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Estimating outputs using an inverse non-radial model with non-discretionary measures: An application for restaurants

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Abstract. Few inverse data envelopment analysis (DEA) models have incorporated non-discretionary measures based on radial efficiency values. However, the efficiency may be miscounted in radial approaches when some non-zero slacks appear. Furthermore, there is scant research on inverse DEA to estimate performance measures in the restaurant industry. Accordingly, this research proposes models based on non-radial DEA to analyze the efficiency and output changes of some Iranian restaurants while also presenting non-discretionary measures. Actually, in the company of non-discretionary factors, a non-radial DEA approach and its inverse problem are introduced to assess the performance and estimate the outputs for the modifications of inputs, respectively, while the inefficiency levels are maintained (and when they are preserved or decreased). The inefficiency of each discretionary input and output is specified using the presented non-radial DEA approach, and output targets are determined through inverse non-radial DEA with non-discretionary inputs. The results show containing non-discretionary data leads to more rational determinations through non-radial DEA-founded problems. This research presents analytic insights into the resources of inefficiency and output targets of entities with non-discretionary data, such as restaurants.

Keywords: data envelopment analysis, non-discretionary measure, inverse DEA, non-radial DEA, restaurant

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1. Introduction

Given the intense competition among different entities, particularly within the restaurant industry, it is vital to assess the performance and input-output indicators. Moreover, non-discretionary factors such as the areas of restaurants affect performance. Conventional data envelopment analysis (DEA) models assume all inputs and outputs are controlled (discretionary measures) by decision-makers. However, in the DEA literature, there are approaches to analyzing the relative efficiency of decision-making units (DMUs) with non-discretionary measures that are not controllable by managers. Banker and Morey [5] provided a DEA-based technique to examine the performance of entities in the short run when some measures are fixed and outside the authority of policymakers. Ruggiero [27] dealt with the correlation between efficiency and non-discretionary situations. Hua et al. [18] introduced a non-radial DEA approach and investigated the ecological efficiency of DMUs containing simultaneous undesirable outputs and

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non-discretionary inputs. Camanho et al. [6] assessed the performance of entities considering internal and external non-discretionary measures. Amirteimoori et al. [3] presented a two-stage DEA method to deal with non-discretionary factors with multiple dimensions. Rashidi et al. [26] introduced a slacks-based measure model in attending non-discretionary factors. The resource allocation problem in the presence of non-discretionary inputs was investigated by Fathi and Izadikhah [10] through a radial inverse DEA model. Khoshfetrat and Ghiyasi [21] also addressed the input changes in the company of non-discretionary factors using radial inverse DEA models. Nevertheless, in the inverse DEA literature, there is no non-radial model to estimate the performance measures of entities with non-discretionary inputs. As mentioned in [11, 32], radial DEA models may overestimate the efficiency of entities when there are non-zero slacks, possibly misleading decision-makers.

Under the efficiency maintenance, inverse DEA problems to assess outputs (inputs) for the input (output) changes were introduced by Wei et al. [30]. Yan et al. [31] developed the inverse DEA technique examining preference cone constraints. Using an inverse DEA method under the variable returns to scale (VRS) supposition, Lertworasirikul et al. [22] examined the resource allocation problem for the increase of some outputs and the decrease of other outputs, whereas the performance levels of all DMUs were maintained. The extension of inverse DEA problems in accordance with the enhanced Russell model was presented by Jahanshahloo et al. [19]. Ghobadi and Jahangiri [14] studied inverse DEA techniques from conceptual and practical aspects. Zhang and Cui [33] developed the existing inverse DEA patterns to tackle the issues that were not investigated. Zhang and Cui [32] suggested an inclusive inverse non-radial DEA framework founded on slacks-based measures. A directorial application of the inverse DEA regarding the definition of activities towards the new product objectives was examined by Lim [23]. Ghiyasi and Zhu [13] presented a directional distance inverse DEA framework incorporating positive and negative factors to determine the performance of some Chinese commercial banks. Hosseininia and Farzipoor Saen [16] evaluated the changes of input-output measures of after-sales units using an inverse slacks-based measure framework. Lim [24], reflecting frontier changes, extended an inverse DEA approach for practical planning. Hu et al. [17] considered certain deficiencies associated with inverse DEA problems founded on radial forms. Soleimani-Chamkhorami et al. [29] ranked extreme efficient firms using an approach based on inverse DEA. Referring to the limitation of inverse model under VRS, Chen and Wang [7] sough to identify the range of input and output changes. Ebrahimzade Adimi et al. [9] proposed an approach to determine the alterations of input-output factors subject to the preservation of the efficiency value and returns to scale. To achieve road transportation safety targets in China, Chen et al. [8] developed an inverse DEA model including undesirable outputs. Sohrabi et al. [28] assessed inputs/outputs using an inverse DEA-R approach with ratio data. Afterwards, Mahla et al. [25] used an inverse DEA ratio technique and assessed input variations when there were negative ratio data. Table 1 displays a comparative examination of some inverse DEA studies.

Study	Discretionary	Non-discretionary	Radial	Non-radial	Approach
Wei et al. [30]	✓	×	✓	×	Inverse problem under CRS, VRS, NIRS and NDRS
Lertworasirikul et al. [22]	✓	×	✓	×	Inverse BCC
Jahanshahloo et al. [19]	✓	×	×	✓	Inverse enhanced Russell forms
Hosseinnia and Farzipour Saen [16]	✓	×	×	✓	Inverse SBM
Hassanzadeh et al. [15]	✓	×	✓	×	Inverse SORM models
Fathi and Izadikhah [10]	✓	✓	✓	×	Inverse VRS model with non-discretionary data
Khoshfetrat and Ghiyasi [21]	✓	✓	✓	×	Inverse VRS model with non-discretionary factors
Zhang et al. [32]	✓	×	×	✓	Inverse SBM
Chen et al. [8]	✓	×	✓	×	Inverse DDF
Ghiyasi and Zhu [13]	✓	×	✓	×	Inverse DDF
This investigation	✓	✓	×	✓	Inverse weighted Russell DDF

Table 1: *Comparative assessment of Inverse DEA*

✓ means that the study considers it and × signifies the opposite. CRS: Constant returns to scale, VRS: Variable returns to scale, NIRS: Non-increasing returns to scale, NDRS:

Non-decreasing returns to scale, SBM: Slacks based measure, DDF: Directional distance function, BCC: Banker, Charnes and Cooper, SORM: Semi-oriented radial measure.

As a review of the inverse DEA literature shows, there is no inverse DEA model based on non-radial forms to estimate outputs (inputs) while presenting non-discretionary inputs. Furthermore, few inverse DEA studies have considered non-discretionary measures in radial models, which may mislead decision-makers due to the presence of non-zero slacks [32]. Besides, there are some measures out of the control of decision-makers in many investigations. For instance, the areas of restaurants cannot be changed in the short run when evaluating restaurants, and it is a non-discretionary measure [3, 6]. Besides, some measures controlled by policymakers, such as the number of seats, expenses, and incomes, are treated as discretionary measures.

Therefore, this assessment presents an alternative weighted Russell directional distance model with non-discretionary inputs and its inverse problem to evaluate the inefficiency sources of entities with non-discretionary inputs, along with the output targets. The input-output dataset of some Iranian restaurants is employed to assess the output projections for the perturbations of discretionary inputs in two cases: when the inefficiency scores are without changes and when they improve or are preserved. In the DEA literature, some studies have addressed the efficiency of restaurants applying the conventional DEA approaches [1, 4, 12, 20]. However, there is no inverse DEA model to measure targets for inputs or outputs in restaurants as far as we know, which is the focus of the current research. Overall, the contribution of this research is threefold:

-Providing a weighted Russell directional distance model with non-discretionary inputs,

-Introducing an inverse non-radial DEA approach concerning non-discretionary inputs, and

-Assessing the efficiency and output changes of some Iranian restaurants using the proposed techniques.

The rest of this paper is organized as follows. The inverse DEA problem is briefly reviewed in Section 2. The non-radial approaches with non-discretionary inputs are suggested in Section 3 to find the inefficiency sources and output modifications. Section 4 addresses the potential of some Iranian restaurants using the designed models in this research. Finally, Section 5 provides concluding remarks.

2. Inverse DEA

Suppose there are n DMUs, $DMU_j, j = 1, ..., n$, that use m inputs $x_{ij}(i = 1, ..., m)$ and produce s outputs $y_{rj}(r = 1, ..., s)$.

The output-oriented BCC technique estimating the maximum increase of outputs while preserving the input values is as follows:

$$\begin{array}{ll} \varphi^* = Max\ \varphi & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro}, r = 1, ..., s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, j = 1, ..., n. \end{array} \tag{1}$$

The optimal value $\frac{1}{\varphi^*}$ indicates the performance score. The DMU under consideration, DMU_o, is called efficient if and only if $\frac{1}{\varphi^*} = 1$. Otherwise, it is inefficient.

Wei et al. [30] and Lertworasirikul et al. [22] provided inverse DEA models under VRS to estimate performance measures, which can be rephrased to respond to the following question:

Given that the efficiency value is maintained, to what extent outputs change to modify inputs as $\alpha_{io} = x_{io} + \Delta x_{io}, \alpha_o = x_o + \Delta x_o = (x_{1o} + \Delta x_{1o}, ..., x_{mo} + \Delta x_{mo}) \geq 0, \Delta x_o \neq 0$?

Considering $\zeta_{ro}(r=1,...,s)$ as the output targets of the DMU under examination, DMU_o, the subsequent problem can be utilized to estimate the maximum output targets for the input modifications $\alpha_{io}(i=1,...,m)$ as long as performance values are maintained.

$$\begin{array}{ll} Max\ (\zeta_{1o},...\zeta_{ro}) & \\ s.t.\ \sum_{j=1}^{n}\lambda_j x_{ij} \leq \alpha_{io}, i=1,...,m, & \\ \quad \sum_{j=1}^{n}\lambda_j y_{rj} \geq \varphi^* \zeta_{ro}, r=1,...,s, & \\ \quad \sum_{j=1}^{n}\lambda_j = 1, & \\ \quad \lambda_j \geq 0, j=1,...,n. & \end{array} \tag{2}$$

In which, φ^* shows the optimal value obtained from model (1).

This approach is radial and does not include non-discretionary inputs. Thus, DEA and inverse DEA frameworks based on the weighted Russell directional distance form are planned in the next section to assess the performance and output targets at the presence of non-discretionary inputs.

3. Methodology

In this section, a weighted Russell DDF model with non-discretionary inputs is primarily developed to address the performance of entities. The inverse model is then presented to estimate output projections for the changes of discretionary inputs when the inefficiency scores are preserved (and when they are maintained or decreased). In so doing, we consider n DMUs, $DMU_j(j=1,...,n)$, that use m discretionary inputs $x_{ij}^{d}(i=1,...,m)$ and m' non-discretionary inputs $x_{i'j}^{nd}(i'=1,...,m')$and also yield s outputs $y_{rj}(r=1,...,s)$. Furthermore, the directional vector $g=(-g_{xi},g_{yr})$ is used to show the directions that discretionary inputs and outputs are scaled. The intensity variables are denoted by $\lambda_j(j=1,...,n)$. The corrections of outputs and the discretionary inputs are indicated by β_r and ρ_i, respectively. The quantities w_i and $\bar{w}_r$ represent the priorities of decision-makers specified for the outputs and discretionary inputs, whose sum equals unity. Subsequently, the following non-radial DEA approach with non-discretionary inputs is provided to examine performance:

$$\begin{array}{ll} E_o^* = Max\ \sum_{i=1}^{m} w_i\rho_i + \sum_{r=1}^{s}\bar{w}_r\beta_r & \\ s.t.\ \sum_{j=1}^{n}\lambda_j x_{ij}^{d} \leq x_{io}^{d} - \rho_i g_{xi},\ i=1,...,m, & \\ \quad \sum_{j=1}^{n}\lambda_j x_{i'j}^{nd} \leq x_{i'o}^{nd},\ i'=1,...,m', & \\ \quad \sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro} + \beta_r g_{yr},\ r=1,...,s, & \\ \quad \sum_{j=1}^{n}\lambda_j = 1, & \\ \quad \rho_i \geq 0, \beta_r \geq 0. & \end{array} \tag{3}$$

Model (3) has been designed under VRS considering $\sum_{j=1}^{n}\lambda_j = 1$. The unit under investigation, DMU_o, is called efficient if and only if the optimal value E_o^* equals zero. It means DMU_o is efficient provided that $\rho_i^*=0$ and $\beta_r^*=0$. Otherwise, it is inefficient. Moreover, the performance values of entities can be determined using the following expression:

$$\bar{E}_o^* = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{\rho_i g_{xi}}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{\beta_r g_{yr}}{y_{ro}}} \tag{4}$$

If $\bar{E}_o^*=1$, DMU_o is efficient and otherwise, inefficient.

At this stage, the output projections are evaluated for the alterations of discretionary inputs while the optimal value E_o^* achieved from model (3) is maintained (improved or maintained). Assume $x_o^d=(x_{1o}^d,...,x_{io}^d)$ is changed to $\mu_o = x_o^d+\Delta x_o^d, \Delta x_o^d \neq 0$. The goal is to identify output projections $\gamma_{ro}=y_{ro}+\Delta y_{ro}$ as long as the inefficiency level E_o^* is preserved. In this way, the subsequent multi-objective linear problem is developed:

$$\begin{aligned}
&Max \ \ (\gamma_{1o},...,\gamma_{so})\\
&s.t. \ \ \textstyle\sum_{j=1}^{n}\lambda_j x_{ij}^{d} \le \mu_{ij} - \rho_i g_{xi}, \ i=1,...,m,\\
&\quad \textstyle\sum_{j=1}^{n}\lambda_j x_{i'j}^{nd} \le x_{i'o}^{nd}, \ i'=1,...,m',\\
&\quad \textstyle\sum_{j=1}^{n}\lambda_j y_{rj} \ge \gamma_{ro} + \beta_r g_{yr}, \ r=1,...,s,\\
&\quad \gamma_{ro} \ge y_{ro},\\
&\quad \textstyle\sum_{j=1}^{n}\lambda_j = 1,\\
&\quad \textstyle\sum_{i=1}^{m} w_i\rho_i + \sum_{r=1}^{s}\bar{w}_r\beta_r = E_o^*\\
&\quad \lambda_j \ge 0, j=1,...,n, \rho_i \ge 0, \forall i, \beta_r \ge 0, \forall r.
\end{aligned} \tag{5}$$

The constraint $\gamma_{ro} \ge y_{ro}$ has been included to determine output projections greater than or equal to the outputs under consideration. This restriction can be omitted according to the plan and managers' perspective.

The constraint $\sum_{i=1}^{m} w_i\rho_i + \sum_{r=1}^{s}\bar{w}_r\beta_r = E_o^*$ shows the whole inefficiency value is maintained while the inefficiency amount of each of discretionary inputs and outputs may be different from those achieved from model (3). This constraint shows more flexibility of the introduced non-radial inverse DEA model compared to considering the constraint $\sum_{i=1}^{m} w_i\rho_i^* + \sum_{r=1}^{s}\bar{w}_r\beta_r^* = E_o^*$. Furthermore, it can be substituted with $\sum_{i=1}^{m} w_i\rho_i + \sum_{r=1}^{s}\bar{w}_r\beta_r \le E_o^*$, indicating the performance of entities is maintained or improved.

The weighted sum approach is applied to tackle the multi-objective linear problem (5). Thus, the objective functions of model (5) can be replaced by:

$$Max \ \ \sum_{r=1}^{s} b_r\gamma_{ro}$$

In which, $b_r(r=1,...,s)$ are weights related to output projections, the sum of which equals one, i.e. $\sum_{r=1}^{s} b_r = 1$. They are user-supplied weights ordinarily assigned to objectives due to their relative significance.

It is noteworthy that the proposed approach can also be developed for two cases:

1. Addressing input changes for output changes when the inefficiency level is preserved.

2. Estimating both inputs and outputs, provided that the inefficiency level is maintained.

The presented approaches in this area are employed in the next section to analyze the performance and output changes of 17 Iranian restaurants.

4. Application

As mentioned in [2], food businesses have attained acceptance these days and grow at a faster rate than any other business sectors, leading to extreme competition among firms operating in this area and necessitating the performance analysis of restaurants to improve operations. Therefore, the dataset of 17 Iranian restaurants with 7 discretionary inputs, one non-discretionary input, and two outputs is considered. Performance measures are selected after reviewing the related literature, consulting with specialists, and taking the data availability into account. These measures are described as follows:

Discretionary inputs

The number of employees, including chef, cooks, cashier, service personnel, etc. (x_1)

The number of seats (x_2)

Assets in billion Toman (x_3)

Labor expenses in million Toman (x_4)

The expenses of foods and beverages in million Toman (x_5)

Current costs, including water, electricity, gas, telephone, and taxes in million Toman (x_6)

The common number of waiters in a shift (x_7)

Non-discretionary input

The area of the restaurant in square meters (x_8)

Outputs

Net income in million Toman (y_1)

Turnover in million Toman (y_2)

The related data set of restaurants is related to the period from April 2021 to May 2022 and presented in Table 2.

Restaurant	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2
1	4	73	3.2	264	282	92	35	95	806	168
2	6	80	4.8	504	630	109	20	115	1710	467
3	3	41	3	261	290	62	22	53	748	180
4	5	62	5.7	590	680	104	25	78	2193	819
5	5	54	4.2	420	758	48	18	73	1790	564
6	9	173	10.3	816	1320	396	30	2103	353000	998
7	4	75	3.5	324	490	38	20	85	1243	391
8	5	80	3.7	432	565	118	25	92	1460	345
9	7	88	4.8	510	703	107	20	104	1712	392
10	4	126	17.56	330	478	142	38	3110	1369	419
11	3	18	1.25	252	322	50	22	44	882	258
12	3	37	2.72	276	398	84	16	68	1215	457
13	6	104	3.1	396	584	182	30	125	1707	545
14	3	48	2	276	410	73	24	58	1242	483
15	2	10	1.5	192	274	45	12	32	686	175
16	5	102	5.166	372	520	102	25	123	1478	484
17	3	38	1.7	284	376	63	30	49	1020	297

Table 2: *Dataset of restaurants*

Figure 1 depicts the descriptive plan of stages to assess performance of restaurants.
Model (3) and expression (4) are computed to assess the performance of restaurants. It should be noted that$w_i = \bar{w}_r = \frac{1}{9}$and $g = (g_{xi}, g_{yr}) = (x^d_{io}, y_{ro})$. To illustrate more, we have assumed all controllable inputs and outputs have a weight equal to 1/9 ($w_i = \bar{w}_r = \frac{1}{9}$), which means they have equal priority, the consequences of which are represented in Table 3. As can be seen from columns 2 and 12, 8 restaurants, including restaurants 4, 5, 6, 7, 11, 12, 14, and 15 are efficient. Restaurant 10 has a much weaker performance than other restaurants. Furthermore, the first output of restaurant 10, net income, has the greatest impact on the inefficiency of this restaurant, as shown in columns 3-11. Management of this restaurant should make more efforts to obtain more net income and reduce inefficiency in this component.

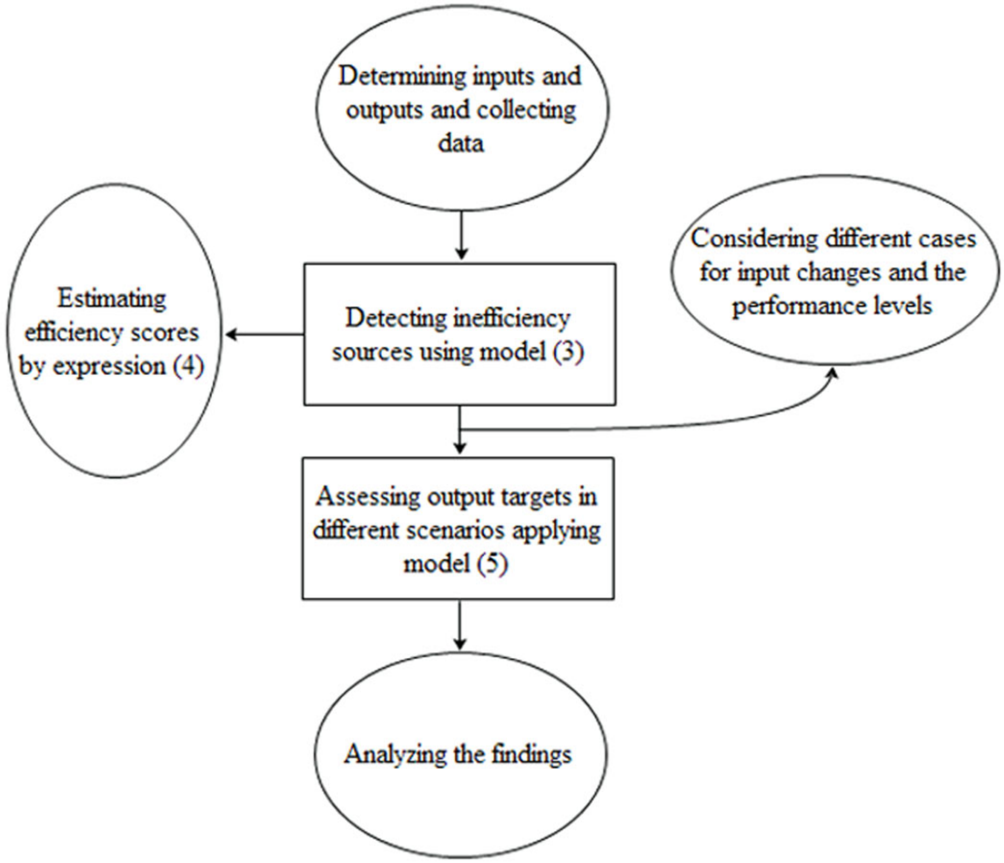

Figure 1: *The graphical framework of stages to examine the performance of restaurants.*

Restaurant	E^*	ρ_1^*	ρ_2^*	ρ_3^*	ρ_4^*	ρ_5^*	ρ_6^*	ρ_7^*	β_1^*	β_2^*	$\bar{E}^*$
1	0.72	0.49	0.85	0.51	0.25	0	0.48	0.65	3.19	0.08	0.20
2	0.91	0.42	0.54	0.27	0.25	0.25	0.26	0.1	6.13	0	0.17
3	0.79	0.31	0.72	0.47	0.24	0.02	0.22	0.45	4.69	0.02	0.20
4	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0	0	0	1
8	0.92	0.43	0.67	0.27	0.31	0.3	0.43	0.38	5.45	0	0.16
9	0.92	0.56	0.66	0.37	0.35	0.39	0.32	0.18	5.44	0	0.16
10	4.27	0.14	0.64	0.81	0.04	0	0.21	0.57	36.06	0	0.03
11	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	1
13	0.93	0.42	0.48	0.11	0.15	0.19	0.52	0.19	6.28	0	0.17
14	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	1
16	1.08	0.3	0.61	0.34	0	0.08	0.19	0.24	7.99	0	0.15
17	0.26	0.18	0.34	0	0.19	0.12	0.1	0.43	0.99	0	0.54

Table 3: *Performance scores*

At this moment, an increase of all discretionary inputs by 1% is assumed, and the output projections, i.e., net income and turnover, are assessed using model (5) in two scenarios:

1. The inefficiency values obtained from model (3) are maintained. Columns 2-3 of Table 4 present the projections of net income and turnover. As a comparison of Tables 2 and 4 shows, for these changes of discretionary inputs and preserving the inefficiency level, it is essential to increase net income in the majority of restaurants, while no changes are needed in turnover.

2. The inefficiency scores of restaurants are maintained or improved, as shown by the findings provided in columns 4 and 5. It is necessary to make minor changes in turnovers in restaurants 1 and 3, while other restaurants need no modifications in this component. Moreover, net income should be increased for most restaurants considering these modifications of discretionary inputs.

An alternative scenario assumes the increase in the expenses of foods and beverages, current costs, and the common number of waiters in a shift by 2%, and the output projections are estimated while the inefficiency scores of restaurants are maintained or improved. The results appear in columns 6 and 7 of Table 4. Similar to scenario 2, in this case, the turnovers of restaurants 1 and 3 need to have slight improvements, while no alterations are necessary in other restaurants. However, the net income of most restaurants must increase in this scenario.

It is noteworthy that restaurant 15 needs no modifications for all scenarios addressed. The output changes of different restaurants can be addressed for various scenarios in the same manner.

The weighted sum approach with equal weights $b_1 = b_2 = \frac{1}{2}$ has been considered to solve model (5). To clarify, they are user-defined weights ordinarily assigned to objectives considering their relative significance. In this case, the equal importance for two output targets has been considered. Nevertheless, the findings strongly depend on the chosen weights for output targets and those showing the priorities of decision-makers for the outputs and discretionary inputs.

Restaurant	Scenario 1		Scenario 2		Scenario 3	
	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2
1	1785.66	168	4330.4	183.51	5280.23	185.73
2	1829.7	467	12190.27	467	12190.27	467
3	800.36	180	4258.47	183.35	4258.47	183.35
4	2193	819	2193	819	2193	819
5	1819.36	564	1819.36	564	1845.91	564
6	353000	998	353000	998	353000	998
7	1390.13	391	1390.13	391	1537.26	391
8	1562.2	345	9417.2	345	9417.2	345
9	1831.84	392	11032.17	392	11032.17	392
10	3854.46	419	53180.97	419	55631.85	419
11	907.69	258	907.69	258	882	258
12	1337.69	457	1337.69	457	1215	457
13	1826.49	545	12634.98	545	12617.36	545
14	1253.37	483	1253.37	483	1242	483
15	686	175	686	175	686	175
16	1589.27	484	13321.16	484	13291.1	484
17	1137.9	297	2068.42	297	2033.48	297

Table 4: *Output targets in some scenarios*

Now, to compare the presented approach in this study with the existing DEA techniques, model (3) is estimated without taking the non-discretionary input, the area of the restaurant. Approaches (1)-(2) that do not incorporate the non-discretionary input are also computed, the results of which are revealed in Tables 5 and 6. As shown, the analogous restaurants are determined as efficient in all there models, including model (3), model (1), and model (3) without considering the non-discretionary input. Nevertheless, there are differences in performance values of all models and the inefficiency levels related to components achieved from two non-radial approaches. The comparison of the output projections of the three models for the changes of inputs is meaningless due to the disparities of the inefficiency scores resulting from approaches. Nevertheless, the output projections resulting from model (2) for the increase of all inputs by 1% and considering equal weights, i.e., $\frac{1}{2}$, are presented in columns 3-4 of Table 6. As can be seen, there are substantial dissimilarities between output targets obtained from the suggested approach and model (2).

Restaurant	E^*	ρ_1^*	ρ_2^*	ρ_3^*	ρ_4^*	ρ_5^*	ρ_6^*	ρ_7^*	β_1^*	β_2^*	E^*
1	0.72	0.49	0.85	0.51	0.25	0	0.48	0.65	3.19	0.08	0.2
2	4.43	0.24	0.03	0.09	0.22	0.03	0	0	39.26	0	0.04
3	1.05	0.3	0.7	0.46	0.23	0	0.19	0.44	7.12	0.04	0.15
4	0	0	0	0	0	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0	0	0	1
8	5.75	0.24	0.31	0	0.2	0.06	0	0.31	50.54	0.12	0.03
9	4.38	0.36	0.11	0.1	0.24	0.15	0	0	38.31	0.16	0.04
10	4.27	0.14	0.64	0.81	0.04	0	0.21	0.57	36.06	0	0.03
11	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	0	0	0	0	0	1
13	3.24	0.36	0.39	0	0.12	0.09	0.36	0.18	27.68	0	0.05
14	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	1
16	3.4	0.24	0.38	0.37	0.08	0	0	0.11	29.39	0	0.05
17	1.28	0	0.32	0	0.08	0.05	0	0.31	10.72	0	0.14

Table 5: *The performance without including the non-discretionary measure*

Accordingly, the performance assessment of entities with non-discretionary inputs, applying the proposed model is beneficial, and decision-makers can be informed about the inefficiency sources among discretionary inputs and outputs. Also, the inclusion of uncontrollable factors affects the efficiency results. The output modifications found by the introduced techniques are more rational when non-discretionary inputs are presented.

Restaurant	$1/\varphi^*$	ζ_1^*	ζ_2^*
1	0.87	3783.41	160.33
2	0.74	52431.15	344.53
3	0.85	6020.35	162.1
4	1	63699.38	400.02
5	1	8948.22	329.33
6	1	353000	998
7	1	1616.37	391.64
8	0.54	41430.55	210.91
9	0.62	42366.97	285.11
10	0.77	55023.21	262.9
11	1	1340.81	258.73
12	1	40971.43	277.49
13	0.94	69277.52	385.17
14	1	26751.14	278.32
15	1	902.14	175.5
16	0.83	50765.05	320.53
17	0.77	12473.21	205.28

Table 6: *The results obtained from models* (1)-(2)

5. Concluding remarks

In many real-world practices, there are performance measures outside control of decision-makers. Therefore, a weighted Russell DDF technique with non-discretionary inputs and expressions were provided in this research to deal with inefficiency components and the performance levels. Then an inverse non-radial DEA approach with non-discretionary inputs was proposed to estimate the maximum outputs for modifications of discretionary inputs when the inefficiency values were maintained (and when they are improved or maintained). The planned multi-objective linear inverse DEA problem was solved using uncomplicated weighted sum approach. The designed approaches were also utilized to examine the performance and the output measures of seventeen Iranian restaurants. The results show that the proposed approaches contribute to exploring the inefficiency factors and output variations when non-discretionary inputs appear. All input-output measures were considered as desirable in this investigation, but it should be kept in mind that there are undesirable factors in some investigations.

Accordingly, the extension of the provided models to occasions presenting undesirable measures is an interesting topic to deal with. Also, estimating performance measures in network systems with different structures at the presence of non-discretionary or semi-discretionary measures can be taken in future research. The proposed methods can be developed to estimate performance measures in production processes with several stages.

References

[1] Alberca, P. and Parte, L. (2018). Operational efficiency evaluation of restaurant firms. International Journal of Contemporary Hospitality Management, 30(3), 1959-1977. doi: 10.1108/IJCHM-09-2016-054

[2] Amidi, A., Darvishmoghaddam, E., Razmfarsa, A. and Binti Othman, R. (2022). Impact of five important factors on restaurant performance and hospitality management: an empirical analysis of technological innovation. Future Technology, 1(2), 1-17. doi: 10.55670/fpll.futech.1.2.1

[3] Amirteimoori, A., Kordrostami, S. and Khoshandam, L. (2013). Multi-dimensional non-discretionary factors in production processes: a data envelopment analysis. IMA Journal of Management Mathematics, 25(40), 435-448. doi: 10.1093/imaman/dpt021

[4] Assaf, A.G., Deery, M. and Jago, L. (2011). Evaluating the performance and scale characteristics of the Australian restaurant industry. Journal of Hospitality & Tourism Research, 35(4), 419-436. doi: 10.1177/1096348010380598

[5] Banker, R.D. and Morey, R.C. (1986). Efficiency analysis for exogenously fixed inputs and outputs. Operations Research, 34(4), 513-521. doi: 10.1287/opre.34.4.513

[6] Camanho, A.S., Portela, M.C. and Vaz, C.B. (2009). Efficiency analysis accounting for internal and external non-discretionary factors. Computers & Operations Research, 36(5), 1591-1601. doi: 10.1016/j.cor.2008.03.002

[7] Chen, L. and Wang, Y.-M. (2021). Limitation and optimization of inputs and outputs in the inverse data envelopment analysis under variable returns to scale. Expert Systems with Applications, 183, 115344. doi: 10.1016/j.eswa.2021.115344

[8] Chen, L., Gao, Y.,Li, M.-J., Wang, Y.-M. and Liao, L.-H. (2021). A new inverse data envelopment analysis approach to achieve China's road transportation safety objectives. Safety Science, 142, 105362. doi: 10.1016/j.ssci.2021.105362

[9] Ebrahimzade Adimi, M., Rostamy-Malkhalifeh, M., Hosseinzadeh Lotfi, F. and Mehrjoo, R. (2021). A model to evaluate the effects of the returns to scale on the inverse data envelopment analysis. Mathematical Sciences, 15(2), 111-121. doi: 10.1007/s40096-020-00353-6

[10] Fathi, N. and Izadikhah, M. (2013). Inverse data envelopment analysis model in the present of non-discretionary and discretionary data to preserve relative efficiency values: The case of variable returns to scale. Journal of Basic and Applied Scientific Research, 3(4), 872-880. Retrieved from: textroad.com

[11] Fukuyama, H. and Weber, W.L. (2009). A directional slacks-based measure of technical inefficiency. Socio-Economic Planning Sciences, 43(4), 274-287. doi: 10.1016/j.seps.2008.12.001

[12] Gharakhani, D., Maghferati, A.P. and Jalalifar, S. (2012). Evaluation of the efficiency of restaurants using DEA method (the case of Iran). Life Science Journal, 9(4), 530-534. Retrieved from: lifesciencesite.com

[13] Ghiyasi, M. and Zhu, N. (2020). An inverse semi-oriented radial data envelopment analysis measure for dealing with negative data. IMA Journal of Management Mathematics, 31(4), 505-516. doi: 10.1093/imaman/dpaa007

[14] Ghobadi, S. and Jahangiri, S. (2015). Inverse DEA: Review, Extension and Application. International Journal of Information Technology & Decision Making, 14(4), 805-824. doi: 10.1142/S0219622014500370

[15] Hassanzadeh, A., Yousefi, S., Farzipoor Saen, R. and Hosseininia, S.S.S. (2018). How to assess sustainability of countries via inverse data envelopment analysis?. Clean Technologies and Environmental Policy, 20(1), 29-40. doi: 10.1007/s10098-017-1450-x

[16] Hosseininia, S.S.S. and Farzipoor Saen, R. (2020). Developing a novel inverse data envelopment analysis (DEA) model for evaluating after-sales units. Expert Systems, 37(5), e12579. doi: 10.1111/exsy.12579

[17] Hu, X., Li, J., Li, X. and Cui, J. (2020). A revised inverse data envelopment analysis model based on radial models. Mathematics, 8(5), 803. doi: 10.3390/math8050803

[18] Hua, Z., Bian, Y. and Liang, L. (2007). Eco-efficiency analysis of paper mills along the Huai River: An extended DEA approach. Omega, 35(5), 578-587. doi: 10.1016/j.omega.2005.11.001

[19] Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Rostamy-Malkhalifeh, M. and Ghobadi, S. (2014). Using enhanced Russell model to solve inverse data envelopment analysis problems. The Scientific World Journal, 2014, 571896. doi: 10.1155/2014/571896

[20] Karakitsiou, A., Kourgiantakis, M., Mavrommati, A. and Migdalas, A. (2020). Regional efficiency evaluation by input-oriented data envelopment analysis of hotel and restaurant sector. Operational Research, 20(4), 2041-2058. doi: 10.1007/s12351-018-0406-1

[21] Khoshfetrat, S. and Ghiyasi, M. (2020). Using Multi-Objective Linear Programming (MOLP) and Data Envelopment Analysis (DEA) models in Non-discretionary Performance Measurement. International Journal of Data Envelopment Analysis, 8(4), 17-38. Retrieved from: ijdea.srbiau.ac.ir

[22] Lertworasirikul, S., Charnsethikul, P. and Fang, S.-C. (2011). Inverse data envelopment analysis

model to preserve relative efficiency values: The case of variable returns to scale. Computers & Industrial Engineering, 61(4), 1017-1023. doi: 10.1016/j.cie.2011.06.014

[23] Lim, D.-J. (2016). Inverse DEA with frontier changes for new product target setting, European Journal of Operational Research 254(2), 510-516. doi: 10.1016/j.ejor.2016.03.059

[24] Lim, D.-J. (2020). Inverse data envelopment analysis for operational planning: The impact of oil price shocks on the production frontier. Expert Systems with Applications, 161, 113726. doi: 10.1016/j.eswa.2020.113726

[25] Mahla, D., Agarwal, S., Amin, G.R. and Mathur, T. (2022). An inverse data envelopment analysis model to consider ratio data and preferences of decision-makers. IMA Journal of Management Mathematics, dpac009. doi: 10.1093/imaman/dpac009

[26] Rashidi, K., Shabani, A. and Farzipoor Saen, R. (2015). Using data envelopment analysis for estimating energy saving and undesirable output abatement: a case study in the Organization for Economic Co-Operation and Development (OECD) countries. Journal of Cleaner Production, 105, 241-252. doi: 10.1016/j.jclepro.2014.07.083

[27] Ruggiero, J. (2004). Performance evaluation when non-discretionary factors correlate with technical efficiency. European Journal of Operational Research, 159(1), 250-257. doi: 10.1016/S0377-2217(03)00403-X

[28] Sohrabi, A., Gerami, J. and Mozaffari, M.R. (2022). A novel inverse DEA-R model for inputs/output estimation. Journal of Mathematical Extension, 16(8), 1-34. doi: 10.30495/JME.2022.2047

[29] Soleimani-Chamkhorami, K., Hosseinzadeh Lotfi, F., Jahanshahloo, G. and Rostamy-Malkhalifeh, M. (2019). A ranking system based on inverse data envelopment analysis. IMA Journal of Management Mathematics, 31(3), 367-385. doi: 10.1093/imaman/dpz014

[30] Wei, Q., Zhang, J. and Zhang, X. (2000). An inverse DEA model for inputs/outputs estimate. European Journal of Operational Research, 121(1), 151-163. doi: 10.1016/S0377-2217(99)00007-7

[31] Yan, H., Wei, Q. and Hao, G. (2002). DEA models for resource reallocation and production input/output estimation. European Journal of Operational Research, 136(1), 19-31. doi: 10.1016/S0377-2217(01)00046-7

[32] Zhang, G. and Cui, J. (2020). A general inverse DEA model for non-radial DEA. Computers & Industrial Engineering, 142, 106368. doi: 10.1016/j.cie.2020.106368

[33] Zhang, M. and Cui, J.-c. (2016). The extension and integration of the inverse DEA method. Journal of the Operational Research Society, 67(9), 1212-1220. doi: 10.1057/jors.2016.2