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NON-PERTURBATIVE HEAVY-LIGHT INTERACTIONS IN QCD

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We assume a Gaussian approximation for QCD and discuss the heavy-light quark system. Spontaneous chiral symmetry breaking is shown to occur quite naturally.

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1. Introduction

The study of heavy-light quark bound states has absorbed in the last years a huge amount of effort on the experimental side, where a lot of new data have been produced (see, for instance, Ref. 1). As is known, the dynamics of the light quark and the glue is inherently nonperturbative, and it is clear that when we deal with light quarks, chiral symmetry breaking and confinement are the two relevant facts. In this paper, we present a simplified model, based on the cumulant expansion of QCD, addressing the interplay between chiral symmetry breaking and quark confinement.

2. The Hamiltonian

The following 4-point Green function $G_{\rm inv}(x, u, y, v)$ describes a system made up by a quark q of mass m and an antiquark $\bar{Q}$ of mass M_Q,

$$G_{\rm inv}(x, u, y, v) = \langle 0|\bar{q}(y)U(y, v)Q(v)\bar{Q}(u)U(u, x)q(x)|0\rangle. \quad (1)$$

The Schwinger string $U(y, x) \equiv {\rm P} \exp\left\{ {\rm i}g \int_0^1 {\rm d}s\,(y-x)^\mu A_\mu(x + s(y-x)) \right\}$ is needed for gauge invariance. P means the path ordering of the color matrices.

Let us consider the limit in which the antiquark is infinitely heavy ($M_Q \to \infty$). The heavy quark behaves then like a static source propagating from $u = (-T/2, \mathbf{0})$ to $v = (T/2, \mathbf{0})$. Let us take $x = (-T/2, \mathbf{x})$ and $y = (T/2, \mathbf{y})$ and choose a gauge $A_\mu(x_0, \mathbf{0}) = 0$; $x^j A_j(x_0, \mathbf{x}) = 0$ [2]. With this gauge choice, we can write the gauge field A_μ as functional of $F_{\mu\nu}$,

$$A_0(x) = \int_0^1 \mathrm{d}\alpha\, x^k F_{k0}(x_0, \alpha\mathbf{x}), \;\; A_j(x) = \int_0^1 \mathrm{d}\alpha\, \alpha\, x^k F_{kj}(x_0, \alpha\mathbf{x}). \tag{2}$$

Next, we assume the approximation of saturating the cummulant expansion on the gluon fields up to two-point correlators. This is an approximation first proposed in Ref. 3, and used in the study of soft high-energy scattering problems (for some recent reviews see Ref. 4). The effective Lagrangian we will use is then,

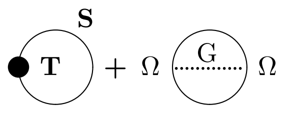
$$\mathcal{L}(x) = \bar{q}(x)(\mathrm{i}\,\partial\!\!\!/ - m)q(x) + \frac{\mathrm{i}}{2}\int \mathrm{d}^4y\, \bar{q}(x)\gamma^\mu T^a q(x)\bar{q}(y)\gamma^\nu T^b q(y)\langle gA^a_\mu(x) gA^b_\nu(y)\rangle, \tag{3}$$

$$\langle gA^a_\mu(x) gA^b_\nu(y)\rangle = x^k y^l \int_0^1 \mathrm{d}\alpha\, \alpha^{n(\mu)} \int_0^1 \mathrm{d}\beta\, \beta^{n(\nu)} \langle gF^a_{k\mu}(x^0, \alpha\mathbf{x}) gF^b_{l\nu}(y^0, \beta\mathbf{y})\rangle, \tag{4}$$

with $n(0) = 0$ and $n(\mathrm{i}) = 1$. The quantities in Eq. (4) are gauge invariant since, due to the gauge choice, all the field strength tensors can be thought to be connected by straight-line Schwinger strings. One part of nonperturbative physics is contained in the non-local gluon condensate $\langle g^2 F^a_{\mu\nu}(x) F^b_{\rho\lambda}(0)\rangle$. The other part is in the quark chiral condensate. We shall see that they are not independent quantities. See also Refs. 5 and 6.

3. The mass gap equation

We can write $H = \mathcal{E} + H_2 + H_4$ where $\mathcal{E}$ is a number and corresponds to the vacuum expectation <H>. H_2 and H_4 denote two- and four-quark operators. The mass-gap equation is $\delta\mathcal{E} = 0$. Using the Feynman rules ($\mathbf{S}$ -quark propagator; G the potential, Ω the vertex), we find

$\delta\mathcal{E} = \delta($  $) =$

$$\mathrm{Tr}\{\int \frac{\mathrm{d}^4k}{(2\pi)^4}\delta\mathbf{S}\, T + \int \frac{\mathrm{d}^4k\, \mathrm{d}^4p}{(2\pi)^8}\delta\mathbf{S}_p\, \Omega\, S_k \Omega\, G_{p-k} + S_p\, \Omega\, \delta\mathbf{S}_k \Omega G_{p-k}\} = 0\,. \tag{5}$$

For an arbitrary variation $\delta\mathbf{S}_p$, we have $T(p) + 2\int \mathrm{d}^4k/\,(2\pi)^4\;\Omega\, S_k \Omega\, G_{k-p} = 0$, the Dyson–Schwinger equation, which is the same as $S^{-1} = S_0^{-1} - \Sigma$. Now, $S = \frac{\mathrm{i}F}{k-E+\mathrm{i}\epsilon}\Lambda^+\beta - \frac{\mathrm{i}F}{-k-E+\mathrm{i}\epsilon}\Lambda^-\beta$, were F stands for the square of the spinors wave function renormalization. Because the $\Lambda^\pm$ are projectors, we have $S^{-1} = \beta\,\frac{k-E}{\mathrm{i}}\;\Sigma\, u\, u^\dagger - \beta\,\frac{-k-E}{\mathrm{i}}\;\Sigma\, v\, v^\dagger$ and two associated equations; the dispersion relation for the **dynamical quark mass** $u^\dagger\beta(S_0^{-1} - \Sigma)u = -\mathrm{i}(k-E)$ and **the mass-gap equation** $u^\dagger\beta S^{-1} v = 0$. In general, $\langle gA_\mu^a(x) gA_\nu^b(y)\rangle = V(r,0) + [V(r,R) - V(r,0)]$ with $r = x-y$; $R = (x+y)/2$. Only $V(r,0)$ contributes to $\mathcal{E}$.

3.1. A simple example

Assume that the leading contribution is given by the electric fields and approximate the exponential fall-off in time with the correlation length T_g, with an instantaneous delta-type interaction $\langle gA_\mu^a(x) gA_\nu^b(y)\rangle \simeq -\mathrm{i}\;\mathbf{x}\cdot\mathbf{y}\;\frac{\delta^{ab}\delta_{\mu 0}\delta_{\nu 0}}{24} T_g \langle g^2E^2(0)\rangle\;\delta(x_0 - y_0)$. A similar assumption can be found in Ref. 7. In this case $\Omega = \gamma_0$ and $F = 1$. In this simple case $\mathcal{E}$ can be read directly from the diagram of Eq.(5) and becomes:

$$\begin{aligned}\mathcal{E} \;=\; & \mathcal{V}\left\{3\int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{Tr}\left[(\alpha\cdot\mathbf{k} + m\beta)\Lambda_-(\mathbf{k})\right]\right. \\ & \left. -2V_0^3\int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3k\,'}{(2\pi)^3}\mathrm{Tr}\left[\Lambda_-(\mathbf{k})\Lambda_+(\mathbf{k}')\right]\int \mathrm{d}^3r e^{-\mathrm{i}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} r^2\right\}\end{aligned} \tag{6}$$

where $V_0^3 \equiv -\frac{T_g}{96}\langle g^2E^2(0)\rangle$. $\mathcal{V}$ is the volume.

Then $\delta\mathcal{E} = 0$ yields the mass-gap equation (and the dynamical quark mass $E(k)$) [8],

$$m\cos\phi(k) - k\sin\phi(k) + \frac{2}{3}V_0^3\left(\phi(k)'' + \frac{2}{k}\phi(k)' + \frac{2}{k^2}\cos\phi(k)\sin\phi(k)\right) = 0. \tag{7}$$

In terms of this variational parameter, we can calculate the quark condensate in terms of $G_2 \equiv \langle\frac{\alpha}{\pi}F^2(0)\rangle$, $\langle 0|\bar{q}q|0\rangle = -\frac{3}{\pi^2}\int_0^\infty \mathrm{d}k k^2\sin\phi(k) = -\frac{1}{24}Tg\,G_2 \times 0.3722$.

In this simple example, substituting T_g by the lattice value 0.35 fm [9] and the gluon condensate G_2 by the value of 0.048 GeV^4 [10], we get $\langle 0|\bar{q}q|0\rangle \simeq$ - (110 MeV)^3 which is a rather low value which is not surprising. Finally, with the help of the mass-gap equation, we turn on the R-dependent interaction and can solve for this simple example the light-heavy quark spectra. For details see Ref. 11.

3.2. A not so simple example

The above sketched formalism is general. It does not depend whether we have an instantaneous potential or not. Recently it has been shown [12] that any model claiming to describe quantitatively the meson spectroscopy must display at the same time and in a consistent way both linear effective confining forces and the mechanism of spontaneous chiral symmetry breaking. So we can apply the above formalism to more realistic correlators [13] displaying asymptotic linear force given by,

$$\begin{aligned} V^{\alpha\beta}(\mathbf{x},\mathbf{y};x_0,y_0) &= [\mathbf{x}\cdot\mathbf{y}\ (I_1\cdot I_2-\alpha\beta\alpha_1\cdot\alpha_2)+\alpha\beta\,(\alpha_1\cdot\mathbf{x})(\alpha_2\cdot\mathbf{y})]\times \\ &\quad\times\int\frac{\mathrm{d}^4k}{(2\pi)^4}\exp\{i[\mathbf{k}(\alpha\mathbf{x}+\beta\mathbf{y})-k_0(x_0-y_0)]\}\ D(k_0^2-\mathbf{k}^2) \\ D(k_0^2-\mathbf{k}^2) &= \frac{48\pi^2}{{T_g}^2}\frac{\mathrm{i}\ (k_0^2-\mathbf{k}^2)}{[k_0^2-\mathbf{k}^2-\frac{1}{T_g}+i\varepsilon]^4} \end{aligned} \tag{8}$$

An alternative is provided by the ansatz of M. D'Elia et al., $D(x) = \exp(-|x|/T_g)$, [9].

Work with N. Brambilla and A. Vairo is in progress and will be reported elsewhere. The main subtlety on an otherwise straightforward application of the above formalism lies in the careful writing of the correlator $\langle gA_\mu^a(x)gA_\nu^b(y)\rangle$ which-because of the associated Wilson loop-must display asymptotic linear rising in both r and R.

4. *Conclusion*

The above sketched formalism is general. It does not depend whether we have an instantaneous potential or not. So we can apply it to more realistic correlators [13,9]. Chiral symmetry breaking and a chiral non-invariant binding interaction emerge quite naturally in our approach and a link is established between chiral symmetry breaking properties and confining interaction. In particular with we establish a relation between the order parameter of chiral symmetry (the quark condensate $\langle 0|\bar{q}q|0\rangle$) and that one which in our framework describes confinement (the gluon correlation length T_g).

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NEPERTURBATIVNA TEŠKO-LAKA MEĐUDJELOVANJA U QCD

Pretpostavlja se Gaussovo približenje za QCD i raspravlja sustav težak–lak kvark. Pokazuje se kako spontano kršenje kiralne simetrije nastupa posve prirodno.