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PION-NUCLEON SIGMA TERM IN THE LINEAR SIGMA MODEL

T. S. T. ALI

Department of Mathematical Sciences, UAE University, P. O. Box. 17551, AL-AIN U.A.E. College of Science, Helwan University, Cairo, Egypt

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We apply the linear sigma model to analyze pion-nucleon sigma-term. Analytic expresions for these quantities are obtained in terms of the parameters of low-energy physics (pion decay constant and the pion mass). Good results have been obtained in comparison with author's previous work and with other models.

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1. *Introduction*

In recent years there has been a growing interest in studying the sigma-term $\sigma(\pi N)$. Many groups made a significant progress towards understanding pion-nucleon sigma-term $\sigma(\pi N)$ using such models. One of these models is the Mean Field Approximation (MFA). In this approach the meson fields are treated as time-independent classical fields.

In a MFA which has the hedgehog property, a similar model has been considered by Kälbermann and Eisenberg [1] and Birse and Banerjee [2], while high-order mesonic interactions in the linear sigma model was considered Sahu and Ohanishi [3] and by M. Rashdan et al. [4, 5]. They used the mean field theory to get better description of the nucleon properties. Ali et al. [6, 7] also used the coherent pair approximation to study the nucleon properties. Hadron models set up to understand the structure of the nucleon should respect the constraints imposed by chiral symmetry. In the first approaches in chiral perturbation theory [8, 9] and lattice QCD [10, 11], also, for example, in the chiral effective theories [12, 13], similarly large values for $\sigma(\pi N)$ were obtained in the (light-cone) QCD sum rule approaches [14, 15]. Spontaneous and explicit chiral symmetry breaking require the existence of the pion whose mass vanishes in the limit of zero current mass. Few solutions for the lagrangian of chiral linear soliton models, when applied to the nucleon, have been suggested. The sigma term is an important quantity in that it measures the degree of chiral symmetry breaking. To achieve these goals, one should not, as we believe, replace the whole system by a boson field (as Skyrme does), nor replace the Dirac sea by a boson field (as in the dispersion-relation techniques). For a system of the size of a nucleon, gradient expansions do not converge well and, hence, one has to treat the Dirac sea "exactly". No experimental method is known to directly measure the pion-nucleon sigma term. In an earlier analysis, Gasser et al. [16] obtained a value of about 45 ± 8 MeV, which was generally accepted until the late 1990's. Recent analyses, however, tend to yield higher values in the range $(80-90)$ MeV (Kaufmann and Hite [17], Pavan et al. [18]) and also using the pion-nucleon scattering data of Mekterović et al. [19].

The mean-field equations are a straightforward extension of that outlined by Goldflam and Wilets [20]. We consider a model based on the idea of strong QCD forces.

The aim of the present work is to investigate and analyze the sigma-term $\sigma\left(\pi N\right)$ depending on different values of the quark and sigma masses, with fixed prameters like the pion decay constant $f_\pi = 91.9$ MeV and the pion mass $m_\pi = 138.04$ MeV, introduced by Struber and Rischke [21]. Description of the model is given in Sec. 2. The results and discussion are given in Sec. 3.

2. *Chiral quark-sigma model*

We describe the interactions of quarks with $\sigma-$ mesons and pions by Birse and Banerjee [2]. The Lagrangian density is

$$L = \mathrm{i}\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}.\partial^\mu\vec{\pi}\right) + g\overline{\Psi}\left(\sigma + \mathrm{i}\gamma_5\tau.\vec{\pi}\right)\Psi - U\left(\sigma,\vec{\pi}\right), \quad (1)$$

where

$$U\left(\sigma,\pi\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \pi^2 - \nu^2\right)^2 + m_\pi^2 f_\pi \sigma, \quad (2)$$

is the meson-meson interaction potential where the Ψ, σ and π are the quark , (scalar, isoscalar) sigma and (pseudoscalar, isovector) pion fields, respectively.
The meson-meson interactions in Eq. (2) lead to hidden chiral $SU(2)\times SU(2)$ symmetry with $\sigma\left(r\right)$ taking on a vacuum expectation value

$$\langle\sigma\rangle = -f_\pi, \quad (3)$$

where f_π is the pion decay constant. The final term in Eq. (2) is included to break the chiral symmetry. It leads to partial conservation of axial-vector isospin current (PCAC). The parameters λ^2, ν^2 can be expressed in terms of f_π ,the mass of the σ and mass of the π meson

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}, \quad (4)$$

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2} \,. \tag{5}$$

Now we expand the extremum, with the shifted field defined as

$$\sigma = \sigma' - f_\pi, \tag{6}$$

substituting by equation(6) into equation (1) we get

$$\begin{aligned} L \;=\; & \mathrm{i}\overline{\Psi}\gamma_\mu\partial^\mu\Psi + \frac{1}{2}\left(\partial_\mu\sigma'\partial^\mu\sigma' + \partial_\mu\overrightarrow{\pi}.\partial^\mu\overrightarrow{\pi}\right) - g\overline{\Psi}f_\pi\Psi + g\overline{\Psi}\sigma'\Psi + \mathrm{i}g\overline{\Psi}\gamma_5\tau.\overrightarrow{\pi}\Psi \\ & -U\left(\sigma',\pi\right), \end{aligned} \tag{7}$$

with

$$U\left(\sigma',\pi\right) = \frac{\lambda^2}{4}\left(\sigma'^2+\pi^2\right)^2 - \lambda^2 f_\pi\sigma'\left(\sigma'^2+\pi^2\right) + \frac{m_\pi^2}{2}\left(\sigma'^2+\pi^2\right) + \lambda^2 f_\pi^2\sigma'^2, \tag{8}$$

the time-independent fields $\sigma'(r)$ and $\pi(r)$ are to satisfy the Euler-Lagrangian equations, and the quark wave function satisfies the Dirac eigenvalue equation. Substituting by equation (7) in Euler-Lagrangian equation we get

$$\left(\Box + m_\sigma^2\right)\sigma' = g\overline{\Psi}\Psi - \lambda^2\sigma'\left(\sigma'^2+\pi^2\right) + \lambda^2 f_\pi\left(3\sigma'^2+\pi^2\right), \tag{9}$$

$$\left(\Box + m_\pi^2\right)\boldsymbol{\pi} = \mathrm{i}g\overline{\Psi}\gamma_5\cdot\tau\Psi - \lambda^2(\sigma'^2+\pi^2-2f_\pi\sigma')\boldsymbol{\pi} \,. \tag{10}$$

where τ refers to Pauli (iso)spin-matrices, $\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

By using the hedgehog Ansatz [2], where

$$\boldsymbol{\pi}(r) = \dot{r}\pi(r) \,. \tag{11}$$

The Dirac wave functions are

$$\Psi\left(r\right) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u\left(r\right) \\ \mathrm{i}w\left(r\right) \end{bmatrix} \quad \text{and} \quad \overline{\Psi}\left(r\right) = \frac{1}{\sqrt{4\pi}}\begin{bmatrix} u\left(r\right) & \mathrm{i}w\left(r\right) \end{bmatrix}. \tag{12}$$

The chiral Dirac equation for the quarks is

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -P\left(r\right)u + \left(E - m_q + S(r)\right)w, \tag{13}$$

where $S(r) = g\left\langle\sigma'\right\rangle, P(r) = \left\langle\boldsymbol{\pi}\cdot\hat{r}\right\rangle, E$ are the scalar potential, the pseudoscalar potential and the eigenvalue of the quarks spinor Ψ, and

$$\frac{\mathrm{d}w}{\mathrm{d}r} = -\left(E - m_q + S(r)\right)u + \left(\frac{2}{r} - P\left(r\right)\right)w. \tag{14}$$

The set of equations(9, 10, 13, 14) is solved following the method used by Goldflam and Wilets [20] and by Birse and Banarjee [2] for the soliton bag model (for more details see Refs. [22, 4]). Including the color degree of freedom, one has $g\overline{\Psi}\Psi \to N_c g\overline{\Psi}\Psi$, where $N_c = 3$ colors and g is the coupling constant. Thus

$$\rho_s = N_c g\overline{\Psi}\Psi = \frac{3g}{4\pi}\left(u^2 - w^2\right), \tag{15}$$

$$\rho_p = \mathrm{i}N_c g\overline{\Psi}\gamma_5\tau\Psi = \frac{3}{4\pi}g\left(-2uw\right), \tag{16}$$

$$\rho_v = \frac{3g}{4\pi}\left(u^2 + w^2\right), \tag{17}$$

where ρ_s, ρ_p and ρ_v are the sigma density, pion density, and vector density, respectively. These equations are subject to the boundary conditions that asymptotically the fields approach their vacuum values,

$$\sigma(r) \sim -f_\pi \qquad \pi(r) \sim 0 \quad \text{at} \quad r \to \infty. \tag{18}$$

Finally, we have solved the Eqs. (13, 14) using fourth-order Rung-Kutta method. Due to the implicit nonlinearly of our Eqs. (9, 10), it is necessary to iterate the solution until self-consisteny is achieved. To start this iteration process, we use the chiral circle form for the meson fields

$$S(r) = m_q(1 - \cos\theta), \tag{19}$$

$$P(r) = -m_q \sin\theta, \tag{20}$$

where

$$\theta = \pi \tanh r. \tag{21}$$

3. Results and conclusions

The field equations (9,10,13,14) have been solved by iteration for different values of the quark and sigma masses. The dependence of the nucleon properties on the sigma and quark masses is listed in Tables 1 and 2. Tables 1 and 2 show the nucleon observables calculated for $m_q \geq 370 - 440$ MeV, $m_\sigma \geqslant 900 - 980$ MeV. As seen from Table 1, increasing the sigma mass decreases the sigma-term $\sigma(\pi N)$ and then gets closer to the experimental data. The experimental values of the pion decay coupling constant and the pion mass are of the order of 93 MeV and 139.6 MeV, respectively, but by decreasing these values (Hemmert et al. [23]) and choosing the values 91.9 MeV and 138.04 MeV decreases the sigma-term $\sigma(\pi N)$. I thus suggest a value of the quark mass to be in the range of $370 - 440$ MeV, depending on the model parameters.

TABLE 1. Values of $\sigma\,(\pi N)$ (MeV), $g_{\pi NN}(0)\dfrac{m_\pi}{2M_B}$ *and* $g_A(0)$, *at* $m_q = 400$ MeV.

m_σ MeV	900	940	960	980
$\sigma\,(\pi N)$	83.03	81.61	80.98	76.99
$g_{\pi NN}(0)\ m_\pi/(2M_B)$	1.35	1.35	1.35	1.32
$g_A(0)$	1.77	1.77	1.77	1.76

TABLE 2. Values of $\sigma\,(\pi N)$ (MeV), *and* $g_A(0)$, *at* $m_q = 1000$ MeV.

m_qMeV	440	420	400	380	370
$\sigma\,(\pi N)$	88.48	88.81	79.86	72.46	65.96
$g_A(0)$	1.82	1.80	1.78	1.74	1.72

In fact, neither m_σ nor m_q have been determined by experiment, since the quark cannot be isolated (because of the confinement principle), and we believe m_q is of the order of one third of the nucleon mass, and from the Nambu and Jona-Lasinio model [24], m_σ is twice m_q. Birse and Banerjee [2] suggested a value 500 MeV for m_q, which is very large. Decreasing this value has the effect of decreasing sigma-term $\sigma\,(\pi N)$ as seen from Table 2. A better result for the sigma term $\sigma\,(\pi N) =$ 65.96 MeV is obtained when $m_q = 370$ MeV and $m_\sigma = 1000$ MeV as seen from Table 2. Furthermore, sigma-term $\sigma\,(\pi N)$ is better reproduced than that of Birse and Banerjee [2] who used standard potential as seen from Table 3.

TABLE 3. Values of the sigma-term $\sigma\,(\pi N)$ MeV *compared with my previous works.*

Observable	[23]	[2]	[4]	[7]	this work
$\sigma\,(\pi N)$	54	92.3	88	88.9	65.96
$g_{\pi NN}(0)\ m_\pi/(2M_B)$	0.61	1.29	1.09	1.35	1.25
$g_A(0)$	0.78	1.85	1.80	1.46	1.72

In particular, $\sigma\,(\pi N)$ is improved because following Refs. [23, 17, 18], recent updated analyzes of pion-nucleon scattering data lead to a dramatic increase of the value of $\sigma\,(\pi N)$. For the sigma form factor $\sigma(t)$ at the Cheng-Dashen point $t = 2m_\pi^2$ it was found $\sigma(2m_\pi^2) = (85 \pm 15)$ MeV by Kaufmann and Hite [23] or $\sigma(2m_\pi^2) = (79 \pm 7)$ MeV by Pavan et al. [18]. With the value $\Delta_\sigma = 15$ MeV from the work by Gasser et al. [16], one concludes for $\sigma\,(\pi N) = \sigma(2m_\pi^2) - \Delta_\sigma$ the experimental results $\sigma\,(\pi N) : 73 \pm 15$ MeV [23] or 64 ± 7 MeV [17]. In this context one should also mension the recent pion-nucleon scattering data by Mekterović et al. [19], along with the axial-vector coupling constant g_A, the pion-nucleon coupling constant $g_{\pi NN}$ depending on the quark and sigma masses. I compared the free skyrmion model by Braghin and Cavalcante [25], Birse and Banerjee [2], Rashdan et al. [4] and Ali et al. [7] (my previous works) with the present work.

Finally, in all of my previous work I made a modification in the potential of the linear sigma model, but in this paper calculations were done differently, using the standard potential of the linear sigma model without any modifications. The new parameterization describes the nucleon properties that have been introduced.

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PIONSKO-NUKLEONSKI ČLAN SIGMA U LINEARNOM SIGMA MODELU

Primijenili smo linearni sigma model u analizi pionsko-nukleonskog člana sigma. Analitički izrazi za te veličine izvode se na osnovi parametara nisko-energijske fizike (konstanta raspada i masa piona). Postigli smo dobre ishode u usporedbi s autorovim ranijim radovima i s drugim modelima.