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Public monopoly versus mixed oligopoly: product differentiation and social efficiency

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ABSTRACT

In this paper, we consider a mixed oligopoly market in which a public firm and private firms compete, in particular, in which private entrants are allowed to enter a monopoly market by a public incumbent who maximizes social welfare. It has been widely believed that the public firm has advantage over private firms because the former who maximizes social welfare can charge a lower price than the latter who maximizes its own profit. However, in a Hotelling model of product differentiation, we obtain the results that both the public firm and private firms charge the same price in equilibrium, and more importantly, that the equilibrium prices may rise as a result of competition, thereby lowering the consumer surplus, if the transportation cost is high enough. We also show that if a private firm enters the market by choosing its own degree of differentiation, it will prefer neither maximum differentiation nor minimum differentiation in the case that the public incumbent is myopic in the sense that it cannot anticipate entry as well as in the case that it is far-sighted enough to anticipate entry. This draws an important policy implication in the market of Korean housing guarantee services.

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1. Introduction

In a modern economy, many industries display mixed oligopoly markets, in which state-owned welfare-maximizing public firms compete against profit-maximizing private firms. Examples of mixed oligopolies are found in sectors such as telecommunications, postal services, banking, education, health care. Most of the mixed oligopoly markets initially start off as purely public markets in which only a single state-owned public firm operates and becomes competitive at later stages by the government's policy to introduce competition. For example, the Korean housing guarantee market has been a monopoly by Korea Housing and Urban Guarantee Corp (HUG) for more than 20 years since the Korean government introduced the housing guarantee system in 1993 for the purpose of managing risks associated with supplying housing and

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enhancing consumer surplus, although the Korean Ministry of Land, Infrastructure and Transport amended the rules of housing supply in 2008 in such a way that the Minister can designate additional housing guarantee firms, because no other firm than HUG has been designated so far. A natural question that arises in this case, which is quite important, is whether introducing competition in the public monopoly market really improves consumer surplus and social welfare through lowering prices. In this paper, we address the issue in a mixed oligopoly market in which new entrants are allowed to enter a monopoly market by a public incumbent that maximizes social welfare.

It has been widely believed that the public firm has advantage over private firms in a mixed oligopoly market, because the former who maximizes social welfare can charge a lower price than the latter who maximizes its own profit. Contrary to the belief, however, we first show in a Hotelling model of product differentiation that both the public firm and private firms charge the same prices in equilibrium, and then show that the prices may rise if entry by private firm(s) is allowed into the monopoly market by the public firm. The intuition for the equal price result is as follows. The public firm has no incentive to undercut the rival private firm, because it cares about the private firm's profit as well as its own profit. It can achieve its goal of maximizing social welfare by minimizing the transportation costs of consumers which can be attained when consumers located between the public firm and the private entrant are served by a firm closer to them. This is possible if and only if the two firms charge the same prices so that they do not distort the purchasing decisions of the consumers.[1] More importantly, competition may raise the equilibrium prices, if the transportation costs of consumers, which can be interpreted as a degree of product differentiation, are very high. A monopolistic public firm has to charge a very low price to satisfy all consumers in the market to offset the high transportation cost, but a public firm in a mixed oligopoly market does not need to do so because the consumers in the niche markets are served by private entrants. So, competition does not guarantee a lower price, thereby possibly lowering the consumer surplus, if the transportation cost is high enough. Note that this result comes from the assumption that the whole market should be fully covered, i.e., all consumers must be served by one of the firms.

We also obtain the result that if a private firm enters the market by choosing its own degree of differentiation, it will prefer neither maximum differentiation nor minimum differentiation. This result holds in the case that the public incumbent is myopic in the sense that it cannot anticipate entry in the future as well as in the case that it is far-sighted enough to anticipate entry. This is sharply contrasted with the widely known result that pure oligopoly leads to maximum differentiation. The intuition for the moderate differentiation result is clear. Since competition in a mixed oligopoly market is less severe than in a pure oligopoly consisting only of private firms, the incentive to avoid price competition by choosing maximum differentiation is weaker in mixed oligopoly.

This result has an interesting policy implication on the Korean housing guarantee market. In Korea, construction companies sell new apartments before they are completed. The buyers make installment payment for the new apartments while they are

being built. Guarantees for completion and distribution are required for the construction companies due to high risks that might occur in case that they go bankrupt before completing the apartment blocks. The market has been a monopoly by a state-run HUG, but the Korean government is considering introducing competition into the market. Also, since it is mandatory for all construction companies, the situation exactly corresponds with the case that the whole market is fully covered, which our result crucially relies on. Accordingly, the analysis in this paper is relevant to the Korean housing guarantee market.

The paper is organized as follows. In Section 2, we briefly review related literature. In Section 3, we present a model of public monopoly as a benchmark case. In Section 4 and Section 5, we analyze the mixed oligopoly models in which only one entrant or two entrants enter the market respectively. Concluding remarks follow in Section 6. Appendix provides proofs for the propositions.

2. Literature review

The idea of mixed oligopoly, more specifically, interaction between a public firm and private firms in an oligopoly situation formally dates back to Merrill and Schneider (1966). Thereafter, many researchers have investigated the issue of whether or not the presence of a public firm can actually increase welfare. Even though a public firm maximizes social welfare by definition, it is not necessarily implied that its interaction with private firms should improve welfare. For example, de Fraja and Delbono (1989) obtains a paradoxical result that the presence of a public firm increases welfare only when there are only a few competitors, but decreases welfare when there are a larger number of competitors. Cremer et al. (1991) also showed that the presence of a public firm improves social welfare if there are only two firms. However, their main focus is to consider a transition from a pure private oligopoly to a mixed oligopoly. To the best of our knowledge, none of them focuses on the effect of a transition from a public monopoly to a mixed oligopoly on social welfare. Beato and Mas-Colell (1984) also obtained some implication of a mixed oligopoly on social welfare in a sequential game by showing that welfare may be higher when the public firm is a follower than when it is a leader.

Most of the mixed oligopoly markets are characterized by either horizontal or vertical differentiation. Cremer et al. (1991) developed a Hotelling-type model of a mixed oligopoly in which private and public firms choose first locations and then prices.[2] So, our paper is similar to Cremer et al. (1991), but distinguished from them in the sense that the private 1991firms' location decisions are made after the location of the public firm is given. Grilo (1994) first considered duopolistic mixed competition when products are vertically differentiated and firms choose first qualities and then prices. Under the assumption of covered markets, she showed that the presence of a public welfare maximizing firm can lead to the social optimum. The assumption of covered market and inelastic demand produces the result that the public firm does not need to deal with any distortion associated with non-optimal consumption due to firms' market power and consequently the mixed duopoly can be socially optimal. Using a model similar to Grilo (1994), Delbono et al.

(1996) introduced the possibility that the market might not be covered. They showed that there exist two equally plausible equilibria in which either firm can be the one producing high quality goods.

There are many authors who treated product differentiation not as exogenous, but endogenized its choice. The pioneering article is d'Aspremont et al. (1979). They established the principle of maximum differentiation that horizontally differentiated firms want to be located as far as possible from the rival in a Hotelling model with quadratic transportation costs. On the other hand, Hamilton et al. (1989) and Anderson and Neven (1991) showed the agglomeration of firms in the center in a quantity competition model (minimum differentiation). Cremer et al. (1991) who considered a model in which all firms choose their locations and then prices simultaneously showed that the resulting product differentiation is neither maximal nor minimal. However, they did not provide a general insight behind the result. Alternatively, Matsushima and Matsumura (2003) analyzed a location-then-quantity choice model by a welfare-maximizing public firm and private firms. As a result, in their model, all private firms agglomerate at a point that is the farthest from the public firm (maximum differentiation). Matsumura and Matsushima (2003) considered a sequential location-then-price choice model as our paper. In this sense, it is closest to our paper. They obtained a similar result that the prices of the public firm and the private firm are the same in a mixed duopoly (not in a mixed oligopoly) but they did not compare the equilibrium prices before and after competition is introduced. Kitahara and Matsumura (2013) considered the case that the demand is elastic. They showed that if the demand is elastic, the degree of product differentiation between the private firm and the public firm is too small compared to social optimum, albeit not zero. Similarly, Matsumura and Tomaru (2014) showed that the degree of product differentiation is too small for social welfare in a mixed duopoly if we consider shadow costs of public funding.

Besides the articles mentioned above, vast literature on mixed oligopoly has been growing recently. While most literature assumes that product differentiation is obtained in a costless way, Liu et al. (2020) considers a model of costly product differentiation and obtains the result that the products are always more differentiated in the mixed duopoly under Cournot competition, while it may be reversed, depending on the differentiation costs under Bertrand competition. Wang et al. (2019) also shows that investments to increase product differentiation improves social welfare.

Nie and Yang (2020) considers a model of process innovation rather than product innovation. They find the result that the private firm invests more in cost-reducing innovation than the public firm in mixed duopoly. This is consistent with the result by Vives (2008) that increasing the degree of product substitutability increases cost reduction expenditures in the sense that firms invest more in R&D as they are more competitive.

Nie (2014) asserts that the performance of a public firm in a mixed duopoly is affected not only by the production cost but also by the capacity constraint, and shows that both the firm-size difference and the price difference between the public firm and the private firm in a mixed duopoly increase with the input capacity when the capacity is scarce. Liu et al. (2020) argues that the performance of the

public firm also depends on institutional variables such as corporate taxation policies. They show that the profit tax rate affects the performance of the partially privatized firm in a mixed duopoly, and therefore affecting the degree of privatization.

3. Public monopoly (benchmark case)

A public firm (firm 1), which is located at z_1 on a unit interval $[0, 1]$, provides services, for example, housing guarantee services with the marginal cost $c(>0)$. Also, an infinite number of customers for this service (construction companies) are uniformly distributed on the interval. We denote a customer's location by $x \in [0, 1]$. We further assume that the location of each customer is not known to a service provider (or service providers).[3]

Each customer is required to travel to the location of the monopolist in order to purchase the service. Thus, the location of a customer reflects its preference for the service provided by the monopolist. The closer a customer is located to the monopolist, the higher satisfaction it gets from the service. The distance between the customer and the monopolist reflects the customer's disutility from the monopolist's service. We assume that the transportation cost that a customer located at x must bear is $t(z_1-x)^2$, where $t(>0)$ is a parameter for the transportation.

Let a customer's reservation utility from purchasing the service be r. We assume that $r > c$.[4] Then, the net benefit of a customer located at x from the service is

$$U(x) = r-t(z_1-x)^2-p,$$

where p is the monopolist's price. The monopolist chooses p to maximize the social welfare, since it is a public firm, not a private firm maximizing its profit.

In this monopoly case, the market can be either covered or not, depending on the size of p. We will first consider the case that the market is not covered.

If we assume that the monopolist is located at the midpoint ($z_1 = \frac{1}{2}$) of the unit interval $[0, 1]$, the location of a customer who is indifferent between purchasing the service and not, which will be denoted by x^M, must satisfy

$$U(x^M) = r-t\left(\frac{1}{2}-x^M\right)^2-p = 0, \tag{1}$$

i.e., $x^M = \frac{1}{2} \pm \sqrt{\frac{r-p}{t}}$ if $\sqrt{\frac{r-p}{t}} \leq \frac{1}{2}$, i.e., $r \leq p + \frac{t}{4}$. Thus, the demand function for the service is $D(p) = 2\sqrt{\frac{r-p}{t}}$.

Let the profit of the monopolist be $\pi(p)$. If this firm maximized the profit, it would choose p to maximize $\pi(p) = 2(p-c)\sqrt{\frac{r-p}{t}}$. The public monopolist is, however, to maximize the social welfare which is defined by the sum of the monopolist's profit and the total customer surplus. If we denote the social welfare and the customer surplus by W and CS respectively, we have

$$\begin{aligned} W &= \pi + CS \\ &= \int_{x_-^*}^{x_+^M} \left[r - c - t(x-\tfrac{1}{2})^2 \right] dx \\ &= \int_{x_-^M}^{x_+^M} (p-c)dx + \int_{x_-^M}^{x_+^M} \left[r - p - t(x-\tfrac{1}{2})^2 \right] dx \\ &= 2(p-c)\sqrt{\frac{r-p}{t}} + \int_{x_-^M}^{x_+^M} \left[r - p - t(x-\tfrac{1}{2})^2 \right] dx \end{aligned}$$

where $x_-^M = \frac{1}{2} - \sqrt{\frac{r-p}{t}}$ and $x_+^M = \frac{1}{2} + \sqrt{\frac{r-p}{t}}$.

Let the optimal price for the public firm be p^M. Then, p^M is determined from the optimization problem:

$$\max_p W = 2(p-c)\sqrt{\frac{r-p}{t}} + \int_{x_-^M}^{x_+^M} \left[r - p - t(x-\frac{1}{2})^2 \right] dx.$$

The first order condition implies that p^M must satisfy

$$\frac{dW}{dp} = \frac{\pi(p)}{dp} + \frac{dCS(p)}{dp} \tag{2}$$

$$= \frac{\pi(p)}{dp} + \frac{d\int_{x_-^M}^{x_+^M} \left[r - p - t(x-\tfrac{1}{2})^2 \right] dx}{dp}. \tag{3}$$

Note that $\frac{dCS(p)}{dp} < 0$ for all p, since $U(x) = r - p - t(x-\frac{1}{2})^2 \geq 0$ for all the customers (x) who purchase the service, $\frac{d(x_+^M - x_-^M)}{dp} < 0$ and $\frac{dU(x;p)}{dp} < 0$. The customer surplus consists of two parts; direct benefit (r – p) and the transportation disutility ($t(x-\frac{1}{2})^2$). So, the price effect on the customer surplus appears to be ambiguous, because a lower price definitely increases the first part but it also increases transportation costs due to a larger demand. However, the total price effect is, in fact, unambiguous. The intuition goes as follows. If p is decreased, the customer surplus is increased because the surplus of each customer who purchases the service is increased and more customers will enjoy the nonnegative net benefit. Therefore, the public firm provides the service at a lower price than the price at which a private firm would provide the service, and as a result, it attains a higher level of social welfare than a private firm.

Now, consider the case that the market is covered, so that all customers purchase the service provided by the public firm. This case occurs when the extreme customers located at $x = 0, 1$ purchase the service, i.e.,

$$U(x=0) = U(x=1) = r - \frac{t}{4} - p \geq 0. \tag{4}$$

Since the market is covered as long as $p \leq r - \frac{t}{4}$, a profit-maximizing private firm who wants to cover the whole market would charge the price as high as possible satisfying the market-coverage condition, i.e., $p = r - \frac{t}{4}$, because its profit would be reduced with maintaining the same customers if it would charge a lower price. Note,

however, that when it lowers the price below $p = r - \frac{t}{4}$, social welfare remains unaffected even if the profit is reduced. The reason is that $r - c = (r - p) + (p - c)$. In other words, as p falls, $p - c$ is reduced but $r - p$ is increased by the equal amount, so that $r - c$ remains constant. This suggests that the public firm could maintain a lower price than the private firm even in the case that the market is covered.

For the purpose of comparison, we will assume that the public firm wants to maximize the profit as far as the social welfare is the same, i.e., charges $p^M = r - \frac{t}{4}$, the highest price satisfying the condition for the market to be covered.[5]

4. Mixed duopoly

In this section, we consider the case that competition is introduced so that a private entrant enters the market. If the government introduces competition into the market by allowing a private firm (firm 2) to enter, the previous monopoly model can be easily modified. We denote the costs of the two firms by c. Also, we assume that the two services are only horizontally differentiated, not vertically differentiated. So, the reservation utility from purchasing either service is assumed to be equally r.[6] To model product differentiation, we follow the standard model of Hotelling (1929).

We can consider the following two-stage game between the two firms after the incumbent's location is chosen. In the first stage, the entrant decides where to enter by choosing its location $z_2 \in [0, 1]$. Once the location of the entrant is determined, in the second stage, the two firms compete for the customers in the prices.

In general, introducing competition into the market is to lower the prices. However, it is true only when the existing firm is private. If the existing firm is public, it is not clear whether the price will fall due to competition, because the price charged by the public monopolist is already lower than the price that a private monopolist would charge. Another possible effect of introducing competition is to provide a more variety of services if the entrant provides service differentiated from the public firm. In this case, customers would be able to get service which can satisfy their tastes better, technically speaking, by saving transportation costs.

For the incumbent's location choice, we consider two cases. The first case is that the public incumbent is naive in the sense that it chooses its location z_1 without anticipating entry in the future. This case can be justified in our motivating example of the Korean housing guarantee market, since the incumbent HUG could hardly expect that the market would be opened when it first started the business in 1993. To the extent that the monopolistic position of the public incumbent firm is protected by the law, this assumption of the naive incumbent seems to be quite reasonable and relevant to reality.[7] The second case is that the public incumbent is far-sighted enough to choose its location in anticipation of a future entrant.

4.1. Purchasing decisions of customers

Let the location of the entrant be z_2. Without loss of generality, we assume that $z_1 \leq z_2$.

Customers located at x will choose between the two firms by comparing the prices (p_1, p_2) and the locations (z_1, z_2), more generally, by comparing their net benefit from the two services,

$$U_i(x) = r - t(z_i - x)^2 - p_i \tag{5}$$

for $i = 1, 2$.

If we are restricted to the case that the market is covered, the location of the borderline customer, i.e., the customer indifferent between the two services, which will be denoted by x^*, can be determined by

$$t(x^* - z_1)^2 + p_1 = t(x^* - z_2)^2 + p_2, \tag{6}$$

which is reduced to

$$x^* = \frac{z_1 + z_2}{2} + \frac{p_2 - p_1}{2t(z_2 - z_1)}. \tag{7}$$

Then, all customers located at $x<x^*$ purchase services from firm 1, while all customers at $x>x^*$ purchase services from firm 2.

4.2. Price decisions

The two firms compete by choosing their prices in this mixed duopoly environment as follows. The public firm chooses p_1 to maximize the social welfare defined by the sum of the profits in the industry and customer surplus:

$$\begin{aligned} W &= \pi_1 + \pi_2 + CS \\ &= \int_0^{x^*} (p_1 - c)dx + \int_{x^*}^1 (p_2 - c)dx + \\ &+ \int_0^{x^*} (r - p_1 - t(x - z_1)^2)dx + \int_{x^*}^1 (r - p_2 - t(x - z_2)^2)dx. \end{aligned}$$

On the other hand, the private firm chooses p_2 to maximize its own profit $\pi_2(p_1, p_2)$. The Nash equilibrium prices can be found by the following two best-response functions;

$$\frac{dW(p_1, p_2)}{dp_1} = t\left[(x^* - z_2)^2 - (x^* - z_1)^2\right]\frac{dx^*}{dp_1} = 0, \tag{8}$$

$$\frac{d\pi_2(p_1, p_2)}{dp_2} = \left[1 - \left(\frac{z_1 + z_2}{2} + \frac{p_2 - p_1}{2t(z_2 - z_1)}\right)\right] - (p_2 - c)\frac{dx^*}{dp_2} = 0. \tag{9}$$

The expression inside the square bracket in (9) is an increase in the profit directly due to a price increase and the remaining term is a decrease in the profit due to a demand decrease ($\frac{dx^*}{dp_2} > 0$). Equation (8) implies that

$$p_1 = p_2, \tag{10}$$

since $\frac{dx^*}{dp_1}<0$ and $(x^*-z_2)^2 \gtreqless (x^*-z_1)^2$ if and only if $p_1 \gtreqless p_2$. On the other hand, rearranging equation (9) by using $\frac{dx^*}{dp_2}=\frac{1}{2t(z_2-z_1)}$ yields

$$p_2=\frac{1}{2}\left[p_1+c+t(z_2-z_1)(2-z_1-z_2)\right]. \tag{11}$$

From equations (10) and (11), we obtain Nash prices

$$p_1^N=p_2^N=c+t(z_2-z_1)(2-z_1-z_2). \tag{12}$$

4.3. Location decisions

Let W^* and π_2^* be obtained by substituting (12) into W and π_2. Then, we get

$$\begin{aligned}\pi_2^*(z_2) &= t(z_2-z_1)(2-z_1-z_2)(1-x^*)\\ &= \frac{t}{2}(z_2-z_1)(2-z_1-z_2)^2.\end{aligned} \tag{13}$$

The indirect profit function of firm 2 given by (13) is maximized when the following first order condition is satisfied;

$$\frac{\partial \pi_2^*(z_2)}{\partial z_2}=(2-z_1-z_2)(2+z_1+z_2)=0. \tag{14}$$

Thus, the optimal location of firm 2 is

$$z_2^*(z_1)=\frac{2+z_1}{3}, \tag{15}$$

which is the best response function of firm 2.

If firm 1 is naive, the monopolist's location obtained in Section 3 is maintained, i.e., $z_1^n=\frac{1}{2}$. This in turn implies that the equilibrium location of firm 2 is $z_2^n=\frac{5}{6}$ from (15). If firm 1 is far-sighted, it will choose its location z_1^f to maximize $W^*(z_1,z_2^*)=W^*\left(z_1,\frac{2+z_1}{3}\right)$ as a Stackelberg leader. We have

$$W^*(z_1,z_2^*(z_1))=r-c-t\left[\int_0^{x^*}(x-z_1)^2dx+\int_{x^*}^1(x-z_2^*(z_1))^2dx\right] \tag{16}$$

where $x^*=\frac{z_1+z_2^*}{2}$. Applying Leibniz rule yields

$$\begin{aligned}\frac{dW^*}{dz_1} &= -t\left[(x^*-z_1)^2\frac{dx^*}{dz_1}+\int_0^{x^*}\frac{\partial(x-z_1)^2}{\partial z_1}dx-(x^*-z_2^*)^2\frac{dx^*}{dz_1}+\int_{x^*}^1\frac{\partial(x-z_2^*)^2}{\partial z_1}dx\right]\\ &= -t\left[\frac{1}{2}(x^*-z_1)^2-\frac{1}{2}(x^*-z_2^*)^2+2\int_0^{x^*}(z_1-x)dx+2\int_{x^*}^1\frac{1}{3}(z_2^*-x)dx\right]\\ &= 0.\end{aligned} \tag{17}$$

It is easy to see that $z_1^f=\frac{1}{4}$ and $z_2^f=\frac{3}{4}$ satisfy (17).[8] The intuition is clear. Since the public incumbent knows that the entrant will respond optimally to z_1 according to its

best response function $z_2^* = \frac{2+z_1}{3}$, firm 1 who maximizes social welfare will choose z_1 that can induce symmetric z_2 in the sense that $z_2 = 1-z_1$ so that the two firms split the market equally. Therefore, firm 1 will choose $z_1^f = \frac{1}{4}$ from solving the equation $1-z_1 = \frac{2+z_1}{3}$.

This result tells us that neither maximum differentiation nor minimum differentiation occurs in equilibrium whether firm 1 is naive or far-sighted. The next proposition summarizes this.

Proposition 4.1. *In a mixed duopoly market that is covered, (i) the equilibrium prices are the same, i.e., $p_1^N = p_2^N$, and (ii) if firm 1 is naive, $(z_1^n, z_2^n) = \left(\frac{1}{2}, \frac{5}{6}\right)$, and if firm 1 is far-sighted, $(z_1^f, z_2^f) = \left(\frac{1}{4}, \frac{3}{4}\right)$.*[9]

Neither result is trivial. The first result is surprising, albeit well-known. The welfare-maximizing public firm always wants to charge the same price as the profit-maximizing private firm. Why doesn't it charge a lower price so that it can easily beat the rival by taking advantage of its strategic advantage of no need to maximize its own profit? The reason is that it cares about the rival's profit as well as its own profit, so it does not want to fare far better than the rival. Also, it cares about the rival's customer surplus as well as its own customer surplus. The effect of a price change on the sum of the quadratic transportation costs of the two firms' customers is exactly offset only when the prices are the same; otherwise, the speed of increasing the transportation cost of the marginal customer to one firm would be faster than the speed of decreasing his transportation cost to the other firm.

The second result is even more surprising. It is well known that a duopoly model between two private firms shows the result of maximum differentiation because of the incentive to avoid severe price competition.[10] In this case of mixed oligopoly, two effects of increasing z_2 coexist just as in the case of pure oligopoly competition between private firms; (i) the effect of avoiding severe price competition and (ii) the effect of reducing demand. In the case of pure oligopoly competition, the former price effect dominates the second demand effect. So, both firms prefer the maximal value of z_2 (maximum differentiation). In the mixed oligopoly, however, the former price effect is not so large because the presence of a public firm softens price competition. Thus, the incentive to be located in extreme points becomes weaker. This leads to neither maximum differentiation nor minimum differentiation.[11] What is surprising in the case of the naive incumbent is that the location decision is made only by the private firm that wants to avoid severe competition. This feature of one-sided location decision may make us conjecture that the private firm prefers maximum differentiation, but nevertheless, it turns out that the private firm still does not prefer maximum differentiation, because the private firm is aware that the public firm has a weak incentive of price competition so it does not need to give up the positive demand effect by differentiating maximally. The intuitive reason for why a far-sighted public incumbent chooses a location left to the midpoint is that the public firm who cares about the other private firm wants the entrant to earn enough profits.

The remaining issues are whether the prices fall as a result of competition and whether social welfare will be actually increased. We have the following proposition.

Proposition 4.2. *If a private entrant enters the public monopoly market, in the case that the market is covered, (i) the price of the public firm does not rise in the case of the naive incumbent but may rise in the case of the far-sighted incumbent if* $t > \frac{4}{3}(r-c)$*, and (ii) the social welfare is increased.*

Non-increase in the price in case of the naive public firm is not a consequence of competition.[12] It comes directly from the market coverage condition. Since the public firm should serve the extreme customer (at least on one side) even after competition is introduced, its price cannot exceed the level that barely satisfies the extreme customer. Thus, the price after competition is introduced cannot be raised. However, the price may rise if the public firm is far-sighted, especially when the transportation cost is high. The public monopolist must lower its price significantly to serve all the customers. But if the public incumbent expects entry to occur, it will focus on a niche market on one side by moving closer to the direction. Thus, even a higher price can serve all the customers in the niche market. Even if there is some competition between the public firm and the private firm, a low degree of substitutability due to a high transportation cost would not lower the price very much. The welfare effect is clear because competition saves the total transportation costs, i.e., customers are served by a firm which is closer.

5. Mixed oligopoly

In this section, for symmetry, we assume that two private firms, firm 2 and firm 3, enter the market and play the same game against a public firm. That is, two entrants first choose their locations simultaneously and then the three firms choose their prices at the same time. We focus only on symmetric equilibria.

The crucial difference of this model from the previous duopoly model is that the public firm need not serve the extreme customer, if each of the entrants enter the opposite niche market.

Let the locations of the entrants be z_2 and z_3 where $z_2 \le z_1 \le z_3$. Also, let the borderline customer who is indifferent between purchasing from firm 1 and firm 2 (or firm 2 and firm 3 *resp.*) be x^* and y^* respectively. Then, social welfare can be computed by

$$\begin{aligned} W &= \pi_1 + \pi_2 + \pi_3 + CS \\ &= \int_0^{x^*} (p_2 - c)dx + \int_{x^*}^{y^*} (p_1 - c)dx + \int_{y^*}^{1} (p_3 - c)dx \\ &+ \int_0^{x^*} (r - p_2 - t(x - z_2)^2)dx + \int_{x^*}^{y^*} (r - p_1 - t\left(x - \frac{1}{2}\right)^2)dx + \int_{y^*}^{1} (r - p_3 - t(x - z_3)^2)dx. \end{aligned}$$

By using symmetry, i.e., $y^* = 1 - x^*$ and $z_3 = 1 - z_2$, we can simplify the first order condition of the public firm which is reduced to

$$(x^* - z_2)^2 - \left(x^* - \frac{1}{2}\right)^2 = \left(\frac{1}{2} - x^*\right)^2 - (z_2 - x^*)^2. \tag{18}$$

This in turn implies that $(x^*-z_2)^2 = \left(x^*-\frac{1}{2}\right)^2$, consequently, $p_1 = p_2(=p_3)$. The first order condition of firm 2 remains unaffected. Therefore, we have the following proposition.

Proposition 5.1. *If two private symmetric entrants enter the public monopoly market, in the case that the market is covered, (i) the price of the public firm falls, if* $t<\frac{36}{17}(r-c)$, *and (ii) the price of the public firm rises, if* $t>\frac{36}{17}(r-c)$, *but (iii) social welfare is always increased*

This proposition says that if very homogeneous service is introduced, the price will fall as a result of competition but if the introduced service is very much differentiated from the existing service, the price will rise after competition is introduced. Since the monopoly price by the welfare-maximizing public firm is $r-\frac{t}{4}$, it is possible that $r-\frac{t}{4}<c+\frac{2}{9}t$, implying that competition may increase the price if t is very large.[13] The intuition behind Proposition 3 is similar to the intuition behind Proposition 2. In the monopoly case, the monopolist public firm must lower its price very much to serve all the customers, i.e., to cover the whole market, since the extreme customer's net valuation is too low due to his remote location. However, in the case of competition, both niche markets are served by two entrants, so the monopolist does not need to keep its price low enough to cater to them. Thus, note that this result is not derived solely by the effect of mixed competition. The requirement that the market should be covered is crucial to deriving the result.

Paradoxically, the price may be increased, because the welfare-maximizing should care about the private firm's profit as well. As a result, competition does not necessarily increase consumer surplus! One possible policy implication would be to permit an entrant only if it can prove that their services are not too much differentiated from the existing service for enough competition.

Meanwhile, social welfare is unambiguously increased by the entry in this model of product differentiation mainly because providing a more variety of products can save transportation costs of consumers. Note that this result depends on the assumption that the fixed entry cost is zero or sunk. With positive fixed costs of the entrants, however, it is clear that additional entries increase total fixed entry costs as well as reduce the transportation costs. Therefore, entries may lower social welfare if entry costs are too high. In general, the socially optimal number of entrants is determined so as to balance the social benefit of saving transportation costs and the social cost of increasing entry costs.

Before we close this section, we will briefly discuss the implication on the privatization policy. Since de Fraja and Delbono (1989)'s finding that social welfare may be higher when a public firm maximizes the profit rather than when it maximizes the welfare, many scholars argued that a public firm should be privatized in some cases. In particular, Matsumura (1998) showed in a homogeneous mixed oligopoly that partial privatization is the optimal policy in the short-run with restricted entry, while Matsumura and Kanda (2005) showed that full nationalization is optimal in the long-run with free entry among private firms. Fujiwara (2007) similarly derived the optimal degree of partial privatization not only in the short run but also in the long run. Xu et al. (2017) and Lee et al. (2018) also showed the optimality of partial privatization in the long run when the timing of privatization policy matters. In particular, Xu

et al. (2017) compared the outcomes of ex ante privatization (privatization before liberalization) and ex post privatization (privatization after liberalization), and showed that ex ante privatization yields higher social welfare because the government can control (reduce) the number of private entrants by choosing a lower level of privatization of the public firm before entry thereby making the public firm more aggressive (i.e., produce more outputs). Lee et al. (2018) generalized this result for a general demand function and a cost function. However, none of them addresses the issue in an endogenous location model. Note that social welfare is determined solely by location decisions if the market is required to be fully covered as in our model, because price decisions simply distributes surplus between firms and customers. Therefore, our result suggests that any privatization policy is unwarranted for improving social welfare further if the marginal costs of the public firm and the private firm are the same,[14] because social optimum is achieved by a far-sighted public firm's location decision.

6. Discussion and conclusion

In this paper, we examined the effects of introducing a private competing firm into a public monopoly market. In such a mixed oligopoly market, prices do not always fall as a result of competition. This is mainly because the public firm does not always want to undercut the price by the new entrant; rather it may want to match its own price to the entrant's price. If prices rise after introducing competition, it may lower customer surplus. Therefore, it may not be always desirable at least to customers to introduce competition in a public monopoly market by allowing private firms to enter the market, especially when the entrants are expected to provide very differentiated services so that the competition effect is minor.

Although this paper sheds some light on the effect of the competition in the monopoly market by the public firm, we admit that it oversimplifies the reality. First, the public firm may not be concerned about the profits of other rival firms. If the public firm does not care about the profits of private firms, the result of equal prices does not need to hold. Second, we may consider the option of the hit-and-run strategy of the entrants in a dynamic model. If the market demand is very fluctuating over time, entrants may have an incentive to enter the market only in the boom period and to quit the market in a recession period, although the option is usually not allowed to the public firm. This *run* strategy may occur when the reservation utility at some period, denoted by r_t, is very low. Since this possibility may jeopardize the sustainability of the public firm, we believe that there should be some regulation preventing the hit-and-run strategy. Also, we focused on the case that the market is covered. In fact, this case corresponds with the real situation in Korea in which it is mandatory for construction companies to buy a housing guarantee service. If it is not mandatory, the market may not be covered. Our conjecture is that if the market is not covered, as far as competition does not guarantee a price fall, a negative effect of a lower sale will overweigh a positive effect of transportation cost saving. Although we believe that it will be worthwhile to compare the effect of competition in both cases more

thoroughly, we must admit that the complete welfare analysis will be quite complicated. We will postpone it as a future research agenda.

Notes

1. In reality, it is empirically observed that public firms charge lower prices than private firms in a mixed oligopoly. Sapienza (2004) observes this phenomenon in the banking industry.
2. Earlier, Harris and Wiens (1980) also analyzed a mixed oligopoly with differentiated products.
3. If the location of each customer is known to a firm, it can price discriminate based on the location. We assume away the possibility of price discrimination. For price discrimination in a mixed oligopoly market, see Heywood and Ye (2009).
4. <collab>Otherwise, no customer purchases service</collab>.
5. Note that $z_1 = \frac{1}{2}$ minimizes the transportation cost of the farthest extreme customer. In other words, it maximizes the price that makes the individual rationality condition given by (4) binding. Therefore, $z_1 = \frac{1}{2}$ can be considered as the optimal location of the public firm that wants the market to be fully covered.
6. If $r_i > r_j$ for $i \neq j$, it could be interpreted as quality difference (vertical difference). We are assuming that there is no quality difference between the two services.
7. <collab>Even if the public incumbent may expect a new firm to enter after a long time, the public firm will not find it in its interest to provide its service at a suboptimal position for such a long period until a new firm enters</collab>.
8. Note that $\int_0^{\frac{1}{2}} \left(\frac{1}{4} - x\right) dx = \int_{\frac{1}{2}}^{1} \left(\frac{3}{4} - x\right) dx = 0$.
9. Matsumura and Matsushima (2003) showed that the outcome in the case of the far-sighted public firm corresponds to social optimum.
10. See d'Aspremont et al. (1979) for a model of horizontal differentiation, and Shaked and Sutton (1982) for a model of vertical differentiation.
11. Cremer et al. (1991) and Matsumura and Matsushima (2003) also obtained similar results, but in Cremer et al. (1991), the public firm and the private firm choose their locations simultaneously, and Matsumura and Matsushima (2003) only considered the far-sighted incumbent.
12. The price of the public firm may rise as a result of competition, if the monopoly public firm charges a price below p^M in order to increase consumer surplus at the expense of its own profit. We exclude this possibility by assuming that the public firm charges the highest price as far as it is indifferent in terms of social welfare.
13. In the hog market in China, Watanabe (2008) provides some data supporting the result that the price is higher in the mixed oligopoly than in the monopoly by the welfare-maximizing public firm. She also showed the result theoretically.
14. 2008 Lee and Hwang (2003) argued that full nationalization may not be optimal if we consider the cost difference between the public firm and the private firm due to managerial production inefficiencies of the public firm.

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