System Identification of Geometrically Nonlinear Structures Using Reduced-Order Models

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ABSTRACT
System identification of engineering structures is an established area in the structural dynamics research community. It is often used to characterise certain physical properties of a structure using the data measured from it. For structures exhibiting nonlinear behaviour, physics-based approaches are used where a form of nonlinearity is synthesised and parameters are estimated using the data, or probabilistic approaches are investigated to tackle the model uncertainty of structures. However, to build reliable models, the estimated parameters from the measurement data must reflect the true underlying physics of the structure. Therefore, Reduced-Order Models (ROMs) can be used as the surrogate models, where the nonlinear parameters of the ROMs are having a meaningful relation with the physical parameters of the system. In this work, we propose nonlinear system identification in the context of using some recently developed ROMs which account for the kinetic energy of unmodelled modes. It is shown how ROMs may be used to represent low-order, accurate models for system identification. Identification of a nonlinear system with strong modal coupling is demonstrated, using simulated data, while the estimated ROM response shows good convergence with that of full order system. Similarly, the estimated parameters match with those of directly computed ROM.

Keywords: Structural Dynamics, Nonlinear Dynamics, System Identification, Reduced-Order Models

INTRODUCTION

To simulate the nonlinear behaviour of a structure, a reliable model is required; this often requires nonlinear system identification to be performed. In this context, measurements are taken from a physical structure and are used to construct a mathematical model. Although system identification is well-established for linear systems, nonlinear system identification is still undergoing significant developments. Here we consider system identification of geometrically nonlinear systems. Mainly, system identification consists of detection, characterisation and parameter estimation. Parameter estimation can be classified into main categories as: time and frequency-domain methods, time-frequency analysis, linearisation, modal methods, black-box modelling and numerical model updating, [1]. Developments have also been made in quantifying uncertainties in parameters. This can mainly be due to numerous candidate models which can be fitted into a set of measured data [1]. This has driven attention towards probabilistic approaches such as Bayesian framework[1]. These approaches can be appealing as they allow for optimum model to be found from a set of candidate models.

Although, for more complex structures (i.e. structures with high number of degrees-of-freedom) synthesising candidate models based on prior knowledge can be a very biased process. Also, the number of nonlinear parameters can drastically increase. This can not only make the parameter estimation process a challenging task, but also increase the uncertainty in the confidence bound and distributions. More recently Reduced Order Models (ROMs) have been investigated as surrogate models for complex structures to accurately represent the desired behaviour of a structure through a smaller sized set of second order differential equations. The usage of ROMs can reduce the computational costs when simulating the response of a structure [2, 3]. Thus, in this work we bridge between system identification and ROMs which can bring a two-fold benefit: first that it will make the system identification process easier and more reliable; secondly, it will bring computational benefits when used to simulate the response of the structure.

In the next section, identification process of a nonlinear system is illustrated. This is followed by the results section and final remarks given in the conclusions at the end.

SYSTEM IDENTIFICATION

Here, we consider identification of a geometrically nonlinear structure - specifically a cantilever beam with spring attached at the free end, as shown in Figure 1. In this beam the lower frequency bending modes of the system are statically coupled with higher frequency transverse modes. This coupling happens due to the membrane stretching in this cantilever type beam. We use some recently developed ROMs which accounts for this coupling [4]. The system is modelled in Abaqus Finite-Element package with parameters $L = 0.3\text{m},\ L_{sp} = 0.1\text{m},\ K_{sp} = 20\text{Nm}^{-1}$, cross sectional area of $A = 0.025\text{m} \times 0.001\text{m}$, Young's modulus $E = 205$ GPa and poisson's ratio $v = 0.3$. Note that the spring is unstretched when its length is L_{sp}

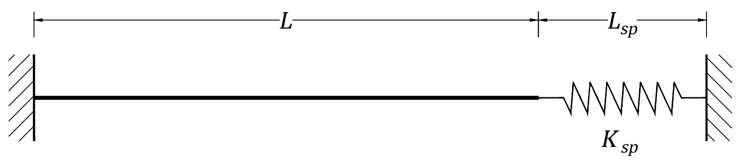

Figure 1: Cantilever beam schematic

The FE model resulted in 1440 DOFs with the first mode being in the bandwidth of interest with $\omega_1 = 57.815$ rad s^{-1}.

The model considered for this system is an inertially compensated ROM (ICROM) [4] as shown in Eq. 1, while, without the 2^{nd} and 3^{rd} terms it would be a standard ROM (SROM). In this Equation the dynamics of governing modes $(\bullet_r)$ is represented and the coupled modes are accounted for in the inertial compensation terms.

$$\ddot{q}_r + \left(\frac{\partial g_i}{\partial q_r}\right)^2 \ddot{q}_r + \left(\frac{\partial g_i}{\partial q_r}\right)\left(\frac{\partial^2 g_i}{\partial q_r^2}\right)\dot{q}_r{}^2 + \omega_r^2 q_r + \sum_{n=2}^{N} \gamma_n q_r^n = 0 \tag{1}$$

$$g_i = \sum_{n=2}^{N} \beta_{i,n} q_r^n \tag{2}$$

where, q represents the response of the system, ω_r is the natural frequency in (rad s^{-1}), γ being the nonlinear parameters of up to N^{th} order. g_i is the coupling function for (i) spanning over coupled modes, with β as the coupling coefficient, shown as Eq. 2.

For SROM without the inertial terms in Eq. 1, a simple linear regression can be applied to estimate the nonlinear parameters γ using the response data of the system. However, the ICROM has to be further developed to be treated as an inverse problem. We consider both the nonlinearity and coupling functions of up to 5^{th} order. By considering a single-mode ICROM $(r = 1)$ and including rest of the modes $(i = [2\ to\ 1440])$ in the coupling function, the IC-terms are derived as outlined in Eq. 3.

$$\begin{aligned}\left(\frac{\partial g_i}{\partial q_1}\right)^2 \ddot{q}_1 + \left(\frac{\partial g_i}{\partial q_1}\right)\left(\frac{\partial^2 g_i}{\partial q_1^2}\right)\dot{q}_1{}^2 =& (4\ddot{q}_1 q_1^2 + 4\dot{q}_1^2 q1)\bar{A} + (12\ddot{q}_1 q_1^3 + 18\dot{q}_1^2 q_1^2)\bar{B} + (\ddot{q}_1 q_1^4 + 2\dot{q}_1^2 q_1^3)\bar{C} + \dots \\ & (4\ddot{q}_1 q_1^5 + 10\dot{q}_1^2 q_1^4)\bar{D} + (2\ddot{q}_1 q_1^6 + 6\dot{q}_1^2 q1^5)\bar{E} + (40\ddot{q}_1 q_1^7 + 140\dot{q}_1^2 q_1^6)\bar{F} + \dots \\ & (25\ddot{q}_1 q_1^8 + 100\dot{q}_1^2 q_1^7)\bar{G}\end{aligned} \tag{3}$$

The above expression representing the inertial terms are substituted into the Eq. 1 and applied in a least squares (LSQ) manner to estimate the unknown parameters. Response data is extracted from the FE model with 0.5% damping ratio for the first mode. An initial condition is applied on the system, such that the structure is given a tip displacement of $(1/3)L$ which is sufficient to activate nonlinearity. The force is removed and the decaying response of the system is captured. Using the resonant decay method (RDM) proposed in [5] the first backbone curve of the system is measured as shown in Figure 2. In the LSQ problem we use the Fourier components of the response data, where certain harmonics within the interest range are included in the computation.

RESULTS

The estimated parameters are used to reconstruct both the 5^{th} order SROM and ICROM, then using numerical continuation technique, in the MATLAB based toolbox - continuation core (COCO) [6], the backbone curves of identified models are constructed and compared with that of full order model in Figure 2.

Backbone curve plots are shown in Figure 2, where the best match with the full model is for ICROM, while the SROM is diverging from the full model at almost all levels of amplitude. The identified ICROM perfectly match the full model backbone at low amplitude levels, however, slight inaccuracies can be because the full model Backbone is constructed from decay response which contains small error due to damping present in the data [5]. This shows the importance of choosing the right model in the identification. It also has to be declared that up to 5^{th} harmonics are included in the identification as higher harmonics' contribution are getting negligible and found to have minimum affect on the response.

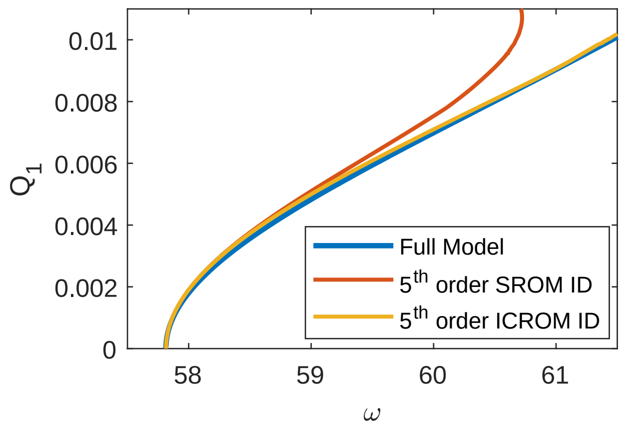

Figure 2: Backbone curve of Full Model, SROM and ICROM

CONCLUSION

Identification of a nonlinear cantilever type structure has been addressed. It was shown how ROMs may be used in the system identification context, while, they can be of manifold advantage. The identification has been applied to an FE model using two types of ROMs while their results were compared. The ICROM has resulted in better response and parameters compared to the SROM. This is due to the significant inertial coupling between the modes of the system, which cannot be accounted for in an SROM. Including additional degrees-of-freedom in SROM can potentially improve the response but will result in additional parameters and computational complexity.

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