UNIVERSIDAD COMPLUTENSE DE MADRID
FACULTAD DE CIENCIAS MATEMÁTICAS

TESIS DOCTORAL

Mathematical programming with uncertainty and multiple objectives for sustainable development and wildfire management

Programación matemática con incertidumbre y múltiples objetivos para desarrollo sostenible y gestión de incendios forestales

MEMORIA PARA OPTAR AL GRADO DE DOCTOR

PRESENTADA POR

Javier León Caballero

Directores

Begoña Vitoriano
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Programa de Doctorado en Ingeniería Matemática, Estadística e Investigación Operativa por la Universidad Complutense de Madrid y la Universidad Politécnica de Madrid

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Tesis Doctoral

Javier León Caballero

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Mathematical programming with uncertainty and multiple objectives for sustainable development and wildfire management

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the requirements for the degree of
Doctor of Philosophy

Javier León Caballero
Bachelor in Mathematics, Autonomous University of Madrid
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School of Science
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UNIVERSIDAD COMPLUTENSE MADRID

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D./Dña. Javier León Caballero,
estudiante en el Programa de Doctorado en Ingeniería Matemática, Estadística e Investigación Operativa,
de la Facultad de Ciencias Matemáticas de la Universidad Complutense de Madrid, como autor/a de la tesis presentada para la obtención del título de Doctor y titulada:

Mathematical programming with uncertainty and multiple objectives
for sustainable development and wildfire management

y dirigida por: Begoña Vitoriano Villanueva y John Hearne

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(Javier León — October 2019)

Agradecimientos

"El trabajo se expande hasta ocupar todo el tiempo del que se dispone"

Ley de Parkinson

Tras más de cuatro años de trabajo, congresos y artículos mirando a *deadlines*, poder escribir los agradecimientos sin una fecha de entrega está resultando más difícil de lo que esperaba. De haber escrito la tesis al ritmo de esta sección hubiese sido completamente imposible defender a tiempo.

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Glossary

CVaR	Conditional value-at-risk
C+NVC	Cost plus net-value-change
DDS	Decision support system
DM	Decision maker
ESCO	Energy service company
FRC	Fire response curve
LP	Linear programming
MIP	Mixed integer programming
MOP	Multiobjective (or multicriteria) programming
MP	Mathematical programming
MSP	Multiobjective stochastic programming
OR	Operational research
OWA	Ordered weighting operator
PVRE	Photovoltaic rural electrification
SDG	Sustainable development goal
SHS	Solar home system
TFI	Tolerable fire interval
VaR	Value-at-risk

Resumen

La Programación Matemática es un campo de la Investigación Operativa bien situado para abordar problemas tan diversos como aquellos que surgen en Logística y en Gestión de Desastres. El objetivo fundamental de la Programación Matemática es la selección de una alternativa *óptima* que cumpla una serie de restricciones.

El criterio por el cual se evalúan las alternativas es tradicionalmente uno solo (por ejemplo minimizar coste), sin embargo es también común que varios objetivos quieran ser considerados simultáneamente, dando así lugar a la Decisión Multicriterio.

En caso de que las condiciones que ha de cumplir una alternativa o la evaluación de dicha alternativa dependan de factores aleatorios (o desconocidos) nos encontramos en un contexto de optimización bajo incertidumbre.

En los primeros capítulos de esta tesis se estudian los campos de decisión multicriterio y optimización con incertidumbre, en dos aplicaciones en el contexto del desarrollo sostenible y la gestión de desastres.

La optimización con incertidumbre se introduce mediante una aplicación a electrificación rural. En zonas rurales es común el acceso a la electricidad mediante sistemas solares instalados en las casas de los consumidores. Estos sistemas han de ser reparados cuando fallen, por lo que la decisión de cómo dimensionar una red de mantenimiento se ve afectada por una gran incertidumbre. Un modelo de programación matemática es desarrollado tratando la incertidumbre de forma no explícita, cuyo objetivo es obtener una red de mantenimiento a mínimo coste. Dicho modelo es posteriormente utilizado como herramienta para la obtención de reglas simples que puedan predecir el coste de mantenimiento utilizando poca información. El modelo es validado mediante información de un programa real implementado en Marruecos.

Al estudiar la Optimización Multicriterio se considera un problema en gestión de incendios forestales. Para mitigar los efectos de los incendios forestales es común la modificación de los bosques, con lo que se conoce como *tratamiento de combustible*. Mediante esta práctica, consistente en la tala o quema controlada de árboles en zonas seleccionadas, se consigue que al producirse inevitablemente incendios estos sean más fáciles de controlar. Desafortunadamente el modificar la flora puede afectar a la fauna existente, con lo que es sensato buscar soluciones que mejoren la situación de cara a un incendio pero sin gran detrimento de las especies existentes. Es decir, hay varios criterios a tener

en cuenta a la hora de optimizar. Se desarrolla un modelo de programación matemática, el cual sugiere qué zonas quemar y cuándo, teniéndose en cuenta estos criterios enfrentados. Este modelo es aplicado a una serie de casos realistas simulados.

A continuación se llega a un estudio teórico del campo de Programación Estocástica Multiobjetivo (MSP, *Multiobjective Stochastic Programming*), en el que son considerados problemas que simultáneamente tienen varios criterios e incertidumbre. En ese capítulo se desarrolla un nuevo concepto de solución para problemas MSP con aversión al riesgo, se estudian sus propiedades y se formula un modelo de programación lineal capaz de obtener dicha solución. También se lleva a cabo un estudio computacional del modelo, aplicándolo a una variación del conocido problema de la mochila.

Finalmente se estudia de nuevo el problema de las quemas controladas, considerando esta vez la incertidumbre existente al no ser posible saber con certeza cuántas quemas controladas pueden ser realizadas en un año, debido a la limitada ventana de tiempo en que estas pueden realizarse. El problema es resuelto mediante la metodología multicriterio y estocástica con aversión al riesgo desarrollada en el capítulo anterior. Por último, el modelo resultante es aplicado a un caso real situado en el sur de España.

Abstract

Mathematical Programming is a field of Operations Research well located for tackling problems as diverse as those arising in Logistics and Disaster Management. The main objective of Mathematical Programming is the selection of an *optimal* alternative satisfying a series of constraints.

Traditionally alternatives are usually judged by a single criterion (for example, minimizing cost); however, it is also common that multiple objectives have to be considered simultaneously, leading to Multicriteria Decision Making.

When the conditions to be satisfied by an alternative, or the evaluation of that alternative relies on random or unknown factors, there is a context of Optimization under uncertainty.

The first chapters of this thesis study the field of Multicriteria Decision Making and Optimization under uncertainty, in two application in the context of sustainable development and disaster management.

Optimization with uncertainty is presented with an application to rural electrification. It is common, especially in rural areas, that the access to electricity is provided via solar systems installed on the homes of the users. These systems have to be repaired when they malfunction. Consequently, the decision of how to size and locate a maintenance network is affected by uncertainty. A mathematical programming model is developed, treating the uncertainty in a non-explicit way, whose goal is to obtain a maintenance network at minimum cost. Such model is then used as a tool for obtaining more straightforward rules that are able to predict maintenance cost using limited information. The model is validated using information from a real program implemented in Morocco.

When studying Multicriteria Decision Making a problem in wildfire management is considered. To mitigate the effect of wildfires, it is common the modification of forest, with what is known as *fuel management*. This technique, consisting in the felling or controlled burns of vegetation in selected areas, results on more manageable fires when they inevitably occur. Unfortunately, modifying flora can affect existing fauna, and thus it is sensible to search for solutions that improve the landscape wildfire-related, without substantial damage to existing species. That is, there are multiple criteria to take into account when optimizing. A mathematical programming model is developed, suggesting which areas to burn and when, taking into account the conflicting criteria. This model is applied to a series of realistic simulated cases.

After that, a theoretical study of the field of Multiobjective Stochastic Programming (MSP) is performed, in which problems which simultaneously have multiple criteria and uncertainty are considered. In that chapter, a new concept of solution for MSP problems with risk-aversion is developed, its properties are studied, and a linear programming model is formulated for obtaining such a solution. A computational study of the model is also performed, applying it to a variant of the well-known knapsack problem.

Finally, prescribed burning is studied again, considering this time the existing uncertainty due to not knowing how many prescribed burns can be completed within a year, caused by the limited time-window in which prescribed burns can be performed. The problem is solved using the risk-averse multiobjective stochastic methodology developed in the previous chapter. Lastly, the resulting model is applied to a real case located in the south of Spain.

Chapter 1

Introduction

1.1 Motivation

Decision making is not easy, but we are making decisions constantly. How should I go to work tomorrow? Public transport or car? Which is faster? Cheaper? More comfortable? What if there is a traffic jam? Or the train is late? How bad would it be if I arrive some minutes late?

Uncertainty is present in our world and the decisions we make must account for it. It comes in many different ways:

- A situation can occur with some given odds. For instance, when throwing a balanced coin.
- Uncertainty might be present due to ignorance of the situation.
- Another possibility is that there is semantic uncertainty, which can occur when people use vague terms or interpret them differently.

Any of these variations, which will be further discussed in Chapter 2, definitely push us to seek for decisions that protect us from the inherent risk. Such is the case when buying insurance: we are open to pay some recurring fee, so in case an accident happens, we are not in the worst possible scenario.

Another concept that complicates our decisions is that most often than not they are conflicting: the bigger the house is, the more expensive it becomes, and the higher profit an asset yields, the riskier it is.

One could argue that without that conflict decisions could be hardly labelled as such: if someone has to choose between to options, and one of those is better than the other in every aspect that matters, there is nothing to be decided.

The development of this PhD thesis is initially motivated as a continuation of the Final Master Project *Cost estimation models and sizing of maintenance service of a rural electrification network* of the *Master in Mathematical Engineering* of Complutense University of Madrid (UCM) under the supervision of Begoña Vitoriano. This work is part of the UCM research group Decision Aid Models for Logistics and Disaster Management (Humanitarian Logistics).

The research has been developed in the framework of the H2020 European-funded action Marie Skłodowska-Curie RISE *GEO-SAFE Geospatial based Environment for Optimisation Systems Addressing Fire Emergencies*, as an *early-stage researcher*. The collaboration has led to the realization of the thesis with a cotutelle agreement between Complutense University of Madrid and the Royal Melbourne Institute of Technology (RMIT), being supervised by Begoña Vitoriano at UCM and John Hearne at RMIT.

The research and project mentioned above motivate that the problems arisen in this thesis are focused on sustainable development and wildfire management.

1.2 Sustainable development and wildfires

According to *The Sustainable Development Agenda* (2015) Sustainable development has been defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs". For achieving this purpose, the Sustainable Development Goals (SDGs) were adopted in 2015 for the period 2015-2030. Table 1.1 shows the SDGs.

1	No Poverty
2	Zero Hunger
3	Good Health and Well-being
4	Quality Education
5	Gender Equality
6	Clean Water and Sanitation
7	Affordable and Clean Energy
8	Decent Work and Economic Growth
9	Industry, Innovation, and Infrastructure
10	Reducing Inequality
11	Sustainable Cities and Communities
12	Responsible Consumption and Production
13	Climate Action
14	Life Below Water
15	Life On Land
16	Peace, Justice, and Strong Institutions
17	Partnerships for the Goals

Table 1.1: Sustainable Development Goals

Among the SDGs, goals 7 and 15 will be of particular interest through this thesis. Goal 7 includes in its targets "ensure universal access to affordable, reliable and modern energy services" as well as "expand infrastructure and upgrade technology for supplying modern and sustainable energy services for all in developing countries".

One of the challenges associated with these targets is, therefore, the design of electrification programs. Such programs can be based on solar home systems, especially in rural areas in developing countries (Ellegård et al., 2004; Wamukonya, 2007). A difficulty that often arises is the assessment of a fair price for consumers accessing the programs. Similarly, cost estimation is necessary for Governments or energy companies that want to create or participate in electrification programs. However studying installations costs is not enough, and maintenance costs have to be considered as well.

Estimating maintenance costs has been missing in part of the programs implemented in developing countries (Lemaire, 2009, 2011), possibly due to the high uncertainty caused by the high number of systems malfunctions (Carrasco, Narvarte, Peral and Vázquez, 2013). This problem is later studied in Chapter 2.

The SDGs do not explicitly mention wildfires. However, Goal 15 does include the following targets:

15.1 By 2020, ensure the conservation, restoration and sustainable use of terrestrial and inland freshwater ecosystems and their services, in particular forests, wetlands, mountains and drylands, in line with obligations under international agreements.

15.5 Take urgent and significant action to reduce the degradation of natural habitats, halt the loss of biodiversity and, by 2020, protect and prevent the extinction of threatened species.

Both of these targets are undeniably affected by wildfires. Furthermore, *Disaster risk reduction and resilience in the 2030 agenda for sustainable development* (2015) argue how disaster risk reduction is closely related to the SDGs, and therefore decreasing the risk of wildfires is, in fact, aligned with Sustainable Development.

Fire is a phenomenon that has a significant impact on the shaping of our planet. Fires can only happen if the following components are present: fuel, heat and oxygen (more generally, an oxidizing agent). Although it can emerge as a natural process, most of them are human-caused, and they have the potential to generate tremendous losses (human lives, monetary losses or environmental ones). Dennison et al. (2014) expose how the severity of wildfires is increasing. Pacheco et al. (2015) review different decision-support tools available for fire management.

Mathematical models for forest fires emerged after Richard Rothermel developed in 1972 a spread model, which is still the base of modern spread models, that predicted fire spread rates (Rothermel, 1972). In addition to that, Operational Research is a branch that is becoming more and more important in forest fire management, especially concerning prevention (Martell, 2007).

Several references can be found stating how disastrous can forest fires become. A fire in Idaho, USA, in 1967 burned down over 20 000 ha (Pyne et al., 1996); in 2013 a series of forest fires in California burned down over 300 000ha and killed 24 people (Hyndman and Hyndman, 2010); in 2007 in Peloponnese, Greece, three consecutive fires took 67 lives (Maditinos and Vassiliadis, 2011).

Statistics about forest fires taking place in Spain since 1968 are provided by the Ministry of Agriculture and Fisheries, Food and Environment[1] on their website. Every five years a summary of the last ten years events is also provided. Among other figures, they provide data on the amount of wildfires and burnt area (Fig. 1.1) and the causes (Fig. 1.2).

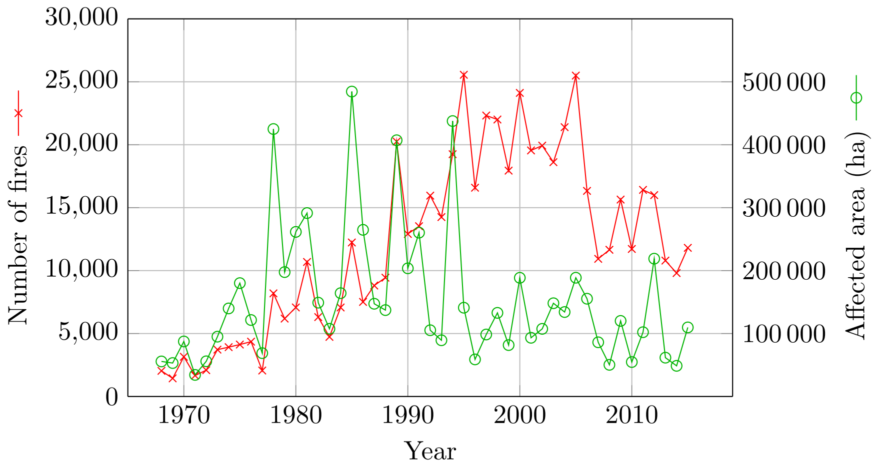


Figure 1.1: Wildfires in Spain, 1968-2015 (López Santalla and López García, 2019)

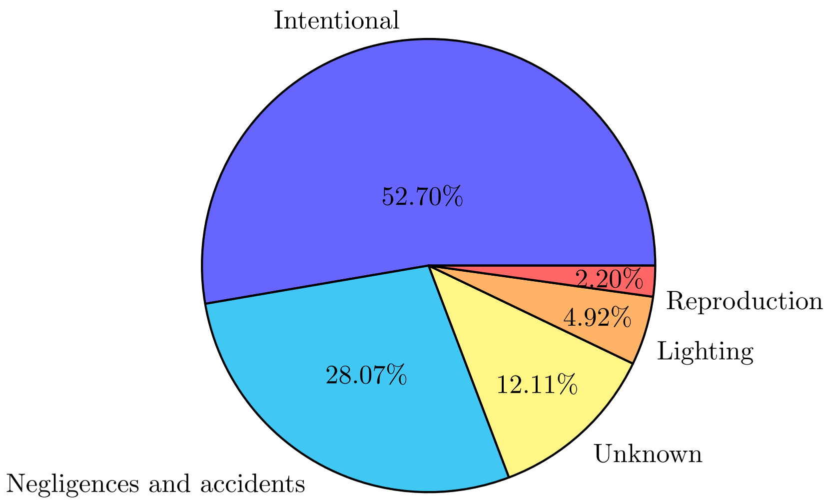


Figure 1.2: Causes of wildfires in Spain, 2006-2015 (López Santalla and López García, 2019)

Mathematics can be helpful for wildfires in a number of issues, such as: estimating how will the fire advance (Sullivan, 2009*a*,*b*,*c*), suggesting decisions on how to contain it, modifying the fuel for modulating the behaviour and effects of fire and improving also the efficiency of fire-fighting

[1] *Ministerio de Agricultura y Pesca, Alimentación y Medio Ambiente* in Spanish.

(Pyne et al., 1996), preallocating fire resources (Wei et al., 2017) or protecting assets (Roozbeh et al., 2018).

Prescribed burning is a fuel management technique, consisting in the large-scale removal of fuel with the aim of reducing the intensity of an eventual wildfire (Pyne et al., 1996). Even if prescribed burning does not eliminate the risk of wildfire, it is regarded as highly important for its mitigation benefits (Burrows, 2008), although it may have unwanted effects if some species rely on old vegetation (Brown et al., 2009). Prescribed burning is studied in Chapters 3 and 5.

1.3 Mathematical programming

Mathematical programming is a useful tool for helping make decisions on problems arising in sustainable development, disaster management and emergencies. Development of effective algorithms as well as technological advances allows to solve in relative small times problems with thousands of constraints and variables, especially linear ones.

Mathematical Programming (MP) is a field of Operations Research (OR), and as such it inherits the main goal of OR: the search for an optimal solution. Hillier and Lieberman (2015) note that the word programming is a synonym for planning rather than referring to computer programming. MP models share the next components (Taha, 2017):

1. Decision variables that we seek to determine.
2. Objective (goal) that we need to optimize (maximize or minimize).
3. Constraints that the solution must satisfy.

After constructing the appropriate model, an algorithm is used to solve for an optimal solution (Hillier and Lieberman, 1995). In particular, Linear Programming (LP) models are those MP models in which the variables appear in the objective function and constraints linearly. Bazaraa et al. (2010) argue that the linearity assumption is not so restrictive, as LP models are among the most used today and they are often used to solve more complex problems.

Other techniques include integer programming (in which the variables assume integer values), dynamic programming (in which the original model can be decomposed into smaller more manageable subproblems), and nonlinear programming (in which functions of the model are nonlinear) (Taha, 2017).

An MP model involves a single objective function to be minimized or maximized, but multiple objectives functions can also be approached with different techniques (Williams, 2013). The LP model in standard form is as follows:

$$\min\ c^t x$$
$$\text{s.t. } Ax = b$$
$$x \geq 0$$

Most authors date the beginning of OR to the military services early in World War II. Taha (2017) single out Leonid Kantorovich as the founder of the theory of linear programming, but it was Dantzig's simplex algorithm, which rendered large LPs solvable in practice, which ignited developments in the area. Moreover, Hillier and Lieberman (2015) consider that the computer revolution gave momentum to the growth of the field, further accelerating in the 1990s and into the 21st century when powerful computers became accessible to everyone.

George Bernard Dantzig (1914-2005) developed in 1947 the (primal) simplex algorithm. The simplex method remains a viable and popular tool for solving LP problems (Bazaraa et al., 2010). Although the complexity of the simplex algorithm is exponential (Klee and Minty, 1972), the number of iterations has been shown to be proportional to the number of variables on average (Smale, 1983).

It is also worth mentioning that every LP problem has another LP problem associated, called *dual*. The original problem is traditionally called *primal* problem (according to Bazaraa et al. (2010) the *primal* term was first used by Dantzig's father). Duality is of interest as the primal and the dual are closely related, as the solution of each of them can be easily inferred from the solution of the other. Bazaraa et al. (2010) also remarks that John von Neumann is credited with the existence of the dual problem.

Based on duality theory, the dual-simplex method solves LP problems by iterating through feasible solutions of the dual problem, until the optimal of the dual (and also the primal) is obtained (Bazaraa et al., 2010). The dual-simplex method is especially useful for performing sensitivity analysis, and it is among the basis of integer programming algorithms.

The simplex method had no serious competition until a new polynomial-time algorithm for linear programming problems was proposed by Karmarkar (Karmarkar, 1984; Bazaraa et al., 2010). Karmarkar's algorithm is classified as a barrier method. While primal and dual simplex methods move in each iteration along vertices of the primal/dual feasible region, barrier methods iterate within the interior of the feasible region. All these methods are currently used (*User's Manual for CPLEX*, 2019), since the best algorithm to use depends on the problem itself.

Occasionally some of the decision variables need to be integer. This can happen by the nature of the problem (for example if a decision variable is the amount of cars purchased) or by modelling needs (binary variables can encode logical relationships such as implications or disjunctions). The two main algorithms for solving linear problems with integer variables, Branch-and-bound (Land and Doig, 1960) and Cutting-planes (Gomory, 1958), rely on solving a series of LP problems. Modern implementations include an integration of these two algorithms, denoted by branch-and-cut (Balas et al., 1996; Mitchell, 2002).

The LP model and its solution methods only consider one objective function. However, it is common that multiple conflicting objectives need to be optimized simultaneously. According to Zeleny (1982), the foundations of decision-making with multiple objectives started in the 1950s.

It is around this time when the foundations of multi-attribute utility theory are set, as well as the methods for obtaining non-dominated solutions: multiobjective optimization (weighting method (Zadeh, 1963), the ε-constraint method (Marglin, 1967) or compromise programming (Zeleny, 1973)). At the same time, based on Simon's satisficing logic (Simon, 1955), the first work on goal programming is published (Charnes and Cooper, 1957).

One possible application for mathematical programming, and in particular with multiple objectives, is prescribed burning. As discussed earlier, reducing the fuel load on a landscape has been proven beneficial to mitigate risk of wildfires, but this could come into conflict with species relying on old vegetation. Chung (2015) recognizes that little research has been done combining multiple concerns that arise with fuel treatment in an optimization framework. The need for reducing fire risk while taking into account the quality of the habitat is addressed in Rachmawati et al. (2018), where a mathematical programming model for fuel management is shown.

The LP model earlier discussed is *certain* in the sense that all involved parameters are known. Optimization with uncertainty deals with problems in which some of the involved parameters are not known. Section 2.1.1 will briefly introduce the two main techniques used in the field of optimization with uncertainty.

Cost estimation in the design of electrification programs, as mentioned before, is a problem profoundly affected by uncertainty. Consequently, the design of the optimal maintenance network in rural electrifications problems could be solved using mathematical programming with uncertainty. Carrasco et al. (2016), serving as a starting point for the work developed in Chapter 2, develops a mathematical programming model suggesting the key decisions that would lead to the minimum overall cost of the maintenance structure. The review Hernández-Callejo et al. (2019) highlights Carrasco et al. (2016) as one of the few studying the design of a photovoltaic system.

Uncertainty is also present in fuel management problems, but until now, little or nothing has been done in this area related to include it. Nevertheless, this problem is very subject to the uncertainty related to the actual time available to perform the prescribed burns (mainly due to weather conditions).

1.4 Objectives

The general objective is to advance in the treatment of the uncertainty and multiple criteria for sustainable development and disaster management through mathematical programming models. This broad objective is split in several specific objectives applying mathematical programming with uncertainty and/or multiple criteria to several problems related to sustainable development and wildfires management, including a new methodology for multiobjective stochastic programming. These specific objectives are the following:

1. To develop a mathematical programming model for sustainable development including uncertainty, although in a non-explicit manner, in particular for rural electrification.

2. To develop a mathematical programming model for wildfire management with multiple criteria, in particular for prevention and mitigation.

3. To develop a new methodology for multiobjective stochastic programming, including a new concept of solution and a mathematical programming model to obtain it.

4. To develop a multiobjective stochastic programming model for wildfire management, using the new methodology for the fuel management problem.

To achieve the established objectives, the thesis is composed of the following chapters. Chapter 2 considers a rural electrification problem, applying a mathematical programming model in which uncertainty is treated in a non-explicit way. Chapter 3 studies for the first time in this thesis the prescribed burning problem, considering multiple objectives at the time of solving it.

Chapter 4 later develops a risk-aversion methodology for the Multiobjective Stochastic Programming problem, which is applied in Chapter 5 to an extended version of the previously considered prescribed burning problem.

Finally, Chapter 6 summarizes the contributions produced in this thesis, and also indicates some lines of future research.

Chapter 2

Mathematical programming with uncertainty applied to sustainable development. Rural electrification

Contents

2.1 Introduction

The first step towards the goal of studying simultaneously uncertainty and multiple criteria in mathematical programming will be the consideration of uncertainty, with an application to sustainable development.

Section 1.2 discussed the Sustainable Development Goals. Goal 7 was related to energy and the access to it in every country, especially in least developed ones. Within this context, rural electrification is the application depicted in this chapter.

The findings of this chapter have been published in:

> **León, J.**, Martín-Campo, F.J., Ortuño, M.T., Vitoriano, B., Carrasco, L.M. & Narvarte, L. (2019), 'A methodology for designing electrification programs for remote areas', *Central European Journal of Operations Research*, doi:10.1007/s10100-019-00649-6.

2.1.1 Optimization with uncertainty

Consider the *linear programming problem*, which was already mentioned in Section 1.3:

$$\min c^t x$$
$$\text{s.t. } Ax = b$$
$$x \geq 0$$

It is often the case that uncertainty is involved, in either A, b or c. The most common ways for dealing with the uncertainty are *stochastic programming* and *robust optimization*, in which *fuzzy optimization* is also included (Rommelfanger, 2004).

The different ways of treating uncertainty do not respond to the desires of the modeller. Instead they reflect the nature of the uncertainty. If the uncertainty comes with an underlying known or estimated probability distribution, then stochastic programming is used. On the other hand, if uncertainty comes from a lack of precision or semantic uncertainty, then robust optimization is used. Robust optimization does not assume a known (or existing) distribution (Ben-Tal and Nemirovski, 1999; Chen et al., 2007; Klamroth et al., 2017).

Stochastic programming

Stochastic programming is the widest used technique when there are historical data or information to infer a probability distribution. Moreover, usually discrete distributions are used, calling scenarios the different values.

Stochastic programming is used when some of the parameters of the MP problem are not known with certainty, but their distributions are known or can be estimated. The textbook example problem of Stochastic Programming is the Newsvendor Problem.

Example 2.1 (Newsvendor problem)**.** Every morning a newsvendor has to buy newspapers from the publisher to be sold during the day, buying as many as u newspapers (publisher limits). The newspapers are bought at cost c, sold at price q and at the end of the day the unsold ones can be returned, recovering r per newspaper (with $r < c$). The demand for a day is not exactly known, but the newsvendor knows it equals d_ω with probability π_ω, with $\omega \in \Omega$. How many newspapers should be bought if profits wants to be maximized?

A typical way of modelling such problem is given in Model (2.1):

$$\min_{X,Y_\omega} \quad cX - \sum_\omega \pi_\omega \left[qY_\omega + r\left(X - Y_\omega\right)\right] \tag{2.1a}$$
$$\text{s.t.} \quad X \leq u \tag{2.1b}$$
$$Y_\omega \leq d_\omega \qquad \forall \omega \in \Omega \tag{2.1c}$$
$$Y_\omega \leq X \qquad \forall \omega \in \Omega \tag{2.1d}$$
$$X, Y_\omega \in \mathbb{Z}^{\geq 0} \qquad \forall \omega \in \Omega \tag{2.1e}$$

X is the first-stage decision variable, representing the amount of newspapers bought, and Y_ω is the number of newspapers sold, which depends on the scenario that holds. The objective function (2.1a) has a deterministic part (cX, the cost of buying the newspapers) and an uncertain part, which depends on which demand scenario occurs. Equation (2.1b) limits the amount of newspapers that can be bought, whereas Equations (2.1c) and (2.1d) establish that the papers sold are less than the demand and the amount bought. Finally, Eq. (2.1e) sets the domain of the constraints.

Among the most common problems studied in stochastic programming are *recourse problems*. In these problems, a set of first stage decisions has to be made considering the different scenarios that could occur. For each of the scenarios, an independent set of actions is decided, which could be taken in case that scenario actually occurs.

It is worth noting that second-stage (or later) decisions are not necessarily taken, even if the scenario they belong to occurs. The only decisions that are intended to be enforced are the ones in the first stage. Second-stage decisions are considered in stochastic programming as a way of including the uncertain future payoffs at the present time. Once the first level of uncertainty is revealed, and knowing which scenario does hold in the second stage, another optimization problem should be solved starting from the current scenario.

For this reason, how to measure second-stage decisions in the objective function is not immediate. In Model (2.1) probabilities of the scenarios were used to weigh them, but risk-averse options can be used.

For a complete introduction to stochastic programming the reader is suggested the work Birge and Louveaux (2011).

Robust optimization

According to Ben-Tal et al. (2009), the first reference of robust optimization is Soyster (1973), and since 2000 the "area is witnessing a burst of research activity in both theory and applications".

Definition 2.1 (Uncertain linear programming problem)**.** Consider the uncertain linear programming problem, in which the parameters $\tilde{A}, \tilde{b}, \tilde{c}, \tilde{d}$ are not known exactly:

$$\min_x \tilde{c}^t x$$
$$\text{s.t. } \tilde{A}x \leq \tilde{b}$$

Definition 2.2 (Uncertainty set)**.** The possible values for the uncertain parameters is called *uncertainty set* and is denoted with $\mathcal{U}$.

One of the concerns of robust optimization is how feasibility is affected by uncertainty. That is, given x, does the constraint $\tilde{A}x \leq \tilde{b}$ hold for every possible value of $\tilde{A}, \tilde{b} \in \mathcal{U}$?

Definition 2.3 (Robust feasible)**.** A solution is *robust feasible* if $\tilde{A}x \leq \tilde{b}$ for all possible values of $\tilde{A}, \tilde{b} \in \mathcal{U}$.

After guaranteeing a solution is feasible however uncertainty reveals itself, a solution is sought that performs well for any uncertainty realization. For doing that, the robust counterpart problem is solved.

Definition 2.4 (Robust counterpart)**.** Let the *robust counterpart* of the problem given in Definition 2.1 be

$$\begin{aligned} &\min_{x,t} t \\ &\text{s.t. } t \geq c^t x,\ Ax \leq b \quad \forall (A,b,c) \in \mathcal{U} \end{aligned}$$

Given an optimal solution (x^*, t^*) of the robust counterpart it is guaranteed that for any realization $(\tilde{A}, \tilde{b}, \tilde{c}) \in \mathcal{U}$:

1. x^* is feasible, that is, $\tilde{A}x^* \leq \tilde{b}$.
2. t^* is an upper bound of the objective value of x^*. That is, $\tilde{c}^t x \leq t^*$.

While this approach might seem pessimistic, is a common methodology for problems in which the uncertainty does not have an associated probability function, or when risk-aversion is of the uttermost importance (for instance in engineering, where the violation of a constraint could be catastrophic).

For a deeper understanding of robust optimization the reader is directed to the book Ben-Tal et al. (2009). A recent review of the field is written in Gabrel et al. (2014).

The two most common ways for dealing with uncertainty in mathematical programming have been introduced in this section. Another way for dealing with uncertainty is used in this chapter (see Section 2.2.3) for solving a rural electrification problem.

2.1.2 Sustainable development: rural electrification

Human development is a concept that has evolved over time, increasingly incorporating concepts beyond the purely economic. On the UN General Assembly of 2015, a set of Sustainable Development Goals (SDGs) was debated and accepted by the participating countries for the period 2015-2030 (Division for Sustainable Development (2015)). The 7th of the 17 established goals, "ensure access to affordable, reliable, sustainable and modern energy for all", contains the targets proposed related to energy:

7.1 by 2030 ensure universal access to affordable, reliable, and modern energy services

7.2 increase substantially the share of renewable energy in the global energy mix by 2030

7.3 double the global rate of improvement in energy efficiency by 2030

7.a by 2030 enhance international cooperation to facilitate access to clean energy research and technologies, including renewable energy, energy efficiency, and advanced and cleaner fossil fuel technologies, and promote investment in energy infrastructure and clean energy technologies

7.b by 2030 expand infrastructure and upgrade technology for supplying modern and sustainable energy services for all in developing countries, particularly LDCs (least developing countries) and SIDS (Small Island Developing States).

Therefore, energy is currently considered one of the basic services, and governments are facing the challenge of providing it, even in remote and sparse areas, in a sustainable way. In rural areas off-grid, where weather conditions are suitable, photovoltaic electrification is a good option to reach these objectives.

Kanagawa and Nakata (2008) analyze quantitatively the access to electricity in rural areas of developing countries showing the importance of providing lighting to vulnerable people living in developing countries. Gómez-Hernández et al. (2019) propose a set of indicators to evaluate different rural electrification plans and apply them to a case study in Mexico.

For some years, the so-called photovoltaic rural electrification (PVRE) programs are being implemented in developing countries as an alternative to the grid, whose installation is too expensive, and to fossil fuels, which are nonrenewable, dirty, expensive, and difficult to be stored in safe conditions.

To achieve the objective of ensuring access to energy, these PVRE programs should include not only the installation of the systems but also support maintenance and access to spare parts when needed. In this way, these programs involve a service over time to beneficiaries.

This kind of programs is frequently developed in the Asian and African continents. Ellegård et al. (2004), Wamukonya (2007), Lemaire (2009) and Lemaire (2011), among others, review the effectiveness in the African continent by analyzing different countries such as Zimbabwe, Uganda, Ghana, etc., where solar home system based rural electrification programs were implemented. On the other hand, Sharif and Mithila (2013) and Borah et al. (2014) analyze the rural electrification using solar home systems (SHSs) in Bangladesh and India, respectively. Reports from some institutions, as the World Bank (see Cabraal et al. (1996)) and International Finance Corporation (International Finance Corporation (2012)) present different case studies where the photovoltaic technologies have been applied in developing countries and their corresponding impact.

Governments or Development Agencies try to facilitate these programs, but usually, economic issues or lack of resources make it impossible to guarantee the success of such projects and then, public-private-partnerships turn into a possible alternative. The project investments are normally taken from the two parts, where the private energy service company (ESCO) is in charge of managing the planning operations involved, and the Government, besides providing the corresponding public investments, is responsible for designing the PVRE program in a fee-for-service model. In this model, customers do not become owners of the systems; instead, the company continues to charge them a fee for keeping the systems in operation.

Fees are usually calculated according to what users pay for traditional lighting (candles, kerosene, etc.) to ensure access as well as the change to the new system. However, this quantity in most of the cases is not enough to cover the ESCO operations and maintenance costs (see Carrasco, Narvarte and Lorenzo (2013)) having to be subsidized by the Government or by Development Agencies. The cause of high maintenance costs in PVRE is the decentralization of the service (remote and sparse areas). As a result of these over costs, many PVRE programs have failed because ESCOs have abandoned the programs due to financial imbalances, thus not achieving the goal of ensuring access to affordable, reliable, sustainable and modern energy for all (see Chaurey and Kandpal (2010) and van der Vleuten et al. (2007) for reviews). Garcia-Bernabeu et al. (2016) propose a multicriteria approach to obtain a fair price to investors constructing photovoltaic power plants, via government support. Domenech et al. (2019) use a multicriteria approach as well, assisting the promoters of wind-photovoltaic electrification projects.

For this reason, designing a PVRE program requires good models to estimate accurately these costs and to support decisions of sizing the maintenance systems for the ESCO at the time of bidding, and especially for the Government for designing the fee-for-service scheme. Maintenance and operation costs are unknown and uncertain depending on different aspects such as the geographical density of the systems, their reliability, road accesses, local costs, etc.

Although there are many studies on the cost estimation for the reliability of system parts, operational costs are not frequently studied in the literature. Carrasco et al. (2016), which serves as a starting point for this chapter, develops a mathematical programming model to obtain the key decisions that would lead to the minimum overall cost of the maintenance structure. An important drawback of that model is that it needs very precise data of the installed systems which are not usually available before operating and that it requires long runtimes as well. The review Hernández-Callejo et al. (2019) highlights Carrasco et al. (2016) as one of the few studying the design of a photovoltaic system.

2.1.3 Structure of chapter

The objective of this chapter is the consideration of not-explicit uncertainty in a rural electrification problem, using mathematical programming. In addition to that, an expert system to estimate the maintenance system cost will be obtained, to be used for designing the PVRE conditions or for suggesting under which conditions the participation on an electrification program is profitable before the program is already running. A statistical approach can be an option to obtain such a procedure, but usually, there are not enough historical cases into the program to estimate costs from them. The mathematical programming model will be used for obtaining a larger data set formed by synthetic cases based on simulated provinces.

The next methodology, whose aim is obtaining an easy-to-use model for maintenance cost estimation from a limited number of real cases, is presented:

1. A mixed-integer linear programming model is calibrated with the real data available, in order to be useful to estimate costs and size the maintenance systems for new cases with similar

conditions to the initial ones.

2. Several simulated cases are created based on the original ones (keeping similar conditions) whose costs and maintenance system sizes are obtained from the mathematical programming model.

3. A rule-based expert system is obtained from the enlarged data set which will be useful for cost estimation before the program being run.

The chapter is organised as follows. Sections 2.2 and 2.3 describe the problem in detail as well as the methodology proposed for parameter estimation and validation. Section 2.4 shows the proposed mathematical model and its application. Finally, in Section 2.5 the rule-based expert system is developed and an illustrative example of application is represented.

2.2 Problem description

In this section, general aspects of PVRE programs are discussed, being particularized for a Moroccan PVRE whose data are known in detail (provided by the ESCO Isofotón). It is described how its characteristics are taken into account in the models developed later.

2.2.1 PVRE programs

PVRE programs are based on the installation and maintenance of SHSs. These systems include several components to transform and store energy and provide home services. There are different schemes depending on the services required. If home appliances are considered necessarily an inverter must be included in the basic requirements as well as bigger batteries, whereas if only lighting services are considered inverter will not be included (see Fig. 2.1). PVRE programs in developing countries are usually focused on the most basic service that is lighting. In this case, SHSs are composed of solar panels, a solar charge controller, a battery and lamps.

Once the SHS has been installed, operations carried out by ESCOs can be classified into three groups:

- Collecting fees: Since a fee-for-service scheme is designed, the customers must pay monthly fees. These fees can be paid at home, if a visit is carried out during the month, at the ESCO agencies which are usually located in main villages, or in some meeting points that can attract the population of the area (markets/souks).

- Preventive maintenance: To avoid blackouts due to failures of some parts of the SHS, preventive maintenance operations must be carried out. The component to be revised more frequently is the battery, which should be filled with distilled water once every six months. This determines the period between preventive maintenance operations.

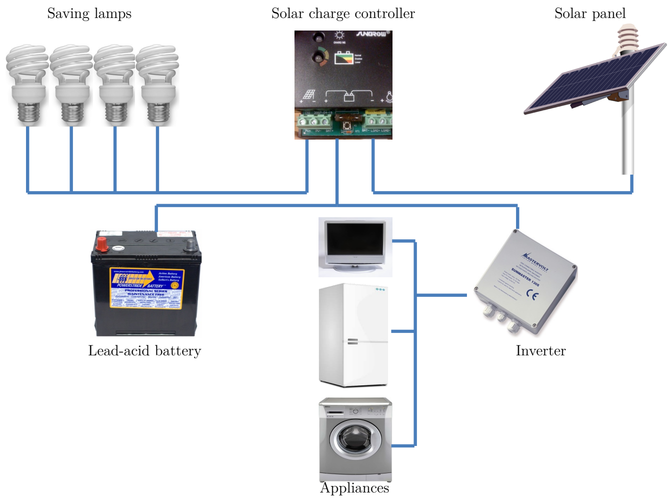


Figure 2.1: General solar home systems

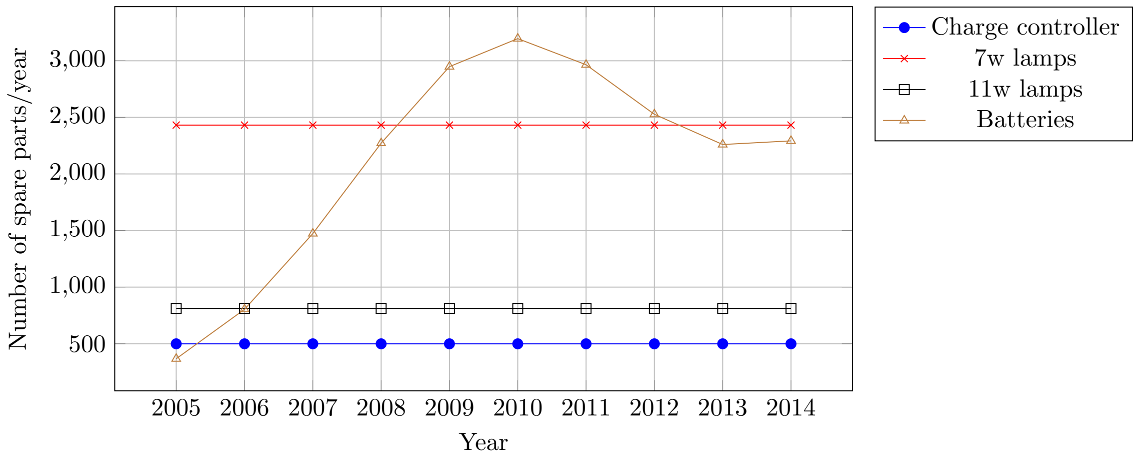


Figure 2.2: Number of spare parts replaced in a PVRE program of 13,600 SHSs along 10 years (Carrasco, Narvarte, Peral and Vázquez (2013))

- Corrective maintenance: Despite preventive maintenance, some parts may fail. These events are subject to uncertainty, as it can never be known in advance when they are going to occur. Solar panels failures are extremely rare. Charge controllers and lamps exhibit a constant failure rate over time, but batteries usually exhibit an increasing failure rate caused by wearing out. Figure 2.2 shows the evolution along 10 years of spare parts consumed in a PVRE program in Morocco implemented in several provinces with 13,600 SHSs (Carrasco, Narvarte, Peral and Vázquez, 2013). These SHSs were formed by battery, charge controller, one 11w lamp and three 7w lamps.

When designing the PVRE program, governments must include precise conditions about the maintenance service to ensure a quality level. At least it should be included:

- A maximum time to respond for corrective maintenance by the ESCO.

- A minimum number of preventive maintenance operations per year.
- The fees to be paid by customers.
- A maximum time to pay fees (if it is exceeded, the ESCO is allowed to dismount the SHS).
- Require the ESCO to maintain a minimum local structure to carry out the operation and maintenance service.

2.2.2 The Moroccan program

The models proposed in this chapter are based on the assumptions and data available of a real PVRE program in Morocco, the so-called *Programme d'Electrification Rurale Global* (PERG). This program was launched in 1995 when the rural electrification rate (TER) in Morocco was only 18%. The PERG implementation was based on two modes of electrification: connection to the interconnected network for the majority of villages, and decentralized rural electrification (mainly photovoltaic equipment) to remote areas of the network or sparsely populated. At the end of 2013, 37,099 villages corresponding to 2,027,120 households were electrified through networks; 51,559 homes in 3,663 villages equipped with photovoltaic kits; and the TER reached 98.51%.

The models have been developed in general terms in order to be easily adapted to other programs, but the authors are aware that some differences can arise in other countries. The basic assumptions are as follows (a more exhaustive discussion can be found in Carrasco et al. (2016)).

- The maximum time to respond for corrective maintenance by the ESCO is 48 hours.
- The minimum number of preventive maintenance operations by year in an SHS is 2.
- The PVRE program is subdivided by provinces, which are managed separately.
- The SHSs are scattered in some minor villages, with the number of SHSs in each village known.
- Villages are scattered in a rural community, with a main center. The average distance from the main center to the villages is known, but the precise location of each village is not provided. It will be assumed that always go from the main center to the villages and back, with length the average distance to a village.
- Location of main centers of rural communities is known with exactitude.
- Meeting points of the population at local markets, which in Morocco are known as souks, are weekly programmed. It is known in which rural communities the meetings are held and the weekday.
- Burnt out lamps are replaced by customers in the souks or agencies.

- Each province will always have a main agency, with a head of the agency, administrative staff and technicians. The head of the agency will have a car available for his own use that can operate as a reserve car if the operative vehicles are on maintenance.
- Other secondary agencies opened in the same province will only include technicians.
- Each car in operation will work with 2 technicians to carry out the maintenance operations.
- Service given in souks will be provided by one of the technicians if an agency is not located in the rural community where the souk is being held, and the other one can develop maintenance operations in villages of the rural community at the same time.
- There is a fixed time to be spent on souks.
- Workday of technicians is fixed, which cannot be exceeded by the daily work.

2.2.3 Uncertainty treatment

Corrective maintenance has to be provided within 48 hours of failure. This need brings uncertainty to the problem of sizing and locating a maintenance structure: how does the ESCO locate its agencies and plans its schedule without knowing when will the systems fail?

Robust optimization does not seem a viable option. For applying robust optimization, an uncertainty set should be defined, including as many uncertain situations as needed. Given that there are numerous situations extremely pessimistic, and likely with low probability, the robust counterpart problem solved would yield a solution with too many agencies located. Such a solution is not realistic, as an ESCO will not consider to install an agency in every province to be able to respond to a disproportionate number of malfunctions in one day, as this would vastly increase the costs.

Another idea could be doing a scenario representation, considering each day and failure of an SHS. Unfortunately, this is not viable given the dimensions of the problem (there are provinces with thousands of systems installed). For this reason, stochastic programming has to be discarded as well.

However, the number of expected corrective maintenance operations is known. The following considerations will be incorporated into the models to treat the uncertainty and approximate the cost and system sizing:

- The expected number of corrective maintenance operations is set at the maximum observed value of operations required during a year (in our case, according to Fig. 2.2, it is the sixth year).
- The number of SHSs of a rural community that will be visited annually is estimated as the total number of SHSs multiplied by 2 (since 2 visits per year are required for preventive maintenance) plus the expected number of corrective maintenance operations in a year.

- The optimization model does not consider the cost of spare parts since that cost is included as a fixed number, estimated in advance.

Some additional conditions on preventive maintenance are included to approximate the solution and the cost. One of these conditions is to set a maximum time between two visits to a rural community, assuming that it will be necessary to visit the community in that period either for preventive or corrective maintenance. Note that this period will be less than the already six months required for preventive maintenance. This period will be introduced in the optimization model as the planning period.

Another assumption will be that visits to rural communities for preventive maintenance will always depart from the local agency, preventing connections between communities. This assumption is made to capture the effect that corrective maintenance has to be provided within 48 hours, often forcing communities to be visited directly from the agencies.

2.3 Methodology

The main objective of this chapter is obtaining an expert system to estimate the maintenance system cost, to be used for designing the PVRE conditions or for suggesting under which conditions the participation on an electrification program is profitable before the program is implemented. If many provinces were already participating in the program it could be possible to obtain simple rules to forecast the cost based on statistical methods. However, the program is generally defined without sufficient information, at most based on some pilot programs with few years in operation.

The methodology developed to obtain the expert system is as follows:

1. Calibrate a mathematical programming model able to size the maintenance system and estimate its cost, based on the information provided by the available data.

2. Create synthetic realistic cases modifying the provinces' features and obtaining the estimated cost for the new cases using the mathematical programming model.

3. Obtain the rule-based expert system based on the information of this enlarged dataset using statistical methods.

The optimization model and the expert system will be used later in the opposite sense, i.e., the expert system will be used initially to design the PVRE program or to make the decision to participate in an already designed program. Afterward, the optimization model will be used for designing the maintenance system once more detailed information about the number of SHS, location, etc., is available.

A summary of the steps of the methodology followed to obtain the models is described below, developed in depth in the following sections illustrated on the application to the Moroccan program.

2.3.1 Mathematical programming model

The methodology followed to calibrate the model is:

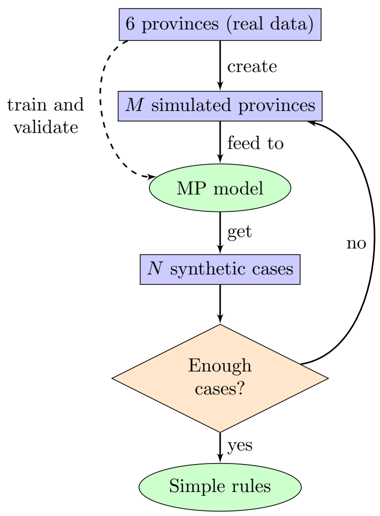
1. Collecting data of representative PVRE programs in operation in terms of number of SHSs, the extension of the province, etc. This includes real (or approximate) costs and structure to be functioning: number of agencies, location, number of cars and technicians.

2. Calibration: Running the model with this data and comparing the obtained costs with the original ones. Some of the model parameters will be fixed. The model is also run with these data but without including anything about the structure currently functioning to know if a better configuration is possible.

3. Validation: Running the model with data from a set of new validation provinces.

2.3.2 Enlarged dataset and rule-based expert system

After developing the mathematical programming model, simpler rules are sought. The steps taken are schematized on Fig. 2.3.



1. A set of simulated realistic cases are created based on the original real data. They are deemed realistic because they share some characteristics with the real ones, instead of generating completely random ones.

2. The mathematical programming (MP) model is used to obtain the optimal infrastructure and cost for each of the new realistic provinces, obtaining an enlarged set of synthetic cases.

3. A model is trained with the enlarged data set for predicting costs without having such detailed information about the provinces as the mathematical programming model requires.

Figure 2.3: Steps followed for obtaining simple rules

2.4 Mathematical programming model

This section presents the mathematical model, whose aim is to determine the necessary local structure and to estimate the costs involved in the operations on a given province. Parts of the development of the model were initially shown in Carrasco et al. (2016), but validation was not performed there. Given the relevance of this model for the development of the rule-based expert system, it is shown below.

Indices sets

$\mathbf{R} = \{1, \ldots, R\}$ Rural communities in the province

$\mathbf{V} = \{1, \ldots, V\}$ Villages in the province

$\mathbf{D} = \{1, \ldots, D\}$ Days in the planning period (maximum time between 2 visits to a rural community)

Parameters

tp	Estimated time between two consecutive visits to a village
$nvil_r$	Number of villages in each rural community $r \in \mathbf{R}$
nv_r	Number of villages of rural community $r \in \mathbf{R}$ to be visited in the planning period. $nv_r = nvil_r \frac{D}{tp}$
$a_{v,r}$	0-1 parameter, 1 if village $v \in \mathbf{V}$ belongs to rural community $r \in \mathbf{R}$, 0 otherwise
$nshs_v$	Total number of SHSs in each village $v \in \mathbf{V}$
$fshs$	Expected percentage of SHS to fail in one year
$nsys_v$	Proportional part of the total SHSs to be visited during the planning horizon. $nsys_v = \left\lceil (2 + fshs) \cdot nshs_v \frac{D}{365} \right\rceil$
tas	Expected time needed to assist an SHS
tw	Technicians' workday's length
nc	Upper bound of the number of vehicles to be distributed among the local agencies
$b_{r,d}$	0-1 parameter, 1 if there is a souk in rural community $r \in \mathbf{R}$ on day $d \in \mathbf{D}$, 0 otherwise
$tb_{r,d}$	Time to be spent in a souk in rural community $r \in \mathbf{R}$ on day $d \in \mathbf{D}$
$dist_{r,r'}$	Distance between the main rural centers $r, r' \in \mathbf{R}$
$td_{r,r'}$	Expected time needed for the displacements between the main rural centers $r, r' \in \mathbf{R}$
$dist'_r$	Average of distances from the main rural center $r \in \mathbf{R}$ to each of its villages
$dist''_r$	Average of distance allocated to each SHS in rural community $r \in \mathbf{R}$ (go and return). Introduction of this parameter avoids using an index for villages, $dist''_r = \frac{2 dist'_r nv_r}{\sum_{v \in \mathbf{V}} a_{v,r} \, nsys_v}$
td'_r	Average time for displacements inside rural community $r \in \mathbf{R}$

td''_r Average time needed to assist an SHS in rural community $r \in \mathbf{R}$ (including displacement time into the rural community). $td''_r = tas + \frac{td'_r nv_r}{\sum_{v \in \mathbf{V}} a_{v,r}\, nsys_v}$

cfp Provincial cost including head of the province (salary, telephone and fixed cost of his car) and administrative staff

cmc_r Cost for locating a local agency on a rural community $r \in \mathbf{R}$. It includes the rent and maintenance cost of the agency

cnc_r Cost related to vehicles assigned to an agency located in rural community $r \in \mathbf{R}$. It includes the fixed cost for vehicles, two technicians' salary and telephone cost (estimated)

ctr Cost of traveling per unit of distance (basically fuel)

Variables

BL_r 0-1 variable, 1 if a local agency is located on rural community $r \in \mathbf{R}$, 0 otherwise

$BA_{r,r'}$ 0-1 variable, 1 if an agency located on $r \in \mathbf{R}$ assists rural community $r' \in \mathbf{R}$, 0 otherwise

$BR_{r,r',d}$ 0-1 variable, 1 if rural community $r' \in \mathbf{R}$ is visited from the agency located on rural community $r \in \mathbf{R}$ on day $d \in \mathbf{D}$, 0 otherwise

$NS_{r',d}$ Number of assisted SHSs in rural community $r' \in \mathbf{R}$ on day $d \in \mathbf{D}$

NR_r Number of vehicles assigned to the local agency located on rural community $r \in \mathbf{R}$

$NT_{r,r',d}$ Number of vehicles travelling to rural community $r' \in \mathbf{R}$ from the agency located on rural community $r \in \mathbf{R}$ on day $d \in \mathbf{D}$

$TM_{r,r',d}$ Time spent in rural community $r' \in \mathbf{R}$ coming from the agency located on rural community $r \in \mathbf{R}$ on day $d \in \mathbf{D}$

Objective function and constraints

$$\min\ cfp + \sum_{r \in \mathbf{R}} cmc_c BL_r + \sum_{r \in \mathbf{R}} cnc_c NR_r + 2ctr \sum_{r,r' \in \mathbf{R}} \sum_{d \in \mathbf{D}} dist_{r,r'} NT_{r,r',d}$$
$$+ ctr \sum_{r \in \mathbf{R}} dist''_r \sum_{v \in \mathbf{V}} a_{v,r} nsys_v$$

s.t.:

$$\sum_{r \in \mathbf{R}} BL_r \geq 1 \tag{2.2}$$

$$\sum_{r \in \mathbf{R}} BA_{r,r'} = 1 \quad \forall r' \in \mathbf{R} \tag{2.3}$$

$$\sum_{r\in\mathbf{R}} BR_{r,r',d} \leq 1 \quad \forall r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.4}$$

$$\sum_{r\in\mathbf{R}} BR_{r,r',d} \geq b_{r',d} \quad \forall r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.5}$$

$$BR_{r,r',d} \leq BA_{r,r'} \leq BL_r \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.6}$$

$$BA_{r,r'} \leq \sum_{d\in\mathbf{D}} BR_{r,r',d} \quad \forall r, r' \in \mathbf{R} \tag{2.7}$$

$$BL_r \leq NR_r \leq nc\, BL_r \quad \forall r \in \mathbf{R} \tag{2.8}$$

$$\sum_{r'\in\mathbf{R}} NR_{r'} \geq \sum_{r'\in\mathbf{R}} b_{r',d}\,(1 - BL_{r'}) \quad \forall d \in \mathbf{D} \tag{2.9}$$

$$NT_{r,r',d} \leq NR_r \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.10}$$

$$BR_{r,r',d} \leq NT_{r,r',d} \leq nc\, BR_{r,r',d} \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.11}$$

$$\sum_{d\in\mathbf{D}} NS_{r',d} = \sum_{v\in\mathbf{V}} a_{v,r'}\, nsys_v \quad \forall r' \in \mathbf{R} \tag{2.12}$$

$$NS_{r',d} \leq \sum_{v\in\mathbf{V}} a_{v,r'}\, nsys_v \sum_{r\in\mathbf{R}} BR_{r,r',d} \quad \forall r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.13}$$

$$TM_{r,r',d} \geq tb_{r,d}\,(1 - BL_r) - M\,(1 - BR_{r,r',d}) \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.14}$$

$$TM_{r,r',d} \geq td''_r\, NS_{r',d} - M\,(1 - BR_{r,r',d}) \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.15}$$

$$\sum_{r'\in\mathbf{R}} (td_{r,r'}\, NT_{r,r',d} + TM_{r,r',d}) \leq tw\, NR_r \quad \forall r \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.16}$$

$$TM_{r,r',d} + 2td_{r,r'}\, NT_{r,r',d} \leq tw\, NT_{r,r',d} \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.17}$$

$$BL_r, BA_{r,r'}, BR_{r,r',d} \in \{0, 1\} \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.18}$$

$$NR_r, NS_{r',d}, NT_{r,r',d} \in \mathbb{Z}^{\geq 0} \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.19}$$

$$TM_{r,r',d} \geq 0 \quad \forall r, r' \in \mathbf{R}, \forall d \in \mathbf{D} \tag{2.20}$$

The objective function is composed of province fixed costs, agencies location costs, cars fixed costs (including costs related to the two technicians) and traveling costs.

Equations (2.2) to (2.7) relate to the location of agencies and the rural communities assisted by those. Equation (2.2) forces at least one agency is opened. Equation (2.3) ensures each community is served by one agency. Equation (2.4) limits the visits that can be done to a community on a single day, and Equation (2.5) makes sure communities are visited on souk days. Equations (2.6) and (2.7) relate the variables BA, BL and BR.

Equations (2.8) to (2.11) model the assignment of vehicles to agencies. Equation (2.8) limits the number of vehicles on a community. As markets are held at the same time of the day, a vehicle can only attend one at a time, and so Eq. (2.9) forces the number of vehicles to be at least the maximum number of markets held on communities without agencies on any single day. Equation (2.10) limits the number of vehicles travelling to any rural community from r to the number of vehicles assigned to r. Equation (2.11) links the trips of the vehicles with the binary variable BR.

Equations (2.12) and (2.13) track the number of visited systems. Equation (2.12) enforces the total number of SHSs are visited throughout the planning period and Equation (2.13) forces a community to be assisted on day d in order to be able to visit any systems that day.

Equations (2.14) to (2.17) keep track of time and connect the previous groups. Equations (2.14) and (2.15) set the time spent in a rural community to be the maximum between the time spent in the souk and the time spent carrying out maintenances. Notice that if a local agency is located in a rural community where a souk is being held, it is not necessary to spend time there since the customers can go directly to the agency. Equations (2.16) and (2.17) ensure the workday of technicians is not be exceeded by the daily work (whole or by community). Equations (2.18) to (2.20) set the domain of the variables.

2.4.1 Calibration and validation

Collecting data

The methodology has been applied to the real PVRE program in Morocco described in Section 2.2.2. Into this program, the ESCO Isofotón is operating since 2005 in 9 provinces with more than 13,000 SHSs grouped on them at the time of the data collection. Such provinces are shown on Fig. 2.4. After discarding 3 provinces in which the system was not working properly (PRG requirements were not being met, such as providing corrective maintenance in less than 48 hours), the other 6 provinces were divided into two groups:

- Training provinces: Al Kalaa des Sraghnas (ALK), Azilal (AZI) and Ben Slimane (BSL) were used before on Carrasco et al. (2016) for fixing parameters of the model.
- Validation provinces: Al Haouz (ALH), Beni Mellal (BME) and Errachidia (ERR) are used to confirm the validity of the model, once all the parameters are fixed.

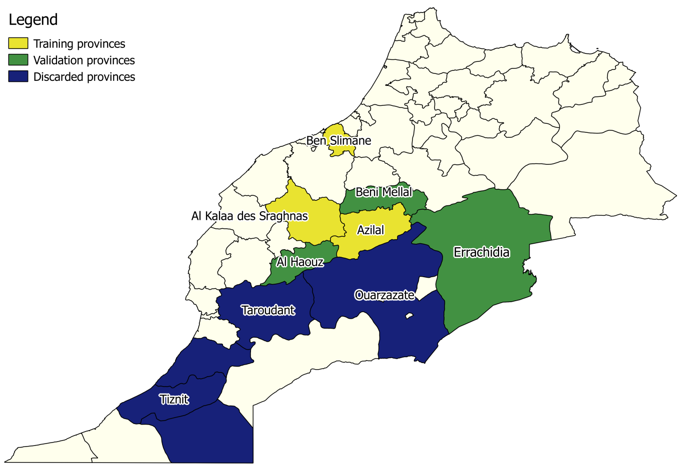


Figure 2.4: Map of Morocco with the provinces where Isofotón operated labelled

The main characteristics of the provinces that will be used hereafter are shown in Table 2.1. Note that the province with more rural communities is Al Kalaa des Sraghnas (55), resulting in an integer programming model with roughly 700,000 constraints and 250,000 variables, one-third of them continuous, another third binary and another third integer.

Table 2.1: Main characteristics of the provinces used for training and validation

	Training			Validation		
Provinces	**ALK**	**AZI**	**BSL**	**ALH**	**BME**	**ERR**
Area (km^2)	10,070	9,800	2,760	7,883	6,638	59,585
Number of SHSs	4,396	1,809	857	862	2,723	959
Density (SHS/km^2)	0.437	0.185	0.311	0.109	0.41	0.016
Rural communities	55	30	9	27	31	29
Villages	1,623	116	51	70	179	106
Souks	17	10	5	2	10	5

Table 2.2 reports the system structure and related costs in the provinces under study. Operation costs have been aggregated into three main groups to compare them with the model results:

- Province fixed costs: fixed costs in each province related to salaries of the head of the agency and administrative staff, fixed cost of the head of the agency's car and communication cost of the main agency.
- Cost related to system sizing: cost of renting buildings for agencies and their functioning (electricity, water, etc.), and cost of vehicles including salaries of the corresponding two technicians working with each vehicle
- Cost related to operation: costs related to the fuel cost of the journeys performed.

It is worth noting that the cost of traveling (fuel and type of vehicles) is province-dependent, as some areas are more mountainous than others and, consequently, different vehicles are used.

Calibration: Training provinces

The second step is to run the model fixing the current system structure, i.e. location of agency and number of technicians hired.

The model has been written in GAMS 23.5.1. and solved using CPLEX 12.2. Table 2.3 reports the obtained results of the training provinces, firstly imposing the implemented structure in reality in order to assess the accuracy of the model; and then without any conditions in order to identify a possible better structure. The last row measures how accurately the mathematical programming model predicts the real costs, which were reported on Table 2.1. Variation is calculated as obtained cost minus real cost, divided by real cost.

The results obtained by the model identify a better structure on the Al Kalaa des Sraghnas province, in which opening a second agency leads to reducing the number of vehicles obtaining

Table 2.2: Real data: System structure implemented in each of the provinces and related costs

	Training			Validation		
Provinces	**ALK**	**AZI**	**BSL**	**ALH**	**BME**	**ERR**
Agencies	1	1	1	1	1	1
Vehicles	4	2	1	1	3	2
Technicians	8	4	2	2	5	3
Fixed costs (€)	14,186	14,186	3,601	14,186	14,186	17,126
System sizing costs (€)	52,689	33,825	15,573	18,379	41,608	23,356
Journey costs (€)	23,280	8,687	1,192	3,008	6,156	12,723
Total yearly cost (€)	**90,155**	**56,698**	**20,366**	**35,573**	**61,950**	**53,205**

Table 2.3: Training results: System structures and costs obtained for training provinces

	Fixing structure			**Model free**		
Provinces	**ALK**	**AZI**	**BSL**	**ALK**	**AZI**	**BSL**
Agencies	1	1	1	2		
Vehicles	4	2	1	3	Same	
Technicians	8	4	2	6	solutions	
Fixed costs (€)	14,186	14,186	3,601	14,186	than fixing	
System sizing costs (€)	52,689	33,825	15,573	43,518	structure	
Journey costs (€)	19,259	10,105	1,218	11,567		
Total yearly cost (€)	86,134	58,116	20,392	69,721		
Variation with data	**-4.46%**	**2.50%**	**0.13%**	**-23.16%**		

a cost reduction of around 20%. On the provinces of Azilal and Ben Slimane the results do not change when infrastructural decisions are not imposed.

These results have been considered good enough by the experts, and valuable since they did not have any tool able to approximate the costs in advance.

Validation: Validation provinces

ESCO Isofotón provided data of three new provinces after having fitted the parameters: Errachidia (ERR), Beni Mellal (BME) and Al Haouz (ALH). The main characteristics of these provinces, which are referred to as validation provinces, were shown in Table 2.1, and the costs arising with the current implemented structure were shown in Table 2.2.

As with the training provinces, the first step is running the developed model mimicking the current structure for each of these provinces, to validate the model. Then, it is run without predefining any structure, in order to look for a better configuration if possible. Results for both of these experiments are shown in Table 2.4.

It can be seen that cost estimation with the real implemented structure results in a maximum of 13.95% of deviation in Beni Mellal. It should be noticed that on the current structure of the provinces Beni Mellal and Errachicia there is one vehicle on each province with only one technician assigned to it, resulting in 3 and 5 technicians hired respectively. An assumption of our model (given by the ESCO) is that two technicians are hired for each vehicle, resulting in an overestimation of one technician for each of these provinces. If the cost of one technician is taken away (4,455.05 €/year), the variation with the original data would be much smaller (6.76% and -1.54%, respectively). Therefore, the model can be considered able to estimate the costs with high accuracy.

Regarding the option of improving costs, the results of Al Haouz are the same in both cases. The agency is moved in Beni Mellal and a new one is opened on Errachidia, showing that the model can provide a better configuration for the maintenance systems.

2.4.2 Application: Economic viability

ESCO Isofotón also reported data about fees and real costs. The fees paid by users in the PERG are 59€/year/SHS, with an initial fee of 70€ (without taxes). The program was planned for 10 years, time for investment amortization and to support the operation and maintenance program. The company claimed that costs are not covered by those fees.

According to the costs provided by the company, the initial cost of this installation minus the initial fee would be 347€, which must be reimbursed in the next 10 years of the program. It will give 34.7€/year/SHS for recovering.

According to raw data of Fig. 2.2, multiplying the number of spare parts by their costs and adding up the costs over the 10 years, the global cost by spare parts replaced is obtained. Dividing by the number of SHSs and the 10 years, it is obtained 23.5€/year/SHS.

Table 2.4: Results of the validation provinces

	Fixing structure			**Model free**		
Provinces	**ALH**	**BME**	**ERR**	**ALH**	**BME**	**ERR**
Agencies	1	1	1	Same than fixing	1	2
Vehicles	1	3	2		2	2
Technicians	2	6	4		4	4
Fixed costs (€)	14,186	14,186	17,126		14,186	17,126
System sizing costs (€)	18,379	46,063	28,811		31,731	30,878
Journey costs (€)	5,275	10,345	11,903		9,132	6,770
Total yearly cost (€)	**37,840**	**70,594**	**56,840**		**55,094**	**54,774**
Variation with data	**6.37%**	**13.95%**	**6.83%**		**-11.14%**	**2.95%**

Adding installation and spare parts cost the result is 58.2€/year/SHS, which is almost the fee that customers are currently paying for the service, even when other costs are not considered. Maintenance costs must be computed to know the uncovered expenses, which will be different for each province. Three data sets will be used: those from the ESCO, those obtained with the model when fixing the structure beforehand and those from the model free.

These results are shown in Table 2.5, where three different costs are calculated:

- Maintenance cost: fixed, system sizing and journey costs, per system.
- Total cost: the sum of the maintenance cost, spare parts, and installation, per system.
- Uncovered expenses: total cost minus income per system, multiplied by the number of systems.

There are important differences between provinces. Figure 2.5 shows that the sparser a province, the higher the maintenance cost. But, anyway, there is a lack of funds in every province enough to produce the undesired effect arising in the PERG companies closing and failing to provide the required service. These results confirm that the model results are in accordance with the reality observed.

2.5 Rule-based expert system

The previous validation of the mathematical programming model shows that the model can be used for accurately predicting the cost and size of the infrastructure needed to perform the maintenance operations. Unfortunately for using the developed mathematical programming a lot of information is needed, such as, the number of rural communities and times needed to go from one another; the number of villages in each rural community and number of installed systems on each of them; or the time needed to visit a village within each rural community.

Table 2.5: Costs for all the studied provinces: costs reported by ESCO, costs obtained by model with fixed structure and costs obtained by model with no predefined structure

		Training			Validation		
	Provinces	**ALK**	**AZI**	**BSL**	**ALH**	**BME**	**ERR**
Maintenance cost (€/SHS/year)	ESCO	20.51	31.34	23.76	41.27	22.75	55.48
	Fixing structure	19.59	32.13	23.79	43.90	25.93	59.27
	Model free	15.86	32.13	23.79	43.90	20.23	57.12
Total cost (€/SHS/year)	ESCO	78.71	89.54	81.96	99.47	80.95	113.68
	Fixing structure	77.79	90.33	81.99	102.10	84.13	117.47
	Model free	74.06	90.33	81.99	102.10	78.43	115.32
Uncovered expenses (€/year)	ESCO	86,638	55,251	19,680	34,884	59,772	52,438
	Fixing structure	82,617	56,669	19,706	37,150	68,416	56,073
	Model free	65,755	56,669	19,706	37,150	52,871	54,007
	Density (SHS/km^2)	0.437	0.185	0.311	0.109	0.41	0.016

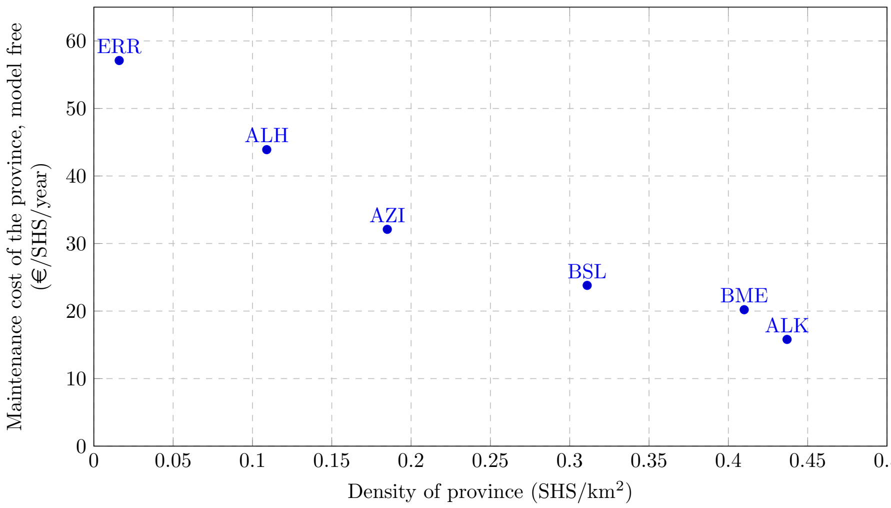


Figure 2.5: Yearly maintenance cost per system and density of each province

All that information will not be available when the program is designed or an ESCO must decide whether to participate in the program. For that reason, it is necessary to obtain simpler rules based on available information that can help with the estimation of size and cost.

Statistics and/or machine learning provide methods for obtaining such simple rules, but, to apply them, a larger number of cases is required. Hence a simulation procedure has been implemented, which is shown below.

2.5.1 Data gathering

For obtaining the aforementioned enlarged data set the next procedure has been followed:

1. A larger data set of simulated realistic provinces is created. To maintain the feasibility of a province, rather than randomly creating new ones, each of the new provinces will be an original one with three of its original characteristics multiplied by a factor. A collection of values for these factors has been defined to obtain different infrastructure sizes than the existing ones, aiming to halve and double the number of systems and areas. For that purpose, the number of systems was multiplied by 0.5 and 2, and since the areas are not an input of the mathematical programming model, a proxy-based on the distances was used instead. If provinces and communities are simplified as circles, their radii should be multiplied by $1/\sqrt{2} \approx 0.7$ and $\sqrt{2} \approx 1.4$ for halving and doubling areas, respectively. Therefore these factors were used for the distances.

   - Number of systems on each village (factors: $0.5, 1, 2$)
   - Mean distance from the center of a rural community to its villages (factors: $0.7, 1, 1.4$)
   - Distances between rural communities (factors: $0.7, 1, 1.4$)

   In other words, a full factorial experiment was designed varying 3 characteristics, each with 3 possible factors, of 6 different provinces. This experiment provides 162 simulated provinces ($6 \cdot 3^3$).

2. The mathematical programming model is run with each of the simulated provinces, limiting the runtime to 2 hours. Only those results with less than 5% of integrality gap are kept, leading to a set of synthetic cases formed by the simulated provinces, coupled with their optimal configurations.

3. A second set of simulated provinces is created, motivated by the results of the first round of synthetic cases. As it can be seen in Fig. 2.6, costs exhibit a clear jump. Trying to reduce such a gap and describe better the area around this jump, provinces with lower costs were multiplied by bigger factors and provinces with higher costs by smaller factors.

   After solving these provinces with the mathematical programming model and runtime limits, a final dataset of 177 synthetic cases was obtained. Each of the new provinces has its information as detailed as the original ones.

Execution for all the provinces accounted for 477.5 hours, or almost 20 days, on a computer with an Intel Core i7 processor and 8gb RAM.

From this dataset, a rule-based expert system will be obtained based on characteristics general enough to be used before the program is run (real or estimated values). A small set of characteristics (explicative variables) will be considered. Such characteristics, which are shown in Table 2.6, are computed from some parameters that also were used in the mathematical programming model (Section 2.4).

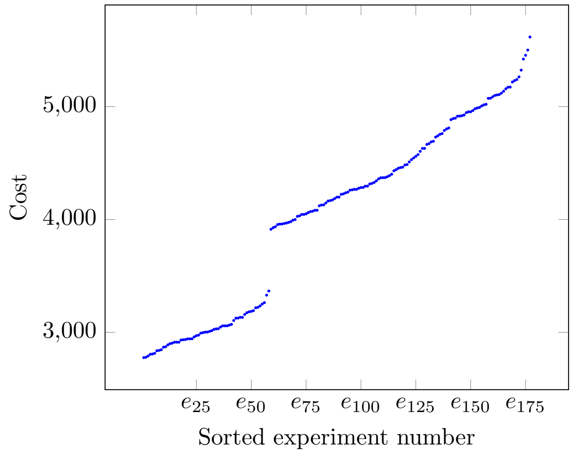


Figure 2.6: Cost of each of the simulated provinces

Table 2.6: Characteristics used as inputs for the rule-based expert system, and their related parameter of the mathematical programming model

Characteristic	Associated parameter
Traveling cost per kilometer	ctr
Number of weekly markets to attend to	$b_{r,d}$
Mean and maximum distance between rural communities	$dist_{r,r'}$
Mean and maximum distance within a rural community	$dist'_r$
Mean and maximum displacement time between rural communities	$td_{r,r'}$
Mean and maximum displacement time within rural community	td'_r
Number of installed systems	$nshs_v$
Number of villages	nv_r
Number of rural communities	$\|\mathbf{R}\|$
Maximum number of systems on a village and on a rural community	$nshs_v, a_{v,r}$
Maximum number of villages on a rural community	$a_{v,r}$

The most direct method to obtain a simple rule is to develop a regression model. Nevertheless, as can be seen in Fig. 2.6, costs show an important shift. That shift might suggest that, before a regression model being fitted, a classification should be performed. Later, a different regression model for each class can be developed, or a dummy variable related to the class can be added.

2.5.2 Classification model

The objective of the classification model is to obtain a simple procedure to determine if a new province should be included in the branch with lower costs or in that one after the jump.

Initially, it was assumed that the most impact variable would be the number of centers to be opened. However, when inspecting the results, it can be seen that the number of vehicles to allocate plays a crucial role in the total cost (Fig. 2.7). This was already observed in the Al Kalaa province: the maintenance cost was cut out by reducing the number of vehicles, even if a new maintenance center was opened. Moreover, regarding Fig. 2.7, a continuity in cost is observed using 2 or 3 vehicles, appearing only discontinuity when changing from one vehicle to multiple vehicles. This number of vehicles does not include the head of the agency's car.

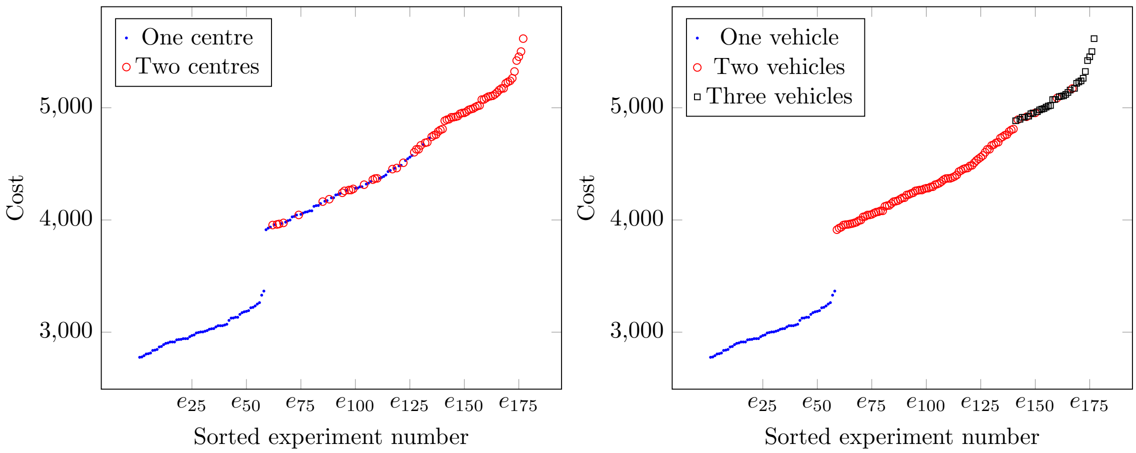


Figure 2.7: Monthly maintenance cost for each of the 177 experiments, sorted by cost and grouped by optimal number of centers (on the left) and grouped by optimal number of vehicles (on the right)

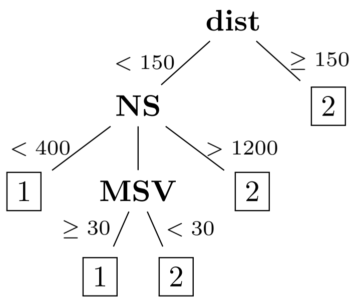
Therefore, the first step for building the expert system is to develop a classification model based on the characteristics previously described to predict if one or multiple vehicles will be needed within the province.

Classification tree has been chosen by its easy interpretation. The obtained tree, trained using all observations, is shown in Fig. 2.8 joined to its confusion matrix. The features on the tree are:

- Mean distance between rural communities, in kilometers (dist)
- Number of installed systems (NS)
- Maximum number of systems on a village (MSV)

The split of the variables MSV is related to the density on a province: for the same number of systems installed, the smaller the maximum number of systems in a village, the greater the dispersion, so it will be more difficult to maintain all the systems with a single vehicle.

The obtained tree, albeit extremely simple and easy to interpret, has great performance: only 3 out of the 177 observations are misclassified (1.69%). Cross-validation was performed to validate the methodology, training 177 different trees leaving each observation out once, and obtaining a mean cross-validation error of 8%.



		predicted	
		1 vehicle	≥ 2 vehicles
observed	1 vehicle	56	2
	≥ 2 vehicles	1	118

Figure 2.8: Classification tree predicting if one vehicle is sufficient (label=1) or more are needed (label=2) for the maintenance of the province, next to its confusion matrix

2.5.3 Regression model

Once the classification model is obtained to predict if a single vehicle will be enough, a linear regression model is sought to predict the maintenance costs including the dummy variable, `multVeh`. Its value will be 1 if more than one vehicle is needed, and 0 otherwise. The rest of the explanatory variables are the same as for the classification tree plus some products of variables included by the possible interaction between them. The cost is obtained for a 4 weeks period (length considered in the mathematical programming model).

The coefficients of the obtained linear regression model are shown in Table 2.7, and the description of the variables in Table 2.8. The adjusted R^2 is 0.9953 when the true class for the number of vehicles is known, and 0.9616 when the number of vehicles class is predicted by the classification tree.

Rounding coefficients and grouping terms, the following rule for estimating cost is obtained:

$$\begin{aligned}\hat{Cost} = 2360 + NV\left(0.4 + distt\left(-0.1 + 2.37ctr\right)\right) + 1.64MSV - 19.03t + \\ + 8.02Mt + 52.83ctr \cdot dist + 1095 multVeh \qquad (2.21)\end{aligned}$$

The cost predicted by this formula is compared with the true cost on Fig. 2.9.

2.5.4 Application: Economic analysis

One of the applications of the developed models is the assessment of a fair user fee. Let T€/year/SHS be the yearly fee users pay. Let C be the maintenance cost for 4 weeks provided by the regression model (Eq. (2.21)). Considering a year with 52 weeks, the cost per system and year

Table 2.7: Coefficients of linear regression model. Description of variables shown in Table 2.8

name	Estimate	SE	tStat	pValue
NV	0.4007	0.021203	18.898	1.9678e-43
MSV	1.6419	0.16187	10.144	3.6789e-19
t	-19.028	0.97193	-19.577	3.296e-45
Mt	8.0224	0.38215	20.993	7.8328e-49
distt × NV	-0.10326	0.014101	-7.3228	9.6231e-12
ctr × dist	52.826	4.08	12.947	4.8945e-27
ctr × distt × NV	2.3706	0.25945	9.1371	2.0227e-16
multVeh	1096.5	20.911	165.23	7.1038e-188
constant	2358.8	19.908	118.49	8.0861e-164

Table 2.8: Description of variables appearing on linear regression model shown in Table 2.7

name	description
NV	Number of villages
MSV	Maximum number of systems on a village
t	Mean displacement time between rural communities (min.)
Mt	Maximum displacement time between rural communities (min.)
dist	Mean distance between rural communities (kms.)
distt	Mean distance within a rural community (kms)
ctr	Traveling cost (euros/km)
multVeh	If more than one vehicle is used

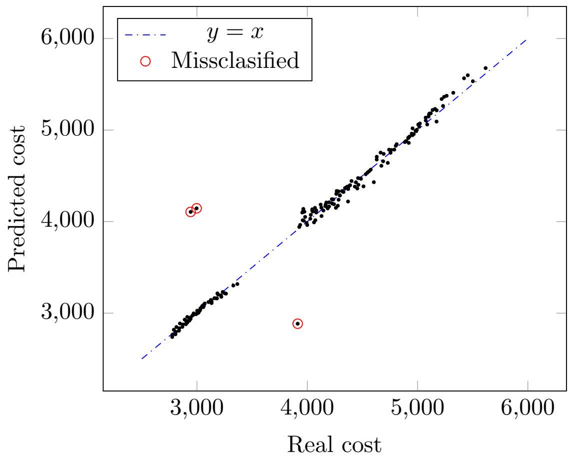


Figure 2.9: Real and predicted cost given by linear regression for each province, including misclassified observations by the classification tree

is $13\frac{C}{NS}$, where NS is the number of installed systems. Adding to this value the installation costs and spare parts given in Section 2.4.2, the annual fee to cover expenses should be at least

$$T \geq 13\frac{C}{NS} + 23.5 + 34.7 = 13\frac{C}{NS} + 58.2 \tag{2.22}$$

As an example, for Azilal case, whose variables values are shown in Table 2.9, the classification tree predicts needing multiple vehicles. Once it has been classified, the linear regression model (Eq. (2.21)) gives an annual cost C of 4475.34€. And finally, from equation (Eq. (2.22)), the yearly fee T should be at least 92.91€. It can be compared with data included in Table 2.5 of Section 2.4.2, where this value was 89.54€ and 90.33€ using ESCO data and mathematical programming model results, respectively.

Table 2.9: Characteristics of a province. All of these quantities can be easily estimated with expert knowledge

name	value	description
NS	1676	Number of installed systems
MSV	56	Maximum number of systems on a village
t	96.724	Mean displacement time between rural communities (min.)
Mt	267	Maximum displacement time between rural communities (min.)
dist	96.724	Mean distance between rural communities (kms)
distt	7.603	Mean distance within a rural community (kms)
ctr	0.093	Traveling cost (euros/km)
NV	116	Number of villages

The expert system can also be used to estimate the effects in case of a variation in the number of systems. Let assume the number of clients from Table 2.9 is doubled:

1. The classification tree still yields multiple vehicles in this case.
2. The regression model does not explicitly include the variable NS, but there are two parameters subject to change:
    - If the number of systems doubles because in every village there are twice as many clients, the parameter MSV should double as well. In this case, the new monthly cost C is 4567.18€, and the lower bound of the fee becomes $T = 75.91$€.
    - If the number of system doubles because the program reaches twice as many places, the parameter NV should double as well. In this case, the new monthly cost C is 4627.94€, and the lower bound on the fee becomes $T = 76.15$€.

Both cases result in a similar decrease in cost per installed system.

2.6 Discussion

A methodology for estimating costs and sizing maintenance systems has been presented in this chapter. The methodology is based on a mathematical programming model and a rule-based expert system. The mixed-integer linear programming model allows to obtain accurate costs and design of the system for a well-described program, or for simulated cases to obtain a larger data set. The rule-based expert system, comprised of a classification tree and a linear regression model, estimates the maintenance cost with little information from a dataset of real or realistic cases. It can be used for designing the program, i.e. knowing the fees to be established in a program, or for an ESCO to decide about participating in the program or not. If the ESCO chooses to participate in the program, already with extended information, the mathematical programming model can obtain the optimal structure of the maintenance system to minimize costs.

The methodology has been applied to and validated with a program in Morocco. The mathematical programming model is parameterized being suitable to be applied in other PVRE programs, and the rule-based model is based on general parameters. Therefore, the full methodology is useful for PVRE programs supporting design decisions and providing realistic cost estimations, in order to achieve the desired Development Goal.

An important limitation of the obtained rule-based expert system is that it cannot be directly applied to other countries. The reason for this is that the system is missing information on the economy of the region: how does the cost of opening a new agency relate to salaries or displacement costs? In this case, a suggested approach is generating new provinces with the mathematical programming model and then retrain the necessary models.

Uncertainty has been included in a non-explicit way since it is not possible to explicitly represent all possible scenarios. However, when a probability distribution can be inferred for the uncertainty, stochastic programming should be used. Chapters 4 and 5 will be focused on stochastic programming with risk aversion, suitable for protecting against the most unfavorable situations.

Chapter 3

Mathematical programming with multiple objectives applied to wildfire management. Fuel management

Contents

3.1 Introduction

Section 1.2 introduced wildfires, and mentioned one possibility for mitigating its effect: fuel modification. This chapter discusses mathematical programming with multiple objectives and a model is developed to aid in the prescribed burning problem.

The findings of this chapter have been published in:

León, J., Reijnders, V.M.J.J., Hearne, J.W., Ozlen, M. & Reinke, K.J. (2019), 'A Landscape-Scale Optimisation Model to Break the Hazardous Fuel Continuum While Maintaining Habitat Quality', *Environmental Modeling & Assessment* **24**(4), 369–379. doi:10.1007/s10666-018-9642-2.

3.1.1 Mathematical programming with multiple objectives

Multicriteria decision making (MCDM) is a field worth of consideration when studying real world problems. Such is the case that different techniques have been recently used for solving problems as varied as: disaster management (Gutjahr and Nolz, 2016; Ferrer et al., 2018), engineering (Sun et al., 2018), finance (Karsu and Morton, 2015; Angilella and Mazzù, 2015), forest planning (Fotakis, 2015), healthcare (Guido and Conforti, 2017), location of waste facilities (Eiselt and Marianov, 2015), police districting (Liberatore and Camacho-Collados, 2016), route planning (Bast et al., 2016), train scheduling (Samà et al., 2015) or urban planning (Della Spina et al., 2015; Carli et al., 2018).

This situation, in which multiple conflicting objectives have to be optimized, has led to the definition of different solution concepts and methodologies. Depending on the problem and the type of solution thought, a specific methodology should be applied.

The concept of efficiency reflects the intuition that for a solution to be acceptable another cannot exist improving in every objective. Multiple notions of efficiency are present. The notation that this thesis follows is the given in Ehrgott (2005).

Definition 3.1 (Efficiency, Ehrgott (2005))**.** Let $f_1(x), \ldots, f_K(x)$ be objective functions to be minimized, and let X be the feasible set. A feasible solution $\hat{x} \in X$ is called:

- *Weakly efficient* if there is no $x \in X$ such that $f(x) < f(\hat{x})$ i.e. $f_k(x) < f_k(\hat{x})$ for all $k = 1, \ldots, K$.

- *Efficient* or *Pareto optimal* if there is no $x \in X$ such that $f_k(x) \leq f_k(\hat{x})$ for all $k = 1, \ldots, K$ and $f_i(x) < f_i(\hat{x})$ for some $i \in \{1, \ldots, K\}$.

- *Strictly efficient* if there is no $x \in X$, $x \neq \hat{x}$ such that $f(x) \leq f(\hat{x})$.

Furthermore, the set of efficient solutions is called the *efficient set*, and the image under f of this set is the *nondominated* set.

Depending on the type of solution provided, the methods can be classified in the following categories (Talbi, 2009).

A priori Methods in which the decision maker (DM) expresses the importances of the objectives before any optimization is carried out. Some methods include utility functions, goal programming or compromise programming, and they provide a point as a solution.

A posteriori Instead of searching for a point satisfying the DM's preferences, the efficient set is sought, and then the DM will choose a point inside it. Some methods for obtaining the efficient set (a field usually called *Multiobjective optimization*) are the weighted sum method and the ε-constraint method.

Interactive In this methods the DM usually establishes some *ideal* solution, and a point similar to that is sought. Afterwards that obtained point is presented to the DM, who later updates their preferences. An example of interactive method is the reference point method.

With a priori methods, it is possible that the importance of the different objectives are not comparable, that is, if even the greatest improvement in one objective does not compensate any loss in another one (that is, there is no trade off between the objectives). When multiple optima exist, a DM could establish priority levels between the objectives, and a solution could be found lexicographically.

3.1.2 Wildfire management: fuel management

For a fire to occur the presence of three factors is essential: oxygen, heat and fuel. Among these factors, the only one that can be acted on for mitigating the effects of forest fires is fuel. According to the Food and Agriculture Organization of the United Nations (FAO), *fuel management* is defined as the "act or practice of controlling flammability and reducing resistance to control of wildland fuels through mechanical, chemical, biological, or manual means, or by fire, in support of land management objectives" (Xanthopoulos et al., 2006). Pyne et al. (1996) explains than modification of fuel consist on modulating the behaviour and effects of fire, improving also the efficiency of fire-fighting. Treatments can be of the following ways (Pyne et al., 1996; Xanthopoulos et al., 2006; Chung, 2015):

- Fuel reduction: Its goal is to reduce waste, both human-caused or natural, in order to reduce the intensity of an eventual wildfire (Coop et al., 2016; Lydersen et al., 2017). Removal of waste can be done manually or mechanically, but the usual practice when wanting to reduce fuel at large scale is by prescribed burns.

- Fuel modification: Replacing flora can led to have a less flammable landscape.
- Fuel isolation: To avoid fire propagation to high-value areas, these can be isolated from fuel.
- Firebreaks: One of the most common techniques on forest fire management is the creation of a firebreak network. Firebreaks are stripes, of different width, in which fuels are completely removed. This responds to both operative needs (they can be used as paths for fire fighting crews) and protection needs (firebreaks can slow down the advance of fires).

The decision of where to effect these treatments can be supported by mathematical programming models. Using fire propagation tools, such as *FARSITE* (Finney, 1998), *FlamMap* (Finney, 2006) or *BehavePlus* (Andrews, 2014), is also common to evaluate which treatment is more effective. For instance, in both Oliveira et al. (2016) and Salis et al. (2016), the minimum travel time fire spread algorithm, which is implemented in *FlamMap*, is used for evaluating different fuel management strategies.

A selection of references in which mathematical models are used, mainly mathematical programming ones, applied to fuel management is presented:

- Richards et al. (1999) studies how to preserve the diversity on a forest, while maintaining some parts of the forest in diverse successional stages, using Markov chains.
- Wei et al. (2008) commences dividing a forest area in cells using a grid. On each cell the probability of an ignition is obtained using FlamMap. Then, by using a mixed integer programming model, the probabilities of propagation from one cell to a neighbouring one are calculated in terms of the wind direction. Finally a series of treatments are considered to reduce the intensity of fires in the are, and decides which to use reducing the risk as much as possible.
- Acuna et al. (2010) develops a MIP model which, together with heuristic techniques, integrates the timber harvesting with the creation of a firebreak network. On a follow up paper it includes an stochastic model considering different scenarios (future prices of timber) (Alonso-Ayuso et al., 2011).
- Minas et al. (2014) starts with a division of a forest area in different cells. Each of theses cells has one type of vegetation which after a set amount of time are deemed to be old. In a sense, the age of a cell is taken as a proxy for the risk it has: the older the vegetation on the cell is, the more intense a fire going through it will be. The article implements a MIP model en which the number of neighboring old cells is minimized during a planning period, deciding in which cell to do prescribed burns and when. It also includes constraints (hard and soft) to keep a diversity of successional stages. In Rachmawati et al. (2015) the developed model is applied on a case study in Australia, where 711 cells were present.

- Minas and Hearne (2016) develops a MIP model that clusters cells in which prescribed burning operations will be carried. At the time of performing prescribed burns it is essential to have resources available all along the perimeter of the burning area. For that reason, the model looks for a way of grouping cells such that the perimeter to be controlled is reduced.

- Driscoll et al. (2016) recognize the conflict when performing prescribed burns, applying a multicriteria decision-making approach to evaluate 22 burn plans while considering 8 objectives.

- Matsypura et al. (2018) propose a MIP model to allocate prescribed burns over a planning horizon. Fuel accumulation is accounted for by using a type of Olson curve (Olson, 1963).

3.1.3 Structure of chapter

The rest of the chapter is organized as follows. Section 3.2 discusses prescribed burning, reviews some works related with the issue and establishes the problem to be solved. In Section 3.3 a mathematical programming model for prescribed burning is developed, with its implementation discussed in Section 3.4. Finally, Section 3.5 shows and comments the results of the conducted experiments.

3.2 Problem description and literature review

Although some negative effects have been noted, positive effects of bush fires on the habitat for native flora and fauna have been recorded (Venn and Calkin, 2011). Reports indicate that areas subject to prescribed burning have more live trees, greater survival, and reduced fire intensity during wildfires compared to untreated areas (Strom and Fulé, 2007). Thinning and burning has also been employed for restoration purposes (Ager et al., 2013). Prescribed burning leads to fuel reduction (Agee and Skinner, 2005) and areas with old vegetation (or areas with excess fuel build-up) are often targeted for treatment (U.S. Government Accountability Office, 2003). Treatments may restrict the spread and intensity of large wildfires and so help mitigate wildfire hazards (Salazar and González-Cabán, 1987; Boer et al., 2009; Carey and Schumann, 2003), and the risk to human life and economic assets (Penman et al., 2011). Thus it has been argued that fuel management is both necessary and important (Burrows, 2008).

The occurrence of catastrophic wildfires decreases with extent treated (King et al., 2006, 2008) but with an optimal landscape mosaic (Finney et al., 2008) hazard reduction can be achieved without excessive costs (Loehle, 2004). Nevertheless the vegetation regenerates, senesces and eventually becomes high fuel load again and treatment extent is always subject to a budget constraint. In Mediterranean climate systems the wet winters and the dry and dangerous conditions in summer further restrict prescribed burning to a narrow time-window. These constraints on treatment extent can lead to desired management plans becoming infeasible in some years. Thus long-term planning is necessary to minimise high-fuel load connections over time (Wei and Long, 2014; Minas et al., 2014; Rachmawati et al., 2015).

Reducing the total fuel load has ecological consequences. Some species rely on vegetation that is classified as high-fuel load. A native Australian bird, the Mallee Emu-wren, is an example (Brown et al., 2009) and the Southern Brown Bandicoot requires heathland that was last burnt between five and fifteen years ago (Southwell et al., 2008). Treatment plans should take into account the habitat needs of species such as these. This might require sufficient areas of high fuel-load to remain after treatment. Prioritising areas for prescribed burning would then involve trying to disrupt fire-spread paths while, for example, maintaining migration paths of fauna (Rayfield et al., 2015). Little research has been done combining multiple concerns that arise with fuel treatment in an optimisation framework (Chung, 2015).

For management purposes states are often divided into large planning areas such as a catchment or national park. Each of these planning areas is then divided into treatment units or potential operational delineations (PODs) (Thompson et al., 2016). These areas are often determined by boundaries such as roads or rivers which facilitates the control of prescribed fire or some other management activities. Formulating a multiperiod schedule of treatments for a landscape comprising such treatment units is a complex spatio-temporal problem (Hof and Omi, 2003; Rönnqvist et al., 2015) and the resulting landscape mosaic is critical for hazard reduction (Fernandes and Botelho, 2003; King et al., 2006) and habitats. An early model addressing the fuel hazard problem only was formulated and illustrated on a regular grid (Minas et al., 2014). The approach was then extended to include multiple vegetation classes in a real landscape (Rachmawati et al., 2016). The computational effort in this work limited the analysis to some extent. More recently a study involving much greater emphasis on the probability of fire occurrence, fire behaviour and assets at risk was published (Alcasena et al., 2018). This work is aimed at prioritising treatment in preparation for the next fire season and does not deal with the multiperiod problem. In another recent work (Rachmawati et al., 2018) a multiperiod model for fuel management was constructed. that comprised ecological constraints. These included the vegetations' "tolerable fire intervals" (Cheal, 2010) as well as the quantity and spatial configuration of habitat for a faunal species. In this chapter the model in Rachmawati et al. (2018) is advanced in the following way:

- the rectangular grid structure is replaced by more realistic polygons
- the concept of connectedness is broadened to recognise the length of a common boundary
- the concept of neighbourhood is changed to reflect the direction of fire spread or migration paths
- fuel accumulation curves are used to categorise hazardous polygons (Keith et al., 2002)
- fire response curves are used to generate a measure of habitat quality of a polygon (MacHunter et al., 2009)

- limitations imposed by computational effort are overcome to some extent by either a rolling horizon or a lexicographic approach to the solution method

The formulated model is a mixed integer programming model with the aim of finding an optimal multiperiod, spatial schedule for prescribed burning that will:

- reduce the connectivity of high fuel load polygons in order to lessen the likelihood of large wildfires
- ensure the sustainability of the vegetation by constraining the timing of prescribed burns within tolerable fire intervals
- ensure adequate levels of habitat quality for fauna are maintained

3.2.1 Multiobjective treatment

As previously mentioned, prescribed burning is a technique with ecological consequences. Consequently, multiobjective optimization is used to obtain solutions for this problem. The first proposed model acknowledges the habitat quality imposing constraints on its level: solutions are sought decreasing fire risk, but without drastically decreasing habitat quality.

Next, these constraints are dropped, and instead, a lexicographical approach is followed, first minimizing fire risk and then maximizing habitat quality. This procedure can accurately reflect situations in which maximizing habitat quality is perceived as necessary, but on a whole different level of importance than minimizing fire risk.

Chapter 5 will later furtherly study this same problem, but it will be solved with a different technique as stochasticity will be considered as well.

3.3 Mathematical programming model

Consider a landscape (i.e. a management area) comprising a mosaic of polygons. In the context of fuel management these polygons are often referred to as 'treatment units' or 'burn units'. The time since the vegetation in each polygon was last burnt determines its fuel load. For convenience for the rest of this thesis we will refer to the time elapsed since the vegetation in each polygon was last burnt as its "age".

On the other hand we also want to take into account the species that live in the landscape. As species have preferences for vegetation of a certain age (Di Stefano et al., 2013), we assign a quality to each polygon according to its area and the relative abundance of species supported by vegetation of that age. We can then only select a polygon for treatment if the habitat quality of at least one of its neighbours is at least as good as the habitat quality of the polygon itself. This way, we take into account the habitat needs of the species, although we realize that individuals might have to migrate from time to time.

Further constraints included in the model relate to the vegetation. To sustain vegetation and the associated ecosystem, fire should not occur more frequently than its 'minimum tolerable fire interval'. On the other hand, for fire-dependent species the 'maximum tolerable fire interval' is also important (Cheal, 2010). An explanation of all constraints is described after the mathematical representation of the model that follows.

Indices sets

I	set of all burn units in the landscape
$\mathcal{N}_i \subset I$	set of burn units connected to burn unit i

Parameters

a_i	initial fuel age of burn unit i
b_t	fuel treatment budget at time t
c_i	area of burn unit i
w_{ij}	relative weighting of the connectivity of burn units i and j
h_{thr}	high-fuel load threshold
h_{targ}	global habitat quality target
$maxTFI$	maximum tolerable fire interval (TFI)
$minTFI$	minimum tolerable fire interval (TFI)
T	number of time periods in the planning horizon
N	number of breakpoints for the piecewise linear function of the fire response curve
r_n	breakpoint of the piecewise linear function or the fire response curve
v_n	value of breakpoint r_n according to the fire response curve
p_{ij}	proportion of boundary of unit i that is shared with unit j
M	big-M coefficient

Variables

X_{it}	1 if burn unit i is treated in time period t, 0 otherwise
H_{it}	1 if burn unit i is classified as high-fuel load in time period t, 0 otherwise
Q_{ijt}	1 if adjacent burn units i and j are both classified as high-fuel load in time period t, 0 otherwise
A_{it}	fuel age of burn unit i in time period t
FRC_{it}	habitat quality of burn unit i in time period t by area (fire response curve)
Z_{itn}	1 if the age of burn unit i in time period t is between r_n and r_{n+1}, 0 otherwise

G_{itn} convex multipliers of burn unit i in time period t for the piecewise linear function

Objective function and constraints

$$\min \quad \sum_{t=2}^{T} \sum_{i \in I} \sum_{\substack{j \in \mathcal{N}_i \\ j>i}} w_{ij} Q_{ijt} \tag{3.1}$$

$$\text{s.t:} \quad \sum_{i \in I} c_i X_{it} \leq b_t \qquad t = 2, \dots, T \tag{3.2}$$

$$A_{it} = a_i \qquad t = 1, \forall i \in I \tag{3.3}$$

$$A_{it} \geq A_{i(t-1)} + 1 - M \cdot X_{it} \qquad t = 2, \dots, T, \forall i \in I \tag{3.4}$$

$$A_{it} \leq A_{i(t-1)} + 1 \qquad t = 2, \dots, T, \forall i \in I \tag{3.5}$$

$$A_{it} \leq maxTFI(1 - X_{it}) \qquad t = 2, \dots, T, \forall i \in I \tag{3.6}$$

$$minTFI \cdot X_{it} \leq A_{i(t-1)} \qquad t = 2, \dots, T, \forall i \in I \tag{3.7}$$

$$A_{it} \leq h_{thr} - 1 + M \cdot H_{it} \qquad t = 2, \dots, T, \forall i \in I \tag{3.8}$$

$$H_{it} + H_{jt} \leq 1 + Q_{ijt} \qquad t = 2, \dots, T, \forall i \in I, \forall j \in \mathcal{N}_i, \ j > i \tag{3.9}$$

$$\sum_{n=1}^{N-1} Z_{itn} = 1 \qquad t = 1, \dots, T, \forall i \in I \tag{3.10}$$

$$Z_{itn} \leq G_{itn} + G_{it(n+1)} \qquad t = 1, \dots, T, \forall i \in I, n = 1, \dots, N-1 \tag{3.11}$$

$$\sum_{n=1}^{N} r_n G_{itn} = A_{it} \qquad t = 1, \dots, T, \forall i \in I \tag{3.12}$$

$$\sum_{n=1}^{N} G_{itn} = 1 \qquad t = 1, \dots, T, \forall i \in I \tag{3.13}$$

$$FRC_{it} = \sum_{n=1}^{N} v_n G_{itn} \qquad t = 1, \dots, T, \forall i \in I \tag{3.14}$$

$$c_i FRC_{i(t-1)} - M(1 - X_{it}) \leq \sum_{j \in \mathcal{N}_i} p_{ij} c_j FRC_{jt} \qquad t = 2, \dots, T, \forall i \in I \tag{3.15}$$

$$\sum_{i \in I} c_i FRC_{it} \geq h_{targ} \qquad t = 2, \dots, T \tag{3.16}$$

$$X_{it} \in \{0, 1\} \qquad t = 2, \dots, T, \forall i \in I \tag{3.17}$$

$$H_{it} \in \{0, 1\} \qquad t = 2, \dots, T, \forall i \in I \tag{3.18}$$

$$Z_{itn} \in \{0, 1\} \qquad t = 1, \dots, T, \forall i \in I, n = 1, \dots, N-1 \tag{3.19}$$

$$Q_{ijt} \geq 0 \qquad t = 2, \dots, T, \forall i \in I, \forall j \in \mathcal{N}_i, \ j > i \tag{3.20}$$

$$A_{it} \geq 0 \qquad t = 1, \dots, T, \forall i \in I \tag{3.21}$$

$$FRC_{it} \geq 0 \qquad t = 1, \dots, T, \forall i \in I \tag{3.22}$$

$$G_{itn} \geq 0 \qquad t = 1, \dots, T, \forall i \in I, n = 1, \dots, N \tag{3.23}$$

The objective function (3.1) minimises the weighted number of connections between high-fuel load burn units. This dimensionless quantity will henceforth be referred to as **fire hazard**. Equation (3.2) limits the amount of land we can burn each year. Equation (3.3) initializes the ages of all burn units and Eqs. (3.4) to (3.6) track the age of all burn units, resetting the age to 0 if we burn the unit. Equation (3.6) forces a unit to be burnt if its age is equal to the maximum TFI and Eq. (3.7) only allows a unit to be burnt if its age is over the minimum TFI. Equation (3.8) ensures a burn unit to be classified as high-fuel if the age of burn unit is equal to or larger than a threshold value h_{thr}. Equation (3.9) sets Q_{ijt} to 1 if both burn units i and j are classified as high-fuel load in time period t.

Equations (3.10) to (3.13) model the piecewise linear function of the fire response curve, which is dependent on the age A_{it}. We break the original curve at breakpoints r_n which have value v_n. Equation (3.10) is used so that the age can only be at one linear function at a time. Equation (3.11) forces the model to only use the convex multipliers of the corresponding linear function. Equations (3.12) and (3.13) makes sure the age is a convex combination of the breakpoints. Then Eq. (3.14) ensures FRC_{it} is given the correct value according to the piecewise linear function.

Equation (3.15) ensures unit i can be burnt at time t only if the habitat quality in the neighbourhood of unit i is at least as good as the quality that unit i had at time $t-1$. To enforce this constraint at $t = 2$ values of FRC_{it} are required at $t = 1$, hence the summations in Equations (3.10) to (3.14) commence at $t = 1$. The quality of the neighbourhood, henceforth referred to as **local quality** takes into account both the areas of neighbouring units as well as the proportion of common boundary. Equation (3.16) ensures that **global habitat quality** is maintained above some target level. We use the term 'global habitat quality' to mean the area-weighted sum of habitat quality over the whole landscape.

Equations (3.17) to (3.19) ensure the decision variables are binaries and Equations (3.20) to (3.23) restrict the age and quality of a unit to only positive values, as well as the breakpoint values.

By definition $Q_{ijt} = Q_{jit}$, and since the weighting of a connection between two cells is determined by the relative length of their common boundary $w_{ij} = w_{ji}$, and hence $w_{ij}Q_{ijt} = w_{ji}Q_{jit}$. Thus the number of binary variables can be reduced by excluding all Q_{ijt} such that $i \leq j$. This saves computational effort and will have no impact on the optimal strategy as it effectively involves dividing the objective function by a constant.

Also note that the initial conditions are set at $t = 1$. The first decision on where to burn is based on these initial conditions but the consequences of that decision is only realised at $t = 2$. Thus fire hazard is tracked and accounted for in the objective function from year 2.

3.4 Model implementation

3.4.1 Implementation

The model is implemented for a heathy woodland vegetation type with 23 randomly generated landscapes (one instance is shown in Fig. 3.1). Each of the landscapes has 45 burn units. The average size of a burn unit in this implementation is taken to be 100 *ha* but within certain limits the representation is scale-free. Experiments are performed with a treatment level of 7 percent of the total area of the landscape each year. The simulations are solved for a planning period of 20 years, with a rolling horizon of 12 years. The problems are solved on a PC using Gurobi 7.5 (Gurobi Optimization, 2019) with the JuMP modeller (Dunning et al., 2017) and the Julia programming language (Bezanson et al., 2017).

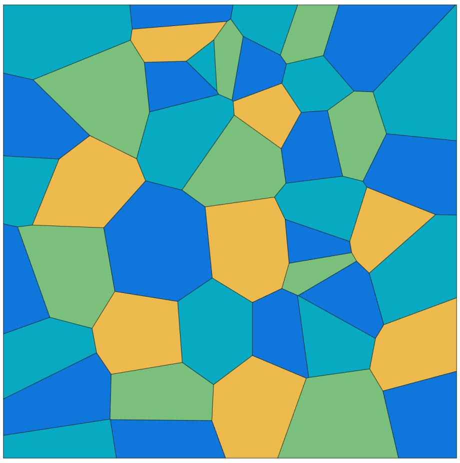


Figure 3.1: Randomly generated landscape with 45 polygons representing a planning area of 45 km^2. On average polygons are 100 ha in extent.

3.4.2 Input data

Based on Cheal (2010) the appropriate parameters for heathland are as follows: $h_{thr} = 10$, $minTFI = 10$, and $maxTFI = 35$. The $maxTFI$ parameter suggested (Cheal, 2010) is 45 rather than 35 years. However, as seen in Fig. 3.2 any unit with a fuel age over 35 years has no habitat value and also represents high fuel hazard. Thus, to improve computation time we have set the parameter $maxTFI$ to be equal to 35.

Habitat considerations are illustrated by the needs of invertebrates in the vegetation. The habitat quality of burn unit i at time t is given by the product of its area and its age-related quality. Based on Di Stefano et al. (2013) the latter is given by the piecewise linear function shown in Fig. 3.2.

The initial value of global habitat quality is chosen as the target value h_{targ} in constraint (16) of the model. This ensures that there is no decline in global habitat quality throughout the simulation

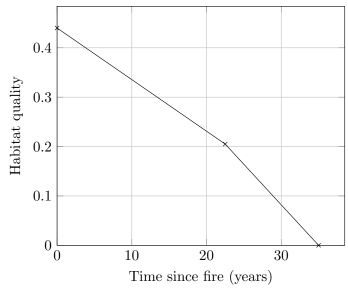


Figure 3.2: Habitat quality as a function of fuel age. Values fitted with data from Di Stefano et al. (2013) for the optimal age distribution for invertebrates on a heathy woodland landscape.

period.

3.4.3 Experiments

Rolling horizon

Solutions are obtained over 20 years using a rolling horizon of 12 years for the following cases:

`rh1`: The model presented above is solved to give the nomimal solution.

`rh2`: The local habitat constraints are relaxed and compared with the nominal solution to determine what effect they have on fire hazard.

`rh3`: The model is solve with a short-term planning period (two-year rolling horizon) and compared with the nominal solution to determine the importance of a longer-term planning. Habitat constraints are dropped to enable a clearer comparison of fire hazard with the two planning period approaches.

Lexicographic approach

The following solutions were obtained for a planning period of 20 years using a lexicographic approach. Using this approach solutions over the full 20-year planning period can be obtained without requiring a rolling horizon. Further changes made include the following:

- The habitat quality curve shown in Fig. 3.2 is changed to create greater tension between reducing fuel load and maintaining habitat quality. The new habitat quality curve is shown in Fig. 3.3.
- In previous experiments fire hazard was defined to be the sum of fire hazard over the full planning period. In this experiment we consider an alternative metric, which might be considered more appropriate by fire managers. We redefine the total fire hazard to be the highest fire hazard in any year. Hence we minimize over the planning period the maximum fire hazard that can occur in any given year (*minimax* approach). As fire hazard in the first

few years is more a reflection of random initial values than any plan, we only consider the second half of the planning horizon.

- As with fire hazard, total habitat value was defined to be a sum throughout the planning horizon. We also redefine total habitat value to be the minimum habitat value in any year, starting in the middle of the planning horizon (*maximin* approach). This new approach is easily justified when we consider that an animal living on the landscape would hardly benefit from a solution in which some years have very high habitat value but other years have a habitat value close to zero.

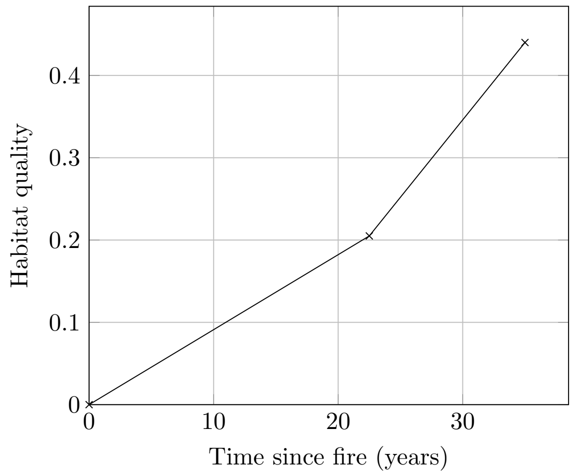


Figure 3.3: Fire response curve: habitat quality of a unit given its age (by hectare)

L1: To understand what range of global habitat values might result when minimising fuel hazard the following sequence was undertaken:

- Relax habitat constraints and then minimise the fuel hazard to yield an optimal fuel objective value f_m.
- Without compromising f_m, maximise the habitat value to yield an optimal habitat value h_M
- Still maintaining the fuel hazard at f_m, *minimise* the habitat value to yield h_m.

If fuel hazard reduction is a priority for landscape managers, the interval $\{h_m, h_M\}$ will show the extent of habitat outcomes possible without compromising fuel hazard objectives.

L2: In the case where a key faunal species requires habitat with a high fuel load it might be important to ascertain differences between fire paths and possible directions of local migration of fauna. To represent these paths the neighbourhood data must be amended accordingly. Our model can easily reflect different neighbourhood definitions. For example a landscape could be located in some place where wind primarily blows in one direction, and hence fire propagation would occur mainly in that direction. If that were the case our model can reflect that information by just changing a neighbourhood matrix. In the model formulation the neighbourhood information is given by the set $\mathcal{N}_i$). An example of this alternative way of defining neighbours is shown in Fig. 3.8a. Another example where fire propagation might occur mainly in one direction (and thus neighbourhoods defined in a similar way) is if the landscape has a high slope and fires are primarily topographical. With the neighbours defined as given by Fig. 3.8a we solve the model lexicographically.

3.5 Results and discussion

For experiment `rh1` the mean fire hazard and global habitat value are shown in Fig. 3.4.

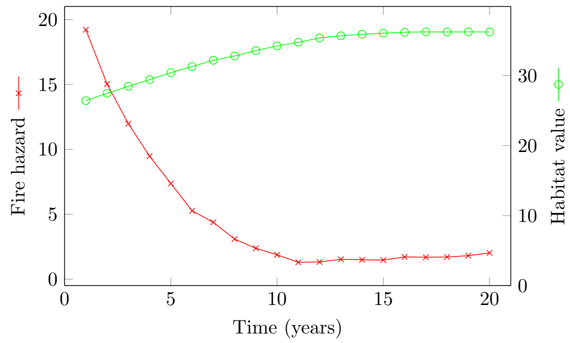


Figure 3.4: Mean fire hazard and mean global habitat value for the 23 scenarios by year

Our objective was to get an overall minimum in the weighted connections between high-hazard burn units. We see that the initial hazard is quickly brought close to 0, while maintaining habitat of good quality (both local and global). Figures 3.5 and 3.6 show the initial conditions (random ages) and the solution after 3 and 19 years. It is clear that the model is achieving its aim.

For experiment `rh2` habitat constraints were relaxed and solutions compared with the nominal results. Figure 3.7 shows that fire hazard is not further reduced significantly. In other words the local habitat constraint has not diminished significantly the fuel reduction objective. Not shown is that collectively for the 23 scenarios, the local habitat connectivity constraint is not satisfied in nearly 50% of cases.

In experiment `rh3`, out of the 23 scenarios three were infeasible when solved with the myopic approach. As habitat constraints are not included, the source of infeasibility is a result of Eq. (3.6) requiring vegetation exceeding the fuel age of the parameter $maxTFI$ to be burnt. In some scenarios situations arise in which the amount to be burnt in one year is greater than that allowed by the budget (Eq. (3.2)).

Table 3.1 reports the results obtained for the last years of the planning horizon, when the effect of initial values have diminished. Even removing the scenarios in which the myopic approach was infeasible, the short-term plans yielded results much worse than that obtained with a longer

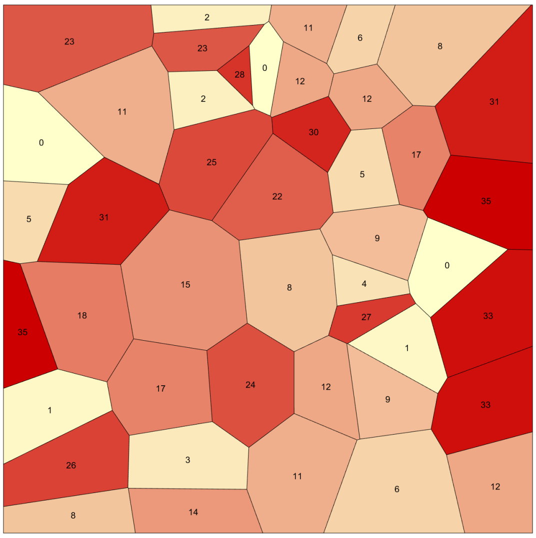
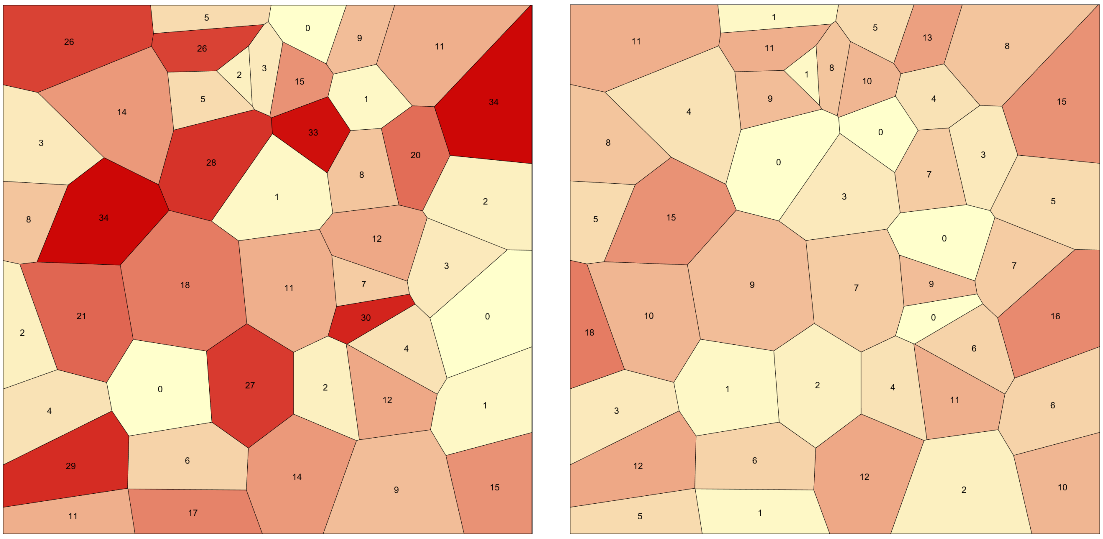


Figure 3.5: Ages of cells on random initial conditions for a given landscape



(a) After 3 years

(b) After 19 years

Figure 3.6: Ages of cells after 3 and 19 years

Year	Long term	Myopic
16	0.936	2.258
17	0.920	2.076
18	0.991	1.899
19	0.815	2.036
20	0.828	2.045

Table 3.1: Mean fire hazard in the last years of simulation, long term rolling horizon window versus myopic approach.

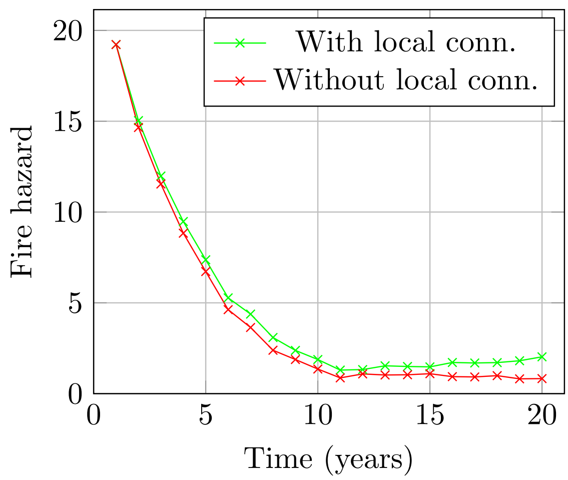


Figure 3.7: Mean fire hazard when local connectivity is not a requirement. Hazard does not change significantly but local connectivity is violated many times (210 times on 437 runs of the model).

planning horizon.

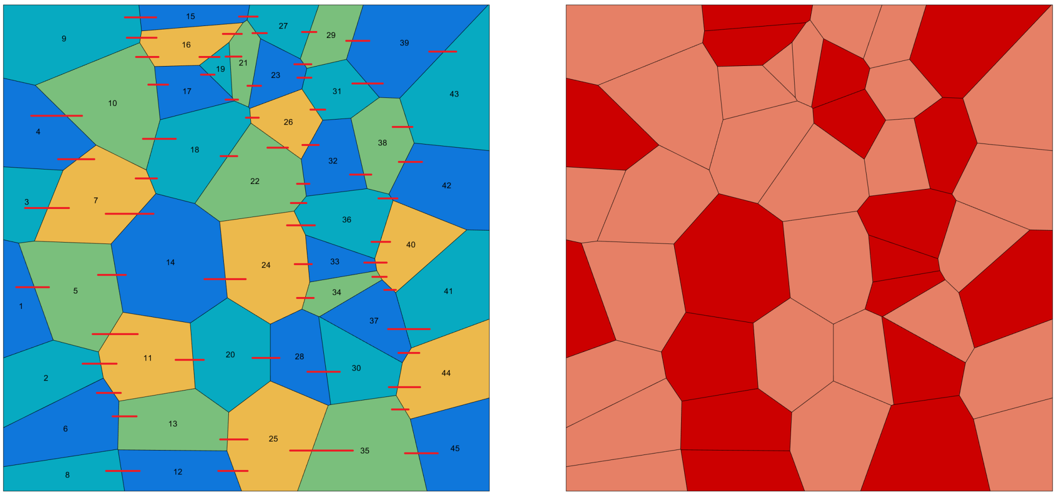
In experiment `L1` all three solutions have fire hazard going to zero. Habitat values are shown in Table 3.2.

Year	Solution 1	Solution 2	Solution 3
10	6.502	7.804	5.675
11	5.539	7.538	5.122
12	4.987	6.449	4.401
13	4.912	6.456	4.311
14	4.699	6.312	**4.122**
15	4.64	6.399	5.033
16	4.604	6.3	4.953
17	4.822	6.327	4.865
18	5.109	6.287	4.785
19	5.174	6.316	4.844
20	5.091	**6.284**	5.025

Table 3.2: Yearly habitat values for three different solutions, all with the same fire hazard. In boldface the minimum habitat value for solutions 2 and 3.

Without compromising the reduction in fuel hazard, a comparison of Solution 2 and Solution 1 reveals that the model can yield significant improvements (more than a third) in habitat quality. In fact, it is possible that a fuel reduction plan such as solution 3 could yield habitat quality more than 50% below its optimal value. There is thus a large interval of habitat values that can result with the same fuel hazard value. This opportunity for optimising habitat should not be neglected.

The state of the landscape for experiment `L2` is shown in Fig. 3.8b for the last year of the planning period. It can be seen that the model makes use of the new definition of neighbours, as fuel load is accumulated in burn units that are geographically adjacent in the north-south direction but were only defined as neighbours in the eat-west direction, and thus do not pose a high fire hazard.



(a) If the landscape has prevailing winds in the west-east direction, and fires are wind-driven, the neighbourhood matrix can reflect this. For this case lines in the figure show which units are defined as neighbours.

(b) Solution in the last year of simulation with dark units reflecting burn units that have old fuel (their age is older than 10).

Figure 3.8: Landscape with neighbourhood matrix redefined

3.6 Discussion

In this chapter, a mixed integer programming model has been presented for a landscape divided into polygons representing realistic treatement units. The model aims to reduce the adjacency of high fuel load areas while trying to maintain the fauna habitat. Habitat quality was considered for invertebrates on a heathland landscape and also a hypothetical species that had a preference for hazardous vegetation.

It has been shown that a significant range of habitat quality outcomes can be obtained without compromising the optimal fuel load goal. It is sensible therefore for habitat considerations to be included in fuel reduction plans.

Computational effort in solving the mixed integer programming model can be considerable and limiting. Both the rolling horizon and the lexicographic approach offer possibilities of achieving solutions in times suitable for using in a realistic situation.

The benefits of including multiple objectives are clear, and those of considering long-term planning horizons as well, even if a rolling horizon approach has to be used. However, the proposed model is making decisions as if the future is known precisely. For instance, it is assumed that the available budget is known for all time periods, or that fuel loads will not be affected by external factors (for instance, by wildfires). If long-term models are to be considered, it is necessary to factor in the uncertain future.

The prescribed burning problem will be resumed in Chapter 5, where some uncertainty will be considered as well as the multiple objectives already studied. The resulting problem, a multiobjective stochastic programming one, will be solved with the risk-averse methodology developed in the next chapter.

Chapter 4

Mathematical programming with uncertainty and multiple objectives. Theoretical approach

Contents

4.1 Introduction

In Chapters 2 and 3 two different problems were presented, one with uncertainty and another with multiple criteria, and different models were discussed to address them.

The remaining of this thesis aims to consider problems in which both uncertainty and multiple criteria are present at the same time.

The main results from this chapter have been submitted to:

> **León, J.**, Puerto, J. & Vitoriano, B. (submitted October 2019), 'A risk-aversion methodology for the multiobjective stochastic programming problem. A real case in Andalucía (Spain)', *Computers & OR*.

4.1.1 Decision making with uncertainty and multiple objectives: motivation

Consider the following example:

Example 4.1 (Multiobjective stochastic knapsack problem)**.** A group of K friends is going on a holiday, but due to budgetary issues they will all share a single suitcase which can carry up to Mkg. There are J different weather scenarios which affect differently to each friend. A total of I items is available for them to bring to the trip, with weights $w_i (i = 1, \dots, I)$ and utility b^i_{jk} (value that person k gives to the object i in case weather j occurs).

Example 4.1 can be formulated as a "Multiobjective stochastic programming" problem, as there are multiple criteria (the utility for each of the travellers) and uncertainty (weather). A possible formulation consists on declaring binary variables X_i which equal 1 if object i is carried and 0 otherwise. The feasible region consists on all vectors $(X_1, \dots, X_I) \in \{0, 1\}^I$ such that $\sum_i w_i X_i \leq M$. With those variables $J \times K$ functions can be defined, one for each scenario and criterion. Namely, $f^k_j(X) = \sum_i b^i_{jk} X_i$.

Example 4.1, which will be given more importance in Section 4.6, has uncertainty only in the objective functions. The next example goes further.

Example 4.2 (Multiobjective newsvendor problem)**.** Every morning a newsvendor has to buy newspapers from the publisher to be sold during the day, buying as many as u newspapers (publisher limits). The newspapers are bought at cost c, sold at price q and at the end of the day the unsold ones can be returned, recovering r per newspaper (with $r < c$). The demand for a day is not exactly known, but the newsvendor knows it equals d_ω with probability π_ω, with $\omega \in \Omega$. How many newspapers should be bought, if profits wants to be maximized and unsatisfied demand minimized?

A way of modelling Example 4.2 is given by Model (4.1):

$$\min_{X, Y_\omega} \left\{ \overbrace{cX - qY_\omega - r\left(X - Y_\omega\right)}^{f_1^\omega}, \overbrace{d_\omega - Y_\omega}^{f_2^\omega} \right\} \tag{4.1a}$$

$$\text{s.t.} \quad X \leq u \tag{4.1b}$$

$$Y_\omega \leq d_\omega \qquad \forall \omega \in \Omega \tag{4.1c}$$

$$Y_\omega \leq X \qquad \forall \omega \in \Omega \tag{4.1d}$$

$$X, Y_\omega \in \mathbb{Z}^{\geq 0} \qquad \forall \omega \in \Omega \tag{4.1e}$$

X is the first-stage decision variable, representing the amount of newspapers bought, and Y_ω is the amount of newspapers sold, which depends on the scenario that holds. The objective function (4.1a) is actually one objective function for each of the criteria and scenario in which f_1^ω represents the expenses (cost of buying newspapers minus income for newspapers sold and returned), and f_2^ω computes the unsatisfied demand.

4.1.2 Structure of chapter

This chapter will study the Multiobjective stochastic programming problem from a theoretical point of view. A new concept of solution will be proposed, especially useful for risk-averse situations, as solutions will be sought that perform well enough in every scenario and criteria. In the context of emergencies this may be relevant if one wants to be protected in complicated situations.

The remaining of the chapter is structured as follows: Section 4.2 shows the state of the art on multiobjective stochastic programming. Section 4.3 suggests the solution concept and its properties are studied. Section 4.4 computes the proposed solution of a very basic example in which the feasible set consists of only 4 alternatives. Section 4.5 formulates a linear programming model that can be used for obtaining the solution, and it is finally applied to a more complex example in Section 4.6.

4.2 State of the art

4.2.1 Stochastic programming and multicriteria decision making

Stochastic programming and multicriteria decision making were discussed in Sections 2.1.1 and 3.1.1 respectively. This section will discuss only the parts that will be relevant for the solution concept proposed.

The concepts of *value-at-risk* (VaR) and *conditional value-at-risk* (CVaR) are widely used for quantifying risk (see for instance Yao et al. (2013); Mansini et al. (2015); Liu et al. (2017); Dixit and Tiwari (2019); Fernández et al. (2019)). They are typically defined for losses distributions in finance, where the right tail of the distributions are of interest.

Definition 4.1 (VaR)**.** Given $F_X(x)$ distribution function, and $\beta \in [0,1]$, then ν is the β-VaR when:

$$\nu = \min \{x \text{ such that } F_X(x) \geq \beta\}$$

Definition 4.2 (CVaR, Rockafellar and Uryasev (2002))**.** Given $F_X(x)$ distribution function, and $\beta \in [0,1]$, the β-CVaR is the conditional expectancy over the β-VaR.

The concept of CVaR allows to aggregate several scenarios by just looking at what happens in the worst ones.

The *ordered weighted averaging* (OWA) operators are defined in Yager (1988), and independently in the field of locational analysis Carrizosa et al. (1994); Nickel and Puerto (1999) under the name of *ordered median function*. These concepts will allow us to aggregate different criteria by looking at the least desirable ones, as a risk-aversion measure.

Definition 4.3 (OWA, Yager (1988))**.** Given $a_1, \ldots, a_n \in \mathbb{R}$, the *ordered weighted averaging* (OWA) operator with weights $\lambda_1, \ldots, \lambda_n$ is defined as:

$$\mathrm{OWA}(a_1, \ldots, a_n) = \sum_i \lambda_i a_{(i)}$$

where $\left(a_{(1)}, \ldots, a_{(n)}\right)$ is the ordered vector from largest to smallest $(a_1, \ldots, a_n)$.

Remark 4.1. For certain weights, the OWA represents a known quantity:

- If $\lambda_i = \frac{1}{n}$, the resulting OWA is the average of a.
- If $\lambda_1 = 1$, and $\lambda_j = 0$ for $j > 1$, the OWA is the maximum of a.
- If $\lambda_n = 1$, and $\lambda_j = 0$ for $j < n$, the OWA is the minimum of a.

Yager and Alajlan (2016) later study how to assign weights for an OWA when criteria have different importances.

Definition 4.4 (OWA with importances, Yager and Alajlan (2016))**.** Given $a_1, \ldots, a_n \in \mathbb{R}$ with importances $u_1, \ldots, u_n$ such that $\sum_i u_i = 1$ the weights λ_j for the OWA can be calculated with f, the *weight generating function* in the following manner:

1. Sort vector a such that $a_{(1)} \geq a_{(2)} \geq \ldots \geq a_{(n)}$.
2. With $(\cdot)$ as the order induced by a, define $T_j = \sum_{k=1}^{j} u_{(k)}$.
3. Let f be a function such that $f : [0, 1] \to [0, 1]$ and $f(0) = 0, f(1) = 1$. This function is called *weight generating function*.
4. Obtain the weights as $\lambda_j = f(T_j) - f(T_{j-1})$.

Example 4.3 (of Definition 4.4)**.** Consider the following weight generating function, for a given $r \in (0, 1]$:

$$f(x) = \begin{cases} \frac{x}{r} & \text{if } x < r \\ 1 & \text{if } x \geq r \end{cases}$$

Let $(\cdot)$ be the order such that $a_{(1)} \geq \cdots \geq a_{(n)}$, $u_{(j)}$ the weight associated to $a_{(j)}$, and also let $T_j = \sum_{k=1}^{j} u_{(k)}$. We shall see know how the weights are obtained from f. Let j^* be such that $T_{j^*-1} < r \leq T_{j^*}$.

- $\lambda_1 = f(T_1) = f(u_{(1)}) = \frac{u_{(1)}}{r}$, assuming $u_{(1)} < r$

- $\lambda_2 = f(T_2) - f(T_1) = f(u_{(1)} + u_{(2)}) - f(u_{(1)}) = \frac{u_{(1)}+u_{(2)}}{r} - \frac{u_{(1)}}{r} = \frac{u_{(2)}}{r}$, assuming $u_{(1)} + u_{(2)} < r$
- ...
- $\lambda_{j^*} = f(T_{j^*}) - f(T_{j^*-1}) = 1 - \left(\frac{u_{(1)}+u_{(2)}+\cdots+u_{(j^*-1)}}{r}\right)$, since $T_{j^*} \geq r$
- $\lambda_{j^*+1} = f(T_{j^*+1}) - f(T_{j^*}) = 1 - 1 = 0$
- ...
- $\lambda_n = f(T_n) - f(T_{n-1}) = 1 - 1 = 0$

Consequently the OWA of $a_1, \ldots, a_n$ with importances $u_1, \ldots, u_n$ is:

$$\begin{aligned} \text{OWA} =& \frac{u_{(1)}}{r} a_{(1)} + \frac{u_{(2)}}{r} a_{(2)} + \cdots + \left[1 - \left(\frac{u_{(1)} + u_{(2)} + \cdots + u_{(j^*-1)}}{r}\right)\right] a_{(j^*)} \\ =& \frac{u_{(1)} a_{(1)} + u_{(2)} a_{(2)} + \cdots + \left(r - u_{(1)} - u_{(2)} - \ldots\right) a_{(j^*)}}{r} \end{aligned}$$

That is, the OWA is the average of the worst a_j, weighted by their importances, with total importance adding up to r.

4.2.2 Multiobjective stochastic programming

The problems to be studied will include multiple objectives and uncertainty simultaneously. Consider the following problem, known as the Multiobjective Stochastic Programming (MSP) problem:

$$\min_{x \in X} \left(f_1(x, \omega), \ldots, f_K(x, \omega)\right)$$

The feasible set X is usually taken as a deterministic set. Otherwise, *chance-constrained* approaches, in which a solution is feasible if it belongs to a set with a given probability, are usually used. $f_1, \ldots, f_K$ are the conflicting functions to be minimised, each of them depending on ω, an uncertainty source. Depending on the nature of this uncertainty there are different contexts:

- If the probability distribution of ω is known $\Rightarrow$ stochastic optimization.
- If the distribution is unknown $\Rightarrow$ robust optimization.
- If ω is not a random variable but a fuzzy number $\Rightarrow$ fuzzy optimization.

In this section we will focus on uncertainty of stochastic nature. Some of the most relevant works for solving the MSP problem are shown below.

In Goicoechea (1980) the PROTRADE method is developed, where multiple non-linear objectives are considered, with objective function and feasible set stochastic, with normal probability distributions. Utilities functions defined to aggregate objectives into a single objective stochastic problem, which is solved with an interactive method, where the decision maker defines an expected solutions and a feasibility probability. A similar problem is considered in Leclercq (1982). The proposed approach consists on reducing the stochasticity by adding some *good* measures to the list of objectives. That is, the MSP problem:

$$\min_{x \in X} (f_1(x,\omega), \dots, f_K(x,\omega))$$

is transformed into the following deterministic multiobjective problem:

$$\Rightarrow \min_{x \in X} \{\mathbb{E}_\omega[f_k(x,\omega)], \mathbb{V}_\omega[f_k(x,\omega)], \mathbb{P}_\omega\left(f_k(x,\omega) \leq g_k\right), \dots\}_{k=1,\dots,K}$$

The resulting problem is solved aggregating the objectives, but it could be solved via other techniques.

This first two example already show a clear distinction in the way of approaching the MSP problem. While Goicoechea (1980) started transforming the MSP problem into a single objective stochastic problem, Leclercq (1982) began with a transformation into a multiobjective deterministic problems. In the MSP jargon these approaches are traditionally called "stochastic transformation" and "multiobjective transformation" respectively. These categories are considered in Ben Abdelaziz (2012), where different solutions methods for the MSP problem are reviewed, categorizing them as stochastic approach or multiobjective approach.

Caballero et al. (2004) also compare the *stochastic approach* with the *multiobjective approach* when using different techniques. On the most trivial example the set of efficient solutions is the same if:

1. Stochasticity is reduced taking expected values and then are aggregated, or

2. Stochastic functions are aggregated and then the expected value of such sum is minimised.

In more complex cases their study *"shows that, when among the stochastic objectives there exists a stochastic dependence, the order of transformations can lead to an efficient solution by following a certain order which is a non-efficient solution according to the inverse order."*. Additionally they show that the stochastic approach is more appropriate when there exist stochastic dependencies between objectives.

Stochastic goal programming is studied in Aouni et al. (2005), where deviation of objective functions to some goals set beforehand to stochastic values is minimized. In Muñoz et al. (2010), an interactive reference point method is developed. The decision maker (DM) gives reference levels u_i and probabilities β_i, hoping to achieve a solution x^* such that $\mathbb{P}\left(f_k(x^*) \leq u_k\right) \geq \beta_k$. If this is infeasible, DM should either increase the reference levels or decrease the probabilities of achievement.

Some fields where MSP models have been developed are: forest management (Álvarez-Miranda et al., 2018), multiple response optimization (Díaz-García and Bashiri, 2014), energy generation (Teghem et al., 1986; Bath et al., 2004), energy exchange (Gazijahani et al., 2018), capacity investment (Claro and de Sousa, 2010), disaster management (Manopiniwes and Irohara, 2016; Bastian et al., 2016), portfolio optimization (Şakar and Köksalan, 2012) and cash management (Salas-Molina et al., 2019).

4.3 A risk-averse solution

Throughout this thesis problems related with emergencies and disaster management are studied. In such problems is reasonable to say that risk-averse solutions might be preferred. Risk-aversion is the attitude for which we prefer to lower uncertainty rather than gambling extreme outcomes (positive or negative).

The MSP is studied in this chapter, and a risk-averse solution concept is proposed in this section. This concept is based on the existing concepts of CVaR and an OWA, presented in the previous section.

4.3.1 Definitions

The starting point of this chapter is the recurrent idea of representing ordered weighted or ordered median operators by means of k-sums. k-sums (or k-centra in the location analysis literature) are sums of the k-largest terms of a vector (Puerto et al., 2017). One can trace back, at least to Kalcsics et al. (2002), the use of k-sums to represent ordered median objectives. More recent references are for instance Blanco et al. (2013, 2014); Ponce et al. (2018) and Filippi et al. (2019). This last reference introduces a normalized version of k-centrum, named β-average that will be used hereafter.

Definition 4.5 (β-average, $g_k^\beta(x)$, Filippi et al. (2019))**.** Given $\beta \in (0,1]$, for each criterion k it can be defined $g_k^\beta(x)$ which measures the average of $f_k(x)$ on the worst scenarios $\left(f_k^1(x), \dots, f_k^J(x)\right)$, with accumulated probability equal to β.

Remark 4.2 (Filippi et al. (2019)). Given a value β, if the sum of the probabilities of the worst scenarios is exactly β, then the β-average is exactly $(1-\beta)$-CVaR.

Example 4.4. Consider a point x, a fixed criterion k and 5 different scenarios with probabilities π_j and values of f_k^j given. Table 4.1 shows the β-averages for different values of β, in which the scenarios have been ordered from largest value of f to smallest.

- For $\beta = 0.2$, the scenario $j = 1$ is the only one needed to obtain the worst scenario with probability 0.2, and hence $g_k^\beta(x) = \frac{0.2 \cdot 10}{0.2} = 0.2$.
- When β equals 0.3 it is necessary to include scenario 2, obtaining a β-average of $\frac{0.2\cdot 10 + 0.1 \cdot 7}{0.3} = 9$.
- Finally if $\beta = 0.5$ scenario 3 needs to be added as well, but only with the probability needed until reaching 0.5: $g_k^\beta(x) = \frac{0.2\cdot 10+0.1\cdot 7+0.2\cdot 4}{0.5} = 7$.

When using the β-average the functions $f_k^j(x)$ were transformed into $g_k^\beta(x)$, a collection of K functions not depending on the scenario. An OWA will be defined now, via its weight generating function, that will reduce the K β-averages into a scalar function.

Table 4.1: Small example of β-average for different values of β

	scenario					β		
	1	2	3	4	5	0.2	0.3	0.5
π_j	0.2	0.1	0.3	0.25	0.15	10	9	7
$f_k^j(x)$	10	7	4	3	2			

Definition 4.6 (r-OWA, $O_r(x)$)**.** Given $x_i \in \mathbb{R}$ with importance w_i ($i = 1, \ldots, K$, $w_i \geq 0, \sum_i w_i = 1$) and $r \in (0, 1]$, the function $O_r(x)$ is defined as the OWA with the following weight generating function:

$$f(x) = \begin{cases} \frac{x}{r} & \text{if } x < r \\ 1 & \text{if } x \geq r \end{cases}$$

Remark 4.3. The definition of $O_r(x)$ is made on a similar manner that the one given of the β-average (Definition 4.5), but it is done on a context with importances rather than probabilities. Example 4.5 shows the similarities between both approaches.

Example 4.5. Consider a point x and let $g_k(x)$ be the evaluation of x under 5 different criteria with importances w_j. Table 4.2 shows the r-OWAs for different values of r, in which the criteria have been ordered from largest values of $g_k(x)$ to smallest. Consider the case $r = 0.5$:

1. As $g_k(x)$ are already ordered for largest to smallest, the values of T_k are:

$$T_1 = 0.2,\ T_2 = 0.2 + 0.1 = 0.3,\ T_3 = 0.6,\ T_4 = 0.85,\ T_5 = 1$$

2. The values of T_k under f:

$$f(T_1) = \frac{0.2}{0.5},\ f(T_2) = \frac{0.3}{0.5},\ f(T_3) = f(T_4) = f(T_5) = 1$$

3. The weights of the OWA:

$$\lambda_1 = \frac{0.2}{0.5},\ \lambda_2 = \frac{0.3 - 0.2}{0.5} = \frac{0.1}{0.5},\ \lambda_3 = 1 - \frac{0.3}{0.5} = \frac{0.2}{0.5},\ \lambda_4 = \lambda_5 = 0$$

4. Consequently the r-OWA is:

$$r\text{-OWA} = \frac{0.2x_{(1)} + 0.1x_{(2)} + 0.2x_{(3)}}{0.5} = \frac{0.2 \cdot 10 + 0.1 \cdot 7 + 0.2 \cdot 4}{0.5} = 7$$

Remark 4.4. Given $x_1, \ldots, x_K$ and its associated importances $w_1, \ldots, w_K$, then the λ_k of the r-OWA are determined in such a way that:

$$O_r(x) = \max\left\{\frac{\tilde{\lambda}_1 x_1 + \cdots + \tilde{\lambda}_K x_K}{r} \mid \tilde{\lambda}_k \leq w_k, \sum \tilde{\lambda}_k = r\right\} \quad \text{with } \lambda_k = \frac{\tilde{\lambda}_k}{r}$$

Table 4.2: Small example of r-OWA for different values of r

	criterion					r		
	1	2	3	4	5	0.2	0.3	0.5
w_k	0.2	0.1	0.3	0.25	0.15	10	9	7
$g_k(x)$	10	7	4	3	2			

Given $r, \beta \in (0, 1]$ and $x \in X$, let us introduce the function $h_r^\beta(x)$ as the r-OWA of the β-averages. That is:

$$h_r^\beta(x) = O_r\left(g_1^\beta(x), \dots, g_K^\beta(x)\right)$$

Remark 4.5. If the importance of all criteria is the same ($w_k = \frac{1}{K}$ for all k) and $r = \frac{n}{K}$ with $n \in \{1, \dots, K\}$, then the $h_r^\beta(x)$ is the average of the n worst β-averages.

Definition 4.7 (Dominance)**.** Let x and y feasible solutions ($x, y \in X$) and $r, \beta \in (0, 1]$. Then x dominates y ($x \succsim y$) if $h_r^\beta(x) \leq h_r^\beta(y)$, where $h_r^\beta(x)$ is the r-OWA of the β-averages.

Definition 4.7 induces a domination relationship with the following properties:

Reflexivity Given x, $h_r^\beta(x) \geq h_r^\beta(x)$, and then $x \succsim x$, so $\succsim$ is reflexive.

Transitiveness Given $x \succsim y$, $y \succsim z$, we have $h_r^\beta(x) \geq h_r^\beta(y)$ y $h_r^\beta(y) \geq h_r^\beta(z)$, and then $h_r^\beta(x) \geq h_r^\beta(z)$, which leads to $x \succsim z$, and we conclude that $\succsim$ is transitive.

Antisymmetry Given $x \succsim y$, $y \succsim x$, we have $h_r^\beta(x) \geq h_r^\beta(y)$ and $h_r^\beta(y) \geq h_r^\beta(x)$, but from $h_r^\beta(x) = h_r^\beta(y)$ it cannot be guaranteed that $x = y$, and hence $\succsim$ is not antisymmetric.

4.3.2 Solution concept and dominance properties

Consider the multiobjective stochastic programming problem:

$$\min_{x \in X} \left(f_1(x, \omega), \dots, f_K(x, \omega)\right)$$

The previously defined concepts of β-average and r-OWA transform the MSP problem into a deterministic multiple objective problem, and then into a deterministic single objective problem.

$$MSP \rightarrow MOP \rightarrow LP(MIP)$$

$$f_k^j(x) \xrightarrow{\beta\text{-average}} g_k^\beta(x) \xrightarrow{r\text{-OWA}} h_r^\beta(x)$$

1. For every $x \in X$ there is a function f_k^j to be minimized which depends on the scenario j and the criterion k.

2. The problem is transformed into a deterministic one with multiple objectives (MOP) using the β-average concept.

3. Computing the r-OWA, each $x \in X$ is assigned a scalar. The problem consists of finding the x which minimizes this $h_r^\beta(x)$.

The solution procedure lies into what is usually called a *scalarization approach*. When obtaining a minimizer of $h_r^\beta(x)$ it is also desired that the optimal solution is efficient for the associated MOP problem:

$$\min_{x \in X} \left(g_1^\beta(x), \ldots, g_K^\beta(x) \right) \qquad \text{(MOP)}$$

Proposition 4.1. *Given x^* minimum of $h_r^\beta(x)$ the following statements hold:*

1. *x^* is not necessarily efficient for the MOP problem.*

2. *x^* is weakly efficient for the MOP problem.*

3. *If x^* is the only minimum of $h_r^\beta(x)$, then x^* is efficient.*

4. *Given x^* non-efficient, an alternative y^* can be found on a second phase such that y^* is efficient and $h_r^\beta(x^*) = h_r^\beta(y^*)$.*

Proof. Each part will be proven independently:

1. Consider the example displayed on Table 4.3, in which there are only two feasible solutions, two equiprobable scenarios ($\pi_1 = \pi_2 = \frac{1}{2}$), three equally important criteria ($w_1 = w_2 = w_3 = \frac{1}{3}$), and consider the values of $\beta = \frac{1}{2}$ and $r = \frac{2}{3}$ are taken.

   The β-averages are $(0.8, 0.4, 0.65)$ for the first alternative and $(0.8, 0.45, 0.65)$ for the second alternative. When computing the function h_r^β, both alternatives have an objective value of 0.725. Consequently, even though the second alternative is an optimal solution of h_r^β, it is not an efficient solution of the original problem as its β-averages are dominated by those of the first alternative.

2. If x^* is not weakly efficient of (MOP), then there exists y such that $g_k^\beta(y) < g_k^\beta(x^*)$ for all k. From that it follows that:

$$h_r^\beta(y) = \sum_k \lambda_k^y g_k^\beta(y) < \sum_k \lambda_k^y g_k^\beta(x^*)$$

Table 4.3: Values of two alternatives for each scenario j and criterion k, together with their β-averages ($\beta = \frac{1}{2}$) and r-OWAs ($r = \frac{2}{3}$)

Table 4.4: Alternative 1

	k_1	k_2	k_3
j_1	0.80	0.40	0.30
j_2	0.60	0.20	0.65
β-average	0.80	0.40	0.65
r-OWA		0.725	

Table 4.5: Alternative 2

	k_1	k_2	k_3
j_1	0.70	0.45	0.65
j_2	0.80	0.30	0.50
β-average	0.80	0.45	0.65
r-OWA		0.725	

where λ_k^y are the appropriate weights for calculating the OWA of y, as defined on Definition 4.6. Realizing that the weights of x^* have to be chosen in order to maximize the value of $\sum_k \lambda_k g_k^\beta(x^*)$ (Remark 4.4), it can be seen that:

$$\sum_k \lambda_k^y g_k^\beta(x^*) \leq \sum_k \lambda_k^{x^*} g_k^\beta(x^*) = h_r^\beta(x^*)$$

and thus, $h_r^\beta(y) < h_r^\beta(x^*)$, contradicting that x^* was an optimal of (MOP).

3. If x^* is not efficient of (MOP), then there exists y such that $g_k^\beta(y) \leq g_k^\beta(x^*)$ for all k, and for at least one k' the strict inequality holds. Following an analogous reasoning than before:

$$h_r^\beta(y) = \sum_k \lambda_k^y g_k^\beta(y) \leq \sum_k \lambda_k^y g_k^\beta(x^*) \leq \sum_k \lambda_k^{x^*} g_k^\beta(x^*) = h_r^\beta(x^*)$$

and hence $h_r^\beta(y) \leq h_r^\beta(x^*)$ which contradicts either that x^* was the optimal solution of h_r^β or that x^* is the only optimal.

4. Let h^* be the optimal value of $h_r^\beta(x)$ for a given r, and let x^* be an optimal solution of the problem:

$$\begin{aligned} \min \ & \sum_k g_k^\beta(x) \\ \text{s.t. } & h_r^\beta(x) \leq h^* \\ & x \in X \end{aligned} \tag{MP2}$$

Then, x^* is an efficient solution of (MOP). Suppose x^* is not an efficient solution. Then there exists $y \in X$ such that $g_k^\beta(y) \leq g_k^\beta(x^*)$ for all k (and for at least one k the strict inequality holds). There are two alternatives:

- $h_r^\beta(y) \leq h^*$. In this case y would be a feasible solution of (MP2) and since y dominating x^* implies $\sum_k g_k^\beta(y) < \sum_k g_k^\beta(x^*)$, that contradicts that x^* is an optimal solution of (MP2).
- $h_r^\beta(y) > h^*$. But this is not possible since $g_k^\beta(y) \leq g_k^\beta(x^*)$ implies $h_r^\beta(y) \leq h_r^\beta(x^*) = h^*$

□

4.4 Computing the minimum: discrete case

The solution concept proposed will be now applied, first with a discrete (and small) case to illustrate properly each step of the process to obtain the solution. When the solution space is discrete, and all feasible solutions can be explicitly enumerated, the steps are as follows:

Step 0 Normalize all objective functions $f_k^j(x)$.

Step 1 Set values for $\beta, r \in (0,1]$.

Step 2 For every $x \in X$ and every criterion define $g_k^\beta(x)$ as:

$$g_k^\beta(x) = \begin{array}{l}\textit{average of worse scenarios for criterion } k\\ \textit{with probabilities adding up to } \beta\end{array}$$

Step 3 Define $h_r^\beta(x)$ as:

$$h_r^\beta(x) = \begin{array}{l}\textit{average of worse } g_k^\beta(x) \textit{ values}\\ \textit{with importances adding up to } r\end{array}$$

Step 4 Search for $x \in X$ minimizing $h_r^\beta(x)$

Assume a decision space with only 4 alternatives ($X = \{x_1, x_2, x_3, x_4\}$), evaluated by under 5 different scenarios with 6 criteria. For each of those alternatives it can be computed the value of the functions $f_k^j(x)$ to be minimized. Table 4.6 shows the values of f, evaluated on feasible point x_1, by each of the scenarios and criteria considered.

Table 4.6: Values of alternative 1 by scenario (j) and criteria (k)

			criteria					
			$w_1 = 0.20$	$w_2 = 0.10$	$w_3 = 0.20$	$w_4 = 0.25$	$w_5 = 0.15$	$w_6 = 0.10$
			k_1	k_2	k_3	k_4	k_5	k_6
scenarios	$\pi_1 = 0.15$	j_1	0.51	0.27	0.39	0.45	0.75	0.76
	$\pi_2 = 0.20$	j_2	0.58	0.65	0.47	0.26	0.90	0.24
	$\pi_3 = 0.30$	j_3	0.48	0.44	0.90	0.50	0.93	0.65
	$\pi_4 = 0.25$	j_4	0.76	0.18	0.01	0.90	0.56	0.02
	$\pi_5 = 0.10$	j_5	0.86	0.36	0.21	0.28	0.63	0.72

The first step consists on calculating the β-averages. Let assume a value of $\beta = 0.3$:

1. For the first criterion the worst scenario is j_5, which has probability 0.1. The second worst is j_4, with a probability of 0.25. As the sum of those probabilities exceeds the β fixed, for computing the β-average just a probability of 0.2 is considered:

$$g_1^\beta(x_1) = \frac{0.1 \cdot 0.86 + 0.2 \cdot 0.76}{0.3} = 0.793$$

2. $g_2^\beta(x_1) = (0.2 \cdot 0.65 + 0.1 \cdot 0.44)/0.3 = 0.580$

3. $g_3^\beta(x_1) = (0.3 \cdot 0.90)/0.3 = 0.900$

4. $g_4^\beta(x_1) = 0.833$, $g_5^\beta(x_1) = 0.930$, $g_6^\beta(x_1) = 0.728$

The last step is calculating the function $h_r^\beta(x)$, that is, the r-OWA of the β-averages. Table 4.7 calculates the r-OWA, and shows as well the information of the previously calculated β-averages, when the value of $r = 0.17$ is taken.

Table 4.7: Values of alternative 1 by scenario (j) and criteria (k)

			criteria					
			$w_1 = 0.20$	$w_2 = 0.10$	$w_3 = 0.20$	$w_4 = 0.25$	$w_5 = 0.15$	$w_6 = 0.10$
			k_1	k_2	k_3	k_4	k_5	k_6
scenarios	$\pi_1 = 0.15$	j_1	0.51	0.27	0.39	0.45	0.75	0.76
	$\pi_2 = 0.20$	j_2	0.58	0.65	0.47	0.26	0.90	0.24
	$\pi_3 = 0.30$	j_3	0.48	0.44	0.90	0.50	0.93	0.65
	$\pi_4 = 0.25$	j_4	0.76	0.18	0.01	0.90	0.56	0.02
	$\pi_5 = 0.10$	j_5	0.86	0.36	0.21	0.28	0.63	0.72
β-average, $\beta = 0.30$			0.793	0.580	0.900	0.833	0.930	0.728
r-OWA, $r = 0.17$			0.927					

Results The values of the functions for the other alternatives, as well as its β-averages and r-OWAs are shown in Tables 4.13 to 4.15, starting on page 79. A summary of the results can be seen in Table 4.8, where all the β-averages and r-OWAs are shown, determining that the optimal alternative for the values of β and r given is Alternative 1.

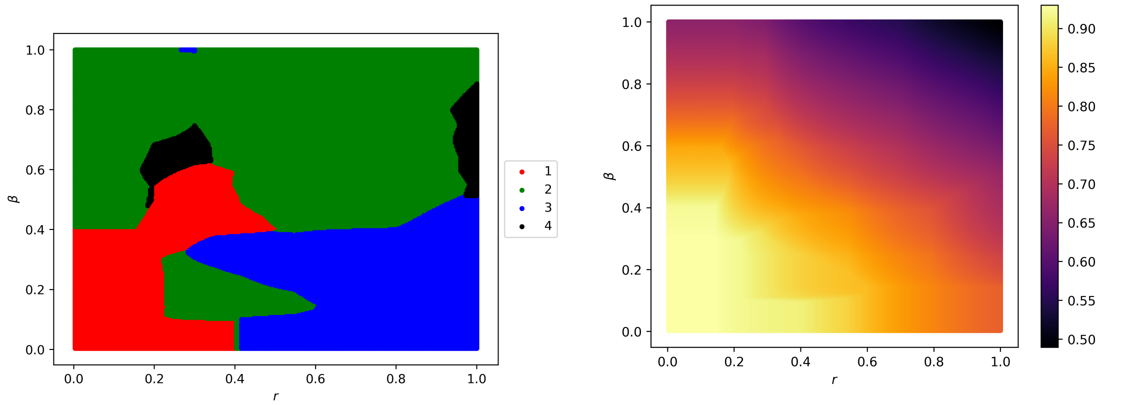
Table 4.8: β-averages and r-OWAs for each of the 4 feasible alternatives of the example

	β-averages						r-OWA
	$g_1^\beta(x)$	$g_2^\beta(x)$	$g_3^\beta(x)$	$g_4^\beta(x)$	$g_5^\beta(x)$	$g_6^\beta(x)$	$h_r^\beta(x)$
Alternative 1	0.793	0.580	0.900	0.833	0.930	0.728	0.927
Alternative 2	0.930	0.832	0.703	0.820	0.660	0.770	0.930
Alternative 3	0.765	0.775	0.468	0.643	0.950	0.883	0.943
Alternative 4	0.993	0.760	0.473	0.773	0.820	0.990	0.993

Variations on β and r yield very different results. Figure 4.1a shows which of the four alternatives has the lowest h value, depending on the values of β and r.

Figure 4.1b shows the optimal objective value when varying the parameters β and r. It can be appreciated how h decreases when β and r increase. This is due to the fact that the original

f_k^j functions are to be minimized, and the larger the parameters β and r are, more favourable scenarios/criteria will take part on the computation of $h_r^\beta(x)$, hence decreasing its optimal value.



(a) Optimal alternative for some values of r and β

(b) Optimal values of function $h_r^\beta(x)$ for some values of r and β

Figure 4.1: Results from illustrative example

4.5 Computing the minimum: continuous case

A concept of solution was proposed with Definition 4.7. When the functions $f_k^j(x)$ to be minimized are given, a new function $h_r^\beta(x)$ to be minimized is defined, with parameters β and r such that $h_r^\beta(x)$ is the r-OWA of the β-averages. If the decision space is sufficiently small, the procedure shown in Section 4.4 obtains such a solution.

In this section, a mathematical programming model will be developed to obtain the minimum of $h_r^\beta(x)$ which allows one to obtain the proposed solution for bigger decision spaces, including continuous ones.

Given k and $x \in X$ we have the vector $\left(f_k^1(x), \ldots, f_k^J(x)\right)$. Let $\left(f_k^{(1)}(x), \ldots, f_k^{(J)}(x)\right)$ be the ordered vector such that $f_k^{(j_1)}(x) \geq f_k^{(j_2)}(x)$ when $j_1 \leq j_2$.

Given $\beta \in (0,1]$, let $\hat{\jmath}$ be the ordered scenario such that:

$$\sum_{j=1}^{\hat{\jmath}} \pi_{(j)} \geq \beta, \qquad \sum_{j=1}^{\hat{\jmath}-1} \pi_{(j)} < \beta$$

Alternatively:

$$f_k^{(1)}(x) \geq f_k^{(2)}(x) \geq \cdots \geq f_k^{(\hat{\jmath})}(x) \geq f_k^{(\hat{\jmath}+1)}(x) \geq \cdots \geq f_k^{(J)}(x)$$

$$1 = \underbrace{\underbrace{\pi_{(1)} + \pi_{(2)} + \cdots + \pi_{(\hat{\jmath}-1)}}_{<\beta} + \pi_{(\hat{\jmath})}}_{\geq\beta} + \cdots + \pi_{(J)}$$

Also let:

$$\hat{\pi}_j = \begin{cases} \pi_j & j \in \{(1), \ldots, (\hat{\jmath}-1)\} \\ \beta - \sum_{j=(1)}^{j=(\hat{\jmath}-1)} \pi_j & j = \hat{\jmath} \\ 0 & \text{otherwise} \end{cases}$$

The definition of $\hat{\pi}_{\hat{\jmath}}$ is made in such a way that $\sum_j \hat{\pi}_j = \beta$. In this way, the average of the β worst values can be computed as $\frac{1}{\beta}\sum_{j=1}^{J} \hat{\pi}_j f_k^{(j)}(x)$, which coincides with the definition of β-average (Definition 4.5). This computation can be written as the following optimization problem:

$$\begin{aligned} \max_{\tilde{u}_j} \quad & \frac{1}{\beta}\sum_{j=1}^{J} \tilde{u}_j \cdot f_k^j(x) \\ \text{s.t.} \quad & \sum_{j=1}^{J} \tilde{u}_j = \beta \\ & 0 \leq \tilde{u}_j \leq \pi_j \qquad j = 1, \ldots, J \end{aligned}$$

A more natural approach would be to consider $u_j = \frac{\tilde{u}_j}{\beta}$. These u_j represent the proportion in which scenario j plays a part on the aggregated β-average. Introducing that change, the model is:

$$\begin{aligned} \max_{u_j} \quad & \sum_{j=1}^{J} u_j \cdot f_k^j(x) \\ \text{s.t.} \quad & \sum_{j=1}^{J} u_j = 1 \\ & 0 \leq u_j \leq \frac{\pi_j}{\beta} \qquad j = 1, \ldots, J \end{aligned}$$

The dual formulation is:

$$\begin{aligned} \min_{z, y_j} \quad & z + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_j \\ \text{s.t.} \quad & z + y_j \geq f_k^j(x) \qquad j = 1, \ldots, J \\ & z \text{ free}, y_j \geq 0 \end{aligned} \tag{4.2}$$

And hence finding the $x \in X$ which minimizes the average of the worst β scenarios for a given k is:

$$\min_{x \in X} \begin{pmatrix} \max_{\tilde{u}_j} & \frac{1}{\beta}\sum_{j=1}^{J} \tilde{u}_j f_k^j(x) & \\ \text{s.t.} & \sum_{j=1}^{J} \tilde{u}_j = \beta & \\ & 0 \leq \tilde{u}_j \leq \pi_j & j = 1, \ldots, J \end{pmatrix}$$

Or alternatively:

$$\min_{x \in X} \begin{pmatrix} \min_{z, y_j} & z + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_j & \\ \text{s.t.} & z + y_j \geq f_k^j(x) & j = 1, \ldots, J \\ & z \text{ free}, y_j \geq 0 & j = 1, \ldots, J \end{pmatrix} \tag{4.3}$$

Which is equivalent to:

$$\min_{z,y_j,x} \quad z + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_j \tag{4.4a}$$

$$\text{s.t.} \quad z + y_j \geq f_k^j(x) \quad j = 1, \dots, J \tag{4.4b}$$

$$z \text{ free}, y_j \geq 0 \qquad j = 1, \dots, J$$

$$x \in X$$

Remark 4.6. Models (4.3) and (4.4) are equivalent, as for any $x \in X$ chosen in (4.4) the values z and y_j will get as small as allowed by constraint (4.4b), as this improves the objective function (4.4a). Consequently for every x, its β-average will be computed appropriately, and thus (4.4) obtains the $x \in X$ with smallest β-average, as desired on (4.3).

For every $k \in \{1, \dots, K\}$ thanks to the problem (4.2) the function $g_k^\beta(x)$ can be defined, which measures for each $x \in X$ the β-average for that criterion, being:

$$\begin{aligned} g_k^\beta(x) \equiv \min_{z_k, y_{kj}} z_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_{kj} & \\ \text{s.t. } z_k + y_{kj} \geq f_k^j(x) \quad & j = 1, \dots, J \\ z_k \text{ free}, y_{kj} \geq 0 \qquad & j = 1, \dots, J \end{aligned} \tag{4.5}$$

The already known approach for single criterion problems ends here. Given that, the next step is finding a "good" solution for all k. That is:

$$\min_{x \in X} \left(g_1^\beta(x), \dots, g_K^\beta(x) \right)$$

Given $r \in (0, 1]$ the r-OWA of the β-averages will be now computed (in accordance with the definition given in Section 4.3). That is, the solution of the following problem is sought:

$$\begin{aligned} \max_{\tilde{t}_k} \quad & \frac{1}{r} \sum_k \tilde{t}_k \cdot g_k^\beta(x) \\ & \sum_k \tilde{t}_k = r \\ & 0 \leq \tilde{t}_k \leq w_k \qquad k = 1, \dots, K \end{aligned}$$

Or equivalently:

$$\begin{aligned} \max_{t_k} \quad & \sum_k t_k \cdot g_k^\beta(x) \\ & \sum_k t_k = 1 \\ & 0 \leq t_k \leq \frac{w_k}{r} \qquad k = 1, \dots, K \end{aligned}$$

Its dual formulation is:

$$\begin{aligned}\min_{z,v_k} \quad & z + \sum_k \frac{w_k}{r} v_k \\ \text{s.t.} \quad & z + v_k \geq g_k^\beta(x) \quad k = 1, \ldots, K \\ & z \text{ free}, v_k \geq 0 \qquad k = 1, \ldots, K\end{aligned}$$

Replacing the value of $g_k^\beta(x)$ given in (4.5) the next model is obtained:

$$\min_{z,v_k} \quad z + \sum_k \frac{w_k}{r} v_k \tag{4.6a}$$

$$\text{s.t.} \quad z + v_k \geq \begin{pmatrix} \min\limits_{z_k, y_{kj}} & z_k + \sum_{j=1}^J \frac{\pi_j}{\beta} y_{kj} & \\ \text{s.t.} & z_k + y_{kj} \geq f_k^j(x) & \forall j \\ & z_k \text{ free}, y_{kj} \geq 0 & \end{pmatrix} \forall k \tag{4.6b}$$

$$z \text{ free}, v_k \geq 0 \qquad \forall k \tag{4.6c}$$

Model (4.6) calculates for a given $x \in X$ the r-OWA of its β-averages, which coincides with the notion of the function $h_r^\beta(x)$ given in Section 4.3. This problem is not explicit in that it contains nested optimization problems in the constraints. For that reason, we propose a single level alternative for $x \in X$ fixed.

Consider the following linear programming model:

$$\min_{z,v_k,z_k,y_{kj}} \quad z + \sum_k \frac{w_k}{r} v_k \tag{4.7a}$$

$$\text{s.t.} \quad z + v_k \geq z_k + \sum_{j=1}^J \frac{\pi_j}{\beta} y_{kj} \qquad \forall k \tag{4.7b}$$

$$z_k + y_{kj} \geq f_k^j(x) \qquad \forall k, j \tag{4.7c}$$

$$y_{kj} \geq 0 \qquad \forall k, j \tag{4.7d}$$

$$z_k \text{ free}, v_k \geq 0 \qquad \forall k \tag{4.7e}$$

$$z \text{ free} \tag{4.7f}$$

Proposition 4.2. *Transformation from model* (4.6) *to model* (4.7) *is valid, in that their optimal solution and objective values coincide.*

Proof. Let $\left(z^*, v_k^*, z_k^*, y_{kj}^*\right)$ be the optimal solution of model (4.7). (z^*, v_k^*) is feasible of model (4.6), and it will be shown that it is also optimal for such model. Assume it exists (z', v_k') feasible of model (4.6) with:

$$z' + \sum_k \frac{w_k}{r} v_k' < z^* + \sum_k \frac{w_k}{r} v_k^*$$

This and constraint (4.7b) implies there exists k_0 such that:

$$z' + v_{k_0}' < z_{k_0}^* + \sum_{j=1}^J \frac{\pi_j}{\beta} y_{k_0 j}^*$$

otherwise $\left(z', v'_k, z^*_k, y^*_{kj}\right)$ would be optimal of model (4.7). Since $z^*_{k_0}$ and $y^*_{k_0 j}$ are feasible of model (4.7) they are also feasible of the model on the right-hand side of constraint (4.6b), and thus z' and v'_{k_0} violate constraint (4.6b). □

Proposition 4.2 showed that the optimal solutions of models (4.6) and (4.7) coincide. Proposition 4.3 goes further showing the connection between their feasible sets.

Proposition 4.3. *The feasible set of model* (4.6) *is a projection of the feasible set of model* (4.7)*.*

Proof.

1. For each feasible solution (z, v_k) of model (4.6) there is at least one feasible solution of model (4.7) with same values (z, v_k), being so the same objective function.

   Let (z^1, v^1_k) a feasible solution of model (4.6), and (z^*_k, y^*_{kj}) the optimal solution where the minimum of the right-hand side of equation (4.6b) is reached for each k. Since constraints (4.7b), (4.7c), (4.7d) and (4.7e) are satisfied in model (4.6), $(z^1, v^1_k, z^*_k, y^*_{kj})$ is a feasible solution or model (4.7).

2. For each feasible solution (z, v_k, z_k, y_{kj}) of model (4.7), (z, v_k) is a feasible solution of model (4.6), being so the same objective function. Let $(z^2, v^2_k, z^2_k, y^2_{kj})$ a feasible solution of model (4.7). Since constraints (4.7b), (4.7c) and (4.7d) are included in model (4.7), (z^2_k, y^2_{kj}) is feasible for the model included in the right-hand side of constraint (4.6b) and therefore greater than or equal to the minimum of that model, verifying:

$$z^2 + v^2_k \geq z^2_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y^2_{kj} \geq \min \left\{ z_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_{kj} \right\}$$

   and so, feasible for model (4.6).

□

Finally after proving the validity of model (4.7) it is possible to let $x \in X$ free, with the purpose of finding the one minimizing the function $h^\beta_r(x)$:

$$\begin{aligned}
\min_{z, v_k, z_k, y_{kj}, x} \quad & z + \sum_k \frac{w_k}{r} v_k \\
\text{s.t.} \quad & z + v_k \geq z_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_{kj} && \forall k \\
& z_k + y_{kj} \geq f^j_k(x) && \forall k, j \\
& y_{kj} \geq 0 && \forall k, j \\
& z_k \text{ free}, v_k \geq 0 && \forall k \\
& z \text{ free} \\
& x \in X
\end{aligned}$$

4.6 Computational experiments: Knapsack problem

The multiobjective stochastic knapsack problem can illustrate the usefulness of the previously defined concept and the MP model.

Example 4.6 (Multiobjective stochastic knapsack problem)**.** Let I be a collection of objects with weights v_i, which can be selected as members of a knapsack with maximum weight V. There is a set of scenarios J, each of them with probability π_j, and a set of criteria K, with importances w_k. For every pair of scenario-criterion, there is a benefit associated with selecting object i, denoted by b^i_{jk}. Which objects should be taken in order to maximize benefit?

The above problem differs with the well-known knapsack problem in that there is stochasticity and multiple objectives to be maximized.

The following MSP model can be adapted to analyze the problem. Note that to preserve the sense of the optimization, rather than to maximize the benefits of the carried objects, it will be minimized the value of the objects not chosen.

$$\begin{aligned}
\min_{x_i} \quad & \left\{ f_k^j(\mathbf{x}) := \sum_i (1-x_i)\, b^i_{kj} \right\} & \\
\text{s.t.} \quad & \sum_i v_i x_i \leq V & \forall i \\
& x_i \in \{0,1\} & \forall i
\end{aligned} \tag{4.8}$$

When using the methodology developed in the previous sections, problem (4.8) is transformed into the following mixed-integer linear programming model:

$$\begin{aligned}
\min_{z,v_k,z_k,y_{kj},x_i} \quad & z + \sum_k \frac{w_k}{r} v_k & \\
\text{s.t.} \quad & z + v_k \geq z_k + \sum_{j=1}^{J} \frac{\pi_j}{\beta} y_{kj} & \forall k \\
& z_k + y_{kj} \geq \sum_i (1-x_i)\, b^i_{kj} & \forall k, j \\
& \sum_i v_i x_i \leq V & \forall i \\
& y_{kj} \geq 0 & \forall k, j \\
& x_i \in \{0,1\} & \forall i \\
& z_k \text{ free}, v_k \geq 0 & \forall k \\
& z \text{ free} &
\end{aligned} \tag{MSP}$$

For given $r, \beta \in (0,1]$, model (MSP) obtains the $\mathbf{x}^*$ minimizing the r-OWA of the β-averages. In order to illustrate the benefits of using model (MSP), a naive way of solving problem (4.8) is considered:

$$\begin{aligned} \min_{x_i} \quad & \sum_{k,j} w_k \pi_j \sum_i (1 - x_i)\, b^i_{kj} \\ \text{s.t.} \quad & \sum_i v_i x_i \le V && \forall i \\ & x_i \in \{0,1\} && \forall i \end{aligned} \tag{MIP}$$

Hence model (MIP) computes the average of the f^j_k, using the importances of the criteria and the probability of the scenarios. It is clear that for "average" criteria-scenarios x^*_{MIP}, the optimal solution of model (MIP), outperforms x^*_{MSP}, the optimal solution of model (MSP). Conversely x^*_{MSP} will improve x^*_{MIP} in unfavourable criteria-scenarios, as expected of a risk-averse solution.

4.6.1 Experiments design

The following sections will show computational experiments, for different values of r and β and different number of objects, scenarios and criteria. Algorithm 1 shows how the random instances are created, given a number of objects, scenarios and criteria.

Algorithm 1 Generating random data, with $\mathcal{U}(a,b)$ the uniform distribution in $[a,b]$

```
function RANDOMINSTANCE(|I|, |J|, |K|)
    p ← U(0.25, 0.75)              ▷ how many objects on average will fit in the knapsack
    W ← 1/p                        ▷ average weight of objects
    for i ∈ I do
        w_i ← U(0.5W, 1.5W)        ▷ weight of each object
        for j, k ∈ J × K do
            b^i_kj ← U(0, 1)       ▷ value of each object
        end for
    end for
end function
```

For each of the solved instances it will be recorded:

- $t_{\text{MSP}}, t_{\text{MIP}}$: Solution time in seconds of models (MSP) and (MIP). With them the following value is calculated:

$$\Delta_{\text{time}} := \frac{t_{\text{MSP}}}{t_{\text{MIP}}} \qquad \textit{(time penalty factor)}$$

  Δ_{time}, the time penalty factor, indicates how many more times does it take to solve the MSP model than the MIP one.

- $z^*_{\text{MSP}}, z^*_{\text{MIP}}$: Optimal values of the models.

- $f_{\text{MSP}}\left(x^*_{\text{MIP}}\right), f_{\text{MIP}}\left(x^*_{\text{MSP}}\right)$: Objective value of x^*_{MIP} in model (MSP) and vice versa.

- To grasp the difference between the MSP and the naive approach, the following will be calculated:

$$\Delta_{\text{avg}} := 100\frac{f_{\text{MIP}}\left(x^*_{\text{MSP}}\right) - z^*_{\text{MIP}}}{z^*_{\text{MIP}}} \qquad \textit{(deteriorating rate)}$$
$$\Delta_{\text{tail}} := 100\frac{f_{\text{MSP}}\left(x^*_{\text{MIP}}\right) - z^*_{\text{MSP}}}{f_{\text{MSP}}\left(x^*_{\text{MIP}}\right)} \qquad \textit{(improvement rate)}$$

  These quantities reflect what is the effect of making decision x^*_{MSP} instead of x^*_{MIP}. Large values of Δ_{avg} indicate high penalties for making decision x^*_{MSP} instead of x^*_{MIP} in average scenarios-criteria. Similarly, the larger Δ_{tail} is, the higher benefit is obtained from making decision x^*_{MSP} in tail events. They will be called *deteriorating rate* and *improvement rate*

Models are solved in GAMS 26.1.0 with solver IBM ILOG CPLEX Cplex 12.8.0.0, using a personal computer with an Intel Core i7 processor and 16Gb RAM.

Experiment 1 First experiment will consist on a full factorial design, in which the values of $|I|$, $|J|$, $|K|$, r and β fall in these sets:

- $|I| \in \{50, 100, 200\}$
- $|J| \in \{5, 25, 100\}$
- $|K| \in \{3, 6, 9\}$
- $r \in \{0.33, 0.5, 0.67\}$
- $\beta \in \{0.05, 0.1, 0.5\}$

For each tuple (I, J, K) random data will be generated, using Algorithm 1, which will then be solved for every pair (r, β). All criteria and scenarios are given same importance and probabilities. That is, $w_k = \frac{1}{|K|}$, $\pi_j = \frac{1}{|J|}$. Time limit was set in two hours by instance, in which all but three of the $3^5 = 243$ configurations were solved to optimality.

Experiment 2 For the next experiment 100 random instances will be created, keeping the values of $|I|, |J|, |K|, r, \beta$ constant and equal to the median value of the previous experiment. That is, $|I| = 100, |J| = 25, |K| = 6, r = 0.5, \beta = 0.1$. All criteria and scenarios are given same importance and probabilities. All 100 instances were solved to optimality.

4.6.2 Results

Experiment 1 Table 4.16 (at the end of the chapter) shows for each of the 243 instances the solution times of the MSP and the MIP models, and the deteriorating and improvement rates of using the MSP solution instead of the MIP solutions (measured in deviation to MIP solution).

Table 4.9 shows the correlations between times and rates with the parameters of the instance. It can be seen how the MSP solution has a higher impact when fewer scenarios are considered. In addition to that, it can be appreciated that the MSP solution times decrease when β increase, that is, when more scenarios are included in the β-average computation.

Table 4.9: Correlations

	|I|	|J|	|K|	r	β
t_{MSP}	0.34	0.09	-0.11	-0.05	-0.19
t_{MIP}	0.51	0.18	-0.14	-0.03	-0.07
Δ_{time}	0.31	0.11	-0.08	-0.02	-0.18
Δ_{avg}	-0.05	-0.57	-0.28	-0.09	-0.36
Δ_{tail}	-0.07	-0.56	-0.18	-0.21	-0.50

This appreciation is confirmed by Table 4.10, in which it can be seen that the median *time penalty factor* (how many more times does it take to solve the MSP model than the MIP one) is much smaller when $\beta = 0.5$ than when $\beta = 0.05$.

Table 4.10: Increase on computing times and MSP solution times, grouped by β

	Δ_{time}					t_{MSP}				
β	min	mean	median	max	std	min	mean	median	max	std
0.05	0.94	3188.96	32.77	50473.04	9472.55	0.12	659.49	6.32	7222.95	1787.07
0.10	0.98	1002.35	11.09	20192.48	3245.85	0.12	212.47	2.23	4765.42	728.49
0.50	1.06	19.14	3.75	414.29	55.51	0.13	3.49	0.67	62.14	9.05

Solution times of the MSP model are alarmingly high for some instances, due to the fact that the admissible integrality gap has been set to zero. If that is relaxed, it can be seen that all of the 243 instances reach an integrality gap smaller than 5% in under 3 seconds, 2% in under 5 seconds and 1% in under 88 seconds.

Table 4.11 groups instances by r and β, and shows the deteriorating and improvement rates. It can be seen that the improvement rate (in the tail) is generally higher than the deteriorating rate (in the average), especially in cases with small r and β.

This claim is also supported with Fig. 4.2, where each of the 243 instances is shown according to the values of Δ_{avg} and Δ_{tail}, and grouped by the values of (r, β). Almost all of the instances ara above the imaginary line $\Delta_{\text{avg}} = \Delta_{\text{tail}}$, which shows that considering the MSP solution improves in the tail more than it loses in the average situations. In addition to that, it can be seen that the largest improvements in the tail are on instances with $\beta = 0.05$ (one of the usual values taken for CVaR), and especially with the smallest values of r. When r and β grow the differences between

Table 4.11: Values of Δ_{avg} and Δ_{tail}, grouped by r and β

		Δ_{avg}					Δ_{tail}				
r	β	min	mean	median	max	std	min	mean	median	max	std
0.33	0.05	0.03	1.94	1.87	5.68	1.42	0.28	4.37	4.21	9.18	2.43
	0.10	0.02	1.70	1.61	5.68	1.44	0.18	3.54	2.85	9.18	2.42
	0.50	0.00	0.93	0.52	4.46	1.08	0.00	1.57	0.92	4.99	1.46
0.50	0.05	0.03	1.87	1.90	4.30	1.30	0.29	3.58	3.30	6.73	1.89
	0.10	0.02	1.65	1.14	4.30	1.37	0.13	2.87	2.47	6.59	1.86
	0.50	0.00	0.72	0.54	3.51	0.75	0.00	1.07	0.79	3.79	1.01
0.67	0.05	0.03	1.64	1.17	3.93	1.24	0.32	3.04	3.06	6.15	1.62
	0.10	0.01	1.50	1.10	3.93	1.31	0.12	2.43	2.02	5.84	1.58
	0.50	0.00	0.60	0.50	3.16	0.66	0.00	0.80	0.59	3.64	0.81

the MIP and MSP solutions are reduced.

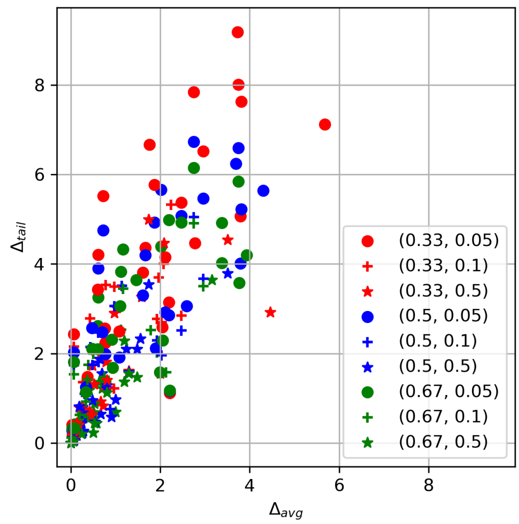


Figure 4.2: Values Δ_{avg} and Δ_{tail} for each of the 243 instances, grouped by values of (r, β)

Experiment 2 Table 4.17 (at the end of the chapter) contains the results for each of the 100 instances, all of them with constant parameters $|I| = 100, |J| = 25, |K| = 6, r = 0.5, \beta = 0.1$.

Table 4.12 contains a summary of the results, where it is again seen that the improvements in the tail are better than the loses in the average situations. Although single instances might take a long computing time, the median MSP solution time (3.74s) is definitely satisfactory. It is worth mentioning that the models were implemented without providing any extra bounds or known cuts that could reduce solution times.

Table 4.12: Summary of set 6

	t_{MSP}	t_{MIP}	Δ_{time}	Δ_{avg}	Δ_{tail}
mean	16.98	0.20	91.31	2.03	3.09
std	46.57	0.03	254.68	1.12	1.49
min	0.53	0.14	2.81	0.16	0.86
25%	1.37	0.17	6.73	1.18	2.09
50%	3.74	0.19	19.72	1.93	2.81
75%	15.50	0.21	86.19	2.52	3.51
max	404.70	0.34	2175.82	5.67	8.57

Finally, Fig. 4.3 (at the end of the chapter) shows the values of $f_k^j(x)$, where $x = x^*_{\text{MIP}}$ in blue squares and $x = x^*_{\text{MSP}}$ in orange circles, for just one of the created instances. It can be appreciated how x^*_{MIP} performs better than x^*_{MSP} in average criteria-scenarios, but x^*_{MSP} is better with unfavourable situations.

4.7 Discussion

A new concept of solution has been proposed for Multiobjective Stochastic Programming problems, focused on suitable risk-aversion situations. This concept can be particularly useful in real-life situations where there exists a great concern with respect to unfavourable situations, such as emergency management (as it will be shown later in wildfire management).

The solution concept is supported by an efficient way to compute it by a Mathematical Programming problem. This model is linear provided that the underlying problem can be linearly representable. Numerical experiments have been conducted for validating this approach, solving a multiobjective stochastic knapsack problem. This problem has been chosen as it is a natural extension of a well-known problem, and the significant part of this chapter is the risk-aversion concept and model rather than the underlying optimization problem.

The research has also shown that the improvements in the tail (unfavourable situations) are consistently higher than loses on average situations, especially when small values of the parameters β and r are chosen. However, although these differences are clearly noticeable, they are not as high as one could expect. This is possibly due to the randomness of the data. It is reasonable to assume that in actual real-life problems there are choices that are more conservative for every scenario and criterion, and thus being preferable for risk-aversion attitudes. Next chapter will be answering this question.

Results have also shown that there is a clear increase in computational time; however, this could be deemed acceptable as a price to pay for risk-aversion as long as the model is still tractable. Moreover, by allowing small integrality gaps (1%), good solutions are achieved in more than acceptable runtimes.

One of the possible applications of this research, in which risk-aversion solutions are desired, is emergencies. Emergencies have multiple conflicting objectives, and it could be decisive to have them all controlled, even the least important ones. This is also true if multiple scenarios are present, as decisions made should be studied for every scenario. The proposed risk-aversion model obtains solutions in which favourable scenarios do not compensate unfavourable ones, or favourable objectives compensate unfavourable ones (which can happen if using a simple average on the scenarios or OWA with the criteria).

This risk-averse methodology for the MSP problem will be applied in the upcoming chapter to the prescribed burning problem studied in Chapter 3.

4.A Extra figures and tables

Table 4.13: Values of alternative 2 by scenario (j) and criteria (k)

			criteria					
			$w_1 = 0.20$	$w_2 = 0.10$	$w_3 = 0.20$	$w_4 = 0.25$	$w_5 = 0.15$	$w_6 = 0.10$
			k_1	k_2	k_3	k_4	k_5	k_6
scenarios	$\pi_1 = 0.15$	j_1	0.40	0.58	0.39	0.45	0.54	0.18
	$\pi_2 = 0.20$	j_2	0.68	0.74	0.70	0.15	0.54	0.72
	$\pi_3 = 0.30$	j_3	0.93	0.52	0.23	0.82	0.21	0.03
	$\pi_4 = 0.25$	j_4	0.37	0.85	0.07	0.42	0.52	0.22
	$\pi_5 = 0.10$	j_5	0.92	0.13	0.71	0.39	0.90	0.87
β-average, $\beta = 0.30$			0.930	0.832	0.703	0.820	0.660	0.770
r-OWA, $r = 0.17$			0.930					

Table 4.14: Values of alternative 3 by scenario (j) and criteria (k)

			criteria					
			$w_1 = 0.20$	$w_2 = 0.10$	$w_3 = 0.20$	$w_4 = 0.25$	$w_5 = 0.15$	$w_6 = 0.10$
			k_1	k_2	k_3	k_4	k_5	k_6
scenarios	$\pi_1 = 0.15$	j_1	0.80	0.90	0.61	0.28	0.94	0.09
	$\pi_2 = 0.20$	j_2	0.29	0.48	0.26	0.23	0.21	0.07
	$\pi_3 = 0.30$	j_3	0.73	0.65	0.32	0.56	0.95	0.65
	$\pi_4 = 0.25$	j_4	0.58	0.39	0.21	0.66	0.70	0.93
	$\pi_5 = 0.10$	j_5	0.73	0.22	0.33	0.31	0.32	0.38
β-average, $\beta = 0.30$			0.765	0.775	0.468	0.643	0.950	0.883
r-OWA, $r = 0.17$			0.943					

Table 4.15: Values of alternative 4 by scenario (j) and criteria (k)

			criteria					
			$w_1 = 0.20$	$w_2 = 0.10$	$w_3 = 0.20$	$w_4 = 0.25$	$w_5 = 0.15$	$w_6 = 0.10$
			k_1	k_2	k_3	k_4	k_5	k_6
scenarios	$\pi_1 = 0.15$	j_1	0.30	0.52	0.12	0.68	0.46	0.73
	$\pi_2 = 0.20$	j_2	1.00	0.57	0.46	0.82	0.90	0.72
	$\pi_3 = 0.30$	j_3	0.18	0.76	0.30	0.34	0.54	0.99
	$\pi_4 = 0.25$	j_4	0.53	0.21	0.13	0.12	0.66	0.86
	$\pi_5 = 0.10$	j_5	0.98	0.46	0.50	0.29	0.27	0.40
β-average, $\beta = 0.30$			0.993	0.760	0.473	0.773	0.820	0.990
r-OWA, $r = 0.17$			0.993					

Table 4.16: All instances of first experiment. The three instances with 200 objects, 100 scenarios, 6 criteria and $\beta = 0.05$ did not reach the optimal solution in 2 hours. The integrality gaps of the solution shown are $0.31\%, 0.24\%$ and 0.19% for $r = 0.33, 0.5$ and 0.67 respectively

		$\beta \rightarrow$	0.05												0.1												0.5											
		$r \rightarrow$	0.33				0.5				0.67				0.33				0.5				0.67				0.33				0.5				0.67			
|I|	|J|	|K|	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}
50	5	3	0.12	0.13	3.75	8.01	0.15	0.12	3.75	8.01	0.18	0.14	3.51	4.54	0.14	0.14	3.75	6.59	0.12	0.12	3.75	6.59	0.20	0.17	3.51	3.79	0.12	0.12	3.75	5.84	0.12	0.12	3.75	5.84	0.13	0.12	3.16	3.64
		6	0.22	0.11	3.79	5.07	0.25	0.11	3.79	5.07	0.18	0.11	1.03	3.01	0.30	0.13	3.79	4.01	0.25	0.13	3.79	4.01	0.23	0.15	1.48	2.10	0.23	0.16	3.77	3.58	0.23	0.14	3.77	3.58	0.18	0.17	1.48	1.47
		9	0.28	0.14	1.76	6.67	0.36	0.14	1.76	6.67	0.20	0.15	1.58	3.26	0.22	0.14	2.02	5.66	0.21	0.14	2.02	5.66	0.22	0.14	1.24	2.10	0.20	0.15	2.02	4.39	0.22	0.13	2.02	4.39	0.25	0.15	1.20	1.37
	25	3	0.76	0.18	2.47	5.37	0.66	0.17	2.47	2.85	0.26	0.17	2.00	1.55	0.64	0.18	2.47	5.08	0.62	0.16	2.47	2.51	0.25	0.15	1.00	0.97	0.60	0.17	2.47	4.93	0.51	0.16	1.79	2.52	0.26	0.14	1.00	0.69
		6	1.20	0.16	1.87	5.77	0.91	0.29	1.96	3.70	0.49	0.18	0.79	1.81	1.15	0.18	1.87	4.93	0.74	0.18	1.14	3.51	0.41	0.18	0.70	1.55	0.93	0.16	1.17	4.33	0.78	0.18	1.17	3.45	0.38	0.16	0.71	1.22
		9	0.69	0.16	0.61	4.21	0.78	0.16	0.43	2.78	0.57	0.16	0.67	0.92	0.52	0.15	0.61	3.90	0.96	0.16	0.44	2.14	0.61	0.16	0.67	0.65	0.69	0.15	0.61	3.25	1.02	0.18	0.39	1.75	0.47	0.18	0.51	0.68
	100	3	1.15	0.15	0.07	2.43	0.78	0.15	0.07	2.15	0.44	0.14	0.07	0.16	1.07	0.14	0.07	2.02	0.83	0.19	0.07	1.74	0.34	0.14	0.07	0.16	1.14	0.15	0.07	1.81	0.85	0.16	0.07	1.53	0.36	0.14	0.07	0.14
		6	2.51	0.20	0.77	2.24	4.45	0.22	0.78	1.19	3.52	0.20	0.23	0.47	2.62	0.25	0.77	1.99	5.09	0.18	0.31	1.10	5.45	0.20	0.19	0.29	2.70	0.22	0.77	1.38	3.31	0.19	0.47	0.63	5.59	0.19	0.23	0.27
		9	4.06	0.18	0.03	0.41	1.47	0.17	0.03	0.30	1.07	0.16	0.00	0.00	2.75	0.17	0.03	0.29	1.12	0.16	0.03	0.30	1.11	0.16	0.00	0.00	2.06	0.19	0.03	0.32	1.10	0.16	0.03	0.18	1.16	0.17	0.00	0.00
100	5	3	1.24	0.26	3.81	7.63	1.29	0.20	3.81	7.63	0.37	0.22	2.08	4.47	0.72	0.18	3.81	5.22	0.66	0.18	3.81	5.22	0.32	0.27	1.56	2.33	0.74	0.19	3.38	4.02	1.13	0.23	3.38	4.02	0.28	0.20	0.63	1.36
		6	8.68	0.22	5.68	7.12	8.69	0.19	5.68	7.12	0.43	0.17	4.46	2.92	0.65	0.20	4.30	5.64	1.06	0.19	4.30	5.64	0.35	0.17	0.81	1.28	1.18	0.22	3.93	4.20	0.64	0.18	3.93	4.20	0.28	0.18	0.53	0.88
		9	3.31	0.18	2.19	3.14	3.26	0.20	2.19	3.14	0.67	0.18	0.97	2.90	1.04	0.20	2.19	2.86	0.96	0.20	2.19	2.86	0.23	0.14	0.64	1.88	1.02	0.17	0.92	2.31	0.90	0.17	0.92	2.31	0.24	0.18	0.41	1.18
	25	3	10.65	0.17	2.96	6.52	3.39	0.18	2.07	4.00	0.29	0.15	0.48	1.73	7.09	0.18	2.96	5.46	1.83	0.19	2.96	3.67	0.30	0.14	0.48	0.95	3.46	0.19	2.19	4.98	1.30	0.16	2.96	3.50	0.34	0.16	0.41	0.59
		6	32.12	0.20	2.78	4.47	9.18	0.19	0.78	3.53	0.44	0.18	0.52	1.31	26.53	0.18	2.59	3.06	3.64	0.22	0.61	2.47	0.32	0.15	0.26	0.79	12.77	0.17	0.60	2.62	0.90	0.17	0.50	2.00	0.41	0.17	0.26	0.65
		9	8.58	0.18	0.72	5.52	1.90	0.17	0.97	3.49	0.42	0.18	0.24	0.82	6.32	0.19	0.72	4.75	1.24	0.16	0.97	3.06	0.51	0.20	0.24	0.44	1.60	0.19	1.12	3.83	0.88	0.19	1.12	2.53	0.59	0.17	0.50	0.23
	100	3	51.23	0.22	2.21	1.12	1.67	0.21	0.27	1.36	0.82	0.18	0.09	0.25	22.70	0.21	2.21	1.16	1.22	0.19	0.34	1.05	0.81	0.18	0.05	0.17	18.75	0.16	2.21	1.17	0.84	0.22	0.34	0.92	0.75	0.18	0.05	0.13
		6	48.25	0.18	0.76	2.56	31.87	0.17	0.62	2.05	62.14	0.15	0.42	0.70	24.73	0.18	0.71	2.48	27.08	0.18	0.62	1.81	42.18	0.19	0.28	0.55	20.26	0.17	0.60	2.10	22.09	0.20	0.75	1.48	7.79	0.20	0.17	0.50
		9	2.16	0.19	0.37	1.48	3.34	0.18	0.29	0.77	1.84	0.17	0.18	0.41	1.80	0.17	0.34	1.25	2.87	0.20	0.28	0.69	2.22	0.19	0.26	0.22	1.67	0.18	0.34	1.14	2.77	0.19	0.20	0.63	3.09	0.16	0.08	0.13
200	5	3	146.24	0.23	1.61	3.81	140.12	0.20	1.61	3.81	7.71	0.23	1.30	1.61	151.22	0.21	1.61	3.30	135.09	0.24	1.61	3.30	4.60	0.21	1.30	1.58	83.44	0.22	1.10	3.06	89.20	0.21	1.10	3.06	4.21	0.22	1.30	1.55
		6	88.70	0.19	1.08	2.50	89.69	0.19	1.08	2.50	5.14	0.17	0.72	0.83	96.44	0.19	1.08	1.91	91.66	0.18	1.08	1.91	2.93	0.18	0.91	0.58	39.26	0.18	0.94	1.68	32.92	0.18	0.94	1.68	0.70	0.17	0.58	0.44
		9	468.37	0.15	3.73	9.18	484.89	0.14	3.73	9.18	29.46	0.16	1.74	4.99	304.04	0.16	3.69	6.24	305.90	0.16	3.69	6.24	2.71	0.16	1.74	3.54	110.03	0.15	3.38	4.92	107.34	0.14	3.38	4.92	0.91	0.17	1.20	2.28
	25	3	5629.58	0.33	2.75	7.84	4765.42	0.24	2.24	5.33	4.86	0.24	0.81	1.40	5430.90	0.25	2.75	6.73	3394.56	0.24	2.75	5.05	5.32	0.28	0.81	1.22	6896.05	0.25	2.75	6.15	2546.43	0.21	2.75	4.91	5.66	0.34	0.81	1.13
		6	2886.13	0.19	1.67	4.36	146.48	0.17	1.93	2.77	0.57	0.17	0.19	0.79	1651.91	0.21	1.67	4.20	15.06	0.22	1.93	2.29	0.71	0.21	0.19	0.81	93.66	0.19	1.46	3.64	19.36	0.19	1.93	2.02	0.55	0.18	0.12	0.40
		9	1235.12	0.32	2.05	2.59	342.32	0.22	0.96	1.22	1.99	0.21	0.22	0.26	404.70	0.29	1.90	2.12	28.09	0.21	0.88	0.76	0.82	0.20	0.13	0.17	99.73	0.21	1.99	1.58	2.23	0.22	0.39	0.58	0.87	0.20	0.06	0.08
	100	3	703.05	0.23	2.11	4.15	373.65	0.22	2.03	2.70	1.42	0.22	0.47	0.63	731.09	0.22	2.11	2.92	157.29	0.20	2.03	1.96	1.11	0.22	0.54	0.47	596.88	0.27	2.06	2.29	349.78	0.30	2.13	1.58	3.22	0.29	0.53	0.44
		6	7222.95	0.22	0.60	3.43	1814.25	0.18	0.48	2.08	22.11	0.21	0.13	0.44	7217.64	0.14	0.47	2.57	916.42	0.21	0.48	1.75	7.04	0.24	0.28	0.27	7216.94	0.15	0.47	2.11	656.48	0.20	0.37	1.41	7.89	0.22	0.24	0.20
		9	3321.23	0.34	0.07	0.28	16.40	0.20	0.02	0.18	2.33	0.17	0.08	0.08	198.14	0.19	0.07	0.32	14.84	0.21	0.02	0.13	2.31	0.21	0.08	0.05	47.16	0.23	0.08	0.33	9.77	0.21	0.01	0.12	2.63	0.20	0.06	0.04

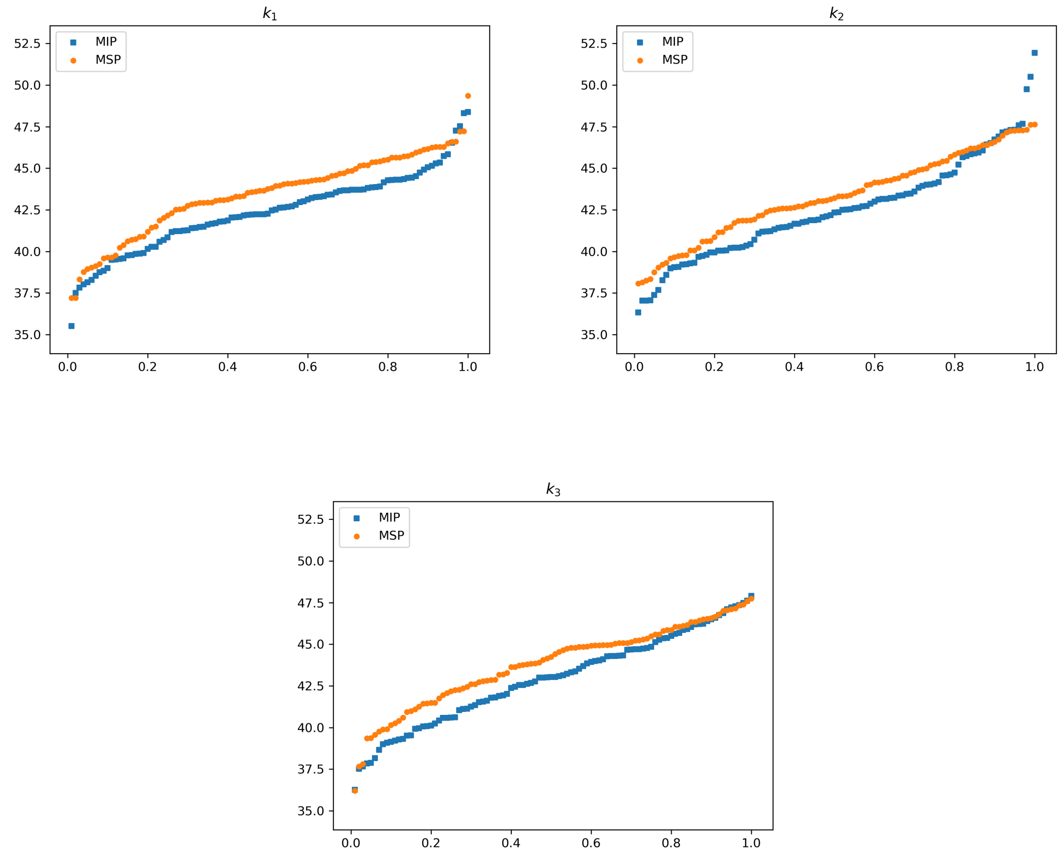


Figure 4.3: Single instance with 100 scenarios and 3 criteria. For each k, sorted values of $f_k^j(x)$, where $x = x^*_{\text{MIP}}$ in blue squares and $x = x^*_{\text{MSP}}$ in orange circles

Table 4.17: All instances of second experiment. $|I| = 100, |J| = 25, |K| = 6, r = 0.5, \beta = 0.1$

t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}	t_{MSP}	t_{MIP}	Δ_{avg}	Δ_{tail}
31.15	0.23	1.53	3.20	0.64	0.20	0.87	1.39	10.48	0.23	2.50	2.29
1.92	0.21	1.66	6.17	0.97	0.23	1.77	2.19	1.63	0.24	2.08	2.29
8.75	0.24	0.52	3.07	0.53	0.18	1.95	2.04	10.78	0.18	0.34	1.80
28.06	0.23	5.08	2.86	7.24	0.22	2.21	1.68	38.80	0.20	1.96	4.69
1.36	0.30	1.00	1.80	0.87	0.25	0.71	1.42	19.74	0.24	0.65	2.20
3.67	0.20	2.27	2.50	8.51	0.20	2.48	4.06	1.37	0.30	2.82	2.92
2.00	0.20	2.51	2.03	13.06	0.20	4.44	2.80	6.28	0.19	2.02	2.08
192.11	0.16	2.61	8.23	59.78	0.20	5.67	4.91	22.27	0.34	1.91	3.13
0.94	0.20	0.43	2.23	67.50	0.19	2.96	3.02	1.69	0.20	2.21	2.42
0.80	0.18	1.64	2.55	3.80	0.17	0.79	1.20	27.77	0.19	0.76	3.28
16.40	0.19	2.23	2.45	3.25	0.20	2.24	1.57	2.00	0.21	2.57	1.93
1.21	0.18	2.82	1.50	5.23	0.16	1.14	4.91	2.61	0.18	2.14	3.33
1.79	0.20	0.72	2.77	0.71	0.17	0.89	3.01	40.93	0.18	1.53	4.61
21.78	0.21	4.50	4.61	4.19	0.17	3.09	2.32	18.84	0.16	0.89	4.37
1.35	0.19	0.69	0.86	3.53	0.18	1.37	6.33	11.26	0.16	3.98	4.76
31.11	0.19	0.98	3.21	19.99	0.14	3.48	5.05	14.41	0.18	1.82	5.87
8.44	0.19	1.82	3.81	2.15	0.16	2.24	2.09	12.14	0.16	2.75	2.75
1.75	0.21	0.88	0.92	20.09	0.16	1.80	6.14	12.58	0.17	1.42	3.46
1.94	0.21	2.18	2.65	7.18	0.16	2.13	1.93	0.84	0.18	0.41	2.15
0.98	0.20	0.87	3.27	1.02	0.16	3.03	3.61	5.20	0.19	3.80	2.02
27.72	0.22	2.03	5.20	3.58	0.24	1.81	6.12	28.16	0.15	4.68	3.56
14.72	0.15	3.34	0.99	3.64	0.19	1.19	3.07	39.10	0.16	3.47	3.59
0.67	0.24	0.81	2.69	128.69	0.23	3.27	2.98	19.22	0.16	3.78	3.13
3.54	0.20	2.64	2.75	0.89	0.18	1.45	0.93	0.56	0.20	0.63	3.22
6.37	0.21	2.79	6.35	1.62	0.23	1.85	3.56	0.68	0.17	1.92	2.44
1.86	0.23	0.93	2.09	4.19	0.22	2.10	1.97	0.70	0.17	1.86	1.40
1.54	0.20	2.00	3.45	2.16	0.19	0.16	1.46	15.20	0.17	0.78	2.66
40.16	0.17	2.06	3.44	1.46	0.24	2.48	2.00	0.88	0.17	2.31	2.13
7.23	0.21	3.17	3.17	0.69	0.20	1.79	2.54	0.59	0.15	1.78	1.68
5.77	0.17	2.84	1.98	20.73	0.20	2.26	3.50	1.08	0.20	2.21	3.14
2.10	0.19	1.39	3.00	1.86	0.24	1.77	2.63	1.14	0.17	0.76	2.48
404.70	0.19	1.50	2.82	14.92	0.17	1.99	8.57	1.58	0.19	1.45	3.14
24.26	0.18	4.81	3.42	0.78	0.20	0.85	1.92	13.48	0.18	1.71	5.41
0.76	0.20	1.28	3.88								

Chapter 5

Mathematical programming with uncertainty and multiple objectives applied to wildfire management. Fuel management

Contents

5.1 Introduction

The previous chapter developed a methodology for obtaining risk-averse solutions in Multiobjective Stochastic Programming problems. The strengths of the solution concept and the model developed are shown now in this chapter, extending the prescribed burning problem studied in Chapter 3 into a Multiobjective Stochastic Programming problem. Such problem consisted of a landscape divided in burn units which were eligible for prescribed burning along a time horizon, with the goal of selecting which units to burn and when to reduce the amount of connections among units with old vegetation, while taking into account the ecological effects.

The following paper is under preparation, including the findings of this chapter:

León, J., Hearne, J. & Vitoriano, B. (under preparation), ‘A risk-aversion solution for the prescribed burning problem’.

5.1.1 Structure of chapter

The chapter is structured as follows. Section 5.2 recalls the previously developed MP model for the prescribed burning problem, and highlights the additional features within this chapter. Section 5.3 discusses the implementation of the model which is applied in Section 5.4 over a randomly generated landscape. Finally, Section 5.5 shows the application to a realistic case study built on real data.

5.2 MSP extension

The model proposed in Chapter 3 was a deterministic one that included two different objectives. To achieve more realism the model is now expanded, including uncertainty in some parameters and four different objectives.

5.2.1 Modification of objectives

Previous model considered a single species living in the landscape, and the model was solved lexicographically. We now consider the following objectives:

1. Minimize number of high-fuel load connections. High-fuel load connections are deemed a proxy of fire-propagation risk.

2. Minimize number of high-fuel load units. High-fuel load units are deemed a proxy of fire-ignition risk.

3. Maximize habitat quality for species 1.

4. Maximize habitat quality for species 2.

(a) Hastings River mouse. Image from https://wetlandinfo.des.qld.gov.au/wetlands/ecology/components/species/?pseudomys-oralis

(b) Mallee emu-wren. Image from https://en.wikipedia.org/wiki/Mallee_emu-wren

Figure 5.1: Species with different vegetation age needs

5.2.2 Including uncertainty

The deterministic model considered that each year t a budget b_t was available for doing the burns. The included budget constraint was measured in terms of area, that is, the total area of units burnt was not to exceed a given quantity. Measuring the budget in terms of area can easily be replaced or complemented with limits on number of units burnt or the perimeters of burnt units (as long as they are independently burnt and not merged together for burning).

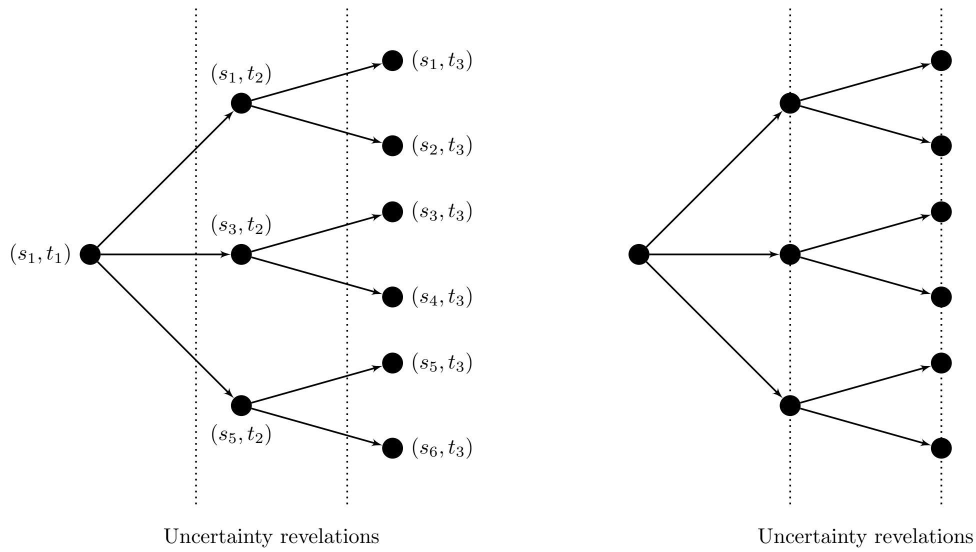
However it is often the case that not every unit scheduled for treatment is burnt . One of the causes for this is weather, as burn opportunities are limited by it, resulting in a different number of burns completed each year (O'Keefe, 1995; Duff et al., 2019).

If the budget probability distribution can be estimated, the problem can be treated with stochastic programming. A simple way of representing the uncertainty is given in Fig. 5.2a. By convention $t = 1$ are the initial conditions, and thus the first decisions are made on $t = 2$. Instead of having a known budget b_2, there are three different budgets b_2^1, b_2^3 and b_2^5, each with their associated probabilities, and decisions are planned accordingly for each scenario independently.

Another possibility, maybe more realistic, is having uncertainty revealed within the decision nodes (Fig. 5.2b), linking the decisions between scenarios at each stage. This situation could arise if some sequence for performing the burns in a year is established, and units are treated one by one until time runs out. Both possibilities for when the uncertainty is revealed are considered in the different implementations.

5.3 MSP model

The mathematical programming model for prescribed burning shown in Chapter 3 will incorporate the previously described aspects, and a multiobjective stochastic programming model will be



(a) Tree with uncertainty revealed before decision nodes

(b) Tree with uncertainty revealed within decision nodes

Figure 5.2: Different compact representation of the scenarios, depending on when uncertainty is revealed

obtained. For that purpose, most parameters have to be redefined, and some new ones introduced.

5.3.1 Indices, parameters and variables

Indices sets

I	set of all burn units in the landscape
$\mathcal{N}_i \subset I$	set of burn units connected to burn unit i
S	set of scenarios
$\Psi_t \subset S$	set of scenarios active at time t
$\Phi_{st} \subset S$	set of children of scenario s at time t

Parameters

T	number of time periods in the planning horizon
Φ_{st}^{-1}	parent of scenario s at time t
Λ_{st}	intermediate stage of final scenario s at time t
a_i	initial fuel age of burn unit $i \in I$
b_t^s	fuel treatment budget at time t in scenario $s \in S$
c_i	area of burn unit $i \in I$
h_i	high-fuel load threshold of burn unit i
$maxTFI_i$	maximum tolerable fire interval (TFI) of burn unit $i \in I$

$minTFI_i$	minimum tolerable fire interval (TFI) of burn unit $i \in I$
N_m	number of breakpoints for the m-th piecewise linear function of the fire response curve
r_{nm}	n-th breakpoint of the piecewise linear function or the fire response curve for the m-th species
v_{nm}	value of breakpoint r_{nm} according to the fire response curve for the m-th species
p_{ij}	proportion of boundary of unit i that is shared with unit j
M	big-M coefficient
π_s	probability of scenario $s \in S$
w_k	importance of k-th objective function, $k = 1, \dots, 4$

Variables

X^s_{it}	1 if burn unit i is treated in time period t in scenario s, 0 otherwise
H^s_{it}	1 if burn unit i is classified as high-fuel load in time period t in scenario s, 0 otherwise
Q^s_{ijt}	1 if adjacent burn units i and j are both classified as high-fuel load in time period t in scenario s, 0 otherwise
A^s_{it}	fuel age of burn unit i in time period t in scenario s
FRC^s_{itm}	habitat quality of burn unit i in time period t for species m in scenario s by area (fire response curve)
Z^s_{itnm}	1 if the age of burn unit i in time period t in scenario s is between r_{nm} and $r_{(n+1)m}$, 0 otherwise
G^s_{itnm}	convex multipliers of burn unit i in time period t in scenario s for the m-th piecewise linear function

5.3.2 Uncertainty treatment

Uncertainty is included in the model, using stochastic programming. S will represent the set of scenarios, with $s \in S$. It is worth noting that every variable depending on time t could be redefined for every single pair (s, t), and then including non-anticipativity constraints.

Let us consider variable $X_{i,t}$, which changes now to $X^s_{i,t}$. If using the tree in Figure 5.2a this approach would lead to defining at time $t = 2$ variables $X^s_{i,2}$, with $s \in \{1, \dots, 6\}$, and then adding the extra constraints $X^1_{i,2} = X^2_{i,2}$, $X^3_{i,2} = X^4_{i,2}$ and $X^5_{i,2} = X^6_{i,2}$ for all i. This is due to the fact that at time $t = 2$ it is not possible to discern between scenarios 1 and 2, 3 and 4, or 5 and 6.

The alternative used is the compact representation, for which for any t only as many pairs (s, t)

will be active as nodes there are in the tree. The following representation will be useful:

$$\Phi_{st} = \{s' \text{ such that } (s', t+1) \text{ is a child of } (s,t)\}$$

$$\Phi_{st}^{-1} = s' \iff s \in \Phi_{s'(t-1)}$$

With this notation, the tree of Figure 5.2a has $\Phi_{1,1} = 1, 3, 5$ and $\Phi_{4,3}^{-1} = 3$. In addition to that, it will be useful to know which scenarios exist for a given time t:

$$\begin{aligned} \Psi_1 &= 1 \\ \Psi_t &= \bigcup_{s \in \Psi_{t-1}} \Phi_{s(t-1)} \qquad (t \geq 2) \end{aligned}$$

That is, Ψ_2 are the children of the root node, Ψ_3 the grandchildren, etc. In the previous example, $\Psi_1 = \{1\}$, $\Psi_2 = \{1, 3, 5\}$ and $\Psi_3 = \{1, 2, 3, 4, 5, 6\}$.

Besides, if uncertainty is revealed within the burn season, the following non-anticipativity constraints should be included:

$$X_{i,t}^{s} \geq X_{i,t}^{s'} \qquad \forall i, t, s, s' \text{ such that } \begin{cases} s, s' \in \Psi_t \\ \Phi_{st}^{-1} = \Phi_{s't}^{-1} \\ b_t^s \geq b_t^{s'} \end{cases}$$

Condition $s, s' \in \Psi_t$ ensures that scenarios s and s' exist at time t, $\Phi_{st}^{-1} = \Phi_{s't}^{-1}$ checks that both nodes come from the same parent, and finally $b_t^s \geq b_t^{s'}$ makes certain that scenario s has more budget than scenario s', and hence whatever is burnt in s' needs to be burnt in s.

It is worth noting that if the non-anticipativity constraints are not enforced, the arising model is not a proper stochastic problem. For instance, if the uncertainty is represented with the tree in Fig. 5.2b, there are three different possible situations at time $t = 2$. As there are not first-stage decisions, one could wait until the uncertainty reveals itself before solving any model.

The last piece of notation introduced will make possible to know all the stages through which a final scenarios went through:

$$\Lambda_{st} = s' \iff \text{final scenario } (s, T) \text{ is a descendant of } (s', t)$$

In the example, $\Lambda_{4,2} = 3$, because final scenario 4 came from scenario 3 at time 2.

5.3.3 Multiobjective treatment

The new model will take into account another species living in the landscape. For that reason another way of measuring habitat quality is considered (representing a different species). Parameters r, v as well as variables G, Z and FRC include now the sub-index m, referring to the values for the m-th species ($m = 1, 2$).

Four different objective functions will be simultaneously considered for all $s \in S$:

1. High fuel load connections:

$$f_1^s = \sum_{t=2}^{T} \sum_{i \in I} \sum_{\substack{j \in \mathcal{N}_i \\ j > i}} w_{ij} Q_{ijt}^{\Lambda_{st}} \qquad \textit{(minimize)} \tag{5.1}$$

2. High fuel load area:

$$f_2^s = \sum_{t=2}^{T} \sum_{i \in I} c_i X_{it}^{\Lambda_{st}} \qquad \textit{(minimize)} \tag{5.2}$$

3. Habitat quality of first species (originally included in Equation (3.16)):

$$f_3^s = \sum_{t=2}^{T} \sum_{i \in I} c_i FRC_{it1}^{\Lambda_{st}} \qquad \textit{(maximize)} \tag{5.3}$$

4. Habitat quality of additional species:

$$f_4^s = \sum_{t=2}^{T} \sum_{i \in I} c_i FRC_{it2}^{\Lambda_{st}} \qquad \textit{(maximize)} \tag{5.4}$$

As habitat quality is now part of the objective function Equations (3.15) and (3.16), enforcing some levels of local and global habitat quality, are removed.

For applying the defined solution technique the functions f_k^s should be normalized. All the scenarios are solved independently, minimizing and maximizing each of the objective functions. For each objective, the minimum objective value under any scenario will be used as a lower bound, and the maximum objective value will be used as an upper bound. If $\underline{f}_k$ and $\overline{f}_k$ are the obtained lower and upper bounds for each objective respectively, the objective functions defined by Eqs. (5.1) to (5.4) are redefined as:

$$\tilde{f}_1^s(x) = \frac{f_1^s(x) - \underline{f}_1}{\overline{f}_1 - \underline{f}_1} \tag{5.5}$$

$$\tilde{f}_2^s(x) = \frac{f_2^s(x) - \underline{f}_2}{\overline{f}_2 - \underline{f}_2} \tag{5.6}$$

$$\tilde{f}_3^s(x) = \frac{\overline{f}_3 - f_3^s(x)}{\overline{f}_3 - \underline{f}_3} \tag{5.7}$$

$$\tilde{f}_4^s(x) = \frac{\overline{f}_4 - f_4^s(x)}{\overline{f}_4 - \underline{f}_4} \tag{5.8}$$

After doing this is guaranteed that the values of $\tilde{f}_k^s(x)$ are in $[0, 1]$ for every feasible solutions, and the direction of optimization of objectives 3 and 4 has changed in order to have all functions to be minimized.

5.3.4 Constraints

The constraints of the model shown in Chapter 3 are modified using the notation previously defined.

$$\sum_{i \in I} c_i X_{it}^s \leq b_t^s \qquad t = 2, \ldots, T, \forall s \in \Psi_t \tag{5.9a}$$

$$A_{it}^s = a_i \qquad t = 1, \forall i \in I, s = 1 \tag{5.9b}$$

$$A_{it}^s \geq A_{i(t-1)}^{s'} + 1 - M \cdot X_{it}^s \qquad t = 2, \ldots, T, \forall i \in I, \forall s \in \Psi_t, s' = \Phi_{st}^{-1} \tag{5.9c}$$

$$A_{it}^s \leq A_{i(t-1)}^{s'} + 1 \qquad t = 2, \ldots, T, \forall i \in I, \forall s \in \Psi_t, s' = \Phi_{st}^{-1} \tag{5.9d}$$

$$A_{it}^s \leq maxTFI(1 - X_{it}^s) \qquad t = 2, \ldots, T, \forall i \in I, \forall s \in \Psi_t \tag{5.9e}$$

$$minTFI \cdot X_{it}^s \leq A_{i(t-1)}^{s'} \qquad t = 2, \ldots, T, \forall i \in I, \forall s \in \Psi_t, s' = \Phi_{st}^{-1} \tag{5.9f}$$

$$A_{it}^s \leq h - 1 + M \cdot H_{it}^s \qquad t = 2, \ldots, T, \forall i \in I, \forall s \in \Psi_t \tag{5.9g}$$

$$H_{it}^s + H_{jt}^s \leq 1 + Q_{ijt}^s \qquad t = 2, \ldots, T, \forall i \in I, \forall j \in \mathcal{N}_i, j > i, \forall s \in \Psi_t \tag{5.9h}$$

$$\sum_{n=1}^{N_m - 1} Z_{itnm}^s = 1 \qquad t = 1, \ldots, T, \forall i \in I, m = 1, 2, \forall s \in \Psi_t \tag{5.9i}$$

$$Z_{itnm}^s \leq G_{itnm}^s + G_{it(n+1)m}^s \qquad t = 1, \ldots, T, \forall i \in I, n = 1, \ldots, N_m - 1, \; m = 1, 2, \forall s \in \Psi_t \tag{5.9j}$$

$$\sum_{n=1}^{N_m} r_{nm} G_{itnm}^s = A_{it}^s \qquad t = 1, \ldots, T, \forall i \in I, m = 1, 2, \forall s \in \Psi_t \tag{5.9k}$$

$$\sum_{n=1}^{N_m} G_{itnm}^s = 1 \qquad t = 1, \ldots, T, \forall i \in I, m = 1, 2, \forall s \in \Psi_t \tag{5.9l}$$

$$FRC_{itm}^s = \sum_{n=1}^{N_m} v_{nm} G_{itnm}^s \qquad t = 1, \ldots, T, \forall i \in I, m = 1, 2, \forall s \in \Psi_t \tag{5.9m}$$

$$X_{it}^s \geq X_{it}^{s'} \qquad t = 2, \ldots, T, \forall i \in I, \forall s, s' \in \Psi_t, \; \Phi_{st}^{-1} = \Phi_{s't}^{-1}, b_t^s \geq b_t^{s'} \tag{5.9n}$$

$$X_{it}^s \in \{0, 1\} \qquad t = 2, \ldots, T, \forall i \in I, s \in \Psi_t \tag{5.9o}$$

$$H_{it}^s \in \{0, 1\} \qquad t = 2, \ldots, T, \forall i \in I, s \in \Psi_t \tag{5.9p}$$

$$Z_{itnm}^s \in \{0, 1\} \qquad t = 1, \ldots, T, \forall i \in I, n = 1, \ldots, N_m - 1, \; m = 1, 2, s \in \Psi_t \tag{5.9q}$$

$$Q_{ijt}^s \geq 0 \qquad t = 2, \ldots, T, \forall i \in I, \forall j \in \mathcal{N}_i, j > i, s \in \Psi_t \tag{5.9r}$$

$$A_{it}^s \geq 0 \qquad t = 1, \ldots, T, \forall i \in I, s \in \Psi_t \tag{5.9s}$$

$$FRC_{itm}^s \geq 0 \qquad t = 1, \ldots, T, \forall i \in I, m = 1, 2, s \in \Psi_t \tag{5.9t}$$

$$G_{itnm}^s \geq 0 \qquad t = 1, \ldots, T, \forall i \in I, n = 1, \ldots, N_m, \; m = 1, 2, s \in \Psi_t \tag{5.9u}$$

The feasible region of the model, X, is given by Eqs. (5.9a) to (5.9u). Equation (5.9n) enforces that uncertainty is revealed within the burn season, as in Fig. 5.2a. If Eq. (5.9n) is not included in the definition of X, the modelled situation is the one shown in Fig. 5.2b.

5.3.5 Risk-averse MSP model

The obtained MSP model is then:

$$\min_{x \in X} \left(\tilde{f}_1^s(x), \ldots, \tilde{f}_4^s(x) \right) \tag{5.10}$$

With the objective functions $\tilde{f}_k^s$ determined by Eqs. (5.5) to (5.8) and X given by Eqs. (5.9a) to (5.9u). This MSP model can be solved with the risk-aversion approach of Chapter 4, and thus the following model is to be solved:

$$\begin{aligned} \min_{z, v_k, z_k, y_{ks}, x} \quad & z + \sum_k \frac{w_k}{r} v_k && \\ \text{s.t.} \quad & z + v_k \geq z_k + \sum_{s=1}^{S} \frac{\pi_s}{\beta} y_{ks} && \forall k \\ & z_k + y_{ks} \geq \tilde{f}_k^s(x) && \forall k, s \\ & y_{ks} \geq 0 && \forall k, s \\ & z_k \text{ free}, v_k \geq 0 && \forall k \\ & z \text{ free} && \\ & x \in X && \end{aligned} \tag{5.11}$$

5.4 Computational experiments

5.4.1 Model parameters

The MSP model is first applied to one of the randomly generated landscapes of Chapter 3, consisting on 45 burn units, and considering a nominal yearly budget for prescribed burning of 7% of the total area. The implemented scenario tree is shown in Figure 5.3. Excluding the time $t = 1$ (initial conditions), the first two burn seasons are split in three equiprobable nodes. In each of these nodes the allocated budget for burnt area used in Equation (3.2) is multiplied by the number in blue. After $t = 3$ each of the nodes has two children, and on the last years each year has just one child. This results in 36 different scenarios.

Concerning the species, the first one considered is the Hastings River mouse (Fig. 5.1a), with a preference for vegetation between 5 and 10 years, and values: $r = [0, 7.5, 35]$, $v = [0.8, 1, 0]$. The second species is the mallee emu-wren (Fig. 5.1b), preferring older vegetation, with values: $r' = [0, 30, 35]$, $v' = [0, 1, 0.8]$. These qualities are reflected in Figure 5.4.

The importances of criteria are respectively $[0.5, 0.2, 0.1, 0.2]$, and the parameters for the risk-averse solution concept are set at $r = 0.5, \beta = 0.1$.

5.4.2 Results

As was done in Section 4.6, the proposed MSP problem will be solved with two different approaches. The first one using the risk-averse function $h_r^\beta(x)$ defined in Chapter 5, and the second

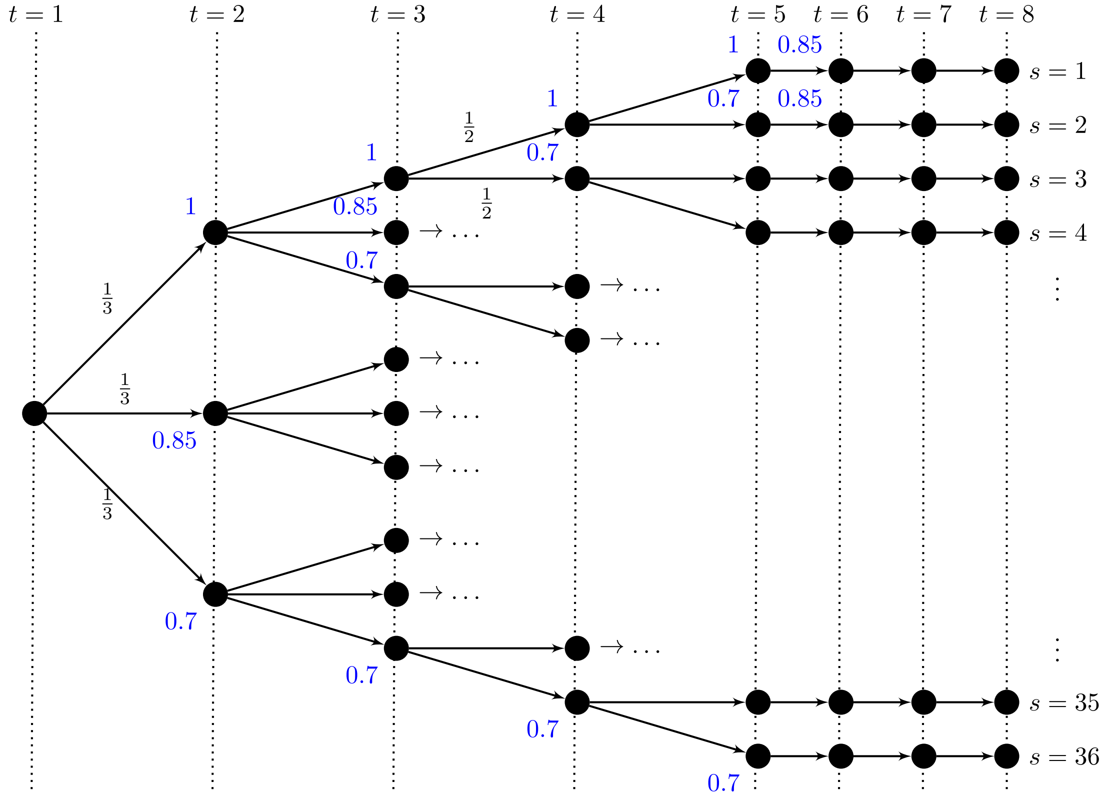


Figure 5.3: Implemented scenario tree, with budget multipliers in blue

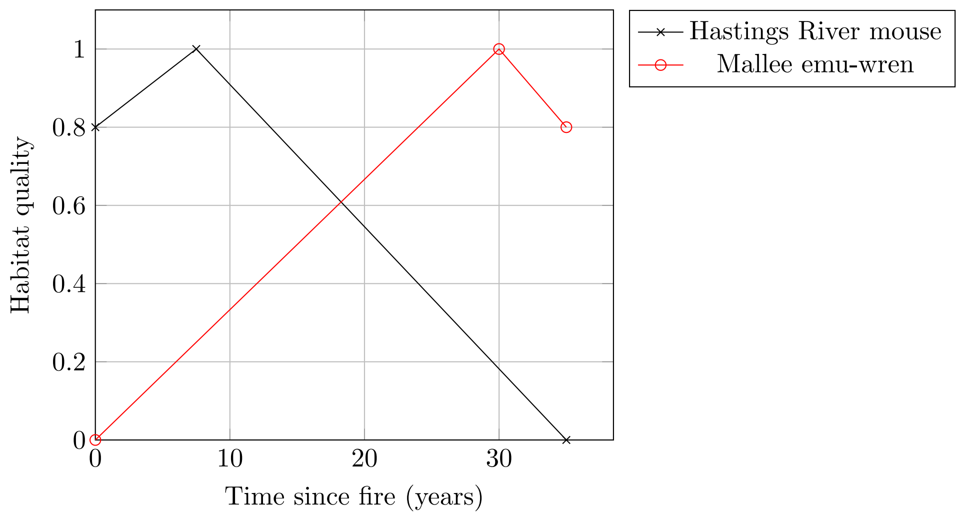


Figure 5.4: Quality of landscape for each of the considered animals

one using a simple weighted average:

$$\min \quad f_{\text{AVG}}(x) = \sum_{k=1}^{4} \sum_{s=1}^{36} w_k \pi_s \tilde{f}_k^s(x) \tag{5.12}$$

The solution obtained with the objective function (5.12) is denoted x^*_{AVG}, in contrast with

x^*_{MSP}, which makes use of the defined risk-averse solution concept. Model has been implemented in Julia, using JuMP, and solved with Gurobi. Models were run until the integrality gap was 1%. The risk-averse model reached x^*_{MSP} in 1905 seconds, with a 0.97% integrality gap, whereas the weighted average model obtained x^*_{MSP} in 2059 seconds, with a 0.97% integrality gap.

Table 5.5 (at the end of the chapter) shows how each solution performs by scenario and criterion, while Figures 5.14 to 5.16 (also at the end of the chapter) show the 36 final landscapes at time $t = 8$, one for each scenario.

Table 5.1 compares both solutions, showing their β-averages, as well as their objective values. Similarly to what was seen in Section 4.6, it can be seen that x^*_{MSP} performs just slightly worse when measuring with the objective function (5.12) (0.234 versus 0.224), but it is instead much better in extreme scenarios/criteria, as measured by function h^r_β. Inspecting the β-averages gives a finer explanation: the fourth criterion, the habitat quality of the mallee emu-wren has a poor value in solution x^*_{AVG}. By using the proposed approach, solution x^*_{MSP} is found, performing slightly worse in criteria one to three, but making up for it in terms of criterion 4.

Table 5.1: Comparison of the solutions x^*_{MSP} and x^*_{AVG}. $\beta = 0.1, r = 0.5$.

	β-averages					
	$w_1 = 0.5$	$w_2 = 0.2$	$w_3 = 0.1$	$w_4 = 0.2$		
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$h^r_\beta(x)$	$f_{\text{AVG}}(x)$
x^*_{MSP}	0.135	0.151	0.704	0.483	0.394	0.234
x^*_{AVG}	0.121	0.148	0.611	0.821	0.510	0.224

Figure 5.5 shows the initial condition, the solution for each of the three scenarios at time $t = 2$, and three additional scenarios at time $t = 3$, starting from the leftmost scenario. Budget multipliers are shown over the edges. Given that the base budget was set at 7%, the scenarios from left to right have an effective budget of $1 \cdot 7\%, 0.85 \cdot 7\%$ and $0.7 \cdot 7\%$, that is, approximately $7\%, 6\%$ and 5%.

Since the selected landscape has 45 burn units, the average burn unit measures $\frac{1}{45} \approx 2\%$. For this reason, changing from an scenario with an affective budget of 5% to another with budget 6% does not necessarily shows any changes: if the plan with budget 5% saturates the budget constraint it might not be possible to find any unit with area $\leq 1\%$ to be burnt in the 6% scenario. Furthermore, it is also possible that even if more burns can be performed without violating the budget constraints they are not in the solution as it can decrease the ecological value.

For $t = 2$, Fig. 5.5 shows the same results for the scenarios with budget multipliers 1 and 0.85. Both of them burn an extra unit than the scenario on the right (unit located at the top part). Same situation arises at time $t = 3$, with an additional unit burnt at the south-west corner in scenarios with multipliers 1 and 0.85.

8 time steps are considered in the model, but only one step is an actual plan. At time $t = 2$ fire managers should start with the "pessimistic" plan on the right (the one with the lowest budget).

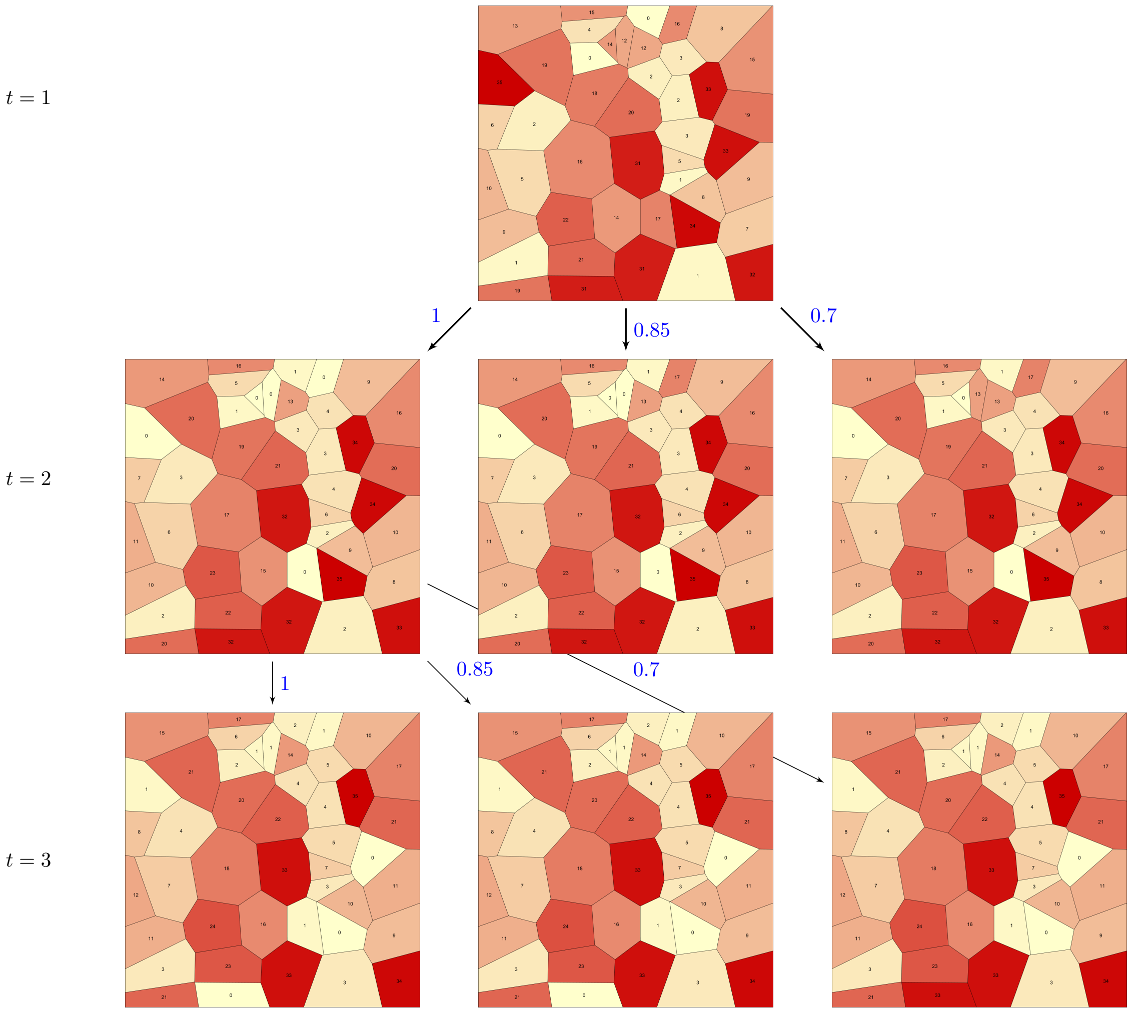


Figure 5.5: Selection of steps of solution

If there is budget/time remaining for performing more burns, then manager should follow with whichever extra units burnt in the middle plan, and finally, with the extra ones in the leftmost plan (none in this case). After burn season is over, it is revealed in which scenario they actually are, and they should have burnt accordingly. For the next years, rather than following the given plan, the model should be run again, updating the initial ages to the actual ones. This situation in which burn plans are contained one within another is a consequence of the imposed Eq. (5.9n), included to achieve more realism.

5.5 Case study

5.5.1 Presentation

A prescribed burning problem with real data will be solved with the previously laid out approach. Data is provided in the context of the GEO-SAFE project by INFOCA, the wildfire prevention and supression service in Andalucía, in the south of Spain. The case study consists of a landscape divided into 193 units. Figure 5.6 shows the area of study, as well as the limitations on where can prescribed burning be conducted. Table 5.2 shows median values of some characteristics of the burn units, grouped by the limitations they have for burning.

Figure 5.6: 193 burn units of the case study in Andalucía. Blue reflects public property where prescribed burns can be performed, green indicates private property and yellow areas where prescribed burning is not allowed due to conservation constraints.

Table 5.2: Median characteristics of landscape in case study, grouped by prescribed burning limitations

Limitation	Area (km^2)	Current age	Minimum TFI	Hazard age	Maximum TFI
No limitation	5.79	2.0	5.0	10.0	15.0
Private property	7.98	2.0	3.0	5.0	8.0
Endangered species	14.01	6.5	3.0	5.0	8.0

Total area of the case accounts for 1820km^2, with an approximate half of it being susceptible of prescribed burning. Some small considerations had to be incorporated into the model to reflect the reality:

- As mentioned before, some land is not eligible for prescribed burning. The parameter $public_i$ is defined, which equals 1 if prescribed burning can be realized in unit i, 0 otherwise. For the units such that $public_i = 0$, burn variables are not defined, and some constraints are dropped (for instance, the maximum tolerable fire interval restriction does not apply). The different objective functions are still measured in such areas, even if it is not possible to intervene there.

Figure 5.7: High fuel load threshold by unit in case study

- Different types of vegetation are present on the landscape, each unit having a single type. This is reflected in that scalar parameters $minTFI, maxTFI$ and h_{thr} now depend on the unit i. Figures 5.7 to 5.10 show the initial conditions in the case study.
- Yearly prescribed burning budget is set around 20km^2. However, fire services in the area do not perform the burns in the entire units, they are rather limited to the boundary of the unit. This is done with the goal of decreasing fire connectivity at a lower expense, and as a means for creating fire attack opportunity areas.

  To integrate this situation with the developed model, the budget constraint is replaced. Instead of limiting the area of units burnt by the budget, only a percentage of the area is limited by this budget. After observing the areas of the units and their maximum TFIs, such percentage is set to 15%.

5.5.2 Model parameters

Two undetermined species will be used in the model, whose habitat qualities are determined by the curves shown in Fig. 5.11. The landscape has a total of 193 burn units, 113 if only the ones in which prescribed burning can be used are counted. This results on a great increase on the dimensions of the model, as the previous example had 45 burn units. To compensate for this a shorter scenario tree has been chosen, although other possibilities such as merging units could be considered. The scenario tree is outlined in Fig. 5.12, in which 5 time periods and 12 scenarios are considered. The yearly budgets for prescribed burning vary from 15 to 25 km^2.

Six different experiments are conducted and compared:

Figure 5.8: Minimum tolerable fire interval by unit in case study

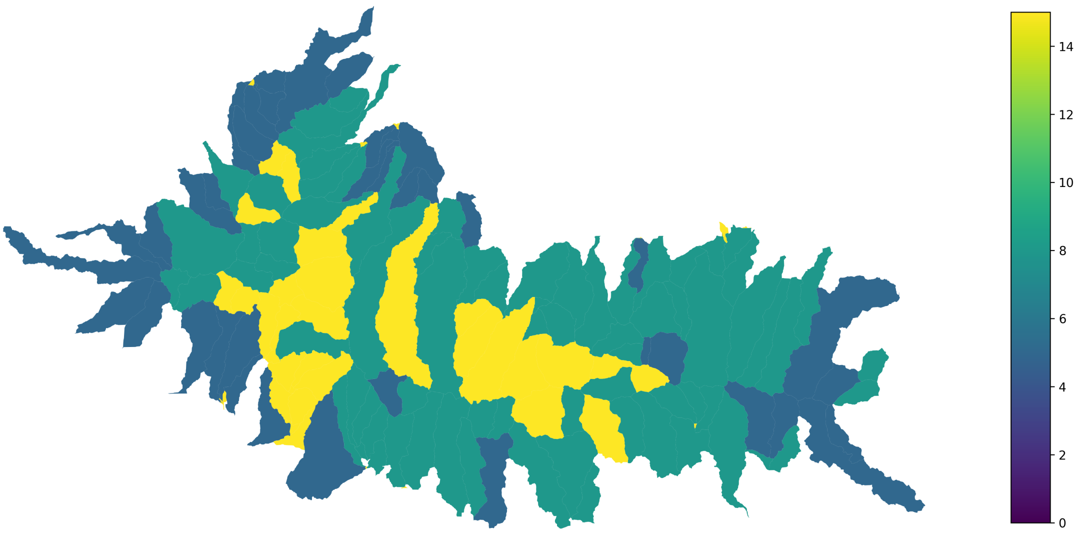


Figure 5.9: Maximum tolerable fire interval by unit in case study

Figure 5.10: Initial age by unit in case study

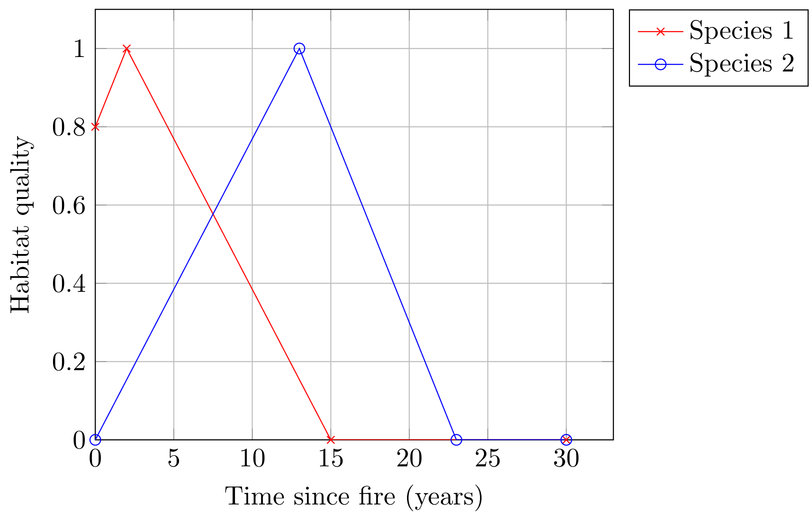


Figure 5.11: Quality of landscape for two undetermined species, one preferring young vegetation and another preferring old vegetation.

- Including the non-anticipativity constraints discussed in Section 5.2.2, that is, if burns are performed sequentially because uncertainty is not revealed until the burns are completed:
  1. Without risk-aversion, minimizing the weighted average function $f_{\text{AVG}}(x)$ (Eq. (5.12))
  2. With risk-aversion, minimizing $h_r^\beta(x)$, with $r = \beta = 0.5$
  3. With risk-aversion, minimizing $h_r^\beta(x)$, with $r = 0.5, \beta = 0.25$
  4. With risk-aversion, minimizing $h_r^\beta(x)$, with $r = 0.25, \beta = 0.5$
  5. With risk-aversion, minimizing $h_r^\beta(x)$, with $r = \beta = 0.25$
- Without including the non-anticipativity constraints. In this case right before burn season it

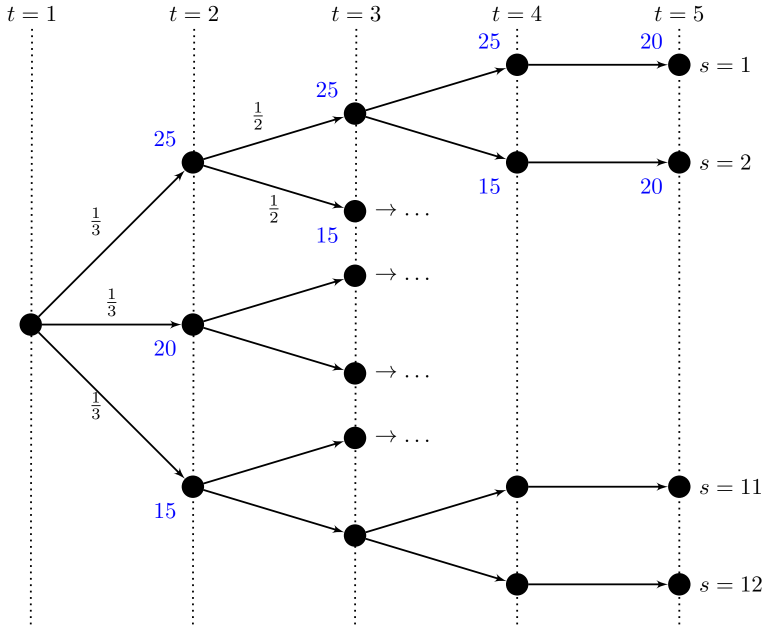


Figure 5.12: Scenario tree in case study. Numbers in blue represents the maximum burnt area in each node, in km^2.

is known the whole budget that will be available:

6. Minimizing $f_{\text{AVG}}(x)$
7. Minimizing $h_r^\beta(x)$, with $r = \beta = 0.5$
8. Minimizing $h_r^\beta(x)$, with $r = 0.5, \beta = 0.25$
9. Minimizing $h_r^\beta(x)$, with $r = 0.25, \beta = 0.5$
10. Minimizing $h_r^\beta(x)$, with $r = \beta = 0.25$

5.5.3 Results

Tables 5.3 and 5.4 show the results for the 10 experiments conducted, with and without the non-anticipativity constraints, respectively. Each row corresponds to one of the experiments. For each of the 10 solutions, it has been measured how well they perform in the other metrics, solving time, final integrality gap, and time until a 1% integrality gap was achieved.

When the non-anticipativity constraints are enforced (Table 5.3), none of the executions was able to reduce the integrality gap to 0 within the time limit (1 hour). However all of them finished under 0.5%, reaching a 1% gap in under 6 minutes in all but one. As observed in the previous chapter, lower values of r and β lead to slower executions.

When the non-anticipativity constraints are not included (Table 5.4) all executions reach the optimal solutions within 1 hour. It is seen once again that the smaller values of r and β are, the slower the execution.

Table 5.3: Results obtained from the different approaches in the case study, when including the non-anticipativity constraints. Each row corresponds to the solution for each of the objective functions considered, and columns show the objective values of each of the optimal solutions in the other objective functions. Columns at the end show solution time, integrality gap, and time until a solution with a 1% gap was found.

	Values of							
Minimizing	f_{AVG}	$h_{0.5}^{0.5}$	$h_{0.5}^{0.25}$	$h_{0.25}^{0.5}$	$h_{0.25}^{0.25}$	**Time (s)**	**Gap (%)**	**T. to 1% (s)**
$f_{\text{AVG}}(x)$	0.203	0.418	0.456	0.606	0.651	3600	0.21	191
$h_{0.5}^{0.5}(x)$	0.269	0.331	0.338	0.444	0.449	3600	0.04	205
$h_{0.5}^{0.25}(x)$	0.266	0.332	0.333	0.451	0.452	3600	0.06	305
$h_{0.25}^{0.5}(x)$	0.292	0.341	0.344	0.426	0.429	3600	0.45	341
$h_{0.25}^{0.25}(x)$	0.296	0.343	0.344	0.426	0.427	3600	0.45	1638

Table 5.4: Results obtained from the different approaches in the case study, when not including the non-anticipativity constraints.

	Values of							
Minimizing	f_{AVG}	$h_{0.5}^{0.5}$	$h_{0.5}^{0.25}$	$h_{0.25}^{0.5}$	$h_{0.25}^{0.25}$	**Time (s)**	**Gap (%)**	**T. to 1% (s)**
$f_{\text{AVG}}(x)$	0.202	0.415	0.452	0.599	0.643	333	0	58
$h_{0.5}^{0.5}(x)$	0.27	0.331	0.334	0.443	0.447	322	0	172
$h_{0.5}^{0.25}(x)$	0.264	0.332	0.333	0.449	0.451	1770	0	190
$h_{0.25}^{0.5}(x)$	0.293	0.343	0.345	0.425	0.427	2126	0	669
$h_{0.25}^{0.25}(x)$	0.289	0.343	0.345	0.427	0.427	2895	0	2665

Both tables show that obtaining risk-averse solutions is slower, regardless of the inclusion or not of the non-anticipativity constraints.

The $h_r^\beta(x)$ do however provide of great solutions if risk-averse solutions are sought. For instance, when looking at the solutions in Table 5.4, if the solution provided by $h_{0.5}^{0.5}(x)$ is taken rather than the obtained with $f_{\text{AVG}}(x)$, a better outcome is expected in every unfavourable situation. Furthermore, as β and r decrease the execution becomes slower. A reasonable explanation for this is that the smaller r and β are, the fewer scenarios and criteria are considered for obtaining the value of $h_r^\beta(x)$, and this can lead to situations with multiple optima which slows down the integer programming solving algorithm. This observation related to solution times, although clearly noticeable, does not have a great impact given that the problem solved is operational. On the other hand, if the problem solved was a response one, solution times would have a great importance.

Figure 5.13 shows the three possible landscapes at time $t = 2$, when the function $h_r^\beta(x)$ with $r = \beta = 0.25$ is minimized, and the non-anticipativity constraints are imposed. A red dot is shown in units that are burnt in scenarios with higher budgets.

5.6 Discussion

The risk-aversion methodology developed in Chapter 4 has been applied to a modified version of prescribed burning problem previously considered in Chapter 3. Four different objectives are considered, and it is taken into account that not every nominated burn is completed, thus adding uncertainty.

Results show that using this risk-averse solution instead of an average-OWA greatly improves the expected outcome of unfavourable conditions, without significantly compromise average outcomes. It has been shown for both approaches, regardless of when the uncertainty is revealed: right before the start of burn season, or at the end of it.

The case study, located in Andalucía, includes almost 200 burn units and an extension of 1820km^2. A limitation of the case study is that the prescribed burning MSP model adapted from the one in Chapter 3 might not accurately reflect how prescribed burning is performed in Andalucía. The developed model burns whole units, while fire services in the area do not perform the burns in the entire units as they are instead limited to the boundary of the unit. A proxy has been done considering that burning the perimeter accounts for burning around 15% of the total area. Future work includes considering the boundaries explicitly and not necessarily for all the perimeter.

Computational times for the different experiments are provided, showing higher but manageable times for the risk-aversion approach. Nevertheless, it should be noted that given that the studied problem is a strategic (long-term) one, computational time is not relevant as long as the solution can be found.

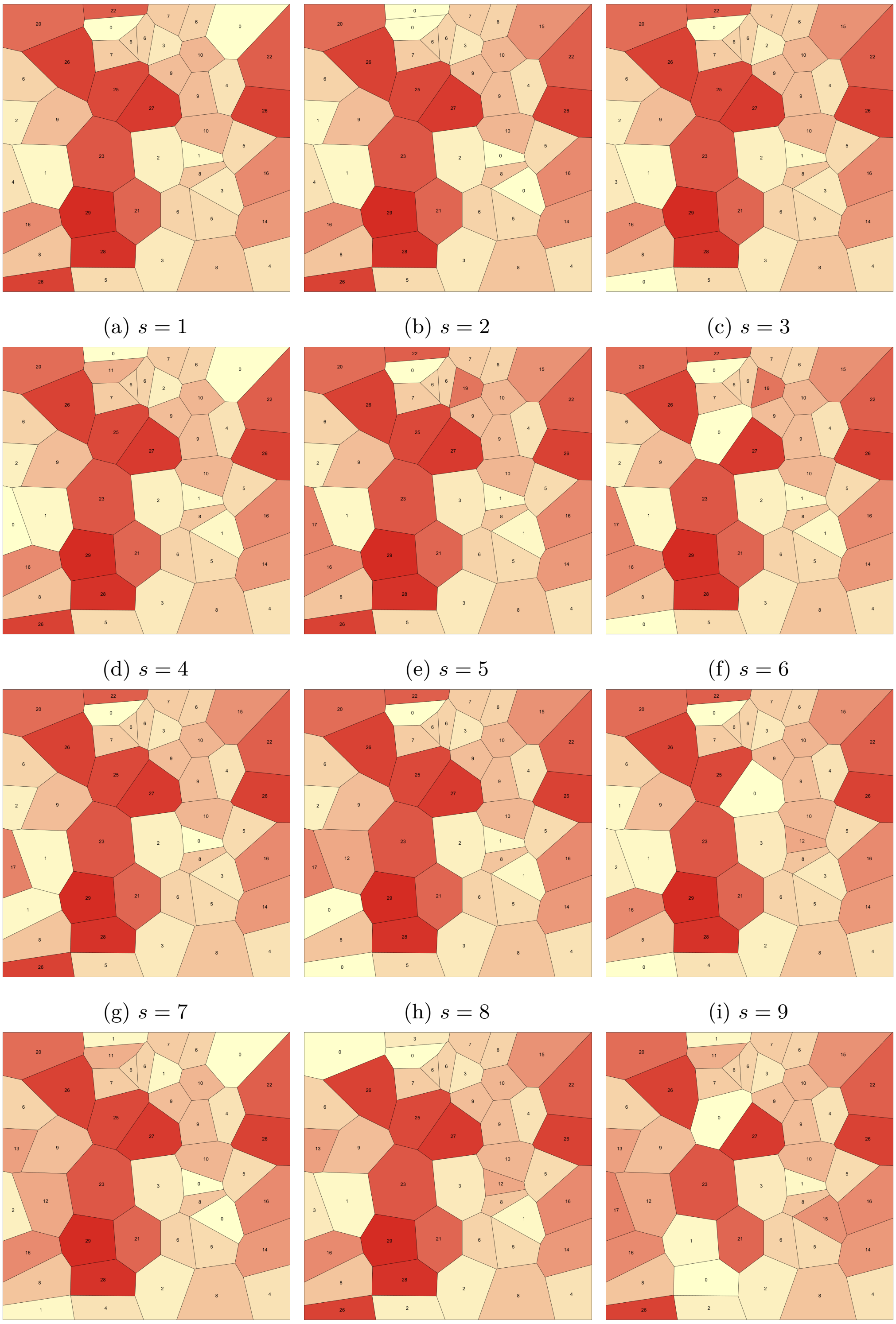
(a) Scenario with smallest budget (15km^2)

(b) Scenario with median budget (20km^2). In red, additional units burnt in this scenario

(c) Scenario with largest budget (25km^2). In red, additional units burnt in this scenario

Figure 5.13: Three scenarios at time $t = 2$, for the experiment in which non-anticipativity constraints are enforced and function $h_r^\beta(x)$, with $r = \beta = 0.25$, is minimized.

5.A Extra figures and tables



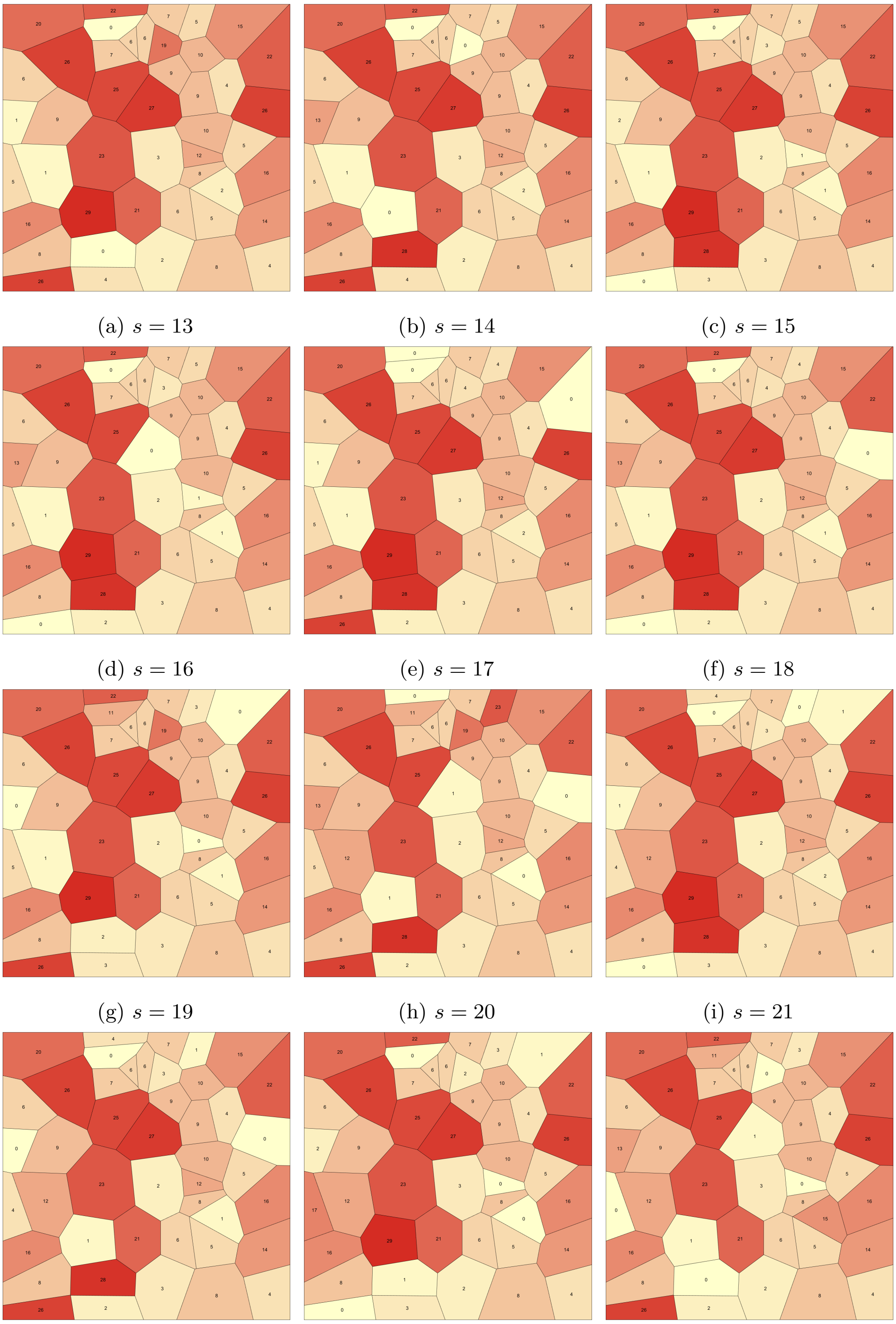
(a) $s = 1$ (b) $s = 2$ (c) $s = 3$

(d) $s = 4$ (e) $s = 5$ (f) $s = 6$

(g) $s = 7$ (h) $s = 8$ (i) $s = 9$

(j) $s = 10$ (k) $s = 11$ (l) $s = 12$

Figure 5.14: Ages of cells after 8 years, scenarios 1 to 12



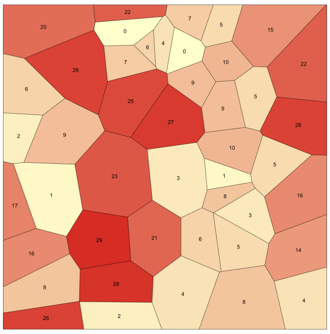
(a) $s = 13$ (b) $s = 14$ (c) $s = 15$

(d) $s = 16$ (e) $s = 17$ (f) $s = 18$

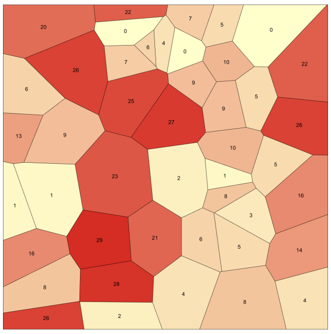
(g) $s = 19$ (h) $s = 20$ (i) $s = 21$

(j) $s = 22$ (k) $s = 23$ (l) $s = 24$

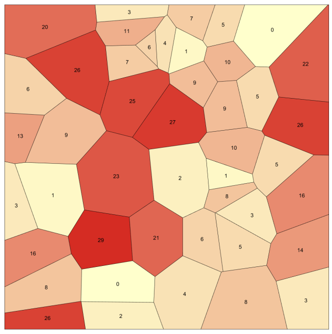
Figure 5.15: Ages of cells after 8 years, scenarios 13 to 24



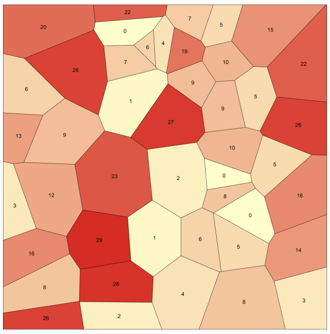
(a) $s = 25$



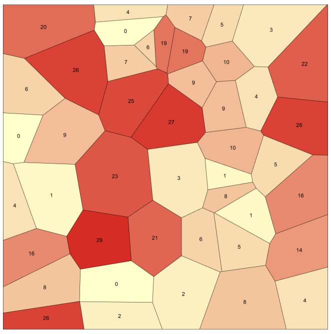
(b) $s = 26$



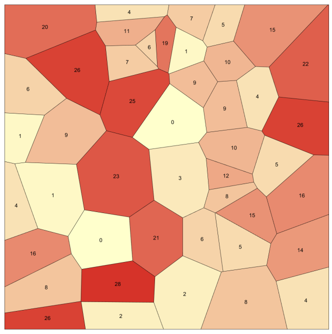
(c) $s = 27$



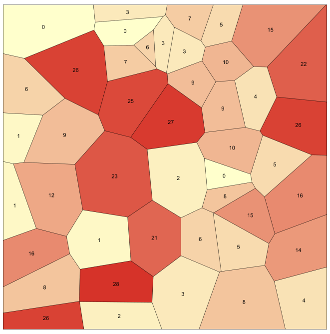
(d) $s = 28$



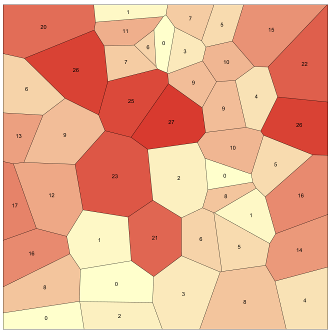
(e) $s = 29$



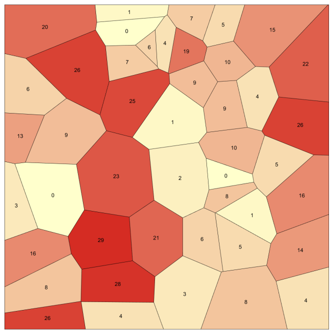
(f) $s = 30$



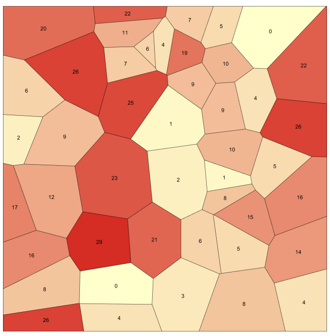
(g) $s = 31$



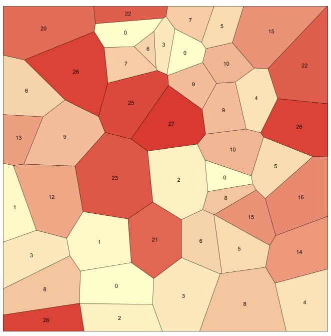
(h) $s = 32$



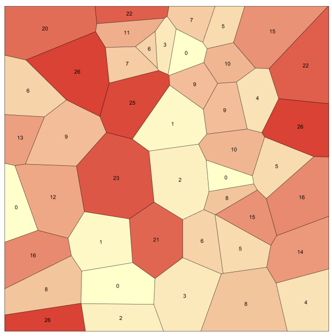
(i) $s = 33$



(j) $s = 34$



(k) $s = 35$



(l) $s = 36$

Figure 5.16: Ages of cells after 8 years, scenarios 25 to 36

Table 5.5: Values of $\left(\tilde{f}_k^s(x^*_{\text{AVG}}), \tilde{f}_k^s(x^*_{\text{MSP}})\right)$. All scenarios are equiprobable.

	$w_1 = 0.5$ $k = 1$	$w_2 = 0.2$ $k = 2$	$w_3 = 0.1$ $k = 3$	$w_4 = 0.2$ $k = 4$
$s = 1$	$(0.021, 0.093)$	$(0.001, 0.076)$	$(0.194, 0.64)$	$(0.906, 0.495)$
$s = 2$	$(0.046, 0.105)$	$(0.04, 0.102)$	$(0.27, 0.642)$	$(0.79, 0.45)$
$s = 3$	$(0.041, 0.1)$	$(0.043, 0.088)$	$(0.267, 0.637)$	$(0.793, 0.48)$
$s = 4$	$(0.052, 0.108)$	$(0.074, 0.106)$	$(0.399, 0.652)$	$(0.65, 0.446)$
$s = 5$	$(0.036, 0.109)$	$(0.035, 0.118)$	$(0.279, 0.627)$	$(0.782, 0.417)$
$s = 6$	$(0.05, 0.109)$	$(0.068, 0.115)$	$(0.413, 0.643)$	$(0.652, 0.436)$
$s = 7$	$(0.059, 0.105)$	$(0.071, 0.096)$	$(0.349, 0.652)$	$(0.698, 0.456)$
$s = 8$	$(0.068, 0.126)$	$(0.089, 0.124)$	$(0.388, 0.641)$	$(0.641, 0.417)$
$s = 9$	$(0.047, 0.103)$	$(0.051, 0.095)$	$(0.396, 0.64)$	$(0.694, 0.479)$
$s = 10$	$(0.075, 0.114)$	$(0.081, 0.131)$	$(0.355, 0.641)$	$(0.672, 0.418)$
$s = 11$	$(0.059, 0.112)$	$(0.066, 0.11)$	$(0.365, 0.7)$	$(0.696, 0.434)$
$s = 12$	$(0.066, 0.124)$	$(0.091, 0.14)$	$(0.405, 0.636)$	$(0.626, 0.43)$
$s = 13$	$(0.056, 0.102)$	$(0.047, 0.107)$	$(0.374, 0.669)$	$(0.726, 0.435)$
$s = 14$	$(0.068, 0.097)$	$(0.078, 0.112)$	$(0.482, 0.659)$	$(0.602, 0.434)$
$s = 15$	$(0.069, 0.113)$	$(0.08, 0.099)$	$(0.463, 0.696)$	$(0.616, 0.441)$
$s = 16$	$(0.072, 0.114)$	$(0.1, 0.109)$	$(0.532, 0.7)$	$(0.536, 0.436)$
$s = 17$	$(0.08, 0.107)$	$(0.094, 0.092)$	$(0.442, 0.662)$	$(0.617, 0.47)$
$s = 18$	$(0.088, 0.119)$	$(0.116, 0.114)$	$(0.559, 0.704)$	$(0.503, 0.424)$
$s = 19$	$(0.089, 0.107)$	$(0.106, 0.105)$	$(0.525, 0.659)$	$(0.554, 0.464)$
$s = 20$	$(0.1, 0.12)$	$(0.134, 0.142)$	$(0.623, 0.652)$	$(0.448, 0.42)$
$s = 21$	$(0.076, 0.116)$	$(0.081, 0.11)$	$(0.456, 0.689)$	$(0.639, 0.435)$
$s = 22$	$(0.087, 0.112)$	$(0.108, 0.121)$	$(0.597, 0.654)$	$(0.507, 0.454)$
$s = 23$	$(0.092, 0.129)$	$(0.112, 0.128)$	$(0.573, 0.673)$	$(0.519, 0.426)$
$s = 24$	$(0.101, 0.129)$	$(0.132, 0.154)$	$(0.621, 0.644)$	$(0.454, 0.414)$
$s = 25$	$(0.074, 0.118)$	$(0.063, 0.115)$	$(0.41, 0.639)$	$(0.673, 0.421)$
$s = 26$	$(0.093, 0.116)$	$(0.084, 0.119)$	$(0.446, 0.679)$	$(0.613, 0.413)$
$s = 27$	$(0.092, 0.103)$	$(0.082, 0.1)$	$(0.42, 0.661)$	$(0.64, 0.473)$
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$s = 33$	$(0.102, 0.11)$	$(0.107, 0.125)$	$(0.455, 0.639)$	$(0.592, 0.438)$
$s = 34$	$(0.109, 0.134)$	$(0.138, 0.14)$	$(0.564, 0.637)$	$(0.465, 0.415)$
$s = 35$	$(0.112, 0.125)$	$(0.139, 0.136)$	$(0.595, 0.67)$	$(0.448, 0.423)$
$s = 36$	$(0.137, 0.135)$	$(0.166, 0.154)$	$(0.584, 0.637)$	$(0.412, 0.419)$

Chapter 6

Conclusions and future work

Contents

6.1 Summary of contributions

The general objective of the thesis, as stated in Section 1.4, was to advance in the treatment of the uncertainty and multiple criteria for sustainable development and disaster management through mathematical programming models. This wide objective was split in several specific objectives applying mathematical programming with uncertainty and/or multiple criteria to several problems related to sustainable development and wildfires management, including a new methodology for multiobjective stochastic programming.

The first step towards achieving such objective was the study of mathematical programming under uncertainty, which was performed in Chapter 2. In this chapter, a problem related to the access to energy in developing countries was studied. Remote areas, especially in such countries, must be serviced by off-grid technologies in a decentralized and sustainable way. Photovoltaic technology is the most popular alternative through photovoltaic rural electrification programs based on solar home systems. These programs are launched by authorities, being carried out usually by private companies in a fee-for-service scheme. These fees can be satisfied by the end-users or subsidized by governments or Development Agencies. The access to the service must be ensured over time, establishing a maintenance program for medium to long term. Fees established must be enough to cover these costs, but they are hard to estimate. Both authorities and private companies need a method to estimate these costs when defining the program or at the bidding time.

Corrective maintenance has to be provided within 48 hours of failure. This need brings uncertainty to the problem of sizing and locating a maintenance structure. The most common ways for treating this uncertainty, stochastic programming and robust optimization, had to be discarded as neither fully reflected the nature of the problem. Instead, given that the number of expected corrective maintenance operations is known, some extra considerations were incorporated into the models to treat the uncertainty, thoroughly discussed in Section 2.2.3.

A methodology for estimating costs and sizing maintenance systems has been presented, successfully handling the uncertainty. The methodology is based on a mathematical programming model and a rule-based expert system. The mixed-integer linear programming model allows to obtain accurate costs and design of the system for a well-described program, or for simulated cases to obtain a larger data set. The rule-based expert system, comprised of a classification tree and a linear regression model, estimates the maintenance cost with little information from a dataset of real or realistic cases. It can be used for designing the program, i.e. knowing the fees to be established in a program, or for a private company to decide about participating in the program or not. If the private company chooses to participate in the program, already with extended information, the mathematical programming model can obtain the optimal structure of the maintenance system to minimize costs.

The methodology has been applied to and validated with a program in Morocco. The mathematical programming model is parameterized being suitable to be applied in other rural electrification programs, and the rule-based model is based on general parameters. Therefore, the full methodology is useful for rural electrification programs supporting design decisions and providing realistic cost estimations in order to achieve the desired Development Goal.

Among the Sustainable Development Goals there is a component of disaster risk reduction.

Wildfires are one type of disaster associated with the Sustainable Development Goals. Chapter 3 is illustrated with a prescribed burning problem. Prescribed burning is an extended technique aiming to mitigate the effects of wildfires by reducing the amount of fuel on a landscape. Unfortunately, reducing the fuel load has ecological consequences, as some species rely on old vegetation. This conflict led to the study of mathematical programming with multiple objectives.

A mixed-integer programming model was developed, suggesting which places should be burnt each year to reduce the adjacency of high fuel load areas. It is shown how adopting a medium-term approach to fuel reduction using the proposed model is much more effective than adopting a myopic approach. In this latter case, it frequently arises that fuel reduction targets cannot be met within budget constraints.

There are ecological consequences from prescribed burning. The model considers habitat quality for invertebrates on a heathland landscape and also a hypothetical species that had a preference for hazardous vegetation. It is shown that a significant range of habitat quality outcomes can be obtained without compromising the optimal fuel load goal, by using a lexicographical approach. Therefore, it is sensible that habitat considerations are included in fuel reduction plans.

After studying optimization with uncertainty and multiple criteria, Chapter 4 considers both characteristics simultaneously, producing research in a field called multiobjective stochastic programming. The applications studied through this thesis, related to emergencies and disaster management, are prone to be associated with risk-averse preferences. For this reason, a new solution concept in multiobjective stochastic programming is introduced based on risk-aversion preferences.

The solution concept is supported by an efficient way to compute it by a Mathematical Programming problem. This model is linear provided that the underlying problem can be linearly representable. Numerical experiments have been conducted for validating this approach, solving a multiobjective stochastic knapsack problem.

The research has also shown that the improvements in unfavourable situations are consistently higher than loses on average situations, especially when small values of the parameters β and r are chosen.

Results showed that there is a clear increase in computational time; however, this is arguably acceptable as a price to pay for being risk-averse. Furthermore, this could also be due to the random nature of the data. Nevertheless, it was also shown that allowing for even rather small integrality gaps (1%) leads to drastic improvement on the computing times.

This technique is used in Chapter 5 for solving the prescribed burning problem studied in Chapter 3, where a more detailed approach is taken for studying the prescribed burning problem. Four different objectives are considered, and it is taken into account that not every nominated burn is completed, thus adding uncertainty.

Results showed that using this concept greatly improves the expected outcome of unfavourable conditions, without significantly compromise average outcomes.

The thesis is finally concluded with a real application of the developed risk-aversion model. A case study is shown, using real data from a landscape in Andalucía, in the south of Spain, including almost 200 burn units and an extension of 1820km^2. Such case study effectively summarizes the main goal of the thesis: the advance in the treatment of the uncertainty and multiple criteria for sustainable development and disaster management through mathematical programming models, including a new methodology for multiobjective stochastic programming.

6.2 Productions

The present PhD dissertation is divided into four main chapters, with the ultimate goal of publishing one article with the findings of each chapter. At the time of writing two articles have been published, one has been submitted and another is under preparation. They can be found in Section 6.2.1.

Additionally, results were presented in 11 conferences. They are shown in Section 6.2.2. Section 6.2.3 enumerates the imparted seminars.

6.2.1 List of publications

Articles published in JCR journals

- **León, J.**, Martín-Campo, F.J., Ortuño, M.T., Vitoriano, B., Carrasco, L.M. & Narvarte, L. (2019), 'A methodology for designing electrification programs for remote areas', *Central European Journal of Operations Research*, doi:10.1007/s10100-019-00649-6.

- **León, J.**, Reijnders, V.M.J.J., Hearne, J.W., Ozlen, M. & Reinke, K.J. (2019), 'A Landscape-Scale Optimisation Model to Break the Hazardous Fuel Continuum While Maintaining Habitat Quality', *Environmental Modeling & Assessment* **24**(4), 369–379. doi:10.1007/s10666-018-9642-2.

Articles submitted to JCR journals

- **León, J.**, Puerto, J. & Vitoriano, B. (submitted October 2019), 'A risk-aversion methodology for the multiobjective stochastic programming problem. A real case in Andalucía (Spain)', *Computers & OR*.

Articles in preparation

- **León, J.**, Hearne, J. & Vitoriano, B. (under preparation), 'A risk-aversion solution for the prescribed burning problem'.

6.2.2 Conferences

Conference FuzzyMAD 2015. 15/12/2015. Madrid, Spain

Title Models under uncertainty for rural electrification programmes

Authors Javier León Caballero, Javier Martín Campo, Begoña Vitoriano Villanueva

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Conference SEIO 2016. 05/09/2016 - 07/09/2016. Toledo, Spain

Title Estimación de costes en programas de electrificación rural (*Cost estimation in rural electrification programs*)

Authors J. León Caballero, B. Vitoriano Villanueva, F. J. Martín Campo, M. T. Ortuño Sánchez, G. Tirado Domínguez, M. Artalejo Álvarez, L. M. Carrasco Moreno, L. Narvarte Fernández

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Conference State of the art in RTD for wildfire prevention and suppression. 06/04/2017. Valabre, France

Title Decision making models for fire suppression

Author Javier León

Conference World Conference on Natural Resource Modeling 2017. 06/06/2017 - 09/06/2017. Barcelona, Spain

Title Multicriteria and Stochastic Optimization applied to wildfires

Authors Javier León, Begoña Vitoriano, John Hearne

Conference 22nd Congress of Modelling and Simulation MODSIM 2017. 03/12/2017 - 08/12/2017. Hobart, Australia

Title Stochastic and multicriteria decision making applied to suppression of wildfires

Authors Javier León, Begoña Vitoriano, John Hearne

Conference SEIO 2018. 29/05/2018 - 01/06/2018. Oviedo, Spain

Title Un modelo de optimización para la planificación de quemas controladas atendiendo a consideraciones ecológicas (*An optimization model for scheduling prescribed burns attending to ecological considerations*)

Authors Javier León, John Hearne

Conference EWG-ORD 2018 Workshop: OR for sustainable development: Establishing Policy and Measuring Global Attainment. 05/06/2018. Madrid, Spain

Title Modeling electrification programs for development of remote areas

Authors Javier León, Begoña Vitoriano, Javier Martín-Campo, Teresa Ortuño

Conference EURO2018. 08/07/2018 - 11/07/2018. Valencia, Spain

Title Stochastic and multicriteria decision making applied to suppression of wildfires

Authors Javier León, Begoña Vitoriano, John Hearne

Conference RSFF 2018, International workshop on robust solutions for fire fighting. 19/07/2018 - 20/07/2018. L'Aquila, Italy

Title A spatial optimisation model for fuel management to break the connectivity of high-risk regions while maintaining habitat quality

Authors Javier León, Victor Reijnders, John Hearne, Melih Ozlen, Karin Reinke

Conference Workshop Towards ecological bushfire management models. 12/11/2018 - 14/11/2018. Melbourne, Australia

Title Prescribed burning and biodiversity: a landscape design problem

Authors Javier León, Victor Reijnders, John Hearne, Melih Ozlen, Karin Reinke

Conference SEIO 2019. 03/09/2019 - 06/09/2019. Alcoi, Spain

Title Un nuevo enfoque de Programación Estocástica Multicriterio con aversión al riesgo para gestión de desastres y emergencias (*A new risk-aversion approach for Multiobjective Stochastic Programming for disaster management and emergencies*)

Authors Javier León, Begoña Vitoriano, Justo Puerto

6.2.3 Seminars

Seminar 1st PhDay Complutense. 29/11/2017. Madrid, Spain

Title Programación matemática aplicada a la extinción de incendios forestales (*Mathematical programming applied to wildfire suppression*)

Author Javier León

Seminar Mathematics PhD students seminars. 31/01/2019. Madrid, Spain

Title ¿Y si no es malo todo el fuego? Un investigador operativo hablando de incendios forestales (*What if not all fire is bad? An operational researcher discussing wildfires*)

Author Javier León

Seminar 3rd PhDay Complutense. 02/07/2019. Madrid, Spain

Title Un nuevo concepto de solución para problemas de Programación Estocástica Multiobjetivo (*A new solution concept for Multiobjective Stochastic Programming problems*)

Author Javier León

6.3 Future work

Mathematical programming has been proven a useful tool throughout this thesis to tackle problems arising in disaster management, especially if multiple objectives and/or uncertainty are also present. The developed mathematical programming model provides an efficient way of computing a risk-averse solution, highly desirable in some contexts.

One of the problems studied during the PhD candidature, but not included in this dissertation, is the fire suppression problem: once a fire has started, how should the available resources be distributed to improve the efficiency of fire-fighting?

When a wildfire is active, there are many sources of uncertainty, varying from weather condition changes to lack of information on the amount of available fuel. Furthermore, incident commanders should consider a range of criteria when devising a plan for attacking the fire: minimizing costs of operation, area burnt or lost, risk for fire crews, etc. For this reason, and due to the risk-averse nature of this problem, the proposed technique for MSP problems seems fitting for solving it.

As a first step for solving such problem, a comprehensive literature review on OR methods for fire suppressions has already been produced, and it is reproduced in Section 6.3.1.

6.3.1 OR for fire suppression

The proposed Mathematical Programming model studied in Chapter 4 opens a window for studying diverse problems in emergencies. One such problem is the wildfire suppression problem. Once a fire has started knowing how it is is going to progress is essential for emergency managers to control it. Pyne et al. (1996) explains that there are two ways in essence for controlling a wildfire:

- Direct control: The one possible only when fire is completely off, and it is not possible for it to reignite again. It is only possible in small fires (small in terms of area and fuel load).
- Perimeter control: Produced when there is a discontinuity between fire and the area that could be affected by it. When fire is completely surrounded by areas free of fuel it is said that fire is *contained* or *confined*. When these areas are wide enough such that fire can not go over them it is said than the fire is *controlled*.

The decision of how to contain a wildfire can be supported by mathematical models that asses which locations are more favourable for controlling the fire. Fire propagation models are more studied and widely used by fire services. However, models that deal with the first-response decisions that need to be made have not had much impact (Martell, 2007; Duff and Tolhurst, 2015).

Previous reviews

Martell (2015) reviews the research on OR for Decision Support Systems (DSS's) for wildfire suppression, especially in North-America, from 2009 to 2014. In his own words: *"One of my greatest concerns, however, is that despite the fact that we expect fire managers to practise science-based* *management, we do not provide them with enough of the science and technology they need to achieve what society expects of them. Forest and wildland fire managers will, in the absence of an adequate understanding of fire and its potential social, economic and ecological impacts and proven fire management decision support systems they can use to enhance their decision-making, increasingly be forced to risk manage by the seat of their pants. We can and must serve their needs much better than we have in the past."*

Dunn et al. (2017) assembles a set of objective functions, decision variables and constraints that a decision support system should have. For each of those, it mentions which of the 17 studied papers include that, but does not mention the solution technique. Pacheco et al. (2015) reviews fire growth simulation models, and also four decision support systems used at the time of the review: LEOPARDS, KITRAL, SINAMI and FPA. Duff and Tolhurst (2015) focuses on reviewing fire simulators that emulate fire suppression, for different purposes, such as determining the appropriate level of investment or for using during the response phase. Sakellariou et al. (2017) reviews DSS's for different phases. Thompson et al. (2017) reviews the key decision factors of fire suppression management, such as contest, complexity, alternatives, consequences and uncertainty. It is illustrated with examples from Andalucía, Spain, and Montana, US.

Martell (1982); Martell et al. (1998) are comprehensive reviews of OR methods applied to forest fire management, later updated on Martell (2007). Minas et al. (2012) also follows this line.

OR methods for fire suppression

Resources preallocation Wei et al. (2017) present a MIP model to examine daily assignments and crews for dispatch zones in Colorado over a fire season. A simulation and optimization procedure assigns crews and engines to meet predicted next-day suppression demands at each dispatch zone, minimizing displacement cost. Fire season defined to be 150 days long. A resource cannot be more than 14 days outside its own base (later decreased to only 4 days). Solved using CPLEX.

Gonzalez-Olabarria et al. (2019) discuss about *Puntos Estratégicos de Gestión*[1] (PEGs), and about Management Areas for Fire Suppression Support (MASSs):

PEG "highly delimited areas, which include a set of infrastructures associated with a predefined fire suppression strategy, based on the study of historic large wildfires and their fire spread patterns"

MASS "areas that, once adequate fuel management is implemented, could reduce the intensity of fires and support fire suppression maneuvers, according to the requirements of the firefighting service"

Two multicriteria decision models are implemented, one for the strategic allocation of MASSs and another for tactical decisions selecting the best fuel management strategies. Criterion DecisionPlus

[1] Strategic management points

(CDP) is used for selecting the best strategies, using a combination of AHP (Analytic Hierarchy Process) and SMART (Simple Multi-Attribute Rating Technique).

Resources deployment In Mees and Strauss (1992) a very basic MIP model is constructed, that selects which resources have to be deployed to build a established fireline (in accordance to their construction rates, and probabilities of success). 41 resources in total, to be used once. For fires that have escaped initial attack.

Donovan and Rideout (2003) develop a MIP model for deploying resources to contain the fire. Fire perimeter is precomputed using Farsite, and resources have to be dispatched on given time periods to build fireline faster than the perimeter growth. Minimization of cost plus net-value change (C+NVC). Example implemented in LINGO, with 6 time periods (of 1 hour each) and 7 deployable resources. Fire perimeter is of 1 hectare on the first period, and of up to 24 on the last.

Wei et al. (2011) build a MIP model which integrates fire growth and suppression decisions. It divides the landscape on cells. Fire advance is simulated using FlamMap, and then that is transformed to fire spread rates between pairs of cells. The MIP model selects some cells to be controlled, stopping or delaying the fire advance. Belval et al. (2015) continues the work and create a more integrated approach, in which beneficial fires can be considered and fire intensities are also calculated within the MIP model (as they are able to be changed by suppression decisions). FlamMap is used to obtain fire spread rates. A basic multiobjective approach, in which the objective function is the weighted sum of the amount of cells burnt and the amount of cells controlled (as a proxy for suppression cost). Two computational results, on a 6-by-6 and a 12-by-11 cells grid. Each of the cells is of 30m by 30m. The large landscape is of just 12 hectares, but they soon ran into performance issues when solving with CPLEX: *"The results shown in Fig. 6 took 40 h to run to optimality on a computer with 32 GB of available RAM."*. Belval et al. (2016) later extends the model making it an stochastic one. Stochasticity comes in the form of uncertainty of fire spread rates, due to different weather scenarios. A multiple stages model is proposed, in which resources have to be deployed attending to non-anticipativity constraints.

Homchaudhuri et al. (2013) uses a genetic algorithm for solving a simulation-optimization approach to optimize fireline construction. The model decides where to deploy some crews (4 to 7 on the examples given), and what is the shape of the line they have to build for containing the fire. Fire growth is simulated at the same time.

In Rodríguez-Veiga, Gómez-Costa, Ginzo-Villamayor, Casas-Méndez and Sáiz-Díaz (2018) two linear integer programming models are proposed to solve decision problems framed in the line of optimal allocation of fire extinction resources. The first model is designed to maximize the output per hour of aerial resources flight time, and the second manages the allocation of aerial resources to refueling bases. Programmed with AMPL, solved with Gurobi. Around 10-15 helicopters are considered on the test cases, with a time-horizon of 30 minutes.Rodríguez-Veiga, Ginzo-Villamayor and Casas-Méndez (2018) proposes an integer linear programming model that optimally selects the resources to be used during a planning period for forest fire extinction, using the C+NVC approach. The formulation addresses maximum flight times and the required rest breaks for air resources and maximum daily operation time for brigades. Without interaction with fire. At the beginning fire perimeter is calculated, and the goal is to deploy resources that can build fireline faster than the perimeter growth. The main real test case has 13 different resources to deploy (between aircraft, engines and crews), and the initial fire perimeter is of 10km. The case includes fourteen 10-minute periods, and is solved with Gurobi. Larger simulated cases are also solved, with 20 to 40 time-periods, and 15 to 30 resources to be deployed.

Preallocation + deployment Haight and Fried (2007) present a two-stage stochastic model for the deployment of resources to contain fire. A set of 22 resources can be deployed to 15 bases. CFES2 (*California Fire Economics Simulator version 2*) is used for simulating 100 fire scenarios, each of them with 4 to 10 fires occurring at different locations. For each of these fire scenarios suppression is also simulated, so it is known which of the 22 resources are needed to contain the fire. Then the two-stage model select where to preallocate the resources, given that they will be dispatched only if they are less than 30-minutes away from the fire (*"We use a 30-minute respond threshold because fast-spreading fires tend to escape initial attack if firefighting is not well underway within 30 minutes following a fire report"*). The objective function is a weighted sum of two objectives: the minimization of preallocated resources and the minimization of the number of fires not able to be controlled with the preallocated resources. Different weights are given to see the tradeoffs. Implemented on GAMS and CPLEX.

Hu and Ntaimo (2009) develop a stochastic extension of the model on Donovan and Rideout (2003). It consists of a two-stage model: a set of fires are simulated with DEVS-FIRE, in which fire perimeter is calculated. The first stage of the SP consists on renting a series of resources, which will be dispatched on the second stage to contain the fire (by having a fireline construction rate higher than the perimeter growth), while minimizing of C+NVC. The exact procedure for dispatching the resourced is later tuned with a simulation model, in which different attack techniques are coded. Experimental results with a time-limit of 6 hours (only initial attack), 6 periods of one hour, a cellular landscape of 3 600 hectares, two fire scenarios, in which fire at the 6th period has burned less than 200 hectares in both scenarios, and 7 resources to be rented and dispatched. CPLEX. The same work is continued on Ntaimo et al. (2012). The model is augmented considering different bases where resources are allocated and can be moved between them before fire occurs. For the application a district in Texas with 7 bases is chosen. Over 13 000 historical fires occurring on that area from 1985 to 2006 are considered. By a clustering technique and an identification of unique patterns, 756 fire scenarios are considered. 18 resources already available on some bases, with 10 extra to be acquired if desired. The SP could not be solved with that many scenarios, so sampling was used, with sample sizes ranging from 25 to 50 scenarios. 6 hours of fire growth simulated with BehavePlus. CPLEX. Later on Ntaimo et al. (2013) the authors include the fire-growth on half an hour intervals to the model, simulated with DEVS-FIRE.

In Chow and Regan (2011) two models for the deployment and dispatching of aerial resources are presented. First one is a MIP to allocate airtankers to bases during a fire season (similar to a p-median problem). Later a stochastic extension is built, including relocation of resources, which is solved with rolling horizon.

Yohan et al. (2014) develop a 2-stage stochastic MIP, for deploying and dispatching resources for initial attack with a case-study in Korea. Fire scenarios are built with historical data, and a sensitivity analysis is performed.

In Gallego Arrubla et al. (2014) the authors develop a stochastic MIP for resource preallocation and deployment, using a C+NVC approach. A chance-constrained approach is used so initial attack is viable for a percentage of fires. Fire manager has to decide its risk-aversion parameter. Different instances are solved, ranging from 5 to 22 fire scenarios, with 38 resources to allocate on 7 bases, and different risk-aversion parameters chosen. CPLEX is used to obtain the optimal solution.

Wei et al. (2015) develop a two-stage stochastic model, in which suppression resources have to be first deployed (for the season) and then they should be sufficient for suppressing a series of scenarios. A chance-constrained approach is used, such that most fires have to be controlled via initial attack. A library of 935 fire scenarios is considered, in which perimeter growth is calculated with FARSITE, and suppression is performed by having a fireline construction rate higher than perimeter growth.

Large fire management Several papers have been found in the literature studying initial attack of wildfires. However, there does not seem to be any OR models for suppression for extended attack:

- Martell (2007): *"Despite the early interest in large fire management there have been few efforts to bring OR to bear on large fire management problems."*

- Finney et al. (2009): *"Nevertheless, the effectiveness of suppression efforts on the progress or containment of large fires has not been modelled or even characterized, and it is presently not known what or how different factors are related to successful containment."*

- Rönnqvist et al. (2015) enumerate a series of open problems in forestries. Their 22nd problem: *"How can we develop large fire management decision support systems that can be used to help determine how to best manage a large fire that has escaped initial attack and may pose significant threats to people, property and forest resources while it enhances natural ecosystem processes under uncertainty?"*

- Thompson et al. (2017): *"Although there exists a large body of research on efficient fire management, most efforts have focused on initial control efforts, whose principal objective is often to keep new ignitions contained as small or as quickly as possible."*

- Wei et al. (2018): *"We therefore argue that a need exists for more contextually relevant decision support to facilitate evaluation of large fire confinement and point protection strategies."*. After identifying this knowledge gap, the authors develop a MIP model to aggregate *Potential wildland fire Operation Delineations (PODs)* into a *response POD* for containing large fires.

Proposed work

Several papers have been found suggesting models how to allocate or preallocate resources for fire suppression. However, the literature review has shown that:

1. Multiple objectives are barely considered, and most papers accounting for various objectives do so by using a cost plus net-value-change approach. With this approach, fire suppression costs are simply added to the losses produced by the fire.

2. Despite the high-risk that is associated with wildfires, the reviewed articles in which uncertainty is included do not search for risk-averse solutions in their proposed models.

A natural progression of this work is to develop a multiobjective stochastic programming model, or adapt one from the literature, to solve it with the risk-aversion methodology discussed in Chapter 4.

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