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**REISSNER-NORDSTRÖM BLACK HOLES IN  
MODIFIED ELECTRODYNAMICS THEORIES**

**AGUJEROS NEGROS DE  
REISSNER-NORDSTRÖM EN TEORÍAS DE  
ELECTRODINÁMICA MODIFICADA**

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## Keywords

**Keywords:** Non-linear Electrodynamics, Dirac monopoles, Modified gravitational theories, Reissner-Nordström Black Holes.

**Palabras clave:** Electrodinámica no lineal, Monopolos de Dirac, Teorías de gravedad modificada, Agujeros negros de Reissner-Nordström.

## Abstract

In the context of modified and gauge invariant Electrodynamics theories minimally coupled with gravitation we look for the  $U(1)$  Lagrangian densities supporting electric and magnetic monopoles which provide static, spherically symmetric and constant curvature Reissner-Nordström black-hole solutions. We achieve a sufficient condition for general Lagrangian densities supporting this kind of solutions, and we propose a simple model which can be interpreted as a small correction to the usual Electrodynamics theory, which is proven to be correct in the asymptotic limit  $r \rightarrow \infty$ . For these models we obtain their correspondent metrics, and then by employing the Euclidean Action approach we perform a thermodynamics analysis and study the existing phases depending on the sign of the heat capacity and the Helmholtz free energy. Thus, we obtain that modified Electrodynamics theories lead to very different thermodynamics properties and, in some particular cases, to a new phase which does not appear in the usual theory.

## Resumen

En el marco de las teorías de electrodinámica modificada, invariantes gauge y mínimamente acopladas a la gravedad, buscamos las densidades lagrangianas con simetría  $U(1)$  que admiten monopolos eléctricos y magnéticos en soluciones de agujero negro de Reissner-Nordström estáticas, esféricamente simétricas y de curvatura constante. En este trabajo obtenemos una condición suficiente para que las densidades Lagrangianas posean este tipo de soluciones, y proponemos un modelo simple que puede ser interpretado como una pequeña corrección a la teoría electrodinámica usual, la cual sabemos que es correcta en el límite asintótico  $r \rightarrow \infty$ . Para dichos modelos obtenemos las correspondientes métricas, y empleando el método de la acción euclídea realizamos un análisis termodinámico y estudiamos las fases de estabilidad presentes, en función del signo de la capacidad calorífica y de la energía libre de Helmholtz. Así, obtenemos que las teorías de electrodinámica modificada conllevan propiedades termodinámicas muy diferentes para las soluciones y, en algunos casos particulares, una nueva fase de estabilidad que no aparece en la teoría usual.

# Chapter 1

## Introduction

General Relativity has been the most successful gravitational theory of the 20th century, and in this frame, Einstein's field equations provide how the spacetime is curved in presence of matter or energy. In this work we are interested in a particular family of solutions of the Einstein's equations: the black-hole (BH) solutions. The concept of BH was introduced for the first time in the second half of the 18th century, when John Michell and Pierre-Simon Laplace proposed in a classical frame the existence of stars massive enough that the escape velocity from their surface was bigger than the speed light, so they would be "invisible" [1]. However, for such intense gravity forces the classical Newtonian gravity theory is not valid, and we have to use the previously appointed General Relativity. In 1916, Karl Schwarzschild found a solution of Einstein's equations for a point mass in a flat space [2]. This solution has a singularity in its center and an horizon surface at  $r = 2M$ , such that any light beam which traverses the horizon cannot escape from the BH. Depending on the parameters which characterize the solutions we define different kinds of BH as for example Schwarzschild BH [2] for a spherically symmetric, static, non rotating and uncharged massive body, Kerr BH [3] for rotating bodies, Reissner-Nordström BH [4, 5] (in which we shall focus) for charged objects and Kerr-Newman BH [6] for rotating and charged bodies.

In the standard theory, with an Electrodynamics Lagrangian density proportional to  $F_{\mu\nu}F^{\mu\nu}$  coupled with gravity, the Reissner-Nordström BH solution has been widely studied (see, for example, Refs. [7, 8, 9]). Nevertheless, other modified Electrodynamics theories have been suggested, mainly due to the divergence of self-energy of point charges (like electrons) in the standard Electrodynamics theory. Some important examples of this kind of theories are the Born-Infeld models [10] and the Euler-Heisenberg models [11]. Born-Infeld models lead to electromagnetic fields which in the asymptotic limit behave as usual, but for which the divergence of the origin is avoided, whereas Euler and Heisenberg obtained a similar result when they studied QED vacuum polarizations in the constant background field limit [12]. These models are really important since some Born-Infeld-like models arise, together with the gravitational field, in the low-energy limit of string theory [13, 14, 15]. Moreover,

in the last years some works studied general modified Electrodynamics models coupled with gravity which provide static and spherically symmetric metrics for electrostatic spherically symmetric fields (see, for example, [16, 17]).

In this project we shall obtain sufficient conditions for the modified Electrodynamics Lagrangian densities in order to provide Reissner-Nordström-like solutions (i.e., spherically symmetric and with constant scalar of curvature) by assuming static and spherically symmetric electric and magnetic fields. Once we get it, we shall propose a simple model of Lagrangian density in order to study its solution. Finally, we shall perform a thermodynamics analysis of the obtained solutions.

The study of BH's thermodynamics started in 1970's with the attainment of the four laws of BH's dynamics [18], which can be summarized in the following way:

- Zeroth Law: the surface gravity  $\kappa$  is constant for a stationary BH over the horizon.
- First Law: the perturbations of mass  $M$ , area  $A$ , angular momentum  $J$  and charge  $Q$  of a stationary BH are fixed by the relation

$$dM = \frac{\kappa}{8\pi}dA + \Omega dJ + \Phi dQ$$

being  $\Omega$  the angular velocity of the BH and  $\Phi$  the electrostatic potential.

- Second Law: the area of the horizon of each BH does not decrease with time provided that the null energy condition holds on [19] ( $(T_{\mu\nu} - R_{\mu\nu})k^\mu k^\nu \geq 0$ , being  $k^\mu$  an arbitrary null vector oriented to the future,  $R_{\mu\nu}$  the Ricci tensor and  $T_{\mu\nu}$  the energy-momentum tensor).
- Third Law: it is impossible by any procedure to reduce the surface gravity to zero by a finite sequence of steps.

These mechanics laws seem very similar to the four laws of Thermodynamics, where the mass of BH's, the area of the horizon and the surface gravity play the roles of the energy, the entropy and the temperature, respectively. However, in a classical frame there is no way to get this relation. First of all, with the classical universal constants (gravitational constant  $G$ , speed of light  $c$ , Boltzmann constant  $k_B$ ) it is not possible to relate these quantities due to dimensional problems. On the other hand, if the BH's had an associated non zero temperature, they would emit radiation. However, Hawking [20] found that due to quantum particle creation effects BH's emit radiation as a black body of temperature  $T = \hbar\kappa/4\pi$ .

In order to obtain the thermodynamical quantities of our BH solutions, we shall use the Euclidean Action Method [21, 22]. This method consists in change the real time coordinate to an imaginary time, so the BH metric becomes Euclidean. Then we can perform a path integral approach in an Euclidean section which avoids the singularity at the origin (since the Euclidean metric corresponds only to the region  $r > r_h$ , being  $r_h$  the external horizon



of the BH) . This Euclidean approach presents some difficulties when is applied to General Relativity. Except in specials cases it is generally impossible to represent an analytic space-time as a Lorentzian section of a four-complex-dimensional manifold with a complex metric which possesses a Euclidean section. So there is not a general prescription for analytically continuing Lorentzian signature metrics to Riemannian metrics. However, in static metrics in which we shall focus we can do it, so we have not this problem. Nevertheless, even if one did, there are not any theorems which guarantee the analyticity of the obtained quantities (for further details, see Ref. [19]). We shall employ this method, and once we obtain the thermodynamics quantities, as the heat capacity, the free energy and the entropy, we shall discuss their admissibility. Moreover, with these thermodynamics quantities we shall study the stability of the solutions, and accordingly we shall define the existing different phases.

The work is divided as follows: in Section 2 we shall show some general results of modified Electrodynamics models coupled with gravity, and we shall obtain a sufficient condition of these models supporting Reissner-Nordström-like solutions, with constant curvature. In Section 3 we shall propose a simple example of these models, and we shall achieve some general results of them. In Section 4 we apply the Euclidean Method in order to distinguish the different thermodynamics phases of the solutions, defined in terms of their stability, and we shall compare the phase diagrams for different Electrodynamics models. On the other hand, in Section 5 we perform a classification of the BH configurations depending on the phase transitions that they present. Finally, in Section 6 we summarize the main conclusions of the work.



# Chapter 2

## General Results

In this section, we shall show some general results of static and spherically symmetric solutions from general relativity and modified Electrodynamics models. We shall follow the Ref. [16], and we shall generalize their results to the case they exist both electric and magnetic fields.

First of all, we have to remark that in all this project we have chosen Planck units,  $G = c = k_B = \hbar = 4\pi\epsilon_0 = 1$ . With this election, the mass shall be expressed in Planck mass ( $1 M_p \simeq 2.18 \cdot 10^{-8}$  kg), the charges in Planck charge ( $1 q_p \simeq 1.9 \cdot 10^{-18}$  C), the temperature in Planck temperature ( $1 T_p \simeq 1.4 \cdot 10^{32}$  K) and the length in Planck length ( $1 l_p \simeq 1.6 \cdot 10^{-35}$  m).

Let start from the action:

$$S = S_g + S_{U(1)}, \quad (2.1)$$

where  $S_g$  and  $S_{U(1)}$  are the gravitational and matter terms of the action, respectively. The usual gravitational action term takes the form:

$$S_g = \frac{1}{16\pi} \int d^4x \sqrt{|g|} (R - 2\Lambda), \quad (2.2)$$

being  $g$  the determinant of the metric  $g_{\mu\nu}$  ( $\mu, \nu = 0, 1, 2, 3$ ),  $R$  the scalar curvature and  $\Lambda$  a cosmological constant.

On the other hand, we shall assume that the matter term of the action  $S_{U(1)}$  depends on a Lagrangian density  $\varphi(X, Y)$  which is an arbitrary function of the Maxwell's invariants  $X$  and  $Y$ , with:

$$X \equiv -\frac{1}{2} F_{\mu\nu} F^{\mu\nu}, \quad Y \equiv -\frac{1}{2} F_{\mu\nu} F^{*\mu\nu}, \quad (2.3)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the usual electromagnetic tensor and  $F^{*\mu\nu} = \frac{1}{2}\sqrt{|g|}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ . In terms of the Lagrangian density  $\varphi(X, Y)$ , the matter term of (2.1) takes the form:

$$S_{U(1)} = - \int d^4x \sqrt{|g|} \varphi(X, Y). \quad (2.4)$$

In this work we shall focus on static and spherically symmetric solution. Thus, for the metric tensor we consider the most general ansatz for static and spherically symmetric scenarios:

$$ds^2 = \lambda(r)dt^2 - \frac{1}{\mu(r)}dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (2.5)$$

where the functions  $\lambda(r)$  and  $\mu(r)$  depend solely on  $r$  in order to ensure staticity and spherical symmetry. On the other hand, with this metric (2.5) we consider an ansatz for the electromagnetic tensor:

$$F_{01} = -F_{10} = E(r), \quad F_{23} = -F_{32} = -B(r)r^2 \sin\theta, \quad (2.6)$$

being identically null the other components, and  $E(r)$  and  $B(r)$  are functions on  $r$ . In Minkowski space, with  $\lambda(r)$  and  $\mu(r)$  equal to 1, (2.6) is the electromagnetic tensor for radial electric and magnetic fields  $E(r)$  and  $B(r)$ , respectively [23]. For this reason, we shall refer to these functions as ‘‘electric’’ and ‘‘magnetic’’ fields.

With the metric (2.5), we can raise or lower the index in (2.6), and then we can rewrite the gauge invariants (2.3) in terms of the electric and magnetic fields:

$$X = \frac{\mu(r)}{\lambda(r)}E(r)^2 - B(r)^2, \quad Y = 2\sqrt{\frac{\mu(r)}{\lambda(r)}}E(r) \cdot B(r). \quad (2.7)$$

Furthermore, with (2.5) we get the scalar curvature  $R$  as function on the coefficients  $\lambda(r)$  and  $\mu(r)$ :

$$\begin{aligned} R(r) = & -\frac{1}{2\lambda(r)^2 r^2} [-\lambda'(r)\mu'(r)\lambda(r)r^2 - 2\mu(r)\lambda''(r)\lambda(r)r^2 \\ & + \lambda'(r)^2 \mu r^2 - 4r\mu(r)\lambda'(r)\lambda(r) - 4r\mu'(r)\lambda(r)^2 \\ & + 4\lambda(r)^2 - 4\lambda(r)^2\mu(r)], \end{aligned} \quad (2.8)$$

where prime denotes derivative with respect to  $r$ .

From (2.4), we define the energy-momentum tensor as:

$$\begin{aligned} T_{\mu\nu} &= -\frac{2}{\sqrt{|g|}} \frac{\delta S_{U(1)}}{\delta g^{\mu\nu}} \\ &= 2F_{\mu\alpha}(\varphi_X F_\nu^\alpha + \varphi_Y F_\nu^{*\alpha}) - \varphi g_{\mu\nu}, \end{aligned} \quad (2.9)$$

where we denote  $\varphi_X \equiv \frac{\partial\varphi}{\partial X}$  and  $\varphi_Y \equiv \frac{\partial\varphi}{\partial Y}$ . By replacing (2.6) and (2.5), we obtain the non null components of (2.9):

$$T_0^0(r) = T_1^1(r) = \varphi_Y Y + 2\varphi_X \frac{\mu(r)}{\lambda(r)} E(r)^2 - \varphi, \quad (2.10)$$

$$T_2^2(r) = T_3^3(r) = \varphi_Y Y - 2\varphi_X B(r)^2 - \varphi. \quad (2.11)$$

By performing variations of (2.1) with respect to the metric tensor, we achieve the Einstein field equations in metric formalism:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} + 8\pi T_{\mu\nu} = 0, \quad (2.12)$$

where  $R_{\mu\nu}$  holds for the Ricci Tensor. By taking the trace in the previous expression, we obtain:

$$R = 4\Lambda + 8\pi T, \quad (2.13)$$

where

$$T \equiv T^\mu{}_\mu = 4(\varphi_X X + \varphi_Y Y - \varphi). \quad (2.14)$$

Besides, in terms of  $\varphi(X, Y)$  and its derivatives, the associated Euler field equations, together with the Bianchi identities for the electromagnetic field, take the form (see Ref. [17]):

$$\nabla_\mu (\varphi_X F^{\mu\nu} + \varphi_Y F^{*\mu\nu}) = 0, \quad \nabla_\mu F^{*\mu\nu} = 0, \quad (2.15)$$

where we denote  $\varphi_X = \frac{\partial\varphi}{\partial X}$  and  $\varphi_Y = \frac{\partial\varphi}{\partial Y}$ .

By replacing the metric tensor (2.5) and the only non-zero components of the energy-momentum tensor (2.10) and (2.11) in the equations (2.12), and defining the quantity  $\zeta(r) \equiv \lambda(r)/\mu(r)$ , we get that the field equation obtained by subtraction of equations with  $\mu = \nu = t$  and  $\mu = \nu = r$  yields:

$$\zeta'(r) = 0, \quad (2.16)$$

i.e., the quantity  $\lambda(r)/\mu(r)$  is a constant, which can be fixed to one by performing a time reparametrization. In others words, equation (2.16) is equivalent to

$$\lambda(r) = \mu(r). \quad (2.17)$$

With this expression, we could simplify the expressions of the gauge invariants (2.7), which read as follows:

$$X = E(r)^2 - B(r)^2, \quad Y = 2E(r) \cdot B(r), \quad (2.18)$$

Moreover, we can replace (2.17) in the rest of field equations (2.12), and we obtain:

$$-r\lambda'(r) - \lambda(r) + 1 + 8\pi T_0^0(r)r^2 = 0, \quad (2.19)$$

$$-16\pi T_2^2(r)r + 2\lambda'(r) + r\lambda''(r) = 0, \quad (2.20)$$

where (2.19) is obtained from the equation (2.12) with  $\mu = \nu = r$  and, on the other hand, equation (2.20) is proportional to (2.12) with  $\mu = \nu = \theta$  or  $\mu = \nu = \phi$  (both equations are in fact equivalent). Additionally, by replacing (2.17) in (2.10) and (2.11), the non-vanishing components of the energy-momentum tensor can be rewritten as:

$$T_0^0(r) = T_1^1(r) = 2\varphi_X E(r)^2 + 2\varphi_Y E(r)B(r) - \varphi, \quad (2.21)$$

$$T_2^2(r) = T_3^3(r) = -2\varphi_X B(r)^2 + 2\varphi_Y E(r)B(r) - \varphi. \quad (2.22)$$

The general solution of this field equations system (2.19, 2.20) reads:

$$\lambda(r) = 1 - \frac{2M}{r} - \frac{8\pi}{r} \int_r^\infty x^2 T_0^0(x) dx + \frac{1}{3} \Lambda r^2, \quad (2.23)$$

where  $M$  is an integration constant, that can be identified as the BH's mass. The metric function (2.23) can be rewritten in terms of an "external energy function", which is defined as:

$$\varepsilon_{ex}(r) = -4\pi \int_r^\infty x^2 T_0^0(x) dx. \quad (2.24)$$

This external energy represents the energy provided by the  $U(1)$  fields  $E(r)$  and  $B(r)$  outside a sphere of radius  $r$  (see Ref. [16]).

In this project we shall focus in constant curvature solutions, i.e., Reissner-Nordstöm-like solutions. From (2.13) we see that in order to obtain constant curvature solutions, the trace of the energy momentum tensor (2.14) cannot depend on  $r$ . By assuming that the energy-momentum tensor has null trace (i.e., by assuming that the scalar of curvature is fully determined by  $\Lambda$ ), we obtain that in order to verify (2.14) equal to zero the Electrodynamics Lagrangian density has to take the form:

$$\varphi(X, Y) = X \cdot \Phi\left(\frac{X}{Y}\right), \quad (2.25)$$

where  $\Phi\left(\frac{X}{Y}\right)$  is an arbitrary function on  $X/Y$  which in the standard Electrodynamics theory is equal to  $\Phi\left(\frac{X}{Y}\right) = b = -1/8\pi$ .

On the other hand, Maxwell's equations (2.15) can be expressed for static and spherically symmetric solutions as

$$B(r) = \frac{Q_m}{r^2}, \quad (2.26)$$

$$r^2 \varphi_X E(r) = -\varphi_Y Q_m + b Q_e, \quad (2.27)$$

with  $Q_m$  and  $Q_e$  integration constants of the Maxwell's equations, which shall be referred to as "magnetic charge" and "electric charge", respectively.

As a summary of this section, by assuming the scalar of curvature is fully determined by a cosmological constant  $\Lambda$  we have obtained that the most general modified Electrodynamics Lagrangian density providing a static, spherically symmetric and constant curvature metric (2.5) is (2.25) for which both metric functions  $\lambda(r)$  and  $\mu(r)$  takes the form (2.23); and the Maxwell's equations in this kind of solutions read as (2.26) and (2.27).





## Chapter 3

# General Results in our proposed model

In this section we shall propose a model supporting Reissner-Nordstöm-like solution. Let us propose an ansatz for the Electrodynamics Lagrangian density:

$$\varphi(X, Y) = bX + a \frac{X^{n+1}}{Y^n}. \quad (3.1)$$

This ansatz satisfies the sufficient condition (2.25) to get a constant curvature solution, and can be seen as a perturbation of the standard Electrodynamics theory  $\varphi(X, Y) \sim X$  for  $a$  small enough. Moreover, (3.1) represents the unique non trivially null terms of a Taylor series of a general modified Electrodynamics theory with constant curvature solutions.

By replacing this ansatz in the Maxwell's equation (2.26), we can rewrite it as:

$$\begin{aligned} r^2 \left[ b + a(n+1) \left( \frac{E(r)^2 - \frac{Q_m^2}{r^4}}{2E(r)Q_m} r^2 \right)^n \right] E(r) \\ = an \left( \frac{E(r)^2 - \frac{Q_m^2}{r^4}}{2E(r)Q_m} r^2 \right)^{n+1} Q_m + bQ_e. \end{aligned} \quad (3.2)$$

It is easy to see that Maxwell's equation (3.2) possesses solutions for electric fields that decrease as  $r^{-2}$ . Thus, provided that we impose  $E(r) = \Theta/r^2$ , we obtain a equation for this parameter  $\Theta$ :

$$\begin{aligned} \Theta \left[ b + a(n+1) \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^n \right] \\ = an \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^{n+1} Q_m + bQ_e. \end{aligned} \quad (3.3)$$

From this equation, one can obtain the parameter  $\Theta$  as function of  $b$ ,  $a$  and the charges  $Q_e$  and  $Q_m$ , and it coincides with  $Q_e$  in the standard Electrodynamics theory  $a = 0$ . The relation between these coefficients for general  $n$  is not trivial, but for small enough  $a$  (3.3) can be expressed as:

$$\Theta = Q_e - a \frac{(2+n)Q_e^2 + nQ_m^2}{2^{1+n}bQ_e^{n+1}Q_m^n} \cdot (Q_e^2 - Q_m^2)^n + \mathcal{O}(a^2). \quad (3.4)$$

In the following, instead of using as charges  $\{Q_e, Q_m\}$  we choose  $\{\Theta, Q_m\}$ . This election has the important advantage that the electric and magnetic fields read directly as  $E = \Theta/r^2$  and  $B = Q_m/r^2$  and it simplifies the next results.

On the other hand, we can obtain the non-vanishing components of the energy-momentum tensor by replacing our model of Lagrangian density (3.1) and the form of the fields  $E(r)$  and  $B(r)$  in equations (2.21) and (2.22):

$$\begin{aligned} T_0^0(r) &= T_1^1(r) \\ &= \frac{1}{r^4} \left[ b + a(n+1) \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^n \right] (\Theta^2 + Q_m^2), \end{aligned} \quad (3.5)$$

$$\begin{aligned} T_2^2(r) &= T_3^3(r) \\ &= -\frac{1}{r^4} \left[ b + a(n+1) \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^n \right] (\Theta^2 + Q_m^2). \end{aligned} \quad (3.6)$$

We can now replace the component of the energy-momentum tensor (3.5) in (2.24), obtaining the external energy:

$$\begin{aligned} \varepsilon_{ex}(r) &= -\frac{4\pi}{r} (\Theta^2 + Q_m^2) \left[ b \right. \\ &\quad \left. + a(n+1) \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^n \right]. \end{aligned} \quad (3.7)$$

We can observe that the external energy diverges at the origin, i.e., the total energy from the  $U(1)$  fields is divergent. However, this divergence also occurs in the standard case ( $a = 0$ ), so this is not such a big problem. Furthermore, we can replace (3.7) in the expression (2.23), and we achieve the form of the metric parameter  $\lambda(r)$ :

$$\lambda(r) = 1 - \frac{2M}{r} + \frac{K}{r^2} + \frac{1}{3}\Lambda r^2, \quad (3.8)$$

where:

$$K = -8\pi \left[ b + a(n+1) \left( \frac{\Theta^2 - Q_m^2}{2\Theta Q_m} \right)^n \right] (\Theta^2 + Q_m^2). \quad (3.9)$$

The obtained metric is a Reissner-Nordström-like with a scalar of curvature  $R = 4\Lambda$ , and a modified charge term equal to  $K(\Theta, Q_m)$  which in the standard case ( $a = 0$ ,  $b = -1/8\pi$ ) recovers the well known sum of squares of charges  $Q_e^2 + Q_m^2$ .

In order to obtain the radii of the horizons, we can follow two equivalent approaches. The first and most usual one is to calculate the roots of  $\lambda(r)$ . The second one, is to obtain the intersection points between the curves

$$M - \frac{r_h}{2} - \frac{1}{6}\Lambda r_h^2 = \varepsilon_{ex}(r_h). \quad (3.10)$$

As we appointed, both approaches are equivalent, and we can obtain only two real roots: the external horizon  $r_h$  (event horizon) and an internal horizon, which could be positive or negative depending on the sign of the external energy (or, equivalently, on the sign of  $K$ ). We are just interested in the anti-de Sitter (AdS) case  $\Lambda > 0$ , since as we shall mention in the Section 4 there is a normalization problem of the Killing vector  $\partial_t$  if  $\Lambda < 0$ . Thus, the value of the external horizon can be represented as (see Ref. [24]):

$$r_h = \frac{1}{2} \left( \sqrt{x} + \sqrt{-\frac{6}{\Lambda} - x + \frac{12M}{\Lambda\sqrt{x}}} \right), \quad (3.11)$$

with

$$x = \left( \frac{1 + 4\Lambda K}{\Lambda} \right) \sqrt[3]{\frac{2}{y}} + \frac{3}{\Lambda} \sqrt[3]{\frac{y}{32}} - \frac{2}{\Lambda}, \quad (3.12)$$

and

$$y = \frac{2 + 36\Lambda M^2 - 24\Lambda K}{+\sqrt{(2 + 36\Lambda M^2 - 24\Lambda K)^2 - 4(1 + 4\Lambda K)^3}}. \quad (3.13)$$

Moreover, with (3.10) we can write the BH mass as a function of the external horizon radius  $r_h$ , the charge term  $K(\Theta, Q_m)$  and the cosmological constant  $\Lambda$  (when at least one horizon is presented):

$$M(r_h) = \frac{r_h}{2} \left( 1 + \frac{K(\Theta, Q_m)}{r_h^2} + \frac{1}{3}\Lambda r_h^2 \right). \quad (3.14)$$

If we assume both  $K(\Theta, Q_m)$  and  $\Lambda$  positive (as in the standard AdS case), the function  $M(r_h)$  has a minimum at  $r_{h \min} = \sqrt{\Lambda \left( \sqrt{1 + 4K(\Theta, Q_m)/\Lambda^2} - 1 \right)}/2$ . This means that provided the mass of the configuration is small enough, there is not any horizon and then such configuration would not be a BH solutions. However, if  $K(\Theta, Q_m)$  takes a negative value and  $\Lambda$  is non negative, the ranges of values of  $M(r_h)$  covers entirely the interval  $[0, \infty)$ , and then for any mass value it is possible a BH solution.



# Chapter 4

## Thermodynamics analysis in AdS space

In this section, we shall apply the so-called Euclidean Action method [21] in order to obtain a thermodynamics analysis of our solution. We shall focus in the AdS space case ( $\Lambda > 0$ ), since otherwise some problems of normalization of the temporal Killing  $\partial_t$  arise (see Ref. [25]). With this method, we shall obtain the thermodynamics properties of the BH solutions, in terms of which their stability could be discussed.

First of all, we shall obtain the BH temperature. It can be defined in terms of the horizon gravity  $\kappa$  as [26]:

$$T = \frac{\kappa}{4\pi}, \quad (4.1)$$

where the horizon gravity is defined as:

$$\kappa = \lim_{r \rightarrow r_h} \frac{\partial_r g_{tt}}{|g_{tt} g_{rr}|}. \quad (4.2)$$

This expression can be simplified by replacing (3.8), so we can express the temperature as:

$$T = \frac{1}{4\pi r_h} \left( 1 - \frac{K(\Theta, Q_m)}{r_h^2} + \Lambda r_h^2 \right). \quad (4.3)$$

For large BH's with  $r_h \rightarrow \infty$  the temperature goes to infinity. On the other hand, near to  $r_h \sim 0$  this temperature diverges, being its sign the opposite of the sign of  $K(\Theta, Q_m)$ . It is important to remark that, by imposing the positivity of the temperature (4.3), we achieve the condition:

$$K(\Theta, Q_m) < r_h^2 (1 - \Lambda r_h^2). \quad (4.4)$$

Once we have obtained the temperature, we can compute the other thermodynamics quantities. The action takes the form (2.1) where the matter term is given by the proposed Lagrangian density (3.1). If in the action we change the time coordinate to an imaginary time  $t \rightarrow i\tau$  the action becomes Euclidean and the metric becomes periodical in this imaginary time  $\tau$ , with a period  $\beta$  which coincides with the inverse of the temperature (4.3). Let us remember that this change to an imaginary time is not trivial: when we perform the coordinate change, we must add a global sign to the action and, since the magnetic field is a pseudo-vector, the magnetic charge becomes imaginary ( $B \rightarrow iB$ ) in the action, so:

$$X = E^2 - B^2 \rightarrow \tilde{X} = E^2 + B^2 = \frac{\Theta^2 + Q_m^2}{r^4}, \quad (4.5)$$

$$Y = 2EB \rightarrow \tilde{Y} = 2iEB = 2i\Theta Q_m. \quad (4.6)$$

So after performing the corresponding changes, the Euclidean action reads:

$$\Delta S_E = -\frac{1}{16\pi} \int d^4x \sqrt{|g|} \left[ R - 2\Lambda - 16\pi\varphi(\tilde{X}, \tilde{Y}) \right]. \quad (4.7)$$

Then, we just need to evaluate the integral in the difference of four-volume of two metrics: the first volume, when there is solely an AdS metric ( $M = 0$ ,  $\Theta = 0$  and  $Q_m = 0$ ); and second one, when there is our metric solution (3.8) (see Ref. [27]). Then, we obtain:

$$\begin{aligned} \Delta S_E &= \beta \left[ -\frac{\Lambda}{12} \left( r_h^3 - \frac{3}{\Lambda} r_h \right) \right. \\ &\quad \left. + \frac{K(\Theta, Q_m) + 2h(\Theta, Q_m)}{4r_h} \right], \end{aligned} \quad (4.8)$$

with

$$h(\Theta, Q_m) = -8\pi \left[ b + a \left( \frac{\Theta^2 + Q_m^2}{2i\Theta Q_m} \right)^n \right] (\Theta^2 + Q_m^2). \quad (4.9)$$

Thus, from (4.8) we can obtain the different thermodynamics parameters. The Helmholtz free energy is just the quotient between the Euclidean Action and the inverse of temperature:  $F = \Delta S_E / \beta$ . Therefore:

$$F = -\frac{\Lambda}{12} \left( r_h^3 - \frac{3}{\Lambda} r_h \right) + \frac{K(\Theta, Q_m) + 2h(\Theta, Q_m)}{4r_h}. \quad (4.10)$$

On the other hand, the total energy is defined as:

$$E = \frac{\partial \Delta S_e}{\partial \beta} = -T^2 \frac{\frac{\partial \Delta S_e}{\partial r_h}}{\frac{\partial T}{\partial r_h}}, \quad (4.11)$$

being  $T$  the temperature of the BH solution (4.3). Then the total energy can be expressed as:

$$\begin{aligned} E &= \left[ 2\Lambda r_h + \Lambda^2 r_h^8 - 3r_h^4 + 6h(\Theta, Q_m)\Lambda r_h^4 \right. \\ &\quad \left. + 3K(\Theta, Q_m)(K(\Theta, Q_m) + 2h(\Theta, Q_m)) \right. \\ &\quad \left. + 6K(\Theta, Q_m)r_h^2 \right] / \left[ 6r_h(r_h^2 - 3K(\Theta, Q_m) - \Lambda r_h^4) \right]. \end{aligned} \quad (4.12)$$

Moreover, the entropy of the BH is defined as:

$$S = \beta E - \beta F, \quad (4.13)$$

so we can express the entropy as:

$$S = \frac{\Lambda r_h^4 - r_h^2 + K(\Theta, Q_m) + 2h(\Theta, Q_m)}{\Lambda r_h^4 - r_h^2 + 3K(\Theta, Q_m)} \pi r_h^2. \quad (4.14)$$

In general, the entropy of the BH is not proportional to the horizon area  $A = 4\pi r_h^2$ . For small  $a$ , we can express this entropy as:

$$S = \frac{1}{4}A - a \frac{2^{6-n}\pi^3 (Q_e^2 + Q_m^2) A (1 + n - i^n)}{384b\pi^3 (Q_e^2 + Q_m^2) + 4\pi A - \Lambda A^2} \cdot \left( \frac{Q_e}{Q_m} - \frac{Q_m}{Q_e} \right)^n + \mathcal{O}(a^2). \quad (4.15)$$

We can see from (4.15) that in the limit  $a \rightarrow 0$  (standard case) we recover the usual result  $S = \frac{1}{4}A$ , as we could expect.

Finally, the heat capacity  $C$  can be defined as

$$C = T \frac{\partial S}{\partial T}, \quad (4.16)$$

so we can replace in this expression (4.14) and (4.3) and we finally obtain

$$C = 2\pi r_h^2 [r_h^2 + \Lambda r_h^4 - K(\Theta, Q_m)] \left\{ -2\Lambda r_h^6 - 6K(\Theta, Q_m)r_h^2 + 8\Lambda K(\Theta, Q_m)r_h^4 + \Lambda^2 r_h^8 + r_h^4 + 3K(\Theta, Q_m) [K(\Theta, Q_m) + 2h(\Theta, Q_m)] - 2h(\Theta, Q_m)\Lambda r_h^4 \right\} / [\Lambda r_h^4 - r_h^2 + 3K(\Theta, Q_m)]^3. \quad (4.17)$$

Once we obtained these quantities, it is possible to discuss the BH stability regions in terms of the sign of the heat capacity (4.17) and the Helmholtz free energy (4.10). BH solutions with  $F > 0$  are more energetic than pure radiation, so they will decay to radiation by tunneling; whereas BH solutions with  $F < 0$  will not decay to radiation since they are less energetic. Furthermore, if the solution has  $C < 0$  it is unstable under acquiring mass, and solutions with  $C > 0$  are stable. For further details, see Ref. [28].

In the following, we discuss the stability regions in the standard Electrodynamics theory, and in modified theories with the  $n$  parameter even (which are parity invariant, since the non parity invariant term  $Y$  is raised to an even power in the Lagrangian density (3.1)) and odd (which are non parity invariant, for the same reason). It is important to remark that provided the parameter  $n$  is odd, then we get complex quantities for the thermodynamics variables (4.10), (4.14) and (4.17). Thus, in our proposed model solely the models invariant under parity shall provide real Helmholtz energy, heat capacity and entropy using the Euclidean Action Method.

## 4.1 Standard case: $a = 0$

In this first case, we shall show the thermodynamics phase regions of the solution in the standard Electrodynamics theory, i.e., taking  $a = 0$  in our proposal Lagrangian density (3.1). As we commented in the previous section, in the standard theory  $\Theta$  coincides with  $Q_e$ . Moreover, in this case the defined quantities  $K(Q_e, Q_m)$  and  $h(Q_e, Q_m)$  coincide, being their expression:

$$h(Q_e, Q_m) = K(Q_e, Q_m) = -8\pi b (Q_e^2 + Q_m^2). \quad (4.18)$$

Using this fact, we can simplify the free Helmholtz energy (4.10) and the Heat capacity (4.17) of the BH solution, and express them as

$$F = -\frac{\Lambda}{12} \left( r_{ext}^3 - \frac{3}{\Lambda} r_{ext} \right) - 6\pi b \frac{Q_e^2 + Q_m^2}{r_{ext}}, \quad (4.19)$$

$$C = 2\pi r_{ext}^2 \frac{\Lambda r_{ext}^4 + r_{ext}^2 + 8\pi b (Q_e^2 + Q_m^2)}{\Lambda r_{ext}^4 - r_{ext}^2 - 24\pi b (Q_e^2 + Q_m^2)}. \quad (4.20)$$

In the Figure 4.1 the phase diagram of a BH solution in the flat limit  $\Lambda \rightarrow 0$  is represented. We see that there are just two different phases: the phase with  $C < 0$  and  $F > 0$  (green) and the phase with  $C > 0$  and  $F > 0$  (blue); the white region is avoided since the temperature is negative there. Models with  $\Lambda \geq r_h^{-2}$  have not any allowed region, since for all charges values they present negative temperature. Finally, in Figure 4.2 we represented again the existing phases of our solution in the electrostatic case  $Q_m \rightarrow 0$  for different values of electric charge and non-null cosmological constant.

Finally, let us remember that in this standard case the BH entropy (4.14) coincides with a quarter of the horizon area, as expected.

## 4.2 General case

In a general Electrodynamics Lagrangian density, the phase diagram can be more complicated. Provided an odd parameter  $n$ , we get complex quantities for the thermodynamics variables (4.10), (4.14) and (4.17). Thus, in non-parity invariant models the thermodynamics quantities obtained in the Euclidean approach are not well defined, and assuming the method is valid we conclude these models lead to unstable solutions.

On the other hand, provided  $n$  even (parity invariant models) the thermodynamics quantities are well defined and we can discuss the stability regions of the solutions. Depending on the parameters  $a$  and  $n$  and the cosmological constant  $\Lambda$  two new phases may appear, corresponding to  $\{C < 0, F < 0\}$  (in the following figures, represented in red) and  $\{C > 0, F < 0\}$  (represented in yellow).



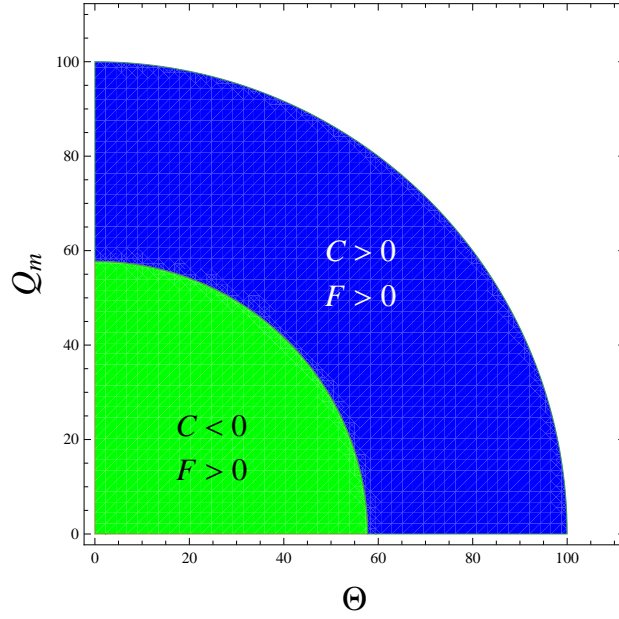


Figure 4.1: Phase diagram of BH solutions with  $b = -1/8\pi$  and  $a = 0$  (usual Electrodynamics Lagrangian) in the flat limit  $\Lambda \rightarrow 0$  corresponding to  $r_h = 100$ . We can see two different phases in the diagram: the blue one corresponds to both  $C$  and  $F$  positive, while the green one corresponds to  $C < 0$  and  $F > 0$ . We avoid the region with negative temperature, coloured in white. The diagram is represented solely for positive values of the charges; however, the diagram is completely symmetric under the change  $\Theta \rightarrow -\Theta$  or  $Q_m \rightarrow -Q_m$ .

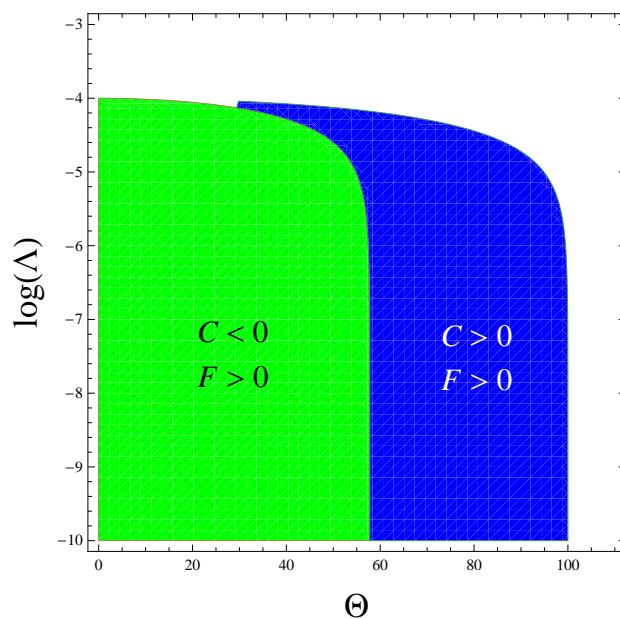


Figure 4.2: Phase diagram of BH solutions with  $b = -1/8\pi$  and  $a = 0$  (usual Electrodynamics Lagrangian) in the electrostatic limit  $Q_m \rightarrow 0$  corresponding to  $r_h = 100$ . The common logarithm of  $\Lambda$  is denoted as  $\log(\Lambda)$ . We can see two phases in the diagram: the blue one corresponds to both  $C$  and  $F$  positive, and the green one corresponds to  $C < 0$  and  $F > 0$ . As in the previous figures, we just colour the regions with positive temperature. The diagram is represented solely for positive values of the charge; however, the diagram is completely symmetric under the change  $\Theta \rightarrow -\Theta$ .

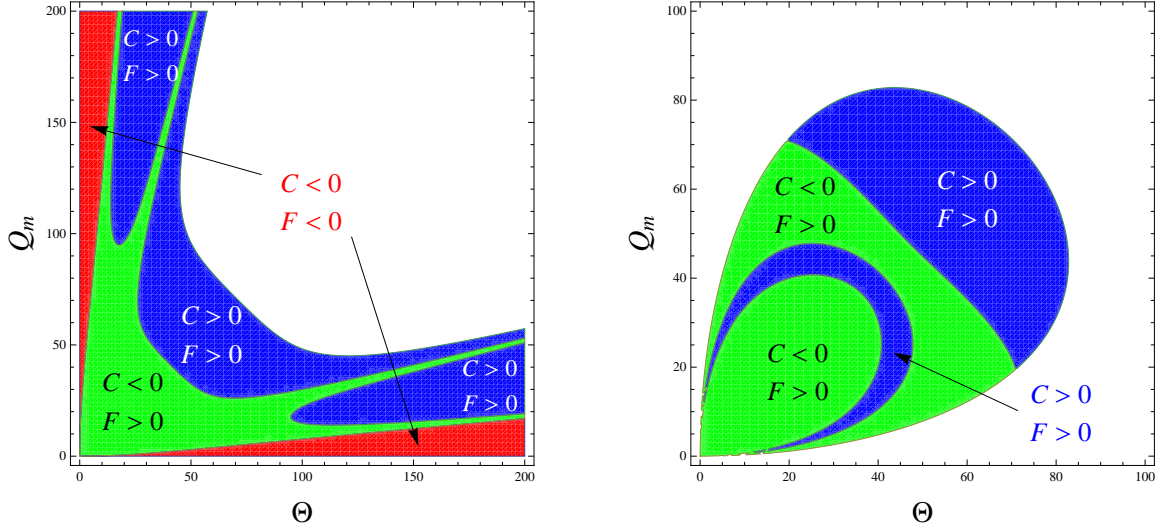


Figure 4.3: Different phase diagrams corresponding with  $b = -1/8\pi$ ,  $|a| = 1/80\pi$ ,  $n = 2$  and  $r_h = 100$  in the flat limit  $\Lambda \rightarrow 0$  are represented. In the left panel, it is represented the phase diagram for positive  $a$ , and they exist three different phases: One with  $C < 0$  and  $F > 0$  (green), other with  $C > 0$  and  $F > 0$  (blue) and a new phase with  $C < 0$  and  $F < 0$  (red). The right panel corresponds to negative  $a$ , and they appear just the phases which are also present in the standard Electrodynamics theory:  $\{C < 0, F > 0\}$  (green) and  $\{C > 0, F > 0\}$  (blue).

In Figure 4.3 we represented different phase diagrams corresponding to  $n = 2$  for different sign of the parameter  $a$  in flat space  $\Lambda \rightarrow 0$ . Moreover, in Figure 4.4 we show the phase diagram for  $n = 2$ ,  $Q_m = 50$  and  $a = 1/80\pi$ . In this plot we see all the stability phases are present. On the other hand, it is easy to prove that in the electrostatic case ( $Q_m \rightarrow 0$ ), if  $n$  is even and positive there is solely one stability region for  $a > 0$ ,  $\{C < 0, F < 0\}$ , which fully covers the plane  $\Lambda - \Theta$ , whereas for  $a < 0$  there is not any stability region since in this case the temperature is always negative.

Finally, we highlight that in these modified theories the entropy (4.14) is not proportional to the horizon area ( $A = 4\pi r_h$ ). In fact, the entropy may decrease with the area, as we shown in Figure 4.5 in the flat case  $\Lambda = 0$  with  $K(\Theta, Q_m) = 100$  and  $h(\Theta, Q_m) = -1000$ . Thus, provided the second law of the BH dynamics is valid, the BH entropy could decrease in some physical process. It does not occur for small corrections to the standard theory, since as we can see from (4.15) the dominant term is still  $A/4$ .

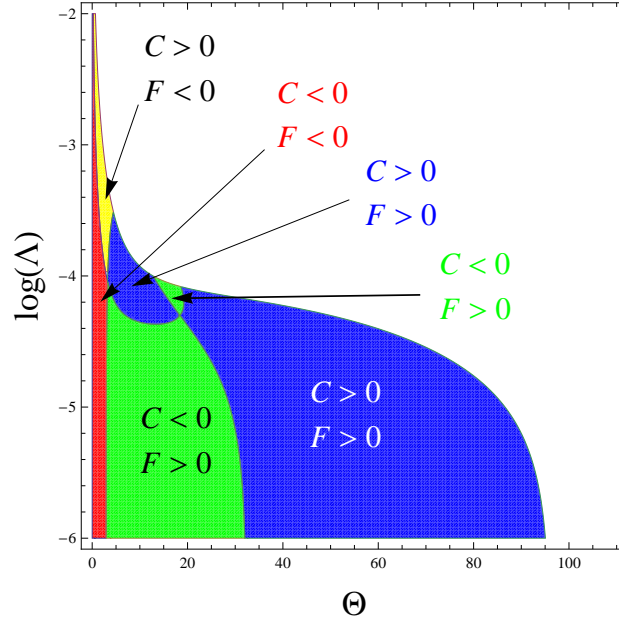


Figure 4.4: Phase diagram corresponding with  $b = -1/8\pi$ ,  $a = 1/80\pi$ ,  $n = 2$ ,  $r_h = 100$  and  $Q_m = 50$ . We denote the common logarithm of  $\Lambda$  as  $\log(\Lambda)$ . In this case, all the stability regions are present:  $\{C > 0, F > 0\}$  (blue),  $\{C < 0, F > 0\}$  (green),  $\{C > 0, F < 0\}$  (yellow) and  $\{C < 0, F < 0\}$  (red). As in the rest of figures, we only represented the regions with positive temperature.

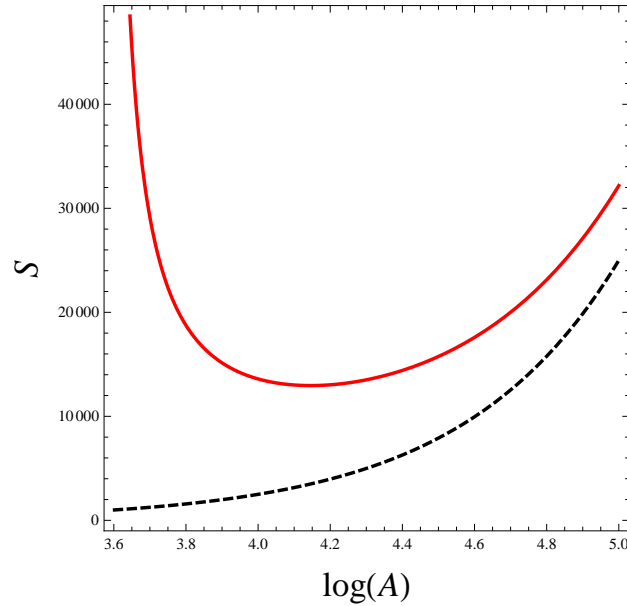


Figure 4.5: In red solid line, BH entropy for  $\Lambda = 0$ ,  $K(\Theta, Q_m) = 100$  and  $h(\Theta, Q_m) = -1000$  as function of the horizon area. For a range of  $A$ , the entropy is a decreasing function of  $A$ . In dashed black line, it is represented the usual result  $S = A/4$ .

# Chapter 5

## Classification of BH solutions in terms of the number of phase transitions

In this section we shall perform a classification of BH solutions based on the number of phase transitions that they present. These phase transitions occur at a set of values of  $\Lambda$ ,  $\Theta$ ,  $Q_m$  and  $M$  for which the denominator of the heat capacity (4.17) goes to zero, i.e., the heat capacity goes through an infinite discontinuity [29]. Thus we have to obtain the parameters for which the derivative of the temperature (4.3) with respect to the external horizon radius is null,  $\left. \frac{\partial T}{\partial r_h} \right|_{\Lambda, \Theta, Q_m} = 0$ , or equivalently, find the parameters for which the relation:

$$r_h^2 = \frac{1}{2\Lambda} \left( 1 \pm \sqrt{1 - 12K(\Theta, Q_m)\Lambda} \right), \quad (5.1)$$

is satisfied. Trying to solve this equation, we can distinguish three different classes of BH solutions:

- Fast BH's. If  $K(\Theta, Q_m) > \frac{1}{12\Lambda}$ , the radicand in (5.1) is negative, so there is not any  $r_h$  for which the expression is satisfied. It means that for these BH configurations there is not any phase transition. We shall refer to this kind of solutions as “fast BH's”. If we are in the flat limit  $\Lambda \rightarrow 0$ , this kind of solution does not hold on.
- Slow BH's. If  $0 < K(\Theta, Q_m) < \frac{1}{12\Lambda}$ , equation (5.1) can be satisfied for both plus or minus sign, since then for both possibilities we get  $r_h^2 > 0$ . It means that for these BH configurations there are present two horizon radii for which a phase transition occurs, i.e., there are two different phase transitions. We shall refer to this kind of solutions as “slow BH's”.
- New BH's. If  $K(\Theta, Q_m) < 0$ , equation (5.1) can be satisfied for plus sign but no for minus sign, since then we would get  $r_h^2 < 0$ . It means that there is solely one phase

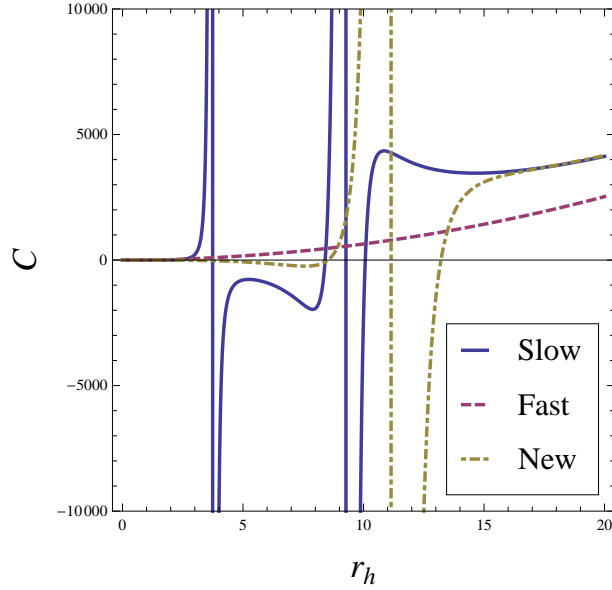


Figure 5.1: Behaviour of the heat capacity for different classes of BH. We can see that slow BH's present two phase transitions (for two horizon radii the heat capacity diverges), new BH's present a unique phase transition and fast BH's has not any phase transitions.

transition. We shall refer to this kind of solutions as “new BH's”, since they do not appear in the standard case.

In Figure 5.1 it is represented the heat capacity for different classes of BH's, from which we check that slow, new and fast BH's present two, one or none phase transitions respectively. In Figure 5.2 we plotted the values of the charge term  $K(\Theta, Q_m)$  and cosmological constant  $\Lambda$  for which each BH class is present. As in the previous section, we just take into account the regions with positive temperature (4.3). On the other hand, in Figure 5.3 we depicted the domain of each class in the case  $n = 2$ ,  $b = -1/8\pi$  and  $a = 1/80\pi$ . For this set of parameters they are present all the classes of BH. However, for other values of these parameters some classes may not appear.

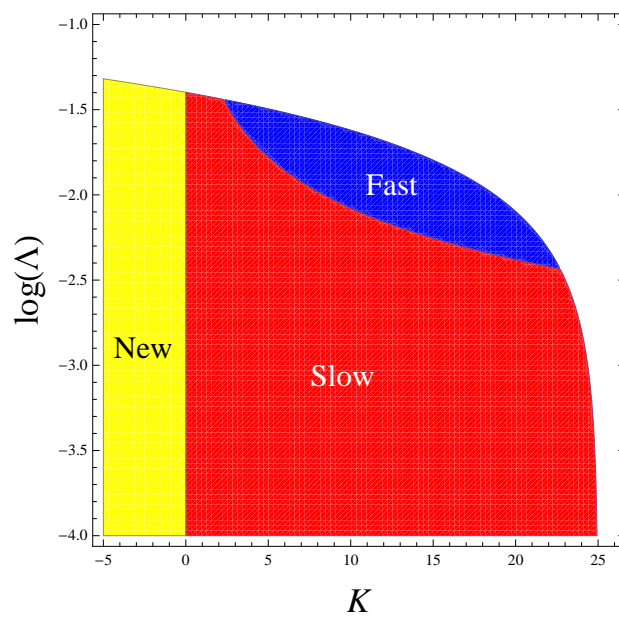


Figure 5.2: Classification of BH solutions with an outer horizon radius of  $r_h = 5$  depending on the number of phase transitions which the solution support for a cosmological constant  $\Lambda$  and a charge term  $K(\Theta, Q_m)$ . The common logarithm of  $\Lambda$  is denoted as  $\log(\Lambda)$ . The slow BH's, with two phase transitions, are represented in red; the fast BH's, with no phase transition, in blue; new BH's, with a unique phase transition, in yellow. The white colored region corresponds to negative temperature. Note that new BH's require  $K(\Theta, Q_m) < 0$ , so not all the the Electrodynamics Lagrangian densities of the form (2.25) support this kind of BH's (for example, the standard case does not support new BH's).

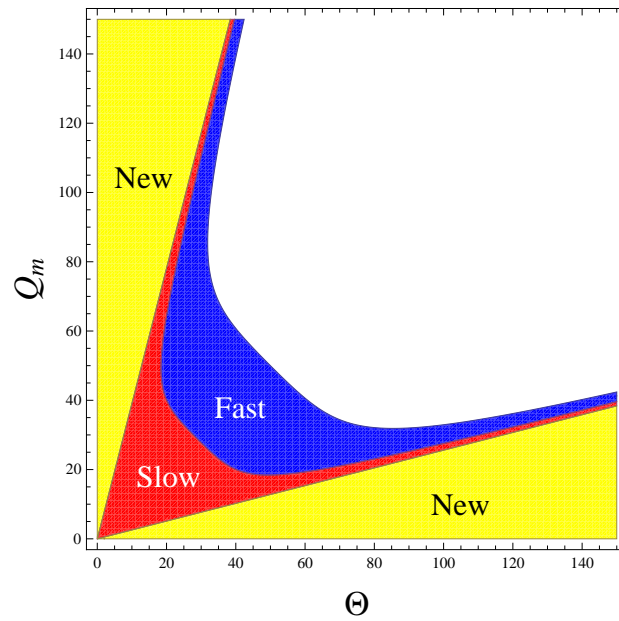


Figure 5.3: Classification of BH's solutions as function of the charges  $\Theta$  and  $Q_m$  for  $b = -1/8\pi$ ,  $a = 1/80\pi$ ,  $n = 2$ ,  $r_h = 100$  and  $\Lambda = 5 \cdot 10^{-5}$ . The colour code is: fast BH's in blue, slow BH's in red and new BH's in yellow. As we can see, they are present all the three classes



# Chapter 6

## Conclusions

In this work we have derived a sufficient condition of modified and gauge invariant Electrodynamics Lagrangian densities for obtaining static and spherically symmetric solutions assuming static and spherically symmetric  $U(1)$  fields.

Once we have obtained this condition, we have proposed one model of Electrodynamics Lagrangian density that could be seen as a perturbation of the standard theory. With this effective Lagrangian model we have derived the metric of the space-time. The obtained result is a Reissner-Nordstöm-like metric with a modified charge term that could be either positive or negative.

After obtaining the metric for our proposed model, we have performed a thermodynamics analysis of the solutions by employing the Euclidean Action approach. Using this method, we have found three important results. The first one is that in our proposed model, just parity-invariant models provide real thermodynamics quantities, whereas in the non-parity-invariant models these quantities are complex. Thus, assuming the Euclidean Action Method is valid we find our non-parity invariant models are unstable.

The second one is related to the phase diagram of the solutions. As we have seen throughout this work, when we represent a phase diagram of the solutions depending on the sign of the heat capacity and the free Helmholtz energy, for some set of values of the parameters of our model new phases which does not appear in the standard electrodynamics theory arise. It means that our modified model could explain the existence of stability phases in the black holes which do not hold on in the usual Electrodynamics theory.

The final result is that in the general case the black-hole entropy is not proportional to the horizon area. Thus, if the second law of the black-hole dynamics is still true ( $dA \geq 0$  in any physical process provided the null energy condition holds), for some sets of parameters the black-hole entropy will decrease.

Experimental tests of modified Electrodynamics models might be done studying astrophysical black holes and, furthermore, micro black holes which would be produced in LHC [30, 31]. The thermodynamics properties and the stability of these produced micro black holes could check our proposed modified model, since as we have seen the thermodynamical quantities of black holes depends on the model we work.

On the other hand, in this work we have studied black holes with both electric and magnetic charges, and for some models the thermodynamics quantities of non-magnetically-charged black holes diverge. The magnetic monopoles have not been observed, so it is a problem if we want to compare the thermodynamics properties of hypothetically observed black holes with the properties of our proposed solutions. Nevertheless, studying the sign of these quantities in the limit  $Q_m \rightarrow 0$  we can compare the thermodynamics phases diagrams of our solutions, which are well defined, with the corresponding of these hypothetically observed black holes.

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