. Without considering the stock effect the trend of the tax follows the ratio between marginal damage and profit and closely follows the ad valorem tax behavior apart from the flow effect of the extraction cost component. But the stock effect substantially changes the optimal rate.

From the analytical results we have shown that increasing taxes can be paradoxical or not depending upon suppliers´ extraction cost and that proven reserves might play a key role in the fiscal policy when the stock effect is taken into account.

We combine the various results on taxes so as to relate to actual policy and provide a quantitative assessment in the next section. But previously we analyze some facts related to oil production and extraction to get a better insight on the issues we will tackle next.

1. Proven reserves´ path is episodic. Some big price increases, particularly in 1980 and the first decade in 21st century resulted in significant jumps in the level of reserves some years later as can be seen in graph 5.1.

2 Oil is simultaneously extracted at very different production costs across the world and this fact is not in line with Herfindhal rule. However countries show homogeneity in extraction costs technology with common slopes for production (flow) and proven reserves (stock) suggesting that the role of scarcity cost is well above the role of marginal cost. Graph 5.2 in the Appendix shows this fact.

Big exceptions are USA, Saudi Arabia, Russia and Venezuela as can be seen in Graph 5.3 in the Appendix. We will take these facts in the next section to understand whether tax policy interacts with extraction costs.
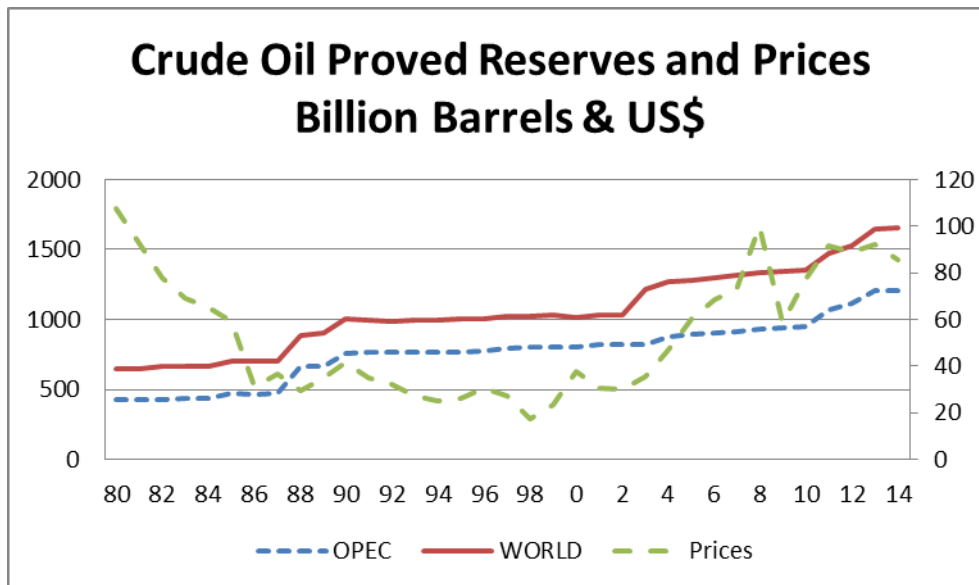
Source: EIA US Energy Information Administration and BP 2015 Statistics

Graph 5.1 Evolution of Proved Reserves and Prices

Although we cannot reject the assumption that extraction costs might be irrelevant for global carbon taxes policies, numerical experiments could be useful to investigate local fiscal policies. In other words, policy for "rich" producers might be affected for the evolution of the stock effect.

Based on the above considerations, a natural question is to check whether it makes sense to take into account the role of proven reserves in fiscal policy. This is the reason why we will explore a shock (with perfect foresight) in R. We want to compare the marginal effect on resource productivity (price) versus the marginal effect on extraction costs, i.e.: to keep track of profits. So in this stage we will focus more in the medium-run, rather than in the long-run described by the analytical case.

5.5 Quantitative assessment of carbon taxes with extraction costs

The quantitative assessment will be divided in three parts. The first part is analytical without extraction costs and we perform two cases with different energy path

consumption. On the first case, energy use is constant while on the second one is decreasing. Within this first part, we check the dynamics of extraction costs in order to understand its quantitative importance to calculate optimal taxes.

The second and third parts are numerical and include a specific extraction cost function. On the second part we analyze the impact on marginal productivity and marginal extraction cost of a shock in proven reserves. On the third part we explore the role of extraction costs, mainly the stock effect, in the optimal taxes setting.

5.5.1 Analytical part: implied extraction costs

The analytical part is discussed with zero extraction cost as in Golosov et al (2009). It is a version of the model that provides a closed form solution to the neoclassical growth model and a benchmark calibration, as long as extraction costs are zero. The main objective of performing this task is to learn from the analytical part on the damage costs and implied parametric extraction costs.

We assume logarithmic preferences and a Cobb-Douglas specification in capital and energy.

$$U(c_t) = \ln c_t \tag{28}$$

$$Y_t = S(S_t) A_t K_t^{\alpha} E_t^{\gamma} \tag{29}$$

Following Golosov et al (2009) we consider $\alpha = 0.3$ and $\gamma = 0.03$. Together with full depreciation and a constant saving rate the following closed form solution for optimal capital is obtained:

$$K_{t+1} = \alpha \beta Y_t \tag{30}$$

As model period is 10 years, it supports the full depreciation assumption. Associated discount factor is $\beta = 0.99^{10}$. The following steps are followed to find the solution for the dynamics:

1. Find shadow price of marginal damage, which is given by

$$\Lambda_t^S = \sum_{s=1}^{\infty} (1-\varphi)^{s-1} \prod_{j=0}^{s} \frac{1}{\rho_{t+j}} \frac{S'(S_{t+s})}{S(S_{t+s})} Y_{t+s} \tag{31a}$$

2. Plug into Hotelling rule: solve for E_t under negligible Q(*) and "simple" damage function.

3. Then solution for R_t and S_t is inmediate.

Note that equation (31a) differs from equation (8) in the general setting of the section 5.2. Both equations measures the marginal cost of a unit of carbon in the atmosphere in terms of the consumption good. But equation (31a) is based in equation (31b) we show below, which is a simple law of motion of the accumulation of carbon between two consecutive periods. Equation (8) is based in equation (3) which states the accumulation of carbon in terms of the amount that is left in the atmosphere a number of periods into the future. That leads to a marginal social damage that is the discounted value of future marginal damages. In the quantitative assessment we follow the approach used by Golosov et al (2009) and in the general setting of Section 5.2 we follow Golosov et al (2011).

A particular solution for the dynamics that we first consider is a constant E_t and it can be shown that

$$E_t = \bar{E} = \gamma \frac{1-\beta(1-\varphi)}{\gamma_S \beta} = E_0$$

is a solution for E_t which implies constant damage S (S_t). We can interpret this solution as if R_t being very big, so it is optimal a constant E_t path.

Beyond the analytical solution of macro variables, the current settings allow analytical solution for climate variables. We do have measures of carbon in the atmosphere. Pre-industrial level is set at 583 GTC and today value is said to be about

783 GTC. To start with, the law of motion for the stock of carbon and the parameter´s value sticks to Golosov et al 2009:

$$S_{t+1} = S_t(1-\varphi) + E_t \qquad (31b)$$

With $\varphi = 1/11.7$, so that the half-life of carbon in the atmosphere is 117 years. We further retain a simple damage function where the climate variable S_t shows exponential γ_S accumulation.

$$S(S_t) = \exp(-\gamma_S S_t) \qquad (32)$$

We calibrate $\gamma_S = 8.6 \times 10^{-5}$, so that $S_t = 783$ in the base case as estimated for current times. We set current proven reserves to $R_0 = 2000$ GTC. With this choice of parameters it turns out that we run out of the resource (depletion) after about 300 years. Figure 5.1 shows the evolution of the resource from the initial value of 2000 GTC.

We parameterized extraction costs implied by the numerical experiment. From the function

$$Q(E_t, R_t) = \chi E_t R_t^\sigma \qquad (33)$$

Q(*) represents about 1/10 of a 1% over Gross output and σ equals -0.8
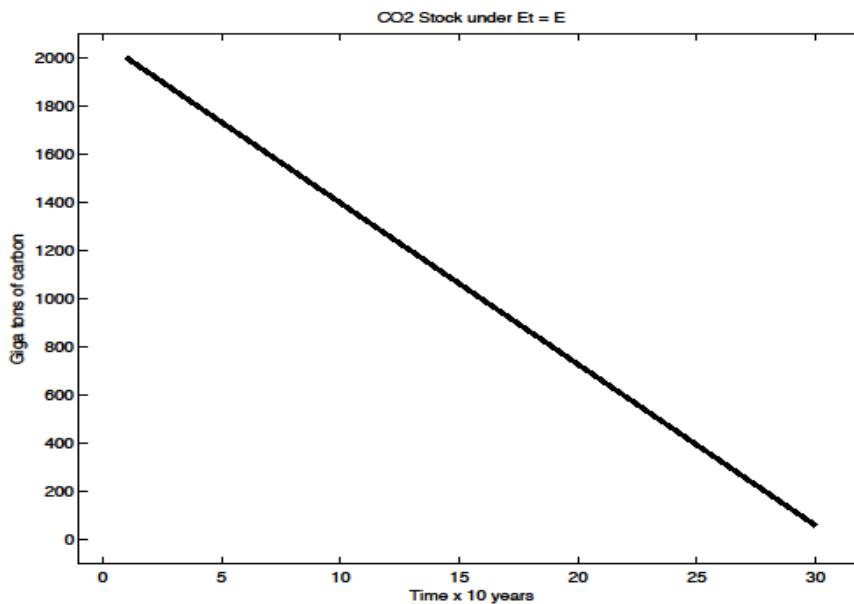
Figure 5.1 Energy Use and Resource Depletion

Figure 5.2 shows the evolution of Extraction Costs over Gross Output for the 300 years period. This is consistent with Mohaddes (2012) estimations.
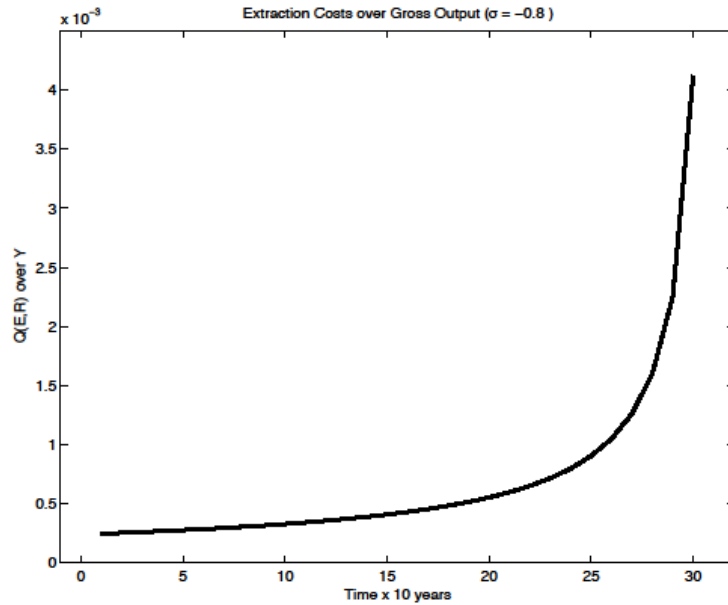
Figure 5.2 Extraction Costs over Gross Output

The second experiment retains the analytical solution but alternatively considers a decreasing E_t path starting at 90% of E_0. Figure 5.3 shows that in such a case the resource is not depleted after 300 years as before.
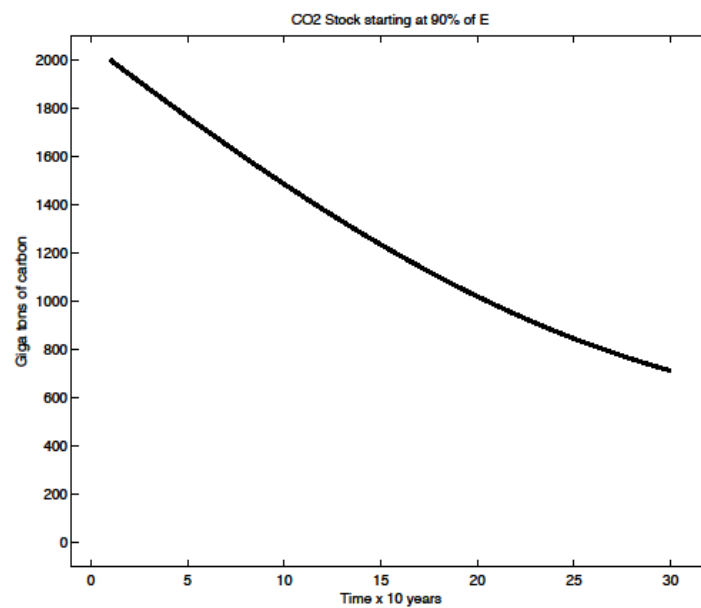
Figure 5.3 Resource Evolution

Damage is increasing as can be seen in Figure 5.4. The stock of carbon in the atmosphere is measured in GTC´s and evolve from 583 GTC which means a value of $S(S_t) = 1$. At this value there is no damage to the production.
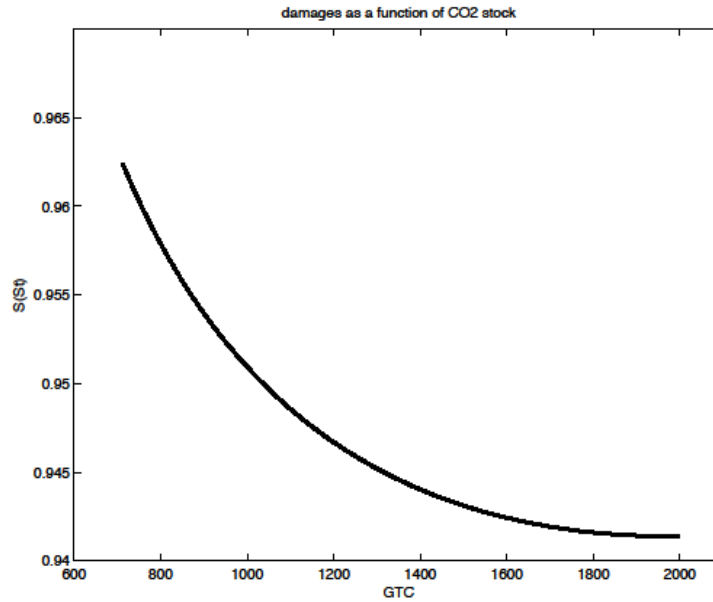
Figure 5.4 Damages as a function of the stock of CO_2

Now we are in a position to evaluate whether the evolution of extraction costs supports our claim that they are relevant for the optimal taxes setting. Figure 5 shows that evolution.
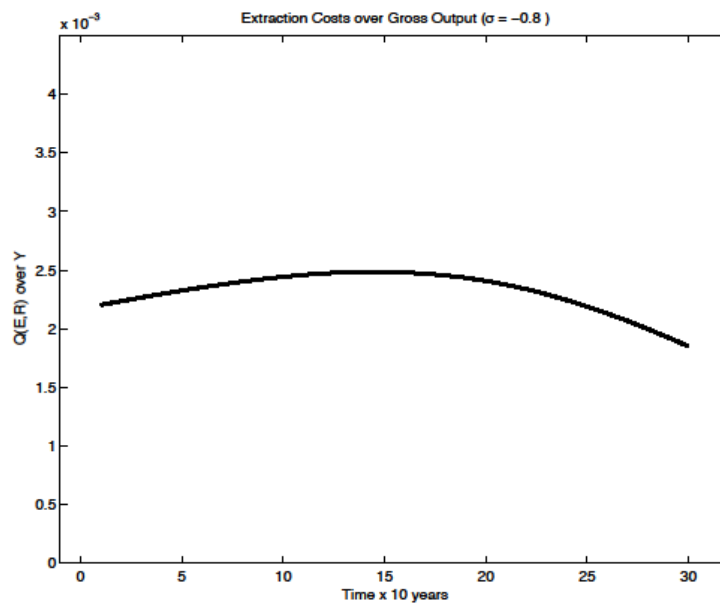
Figure 5.5 Extraction Cost Evolution

The extraction costs over gross output indicate that for relevant parameterizations extraction cost might be hump-shaped. However there is no exponential behavior when we consider a decreasing extraction path. It means that if we start from an environment in which extraction costs are small, then they might remain small.

5.5.2 Numerical Statics in R_t A Shock in Proven Reserves

Now we depart from the analytical case and consider a case in which the planner takes R_t as exogenous with no dynamics, so the optimality conditions are specified accordingly, and considers a proven reserves shock from $R_0 = 2000$ GTC to $R_0 = 3000$ GTC. We make this simplifying assumption on the dynamics of the model but incorporating extraction costs. There are no changes in parameters apart from the exponential damage (now it takes the value $\gamma_S = 3.54 \times 10^{-4}$) and the extraction cost function which takes the following values:

$$- \quad Q\left(E_t, R_t\right) = \chi E_t^{\mu} R_t^{\sigma} = 0.1 E_t^{1.1} R_t^{-0.8}$$

The shock yields some significant changes in the marginal product and marginal extraction costs.

As it can be seen in Figure 5.6, gross output sharply increases after the shock, due to the increase in energy consumption, which implies a decrease in energy productivity, that is to say in energy prices as Figure 5.7 shows. After around 100 years both variables partly recover the initial values showing a flat trend to the end of the period considered.
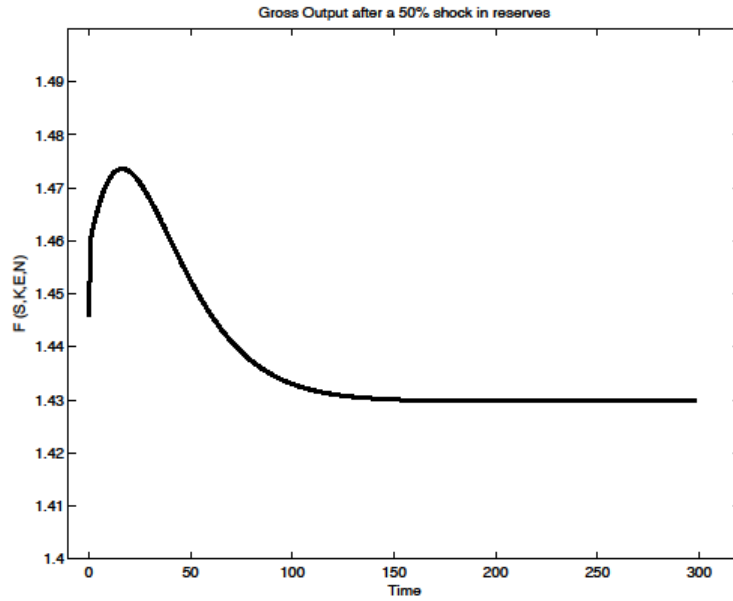
Figure 5.6 Gross Output Evolution after Reserves Shock
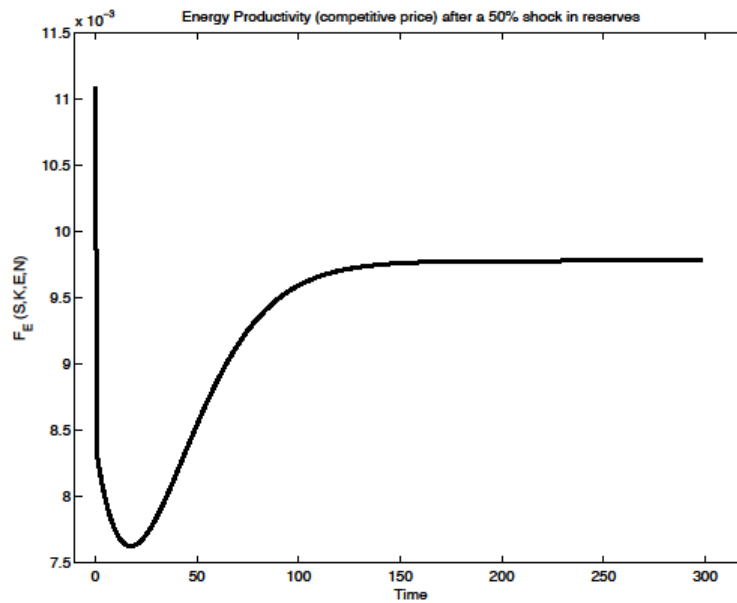

Figure 5.7 Competitive Price Evolution after Reserves Shock

Total extraction cost sharply increases due to the volume increase, but marginal extraction cost sharply decrease, mainly due to the stock effect component as figures 5.8 and 5.9 show. As output decreases, total extraction cost do, but around 100 years after the shock, as output trend is flat, total extraction cost increases due to the continuous increase of the marginal cost
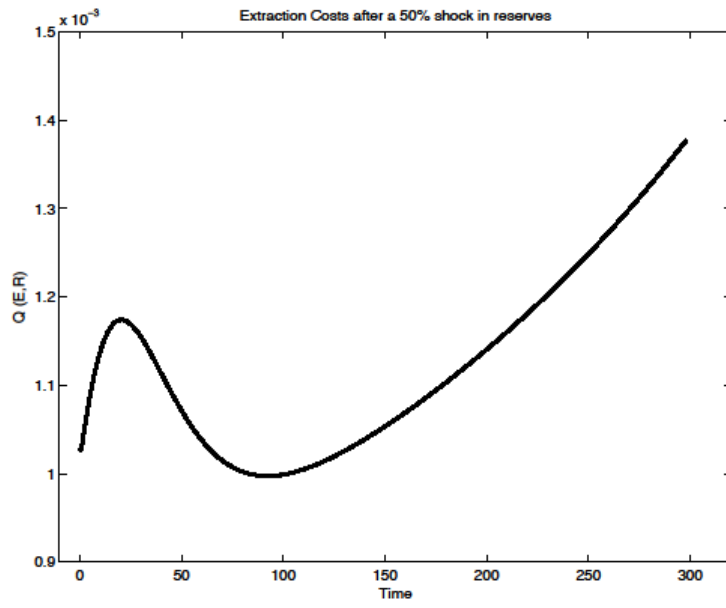
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Figure 5.8 Extraction Costs after Reserves Shock
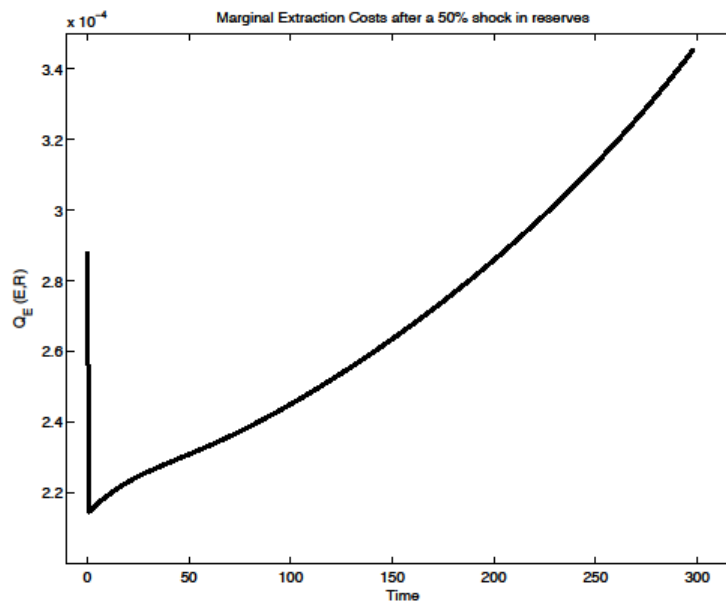
Figure 5.9 Marginal Extraction Costs after Reserves Shock

5.5.3 Full Numerical Case

A full numerical case requires a commitment to alternative BGP. Future research should address whether the first best optimal tax can be obtained by one specific policy instrument as discussed for Equation (21a) or an additional policy instrument would be needed.

5.6 Concluding Remarks

In this chapter we formulate a neoclassical growth model with two important features: an exhaustible natural resource (fossil fuel) and a global externality created by the combustion of the resource. The model is based in Golosov et al (2009, 2011 and 2014) and we particularly focused on the role that extraction costs plays on optimal Pigouvian taxes.

We solve the model to obtain the optimal tax formula considering two different effects on the extraction cost. The effect based on the flow of extraction and the scarcity effect (the cost based on the stock not yet extracted). Our main finding is that this stock effect generates significant inter-temporal distortions in an optimal profit tax while it has no impact in a per-unit tax. The optimal profit tax follows a trending rule that could be affected by shocks in the amount of available resource (proven reserves). Increases of proven reserves mean increases of the optimal profit tax.

We also analyze the stock effect impact over the so called green paradox and find that under rising pricing of energy (sufficiently strong stock effect) an increasing ad valorem tax can postpone extraction. The optimal ad valorem tax could be increasing or decreasing based on the ratio between growths of the marginal damage and the resource price.

We perform a quantitative assessment to check on several issues, like the role of the scarcity cost vs. the role of marginal cost and the interaction of extraction costs and tax policies. We find that for relevant parameterization, extraction cost might be hump-shaped, but there is no exponential behavior under a decreasing extraction path. It means that if extract cost were small they might remain small, so we cannot reject the assumption that extraction costs might be irrelevant for carbon taxes policy at a global level

Still some important open issues have not being covered in this chapter and remain for further research:

1. The theoretical results on proven reserves merit further investigation to quantify their importance on optimal taxes.

2. The stock effect might be quantitatively important

3. A time inconsistency problem may ocurr

4. Numerical experiments are useful to investigate local fiscal policies.[33]

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[33] Some work has already being done on Norway fiscal policy to analyze the evolution of proven reserves and carbon tax rate. In general it seems that OCDE (rich) countries extract resources at a higher speed than OPEP and third world countries. It is like they are not considering the scarcity effect.

APPENDIX

Optimality Conditions Social Planner

The Lagrangian of the social planner´s program is (A.1). There are two control variables (C, E) and three state variables (K, S, R), where λ_t^i (i = K, S, R) are the co-state variables.

$$L = \sum_0^\infty \beta^t u(c_t) + \beta^t \lambda_t^K \left[F_t - c_t + (1-\delta)K_t - Q_t(E_t, R_t) - K_{t+1} \right] +$$
$$+ \beta^t \lambda_t^S \left(S_t - \sum_0^{t+T}(1-d_s)E_{t-s} \right) + \beta^t \lambda_t^R \left(R_t - E_t - R_{t+1} \right)$$
(A1)

The first order conditions yield (A.2) to (A.5).

$$FOC(c_t) \Rightarrow \beta^t u'(c_t) = \beta^t \lambda_t^K \Rightarrow u'(c_t) = \lambda_t^K$$
(A2)

$$FOC(E_t) \Rightarrow \beta^t \lambda_t^K (F_{E,t} - Q_{E,t}) - \sum_{j=0}^\infty \beta^{t+j} \lambda_{t+j}^S \frac{\partial S_{t+j}}{\partial E_t} - \beta^t \lambda^R = 0 \Rightarrow$$
$$\Rightarrow \lambda_t^K (F_{E,t} - Q_{E,t}) - \sum_{j=0}^\infty \beta^j \lambda_{t+j}^S \frac{\partial S_{t+j}}{\partial E_t} - \lambda^R = 0$$
(A3)

$$FOC(K_{t+1}) \Rightarrow -\beta^t \lambda_t^K + \beta^{t+1} \lambda_{t+1}^K \left(F_{K,t+1} + (1-\delta) \right) = 0$$
(A4)

$$FOC(S_{t+1}) \Rightarrow \beta^t \lambda_t^S + \beta^t \lambda_t^K F_{S,t} = 0$$
(A5a)

$$FOC(R_{t+1}) \Rightarrow -\beta^t \lambda_t^R - \beta^{t+1} \lambda_{t+1}^K Q_{R,t+1} + \beta^{t+1} \lambda_{t+1}^R = 0$$
(A5b)

Considering (A.2) at time t+1 and (A.4) we get (A.6) which gives us the real interest rate of the economy. We define ρ as the capitalization factor (1 + the net return on capital).

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left[F_{K,t+1} + (1-\delta) \right] \equiv \rho_{t+1} = \frac{\lambda_t^K}{\beta \lambda_{t+1}^K}$$
(A6)

Differentiating (A.3) with respect to time give us (A.7)

$$\beta^{t+1} \lambda_{t+1}^K (F_{E,t+1} - Q_{E,t+1}) - \sum_{j=0}^\infty \beta^{t+1+j} \lambda_{t+1+j}^S \frac{\partial S_{t+1+j}}{\partial E_{t+1}} - \beta^{t+1} \lambda_{t+1}^R = 0$$
(A7)

The difference between (A7) and (A3) yields (A8)

$$\beta^{t+1}\lambda_{t+1}^{K}\left(F_{E,t+1}-Q_{E,t+1}\right)-\sum_{j=0}^{\infty}\beta^{t+1+j}\lambda_{t+1+j}^{S}\frac{\partial S_{t+1+j}}{\partial E_{t+1}}=$$

$$=\beta^{t}\lambda_{t}^{K}\left(F_{E,t}-Q_{E,t+1}\right)-\sum_{j=0}^{\infty}\beta^{t+j}\lambda_{t+j}^{S}\frac{\partial S_{t+j}}{\partial E_{t}}+\beta^{t+1}\lambda_{t+1}^{R}-\beta^{t}\lambda_{t}^{R}$$

(A8a)

Consider (A5a) yields

$$\beta^{t}\lambda_{t}^{K}\left(F_{E,t}-Q_{E,t}\right)+\sum_{j=0}^{\infty}\beta^{t+j}\lambda_{t+j}^{K}F_{S,t+j}\frac{\partial S_{t+j}}{\partial E_{t}}=$$

$$\beta^{t+1}\lambda_{t+1}^{K}\left(F_{E,t+1}-Q_{E,t+1}\right)+\sum_{j=0}^{\infty}\beta^{t+1+j}\lambda_{t+1+j}^{K}F_{S,t+1+j}\frac{\partial S_{t+1+j}}{\partial E_{t+1}}-\beta^{t+1}\lambda_{t+1}^{K}Q_{R,t+1}$$

(A8b)

Denote the climate change externality measured in consumption units as:

$$\Lambda_{t}^{S}\equiv\frac{-\sum_{j=0}^{\infty}\beta^{j}\lambda_{t+j}^{K}F_{S,t+j}\frac{\partial S_{t+j}}{\partial E_{t}}}{u'(c_{t})}>0$$

Take the above expression on equation (A8b) and consider (A5b) to get the optimal extraction path (A9)

$$\frac{\beta^{t}\lambda_{t}^{K}}{\beta^{t+1}\lambda_{t+1}^{K}}\left[\left(F_{E,t}-Q_{E,t}\right)-\Lambda_{t}^{S}\right]=F_{E,t+1}-Q_{E,t+1}-Q_{R,t+1}-\Lambda_{t+1}^{S}$$

$$\rho_{t+1}=\frac{F_{E,t+1}-Q_{E,t+1}-Q_{R,t+1}-\Lambda_{t+1}^{S}}{F_{E,t}-Q_{E,t}-\Lambda_{t}^{S}}$$

(A9)

This is Equation (7).

Optimality conditions Consumers

The Lagrangian of the consumer´s problem is

$$L=\beta^{t}U\left(c_{t}\right)+\lambda_{t}^{K}\left(K_{t}+rK_{t}+\Pi_{t}^{f}+\Pi_{t}^{E}+T_{t}-c_{t}-K_{t+1}\right)$$

(A10)

FOC on capital and consumption yields:

$$FOC(c_{t})\Rightarrow\beta^{t}U'(c_{t})-\lambda_{t}^{K}=0\Rightarrow\beta^{t+1}U'(c_{t+1})-\lambda_{t+1}^{K}=0$$

$$FOC(K_{t+1})\Rightarrow\lambda_{t+1}^{K}\left(1+r_{t+1}\right)-\lambda_{t}^{K}=0$$

From the above equations it is trivial to get the standard optimal condition

$$\beta^{t+1}U'(C_{t+1})(1+r_{t+1})-\beta^{t}U'(C_{t})=0 \Rightarrow \frac{U'(C_{t})}{\beta U'(C_{t+1})}=1+r_{t+1} \qquad (A11)$$

Optimality Conditions Resource Extraction

The Lagrangian of the Resource Extraction Problem is:

$$L=\Gamma_{t}^{s}\left[(P_{E,s}-\theta_{s})(E_{s})-Q(E_{s},R_{s})\right](1-\tau_{s}^{e})+\mu(-E)+\lambda_{s}^{R}(R_{t}-E_{t}-R_{t+1}) \qquad (A12)$$

And the FOC´s on energy and resource

$$FOC(E_{t})\Rightarrow\Gamma_{t}^{s}\left[(P_{E,t}-\theta_{t})-Q_{E,t}\right](1-\tau_{t}^{e})-\mu-\lambda_{t}^{R}=0$$

$$FOC(R_{t+1})\Rightarrow-\Gamma_{t+1}^{s}Q_{R,t+1}(1-\tau_{t+1}^{e})+\lambda_{t+1}^{R}-\lambda_{t}^{R}=0 \qquad (A13)$$

Now we combine the above expressions to get the optimal extraction rate

$$\Gamma_{t}^{s}\left[P_{E,t}-\theta_{t}-Q_{E,t}\right](1-\tau_{t}^{e})=\Gamma_{t+1}^{s}\left[P_{E,t+1}-\theta_{t+1}-Q_{E,t+1}-Q_{R,t+1}\right](1-\tau_{t+1}^{e})$$

$$\frac{\Gamma_{t}^{s}}{\Gamma_{t+1}^{s}}=\rho_{t+1}\Rightarrow\rho_{t+1}=\frac{\left[P_{E,t+1}-\theta_{t+1}-Q_{E,t+1}-Q_{R,t+1}\right](1-\tau_{t+1}^{e})}{\left[P_{E,t}-\theta_{t}-Q_{E,t}\right](1-\tau_{t}^{e})}=\frac{U'(C_{t})}{\beta U'(C_{t+1})} \qquad (A14)$$

This is the case of variable taxes that correspond to Equation (17). The case without taxes is trivial from the above expression. In the case of fixed taxes the Lagrangian is:

$$L=\Gamma_{t}^{s}\left[(P_{E,s}-\theta)(E_{s})-Q(E_{s},R_{s})\right](1-\tau)+\mu(-E)+\lambda_{t}^{R}(R_{t}-E_{t}-R_{t+1})$$

And the corresponding FOC´s

$$\frac{\partial L}{\partial E_{t}}=0\Rightarrow\Gamma_{t}^{s}\left[(P_{E,t}-\theta)-Q_{E,t}\right](1-\tau)-\mu-\lambda_{t}^{R}=0\Rightarrow$$

$$\Rightarrow\Gamma_{t+1}^{s}\left[(P_{E,t+1}-\theta)-Q_{E,t+1}\right](1-\tau)-\mu-\lambda_{t+1}^{R}=0$$

$$\frac{\partial L}{\partial R_{t+1}}=0\Rightarrow\Gamma_{t+1}^{s}Q_{R,t+1}(1-\tau)=\lambda_{t+1}^{R}-\lambda_{t}^{R}$$

From the above expressions we can get Equation (16) considering $P_{E,t}-Q_{E,t}=NP_{E,t}$

$$\Gamma_t^s \left[P_{E,t} - \theta - Q_{E,t} \right](1-\tau) = \Gamma_{t+1}^s \left[P_{E,t+1} - \theta - Q_{E,t+1} \right](1-\tau)$$

$$\frac{\Gamma_t^s}{\Gamma_{t+1}^s} = \rho_{t+1} = \frac{\left[NP_{E,t+1} - \theta \right]}{\left[NP_{E,t} - \theta \right]} - \frac{Q_{R,t+1}}{NP_{E,t} - \theta}$$

Evolution of Reserves and Production by Countries

We first take all significant countries but the "four majors" in terms of proven reserves and production amounts. USA is the main producer in the world and Saudi Arabia comes second. Venezuela and Saudi Arabia have the most important reserves. And Russia is the third most important producer.

The graphic 5.2.a shows the homogeneity of countries in the relationship between their proven reserves and production amounts. Only China is a clearly outsider. Like the USA and Russia, China production exceeds what is the OPEP countries behavior.

The graphic 5.2.b shows a similar situation, but in this case China is not included.
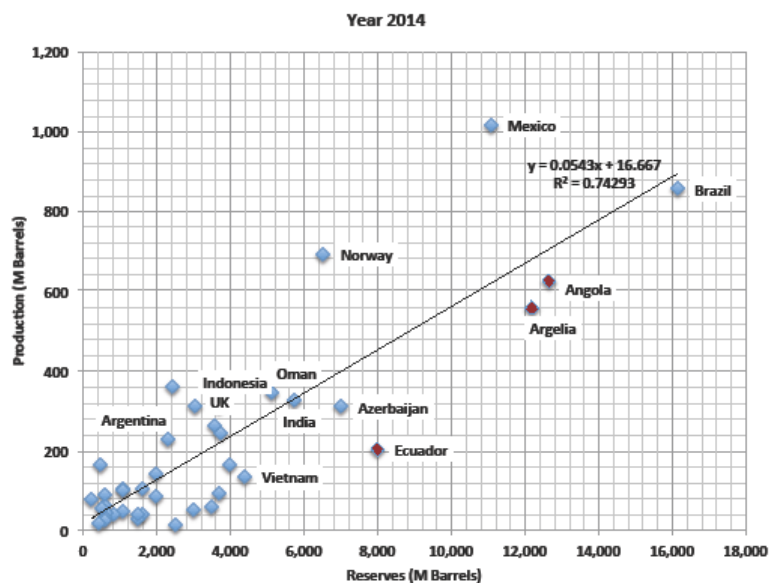
The graphic 5.2.c excludes China and OPEP countries and still shows a remarkable homogeneity among countries.
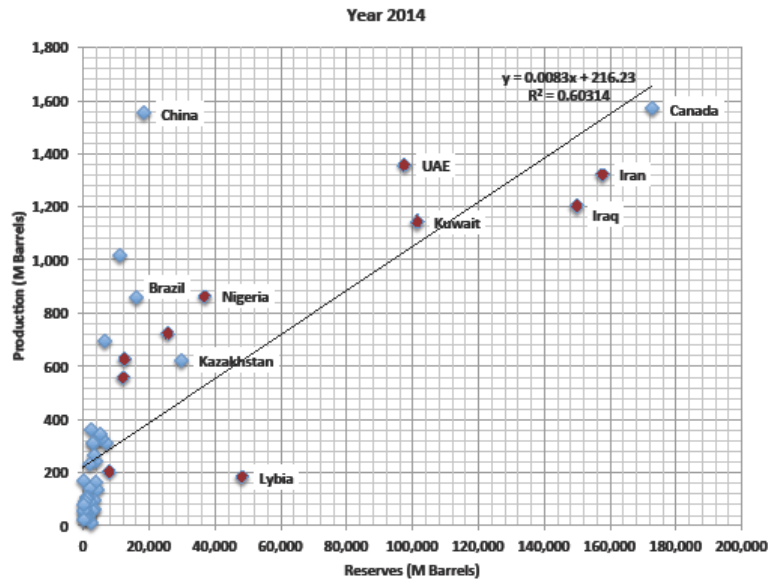
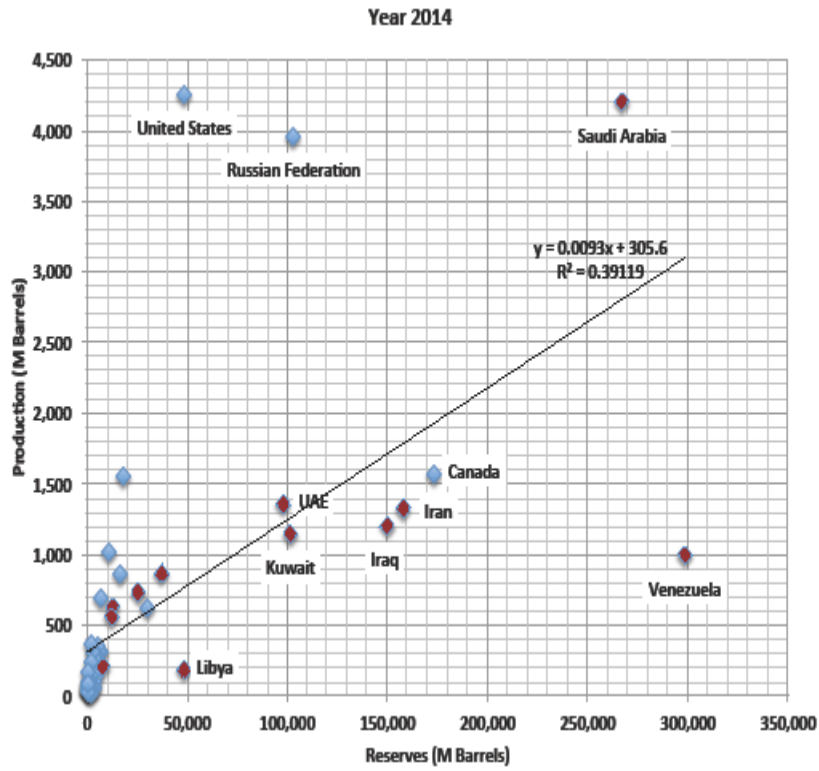
Graph 5.2.a Proven Reserves and Production
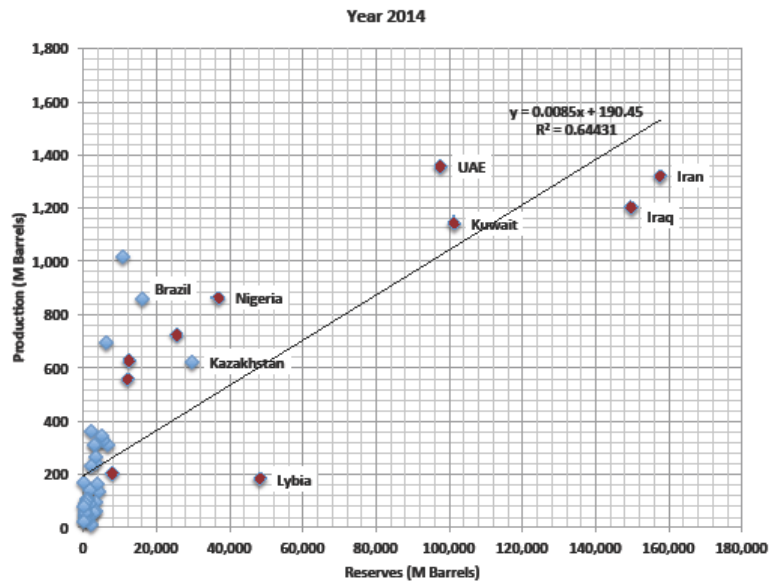
Graphic 5.2.b Proven Reserves and Production W/O China

Graphic 5.2.c Proven Reserves and Production W/O China and OPEP countries

Graphic 5.3 consider all significant countries and clearly show how "big producers" are extracting faster than the rest of countries. It is like they are not taking into account the scarcity effect. On the other hand Venezuela shows a completely different approach, although this could be a temporary situation.

Graph 5.3 Relationships between Reserves and Production all countries

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