ISSN 1330–0016
CODEN FIZBE7

PARTICLE YIELDS, ANTIPROTON SCALING AND THE AVERAGE TRANSVERSE MOMENTA IN HIGH ENERGY LEAD-LEAD COLLISIONS: A MODEL-BASED STUDY

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Received 11 December 2000; revised manuscript received 27 March 2001
Accepted 26 June 2001

The study aims at explaining the behaviour of some of the very important observables measured in the latest lead-lead collisions at CERN in the light of a variety of the sequential chain model. Calculated values, to our surprise, are in excellent agreement with the measurements, especially when the effect of cascading and rescattering is empirically introduced in the calculations of the average transverse momenta. Implications of the results are discussed.

PACS numbers: 25.75.-q, 13.85.Ni, 12.38.Mh UDC 539.125, 539.126

Keywords: lead-lead collisions, sequential chain model, cascading and rescattering, transverse momenta

1. *Introduction*

Relativistic nucleus-nucleus collisions are believed to offer the unique opportunity to produce and thus 'observe' hadronic matter at high temperatures and densities in the laboratory. In recent experiments at the CERN-SPS, a variety of reactions ranging from p+p via p+A to finally Pb+Pb collisions with controlled impact parameter and with different centralities are studied [1–3]. The ultimate

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goal is supposed to be the observation of the exotic states of strongly interacting matter, e.g. the formation of the quark-gluon plasma (QGP) [4]. Before one can conclude that new effects are a convincing proof of the QGP formation, one has to study carefully the consequences of the interaction mechanism.

There are various experimental observables which give access to the physics of both hadronic and nuclear collisions. The nature of multiplicity ratios and the behaviour of the average transverse momenta of emitted secondaries, which will come under focus of the present study, constitute a very significant cornerstone. Secondly, electromagnetic radiation and lepton pairs are also interesting because they provide very penetrating probes, but they are strongly suppressed relative to the hadronic signals. Newly produced hadrons dominate the final state at high energies. However, near the production threshold, i.e. at relatively low energies, they are interesting observables because their emission characteristics are strongly affected by the interplay between the medium and the threshold effects [5].

Our attempt here is to interpret a part of the data observed, measured and reported by the NA49 group [1–3] on Pb-Pb collisions that can be understood with the help of a model based on a non-QGP approach, with some other ancillary physical ideas. We regard it to be an important task for the following reasons: firstly, several possible signatures have been proposed for the observations of a QGP-state of matter, but no unambiguous proof of its existence has yet been ascertained or confirmed [6]. Secondly, Gorenstein et al. [7] showed in a recent paper that $\bar{p}/\pi$ ratios could be explained nicely by using statistical models of hadron production in heavy-ion collisions. So, it is imperative that we apply our model to confront the latest available data, and wish the model to pass the acid test of the up-to-date results. All this might enable us to arrive at a conclusion of the sort that the diagnostics so far proposed in the field would not suffice to indicate what they are actually supposed to represent.

The plan of this work is as follows. Subsection 2.1 deals with the model we intend to apply. In Subsect. 2.2, we focus on the average transverse momenta, followed by the Sect. 3 wherein the concept of wounded nucleons is taken into account. Lastly, we present the results we have obtained and the relevant conclusions in Sections 4 and 5, respectively.

2. *Nucleon-nucleon collisions at high energies*

2.1. *A specific multiple production model*

We use here a variety of the sequential chain model (SCM) [8–10] to study some of the main characteristics and the crucial observations in Pb-Pb collisions. A standard connector for the transitions from the results of nucleon-nucleon collisions to the relationships for Pb-Pb (nucleus-nucleus, in general) collisions will be introduced in the next section. The expressions derived on the basis of rigorous field-theoretic considerations for the inclusive production cross-sections and average multiplicity values of the various types of secondary pions (of any variety), kaons (of each type) and antiprotons produced in the chain are given by the following sets of relations.

For any variety of secondary pions (π^+, π^- or π^0), we have

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\pi^-\mathrm{x}} \cong C_{\pi^-}\ \exp\left(\frac{-26.88\,p_T^2}{< n_{\pi^-} >_{\mathrm{pp}}\ (1-x)}\right)\ \exp(-\ 2.38\ \ < n_{\pi^-} >_{\mathrm{pp}}\ x)\ , \tag{1}$$

where $|C_\pi| \cong 90$ for the ISR energy region, but for $p\bar{p}$ collider energy it will increase as the inelastic cross-section increases, when

$$< n_{\pi^+} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\pi^-} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\pi^0} >_{\mathrm{pp}}\ \ \cong\ \ 1.1\,s^{1/5} \tag{2}$$

for very high energies, and at low p_T

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\pi^-\mathrm{x}}\ \cong\ E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\pi^+\mathrm{x}}\ \cong\ E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\pi^0\mathrm{x}}\ . \tag{3}$$

Similarly, for kaons of any specific variety (K^+, K^-, K^0 or $\bar{\mathrm{K}}^0$), we have

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3p}|_{\mathrm{pp}\to\mathrm{K}^-\mathrm{x}}\ \cong\ C_{\mathrm{K}^-}\ \exp\left(\frac{-\ 1.329\ p_T^2}{< n_{K^-} >_{\mathrm{pp}}^{3/2}}\right)\ \exp(-6.55\ \ < n_{\mathrm{K}^-} >_{\mathrm{pp}}\ x)\ , \tag{4}$$

with $|C_{K^-}| \cong 11.22$ for ISR energies, and with

$$< n_{\mathrm{K}^+} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\mathrm{K}^-} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\mathrm{K}^0} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\bar{\mathrm{K}}^0} >_{\mathrm{pp}}\ \ \cong\ \ 5\times 10^{-2}\ s^{1/4}\ , \tag{5}$$

and at low p_T

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\mathrm{K}^-\mathrm{x}}\ \cong\ E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\mathrm{K}^+\mathrm{x}}\ \cong\ E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}|_{\mathrm{pp}\to\mathrm{K}^0\mathrm{x}\ \mathrm{or}\ \bar{\mathrm{K}}^0\mathrm{x}}\ . \tag{6}$$

For low-p_T antiproton production in pp scattering at high energies, the derived expression for inclusive cross-section is

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}\,|_{\mathrm{pp}\to\bar{\mathrm{p}}\mathrm{x}}\ \ \cong C_{\bar{\mathrm{p}}}\ \exp\left(\frac{-\ 0.66\ ((p_T^2)_{\bar{\mathrm{p}}}+\mu_{\bar{\mathrm{p}}}^2)}{< n_{\bar{\mathrm{p}}} >_{\mathrm{pp}}^{3/2}\ \ (1-x)}\right)\ \exp(\,-\,25.4\ < n_{\bar{\mathrm{p}}} >_{\mathrm{pp}}\ x)\,, \tag{7}$$

with $|C_{\bar{\mathrm{p}}}|\ \cong\ 1.87\times 10^3$, for ultrahigh energies

$$< n_{\bar{\mathrm{p}}} >_{\mathrm{pp}}\ \ \cong\ \ < n_{\mathrm{p}} >_{\mathrm{pp}}\ \ \cong\ 2\times 10^{-2}\ s^{1/4}\ , \tag{8}$$

and at low p_T

$$E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}\,|_{\mathrm{pp}\to\mathrm{px}}^{nls}\ \cong\ E\frac{\mathrm{d}^3\sigma}{\mathrm{d}p^3}\,|_{\mathrm{pp}\to\bar{\mathrm{p}}\mathrm{x}}{}^{nls}\ . \tag{9}$$

Here nls stands for *nonleading-secondaries.*

2.2. Rise of average transverse momentum and the values of average transverse momentum for various secondaries

The average transverse momentum for any secondary (c) is defined as

$$< p_T >_c \; = \; \frac{\int F(x, p_T)\; p_T \; \mathrm{d}p_T^2}{\int F(x, p_T)\; \mathrm{d}p_T^2} \; . \tag{10}$$

Using the above definition and putting into use the altered forms of inclusive cross-sections for respective particles, we arrive at

$$< p_T >_{\mathrm{pp}}^{\pi^+} \; = \; < p_T >_{\mathrm{pp}}^{\pi^-} \; = \; < p_T >_{\mathrm{pp}}^{\pi^0} \; \cong \; 3.15 \times 10^{-1} \; s^{1/10} \; \mathrm{GeV}/c \; , \tag{11}$$

$$< p_T >_{\mathrm{pp}}^{\mathrm{K}^+} \; = \; < p_T >_{\mathrm{pp}}^{\mathrm{K}^-} \; = \; < p_T >_{\mathrm{pp}}^{\mathrm{K}^0 \text{ or } \bar{\mathrm{K}}^0} \; \cong \; 6.2 \times 10^{-2} \; s^{3/16} \; \mathrm{GeV}/c \; , \tag{12}$$

$$< p_T >_{\mathrm{pp}}^{\bar{\mathrm{p}}-nls} \; = \; < p_T >_{\mathrm{pp}}^{\mathrm{p}-nls} \; \cong \; 7.5 \times 10^{-2} \; s^{3/16} \; \mathrm{GeV}/c \; . \tag{13}$$

They were, of course, derived here on the basis of the assumption of approximate energy independence of inclusive cross-sections which, in actuality, does not represent reality. But, as the correction factor is not very large, we neglect the contribution here.

3. Nucleus-nucleus collisions and the nucleon-nucleon reactions: the overbridge

Basically, the philosophy is rooted in the work of Glauber and Matthiae [11]. For the purpose of calculations and operational considerations, we proceed in the route built up by Kadija et al. [12] through the process of computational reckoning of the number of 'wounded' nucleons and the number of wounded partons in nuclear collisions.

The inelastic nucleus-nucleus collisions can be described as an incoherent superposition of the collisions of individual nucleons. In collisions of a high-energy nucleon N with a target nucleus A of mass number A, the wounded nucleon number $W^N(A)$ comes into play. Similarly, $W_{A_1}^N(A_2)$ denotes the number of wounded nucleons in the target nucleus of mass number A_2, in a collision with a projectile nucleus of mass number A_1. In experiments, the NA49 group [1,2] indicated a correspondence and relationship of these factors with the number of participant nucleons (N_{part}) representing nucleons that had suffered an 'interaction'. It was further suggested that this number, N_{part}, could be obtained by the direct measurements of the net proton and kaon spectra.

In calculating the number of wounded nucleons, Kadija et al. [12] suggested that in collisions NA, the projectile nucleon is assumed to move on a straight line, defined by the impact parameter b, through the nucleus, making an interaction whenever it comes close enough to a target nucleon N_j

$$(x_0^p - x_j)^2 + (y_0^p - y_j)^2 \leq \frac{\sigma_{\mathrm{NN}}}{\pi}, \qquad j = 1, ..., A; \quad x_0^p = b_x; \quad y_0^p = b_y \tag{14}$$

Here the coordinates of the projectile nucleon x_0^{p}, y_0^{p} are taken to be the x and y components of the impact parameter vector $\vec{b} = (b_x, b_y)$ defined in the centre of mass of the target system.

The extrapolation to nucleus–nucleus ($\mathrm{A_1A_2}$) collisions is obvious. The number of projectile and target nucleons (with the coordinates $x_i^p = x_i + b_x$, $y_i^p = y_i + b_y$ ($i = 1, ..., A_1$) and x_j, y_j ($j = 1, ..., A_2$)) which will participate in the collisions can be calculated, demanding that

$$({x_i}^{\mathrm{p}} - x_j)^2 + ({y_i}^{\mathrm{p}} - y_j)^2 \leq \frac{\sigma_{\mathrm{NN}}}{\pi} \qquad i = 1, ..., A_1; \quad j = 1, ..., A_2 . \tag{15}$$

Following Kadija et al. [12], we propose here that the number of wounded nucleons is to be expressed in the form

$$W_{A_1}^{\mathrm{N}}(A_2) \propto [(A_1 A_2)^{1/3} \, \sigma_{\mathrm{NN}}]^{\beta}, \tag{16}$$

where $\beta \approx 0.81$, with the assumption that, although the nucleon becomes excited (wounded) as a result of consecutive collisions, it remains essentially in nucleonic state and thus it interacts with the same cross-sections as the original nucleon [13].

The hints for practical calculations from above are: a) the number of wounded nucleons would have a certain dependence on the basic nucleon(N)-nucleon(N) total cross-sections through the involvement of impact parameter; b) the number of wounded nucleons does have a simple proportionality relation with the mass numbers (A_1A_2) of the nuclei, in the form of $(A_1A_2)^{1/3}$.

Now, for the calculation of σ_{NN}, there is a well known formula given by the Particle Data Group [14] and by Cudell et al. [15], which correlate σ_{NN} with the square of c. m. energy, i.e. with s. Actually, it is of the form $\sim s^{\epsilon}$. In other words, σ_{NN} varies as s^{ϵ}, or more precisely,

$$\sigma_{\mathrm{NN}} \sim k' s^{\epsilon} . \tag{17}$$

The value of σ_{NN} for Pb-Pb collisions is calculated according to this formula and it is found to be ≈ 38.66 mb. We can write

$$s \cong k(W_{A_1}^{\mathrm{N}}(A_2))^{1/0.5103} . \tag{18}$$

Here, k is the product of two terms of which one depends on the nature of the secondary particles produced and the other on $1/(A_1A_2)^{0.27}$.

4. *Results and discussion*

A. Pb+Pb *collisions and the average multiplicities of pions, kaons and antiprotons*

The multiplicity in an event involving nuclear collisions is closely related to the number of participating nucleons, and the fluctuations in the number of participants is large compared to the multiplicity fluctuations from a single collision. The produced particle multiplicities are to a large extent dominated by the impact parameter rather than the detailed dynamics and the structure of the hadrons involved in the collisions [12].

On the basis of the expressions stated above, we write the following relations

$$< n_{\pi^+} > \;=\; < n_{\pi^-} > \;=\; < n_{\pi^0} > \;\cong\; 0.1794\;(W^{\rm N}_{A_1}(A_2))^{0.3919}\;, \qquad (19)$$

$$< n_{\rm K^+} > \;=\; < n_{\rm K^-} > \;=\; < n_{\rm K^0\ or\ \bar{K}^0} > \;\cong\; 0.0123\;(W^{\rm N}_{A_1}(A_2))^{0.4899}\;, \qquad (20)$$

$$< n_{\rm p} > \;=\; < n_{\bar{\rm p}} > \;\cong\; 0.0024\;(W^{\rm N}_{A_1}(A_2))^{0.4899}\;. \qquad (21)$$

The results are plotted for different numbers of participating nucleons and are shown by the lines in Figs.1 and 2. For the pion production, an empirical rescattering contribution term has been introduced in Fig.1.

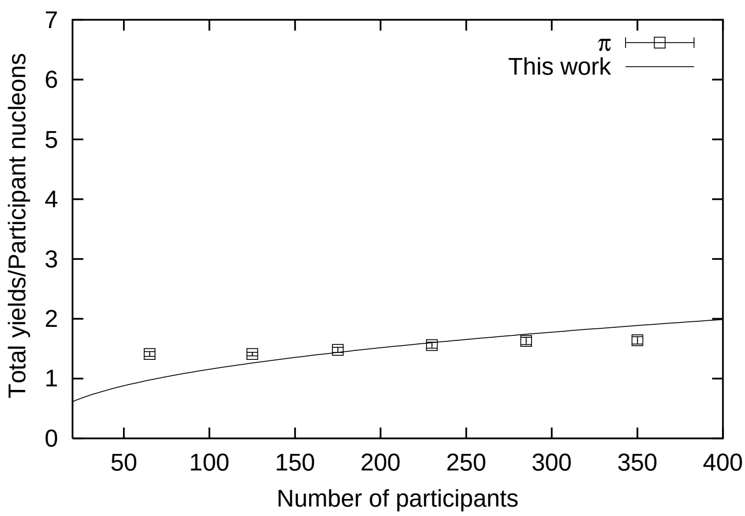

Fig. 1. Dependence of the total yields per participant nucleon on the number of participants in Pb+Pb *collisions. Values for average charged pions are taken from NA49 data. The line shows the theoretical fit.*

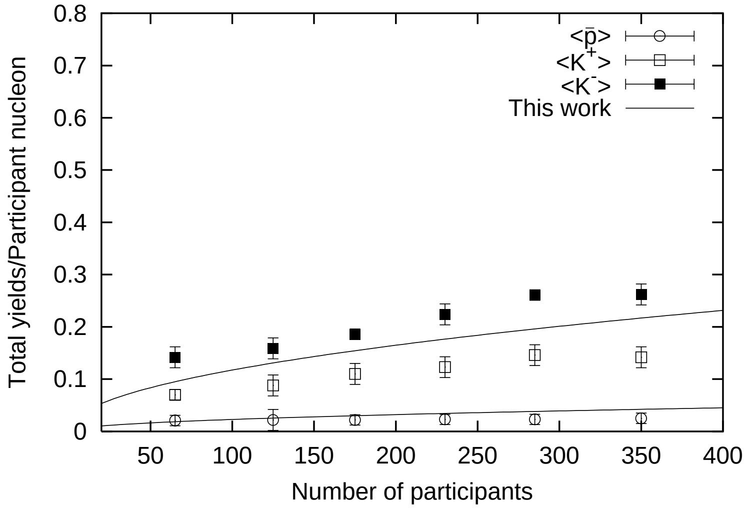

Fig. 2. Dependence of the total yields per participant nucleon on the number of participants in Pb+Pb *collisions. Values for average kaons and antiprotons are taken from NA49 data. The lines show the theoretical fits.*

B. Multiplicity ratios of $\bar{p}/\pi$

The ratios of $\bar{\mathrm{p}}/\pi$ for different numbers of participating nucleons are shown in the Table 1 and are plotted in Fig. 3. Introduction of the rescattering contribution in pion production has yielded fair agreement with the data.

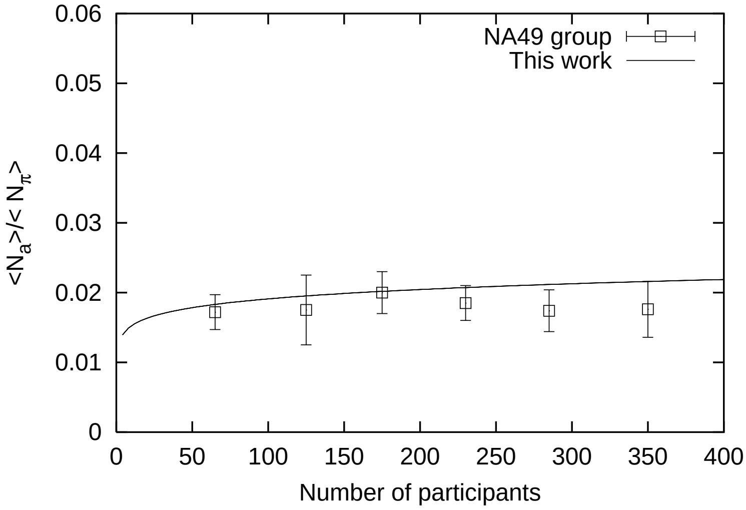

Fig. 3. The preliminary NA49 data on the antiproton to pion ratio in centrally selected Pb+Pb *collisions at 158A* GeV *are plotted as a mean number of wounded nucleons. The line shows the expected theoretical fit within SCM.*

Table 1. Dependence of the ratio $\bar{p}/\pi$ on the number of participants.

No. of participants	65	125	175	230	285	350
$\bar{p}/\pi$	0.018	0.019	0.020	0.020	0.021	0.021

C. Average transverse momentum for pions, kaons and antiprotons

By applying the same procedure, we get the expressions for average transverse momenta. We can write

$$< p_T >^{\pi^+} \;=\; < p_T >^{\pi^-} \;=\; < p_T >^{\pi^0} \;\cong\; 0.0747\;(W^{\rm N}_{A_1}(A_2))^{0.1959}\;, \qquad (22)$$

$$< p_T >^{\rm K^+} \;=\; < p_T >^{\rm K^-} \;=\; < p_T >^{\rm K^0\ or\ \bar{K}^0} \;\cong\; 0.0791\;(W^{\rm N}_{A_1}(A_2))^{0.3674}\;, \qquad (23)$$

$$< p_T >^{\bar{\rm p}-nls} \;=\; < p_T >^{{\rm p}-nls} \;\cong\; 0.0955\;(W^{\rm N}_{A_1}(A_2))^{0.3674}\;. \qquad (24)$$

Now we would consider and introduce empirically the effect of cascading and rescattering on average transverse momenta of the secondaries produced. As in ultrarelativistic heavy-ion collisions, there is a certain degree of cascading and rescattering processes [16] which cannot be ignored altogether. We assume a fully statistical and charge-independent behaviour for cascading and rescattering effects as a first approximation. As we have accepted the collisions to be nearly central, we cannot ignore the role of cascading and rescattering in the overlap regions of the collid-

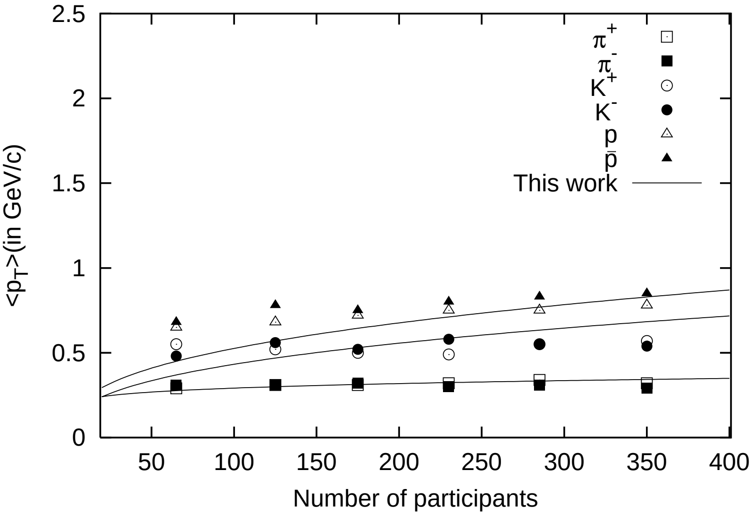

Fig. 4. Dependence of the average transverse momentum ($x_F = 0$) on the number of participants done by NA49 Collaboration for the Pb+Pb *collisions. The lines show the values for the theoretical SCM.*

ing nuclei which will contribute independently to particle production. The role of these two effects is here computed empirically with the reckoning of the factor $\approx (\bar{\nu}_k(A))^\epsilon$, with $\epsilon \ll 1$. Here, $\bar{\nu}_k(A)$ is the number of secondary collisions of the produced particles.

The average transverse momenta $< p_T >_x$ of particles of type x is assumed to be a sum of primary and secondary interactions. With this assumption, we can write

$$< p_T >_x \;=\; C_x \; (W^{\rm N}_{A_1}(A_2))^{B_x} \;+\; \alpha_x \; (\bar{\nu_k}(A))^\epsilon \; {\rm GeV}/c \; . \qquad (25)$$

$\bar{\nu_k}(A)$ is taken ≈ 14 from Ref. [12], $\epsilon \approx 1/15$ and values of α_x for π's, K's and p's are taken as 0.09, 0.002 and 0.006, respectively. The results for various hadronic secondaries are shown in Fig. 4 versus the number of participants.

5. *Summary and conclusions*

The results of our calculations are based on the derived values of the average pion, kaon and antibaryon multiplicity. While the agreement for pions is excellent the obtained values for the kaons and antiprotons show a certain degree of deviation. This was expected because the production of strange mesons and also of baryon-antibaryons is influenced by various resonance decays which we did not take into account in the calculations. The errors of the measurements are also larger in these cases. Secondly, the model supports the moderate observance of the antiproton scaling ($\bar{\rm p}/\pi$ ratio behaviour), though there is a very slow rise of the $\bar{p}/\pi$ ratio with the number of participating nucleons in the nuclei. Thirdly, the slopes of the average transverse momentum curves are well-reproduced by the model, and the qualitative agreement in the behaviour is certainly a product of our derived expressions. But in so far as the absolute values of $< p_T >$s are concerned, there is an element of arbitrariness in the case of nucleus-nucleus collisions, and this cannot be eliminated now. The assumptions accompanying the naturalisation of comparisons between nucleon-nucleon reactions and the nucleus-nucleus interactions introduce other elements of uncertainty and error.

Further, a comment is in the order here on the specific nature of the average transverse momentum. It is to be noted that the range of the average transverse momentum values for high energy nucleus-nucleus collisions is nearly the same as that in the nucleon-nucleon collision at high energies, as was reported by UA(5) collaboration [17]. In our opinion, this is a strong indicator to a suspect physical reality that the physics of nucleus-nucleus and of nucleon-nucleon collision should have some basic and intrinsic resemblance, especially with regard to the production of secondary particles in high energy collisions.

Despite these limitations, the most significant implication of the study is that we have succeeded modestly in building, at least, a footbridge between heavy-ion collision studies and the basic mechanism of nucleon-nucleon interactions at high energies.

Acknowledgements

The authors are very grateful to the two anonymous referees for some valued comments, constructive criticisms and helpful suggestions. PGR, one of the authors, is thankful to the UGC (India), for its financial support through the Faculty Development Programme and the authorities of the host (sponsoring) institutes like IACS, Kolkata-700032 and ISI, Kolkata-700035, India.

References

[1] NA49 Collab.: F. Sikler et al., Nucl. Phys. A **661** (1999) 45c.

[2] NA49 Collab.: G. I. Veres et al., Nucl. Phys. A **661** (1999) 383c.

[3] NA49 Collab.: G. E. Cooper et al., Nucl. Phys. A **661** (1999).

[4] S. A. Bass, M. Gyulassy, H. Stocker and W. Greiner, J. Phys. G **25** (1999) R1.

[5] P. Senger and H. Strobele, J. Phys.G **25** (1999) R59.

[6] S. Soff et al., Phys. Lett. B **471** (1999) 89.

[7] M. I. Gorenstein, M. Gazdzicki and W. Greiner, Phys. Lett. B **483** (2000) 60.

[8] S. Bhattacharyya, J. Phys. G **14** (1988) 9.

[9] P. Bandyopadhyay, R. K. Roychoudhuri, S. Bhattacharyya and D. P. Bhattacharyya, Nuovo Cimento A **50** (1979) 133.

[10] S. Bhattacharyya, Nuovo Cimento C **11** (1988) 51.

[11] R. J. Glauber and G. Matthiae, Nucl. Phys. B **21** (1970) 135.

[12] K. Kadija, I. Derado, N. Schmitz and P. Seyboth, Z. Phys. C **66** (1995) 373.

[13] WA97 and NA57 Collab.: F. Antinori et al., Nucl. Phys. A **661** (1999) 357c.

[14] Particle Data Group, Eur. Phys. J. C **15** (2000) 231.

[15] J. R. Cudell et al., Phys. Rev. D **61** (2000) 034019.

[16] J. Geiss, W. Cassing and C. Greiner, Nucl. Phys. A **644** (1998) 107.

[17] UA(5) Collab.: Phys. Rep. **154** (1987) 247.

PRINOSI ČESTICA, SKALIRANJE ANTIPROTONA I PROSJEČNI POPREČNI IMPULSI U SUDARIMA OLOVO–OLOVO NA VISOKOJ ENERGIJI: MODELSKA STUDIJA

Pokušavamo objasniti svojstva nekih vrlo važnih opservabli koje su u CERNu mjerili u najnovijim sudarima olovo–olovo i to na osnovi inačice modela lančastih nizova. Na naše iznenađenje, izračunate su vrijednosti u izvrsnom skladu s ishodima mjerenja, posebice kada se kaskade i višestruko raspršenje iskustveno uvedu u račun poprečnih impulsa. Raspravljaju se posljedice usporedbi.