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A SIMPLE EXPERIMENTAL CHECK OF HEISENBERG'S UNCERTAINTY RELATIONS

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We show that the quantum mechanical interpretation of the diffraction of light on a slit, when a wave function is assigned to a photon, can be used for a direct experimental study of Heisenberg's position-momentum and equivalent position-wave vector uncertainty relation for the photon. Results of an experimental test of the position-wave vector uncertainty relation, where the wavelength is used as the input parameter, are given and they very well confirm our approach. The same experimental results can also be used for a test of the position-momentum uncertainty relation when the momentum p_0 of a photon is known as the input parameter. We show that a measurement of p_0, independent of the knowledge of the value of the Planck's constant, is possible. Using that value of p_0, a test of the position-momentum uncertainty relation could be regarded as a method for a direct measurement of the Planck's constant. This is discussed, since the diffraction pattern is also well described by classical electrodynamics in the considered experimental conditions. This approach for testing the Heisenberg's uncertainty relations is very simple and consequently suitable as a quantitative exercise in undergraduate experimental courses, as well as a visual and attractive demonstration of the Heisenberg's uncertainty principle in courses of quantum mechanics.

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1. Introduction

Diffraction of light due to sources of conventional strength is well described by classical electrodynamics. In 1909, four years after the concept of photon was introduced, Taylor [1] showed that the diffraction pattern produced by an extremely

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feeble light and using very long exposure has the same structure as the pattern produced by light of a strong intensity. In such type of experiments, modern detectors at the screen can reveal the discrete nature of the diffraction pattern. From the standpoint of quantum mechanics [2], it is a process in which one photon interferes (diffracts) with itself. A large number of such events accumulated on a screen results in the diffraction pattern, which can be observed on a macroscopic scale.

In 1927, Davisson and Germer [3], and independently Thomson [4], were first to observe the diffraction of electrons. While for the explanation of diffraction of feeble light, the concept of photon (the particle of the light) was necessary, an explanation of the electron diffraction required introducing of wave properties for the electron. In that manner, a basic explanation for the diffraction of electrons became the same as that of the photon. Also, the diffraction became a common point of classical wave and particle concept, which unified them into the object known as a quantum particle. As long as we believe in the wave-particle duality of the matter, we should believe that the diffraction is a common feature of all quantum particles.

In well established courses of quantum mechanics (like Feynman's [5], Messiah's [6] and Shiff's [7]), the diffraction of a quantum particle on a slit has been exploited for the purpose of a qualitative demonstration of the position-momentum uncertainty relation [8] given by

$$\Delta x \Delta p_x \geq \hbar/2\,, \tag{1}$$

where Δx is the uncertainty of the position and Δp_x is the uncertainty of the momentum of the quantum particle in the direction of x-axis (the uncertainties in relation (1) are defined as root-mean-square deviations of x and p_x) from their average values. These are descriptions of basic experiments (idealized experiments), which are difficult to realize in the in reality. It seems that for many undergraduates, the lack of basic experiments and indirect proofs of the position-momentum uncertainty relation are not a sufficient for its complete understanding. Thus, some authors, in their courses of quantum mechanics [9,10], point out at several common misleading or erroneous statements regarding the position-momentum uncertainty relation (1). Our opinion is that one of origins of these misunderstandings is a lack of demonstration and quantitative laboratory exercise about this uncertainty relation in undergraduate study. Namely, without a suitable, simple and direct quantitative experiment, an average undergraduate can hardly obtain a complete feeling about the limitation in simultaneous precise determination of the position and momentum of a quantum particle as uncertainty relation (1) imposes. The aim of the present paper is to present and to establish a good demonstration of the uncertainty relations.

In the following section, we give a simple quantitative description of diffraction of a photon by assigning to it a wave function with the ordinary meaning as it has in quantum mechanics. Then, we quantitatively apply formalisms of diffraction of a wave function well known in quantum mechanics. Our treatment of the photon is not in the framework of the quantum electrodynamics [11], in which it is hard to consider the position and momentum of a photon in ordinary fashion as in quantum mechanics of a single particle [12]. For our treatment of the photon, we find a motivation and justification in the following classic Dirac's text on quantum mechanics [2]: "If we are given a beam of roughly monochromatic light, then we know something about the location and momentum of the associated photons. We know that each of them is located somewhere in the region of the space through which the beam is passing and has a momentum in the direction of the beam of magnitude given in terms of the frequency of the beam by Einstein's photo-electric law – momentum equals frequency multiplied by a universal constant."

From the theoretical treatment that we present in this work follows that a very simple experimental set up, as that used for the diffraction of the light, provides a direct quantitative check of the position-momentum and, from the standpoint of wave–particle duality, the equivalent position–wave vector Heisenberg uncertainty relation for visible photon. The latter uncertainty relation is given by

$$\Delta x \Delta k_x = 1/2\,, \tag{2}$$

where Δk_x is the uncertainty of the wave vector of a photon in the direction of the x-axis defined by $\Delta k_x = \Delta p_x/\hbar$. The presented approach is technically very similar to the mentioned idealized experiments from literature and to recently performed Heisenberg's uncertainty experiments using particles with mass (see Ref. [13] and references therein). However, there is an essential difference: it is an extremely simple one and, therefore, very suitable in education of physics at the university level and even in advanced high-school classes.

2. *Diffraction of photons*

The experimental setup, which we used for the quantitative study of Heisenberg's uncertainty relations for the photon, consists of a laser as a source of visible photons, an adjustable slit, a simple screen of white paper or white wall and a simple meter rule, as is schematically shown in Fig. 1. For measuring the slit opening, an optical micrometer, or pieces of steel with calibrated width or micrometer scale mounted on the slit can be used. Such a set up in any practical application satisfies the Fraunhofer's approximations $L \gg d \gg \lambda$, where L is the distance slit-screen, d is a width of the slit opening and λ is the wave length of the laser beam.

Let the laser be positioned in such a way that the axis of the photon beam, presumed to be the z-axis, falls on the centre of the slit. The laser beam is very wide compared to the width of the slit and the photons are highly monochromatic. Therefore, each photon in its way to the slit can be represented by the plane wave function

$$\Psi_0(x, y, z) = C \exp(-\mathrm{i} p_0 z/\hbar)\,, \tag{3}$$

where C is a normalization constant (its value is not important for our consideration), p_0 is the photon momentum in the direction of the z-axis, $\hbar = h/2\pi$ and h is

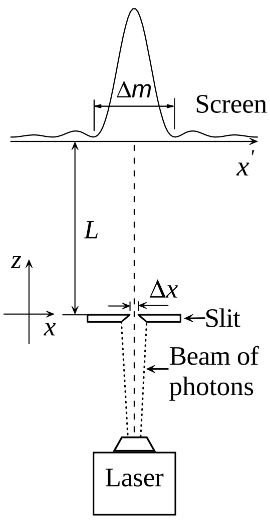

Fig. 1. Experimental arrangement.

the Planck's constant. The photons also have definite momenta $\Delta p_x = \Delta p_y = 0$ in the two perpendicular directions with the values of $p_x = p_y = 0$. Of course, uncertainties of their position in these directions are indefinite. At the moment of reaching the slit, the photon wave is suddenly changed (cut) by slit edges and can be described by the simple normalized wave function[2] given by

$$X(x) = \begin{cases} 1/\sqrt{d} & \text{for } |x| \le d/2\,, \\ 0 & \text{for } |x| > d/2\,. \end{cases} \tag{4}$$

That means that photons are uniformly distributed along the x-axis (Fig. 2a) inside

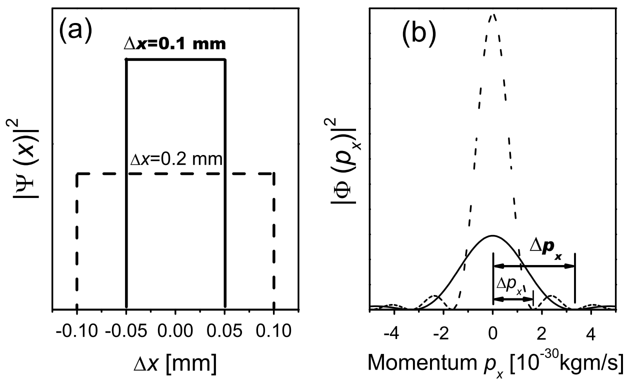

Fig. 2. a) Absolute square of the photon wave function and b) Momentum distribution at the slit.

[2]From now on, we describe the properties of photons only along the x-axis. An arbitrary phase factor can be omitted in the wave function $X(x)$.

the slit of width d. The Fourier transformation of the wave function $X(x)$ is the following wave function in the momentum space

$$\Phi(p_x) = \sqrt{\frac{2\hbar}{\pi d}}\,\frac{1}{p_x}\,\sin\left(\frac{p_x d}{2\hbar}\right). \tag{5}$$

Its absolute square determines the photon momentum distribution along x-axis in the slit, as shown in Fig. 2b. As seen at the distant screen ($L \gg d$) in the x direction, the slit looks like a point source of photons with the momentum distribution $|\Phi(p_x)|^2$. Photons with a momentum p_x in the slit will travel to the screen in the direction defined by the angle

$$\alpha(p_x) \approx \frac{p_x}{p_0} \tag{6}$$

with respect to the z-axis (α is small since $p_0 \gg p_x$). From geometry of the setup follows that the angle α can also be expressed via the screen coordinate x' (see Fig. 1) as

$$\alpha(p_x) \approx \frac{x'}{L}\,, \tag{7}$$

what gives the connection between the momentum p_x and the screen coordinate x'

$$x' = L\frac{p_x}{p_0} \tag{8}$$

Equation (8) means that a photon momentum distribution in the slit is linearly mapped into the spatial distribution on the screen. In other words, the same number of photons in a momentum interval $\mathrm{d}p_x$ around the value p_x in the slit will be found on the screen in an interval $\mathrm{d}x'$ around the position x'. Thus, it is valid

$$|\Phi(p_x)|^2\mathrm{d}p_x = |\Psi(x')|^2\mathrm{d}x' \tag{9}$$

what gives

$$|\Psi(x')|^2 = \frac{p_0 d}{hL}\left(\frac{\sin\big(\pi d p_0 x'/(hL)\big)}{\pi d p_0 x'/(hL)}\right)^2. \tag{10}$$

Equation (10) defines the diffraction pattern on the screen. A photon detector placed around the x' coordinate on the screen with spatial resolution $\mathrm{d}x'$, would count $N|\Psi(x')|^2$ photons per unit time, where N is total number of photons per unit time which reached the screen. When N is a large number, it is more convenient to express the diffraction pattern via the intensity distribution $I(x')$ expressed in W/m. In this sense, the phenomenological description of classical electrodynamics is valid. Since each photon gives an amount of energy E_0 to the screen, which is connected with momentum p_0 via the well known relativistic expression

$$p_0 = E_0/c\,, \tag{11}$$

then $I(x')$ is given by

$$I(x') = E_0 N |\Psi(x')|^2 . \tag{12}$$

If one takes Einstein's relation between energy and frequency of a photon, Eq. (12) becomes identical to the result obtained by classical electrodynamics in Frauenhofer's approximations [14].

3. *Heisenberg's uncertainty relations*

The frequently used definition for the uncertainty of an observable based on root-mean-square deviation is not suitable for a consideration of a wave packet shaped like the diffraction pattern. In that case, it gives $\Delta x = d/\sqrt{12}$, which is very difficult to measure, and $\Delta p_x = \infty$[3]. The meaning of the Heisenberg's uncertainty relation (1) does not depend on a definition but only of meaning of the uncertainties. Hence, it is a matter of definition and convenience which quantity one should choose as a measure of position and momentum uncertainties. In our case, it is very convenient to take $\Delta x = d$ and Δp_x to be equal to the difference between momenta in the first minimum and in the central maximum of the momentum distribution $|\Phi(p_x)|^2$. For the first minimum of that distribution, we have

$$\Delta p_x = 2\pi\hbar/d . \tag{13}$$

Taking this result and $\Delta x = d$, we obtain

$$\Delta x \, \Delta p_x = h . \tag{14}$$

Equation (14) tells us that a photon in the slit can not be in a state of well defined position and momentum at the same time. This is the well-known Heisenberg's position-momentum uncertainty relation for the diffraction pattern [6, 13]. Note that the obtained constant for the product of the two uncertainties depends on the definition for the uncertainties.

To experimentally test the uncertainty relation (14), measurements of the uncertainties Δx and Δp_x are necessary. Obviously, Δx can be directly determined by a measurement of the slit opening. But, the uncertainty Δp_x can not be directly determined. For its measurement, we use our result about mapping of the photon momentum distribution in the slit into the spatial photon distribution on the screen. That means that the spatial distribution on the screen reveals the photon momentum distribution in the slit, Δp_x. Accordingly, the distance between the first

[3] There are several formulations of the uncertainty relation with the aim of circumventing the infinite value of Δp_x in the case of the diffraction pattern (see Ref. [15] and references therein). A very interesting and instructive example is to look at the diffraction pattern as Fisher information as given in Ref. [15]. In that way, it is possible to obtain the uncertainty relation formally identical to the classical Heisenberg relation (1) without the problem of infinity. Despite that, the reader should not confuse these two uncertainty relations. They are different because of different meaning of p (and Δp).

minimum and the central maximum on the screen $\Delta m/2$ (Δm is distance between the two first minima) gives

$$\Delta p_x \approx p_0 \alpha(\Delta p_x) = p_0 \frac{\Delta m/2}{L} \,. \tag{15}$$

The momentum p_0 of a laser photon appears in Eq. (15) and, consequently, in the uncertainty relation (14) as an input (external) parameter. Instead of p_0, laser suppliers or manufacturers almost always give a value for the wavelength, i.e., the wave vector $k_0 = 2\pi/\lambda$ for a laser photon. Then, the position-momentum uncertainty relation (14) is transformed into the equivalent position-wave vector uncertainty relation

$$\Delta x \Delta k_x = 2\pi \,, \tag{16}$$

which can be experimentally tested.

When one knows the momentum p_0 of a laser photon as the input parameter, which is obtained without the knowledge of the value for h, the uncertainty relation (14) can be experimentally checked. That check can also be used as a method for a direct measurement of the Planck's constant h. Such a value of the momentum p_0 can be obtained via its energy E_0 using the relation (10). From the basic definition of a photon as a quantum of light follows

$$E_0 = P/n \tag{17}$$

where P is the power and n the number of photons per unit time of the laser beam. A process of the measurement of P and n, could be done in the following experimental procedure. The laser beam should be slightly spread by a divergent lens and its intensity should be lowered, e.g., by crossed linear polarizers until a very fast counter can normally count photons (n). The power P of such an adjusted laser beam can be measured using a classical calorimeter such as the isoperibol laser calorimeter [16], which provides a measurement of a power level of 1 nW and higher. At 1nW, expected counting rate is about $2 \cdot 10^9$ photons per second in the visible. That is in range of today's fast counters (their maximum counting rate is about 10^{10} counts/s per cm^2 of detector area [17]), what makes such a measurement possible.

4. *Measurement*

We made measurements of the position-wave vector uncertainty relation using a helium-neon gas laser (supplied by Uniphase, Eindhoven, Netherlands) with output power of 1 mW, which gave photons of red light with wavelength $\lambda = 632.8$ nm (given by laser supplier), an adjustable slit, that was made of steel with very sharp edges, and a white paper as a screen. On the slit, a micrometer was mounted which provided measurement of the slit opening with an accuracy of about 0.001 mm. The laser beam was slightly divergent, and therefore, we checked dependence of results on the distance between the laser and the slit. The results were independent of the distance, when it was lager then about 3 m. The distance between the slit and the screen was $L = (4.500 \pm 0.005)$ m. We made eleven series of measurements. The slit width Δx was changed from 0.1 mm to 0.8 mm in steps of 0.05 mm. The distance between the first minima Δm in the photon distribution on the screen was measured by a simple meter rule, and it ranged from 56.2 mm, when $\Delta x = 0.1$ mm, to 7.0 mm, when $\Delta x = 0.8$ mm. The results for product $P_{\rm rel} = \Delta x \Delta k_x$ have been averaged out and they are shown versus Δx in Fig. 3. We see that the product

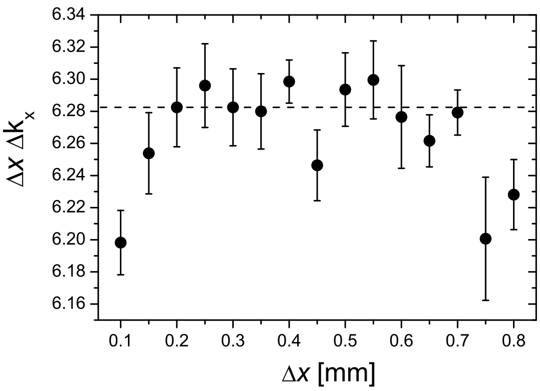

Fig. 3. Results for the product of uncertainties in position (Δx) and wave vector (Δk_x) in the slit.

$P_{\rm rel}$ is independent of the uncertainty Δx, what one can expect according to the relation (16). The average value obtained from the data is

$$P_{\rm rel} = (6.264 \pm 0.006)\,,$$

what is in a very good agreement (within 0.3 %) with the expected value $P_{\rm rel} = 2\pi$ indicated by the dashed line in Fig. 3. The high accuracy of the result has been achieved due to accurate workmanship of the slit and high accuracy in measuring of the slit width. The accuracy of the experimental points is limited by the accuracy of the measurement of Δx at small values and by the accuracy of the measurement of Δm at large values of the slit opening.

5. *Discussion*

We want to emphasize that simplicity is one of the important features of our study of the uncertainty relations for a photon. First, the given quantum description of photon diffraction is very simple and can be applied to other problems, which can be as simple to explain using of the concept of photon as using the classical electrodynamics (for instance radiation pressure, Doppler effect, see Ref. [18]). Because of its complexity, a quantum electrodynamical approach of the photon interference (diffraction), even in its simple version as in Ref. [11], is not suitable for the education of undergraduates. The use of visible photons gives two key benefits: a very simple experimental set up: eyes are used as instruments (detectors) and visible photons have an adequate momentum what permits the experiment to be performed on a suitable scale. An application of photons possessing considerable smaller momentum would require a huge distance L, On the other hand, an application of photons with a much larger momentum or any other kind of particle possessing mass would need an extremely narrow width of the slit d, and its control would require sophisticated equipment [13]. For this reason, a simple experiment of the Heisenberg's position-momentum uncertainty relation, like presented one, is not possible using any non-zero rest-mass particles, even the lightest ones, the electrons.

One could wonder how the Planck's constant h can be measured using an experiment which is well described by classical electrodynamics. First, we remind that classical description of diffraction of light is an approximation. It is a phenomenological approach valid only for a large number of photons. Real nature of the diffraction pattern is diffraction of individual photons independently of the number of photons accumulated on the screen. This apparent paradox is basically caused by differences between limiting criteria for validity of classical mechanics and classical electrodynamics. Namely, in our particular example, we describe the photon by a wave function (probability amplitude), what is a description analogue to that of any particle (electron, proton etc.) in quantum mechanics and different from that of the photon in quantum electrodynamics[4]. A classical mechanical description is valid whenever the commutativity of dynamical variables can be neglected, i.e.,

$$\left[x, p_x\right] = \mathrm{i}\,\hbar, \quad \text{when } h \to 0\,. \tag{18}$$

Obviously, the Planck's constant h can not be neglected in our quantum mechanical description of individual photons. On the other hand, a description of classical electrodynamics, as seen from the view point of the quantum electrodynamics, is trustworthy whenever the commutativity of creation a^+ and annihilation operator a can be ignored [12], i.e.,

$$\left[a, a^+\right] = 1 \to 0 \quad \text{when } n \to \infty\,. \tag{19}$$

Thus, the classical electrodynamics is valid when the number of photons n is large compared to 1, what is very well fulfilled in our experiment (the laser emitted about 10^{18} photons per second). In conclusion, in the description of a photon by the wave function, like for any particle in quantum mechanics, the Planck's constant can not be neglected even when the classical electrodynamical description is reliable because of a large number of photons in the beam.

[4]The basic uncertainty relation in quantum electrodynamics is $\Delta N \Delta\phi \geq 1$, where ΔN is the uncertainty in the number of photons and $\Delta\phi$ is the uncertainty in phase differences between the plane-wave components. Hence, the momentum uncertainty of the light beam arises from the uncertainty in the number of photons, i.e., $\Delta p_x = \Delta N \hbar k_x$. Taking into account $\Delta\phi = k_x \Delta x$, one can obtain the uncertainty relation which is formally identical to the Heisenberg's relation (1), but the uncertainties refer to the light beam (not to a photon) (see Ref. [12]).

6. Conclusions

A quantum mechanical description of diffraction of light on a slit, when a wave function is assigned to a photon, provides that the diffraction pattern on the screen is a consequence of the momentum distribution of photons in the slit. That is used in a simple direct measurement of Heisenberg's position–momentum uncertainty relation, assuming as the input parameter the momentum p_0 of a photon, or the equivalent position–wave vector uncertainty relation, when as the input parameter the wavelength λ, i.e. wave vector k_0, of a photon is known. This approach is very well confirmed by results of a test of the position–wave vector uncertainty relation made using a slit with a micrometer, whose opening was changed from 0.1 mm to 0.8 mm, and a laser as a source of photons of wavelength of 632.8 nm. Since the laser suppliers specify the value of λ (k_0), we give a procedure for a measuring of p_0, which does not require the knowledge of the value of Planck's constant h. The value of p_0 permits not only a test of the position–momentum uncertainty relation but also a direct measurement of the value for h. Although the diffraction pattern is also well described by the classical electrodynamics in the approximation of large number of incident photons, a possibility of a measurement of h in such an experiment arises from the model of photon used and from differences between limiting criteria for validity of the classical mechanics and the classical electrodynamics. The presented experiment is as simple as possible since eyes are used as the detector and visible photons have such adequate momenta that a reasonable distance slit-to-screen and of the slit opening may be used. The experiment can be used as a quantitative exercise for both uncertainty relations in experimental courses for undergraduates. It can also be used as their simple demonstration. Namely, a continuous narrowing of the slit opening causes a continuous spreading of the diffraction pattern on the screen and vice versa. That process can be demonstrated to an auditorium if the laser used has sufficient power. It can be regarded as a visualization of the Heisenberg's uncertainty principle if one has in mind the presented consideration.

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JEDNOSTAVNA EKSPERIMENTALNA PROVJERA HEISENBERGOVIH RELACIJA NEODREĐENOSTI

Pokazujemo da se kvantno-mehaničko tumačenje ogiba svjetlosti na pukotini, u kojem smo fotonu pridijelili uobičajenu valnu funkciju, može upotrijebiti za izravnu eksperimentalnu provjeru Heisenbergovih relacija neodređenosti položaj–impuls i ekvivalentnih položaj–valni vektor. Rezultati testiranja relacije neodređenosti položaj–valni vektor, u kojem smo uzeli valnu duljinu lasera kao ulazni parametar, dobro potvrđuju na pristup. Na istovjetan način se može napraviti provjera relacija neodređenosti položaj–impuls ako je impuls p_0 laserskih fotona poznat kao ulazni parametar. Dokazujemo da je moguće mjeriti p_0 neovisno o poznavanju vrijednosti Plankove konstante. S tako dobivenom vrijednošću p_0 opisani eksperiment se može promatrati i kao način mjerenja Plankove konstante. To smo detaljnije pojasnili s obzirom da ogibnu sliku u danim eksperimentalnim uvjetima možemo također odlično opisati i klasičnom elektrodinamikom. Opisani eksperimentalni pristup za provjeru Heisenbergovih relacija neodređenosti je vrlo jednostavan, te je stoga pogodan kao vježba u studentskim praktikumima ali i kao vizualno atraktivna demonstracija na predavanjima iz kvantno-mehaničkih kolegija.