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Three Essays on Residential Mortgage Payment Behavior

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ABSTRACT

This dissertation is comprised of three essays on residential mortgage payment behavior. The first chapter analyzes the simultaneous mortgage-termination risks of 90-day delinquency and prepayment, the second and third chapters study borrower mortgage payment and exiting behavior in the CARES Act Mortgage Forbearance program.

Ding, Tian, Yu, and Guo (2012) analyze transformations of the binomial logit duration model for which the results are an exact binomial logit duration model when the transformation parameter equals one and an interval censored proportional hazard model as the transformation parameter limits to zero. In the first chapter, which is co-authored with Ran An and Jan Ondrich, we incorporate one of the Ding et al. transformations into a model with more than one mortgage-termination risk. In this case the resulting model is multinomial logit when the transformation parameter equals one. The resulting model as the transformation parameter approaches zero is not an interval censored competing risk proportional hazard model (see An and Qi 2012). However, it may approximate one and is in any case a valid statistical model. We analyze the simultaneous mortgage-termination risks of 90-day delinquency and prepayment for single-family 30-year fixed-rate mortgages securitized by Fannie Mae using the Fannie Mae public use data. We show that the transformation can control for over-dispersion in the data and that transformed models perform better than the corresponding models without the transformation.

The second chapter uses borrower mortgage payment behavior in the CARES Act Mortgage Forbearance program to predict the mode of exit from the program. The CARES Act permits borrowers to postpone mortgage payments without penalty. In the empirical work, this chapter extends the beta-logistic model in Heckman and Willis (1977) to the Dirichlet nested logit model, which allows the state dependence of choices to vary across different nests. The

results show that the beta distribution of probabilities of choices within the nest and between nests are both J shaped, which indicates that the payment behavior probability of relatively few borrowers is near the average. Moreover, borrowers who make curtailment payments are more likely to exit forbearance with prepayment or reinstatement. In comparison, borrowers who frequently forbear payments are more likely to leave with payment deferral or trial/modification.

The models in the second chapter estimate the effect of payment behavior in the CARES Act forbearance program as the program continues through time. For a given exit time, the likelihoods contained information on only those mortgages that failed at that exit time. Results were presented for three exit times: 6, 12, and 18 months. A two-step estimation technique was used and standard errors were corrected in the second step. The first improvement in the final chapter incorporates information on all mortgages that survive until a given time into the likelihood functions. I show that the estimation can be accomplished in a single step. The accuracy of the two-step estimation and single-step estimation results are compared. The second improvement in the final chapter is to construct a single model, estimated in a single step, that uses information for all of the first six months. The accuracy rate of the estimation for this new model is substantially higher than the accuracy rate of the estimation for the model with a single survival time of six months. Future work is to extend the estimation to cover the entire length of the program.

THREE ESSAYS ON RESIDENTIAL MORTGAGE PAYMENT BEHAVIOR

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1.1 Introduction

Ding, Tian, Yu, and Guo (2012) introduce a discrete time transformation family of the binary logit duration model and apply it to bankruptcy probability prediction with time-varying covariates. The transformation model family contains the Shumway (2001) model and Cox proportional hazard model (Cox 1972). In this chapter, we construct a way to apply the Ding-Tian-Yu-Guo transformation to multinomial logit duration models, which allow for termination into multiple states, and add controls for unobserved heterogeneity. Our empirical application uses the Fannie Mae public use data to analyze the simultaneous mortgage-termination risks of 90-day delinquency and prepayment for single-family 30-year fixed-rate mortgages. Both in-sample and out-of-sample validation statistics show the models with the Ding-Tian-Yu-Guo transformation improve the performance over the corresponding models without the transformation.

As of the first quarter of 2017, there was about $14.4 trillion in total outstanding US mortgage debt, more than half of which was held in mortgage-backed securities (MBS's), mortgage bundles sold to investment banks or other investors. MBS's allow millions of Americans to own homes by effectively connecting the needs of investors and borrowers. However, the borrowers make individual decisions based on current circumstances that cannot be accurately predicted by MBS investors. Each month borrowers decide whether to prepay, go into delinquency, or remain current. Prepayment occurs when the borrower pays off the loan before the maturity date. It can be viewed as the exercise of a financial option to buy the mortgage (call option). Delinquency occurs when the borrower fails to make a payment; 90-day delinquency will always be the first step in a default. Default can be viewed as the exercise of a financial option to sell the mortgage (put option).

Kau, Donald, Walter, and James (1992) use option-pricing theory to rationally price mortgages for both default and prepayment risks, and show that the decisions to prepay or default are substitutes. Default behavior for residential mortgages has been studied by, among others, Ambrose, Capone, and Deng (2001), Lacour-Little and Malpezzi (2003), Agarwal, Deng, and He (2014), and An, Deng, and Gabriel (2019). Follain, Ondrich, and Sinha (1997) and Archer, Ling, and McGill (2002) studied residential prepayment behavior.

Lancaster (1979), in a study of unemployment duration, shows that parameter estimates may be inconsistent when there are omitted variables, even when the omitted variables are orthogonal to the included variables. He proposes a mixed duration model that incorporates a parametric random effect to control for the unobserved heterogeneity. Heckman and Singer (1984) develop a method for estimating a mixed model with a non-parametric random effect to overcome the "over-parametrization" inherent in parametric forms. An alternative method, suggested by Trussell and Richards (1985), is to keep increasing the number of mass points until the likelihood improvement becomes insignificant. Deng, Quigley, and van Order (2000) introduce mass-point corrections for unobserved heterogeneity in a competing risks proportional hazard model for prepayment and default. Clapp, Deng, and An (2006) compare such corrections for unobserved heterogeneity across proportional hazard and multinomial logit models. They find that the proportional hazard versions of their models outperform the multinomial logit versions both in-sample and out-of-sample. Other studies that use a competing risks framework include Calhoun and Deng (2002) and Pennington-Cross (2003).

In this chapter we estimate models that predict prepayment and 90-day delinquency rates given economic scenario, loan information, and borrower characteristics. The remainder of this chapter is organized as follows. Section 1.2 provides the summary of the data and definitions of explanatory variables. Section 1.3 discusses the models with and without non-parametric random effect controls for over-dispersion, and for each of these, with and without the Ding-Tian-Yu-Guo transformation. Section 1.4 presents the empirical results. Section 1.5 presents the conclusions.

1.2 Data

The data in this chapter come from the public use Fannie Mae single-family loan performance data for 30-year fixed-rate mortgages. The final data set represents a randomly selected sub-sample of the 466,700 mortgages originated from January 1, 2000 to December 31, 2016 for the city of Miami, Florida. Half of the mortgages are held back for validation. The observations for each mortgage are annual. The final estimation data set has 11,012 loans and 49,799 loan years comprised of all the years up to and including the year of prepayment or the year of the first 90-day delinquency.

Accordingly, there are three risks in each year of the mortgage: prepayment, 90-day delinquency, and remaining current. The explanatory variables for the three risks are similar to those used in other studies. Prepayment and 90-day delinquency each have mass-point intercepts and probabilities in the models correcting for unobserved heterogeneity and have the following four variables in common: FICO score, log loan size, an indicator variable for having more than one mortgage, and debt-to-income ratio. Additionally, the prepayment risk is determined by the value of the call option and the 90-day delinquency risk is determined by the amount of negative equity in the home, an indicator of the Great Recession, and the original loan-to-value ratio. The remaining current is the baseline risk. A full description of the explanatory variables is given in Appendix.

1.3 Model

Our application uses annual data on mortgages. In any year, the mortgage can be one of three possible states: A^p, A^d, and C. A^p represents the act of prepayment, A^d represents the act of going into 90-day delinquency, and C represents the act of remaining current. We sometimes use a state $A = A^p \cup A^d$, which means that either prepayment or 90-day delinquency happens in that year.

1.3.1 Multinomial Logit Model

The binary and multinomial logit are among the most widely used discrete choice models. These models imply proportional substitution across alternatives. The independence of irrelevant alternative (IIA) property of the multinomial logit model means that for any two alternatives i and j, the ratio of the probabilities does not depend on any alternatives other than i and j. Chipman (1960) and Debreu (1960) point out that the IIA property is clearly inappropriate in many choice situations.

The probability functions for a multinomial logit model are

$$\begin{aligned} P(A^p_{mt}) &= \pi_{pmt} = \tfrac{\exp(z'_{pmt}\beta)}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)} \\ P(A^d_{mt}) &= \pi_{dmt} = \tfrac{\exp(z'_{dmt}\beta)}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)} \\ P(C_{mt}) &= \pi_{cmt} = \tfrac{1}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)} \end{aligned} \quad .$$

The likelihood function for the multinomial logit has the form:

$l(\beta) = \prod_{m=1}^{M} \prod_{t=1}^{T_m} \pi_{pmt}^{\delta^p_{mt}} \pi_{dmt}^{\delta^d_{mt}} \pi_{cmt}^{\delta^c_{mt}}$

$=\prod_{m=1}^{M} \prod_{t=1}^{T_m} (\frac{\exp(z'_{pmt}\beta)}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)})^{\delta^p_{mt}} (\frac{\exp(z'_{dmt}\beta)}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)})^{\delta^d_{mt}} (\frac{1}{1+\exp(z'_{pmt}\beta)+\exp(z'_{dmt}\beta)})^{\delta^c_{mt}}$,

where m is the indicator of the mortgage, t is the age of the mortgage, T_m is the age of mortgage m at termination, the π's are the probabilities, and the δ_{mt}'s are the out-come indicator variables.

1.3.2 The Transformed Mutinomial Logit Model

Ding, Tian, Yu, and Guo (2012) analyze the risk of bankruptcy by constructing transformed binary logit models. In this chapter, we use one of their two transformation functions. This transformation function gives as the probability of bankruptcy in a given year

$$\pi_{mt} = \begin{cases} 1 - \frac{1}{\left(1+c\exp\left(z_{bmt}'\beta\right)\right)^{1/c}} & , \quad c > 0 \\ 1 - \exp\left(-\exp\left(z_{bmt}'\beta\right)\right) & , \quad c = 0 \end{cases} \quad .$$

The probability of continuing another year without bankruptcy is $1 - \pi_{mt}$.

The effect of c in the present case is similar to the effect of transformation parameters in the Box-Cox model (see Box and Cox 1964). In the present case, when c limits to 0 from the right, the discrete time survival model is the interval censored proportional hazard model. When c limits to 1, the discrete time survival model is a binary logit.

We incorporate the Ding-Tian-Yu-Guo transformation into the multinomial logit model in section 3.2.1 and present full information maximum likelihood estimation. In section 3.2.2, we present a two-step method that may be useful in finding the starting values for the parameters.

Full Information Maximum Likelihood

In the transformed multinomial logit model, the probabilities for each year are

$$P(A_{mt}^p|A_{mt}) = \frac{\exp(z_{pmt}'\beta)}{\exp(z_{pmt}'\beta)+\exp(z_{dmt}'\beta)}$$
$$P(A_{mt}^d|A_{mt}) = \frac{exp(z_{dmt}'\beta)}{\exp(z_{pmt}'\beta)+\exp(z_{dmt}'\beta)}$$

$$P(A_{mt}) = 1 - \frac{1}{(1+c\exp(I_{mt}^A))^{1/c}}$$
$$P(C_{mt}) = \frac{1}{(1+c\exp(I_{mt}^A))^{1/c}} \quad ,$$

where I_{mt}^A is called inclusive value for event A at age t of mortgage m. Its formula is given by:

$$I_{mt}^A = \log(\exp(z_{pmt}'\beta) + \exp(z_{dmt}'\beta)) \quad .$$

The marginal probabilities of A^p_{mt} and A^d_{mt} are given by

$$P(A^p_{mt}) = P(A^p_{mt}|A_{mt})P(A_{mt}) = \frac{\exp(z'_{pmt}\beta)}{\exp{(I^A_{mt})}}(1 - \frac{1}{(1+c\exp(I^A_{mt}))^{1/c}})$$
$$P(A^d_{mt}) = \frac{\exp(z'_{dmt}\beta)}{\exp{(I^A_{mt})}}(1 - \frac{1}{(1+c\exp(I^A_{mt}))^{1/c}}) \quad .$$

Therefore the likelihood function is

$$\begin{aligned} l(\beta, c) = \prod_{m=1}^{M} \prod_{T=1}^{T_m} & (\frac{\exp(z'_{pmt}\beta)}{\exp{(I^A_{mt})}}(1 - \frac{1}{(1+c\exp{(I^A_{mt})})^{1/c}}))^{\delta^p_{mt}} \\ & (\frac{\exp(z'_{dmt}\beta)}{\exp{(I^A_{mt})}}(1 - \frac{1}{(1+c\exp{(I^A_{mt})})^{1/c}}))^{\delta^d_{mt}} \\ & (\frac{1}{(1+c\exp(I^A_{mt}))^{1/c}})^{1-\delta^p_{mt}-\delta^d_{mt}} \quad . \end{aligned}$$

A Two-Step Estimation Method

Amemiya (1978) shows that the two-step estimation of a multivariate logit model can be considerably simpler than full information maximum likelihood estimation, especially for the model with many dependent variables. Moreover, the two-step estimation model may be more helpful in finding the starting values.

In the first step, we use only the year observations for which there is prepayment or 90-day delinquency. We run a binary logit for $P(A^p|A)$ versus $P(A^d|A)$. The first stage estimating $P(A^p|A)$ and $P(A^d|A)$ is a binary logit, so we have

$$P(A^d|A) = \frac{\exp(z'_{mt}\beta_1)}{1+\exp(z'_{mt}\beta_1)}$$
$$P(A^p|A) = \frac{1}{1+\exp(z'_{mt}\beta_1)} \quad ,$$

where z_{mt} contains all the variables, β_1 is the coefficient vector[1] from step 1, and the likelihood function can be written as

$$l(\beta) = \prod_{m=1}^{M} \prod_{T=1}^{T_m} (\frac{\exp(z'_{mt}\beta_1)}{1+\exp(z'_{mt}\beta_1)})^{\delta^d_{mt}} (\frac{1}{1+\exp(z'_{mt}\beta_1)})^{\delta^p_{mt}} \quad .$$

[1] $\beta_1 = \beta_d - \beta_p$, because we set prepayment as the reference group.

In the second step, we bring in the result from the first stage and calculate the marginal probabilities of A and C.[2]

$$P(A) = 1 - \frac{1}{(1+c\exp(I^A))^{1/c}}$$
$$P(C) = \frac{1}{(1+c\exp(I^A))^{1/c}} \quad ,$$

where I^A is the inclusive value $\log(\exp(z'_{pmt}\beta) + \exp(z'_{dmt}\beta))$, and the likelihood can be written as

$$l(\beta, c) = \prod_{m=1}^{M} \prod_{T=1}^{T_m} (1 - \frac{1}{(1+c\exp(I^A))^{1/c}})^{\delta^A_{mt}} (\frac{1}{(1+c\exp(I^A))^{1/c}})^{\delta^C_{mt}} \quad .$$

1.3.3 Models with Unobserved Heterogeneity

Heckman and Singer (1984) suggest using finite mixture random effects in a continuous time duration model. Follmann and Lambert (1989) discuss generalizing panel binary logistic regression using non-parametric mixing. In the present case, we use non-parametric mixing for panel multinomial logit regression across mortgage age. In Follmann and Lambert, the number of trials is fixed, whereas in our case the number of trials is determined by the number of sample years for each mortgage. Follman and Lambert argue that over-dispersion relative to the binomial distribution is possible if the trials are positively correlated, perhaps because an important covariate is omitted.

Multinomial Logit Model with Unobserved Heterogeneity

The probability functions for mass point j of the P distinct mass points in the finite mixture distribution of random effects are:

[2] In the second step, we estimate the coefficients for the common variables: FICO, loan size, multiple units dummy, DTI, and age dummies for prepayment. The coefficient for delinquency for those common variables equal the coefficient (β_1) from step 1 plus the coefficient for prepayment. For the coefficients of the variables which exist in only one of the delinquency or prepayment risks, we use the corresponding coefficient or negative of the corresponding coefficient from β_1 in step 1.

$$\pi_{pmt,j} = \frac{\eta_{p,j}\exp(z'_{pmt}\beta)}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)}$$
$$\pi_{dmt,j} = \frac{\eta_{d,j}\exp(z'_{dmt}\beta)}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)}$$
$$\pi_{cmt,j} = \frac{1}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)} \quad ,$$

where $\eta_{p,j}$ is the scale parameter associated with the risk of prepayment for the j^{th} mass point, and $\eta_{d,j}$ is the scale parameter associated with the risk of 90-day delinquency for the j^{th} mass point.

Then, the log-likelihood function for the mixed multinomial logit model is:

$$L(\beta, p, \eta) = \sum_{m=1}^{M} \log(\sum_{j=1}^{P} p_j (\prod_{t=1}^{T_m} (\pi_{pmt,j})^{\delta^p_{mt}} (\pi_{dmt,j})^{\delta^d_{mt}} (\pi_{cmt,j})^{\delta^c_{mt}}))$$
$$= \sum_{m=1}^{M} \log(\sum_{j=1}^{P} p_j (\prod_{t=1}^{T_m} (\frac{\eta_{p,j}\exp(z'_{pmt}\beta)}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)})^{\delta^p_{mt}}$$
$$(\frac{\eta_{d,j}\exp(z'_{dmt}\beta)}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)})^{\delta^d_{mt}}$$
$$(\frac{1}{1+\eta_{p,j}\exp(z'_{pmt}\beta)+\eta_{d,j}\exp(z'_{dmt}\beta)})^{\delta^c_{mt}})) \quad ,$$

where p_j is the probability of the j^{th} mass point.

The Transformed Multinomial Logit Model with Unobserved Heterogeneity

The probability functions for mass point j are:

$$\pi_{pmt,j} = \frac{\eta_{p,j}\exp(z'_{pmt}\beta)}{\exp(I^A_{mt,j})}(1 - \frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}})$$
$$\pi_{dmt,j} = \frac{\eta_{d,j}\exp(z'_{dmt}\beta)}{\exp(I^A_{mt,j})}(1 - \frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}})$$
$$\pi_{cmt,j} = \frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}} \quad ,$$

where

$$I^A_{mt,j} = \log(\eta_{p,j}\exp(z'_{pmt}\beta) + \eta_{d,j}\exp(z'_{dmt}\beta)).$$

The log-likelihood function has the form

$$L(\beta, p, \eta) = \sum_{m=1}^{M} \log(\sum_{j=1}^{P} p_j (\prod_{t=1}^{T_m} (\pi_{pmt,j})^{\delta^p_{mt}} (\pi_{dmt,j})^{\delta^d_{mt}} (\pi_{cmt,j})^{\delta^c_{mt}}))$$
$$= \sum_{m=1}^{M} \log(\sum_{j=1}^{P} p_j (\prod_{t=1}^{T_m} (\frac{\eta_{p,j}\exp(z'_{pmt}\beta)}{\exp(I^A_{mt,j})}(1 - \frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}}))^{\delta^p_{mt}}$$
$$(\frac{\eta_{d,j}\exp(z'_{dmt}\beta)}{\exp(I^A_{mt,j})}(1 - \frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}}))^{\delta^d_{mt}}$$
$$(\frac{1}{(1+c\exp(I^A_{mt,j}))^{1/c}})^{\delta^c_{mt}})) \quad .$$

1.4 Empirical Results

To clearly assess the effect of Ding-Tian-Yu-Guo transformation, we present the transformed and untransformed empirical results in a single table. Table 1.1 compares the untransformed multinomial logit model (MNL) with the transformed multinomial logit model, estimated respectively without unobserved heterogeneity in models 1 and 2 and with unobserved heterogeneity in models 3 and 4. Trussell and Richards (1985) suggest adding mass points until there is no significant increase in the log-likelihood. Using this approach, we stop at three mass points. We present the results of four mass points in the Appendix.

Table 1.1 shows that the likelihood increases from the untransformed model to the transformed model. Moreover, both forms of model are improved by incorporating unobserved heterogeneity, given the log likelihood increases so much.

The results for the coefficient estimates are uniform across models and consistent with the predictions of option theory. The probability of prepayment increases when the call option is positive (in the money); similarly higher negative equity increases the risk of 90-day delinquency. The results suggest that the behavior of borrowers may be affected by other factors. The estimates show that the 90-day delinquency risk is associated with a higher original loan-to-value ratio. The estimates also show that a higher unemployment rate will increase the 90-day delinquency risk. The debt-to-income ratio is positive in the 90-day delinquency risk and negative in the prepayment risk. This means that for the same monthly mortgage payment, a borrower with a higher income is more likely to prepay and less likely to become delinquent. Having more than one mortgage negatively affects the prepayment risk. A higher FICO score negatively affects both termination risks. The FICO coefficient estimate in the 90-day delinquency risk is 10 times larger than the corresponding coefficient estimate in the prepayment risk.

1.4.1 Comparison of the Specifications

The model with both unobserved heterogeneity and the Ding-Tian-Yu-Guo transformation have the largest absolute coefficient values and lowest p-values for several important variables, among them the negative equity variables and the unemployment rate in the delinquency risk, the call option in the prepayment risk, and the FICO score, the debt-to-income ratio, and the loan-to-value ratio in both risks. Moreover, the model controlling for unobserved heterogeneity does better than the model without controls for unobserved heterogeneity and the transformed model does better than the untransformed model. We conjecture that this is because mixing and transforming the model are alternative and complementary ways to control for over-dispersion. The model that is both mixed and transformed works best because the transformation controls for residual mixing not captured by the Trussell and Richards procedure. In Appendix we show that for any multinomial outcome, except for one special case, there exists a valuable transformation parameter that increases the variance of the outcome and therefore controls for over-dispersion.

The multinomial logit specifications with and without unobserved heterogeneity both have a larger log-likelihood with the Ding-Tian-Yu-Guo transformation than without it. McFadden, Train, and Tye (1977), Hausman and McFadden (1984), and McFadden (1987) develop tests for the IIA property. A Wald test for c equals 1 in a transformed model also tests IIA. The estimated value of the transformation parameter c is 5.765 for the transformed multinomial logit without unobserved heterogeneity and 3.980 for the transformed multinomial logit with unobserved heterogeneity. The null hypothesis that c equals 1 is rejected in the both cases at the 1 percent level.

1.4.2 Out-of-Sample Predictive Accuracy

For the cross-sample validation we use a method similar to that of Clapp, Deng, and An (2006), but instead of holding back a 10 percent sub-sample for validation, we hold back a 50 percent sub-sample. For both sub-samples we use the estimation results to predict the delinquency and prepayment probabilities in the final year for each mortgage. For each sub-sample and each risk we then regress the indicators for delinquency and prepayment on their respective predicted probabilities and compare R-squares.

Table 1.2 presents the results. The models with the Ding-Tian-Yu-Guo transformation provide a better fit than corresponding untransformed models for both risks. Moreover, the models without unobserved heterogeneity are out-performed by the corresponding unobserved heterogeneity specifications.

An alternative criterion is McFadden's Pseudo R-square, which compares an estimated unrestricted log-likelihood from a specification with both covariates and loan age dummies to an estimated restricted log-likelihood with loan age dummies only:

$$\rho = 1 - LL_U/LL_R \quad ,$$

where LL_U is the unrestricted log-likelihood and LL_R is the restricted log-likelihood. The Pseudo R-squares do not change very much across sub-samples within models. The results are presented in Table 1.3 and are qualitative similar to the results in Table 1.2. For each of the four models in-sample and out-of-sample Pseudo R-squares are similar with the preferred specification being the transformed multinomial logit with unobserved heterogeneity.

1.5 Conclusions

Ding, Tian, Yu, and Guo (2012) introduce transformed binary logit models that they apply to a univariate duration analysis of time to bankruptcy for banks. We extend the Ding-Tian-Yu-Guo transformation to cover multinomial logit duration models which we apply to time to prepayment or delinquency for Fannie Mae mortgages. We show that a Wald test on the transformation parameter provides a test of the multinomial logit specification, like tests previously developed by McFadden, Train and Tye (1977), Hausman and McFadden (1984), and McFadden (1987).

An additional contribution of our work is exploring the relationship between the Ding-Tian-Yu-Guo transformation and over-dispersion in the data due to unobserved heterogeneity. We show that except for one special case, there is a value of the transformation parameter for which the outcomes are over-dispersed relative to the multinomial logit model. Follmann and Lambert (1989) address over-dispersion by generalizing binary logistic regression using non-parametric mixing for a balanced panel. In the present case, we use non-parametric mixing in a transformed multinomial logit duration model, which results in an unbalanced panel.

We use Fannie Mae public use data for single-family, 30-year fixed-rate mortgages to analyze 90-day delinquency and prepayment risks. Our empirical results generally support the hypothesis that prepayment and delinquency can be treated as financial options.

Our empirical results clearly reject the standard multinomial logit specifications. Including corrections for unobserved heterogeneity, as in Deng, Quigley and van Order (2000), Pennington-Cross (2003), and Clapp, Deng, and An (2006), improve both transformed and untransformed specifications. Similarly, transforming the model improves the specifications with and without controls for unobserved heterogeneity. We conjecture that the model that is both mixed and transformed works best because the transformation controls for residual mixing that cannot be precisely measured.

1.6 Tables

Table 1.1: Multinomial Logit with and without Transformation
(standard errors in parenthesis)

Estimate	Model 1	Model 2	Model 3	Model 4
Delinquency				
Baseline Intercept (Group 1)			2.246	4.283
			(0.224)	(1.076)
Baseline Intercept (Group 2)			0.177	-0.783
			(0.106)	(1.164)
Baseline Intercept (Group 3)			-1.902	1.283
			(0.209)	(1.128)
Recession Indicator	1.017	1.248	0.923	1.142
	(0.108)	(0.107)	(0.114)	(0.125)
Negative Equity	4.405	5.767	6.425	7.292
	(0.287)	(0.308)	(0.362)	(0.305)
Negative Equity Square	-2.022	-2.838	-2.635	-3.286
	(0.214)	(0.228)	(0.227)	(0.229)
Negative Equity * Recession	-0.361	-0.038	-0.751	-0.426
	(0.222)	(0.221)	(0.255)	(0.265)
FICO	-1.057	-1.329	-1.461	-1.713
	(0.057)	(0.061)	(0.046)	(0.088)
Log Loan Size	1.926	0.303	0.970	-0.389
	(1.731)	(0.837)	(0.207)	(2.161)
Dummy Units>1	0.124	0.168	0.101	0.033
	(0.230)	(0.030)	(0.053)	(0.194)
Debt-to-Income	1.052	1.238	1.404	1.485
	(0.277)	(0.200)	(0.128)	(0.310)
Unemployment Rate	0.211	0.561	0.547	0.747
	(0.144)	(0.158)	(0.064)	(0.199)
Original Loan-to-Value	2.468	3.081	3.127	3.542
	(0.240)	(0.139)	(0.074)	(0.295)
Prepayment				
Baseline Intercept (Group 1)			-1.735	-1.296
			(0.240)	(0.538)
Baseline Intercept (Group 2)			-5.096	-2.576
			(0.244)	(0.550)
Baseline Intercept (Group 3)			-2.765	-6.121
			(0.220)	(0.857)
Call Option	5.822	8.275	7.144	8.929
	(0.174)	(0.307)	(0.043)	(0.320)
FICO	-0.059	-0.176	-0.163	-0.273
	(0.024)	(0.028)	(0.022)	(0.040)
Log Loan Size	3.088	4.540	1.835	2.749
	(0.575)	(0.690)	(0.293)	(0.809)
Dummy Units>1	-0.416	-0.599	-0.575	-0.733
	(0.105)	(0.131)	(0.030)	(0.144)
Debt-to-Income	-0.663	-0.699	-0.619	-0.619
	(0.112)	(0.132)	(0.111)	(0.167)
Mass Point (Group 1)			0.345	2.367
			(0.106)	(0.258)
Mass Point (Group 2)			-1.426	1.790
			(0.113)	(0.269)
c		5.765		3.980
		(0.494)		(0.345)
Log-Likelihood	-22181.48	-21974.54	-21947.47	-21922.38

Notes: The four models 1-4 are multinomial logit model, multinomial logit model with transformation, multinomial logit with unobserved heterogeneity, and multinomial logit model with transformation and unobserved heterogeneity.

Table 1.2: Cross-Sample Validation for the Four Models

Models	In Sample	Out of Sample
Delinquency		
Multinomial Logit	0.272	0.267
Transformed Multinomial Logit	0.282	0.275
Multinomial Logit With Unobserved Heterogeneity	0.293	0.285
Transformed Multinomial Logit With Unobserved Heterogeneity	0.295	0.289
Prepayment		
Multinomial Logit	0.042	0.054
Transformed Multinomial Logit	0.055	0.068
Multinomial Logit With Unobserved Heterogeneity	0.104	0.116
Transformed Multinomial Logit With Unobserved Heterogeneity	0.120	0.133

Table 1.3: McFadden Pseudo R-square Results

Models	In Sample	Out of Sample
Multinomial Logit	0.0601	0.0579
Transformed Multinomial Logit	0.0688	0.0684
Multinomial Logit With Unobserved Heterogeneity	0.0700	0.0694
Transformed Multinomial Logit With Unobserved Heterogeneity	0.0710	0.0706

1.7 Appendix

1.7.1 Explanatory Variables

A description of the explanatory variables follows.

Original Loan-to-Value Ratio (LTV): Many mortgages require a minimum down payment. The loan-to-value ratio is the loan amount divided by property value at origination. A higher LTV will increase the risk of 90-day delinquency since it means a lower down payment, all else equal.

Original Debt-to-Income Ratio (DTI): Banks may require a debt-to-income ratio below a certain level. We expect that a higher DTI will cause the risk of 90-day delinquency to increase and the risk of prepayment to decrease.

Original Fair Isaac Corporation Score (FICO): This is the borrower credit score at origination. It is a number between 300 and 850, and it is used to evaluate the quality of borrower credit. We expect that a higher FICO score will have a negative effect on the risk of 90-day delinquency.

Call Option Value: This is the value of the option to prepay. It is computed as the ratio of the difference between the market value of the mortgage and the book value of the mortgage to the market value of the mortgage.

$$Call\,Option_t = \frac{Market\,Value_{tj} - Book\,Value_{tj}}{Market\,Value_{tj}} \quad .$$

Market value and book value are calculated as follows:

$$Market\,Value_t = \sum_{t=1}^{360-t} \frac{Monthly\,Payment}{(1+market\,rate/12)^i}$$
$$Book\,Value_t = \sum_{t=1}^{360-t} \frac{Monthly\,Payment}{(1+contract\,rate/12)^i} \quad .$$

In calculating book value, the contract rate is the interest rate at origination, provided by the Fannie Mae dataset. The market interest rate in a given calendar year is calculated as the average note rate of mortgages originated in that year. If the call option variable is positive, the market value is greater than the book value and the probability of prepayment increases.

Negative Equity: This is an important determinant of default risk. The functions used to calculate this are:

$$X_i = \text{ current property value}_i \text{ - remaining balance}_i$$
$$\text{negative equity}_i = \begin{cases} \text{absolute value of } X_i & \text{if } X_i < 0 \\ 0 & \text{if } X_i \geq 0 \end{cases} .$$

The current property value is calculated as following:

$$\text{current property value}_i = \left(\frac{\text{Case Schiller index}_i}{\text{Case Schiller index}_0}\right) \times \text{ original property value} \quad ,$$

where Case-Schiller index_0 is the Case-Shiller index at the origination date of the mortgage and Case-Schiller index_i is the Case-Shiller index at year i. The remaining balance at month i is provided in the data. If the current property value is less than the remaining balance, it is more likely that the borrower will default. So we expect a positive effect of negative equity on 90-day delinquency.

Unemployment Rate: This is the annual unemployment rate for Miami. The higher the unemployment rate in Miami, the higher should be the risk of 90-day delinquency.

Original Loan Size: This is the original loan amount. We expect that the larger the loan amount, the less likely is prepayment, and the more likely is delinquency.

Table 1.4 presents the definitions of our explanatory variables, and Table 1.5 presents the means and standard deviations.

Table 1.4: Definition of Explanatory Variables

Variables	Definition
Call Option	(Market Value-Book Value)/Book Value.
Negative Equity	The absolute value of current property value minus current unpaid principal if the current unpaid principal is greater than the current property value, and zero otherwise. The variable is divided by a hundred thousand in the estimation.
Recession Indicator	The recession indicator equals 1, when the calendar year is 2008, 2009, or 2010.
Original FICO Score	The score has a minimum value of 300 and a maximum value of 850. The variable is divided by a thousand in the estimation.
Log Loan Size	Log of the original loan amount. The variable is divided by 10 in the estimation.
Original Debt-to-Income	Monthly Payment/Stable Monthly Income.
Original Loan-to-Value	The original loan amount over the value of the mortgaged property.
Unemployment Rate.	The annual unemployment rate of Miami. The variable is divided by a hundred in the estimation
Dummy Units > 1	The number of units comprising the related mortgaged property. If the number of units is greater than 1, the dummy equals 1, and zero otherwise.
Loan Age Dummy 1 to 16	The loan age dummy i equals 1, when loan age in years equals i. The variables are multiplied by 10 in the estimation.

Table 1.5: Descriptive Statistics on Mortgage Loans

Variables	Mean	Standard Deviation
Call Option	0.0377	0.0883
Negative Equity	7,210.28	22,021.01
Recession Indicator	0.1767	0.3813
Original FICO	724.40	56.36
Loan Size	172,007.34	92,202.62
Original Debt-to-Income	35.81	12.33
Original Loan-to-Value	72.76	16.16
Unemployment Rate	5.773	2.362
Dummy Units>1	0.0195	0.1381
Loan Age 1	0.2220	0.4158
Loan Age 2	0.1921	0.3939
Loan Age 3	0.1423	0.3493
Loan Age 4	0.1030	0.3039
Loan Age 5	0.0785	0.2690
Loan Age 6	0.0622	0.2416
Loan Age 7	0.0487	0.2153
Loan Age 8	0.0377	0.1906
Loan Age 9	0.0300	0.1706
Loan Age 10	0.0236	0.1517
Loan Age 11	0.0169	0.1289
Loan Age 12	0.0130	0.1132
Loan Age 13	0.0105	0.1018
Loan Age 14	0.0084	0.0912
Loan Age 15	0.0064	0.0799
Loan Age 16	0.0045	0.0671
Num. Observations		49,799

Table 1.6: Descriptive Statistics on Mortgage Loans

	Prepayment		90-day Delinquency	
	Total	Call-Option> 0	Total	Neg Equity> 0
Loan Age 1	754	441	103	55
Loan Age 2	1,707	1,302	191	122
Loan Age 3	1,224	925	204	152
Loan Age 4	700	445	148	108
Loan Age 5	460	322	121	83
Loan Age 6	336	255	87	59
Loan Age 7	263	236	65	34
Loan Age 8	246	242	44	17
Loan Age 9	223	221	23	8
Loan Age 10	225	224	20	1
Loan Age 11	146	146	13	0
Loan Age 12	88	88	13	0
Loan Age 13	73	73	9	0
Loan Age 14	50	50	2	0
Loan Age 15	36	36	6	0
Loan Age 16	22	22	2	0
Num. Observations				49,799

Table 1.7: Descriptive Statistics on Mortgage Loans for Validation Sample

Variables	Mean	Standard Deviation
Call Option	0.0393	0.0897
Negative Equity	6,829.91	2,0769.93
Recession Indicator	0.1767	0.3813
Original FICO	724.33	57.00
Loan Size	169,560.21	91,380.94
Original Debt-to-Income	36.04	12.38
Original Loan-to-Value	72.29	16.51
Unemployment Rate	5.789	2.364
Dummy Units>1	0.0169	0.1290
Loan Age 1	0.2186	0.4133
Loan Age 2	0.1893	0.3917
Loan Age 3	0.1398	0.3468
Loan Age 4	0.1031	0.3041
Loan Age 5	0.0795	0.2706
Loan Age 6	0.0640	0.2448
Loan Age 7	0.0497	0.2173
Loan Age 8	0.0387	0.1930
Loan Age 9	0.0309	0.1730
Loan Age 10	0.0239	0.1527
Loan Age 11	0.0177	0.1319
Loan Age 12	0.0137	0.1164
Loan Age 13	0.0111	0.1045
Loan Age 14	0.0090	0.0945
Loan Age 15	0.0064	0.0800
Loan Age 16	0.0045	0.0670
Num. Observations		49,676

Table 1.8: Descriptive Statistics on Mortgage Loans for Validation Sample

	Prepayment		90-day Delinquency	
	Total	Call-Option> 0	Total	Neg Equity> 0
Loan Age 1	694	377	108	63
Loan Age 2	1,755	1,310	198	118
Loan Age 3	1,184	921	201	138
Loan Age 4	664	454	142	102
Loan Age 5	449	282	111	70
Loan Age 6	362	242	105	64
Loan Age 7	285	262	76	44
Loan Age 8	240	236	57	22
Loan Age 9	248	245	33	13
Loan Age 10	212	211	26	6
Loan Age 11	149	149	14	1
Loan Age 12	100	99	8	0
Loan Age 13	66	66	10	0
Loan Age 14	61	61	4	0
Loan Age 15	40	40	2	0
Loan Age 16	18	18	0	0
Num. Observations				49,676

1.7.2 Theorem 1

Theorem 1. *Suppose we have a transformed multinomial logit model with J outcomes or choices where $\beta_J = 0$. Let Y_j, $j = 1, ..., J-1$ be the indicator variable for outcome or choice j. Define $\rho_j = \exp(z_j\beta)$ and $S = \sum_{k=1}^{J-1} \exp(z_k\beta)$. Then, for each j, there exists a transformation parameter value c such that the value of var(Y_j) increases above its value in the corresponding untransformed model (the case where $c = 1$), except when $\frac{\rho_j}{1+S} = \frac{1}{2}$.*

Proof. With or without the Ding-Tian-Yu-Guo transformation in a multinomial logit model, the probability function for the indicator variable, Y_j, $j = 1, ..., J-1$, is the same as for the binomial indicator variable:

$$P(Y_j = y_j) = \pi_j^{y_j}(1-\pi_j)^{1-y_j} \quad .$$

The variance of Y_j is $\pi_j(1-\pi_j)$. We now show that for each j, there exists a value of c such that the variance of Y_j in a transformed multinomial logit has a larger variance than the corresponding Y_j in an untransformed model.

Let σ_{jc}^2 be the variance in a transformed model, and σ_j^2 be the variance in an untransformed model. Then $f_j = \sigma_{jc}^2 - \sigma_j^2$ is equivalent to

$$f_j = [(1 - \frac{1}{(1+cS)^{1/c}})\frac{\rho_j}{S}][1 - (1 - \frac{1}{(1+cS)^{1/c}})\frac{\rho_j}{S}] - (\frac{\rho_j}{1+S})(\frac{1+S-\rho_j}{1+S}) \quad , \tag{1}$$

where $\rho_j = \exp(z_j\beta)$ and $S = \sum_{k=1}^{J-1} \exp(z_k\beta)$. Define $l_j = (1 - \frac{1}{(1+cS)^{1/c}})\frac{\rho_j}{S}$. Equation (1) can now be rewritten as:

$$f_j = l_j(1-l_j) - (\frac{\rho_j}{1+S})(\frac{1+S-\rho_j}{1+S}) \quad . \tag{2}$$

Taking the first derivative with respect to c, we get

$$\frac{\partial f_j}{\partial c} = \frac{\partial l_j}{\partial c} - 2l_j\frac{\partial l_j}{\partial c} = (1-2l_j)\frac{\partial l_j}{\partial c} \quad . \tag{3}$$

Define $g = \frac{1}{(1+cS)^{1/c}}$. Then

$$\frac{\partial g}{\partial c} = \frac{\partial \log(g)}{\partial c} g = \frac{\partial(-\frac{1}{c}\log(1+cS))}{\partial c}\frac{1}{(1+cS)^{1/c}} = (\frac{1}{c^2}\log(1+cS) - \frac{1}{c}(\frac{S}{1+cS}))\frac{1}{(1+cS)^{1/c}} \quad , \tag{4}$$

and

$$\frac{\partial l_j}{\partial c} = -\frac{\rho_j}{S}\frac{\partial g}{\partial c} = -\frac{\rho_j}{S}(\frac{1}{c^2}\log(1+cS) - \frac{1}{c}(\frac{S}{1+cS}))\frac{1}{(1+cS)^{1/c}} \quad . \tag{5}$$

Plugging equation (5) into equation (3) yields:

$$\frac{\partial f_j}{\partial c} = -(1-2l_j)\frac{\rho_j}{S}\frac{1}{c^2}(\log(1+cS) - \frac{cS}{1+cS})\frac{1}{(1+cS)^{1/c}} \quad . \tag{6}$$

It is straightforward to show that $\log(1+cS) - \frac{cS}{1+cS} > 0$. Therefore, we have an internal solution only if $l_j = \frac{1}{2}$, which in turn requires that $\frac{\rho_j}{S} > \frac{1}{2}$. Moreover,

$$\frac{\partial^2 f_j}{\partial c^2} = \frac{\partial^2 l_j}{\partial c^2} - 2(\frac{\partial l_j}{\partial c})^2 - 2l_j\frac{\partial^2 l_j}{\partial c^2} = -2(\frac{\partial l_j}{\partial c})^2 < 0 \tag{7}$$

at $l_j = \frac{1}{2}$.

Denote the maximum value of f_j as f_j^*. Then

$$f_j^* = \begin{cases} > 0, & \frac{\rho_j}{1+S} \neq \frac{1}{2} \\ = 0, & \frac{\rho_j}{1+S} = \frac{1}{2} \end{cases} \quad . \tag{8}$$

To see this, substitute $l_j = \frac{1}{2}$ into equation (2):

$$\begin{aligned} f_j &= \frac{1}{2}(1-\frac{1}{2}) - (\frac{\rho_j}{1+S})(\frac{1+S-\rho_j}{1+S}) \\ &= (\frac{\rho_j}{1+S})^2 - (\frac{\rho_j}{1+S}) + \frac{1}{4} \end{aligned} \quad . \tag{9}$$

We want to show that when $\frac{\rho_j}{S} \leq \frac{1}{2}$, there always exits a value of c for which $f_j > 0$. For this, we need to demonstrate two facts: (1) $\frac{\partial f_j}{\partial c} < 0$ for $\frac{\rho_j}{S} \leq \frac{1}{2}$, and (2) $f_j > 0$

as c approaches 0 from the right. For the first fact, note that all of the factors in $\frac{\partial f_j}{\partial c}$ in equation (6) are positive, except $-(1-2l_j)$. But this factor is negative because

$$1 - 2l_j = 1 - 2\left(1 - \frac{1}{(1+cS)^{1/c}}\right)\frac{\rho_j}{S} > 1 - 2\frac{\rho_j}{S} > 0 \quad . \tag{10}$$

To see the second fact, note that $\frac{\partial f_j}{\partial c} < 0$ implies that $f_j(c) > 0$ for c in the interval $(0, 1)$, because the transformed and standard multinomial logit models are equivalent when $c = 1$. $\square$

1.7.3 The Results of Four Mass Points

Table 1.9: Multinomial Logit with and without Transformation
(standard errors in parenthesis)

Estimate	Model 1	Model 2
Delinquency		
Baseline Intercept (Group 1)	3.301	6.596
	(0.242)	(0.694)
Baseline Intercept (Group 2)	0.302	4.126
	(0.263)	(0.635)
Baseline Intercept (Group 3)	0.967	-2.338
	(0.253)	(2.182)
Baseline Intercept (Group 4)	-7.859	0.832
	(0.145)	(0.643)
Recession Indicator	0.942	1.325
	(0.114)	(0.176)
Negative Equity	6.714	8.750
	(0.389)	(0.738)
Negative Equity Square	-2.749	-3.973
	(0.290)	(0.432)
Negative Equity * Recession	-0.727	-0.433
	(0.271)	(0.334)
FICO	-1.526	-2.132
	(0.067)	(0.186)
Log Loan Size	-0.039	2.192
	(0.298)	(1.132)
Dummy Units>1	0.004	0.095
	(0.142)	(0.151)
Debt-to-Income	1.402	2.109
	(0.127)	(0.500)
Unemployment Rate	0.582	0.823
	(0.169)	(0.222)
Original Loan-to-Value	3.248	4.719
	(0.233)	(0.541)
Prepayment		
Baseline Intercept (Group 1)	-1.594	-4.203
	(0.175)	(1.606)
Baseline Intercept (Group 2)	-5.219	-0.708
	(0.375)	(0.299)
Baseline Intercept (Group 3)	-2.579	-2.375
	(0.286)	(0.284)
Baseline Intercept (Group 4)	-2.839	-6.297
	(0.285)	(0.477)
Call Option	7.176	10.46
	(0.105)	(0.637)
FICO	-0.178	-0.312
	(0.018)	(0.048)
Log Loan Size	2.024	2.469
	(0.424)	(0.534)
Dummy Units>1	-0.595	-0.871
	(0.120)	(0.176)
Debt-to-Income	-0.602	-0.711
	(0.126)	(0.199)
Mass Point (Group 1)	0.572	-0.441
	(0.182)	(0.426)
Mass Point (Group 2)	-1.049	2.445
	(0.149)	(0.202)
Mass Point (Group 3)	0.057	1.690
	(0.364)	(0.255)
c		5.473
		(0.399)
Log-Likelihood	-21944.80	-21909.46

Chapter 2: A Dirichlet Nested Logit Model of Household Mortgage Payment in the CARES Act Forbearance Program

2.1 Introduction

To reduce the risk of widespread foreclosure, Congress passed the Coronavirus Aid, Relief, and Economic Security Act (CARES Act) on March 27, 2020. The CARES Act Mortgage Forbearance program allows borrowers to postpone mortgage payments without penalty. As of May 9, 2022, about 8.2 million borrowers with $1.7 trillion in mortgage loans entered the forbearance program (An, Cordell, Geng, and Lee, 2022). As the economy emerges from the pandemic, a key problem for economists and policymakers is understanding how borrowers leave the CARES forbearance program and resume regular payments.

To understand how borrowers leave the CARES forbearance program, we must first understand the composition of borrower risk types entering the program. In principle, the standard economic approach would be to target the forbearance program to those truly experiencing a Covid-19 economic hardship. But according to the legislation no documentation is required. Mortgagors may have entered the program as a way to defer payments for reasons other than hardship. Indeed, Anderson, Harrison, and Seiler (2022) show that borrowers who entered the program may not have experienced a hardship associated with or due to Covid-19, and Farrell, Greig, and Zhao (2020) show that many borrowers in the forbearance program continued making all their payments. This suggests that there is substantial heterogeneity in risk type among CARES enrollees.

This heterogeneity is generally unobservable to the econometrician, which raises fundamental issues in estimation. Borrowers in the forbearance program must choose how much of their budget to allocate to mortgage payments each month: not making a payment, making a curtailment payment (over-payment), or paying a scheduled monthly payment. A borrower's decision on payment behavior is a discrete choice. The binary or multinomial logit model is a commonly used framework for analyzing mortgage performance (e.g., Capozza, Kazarian, and Thomson, 1997; Follain, Ondrich, and Sinha, 1997; Archer, Elmer, Harrison, and Ling, 2002; Ding, Quercia, Li, and Ratcliffe, 2011; Fitzpatrick and Mues, 2016). The traditional multinomial logit model generates mean response probabilities based solely on the values of exogenous variables without information about higher moments of the distribution of the probabilities among individuals. This may lead to bias in panel data because of unobserved heterogeneity. Accounting for unobserved heterogeneity by mass-point corrections in the analysis of mortgage performance (e.g., Deng, Quigley, and van Order, 2000; Pennington-Cross, 2003; Clapp, Deng, and An, 2006; and An, Lin, Ondrich, 2022) increases in difficulty as the number of choices grows. This chapter proposes a new discrete choice estimator.

Specifically, to solve the unobserved heterogeneity problem in the forbearance program, higher moments of the distribution of borrower payment behavior probabilities are needed, which allows for the identification of groups of borrowers by their distribution of response probabilities. To deal with quantal response problems in panel data from heterogeneous observations, Heckman and Willis (1977) assumes that the distribution of response probabilities is beta distributed. Because there are more than two choices for borrower payment in the forbearance program, this chapter extends the beta-logistic model of Heckman and Willis to the Dirichlet multinomial logit model. The new estimator allows state dependence to vary across individuals and time. However, the probability of each type of payment depends only on the number of such payments and the total number of payments of all types. To relax this restriction, this chapter combines the Dirichlet distribution with a nested logit model to allow different observed payment types to be more closely related than others. We call this model the Dirichlet nested logit model.

This chapter applies the estimator to rich panel data on the mortgage payment behavior of borrowers enrolled in the CARES program from the public-use Fannie Mae Credit Insurance Risk Transfer (CIRT)/Connecticut Avenue Securities (CAS) data set. The sample consists of 68,313 loans in the CARES Act Mortgage forbearance program from March, 2020, to March, 2022. The data include characteristics and financial conditions of borrowers and mortgages, such as the FICO credit score, loan-to-value ratio, loan amount, mortgage rate, debt-to-income ratio, year, occupancy status, and property location (three-digit ZIP code), as well as time-varying information on loan performance, such as current unpaid balance, payment status, forbearance indicator, and servicer and seller names. In contrast to other loan-level mortgage-backed securities data, the CIRT data include the current FICO score. Finally, the data provide forbearance information, including forbearance start date, exit date, and exit type.

Borrowers can exit the forbearance program in one of five ways: reinstatement, repayment plan, payment deferral, modification, or prepayment. Under reinstatement, the borrower pays the forborne amount before they exit forbearance; a repayment plan enables the borrower to pay off the forborne amount over a period of time; payment deferral is when the forborne balances are placed into a balloon loan payable at the liquidation date of the loan; loan modification reduces the size of the monthly payment by extending the term of the loan or reducing the mortgage rate; finally, the exit is defined as prepayment if the borrower pays off the entire remaining balance, generally by refinancing the mortgage. As of March, 2022, 23,775 loans (34.8 percent of all forborne loans in the sample) had exited forbearance, 12,502 loans (18.3 percent) had exited by reinstatement, 690 loans (1.01 percent) had exited by repayment plan, 19,237 loans (28.1 percent) had exited by payment deferral, and 3,782 loans (5.54 percent) had exited by trial/modification.

Following Smith and Lawrence (1995), who show that recent payment history predicts transitions out of delinquency, a key feature of my approach is that probability of borrower payment behavior at time t is predicted not only by a function of exogenous variables but also by the previous payment behavior. Therefore, we allow borrowers with more non-payment during forbearance to be more likely to exit the program with a payment deferral or trial/modification, and borrowers who always make scheduled or curtailment payments to be more likely to exit with a prepayment or a reinstatement. I use a two-stage estimation technique. In the first stage a Dirichlet multinomial logit model and a Dirichlet nested logit model are used to calculate borrower payment behavior in the termination period. In the second stage the calculated predictive probabilities from the first stage are used as controls in a multinomial logit model for the choice of exit type in the termination period.

There are four primary findings from the empirical analysis. First, the empirical results of the Dirichlet multinomial logit model and the Dirichlet nested logit model demonstrate considerable unobserved heterogeneity in borrower payment behavior. Furthermore, after dealing with the unobserved heterogeneity problem, the accuracy rates for my models are about twice as high as those for the traditional multinomial logit model. Second, borrowers with a low FICO score, a high loan-to-value ratio at entry into the program, and a high original debt-to-income ratio are less likely to make payments while in the forbearance program. Third, borrowers with a higher probability of making curtailment payments are more likely to exit forbearance with prepayment or reinstatement. In comparison, the probability of payment deferral and trial/modification increases with the predictive probability of not paying. Finally, after separating borrowers into two groups by the estimated mean probability of not making payment, I find that the estimated effect of the predictive probabilities of payment type on the probability of forbearance exit type changes among groups. The estimated marginal effect of a 1 percent increase in the probability of not paying on the probability of trial/modification is more than five times higher for the borrowers more likely to not pay. This suggests that the effect of the probability of not paying increases nonlinearly. On the other hand, the probability of not paying has a smaller estimated marginal effect on the probability of payment deferral for the borrowers more likely to not pay. This is interesting because trial/modification requires documentation of hardship, while payment deferral does not. It may be the case that estimating a strong effect of the predictive probability of not paying requires the non-payment to be due to actual hardship rather than due to precautionary motives. These results confirm the importance of using payment behavior in the forbearance program to predict the exit type.

From a policy perspective, there is a trade-off between providing relief immediately and providing relief efficiently. Immediate relief helps borrowers who have hardship caused by Covid-19 to begin forbearance and reduce the risk of foreclosure. Efficient relief requires a separating mechanism to determine borrower risk types. Exploring the economic implications of this policy, Farrel, Greig, and Zhao (2020), who use checking account balance data to control the changes in income argue that there is little evidence of significant moral hazard, since borrowers with non-payment in the forbearance program are found to have had larger drops in income. On the other hand, Anderson, Harrison, and Seiler (2022) suggest strategic mortgage forbearance can be significantly reduced by requiring a 1-page attestation. Those papers focus on policy implications on how people enter the program. This chapter looks into how borrowers exit the forbearance program since the success of the mortgage forbearance policies will depend on the results of exit options. The distribution of borrower payment behavior probabilities is used as a separating mechanism to determine borrower risk type.

The remainder of the chapter is organized as follows. In section 2.2, a simple model of sequential payment behavior in the forbearance program during the pandemic is presented. In section 2.3, this chapter assumes that payment behavior probabilities are governed by a Dirichlet distribution. Under a plausible parameterization of this distribution, I derive a likelihood function for sequential payment behaviors for borrowers, which reduces to the likelihood function of the conventional multinomial logit and nested logit model in the case of cross-section data. Section 2.4 presents the empirical analysis. There is a brief conclusion.

2.2 Empirical Framework

2.2.1 Background

The unemployment rate in the United States increased from 4.4 percent in March, 2020, to 14.7 percent in April, 2020, as a result of the Covid-19 pandemic. Many borrowers who lost their jobs or faced income reduction experienced difficulty paying their mortgages. Moreover, individuals were not uniformly impacted by the Covid-19 pandemic. Indeed, many studies showed that the Covid-19 pandemic has had a stronger impact on minority and lower-income groups (see, e.g., Chakrabarti and Nober 2020; van Dorn, Cooney, and Sabin 2020; Polyakova, Kocks, Udalova, and Finkelstein 2020; An et al. 2022).

To reduce the risk of widespread foreclosure, Congress passed the Coronavirus Aid, Relief, and Economic Security Act (CARES Act) on March 27, 2020. Section 4022 of this Act expanded the use of forbearance on federally backed loans to assist mortgagors during the Covid-19 pandemic.[3] The new law enabled borrowers, regardless of delinquency status, to forbear mortgage payments while "...experiencing a financial hardship during the Covid-19 emergency." Servicers only required a borrower's attestation to a financial hardship caused by the Covid-19 emergency to begin forbearance. The initial forbearance period was 180 days and could be extended to one full year. The FHFA extended the forbearance period to a maximum of 18 months. The law offers no requirements or a path

[3] Federally backed loans involve mortgages that are insured or guaranteed by federal agencies, including the Department of Housing and Urban Development (HUD) through the Federal Housing Administration (FHA), the Department of Veterans Affairs (VA), and the U.S. Department of Agriculture (USDA). Additionally, CARES Act also offers relief for borrowers with mortgages backed by government-sponsored enterprises (GSEs), including Fannie Mae and Freddie Mac. The CARES Act covered mortgages representing about 75 percent of all mortgages (Pendleton 2021).

for the borrower to exit forbearance.

On May 13, 2020, the FHFA announced payment deferral as a new repayment plan option for borrowers to exit forbearance programs when they are able to make their originally scheduled mortgage payment. Under the payment deferral option, the borrower's forborne payments are placed into a zero interest bearing account payable when the loan is refinanced or the home is sold. The FHFA announcement lists the hierarchy of repayment plan options for the borrower to exit forbearance.

- Reinstatement requires the borrower to pay all forborne amounts with or before the next scheduled payment. This option is ideal for borrowers who continued to make payments while in forbearance or borrowers who have cash available to repay the forborne amount.
- Repayment plan enables the borrower to pay off the forborne amount over a period of time, perhaps 6 or 12 months. This option works for borrowers who cannot immediately pay off their forborne amount yet are able to make their regularly scheduled payment plus a fraction of the forborne amount. For example, if the repayment plan is for 9 months, the borrower would pay 1/9th of the forborne balance plus their regularly scheduled mortgage payment for 9 months.
- Payment Deferral is when the forborne balances are placed into a balloon loan payable at the liquidation date of the loan. Before a borrower with a payment deferral can refinance, they must make three full payments after the effective date of the payment deferral. The balloon loan could either be rolled into the new loan or paid off.
- Loan Modification is available for borrowers who are unable to make their regularly scheduled monthly mortgage payment. Borrowers who need a loan modification will make three trial payments before the loan is permanently modified. Note that forborne balances are first capitalized into the loan and then the rate on the loan is reduced (if currently above the market rate) and the term of the loan is extended to

40 years in an effort to reduce the size of the monthly payment.

Borrowers can also choose to exit forbearance with prepayment, a termination event, which requires the borrower to pay off the entire remaining balance, generally by refinancing the mortgage. The timing of when the borrower exits forbearance and the exit choice are influenced by the financial strength of the borrower. Some borrowers' finances during the pandemic benefited from three rounds of direct government stimulus payments totaling in excess of $850 billion. For borrowers out of work, unemployment benefits were enhanced during the pandemic by three temporary programs in the CARES Act (Federal Pandemic Unemployment Compensation (FPUC), Pandemic Unemployment Assistance (PUA), and Pandemic Emergency Unemployment Compensation (PEUC) Programs). The pandemic temporarily modified household consumption patterns where expenditures outside of the house (e.g., travel, entertainment, eating out) were drastically reduced while other expenses increased (e.g., home office expenses). Many households paid off credit card debt and/or added household savings. Real disposable income after March, 2020, increased dramatically and remained above the pre-Covid level for two years. In addition to the fiscal stimulus, the Federal Reserve embarked on a historic level of intervention.

Clarida, Duygan-Bump, and Scotti (2021) discuss the response of the Federal Reserve to the pandemic. The efforts of the Federal Reserve directly created the environment for mortgage interest rates to decline to historically low levels by December, 2020. The low mortgage interest rate environment in conjunction with the flexibility to close loans virtually without weakening underwriting standards increased the value of the borrower's option to refinance.

2.2.2 A Model of Sequential Borrower Payment Behavior and the Problem of Heterogeneity

A borrower must choose how much money to allocate to mortgage payments each month: not making payments, paying the scheduled monthly payment, or making curtailment payments.[4] Borrowers' decisions on making payments are a function of socio-economic characteristics (e.g., income, FICO score, age), loan characteristics (e.g., loan amount, loan-to-value ratio, note rate), and financial market conditions (e.g., unemployment rate).

The stochastic utility of person m choosing alternative j at month t is specified as

$$U_{jmt} = V_{jmt} + S_{jmt}, \quad j = 1, 2, 3,$$

where U_{jmt} is the utility that the decision-maker actually obtains while in forbearance, V_{jmt} is the deterministic utility (the utility based on observed variables), S_{jmt} are error terms. The decision maker's choice maximizes utility. In the case of a single cross-section for each individual, McFadden (1974) assumes the S_{jmt} is distributed independently, identically standard extreme value. The associated likelihood function is the conditional logit likelihood.

With a panel dataset, however, the assumption that the error term, S_{jmt}'s, are statistically independent across time is not true empirically. To show this, I look at the subset of $5,224$ borrowers who left the forbearance program after six months. In the first month of the forbearance program, 55 percent of borrowers chose to not pay the payment. Assuming independence, I expect that $(0.55)^2 = 0.303$ of borrowers would not make payments in either of the first two months. However, the number of mortgagors who actually did not make payments in either of the first two months was 43 percent.

[4]Paying less than the scheduled monthly payment is also defined as the choice of not making payments, and most borrowers simply choose not to pay instead of making partial payments.

The difference between predicted and observed behavior might be explained by the unobserved variables. The unobserved factors that affect decision-makers are work status, health situation, attitude toward risk, age, gender, and so on. For example, researchers are not able to observe borrowers' job status, and they may lose their job due to Covid-19. This job loss effect would not only affect the current period but also have a continuous future impact on the borrower. A borrower, who does not pay a payment because of job loss during Covid-19, are also less likely to pay future payments while in the forbearance program. Heterogeneity implies that the sequential payment behavior of any individual within a group of observationally identical borrowers differs from the average behavior of the group. In cross-section data it is not possible to distinguish the heterogeneous case from the homogeneous case because the average probability is invariant among individuals. In a given month, the expected fraction of borrowers who make their regular payments is simply the average of the probabilities of making regular payments in the population.

Following a convention in the analysis of unobserved variables, this chapter follows the structure in Heckman and Willis (1977) to decompose the S_{jmt} for m^{th} borrower in year t into a "permanent component," σ_{jm}, and a "transitory component," ϵ_{jmt}. The S_{jmt} is given by

$$S_{jmt} = \sigma_{jm} + \epsilon_{jmt} \tag{11}$$

where the ϵ_{jmt} is distributed independently and identically. This chapter assumes the σ_{jm} are serially independent and independent of ϵ_{jmt}. This existence of unobserved permanent components causes borrowers who are homogeneous in terms of their observed characteristics to be heterogeneous in their payment behavior probabilities.

It is plausible to assume a multinomial distribution for the payment probabilities. The probability that a given borrower does not make a payment is given by π_1; the probability that a given borrower makes a curtailment payment is π_2; and the probability that a given borrower makes a regular payment is $\pi_3 = 1 - \pi_1 - \pi_2$.[5] Then, the probability that a given borrower does not make a payment for x_1 months, makes a curtailment payment for x_2 months, and makes a regular payment for x_3 months out of T months in the forbearance program is given by:

$$p(x_1, x_2, x_3, T) = \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}(1 - \pi_1 - \pi_2)^{x_3} \ . \tag{12}$$

The expected fraction of borrowers with payment behavior $(x_1 \ x_2 \ x_3)$ is given by [6]

$$E[p(x_1, x_2, x_3, T)] = \int \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}(1 - \pi_1 - \pi_2)^{x_3} f(\boldsymbol{\pi})d\boldsymbol{\pi} \ . \tag{13}$$

The expectation $E[p(x_1, x_2, x_3, T)]$ for a group of individuals is not necessarily equal to $p(x_1, x_2, x_3)$. Specifically, the distribution of months of borrower payment behavior in the heterogeneous population will tend to have fatter tails than a multinomial distribution. For example, the probability that a representative borrower did not pay in each of T months is $E(\pi_1)^T$, and the fraction of borrowers who did not pay the payment in each of T months is

$$E(\pi_1^T) = \int_0^1 \pi_1^T f(\boldsymbol{\pi})d\boldsymbol{\pi} \tag{14}$$

Since π_1^T is a convex function of π_1 for $T > 1$, it follows from Jensen's inequality that $E(\pi_1^T) > E(\pi_1)^T$. So the fraction of borrowers who not pay the payment for T months is greater than the probability that the representative borrowers will not pay the payments for T months.

[5] In year t, the probabilities of choosing choice j of the m^{th} borrower can be written as

$$\pi_{jmt} = P(U_{jmt} > U_{rmt}, \forall r \neq j) = P(V_{jmt} + \sigma_{jm} - (V_{rmt} + \sigma_{rm}) > \epsilon_{rmt} - \epsilon_{jmt}, \forall r \neq j) \ .$$

Assuming V_{jmt} for all j are constant over time, and ϵ_{jmt} is serially independent, rewrite this probability as

$$\pi_{jm} = P(V_{jm} + \sigma_{jm} - V_{rm} - \sigma_{rm} > \epsilon_{rmt} - \epsilon_{jmt}, \forall r \neq j) \ ,$$

where $\pi_{jm} = \pi_{jmt}$ for all t.

[6] It is important to point out that the assumption of stationarity is needed to derive equation (13). The reason for this is that $f(\boldsymbol{\pi})$ must be stable over time.

In the next section the model of borrower payment behavior shows that the conditional probability of remaining in a given state tends to increase the longer the mortgage has been in that state when the population is heterogeneous. Thus, for example, the conditional probability that a borrower does not pay increases the more frequent the non-payment.

2.3 Models

2.3.1 The Dirichlet Multinomial Logit Model

The Dirichlet distribution for a random vector [π_1,..., π_K] has probability density function

$$D(\pi_1, ..., \pi_K) = \frac{1}{B(\boldsymbol{\alpha})}\pi_1^{\alpha_1 - 1}...\pi_K^{\alpha_K - 1} \tag{15}$$

over the unit simplex. The parameter vector $\boldsymbol{\alpha} = (\alpha_1, ...\alpha_K)$ has positive components, $B(\boldsymbol{\alpha}) = \frac{\Gamma(\alpha_1)...\Gamma(\alpha_K)}{\Gamma(\alpha_1 + ... + \alpha_K)}$ is the beta function, where $\Gamma(z)$ is the gamma function and $\Gamma(z) = \int_0^1 t^{z-1}e^{-t}dt$.

The Dirichlet distribution is an attractive choice of functional form for the responding probability (π_1 π_2 π_3) for several reasons. First, the Dirichlet distribution is appropriate for a distribution of probabilities because the range of the distribution is from 0 to 1. Second, the parameters $\alpha_1, \alpha_1, \alpha_3$ govern the shape of the distribution given $K = 3$. If $\alpha_1 = \alpha_2 = \alpha_3 = 1$, independent draws from the distribution are uniform over the simplex (Figure 2.1 (a)); if $\alpha_1 < 1$, $\alpha_2 < 1$, $\alpha_3 < 1$, there is the greater frequency at the corners of the simplex (Figure 2.1 (b)); and if $\alpha_1 > 1$, $\alpha_2 > 1$, $\alpha_3 > 1$, there is a concentration at the center of the simplex (Figure 2.1 (c)). If $\alpha_1 < \alpha_2 = \alpha_3$, there is a concentration around the side of the simplex joining π_2 and π_3 (Figure 2.1 (d)). If α_1 and α_2 are both substantially less than α_3, there is a concentration at the π_3 corner of the simplex (Figure 2.1 (e)).

I now derive the expected probability of the payment path under the assumption that the responding probabilities (π_1 π_2 π_3) follows a Dirichlet distribution. Substituting the Dirichlet density in equation (15) for $f(\boldsymbol{\pi})$ in equation (13), the expected probability of not making a payment for x_1 months, making curtailment payments for x_2 months, and making regular payments for x_3 months within T months in the forbearance program are obtained by:

$$\begin{aligned}
E[p(x_1, x_2, x_3, T)] &= \int \frac{T!}{x_1!x_2!x_3!}\pi_1^{x_1}\pi_2^{x_2}(1-\pi_1-\pi_2)^{x_3} f(\boldsymbol{\pi})d\boldsymbol{\pi} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{1}{B(\alpha_1,\alpha_2,\alpha_3)}\int \pi_1^{x_1+\alpha_1-1}\pi_2^{x_2+\alpha_2-1}(1-\pi_1-\pi_2)^{x_3+\alpha_3-1}d\boldsymbol{\pi} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{B(x_1+\alpha_1, x_2+\alpha_2, x_3+\alpha_3)}{B(\alpha_1,\alpha_2,\alpha_3)} \\
&= \frac{T!}{x_1!x_2!x_3!}\frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)}\frac{\Gamma(x_1+\alpha_1)\Gamma(x_2+\alpha_2)\Gamma(x_3+\alpha_3)}{\Gamma(T+\alpha_1+\alpha_2+\alpha_3)}
\end{aligned} \tag{16}$$

where $T = x_1 + x_2 + x_3$ is the total number of months spent in the forbearance program.

The properties of the model are derived from equation (16) using the recurrence relationship $\Gamma(x+1) = x\Gamma(x)$. The average probability of making curtailment payments in one month is

$$E[p(1,0,0,1)] = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}, \tag{17}$$

with variance

$$\begin{aligned}
\sigma^2 &= E[(p(1,0,0,1))^2] - E[(p(1,0,0,1))]^2 \\
&= \frac{\alpha_1(\alpha_2+\alpha_3)}{(\alpha_1+\alpha_2+\alpha_3)^2(1+\alpha_1+\alpha_2+\alpha_3)}
\end{aligned} \tag{18}$$

which is a decreasing function of α_1, α_2, and α_3.

The state dependence caused by heterogeneity can be discovered by comparing the conditional probability of not paying in month t for borrowers who did not pay in month $t-1$ with that of borrowers who paid in month $t-1$ (made a curtailment payment or scheduled payment in month $t-1$):

$$
\begin{aligned}
&P(y_{1t}=1|y_{1,t-1}=1)-P(y_{1t}=1|y_{2,t-1}=1) \\
&=\frac{P(y_{1t}=1,y_{1,t-1}=1)}{P(y_{1,t-1}=1)}-\frac{P(y_{1t}=1,y_{2,t-1}=1)}{P(y_{2,t-1}=1)} \\
&=\frac{1}{1+\alpha_1+\alpha_2+\alpha_3}\ ,
\end{aligned}
\tag{19}
$$

and

$$
P(y_{1t}=1|y_{1,t-1}=1)-P(y_{1t}=1|y_{3,t-1}=1)=\frac{1}{1+\alpha_1+\alpha_2+\alpha_3}\ , \tag{20}
$$

where y_{1t}, y_{2t}, and y_{3t} are indicator variables for not paying, making payments, and making scheduled payments in month t, respectively.

The conditional probability of not paying rises the more frequently the previous non-payment is, since

$$
\begin{aligned}
P(y_{1t}=1|y_{1,t-1}=1,\cdots,y_{11}=1)&=\frac{P(y_{1t}=1,y_{1,t-1}=1,\cdots,y_{11}=1)}{P(y_{1,t-1}=1,\cdots,y_{11}=1)} \\
&=\frac{\alpha_1+t-1}{\alpha_1+\alpha_2+\alpha_3+t-1}\ ,
\end{aligned}
\tag{21}
$$

which is a monotonic increasing function of t. As t increases to infinity, the conditional probability limits to 1.

If all factors resulting in differences in the mortgagor's decisions are unobserved, the α_i's are scalars. However, the data contain observed variables, such as debt-to-income ratio, FICO score, loan-to-value ratio, and loan size, which influence these decisions. This chapter therefore parameterizes α_1, α_2, α_3 as follows:

$$
\begin{aligned}
\alpha_1&=e^{z'\beta_1} \\
\alpha_2&=e^{z'\beta_2} \\
\alpha_3&=e^{z'\beta_3}\ .
\end{aligned}
\tag{22}
$$

Substituting equation (22) into (17), the conditional probability of not paying at $T = 1$ can be written as

$$
\begin{aligned}
E[p(1,0,0,1)] &= \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3} \\
&= \frac{e^{z'(\beta_1 - \beta_3)}}{e^{z'(\beta_1 - \beta_3)} + e^{z'(\beta_2 - \beta_3)} + 1} \ ,
\end{aligned}
\tag{23}
$$

which is the traditional multinomial logit model. With cross-section data, β_1, β_2, and β_3 cannot be identified separately. Therefore, the traditional multinomial logit model can be used only to predict the mean payment rate in the population, but we need higher moments of the distribution of response probabilities to solve the unobserved heterogeneity problem. Moreover, we need at least two periods of data on the same individuals to identify β_1, β_2, and β_3.

Given a vector of independent variables, z, parameters β_1, β_2, and β_3 can be estimated by maximum likelihood. The likelihood function is

$$
\begin{aligned}
l(\beta_1, \beta_2, \beta_3) &= \prod_{m=1}^{M} \frac{T_m!}{x_{1m}! x_{2m}! x_{3m}!} \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} \frac{\Gamma(x_{1m} + \alpha_1)\Gamma(x_{2m} + \alpha_2)\Gamma(x_{3m} + \alpha_3)}{\Gamma(T_m + \alpha_1 + \alpha_2 + \alpha_3)} \\
&= \prod_{m=1}^{M} \frac{T_m!}{x_{1m}! x_{2m}! x_{3m}!} \frac{\Gamma(e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3})}{\Gamma(e^{z'\beta_1})\Gamma(e^{z'\beta_2})\Gamma(e^{z'\beta_3})} \\
&\quad \times \frac{\Gamma(x_{1m} + e^{z'\beta_1})\Gamma(x_{2m} + e^{z'\beta_2})\Gamma(x_{3m} + e^{z'\beta_3})}{\Gamma(T_m + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3})}
\end{aligned}
\tag{24}
$$

The predictive probability of borrower payment behavior at time t given the previous pattern of borrower payment behavior to time $t-1$ is

$$\begin{aligned} f_{1t} = P(y_{1t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1+1, x_2, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_1 + \alpha_1}{t - 1 + \alpha_1 + \alpha_2 + \alpha_3} \\ &= \frac{x_1 + e^{z'\beta_1}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \end{aligned}$$

$$\begin{aligned} f_{2t} = P(y_{2t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1, x_2+1, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_2 + e^{z'\beta_2}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \end{aligned} \tag{25}$$

$$\begin{aligned} f_{3t} = P(y_{3,t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1, x_2, x_3+1, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_3 + e^{z'\beta_3}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \ , \end{aligned}$$

where $y_{1,t}$, $y_{2,t}$, and $y_{3,t}$ are binary variables, and $y_{i,t}$ equals 1 if choice i has been chosen at time t. $x_{i,t-1}$ is the cumulative number of times that choice i has been chosen in the previous $t-1$ periods, where $t-1 = \sum x_{i,t-1}$. The predictive probability of borrower payment behavior at time t is not only a function of exogenous variables but is also affected by the borrower's previous payment behavior. Note that the probability of each type of payment or non-payment is determined only by the number of previous instances of the same type. In the Dirichlet nested logit model, this restriction is relaxed.

2.3.2 The Dirichlet Nested Logit Model

The Dirichlet multinomial logit model allows state dependence to vary across individuals and time. However, in the Dirichlet multinomial logit model equation (19) and (20) produce the same value for the difference of conditional probabilities, whether or not borrowers make the curtailment payment or the scheduled monthly payment in month

$t-1$. I now derive the Dirichlet nested logit model to allow for the difference of state dependence for choices from different nests for the month $t-1$. To derive the Dirichlet nested logit model, we first show that the Dirichlet multinomial logit model can be written as the product of two beta-logistic models. Then I use this fact to obtain the final model.

In the application, there are two groups of payments. Group G_1 includes not paying and making a curtailment payment, while group G_2 contains only making the scheduled payment. Figure 2.2 presents payment proportions for the month $t-1$ for mortgages with curtailment payments in month t for 2 of the 3 subsamples. One reason for including not paying and making curtailment payments together in the group G_1 can be seen in the bar chart. Borrowers who currently make curtailment payments are less likely to make payments in the months immediately before. This suggests that borrowers with recent arrears may want to move back to being current when they have more options for how to exit forbearance.

The expected probability after forming two groups of borrower payment behavior can be written as

$$\begin{aligned} E[p(x_1, x_2, x_3, T)] &= \int \frac{T!}{x_1!x_2!x_3!} \pi_1^{x_1} \pi_2^{x_2} (1-\pi_1-\pi_2)^{x_3} f(\boldsymbol{\pi}) d\boldsymbol{\pi} \\ &= \int \frac{T!}{x_1!x_2!x_3!} (\frac{\pi_1}{\pi_1+\pi_2})^{x_1} (\frac{\pi_2}{\pi_1+\pi_2})^{x_2} (\pi_1+\pi_2)^{x_1+x_2} (1-\pi_1-\pi_2)^{x_3} f(\boldsymbol{\pi}) d\boldsymbol{\pi} \end{aligned} \tag{26}$$

Let $p_1 = \frac{\pi_1}{\pi_1+\pi_2}$ and $p_2 = \pi_1 + \pi_2$, the expected probability becomes

$$E[p(x_1, x_2, x_3, T)] = \int \frac{T!}{x_1!x_2!x_3!} p_1^{x_1} (1-p_1)^{x_2} p_2^{x_1+x_2} (1-p_2)^{x_3} f(\mathbf{p}) d\mathbf{p} \tag{27}$$

where $f(\mathbf{p})$ is obtained from $f(\boldsymbol{\pi})$ by applying the Jacobian transformation. The expression for $f(\mathbf{p})$ is

$$f(\mathbf{p}) = \frac{1}{B(\boldsymbol{\alpha})} p_1^{\alpha_1 - 1}(1-p_1)^{\alpha_2 - 1} p_2^{\alpha_1+\alpha_2-1}(1-p_2)^{\alpha_3 - 1} \quad . \tag{28}$$

From equation (27) and (28), we have

$$\begin{aligned} E[p(x_1, x_2, x_3, T)] &= \int \frac{T!}{x_1!x_2!x_3!}(p_1)^{x_1}(1-p_1)^{x_2}(p_2)^{x_1+x_2}(1-p_2)^{x_3} f(\mathbf{p}) d\mathbf{p} \\ &= \int \frac{(x_1+x_2)!}{x_1!x_2!} \frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}(p_1)^{x_1+\alpha_1-1}(1-p_1)^{x_2+\alpha_2-1} dp_1 \\ &\times \int \frac{T!}{(x_1+x_2)!x_3!} \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1+\alpha_2)\Gamma(\alpha_3)}(p_2)^{x_1+x_2+\alpha_1+\alpha_2-1}(1-p_2)^{x_3+\alpha_3-1} dp_2 \quad . \end{aligned} \tag{29}$$

Equation (29) shows that the Dirichlet multinomial logit model is the product of two beta-logistic models under the assumption that the response probability p_1 within G_1 and the response probability of p_2 of G_1 both have a beta distribution.

A nested logit model is appropriate when the alternatives faced by a decision-maker can be partitioned into nests. The independence of irrelevant alternatives (IIA) does not necessarily hold for alternatives in different nests. Therefore, it allows for the correlation across the choices within a nest. The nested logit probability (see McFadden 1977) can be rewritten as the product of two standard logit probabilities: the probability of choosing alternative i, given the choice of the nest containing alternative i, and the probability of choosing the nest. The probability of borrower payment behavior in each month can be written as

$$
\begin{aligned}
P(y_{1t}) &= P(y_{1t}|G_1) \cdot P(G_1) \\
&= \frac{\exp(z^{'}\beta_1/\theta)}{\exp(I^{G_1})} \cdot \frac{\exp(\theta I^{G_1})}{1+\exp(\theta I^{G_1})}
\end{aligned}
$$

$$
\begin{aligned}
P(y_{2t}) &= P(y_{2t}|G_1) \cdot P(G_1) \\
&= \frac{\exp(z^{'}\beta_2/\theta)}{\exp(I^{G_1})} \cdot \frac{\exp(\theta I^{G_1})}{1+\exp(\theta I^{G_1})}
\end{aligned} \tag{30}
$$

$$
P(y_{3t}) = \frac{1}{1+\exp(\theta I^{G_1})}
$$

where θ is the dissimilarity parameter for the nest,[7] and the inclusive value

$I^{G_1} = \ln(\exp(z'\beta_1/\theta){+}\exp(z'\beta_2/\theta))$.

This chapter now derives the expected probability of the payment path of not making a payment for x_1 months and making curtailment payments for x_2 months under the assumption that (π_1 , π_2) follows a Beta distribution.[8] The expected probability of not making a payment for x_1 months and making curtailment payments for x_2 months within group G_1 is given by

$$
E(p(x_1, x_2, x_{G_1})) = \frac{x_{G_1}!}{x_1!x_2!} \frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \frac{\Gamma(\alpha_1+x_1)\Gamma(\alpha_2+x_2)}{\Gamma(\alpha_1+\alpha_2+x_1+x_2)} \tag{31}
$$

where $\alpha_1 = \exp \frac{z'\beta_1}{\theta}$, and $\alpha_2 = \exp \frac{z'\beta_2}{\theta}$.

[7] If $\theta = 1$, the nested logit model and the traditional multinomial logit model are equivalent.

[8] Dirichlet distribution is a multivariate generalization of the beta distribution. The probability density function for not making the payment within group G_1 for the Beta distribution is given

$$
f(\pi_1) = \frac{1}{B(\alpha_1, \alpha_2)} \pi_1^{\alpha_1 - 1}(1-\pi_1)^{\alpha_2 - 1} \qquad 0 \le \pi_1 \le 1, \quad \alpha_1, \alpha_2 > 0
$$

where

$$
B(\alpha_1, \alpha_2) = \int_0^1 y^{\alpha_1 - 1}(1 - y^{\alpha_2 - 1})dy = \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)}{\Gamma(\alpha_1+\alpha_2)} \ .
$$

The expected probability of choosing among groups under the assumption that (π_1+π_2 , π_3) follows a Beta distribution can also be written as:[9]

$$E(p(x_{G_1}, x_{G_2}, T)) = \frac{T!}{x_{G_1}!x_{G_2}!} \frac{\Gamma(\alpha_{G_1} + \alpha_{G_2})}{\Gamma(\alpha_{G_1})\Gamma(\alpha_{G_2})} \frac{\Gamma(\alpha_{G_1} + x_{G_1})\Gamma(\alpha_{G_2} + x_{G_2})}{\Gamma(\alpha_{G_1} + \alpha_{G_2} + T)} \tag{32}$$

where $\alpha_{G_1} = (\exp \frac{z'\beta_1}{\theta} + \exp \frac{z'\beta_2}{\theta})^{\theta}$, $\alpha_{G_2} = \exp z'\beta_3$, $x_{G_1} = x_1 + x_2$, $x_{G_2} = x_3$, and $T = x_{G_1} + x_{G_2}$. From equation (31) and equation (32), the expected probability of borrower payment of a specific behavior can be written as

$$\begin{aligned} E[p(x_1, x_2, x_3, T)] = {} & \frac{(x_1 + x_2)!}{x_1!x_2!} \frac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \frac{\Gamma(\alpha_1 + x_1)\Gamma(\alpha_2 + x_2)}{\Gamma(\alpha_1 + \alpha_2 + x_1 + x_2)} \\ & \times \frac{T!}{x_{G_1}!x_{G_2}!} \frac{\Gamma(\alpha_{G_1} + \alpha_{G_2})}{\Gamma(\alpha_{G_1})\Gamma(\alpha_{G_2})} \frac{\Gamma(\alpha_{G_1} + x_{G_1})\Gamma(\alpha_{G_2} + x_{G_2})}{\Gamma(\alpha_{G_1} + \alpha_{G_2} + T)} \end{aligned} \tag{33}$$

The properties of the model can be derived from equation (33) using the recurrence relationship $\Gamma(x + 1) = x\Gamma(x)$. The average probabilities of not paying, making a curtailment payment, and making a scheduled payment for $T = 1$ are

$$E[p(1, 0, 0, 1)] = \frac{\alpha_1}{\alpha_1 + \alpha_2} \frac{(\alpha_1 + \alpha_2)^{\theta}}{(\alpha_1 + \alpha_2)^{\theta} + \alpha_3}, \tag{34}$$

$$E[p(0, 1, 0, 1)] = \frac{\alpha_2}{\alpha_1 + \alpha_2} \frac{(\alpha_1 + \alpha_2)^{\theta}}{(\alpha_1 + \alpha_2)^{\theta} + \alpha_3}, \tag{35}$$

$$E[p(0, 1, 0, 1)] = \frac{\alpha_3}{(\alpha_1 + \alpha_2)^{\theta} + \alpha_3}. \tag{36}$$

With $\alpha_1 = \exp \frac{z'\beta_1}{\theta}$, $\alpha_2 = \exp \frac{z'\beta_2}{\theta}$, and $\alpha_3 = \exp z'\beta_3$, we have a nested logit model.

I now examine the difference in state dependence between the Dirichlet multino-

[9] The probability density function for choosing among groups for the Beta distribution is given

$$f(\pi_1 + \pi_2) = \frac{1}{B(\alpha_{G_1}, \alpha_{G_2})} (\pi_1 + \pi_2)^{\alpha_{G_1} - 1} (\pi_3)^{\alpha_{G_2} - 1} \qquad 0 \le \pi_1 + \pi_2 \le 1, \quad \alpha_{G_1}, \alpha_{G_2} > 0$$

where

$$B(\alpha_{G_1}, \alpha_{G_2}) = \int_0^1 y^{\alpha_{G_1} - 1}(1 - y^{\alpha_{G_2} - 1})dy = \frac{\Gamma(\alpha_{G_1})\Gamma(\alpha_{G_2})}{\Gamma(\alpha_{G_1} + \alpha_{G_2})} \ .$$

mial logit model and the Dirichlet nested logit model. State dependence in the Dirichlet multinomial logit model was derived above in equations (19) and (20). A borrower who did not pay at time $t-1$ would have a higher probability of not making a payment at time t than would a borrower who made a curtailment payment at time $t-1$ or a borrower who made a scheduled payment at time $t-1$. Moreover, the latter two probabilities are restricted to be equal. In the Dirichlet nested logit model, this restriction is relaxed.

The difference of the probability of not paying at time t conditional on not paying at time $t-1$, and the probability of not paying at time t conditional on making a curtailment payment at time $t-1$ is:

$$\begin{aligned}
&P(y_{1,t}=1|y_{1,t-1}=1)-P(y_{1,t}=1|y_{2,t-1}=1)\\
&=\frac{(1+\alpha_1)(1+(\alpha_1+\alpha_2)^\theta)}{(1+\alpha_1+\alpha_2)(1+(\alpha_1+\alpha_2)^\theta+\alpha_3)}-\frac{(\alpha_1)(1+(\alpha_1+\alpha_2)^\theta)}{(1+\alpha_1+\alpha_2)(1+(\alpha_1+\alpha_2)^\theta+\alpha_3)}\\
&=\frac{1+(\alpha_1+\alpha_2)^\theta}{(1+\alpha_1+\alpha_2)(1+(\alpha_1+\alpha_2)^\theta+\alpha_3)}\ .
\end{aligned}\tag{37}$$

Equation (37) is an increasing function of θ if $\alpha_1+\alpha_2>1$. The higher the correlation between the choice of not making a payment and curtailing a payment ($\theta\to 0$), the lower the difference in the conditional probabilities. As θ approaches 1, I end up with the same value as the Dirichlet multinomial logit model.

The difference of the probability of not paying at time t conditional on not paying at time $t-1$ and the probability of not paying at time t conditional on making a scheduled payment at time $t-1$ is:

$$\begin{aligned}
&P(y_{1t}=1|y_{1,t-1}=1)-P(y_{1t}=1|y_{3,t-1}=1)\\
&=\frac{1+\alpha_1+\alpha_2(\alpha_1+\alpha_2)^{\theta-1}}{(1+\alpha_1+\alpha_2)(1+(\alpha_1+\alpha_2)^\theta+\alpha_3)}\ .
\end{aligned}\tag{38}$$

As $\theta\to 1$, I have the same value as in equation (20).

Subtracting the right hand side of equation (37) from right hand side of equation

(38), I obtain the difference between the probability of not paying at time t conditional on making a scheduled payment at time $t-1$ and the probability of not paying at time t conditional on making a curtailment payment at time $t-1$:

$$\begin{aligned} &P(y_{1t}=1|y_{3,t-1}=1)-P(y_{1t}=1|y_{2,t-1}=1) \\ &\quad= \frac{\alpha_1((\alpha_1+\alpha_2)^{\theta-1}-1)}{(1+\alpha_1+\alpha_2)(1+(\alpha_1+\alpha_2)^{\theta}+\alpha_3)} \ . \end{aligned} \tag{39}$$

As $\theta \to 1$, equations (37) and (38) coincide. For $\alpha_1+\alpha_2>1$, the probability of not paying at time t conditional on making a scheduled payment at time $t-1$ is less than or equal to the probability of not paying at time t conditional on making a curtailment payment at time $t-1$. The intuition is that not making a payment and making a curtailment payment are closer substitutes than not making a payment and making a scheduled payment.

The parameters β_1, β_2, β_3, and θ can be estimated by maximum likelihood. The likelihood function is

$$\begin{aligned} l(\beta_1,\beta_2,\beta_3,\theta) &= \prod_{m=1}^{M} \frac{(x_{1m}+x_{2m})!}{x_{1m}!x_{2m}!}\frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\frac{\Gamma(\alpha_1+x_{1m})\Gamma(\alpha_2+x_{2m})}{\Gamma(\alpha_1+\alpha_2+x_{1m}+x_{2m})} \\ &\quad\times \frac{T_m!}{x'_{1m}!x'_{2m}!}\frac{\Gamma(\alpha_{G_1}+\alpha_{G_2})}{\Gamma(\alpha_{G_1})\Gamma(\alpha_{G_2})}\frac{\Gamma(\alpha_{G_1}+x'_{1m})\Gamma(\alpha_{G_2}+x'_{2m})}{\Gamma(\alpha_{G_1}+\alpha_{G_2}+T_m)} \\ &= \prod_{m=1}^{M} \frac{(x_{1m}+x_{2m})!}{x_{1m}!x_{2m}!}\frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\frac{\Gamma(\alpha_1+x_{1m})\Gamma(\alpha_2+x_{2m})}{\Gamma(\alpha_1+\alpha_2+x_{1m}+x_{2m})} \\ &\quad\times \frac{T_m!}{(x_{1m}+x_{2m})!x_{3m}!}\frac{\Gamma((\alpha_1+\alpha_2)^{\theta}+\alpha_3)}{\Gamma((\alpha_1+\alpha_2)^{\theta})\Gamma(\alpha_3)} \\ &\quad\times \frac{\Gamma((\alpha_1+\alpha_2)^{\theta}+x_{1m}+x_{2m})\Gamma(\alpha_3+x_{3m})}{\Gamma((\alpha_1+\alpha_2)^{\theta}+\alpha_3+T_m)} \ , \end{aligned} \tag{40}$$

where $\alpha_1=\exp\frac{z'\beta_1}{\theta}$, $\alpha_2=\exp\frac{z'\beta_2}{\theta}$, and $\alpha_3=\exp z'\beta_3$.

The predictive probability of borrower payment behavior at time t given the previous pattern of borrower payment behavior to time $t-1$ are

$$
\begin{aligned}
f_{1t} &= P(y_{1t} = 1 | x_1, x_2, x_3, t-1) \\
&= \frac{E[p(x_1+1, x_2, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\
&= \frac{(x_1+\alpha_1)((\alpha_1+\alpha_2)^\theta + x_1 + x_2)}{(t-1+(\alpha_1+\alpha_2)^\theta + \alpha_3)(\alpha_1+\alpha_2+x_1+x_2)} \\
&= \frac{(x_1+e^{z'\beta_1/\theta})((e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta})^\theta + x_1 + x_2)}{(t-1+(e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta})^\theta + e^{z'\beta_3})(e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta}+x_1+x_2)}
\end{aligned}
$$

$$
\begin{aligned}
f_{2t} &= P(y_{2,t} = 1 | x_1, x_2, x_3, t-1) \\
&= \frac{E[p(x_1, x_2+1, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\
&= \frac{(x_2+\alpha_2)((\alpha_1+\alpha_2)^\theta + x_1 + x_2)}{(t-1+(\alpha_1+\alpha_2)^\theta + \alpha_3)(\alpha_1+\alpha_2+x_1+x_2)} \\
&= \frac{(x_2+e^{z'\beta_2/\theta})((e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta})^\theta + x_1 + x_2)}{(t-1+(e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta})^\theta + e^{z'\beta_3})(e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta}+x_1+x_2)}
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
f_{3t} &= P(y_{3,t} = 1 | x_1, x_2, x_3, t-1) \\
&= \frac{E[p(x_1, x_2, x_3+1, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\
&= \frac{x_3+\alpha_3}{t-1+(\alpha_1+\alpha_2)^\theta + \alpha_3} \\
&= \frac{x_3+e^{z'\beta_3}}{t-1+(e^{z'\beta_1/\theta}+e^{z'\beta_2/\theta})^\theta + e^{z'\beta_3}}
\end{aligned}
$$

where y_{1t}, $y_{2,t}$, $y_{3,t}$, $x_{i,t-1}$, and $t-1$ are as defined at the end of the previous section. Note that, unlike in the Dirichlet multinomial logit model, the probability of each type of payment or non-payment is determined by the number of previous instances of types in the same nest.

2.3.3 How Do Borrowers Exit Forbearance?

The pattern of payment behavior during the forbearance program predicts how borrowers choose to exit forbearance. For example, borrowers with more forborne payments are more likely to exit the forbearance program with a payment deferral or a trial or modification. In contrast, borrowers who always make scheduled or curtailment payments are more likely to exit with a prepayment or a reinstatement. This chapter uses a two-step estimation technique to study how the pattern of borrower payment behavior during the forbearance program affects borrowers' choice of exit types. In the first step, the Dirichlet multinomial or nested logit model estimates the patterns of payment behavior and predicts the marginal probability of each borrower's pattern at the end by equation (25) and (41). These predictions are used in a multinomial logit model to estimate the probability of forbearance exit type in the second step. The multinomial logit model that estimates the probability of forbearance exit type at the termination period (time T) with the predicted probabilities of payment behaviors as explanatory variables can be written as

$$P(C_{jmT}) = \begin{cases} \frac{\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})}{1+\sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & , j = 1, ...J-1 \\ \frac{1}{1+\sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & , j = J \end{cases} \tag{42}$$

where C_{jmT} for $j = 1, \ldots, J$ represents the exit type, and W_{jT}'s are vectors of exogenous variables that affect how borrowers exit forbearance for the exiting type j. $\hat{f}_{1T}$ is the estimated predictive probability of not making a payment at terminate time T, and $\hat{f}_{2T}$ is the estimated predictive probability of making curtailment payment at terminate time T.[10]

[10] The two-step estimation is consistent but asymptotically less efficient than the maximum likelihood estimator. The standard errors have to be adjusted for the first step estimation error, as in Murphy and Topel (2002). More details are presented in Appendix.

2.4 The Empirical Application

2.4.1 Data

The loan-level data in this chapter are from the public use Fannie Mae Credit Insurance Risk Transfer (CIRT)/Connecticut Avenue Securities(CAS) data. Starting in 2013, the government-sponsored enterprises (GSEs) in consultation with the Federal Housing Finance Agency (FHFA) began selling credit risk to private investors through the credit risk transfer (CRT) programs. According to Fannie Mae, $681B of the unpaid principal balance of single-family mortgage loans has been partially covered through CIRT transactions as of Q2:2022. CIRT helps reduce credit risk by bringing additional private capital to the single-family housing market; in other words, CIRT transfers credit risk on a pool of loans to an insurance provider.

This chapter selects CIRT loans that participated in the CARES Act Mortgage forbearance program from March, 2020, to March, 2022 (68,313 loans). CIRT data provides characteristics and financial conditions of borrowers and mortgages. The data include standard information about loans at origination, such as credit score, loan-to-value, loan amount, mortgage rate, debt-to-income, year, occupancy status, and property location (three-digit ZIP code). The data also include time-varying information on loan performance: current unpaid balance, payment status, forbearance indicator, and servicer and seller names. In contrast to loan-level mortgage-backed securities data, the CIRT data include the current FICO score. Very few academic papers consider the application of updated credit scores. The current FICO score is one of the essential determinants of borrowers' payment behavior and termination events; for example, borrowers with a high current FICO score will have a low risk of delinquency.

The CIRT data provides forbearance information, including forbearance start date, and exit date and type. Borrowers can exit the forbearance program in one of five ways:

reinstatement, repayment plan, payment deferral, modification, or prepayment. The exit is defined as reinstatement if the borrower is not delinquent at the time of exit. A borrower can end a spell of forbearance by reinstatement and prepay within 18 months from the start of forbearance. In such a case, I consider the reinstatement to be transitional and the prepayment event to be terminal. As of March 2022, 23,775 loans (34.8 percent of all forborne loans in the sample) had exited forbearance by prepayment, 12,502 loans (18.3 percent) had exited by reinstatement, 690 loans (1.01 percent) had exited by repayment plan, 19,237 loans (28.1 percent) had exited by payment deferral, and 3,782 loans (5.54 percent) had exited by trial/modification.

Figure 2.3 shows the conditional rate of exit from the forbearance program. The figure shows that prepayment and payment deferral are the two main exit types, both of which increase with forbearance duration. In comparison, exiting forbearance with reinstatement happens more frequently in the first six months. Moreover, borrowers who exit by trial/modification are more likely to stay in the forbearance program until month 18.

The CIRT data are augmented with external data to control for the local economy and the macroeconomy. The All-Transactions[11] House Price Index (HPI) at three-digit ZIP Code level from FHFA is used to measure the movement of single-family house prices at the most granular level possible. The FHFA HPI is a weighted repeat-sales index that measures average price changes in sales or refinancing on the same properties.

As part of the CARES Act, Congress enacted the Economic Impact Payment (EIP) program to provide broad relief to Americans from the economic shock of Covid-19. Eligibility requires U.S. citizenship or U.S. residency with a valid Social Security number and income up to $\$75,000$ for individuals and $\$150,000$ for couples filing jointly. The U.S. Department of the Treasury publishes the disbursement of the EIP payments at the national level over calendar time. The IRS provides the total dollar amount and the number of receipts in aggregate at the state level for the first, second, and third rounds

[11]Appraisal values from refinancing mortgages are included in the purchase-only data.

of payment. Figure 2.4 shows the timeline of Economic Impact Payments.

Summary Statistics

Table 3.1 presents the definitions of the explanatory variables. This chapter defines the monthly benefit of refinancing as the monthly decrease in payment on refinancing in the current month. I expect a positive effect of the monthly benefit of refinancing on the probability of prepayment. The value of payment deferral is defined as the gains of investing the values of the missed payments into a 10-year Treasury bond until the mortgage maturity date. I expect the value of payment deferral to affect the probability of payment deferral positively. Table 2.11 provides summary statistics for the explanatory variables at origination and across time from March 2020 to March 2022 for all forborne loans.

Each month in the forbearance program, borrowers choose how to allocate mortgage payments. This chapter uses the Dirichlet multinomial logit and the Dirichlet nested logit to analyze borrower payment behavior separately for borrowers with the forbearance duration of 6, 12, and 18 months. Table 2.2 summarizes the explanatory variables by payment behaviors and forbearance duration. The table shows that higher credit risk loans, characterized by a longer proportion of time in delinquency, lower current FICO score, higher loan-to-value ratio, and higher debt-to-income ratio, are more likely to forbear their monthly payments.

Table 2.3 provides summary statistics of the explanatory variables by forbearance exit type. The table shows that borrowers with a higher monthly benefit of refinancing are more likely to exit forbearance with prepayment. Meanwhile, the value of deferral increases the probability of exit by payment deferral or modification. Loans with delinquency spells, lower current FICO scores, higher loan-to-value ratios, and higher debt-to-income ratios are less likely to exit the forbearance program by prepayment and reinstatement.

2.4.2 Results for the Dirichlet Multinomial and Dirichlet Nested Logit Models

This section presents estimates of the models for the sequential payment behavior of mortgage borrowers based on Fannie Mae Credit Insurance Risk Transfer (CIRT) data. The sample consists of 68313 for Covid-19 forbearance borrowers with up to 18 months in the forbearance program. Each month in the forbearance program borrowers can choose to not make payments, make curtailment payments, or make scheduled monthly payments.

Assuming that borrower payment behavior probabilities while in the forbearance program follow a Dirichlet distribution, coefficient vectors β_1, β_2, β_3, and θ can be estimated by maximum likelihood from equations (24) and (29) separately. Table 2.4 presents the regression results of the traditional multinomial logit (MNL), the Dirichlet multinomial logit model (DMNL), and the Dirichlet nested logit model (DNL) for three groups of borrowers who exit forbearance at the 6^{th}, 12^{th}, or 18^{th} month.[12]

The values of covariates are set equal to their values of the first month of borrowers participating in the forbearance program.[13] This assumption will not substantially affect the final results given that the length of the program is relatively short compared to the term of the loan.

The pattern of borrower payment behavior for borrowers in the program is determined by borrower characteristics, e.g., income, FICO, and age, by loan characteristics, e.g., loan amount, loan-to-value ratio, and note rate, and by financial market conditions, e.g., unemployment rate and house price. From Table 2.4, a high FICO score tends to reduce non-payment and to increase the probability of curtailment and scheduled payments. The borrower's previous delinquency status, taken to be the number of delinquency spells and average time spent delinquent, reduces scheduled payments and increases non-payment. Similarly, borrowers with a high marked-to-market loan-to-value

[12] The full results are presented in Table 2.12 in Appendix.

[13] 76.87 percent of borrowers entered the forbearance program in March, April, and May 2020.

ratio and original debt-to-income ratio are more likely not to make payments.

Dirichlet Distribution

The presence of considerable heterogeneity could be discovered in the distribution of response probability. To show that, the shape parameters (α_1, α_2, and α_3) of the Dirichlet multinomial distribution are evaluated as an exponential function of the mean values of the exogenous variables: $\alpha_i = e^{\bar{z}_i^{'}\beta_i}$, which implies that the distributions of borrowers' payment behaviors with average characteristics are in Figure 2.5. I have an asymmetric Dirichlet distribution tightly concentrated at one side of the simplex (π_1). Moreover, the distribution becomes more likely located at the corners as the duration of the forbearance program increases. This corner distribution indicates relatively few borrowers have probabilities of payment behaviors near the mean. Borrowers who stay in the forbearance program until the 18^{th} months are more likely to have a longer duration of not making payments.

The Beta Distribution within the Nest and between Nests

This subsection examines the role of heterogeneity in the models and presents descriptive statistics of the results. The DMNL and DNL models can be written as the product of two parts: the probability of choices within the nest and between nests.

Figure 2.6(a) presents graphs of the pdf of the expected probability of not making a payment for the DMNL using the estimated shape parameters and the mean values of the exogenous variables to study the choices within the set composed of not making a payment and making curtailment payments ($\alpha_1 = \exp\{\bar{z}_1^{'}\beta_1\}$ and $\alpha_2 = \exp\{\bar{z}_2^{'}\beta_2\}$). The beta distribution for this set is J shaped because relatively few borrowers have a probability of not making a payment and making a curtailment payment near the mean. Within this set the average expected probability of not making a payment, $\alpha_1/(\alpha_1 + \alpha_2)$, is 0.817 (the vertical red line in Figure 2.6(a)) for borrowers who stay in the forbearance program for 6 months and 0.892 (the vertical green line in Figure 2.6(a)) for borrowers who stay in the forbearance program for 18 months. The results show that the average expected probability of not making a payment tends to increase over time in the forbearance program. The variance of the probability of not making a payment, $(\alpha_1\alpha_2)/((\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1))$, is 0.058 for borrowers who stay in the forbearance program for 6 months and 0.045 borrowers who stay in the forbearance program for 18 months. This decline in variance lowers the probability mass in the tails.

Figure 2.6(b) presents graphs of the pdf of the expected probability of choosing G_1 for the DMNL using the estimated shape parameters and the mean values of the exogenous variables to study the choices between nests ($\alpha_{G_1} = \exp\{\bar{z}_1'\beta_1\} + \exp\{\bar{z}_2'\beta_2\}$ and $\alpha_{G_2} = \exp\{\bar{z}_3'\beta_3\}$). The beta distribution of probabilities for G_1 (not paying and making curtailment payment) and G_2 (making scheduled payment) is also J shaped. The average expected probability of choosing G_1, $\alpha_{G_1}/(\alpha_{G_1} + \alpha_{G_2})$, is 0.647 (the vertical red line in Figure 2.6(b)) for loans that stay in the forbearance program for 6 months and 0.731 (the vertical green line in Figure 2.6(b)) for loans that stay in the forbearance program for 18 months. Moreover, the variance of the probability of choosing G_1, $(\alpha_{G_1}\alpha_{G_2})/((\alpha_{G_1} + \alpha_{G_2})^2(\alpha_{G_1} + \alpha_{G_1} + 1))$, is 0.067 for borrowers who stay in the forbearance program for 6 months and 0.076 borrowers who stay in the forbearance program for 18 months. This increase in variance raises the probability mass in the tails.

Comparable curves for the DNL case are presented in Figure 2.7. Figure 2.7(a) presents graphs of the pdf of the expected probability of not making a payment for the DNL using the estimated shape parameters and the mean values of the exogenous variables to study the choices within the nest composed of not making a payment and making curtailment payments ($\alpha_1 = \exp\{\bar{z}_1'\beta_1/\theta\}$ and $\alpha_2 = \exp\{\bar{z}_2'\beta_2/\theta\}$). The beta distribution for this nest is J shaped. Within this nest, the average expected probability of not making a payment, $\alpha_1/(\alpha_1 + \alpha_2)$, is 0.876 (the vertical red line in Figure 2.7(a)) for borrowers who stay in the forbearance program for 6 months and 0.911 (the vertical green line in Figure

2.7(a)) for borrowers who stay in the forbearance program for 18 months. The average expected probability of not making a payment increased by about 0.05 from the DMNL to the DNL after allowing for the correlation between choices. And the variance of the probability of not making a payment is 0.0279 for borrowers who stay in the forbearance program for 6 months and 0.0283 for borrowers who stay in the forbearance program for 18 months. The variance of the probability for not making payments tends to decrease by using the DNL instead of using the DMNL.

Figure 2.7(b) presents graphs of the pdf of the expected probability of choosing G_1 for the DNL using the estimated shape parameters and the mean values of the exogenous variables to study the choices between nests ($\alpha_{G_1} = \exp\{\theta ln(\exp\{\bar{z}_1'\beta_1/\theta\}+\exp\{\bar{z}_2'\beta_2/\theta\})\}$ and $\alpha_{G_2} = \exp\{\bar{z}_3'\beta_3\}$). The average expected probability of choosing G_1, $\alpha_{G_1}/(\alpha_{G_1} + \alpha_{G_2})$, is 0.633 (the vertical red line in Figure 2.7(b)) for loans that stay in the forbearance program for 6 months and 0.722 (the vertical green line in Figure 2.7(b)) for loans that stay in the forbearance program for 18 months. The variance of the probability of choosing G_1 is 0.0824 for borrowers who stay in the forbearance program for 6 months and 0.0823 for borrowers who stay in the forbearance program for 18 months. The variances of the probability of choosing G_1 are higher than the beta distribution under the DMNL.

The Accuracy Rate

The MNL may be misleading in panel data because of heterogeneity. The distribution of the payment probabilities is assumed to be Dirichlet distributed to solve the heterogeneity problem. The DNL additionally relaxes restrictions on state dependence and correlation between choices. The fit of the estimated models can be examined by comparing the accuracy rate. The monthly accuracy rates for the three models, defined as the percentage of times that the behavior predicted with the highest probability is the behavior chosen by the borrower, are shown in Figure 2.8. The figure shows that the accuracy rates for the DNL model and DMNL are about twice as high as those for the MNL. The rates increase dramatically in the first three months for the DNL and DMNL. This increase is larger the higher is the forbearance age.

In-sample and Out-of-sample Tests

For cross-sample validation, this chapter uses a method similar to that of Clapp, Deng, and An (2006), but instead of holding back a 10 percent sub-sample for validation, this chapter holds back a 20 percent sub-sample. An in-sample and an out-of-sample test are performed to compare the DMNL and the DNL. For the out-of-sample test, the method is as follows:

(a) Randomly draw 80 percent of the loans from the CIRT forbearance sample as the estimation sub-sample.

(b) Use the remaining 20 percent of the loans as the validation sub-sample.

(c) Use the estimation sub-sample to estimate the DMNL and DNL.

(d) Calculate the predictive probabilities of not making payments, making curtailment payments, and making scheduled payments each month using equation (37) and equation (40).

(e) Regress the indicators for the outcomes on the respective predictive probabilities for not making payments, making curtailment payments, and making scheduled payments.

(f) Compare the value of the R-squared of the above regressions.

Table 2.5 and Table 2.6 present the in-sample and out-of-sample test results. The R-squared for each model's in-sample and out-of-sample prediction is very high, especially for the predictive probabilities of making scheduled payments (around 0.5) and not making payments (around 0.7). Moreover, the coefficient of the predictive probabilities is always very close to 1. For the in-sample test, the R-squared for curtailment payments is

10.85 percent higher in the DNL. For the other two payment behaviors, the DMNL has a higher R-squared by 0.41 percent for scheduled payment and by 0.94 percent for non-payment. For the out-of-sample test, the R-squared for curtailment payments is 8.1 percent higher in the DNL. For the other two payment behaviors, the DMNL has a higher R-squared by 0.2 percent for scheduled payment and by 0.95 percent for non-payment. The results show that the DNL performs much better than the DMNL in predicting the probability of making curtailment payments, and the DMNL is slightly superior to the DNL for scheduled payments and non-payment.

2.4.3 How Do Borrowers Exit Forbearance?

Borrowers who enter the forbearance program are not required to provide documentation giving their reasons for wanting to be part of the program. Yet, these reasons are an important part of the explanation of the different behavior patterns across borrowers. This missing piece of data as well as other possibly important unobserved factors are the reason for introducing controls for unobserved heterogeneity in the payment behavior probabilities. The DMNL and DNL accomplish this by giving the payment behavior probabilities a Dirichlet distribution. In these models, current payment behavior is correlated with past payment behavior. Borrowers, directly or indirectly affected by Covid-19, are more likely to not pay their payments after enrolling in the forbearance program. Moreover, these borrowers have difficulty paying back the forborne amount before they exit the program and are therefore more likely to exit forbearance with payment deferral or trial/modification. On the other hand, borrowers who join the forbearance program but are not experiencing a Covid-19 related hardship are likely to make their payments and exit forbearance with reinstatement or prepayment. This can be seen in Figure 3.3, which shows the average proportion of making curtailment payments, making scheduled payments, and not making payments for each of the exit types.

A two-step estimation technique is used to study how borrowers exit the forbearance program. In the first step, the results of the DMNL and DNL discussed in section 2.4.2 are used to calculate borrower payment behavior in the termination period. These predictive probabilities are used as controls in an MNL for choices of exit type in the termination period. Table 2.7 shows the results of the MNL with and without controlling for the predictive probability of borrower payment behavior.[14] Controlling the borrower payment behavior increases the likelihood substantially. Table 2.8 presents the results of the marginal effects of payment behavior on forbearance exit type probabilities.[15] As expected, the borrowers who have a higher probability of making curtailment payments are more likely to exit forbearance with prepayment or reinstatement. The magnitude of the effects decreases with the duration of the forbearance program. A 1 percent increase in the probabilities of making a curtailment payment will increase the probability of prepayment (reinstatement) by about 1.38 percent (1.79 percent) for borrowers who exit forbearance at the 6^{th} month.[16] The effect drops to 0.18 percent (0.08 percent) for borrowers who exit at the 18^{th} month. On the other hand, the probability of not paying increases the probability of payment deferral or trial/modification. A 1 percent increase in the probability of not paying will increase the probability of payment deferral (trial/-modification) by about 1.26 percent (0.12 percent) for borrowers who exit forbearance at the 6^{th} month, and by 0.32 percent (0.41 percent) for borrowers who exit at the 18^{th} month.

The results suggest that the exit behavior of borrowers may be affected by other factors as well. The probability of prepayment and reinstatement increases when the benefit of refinancing is positive; similarly a higher benefit of payment deferral decreases the probability of prepayment and reinstatement. The estimates suggest that the probability of prepayment increases with the FICO score (also shown in Capponi, Jia, and Rios

[14]The full results are presented in Table 2.13 in Appendix.

[15]The formula for the average marginal effects are presented in appendix 3. The Delta method is used to calculate the standard error for the average marginal effect.

[16]This positive effect on the probability of prepayment and reinstatement can be explained by Adelman, Cross, Shrider (2010). They find the higher propensities to save are strong positively correlated with the probability of curtailing the mortgage.

2021), while the probability of trial/modification decreases with the FICO score. Borrowers, who received stimulus checks, are more likely to exit forbearance with prepayment, reinstatement, or repayment plan in the 6^{th} month, but not in the 12^{th} or 18^{th} month.

2.4.4 Types of Borrowers

Mortgagors are not uniformly impacted by the Covid-19 pandemic and may have different motivations or incentives, many of which are unobserved, for the same mortgage payment behavior. The method for dealing with the unobserved heterogeneity problem in the forbearance program is to assign a probability distribution to the borrower payment behavior probabilities.

The DNL models assign a Dirichlet distribution to the borrower payment behavior probabilities and allow different observed payment behaviors to be more closely related than others. The DNL models can be written as the product of two beta functions: the first defines the probability of choices within the nest, and the second defines the probability of choices across nests, and corresponding probabilities both for the choices within the nest and the choices of nests follows a beta distribution. The estimates of the DNL discussed in section 2.4.2 are used to determine the shape of the beta distribution for the payment behavior probabilities for each borrower. The shape of the beta distribution for the choices within the nest composed of not making a payment and making curtailment payments is determined by the estimated shape parameters from Table 2.4 and the values of the exogenous variables. Within this nest (G_1) the expected probability of not making a payment for i^{th} borrower is

$$E(\hat{\pi}_{1i}) = \hat{\alpha}_{1i}/(\hat{\alpha}_{1i} + \hat{\alpha}_{2i}) \ , \tag{43}$$

where $\hat{\alpha}_{1i} = \exp\{z_i^{\prime}\hat{\beta}_1/\hat{\theta}\}$ and $\hat{\alpha}_2 = \exp\{z_i^{\prime}\hat{\beta}_2/\hat{\theta}\}$. The beta distribution for the choices between nests has the expected probability of choosing G_1 is

$$E(\hat{\pi}_{G_1}) = \hat{\alpha}_{G_1}/(\hat{\alpha}_{G_1} + \hat{\alpha}_{G_2}) \ . \tag{44}$$

where $\hat{\alpha}_{G_1} = \exp\{\hat{\theta} ln(\exp\{z_i'\hat{\beta}_1/\hat{\theta}\} + \exp\{z_i'\hat{\beta}_2/\hat{\theta}\})\}$ and $\hat{\alpha}_{G_2} = \exp\{z_i'\hat{\beta}_3\}$. In this section, borrowers are divided into two classes, those with the expected probability of paying (P) less than 50 percent and those with a probability of at least 50 percent. Then I reestimate the MNL for exits from the program for each of these two classes.

Table 2.9 provides summary statistics of the explanatory variables by borrower class. The table shows that the proportion of borrowers less likely to pay increases with the final duration of participation in the forbearance program. This class of borrowers has a higher average benefit of deferral, a longer average time in delinquency before entering the program, a lower average FICO score, a higher average loan-to-value ratio, and a higher average debt-to-income ratio. Borrowers in this class are more likely to be a first-time home buyer and less likely to have a co-borrower.

Table 2.10 presents the regression results and marginal effects. The results show how borrowers' choice of exit types may be affected differently by the payment behavior of different classes of borrowers. The probability of making a curtailment payment has a larger estimated impact on the choices of prepayment and reinstatement for the borrowers more likely to pay. For this class of borrower, the marginal effect of a 1 percent increase in the probability of making a curtailment payment will increase the probabilities of both prepayment and reinstatement by about 0.15. On the other hand, the estimated marginal effect of a 1 percent increase in the probability of not paying on the probability of trial/modification, an exit type that requires documentation, is more than five times higher for the borrowers less likely to pay. In comparison, the probability of not paying has a smaller estimated marginal effect on the probability of payment deferral, an exit type that does not require documentation, for the borrowers less likely to pay (0.76 percent) than those more likely to pay (1.09 percent). These results confirm the importance of using payment behavior in the forbearance program to predict the exit type.

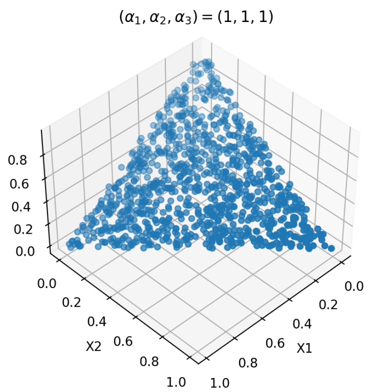
2.5 Conclusions

The results indicate that there are important unobserved factors, such as work status, health situation, attitude toward risk, age, and gender, that lead to differences between observed behavior and behavior predicted by MNL. To solve this unobserved heterogeneity problem, this chapter extends the work of Heckman and Willis (1977) to construct the DMNL and the DNL. For these models, the predictive probabilities of payment behavior are determined not only by borrower and loan characteristics, but also by past payment behavior. The DNL differs from the DMNL by allowing different degrees of substitutability across payment types. The accuracy rates for the DMNL and the DNL are found to be about twice as high as those for the MNL. The empirical results demonstrate considerable unobserved heterogeneity. In the DNL, the beta distribution for the probabilities of choices within the nest and the probabilities of choice of the nest are both J shaped because relatively few borrowers have probabilities near the mean. And the average expected probability of not making a payment tends to increase with the duration of the forbearance program.

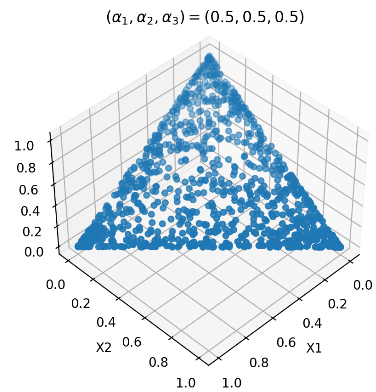
A two-step estimation technique is used to study how borrowers exit the forbearance program. The predictive payment behavior probabilities from the DMNL and DNL are used as controls in an MNL for choices of exit type in the termination period. The results show that the probability of making curtailment payments has a positive and significant effect on prepayment and reinstatement, and the probability of not making a payment has a positive and significant effect on trial/modification and payment deferral. The result shows that a 1 percent increase in the probability of making curtailment payment will increase the probability of prepayment by about 1.38 percent (0.18 percent) for borrowers who exit forbearance at the 6^{th} (18^{th}) month. And a 1 percent increase in the probability of not paying will increase the probability of payment deferral by about 1.26 percent (0.32 percent) for borrowers who exit forbearance at the 6^{th} (18^{th}) month.

2.6 Figures

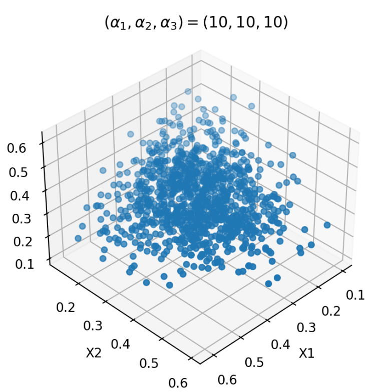
Figure 2.1: Scatter Plots of the Dirichlet Distribution



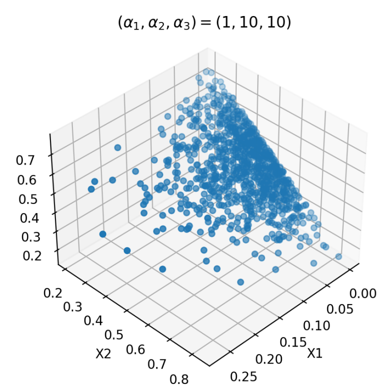
(a)



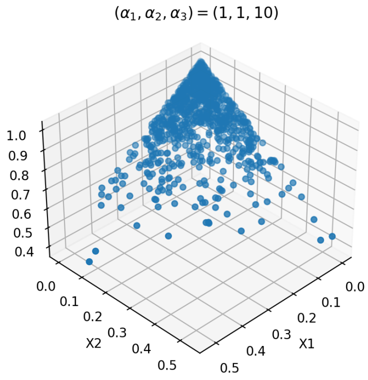
(b)



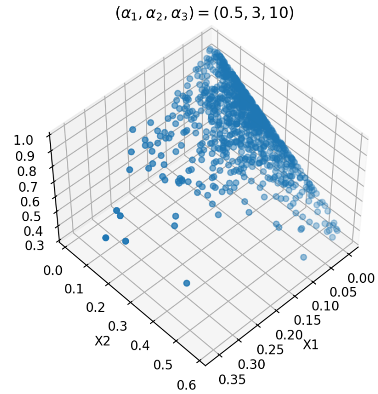
(c)



(d)



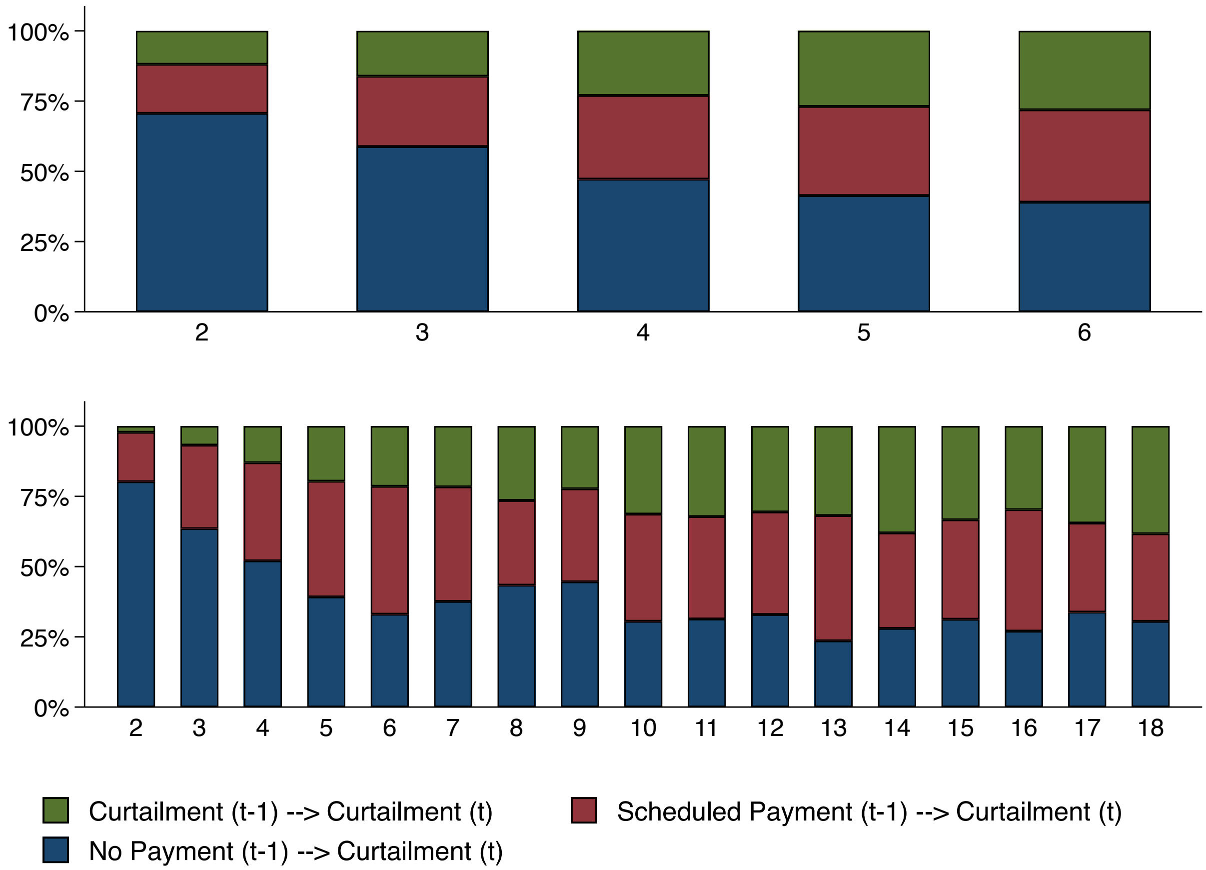
(e)



(f)

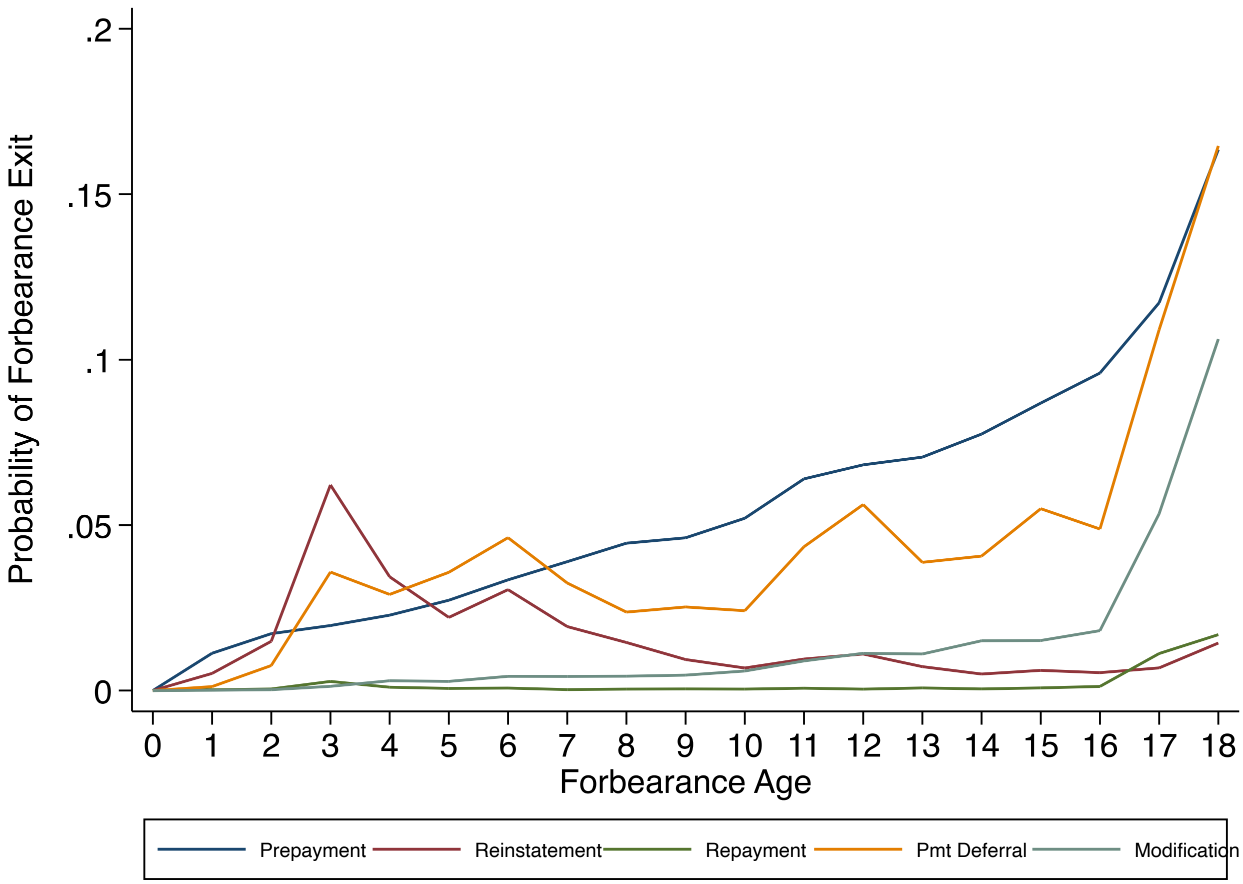
Notes: This figure shows the Dirichlet distributions under different shape parameters $(\alpha_1\ \alpha_2\ \alpha_3)$ for $K = 3$. Those shape parameters govern the shape of the distribution.

Figure 2.2: Payment Proportion for Month $t-1$ Given Curtailment in Month t



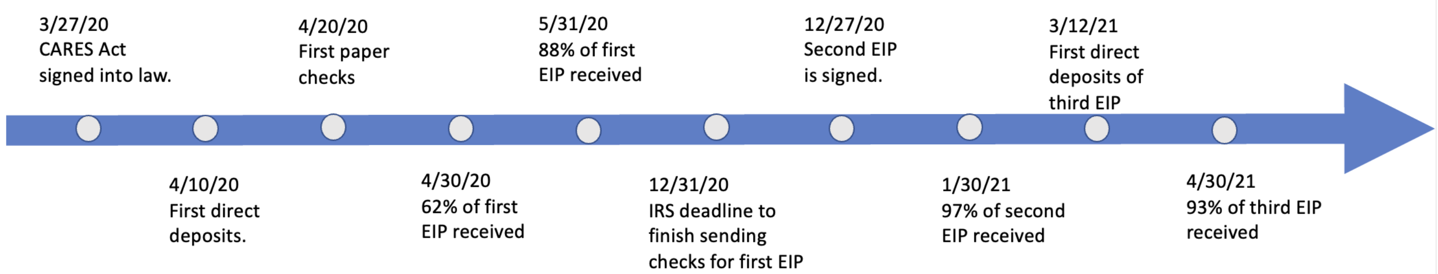
Notes: This figure presents payment proportions for the month $t-1$ for mortgages with curtailment payments in month t for borrowers with the forbearance duration of 6 and 18 months.

Figure 2.3: Forbearance Exits



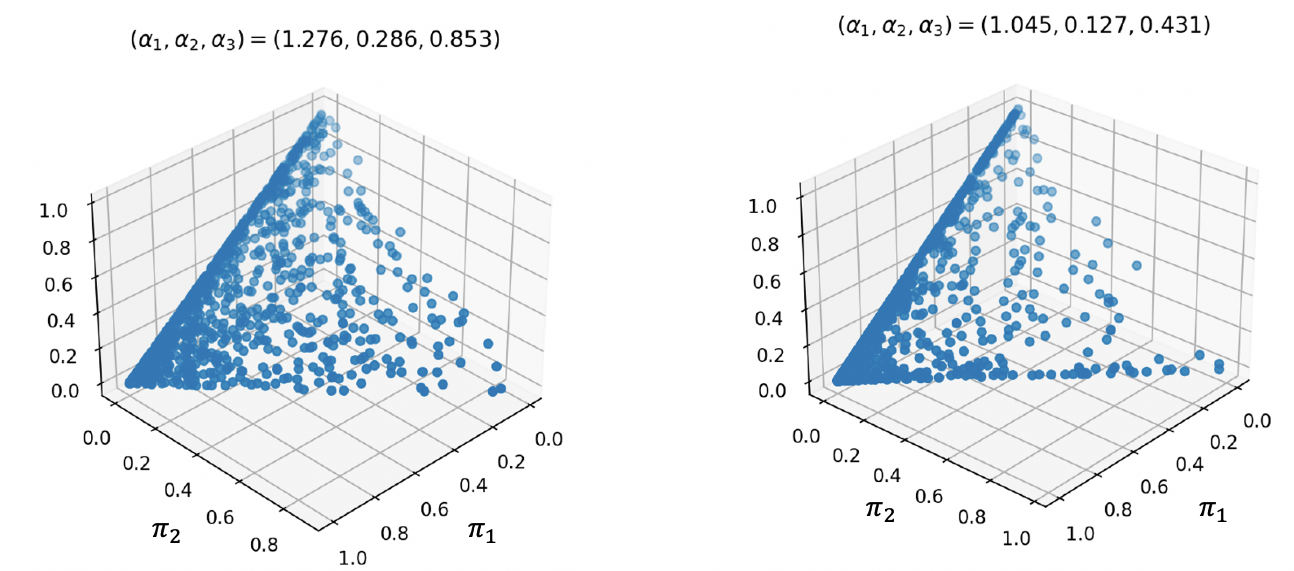
Notes: This figure shows the conditional rate of exit from the forbearance program from March 2020 to March 2022 for all forborne loans in the CIRT data.

Figure 2.4: Timeline of Economic Impact Payments



Notes: This figure shows the timeline of Economic Impact Payments. The disbursement of the EIP payments over time is based on the US Department of the Treasury.

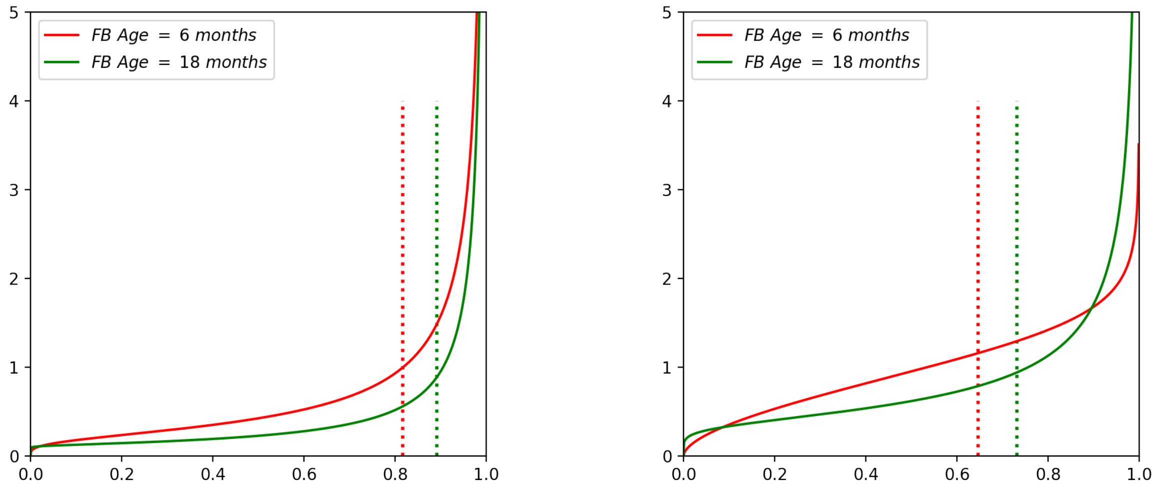
Figure 2.5: The Dirichlet Distribution for the DMNL



(a) FB age = 6 (b) FB age = 18

Notes: This figure shows the Dirichlet distributions of borrowers' payment behaviors with average characteristics for borrowers with a forbearance duration of 6 and 18 months. And the shape parameters in the Dirichlet distribution are evaluated as an exponential function of the mean values of the exogenous variables: $\alpha_i = e^{\bar{z}_i^{'}\beta_i}$.

Figure 2.6: The Beta Distribution for the DMNL

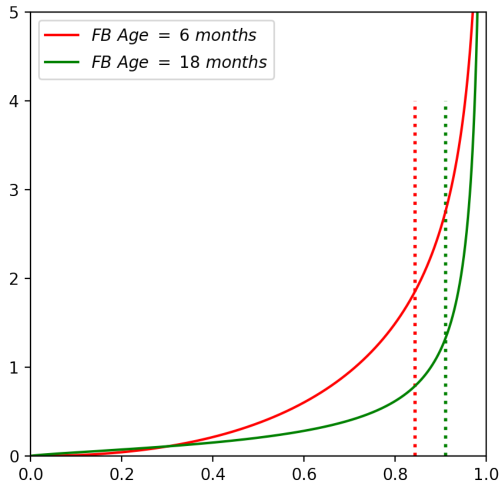


(a) Within the Nest

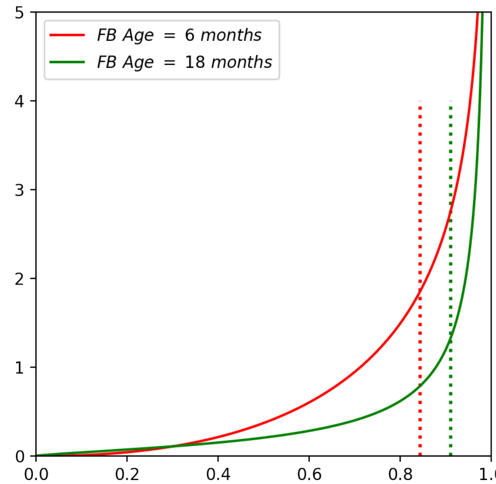
(b) Between Nests

Notes: Figure 2.6(a) presents graphs of the Beta distribution of the expected probability of not making a payment for the DMNL. And the vertical line shows the average expected probability of not making payments. Figure 2.6(b) presents graphs of the Beta distribution of the expected probability of choosing G_1 for the DMNL. And the vertical line shows the average expected probability of choosing G_1.

Figure 2.7: The Beta Distribution for the DNL



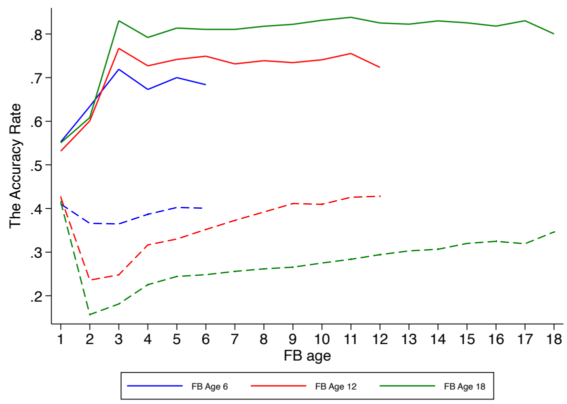
(a) Within the Nest



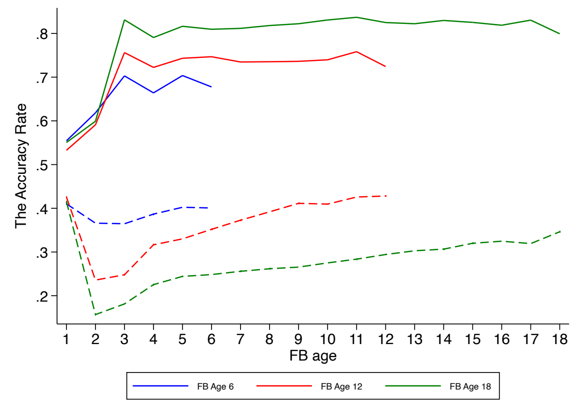
(b) Between Nests

Notes: Figure 2.7(a) presents graphs of the Beta distribution of the expected probability of not making a payment for the DNL. And the vertical line shows the average expected probability of not making payments. Figure 2.7(b) presents graphs of the Beta distribution of the expected probability of choosing G_1 for the DNL. And the vertical line shows the average expected probability of choosing G_1.

Figure 2.8: The Accuracy Rate



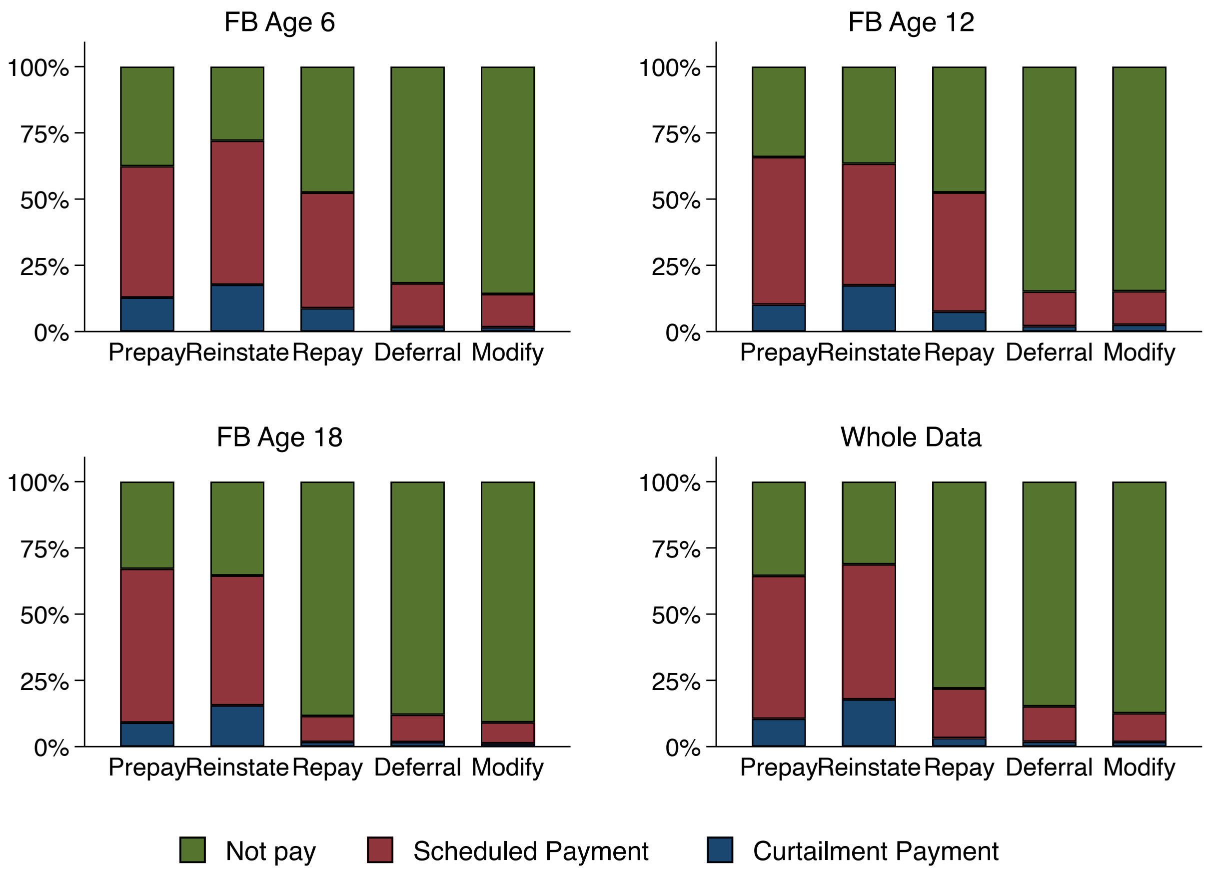
(a) MNL vs DMNL



(b) MNL vs DNL

Notes: This figure shows the monthly accuracy rates for three models. The monthly accuracy rates are calculated as the percentage of times that the behavior predicted with the highest probability is the behavior chosen by the borrower. The MNL has the dashed lines, and the DMNL has the solid lines in Figure 2.8(a). The MNL has the dashed lines, and the DNL has the solid lines in Figure 2.8(b).

Figure 2.9: Payment Behavior by Forbearance Exits



Notes: This figure shows the average proportion of making curtailment payments, making scheduled payments, and not making payments for each exit type.

2.7 Tables

Table 2.1: Explanatory Variables and Definition

Variables	Definition
Monthly Benefit of Refinance	The difference between the current monthly payment and the monthly payment after refinancing. The variable is divided by a thousand in the estimation.
Value of Payment Deferral	The gains of investing the values of missed payments into a 10-year treasury bond until the end of the loan[a]. The variable is divided by a thousand in the estimation.
Current FICO Score Before Covid	The score has a minimum value of 300 and a maximum value of 850. Group mortgages by FICO in Feb 2020 (before Covid-19) into: poor (300-579), fair (580-669), good (670-739), very good (740-799), and exceptional (800-850).
Marked-to-Market Loan-to-Value (MLTV)	It is calculated by $\frac{(Original\,Loan{-}to{-}Value)(Current\,Unpaid\,Balance)}{(Original\,Unpaid\,Balance)(\frac{House\,Price\,Index_t}{House\,Price\,Index_0})}$[b]. The variable is divided by a hundred in the estimation.
First-Time Home Buyer Dummy	If the borrower or co-borrower bought the house as a first-time home buyer, the dummy equals 1, and zero otherwise.
Co-Borrower Dummy	If the borrower has a co-borrower, the dummy equals 1, and zero otherwise.
Original Debt-to-Income	It is calculated by Monthly Debt/Stable Monthly Income.
Original Loan Size	The original loan amount has been grouped by $0 - 33^{th}$, $33^{th} - 66^{th}$, and $66^{th} - 100^{th}$ quantiles.
The Number of Delinquency Spell	The number of times the mortgage had been delinquent within 18 months before Covid-19 started.
Average Duration of Delinquency Spell	Define as (number of delinquent months within 18 months before Covid-19 started)/18 if the loan age is greater than 18 when Covid-19 started; (number of delinquent months within 18 months before Covid-19 started)/(loan age) if the loan age was less than 18 when Covid-19 started.
Economic Impact Payment	The average amount of receipts for Economic Impact Payments authorized by CARES Act[c]. The variable is divided by a thousand in the estimation
High Covid Rate US Dummy	An indicator that denotes if $\frac{Covid{-}19\,Cases\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the national level $\frac{Covid{-}19\,Cases\,in\,the\,US}{US\,Population}$. And $\frac{Covid{-}19\,Deaths\,in\,the\,MSA}{MSA\,Population}$ in the MSA is also greater than the national level ($\frac{Covid{-}19\,Deaths\,in\,the\,US}{US\,Population}$).
High Covid Rate State Dummy	An indicator that denotes if $\frac{Covid{-}19\,Cases\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the State level $\frac{Covid{-}19\,Cases\,in\,the\,State}{State\,Population}$, and $\frac{Covid{-}19\,Deaths\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the State level ($\frac{Covid{-}19\,Deaths\,in\,the\,State}{State\,Population}$).

Variables	Definition
High Unemployment Rate US Dummy	An indicator that denotes if the monthly unemployment rate in the MSA is greater than the national level.
High Unemployment Rate State Dummy	An indicator that denotes if the monthly unemployment rate in the MSA is greater than the State level.
Income Dummy	An indicator that denotes the income[d] thresholds of $75000 for individuals and $150000 for couples filing jointly.
The Number of Other Disaster[e]	Number of disaster types is happening at the same time when the borrower joints the forbearance program.
Originate Year Dummy	Fannie Mae and Freddie Mac implemented CRT programs in 2013. CRT data includes loans originating from 2013 to 2021.
Forbearance Duration	The duration of the loans stay in the forbearance program with a maximum duration time of 18 months.

[a] This chapter assumes the average life of a mortgage is ten years.
[b] Three-Digit ZIP Codes House Price Index (HPI) from FHFA has been used to calculate MLTV.
[c] The disbursement of the EIP payments over time is based on the US Department of the Treasury.
[d] Approximate monthly income is calculated by (Monthly Mortgage Payment)*12/(Original Debt-to-Income).
[e] Other types of disaster include coastal storm, dam/levee break, earthquake, fire, flood, hurricane, mud/landslide, severe ice storm, severe storm, snow, and tornado.

Table 2.2: Summary Statistics by Borrower Payment Behavior

	FB Age 6			**FB Age 12**			**FB Age 18**		
	No Pay	Curtail	Regular	No Pay	Curtail	Regular	No Pay	Curtail	Regular
Num. Delinquency Spell	0.243	0.188	0.150	0.295	0.211	0.157	0.227	0.234	0.166
	(0.707)	(0.662)	(0.546)	(0.786)	(0.671)	(0.580)	(0.658)	(0.661)	(0.555)
Avg DUR of Delinq	0.408	0.210	0.223	0.517	0.316	0.248	0.381	0.434	0.297
	(1.564)	(0.845)	(1.051)	(1.794)	(1.383)	(1.218)	(1.458)	(1.687)	(1.235)
Other Disaster	0.247	0.248	0.247	0.430	0.451	0.495	0.601	0.614	0.629
	(0.508)	(0.490)	(0.480)	(0.602)	(0.588)	(0.591)	(0.705)	(0.711)	(0.712)
High Covid US	0.233	0.254	0.224	0.213	0.233	0.211	0.225	0.221	0.219
	(0.423)	(0.435)	(0.417)	(0.410)	(0.423)	(0.408)	(0.418)	(0.415)	(0.413)
High Covid State	2.244	1.879	1.764	3.881	4.169	4.144	5.330	6.007	5.972
	(3.314)	(2.630)	(2.391)	(3.866)	(3.774)	(3.755)	(4.550)	(4.632)	(4.684)
High UNRATE US	0.497	0.543	0.523	0.506	0.505	0.533	0.507	0.497	0.515
	(0.500)	(0.498)	(0.499)	(0.500)	(0.500)	(0.499)	(0.500)	(0.500)	(0.500)
High UNRATE State	0.416	0.446	0.431	0.430	0.447	0.428	0.404	0.412	0.4262
	(0.493)	(0.497)	(0.495)	(0.495)	(0.497)	(0.495)	(0.491)	(0.492)	(0.495)
Current FICO	692.019	718.302	714.512	689.026	712.066	713.909	686.484	701.064	704.505
	(84.973)	(76.805)	(76.782)	(85.444)	(81.884)	(73.657)	(85.049)	(82.721)	(75.830)
MLTV	69.441	65.536	68.006	70.519	69.465	70.437	72.393	72.531	73.060
	(12.940)	(14.699)	(12.679)	(12.087)	(13.039)	(11.952)	(11.483)	(11.707)	(11.640)
OFICO	723.313	735.183	730.220	722.722	730.287	730.132	722.134	726.405	725.492
	(46.601)	(46.963)	(46.883)	(46.794)	(48.068)	(47.306)	(47.243)	(45.146)	(45.720)
ODTI	38.996	37.852	38.708	39.179	38.554	39.026	39.815	39.103	39.658
	(7.829)	(8.745)	(8.025)	(7.693)	(8.262)	(7.676)	(7.783)	(8.090)	(7.528)
Loan Size	251044	252925	263928	266175	256491	277072	277553	245084	263346
	(133112)	(138750)	(133769)	(140490)	(132876)	(137743)	(141002)	(129434)	(134699)
First-Time Home Buyer	0.326	0.297	0.283	0.321	0.314	0.291	0.302	0.336	0.300
	(0.469)	(0.457)	(0.451)	(0.467)	(0.464)	(0.454)	(0.459)	(0.472)	(0.458)
Co-borrower	0.405	0.421	0.426	0.389	0.418	0.435	0.368	0.413	0.419
	(0.491)	(0.494)	(0.494)	(0.488)	(0.493)	(0.496)	(0.482)	(0.492)	(0.493)
Enter FB <3 Months	0.635	0.696	0.697	0.635	0.722	0.719	0.738	0.732	0.723
	(0.482)	(0.460)	(0.460)	(0.481)	(0.448)	(0.450)	(0.440)	(0.443)	(0.448)
Enter FB $3-6$ Months	0.210	0.212	0.217	0.237	0.213	0.211	0.237	0.252	0.259
	(0.407)	(0.409)	(0.413)	(0.426)	(0.410)	(0.408)	(0.425)	(0.434)	(0.438)
Enter FB >6 Months	0.155	0.092	0.086	0.128	0.064	0.070	0.026	0.016	0.018
	(0.362)	(0.289)	(0.280)	(0.334)	(0.245)	(0.255)	(0.159)	(0.126)	(0.134)
Num. Loans		5224			3396			2919	

Notes: This table summarizes the explanatory variables by payment behaviors for borrowers with the forbearance duration of 6, 12, and 18 months. Standard errors are in parentheses.

Table 2.3: Summary Statistics by Forbearance Exits

	Continue	Forbearance Exits				
		Prepay	Reinstate	Repay	Defer	Modify
	(1)	(2)	(3)	(4)	(5)	(6)
Monthly Benefit of Refinance	214.130	238.128	197.063	181.647	156.213	210.356
	(149.999)	(160.741)	(143.570)	(134.597)	(130.245)	(157.100)
Value of Deferral	392.347	116.678	51.408	775.681	876.142	1629.340
	(750.410)	(436.785)	(221.126)	(1228.799)	(1053.404)	(1472.079)
Economic Impact Payment	237.139	241.221	93.600	75.633	193.489	149.956
	(457.802)	(509.608)	(333.557)	(283.576)	(496.957)	(419.760)
Delinquency Spell	0.232	0.112	0.159	0.306	0.234	0.519
	(0.693)	(0.475)	(0.582)	(0.770)	(0.694)	(0.981)
Avg Duration of Delinquency Spell	0.412	0.167	0.1797	0.663	0.362	1.159
	(1.607)	(0.936)	(1.169)	(2.111)	(1.428)	(2.760)
Num. of Other Disaster	0.455	0.694	0.428	0.613	0.637	0.851
	(0.631)	(0.695)	(0.614)	(0.767)	(0.711)	(0.769)
Covid Cases Rate	4.232	6.741	3.594	6.666	6.564	9.825
	(4.527)	(4.938)	(4.183)	(5.805)	(5.366)	(5.911)
Covid Deaths Rate	0.079	0.111	0.073	0.114	0.112	0.151
	(0.071)	(0.077)	(0.065)	(0.087)	(0.080)	(0.086)
Unemployment Rate	8.832	7.027	8.786	7.080	6.988	5.793
	(4.018)	(2.899)	(3.425)	(3.181)	(2.846)	(2.244)
Current FICO	699.826	730.584	716.340	683.783	695.683	659.226
	(83.536)	(64.213)	(76.095)	(85.740)	(83.246)	(92.520)
Marked-to-Market LTV	67.165	64.896	64.571	65.174	67.905	63.248
	(12.479)	(12.279)	(13.200)	(12.874)	(12.521)	(11.745)
Original FICO	725.605	733.454	731.489	721.822	722.499	712.853
	(46.749)	(45.343)	(47.584)	(47.575)	(46.876)	(45.526)
Original Debt-to-Income	39.094	38.883	38.326	37.506	39.055	39.817
	(7.840)	(7.886)	(8.158)	(8.146)	(7.769)	(7.479)
Original Loan Size	265,943	285,880	233,060	229,385	251,464	265,188
	(138,581)	(136,618)	(129,837)	(130,679)	(132,567)	(142,572)
First-Time Home Buyer	0.312	0.276	0.328	0.381	0.337	0.332
	(0.463)	(0.447)	(0.469)	(0.486)	(0.473)	(0.471)
Co-Borrower	0.396	0.439	0.395	0.377	0.394	0.352
	(0.489)	(0.496)	(0.489)	(0.485)	(0.489)	(0.478)
N	620,659	23775	12502	690	19237	3782

Notes: This table provides summary statistics of the explanatory variables by forbearance exit type.
Column (1) shows the summary statistics for mortgages that are still in the forbearance plan.
Columns (2)-(6) show the corresponding summary statistics mortgages at the exiting period for each exit type: prepayment, reinstatement, repayment plan, payment deferral, and trial/modification. Standard errors are in parentheses.

Table 2.4: Results for the DMNL and DNL
(standard errors are in parentheses)

	FB Age 6			FB Age 12			FB Age 18		
	MNL	DMNL	DNL	MNL	DMNL	DNL	MNL	DMNL	DNL
Not Making Payment									
Num. Delinquency Spell	0.1715	0.2923	0.0464	0.2791	0.4352	0.0684	-0.0037	0.3577	0.0308
	(0.0496)	(0.1179)	(0.0240)	(0.0412)	(0.0410)	(0.0351)	(0.0354)	(0.0605)	(0.0360)
Avg DUR of Delinq Spell	0.0330	-0.0265	-0.0042	0.0134	-0.0333	-0.0010	-0.0221	-0.0508	-0.0048
	(0.0123)	(0.0304)	(0.0098)	(0.0092)	(0.0186)	(0.0068)	(0.0086)	(0.0221)	(0.0057)
Fair FICO	-0.1765	-0.1041	0.0012	-0.2910	-0.1183	-0.0065	-0.3937	-0.3831	-0.0125
	(0.0469)	(0.0784)	(0.0152)	(0.0423)	(0.0485)	(0.0228)	(0.0364)	(0.0535)	(0.0189)
Good FICO	-0.2012	-0.0451	0.0183	-0.4066	-0.1548	0.0001	-0.5576	-0.4727	-0.0186
	(0.0467)	(0.0815)	(0.0174)	(0.0422)	(0.0521)	(0.0252)	(0.0364)	(0.0556)	(0.0248)
Very Good FICO	-0.4941	-0.3428	-0.0116	-0.6418	-0.3219	-0.0297	-0.7250	-0.6280	-0.0261
	(0.0482)	(0.0866)	(0.0194)	(0.0435)	(0.0552)	(0.0281)	(0.0379)	(0.0601)	(0.0328)
Exceptional FICO	-0.5657	-0.3823	-0.0216	-0.6528	-0.2640	-0.0513	-0.8670	-0.7342	-0.0399
	(0.0603)	(0.0993)	(0.0230)	(0.0553)	(0.0666)	(0.0339)	(0.0524)	(0.0727)	(0.0467)
MLTV	0.9116	1.7118	0.3459	0.0544	0.6637	0.1199	-0.6743	0.4227	0.0571
	(0.1215)	(0.2649)	(0.0856)	(0.1112)	(0.1780)	(0.0719)	(0.1055)	(0.1773)	(0.0691)
Medium ODTI	0.0375	0.0213	0.0074	0.0332	-0.1763	-0.0316	0.0486	-0.0911	0.0055
	(0.0287)	(0.0577)	(0.0170)	(0.0254)	(0.0533)	(0.0220)	(0.0241)	(0.0444)	(0.0113)
High ODTI	0.0702	0.0070	0.0078	0.0292	-0.0221	0.0149	0.0720	0.0127	0.0006
	(0.0262)	(0.0506)	(0.0158)	(0.0234)	(0.0491)	(0.0185)	(0.0222)	(0.0425)	(0.0098)
Curtailment Payment									
Num. Delinquency Spell	0.3231	0.3871	0.0594	0.3688	0.4364	0.0745	0.2334	0.4574	0.0365
	(0.0872)	(0.1103)	(0.0237)	(0.0796)	(0.0403)	(0.0325)	(0.0495)	(0.0643)	(0.0387)
Avg DUR of Delinq Spell	-0.0544	-0.1009	-0.0130	-0.0070	-0.0580	-0.0069	0.0356	0.0116	-0.0006
	(0.0280)	(0.0325)	(0.0083)	(0.0187)	(0.0209)	(0.0063)	(0.0134)	(0.0215)	(0.0026)
Current FICO	1.3208	1.5068	0.2567	0.9439	1.8055	0.3002	0.6608	0.8993	0.0796
	(0.2779)	(0.3662)	(0.0791)	(0.2805)	(0.3764)	(0.1076)	(0.2899)	(0.2525)	(0.0786)
MLTV	-1.5225	-0.2499	0.0975	-0.7941	0.0235	0.0054	-0.5846	0.2122	0.0236
	(0.1552)	(0.2192)	(0.0526)	(0.1715)	(0.1844)	(0.0540)	(0.1977)	(0.2066)	(0.0375)
Medium ODTI	-0.0798	-0.0757	-0.0050	-0.0991	-0.2854	-0.0443	-0.2758	-0.2142	-0.0073
	(0.0455)	(0.0584)	(0.0152)	(0.0489)	(0.0561)	(0.0197)	(0.0520)	(0.0451)	(0.0119)
High ODTI	-0.0773	-0.1118	-0.0079	-0.0223	-0.0436	0.0066	-0.0580	-0.0980	-0.0058
	(0.0414)	(0.0553)	(0.0144)	(0.0429)	(0.0514)	(0.0150)	(0.0456)	(0.0474)	(0.0095)
Regular Payment									
Num. Delinquency Spell		0.1170	-0.0771		0.1294	-0.1262		0.3108	0.1030
		(0.1126)	(0.0459)		(0.0396)	(0.0576)		(0.0606)	(0.0899)
Avg DUR of Delinq Spell		-0.0523	-0.0305		-0.0444	-0.0188		-0.0138	0.0052
		(0.0277)	(0.0165)		(0.0185)	(0.0170)		(0.0196)	(0.0194)
Fair FICO		0.1262	0.1652		0.2070	0.2490		0.0818	0.2525
		(0.0811)	(0.0526)		(0.0479)	(0.0862)		(0.0402)	(0.0845)
Good FICO		0.2149	0.2083		0.3016	0.3513		0.1384	0.3406
		(0.0873)	(0.0498)		(0.0504)	(0.0886)		(0.0426)	(0.0845)
Very Good FICO		0.2109	0.3630		0.3639	0.4836		0.1478	0.4180
		(0.0911)	(0.0517)		(0.0596)	(0.0890)		(0.0535)	(0.0891)
Exceptional FICO		0.1987	0.3575		0.4166	0.4802		0.1786	0.4970
		(0.1076)	(0.0689)		(0.0804)	(0.1011)		(0.0638)	(0.1273)
MLTV		0.8822	0.1100		0.6266	0.3325		0.9590	0.7801
		(0.2540)	(0.1719)		(0.1135)	(0.2243)		(0.2009)	(0.2307)
Medium ODTI		-0.0143	-0.0139		-0.2004	-0.0790		-0.1364	-0.0706
		(0.0568)	(0.0390)		(0.0508)	(0.0545)		(0.0395)	(0.0603)
High ODTI		-0.0576	-0.0372		-0.0569	-0.0250		-0.0366	-0.0358
		(0.0500)	(0.0346)		(0.0439)	(0.0450)		(0.0428)	(0.0551)
$\boldsymbol{\eta}$			-2.1150			-1.8002			-2.7021
			(0.2022)			(0.2491)			(1.0224)
Log Likelihood	-28183.40	-25094.22	-24940.86	-44770.65	-28159.85	-28085.35	-57723.28	-27935.44	-27905.20

Table 2.5: In-sample Tests
(standard errors are in parentheses)

	FB Age 6		**FB Age 12**		**FB Age 18**	
	DMNL	DNL	DMNL	DNL	DMNL	DNL
Making Curtailment Payments						
β	0.8168	0.9772	0.8506	0.9614	0.8649	0.9674
	(0.0119)	(0.0135)	(0.0102)	(0.0112)	(0.0091)	(0.0099)
R-squared	0.129	0.143	0.146	0.152	0.147	0.154
Making Scheduled Payments						
β	1.0691	1.0258	1.1058	1.0816	1.0901	1.0757
	(0.0062)	(0.0060)	(0.0052)	(0.0051)	(0.0044)	(0.0043)
R-squared	0.484	0.482	0.531	0.528	0.542	0.540
Making No Payments						
β	1.0138	1.0121	0.9953	0.9949	1.0103	1.0102
	(0.0042)	(0.0042)	(0.0032)	(0.0032)	(0.0021)	(0.0021)
R-squared	0.646	0.640	0.710	0.708	0.812	0.811

Table 2.6: Out-of-sample Tests
(standard errors are in parentheses)

	FB Age 6		FB Age 12		FB Age 18	
	MNL	NL	MNL	NL	MNL	NL
Making Curtailment Payments						
β	0.7425	0.8774	0.8393	0.9529	0.8194	0.9092
	(0.0269)	(0.0303)	(0.0235)	(0.0266)	(0.0214)	(0.0233)
R-squared	0.110	0.119	0.134	0.136	0.119	0.124
Making Scheduled Payments						
β	1.1065	1.0578	1.1158	1.0916	1.0925	1.0761
	(0.0141)	(0.0135)	(0.0117)	(0.0115)	(0.0097)	(0.0095)
R-squared	0.499	0.498	0.525	0.523	0.542	0.541
Making No Payments						
β	0.999	1.001	0.9850	0.9842	1.0130	1.0144
	(0.0100)	(0.0097)	(0.0070)	(0.0070)	(0.0046)	(0.0046)
R-squared	0.638	0.632	0.707	0.705	0.817	0.817

Table 2.7: Results of Step 2 MNL: Determinants of Exits
(standard errors are in parentheses)

	FB Age 6			FB Age 12			FB Age 18		
	MNL	MNL_{DMNL}	MNL_{DNL}	MNL	MNL_{DMNL}	MNL_{DNL}	MNL	MNL_{DMNL}	MNL_{DNL}
Prepayment									
Monthly Benefit of Refinance	6.5085	3.9223	4.0451	10.3030	7.8023	7.7791	10.5656	7.8980	7.8969
	(0.5379)	(0.4997)	(0.4990)	(0.7996)	(0.8060)	(0.7955)	(0.8576)	(0.8934)	(0.8940)
Value of Deferral	-6.6412	-1.6609	-1.9690	-2.9804	-1.8614	-1.8588	-2.6786	-1.8511	-1.8507
	(0.2317)	(0.2830)	(0.2810)	(0.1089)	(0.1338)	(0.1326)	(0.1065)	(0.1258)	(0.1250)
Economic Impact Payment	0.3240	0.1786	0.2562	-0.0290	-0.0941	-0.0974	0.0000	0.0000	0.0000
	(0.1618)	(0.1564)	(0.1604)	(0.0625)	(0.0706)	(0.0754)	(0.0000)	(0.0000)	(0.0000)
Delinquncy Spell Dummy	-0.4048	-0.6371	-0.6914	-0.9145	-0.9050	-0.9278	-0.5573	-0.6444	-0.6568
	(0.1868)	(0.1965)	(0.1936)	(0.2282)	(0.2453)	(0.2372)	(0.2667)	(0.2887)	(0.2870)
Avg Duration of Delinquency Spell	0.1571	0.1854	0.1979	0.1252	0.1269	0.1284	0.0280	0.0095	0.0079
	(0.0467)	(0.0462)	(0.0463)	(0.0519)	(0.0630)	(0.0566)	(0.0606)	(0.0711)	(0.0711)
Current FICO	5.3729	4.3983	3.3841	3.3474	3.1503	3.0280	2.3750	2.2463	2.2735
	(0.6231)	(0.6712)	(0.6725)	(0.8048)	(0.8706)	(0.8540)	(0.9714)	(1.0293)	(1.0164)
Marked-to-Market LTV	-2.2509	-1.4714	-1.2475	0.2609	0.4338	0.4621	-3.3577	-2.3607	-2.3557
	(0.4606)	(0.4929)	(0.4891)	(0.6497)	(0.7072)	(0.6709)	(0.8225)	(0.8881)	(0.8879)
Original Debt-to-Income	-1.3768	-0.8709	-0.7013	-0.0892	-0.0335	-0.0239	1.2679	1.0209	1.0134
	(0.5106)	(0.5483)	(0.5394)	(0.7165)	(0.7762)	(0.7723)	(0.9048)	(0.9473)	(0.8513)
Prob. Curtailment		0.0995	0.1215		0.0247	0.0320		-0.0088	-0.0087
		(0.0068)	(0.0084)		(0.0109)	(0.0114)		(0.0106)	(0.0110)
Prob. Not paying		-0.0460	-0.0480		-0.0338	-0.0338		-0.0310	-0.0311
		(0.0027)	(0.0026)		(0.0035)	(0.0035)		(0.0038)	(0.0037)
Reinstatement									
Monthly Benefit of Refinance	6.7687	3.8499	3.9665	10.1387	6.9504	7.0066	13.6590	10.3401	10.3489
	(0.5759)	(0.5443)	(0.5435)	(0.9189)	(0.9721)	(0.9296)	(1.2270)	(1.2638)	(1.2640)
Value of Deferral	-10.3606	-2.0719	-2.5557	-3.3858	-2.2497	-2.2925	-1.6025	-0.5381	-0.5429
	(0.3587)	(0.4077)	(0.4076)	(0.2284)	(0.2819)	(0.2559)	(0.1683)	(0.1842)	(0.1852)
Eeconomic Impact Payment	0.3245	0.2522	0.3145	0.0080	-0.0469	-0.0498	0.0000	0.0000	0.0000
	(0.1667)	(0.1659)	(0.1710)	(0.0865)	(0.0928)	(0.1004)	(0.0000)	(0.0000)	(0.0000)
Delinquncy Spell Dummy	0.0697	-0.1549	-0.1906	-0.5712	-0.6653	-0.6585	0.1698	-0.0246	-0.0316
	(0.1757)	(0.1967)	(0.1930)	(0.2902)	(0.3080)	(0.3095)	(0.3774)	(0.3935)	(0.3873)
Avg Duration of Delinquency Spell	0.0606	0.0826	0.0905	0.1682	0.1768	0.1776	0.0654	0.0478	0.0457
	(0.0506)	(0.0534)	(0.0537)	(0.0558)	(0.0606)	(0.0612)	(0.0701)	(0.0833)	(0.0892)
Current FICO	0.9683	-0.4683	-1.5548	-1.5771	-1.9958	-2.1464	-0.6227	-1.3443	-1.3038
	(0.6067)	(0.6742)	(0.6814)	(1.1027)	(1.1459)	(1.1156)	(1.5855)	(1.6557)	(1.6024)
Marked-to-Market LTV	-4.1488	-2.9136	-2.4572	-3.8904	-3.2101	-3.1679	-6.3603	-4.6707	-4.6546
	(0.4826)	(0.5369)	(0.5337)	(0.8880)	(0.9363)	(0.9231)	(1.2903)	(1.4205)	(1.4201)
Original Debt-to-Income	0.3362	1.0180	1.2580	-0.7518	-0.5903	-0.5732	3.1402	3.1180	3.0864
	(0.5358)	(0.5963)	(0.5896)	(1.0192)	(1.0689)	(1.0691)	(1.6254)	(1.6194)	(0.8511)
Prob. Curtailment		0.1152	0.1415		0.0520	0.0582		0.0149	0.0148
		(0.0067)	(0.0084)		(0.0115)	(0.0122)		(0.0120)	(0.0128)
Prob. Not paying		-0.0630	-0.0653		-0.0243	-0.0242		-0.0393	-0.0393
		(0.0028)	(0.0028)		(0.0046)	(0.0047)		(0.0064)	(0.0063)
Trial/Modification									
Monthly Benefit of Refinance	3.1190	3.3386	3.3307	4.7824	4.6258	4.6368	8.9132	9.1884	9.1958
	(0.6906)	(0.6731)	(0.6858)	(0.7079)	(0.7288)	(0.7668)	(0.5633)	(0.6230)	(0.6233)
Value of Deferral	0.9872	1.0030	0.9933	0.2635	0.3017	0.2992	0.2454	0.1028	0.1011
	(0.2297)	(0.2481)	(0.2511)	(0.0986)	(0.1096)	(0.1199)	(0.0562)	(0.0667)	(0.0674)
Eeconomic Impact Payment	-0.0684	-0.0623	-0.0561	-0.4074	-0.4082	-0.4076	0.0000	0.0000	0.0000
	(0.1985)	(0.1949)	(0.1962)	(0.0813)	(0.0879)	(0.0860)	(0.0000)	(0.0000)	(0.0000)
Delinquncy Spell Dummy	0.3001	0.2887	0.2878	0.4903	0.4841	0.4894	0.3003	0.3592	0.3638
	(0.2338)	(0.2302)	(0.2321)	(0.2208)	(0.2202)	(0.2326)	(0.1821)	(0.1875)	(0.1876)

Table 2.7: Results of Step 2 MNL: Determinants of Exits, cont'd
(standard errors are in parentheses)

	FB Age 6			**FB Age 12**			**FB Age 18**		
	MNL	MNL$_{DMNL}$	MNL$_{DNL}$	MNL	MNL$_{DMNL}$	MNL$_{DNL}$	MNL	MNL$_{DMNL}$	MNL$_{DNL}$
Avg Duration of Delinquency Spell	0.1397	0.1413	0.1421	0.0952	0.0942	0.0935	0.0110	0.0168	0.0168
	(0.0421)	(0.0369)	(0.0372)	(0.0426)	(0.0425)	(0.0449)	(0.0390)	(0.0451)	(0.0452)
Current FICO	-1.4972	-1.6121	-1.6914	-3.5581	-3.6200	-3.6268	-3.5107	-3.4364	-3.4196
	(0.9590)	(0.9388)	(0.9526)	(0.9136)	(0.9264)	(0.9243)	(0.6954)	(0.7052)	(0.7053)
Marked-to-Market LTV	-2.4023	-2.4182	-2.4073	-4.4949	-4.4902	-4.5043	-6.1439	-6.1655	-6.1697
	(0.8107)	(0.8204)	(0.8152)	(0.7858)	(0.7968)	(0.8046)	(0.6175)	(0.6528)	(0.6508)
Original Debt-to-Income	-0.4542	-0.4998	-0.4770	1.4740	1.5026	1.5023	0.3328	0.3127	0.3081
	(1.0173)	(0.3765)	(0.3202)	(0.9870)	(0.9828)	(0.9849)	(0.6955)	(0.7025)	(0.6947)
Prob. Curtailment		0.0118	0.0120		0.0038	0.0005		-0.0434	-0.0458
		(0.0187)	(0.0209)		(0.0174)	(0.0199)		(0.0176)	(0.0186)
Prob. Not paying		0.0018	0.0011		-0.0036	-0.0039		0.0087	0.0087
		(0.0054)	(0.0050)		(0.0055)	(0.0053)		(0.0047)	(0.0047)
Repayment									
Monthly Benefit of Refinance	0.6536	-2.2402	-2.3083	5.2352	2.3391	2.3267	12.0172	12.6129	12.6054
	(0.9633)	(2.0419)	(2.0402)	(3.2340)	(1.3174)	(3.2946)	(0.8700)	(0.9278)	(0.9262)
Value of Deferral	-5.8259	-2.3586	-2.1651	-1.9318	-0.6349	-0.6434	-0.3135	-0.4978	-0.4974
	(0.4952)	(1.2542)	(1.2312)	(0.6458)	(0.6742)	(0.8079)	(0.0773)	(0.0967)	(0.0941)
Economic Impact Payment	0.6622	0.5714	0.6331	-0.5018	-0.5554	-0.5572	0.0000	0.0000	0.0000
	(0.3928)	(0.3831)	(0.3828)	(0.3597)	(0.3623)	(0.3858)	(0.0000)	(0.0000)	(0.0000)
Current FICO	-2.8371	-3.9109	-4.7013	-6.5891	-7.7620	-7.7930	-2.2734	-2.1042	-2.0876
	(1.9006)	(1.8736)	(1.8764)	(2.0064)	(1.8183)	(3.4079)	(1.2460)	(1.2642)	(1.2381)
Marked-to-Market LTV	-1.2241	-2.1458	-1.9999	-0.3211	-0.9199	-0.9387	-0.9601	-0.6407	-0.6428
	(1.1397)	(1.3634)	(1.3688)	(2.2361)	(2.4050)	(2.7782)	(1.0020)	(1.0119)	(0.9953)
Prob. Curtailment		0.0662	0.0904		0.0106	0.0184		-0.0269	-0.0278
		(0.0211)	(0.0258)		(0.0392)	(0.0434)		(0.0300)	(0.0301)
Prob. Not paying		-0.0378	-0.0416		-0.0444	-0.0432		0.0229	0.0227
		(0.0088)	(0.0089)		(0.0159)	(0.0160)		(0.0085)	(0.0083)
Original Year	Y	Y	Y	Y	Y	Y	Y	Y	Y
Other Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y
Log Likelihood	-4734.41	-4328.46	-4368.06	-2376.52	-2279.17	2281.33	-2243.16	-2154.43	-2154.61

Table 2.8: Marginal Effect of Borrower Payment Behavior
(standard errors are in parentheses)

	FB Age 6		**FB Age 12**		**FB Age 18**	
	MNL_{DMNL}	MNL_{DNL}	MNL_{DMNL}	MNL_{DNL}	MNL_{DMNL}	MNL_{DNL}
Prepayment						
Prob. Curtailment	1.1404	1.3854	0.4231	0.5895	0.1650	0.1881
	(0.0970)	(0.1175)	(0.2095)	(0.2349)	(0.1794)	(0.2587)
Prob. Not paying	-0.4590	-0.4806	-0.7391	-0.7363	-0.7630	-0.7646
	(0.0438)	(0.0437)	(0.0779)	(0.0757)	(0.0615)	(0.0831)
Reinstatement						
Prob. Curtailment	1.4534	1.7933	0.2663	0.2921	0.0875	0.0892
	(0.0858)	(0.1071)	(0.0451)	(0.0509)	(0.2010)	(0.0148)
Prob. Not paying	-0.8670	-0.8957	-0.0470	-0.0475	-0.0925	-0.0925
	(0.0429)	(0.0438)	(0.0278)	(0.0270)	(0.1012)	(0.0084)
Payment Deferral						
Prob. Curtailment	-2.4168	-2.9589	-0.5969	-0.7374	0.4781	0.4977
	(0.1539)	(0.1928)	(0.2338)	(0.2663)	(0.1945)	(0.2774)
Prob. Not paying	1.2114	1.2615	0.6840	0.6837	0.3277	0.3294
	(0.0568)	(0.0565)	(0.0798)	(0.0739)	(0.0582)	(0.0810)
Trial/Modification						
Prob. Curtailment	-0.1810	-0.2307	-0.0911	-0.1440	-0.6823	-0.7252
	(0.0662)	(0.0740)	(0.1289)	(0.1263)	(0.2997)	(0.2362)
Prob. Not paying	0.1195	0.1213	0.1100	0.1075	0.4109	0.4112
	(0.0195)	(0.0181)	(0.0388)	(0.0392)	(0.0805)	(0.0609)
Repayment						
Prob. Curtailment	0.0039	0.0108	-0.0015	-0.0003	-0.0484	-0.0497
	(0.0134)	(0.0161)	(0.0107)	(0.0116)	(0.4752)	(0.4018)
Prob. Not paying	-0.0049	-0.0066	-0.0079	-0.0075	0.1169	0.1165
	(0.0056)	(0.0057)	(0.0048)	(0.0045)	(0.1310)	(0.1097)

Table 2.9: Summary Statistics by Borrower Types
(standard errors are in parentheses)

	FB Age 6		**FB Age 12**		**FB Age 18**	
	$P < 0.5$	$P \geq 0.5$	$P < 0.5$	$P \geq 0.5$	$P < 0.5$	$P \geq 0.5$
Monthly Benefit of Refinance	188.2	223.1	206.1	204.5	142.1	61.22
	(133.7)	(158.7)	(154.9)	(144.7)	(138.1)	(24.4)
Value of Deferral	283.7	125.3	818.3	508	1503	236.8
	(399.3)	(223.6)	(1114)	(937.0)	(1685)	(473.5)
Delinquncy Spell	0.1865	0.02022	0.1664	0.0048	0.1331	0.500
	(0.3896)	(0.1408)	(0.3725)	(0.0692)	(0.3397)	(0.5774)
Avg Duration of Delinquency Spell	0.5134	0.02244	0.4989	0.0048	0.3589	1.250
	(1.688)	(0.1628)	(1.748)	(0.0692)	(1.412)	(1.5)
Num. of Other Disaster	0.4978	0.5893	0.6608	0.4816	0.7043	0.250
	(0.5001)	(0.4921)	(0.4735)	(0.5001)	(0.4564)	(0.5)
Covid Cases Rate	4.546	2.746	9.307	7.900	12.40	15.03
	(4.377)	(1.987)	(4.227)	(2.931)	(5.192)	(2.501)
Unemployment Rate	6.722	7.485	5.991	6.36	4.532	4.075
	(2.114)	(2.164)	(1.939)	(1.911)	(1.356)	(1.144)
Current FICO	669.9	766.0	694.9	77.73	703.3	775.2
	(77.58)	(41.19)	(77.50)	(33.42)	(80.14)	(31.71)
Marked-to-Market LTV	69.76	60.92	64.54	64	61.55	71.95
	(12.479)	(12.27)	(12.27)	(11.93)	(11.09)	(53.64)
Original Debt-to-Income	39.76	37.25	39.54	37.05	39.75	28.25
	(7.840)	(8.466)	(7.457)	(8.561)	(7.722)	(5.377)
First-Time Home Buyer	0.3717	0.2071	0.3327	0.2064	0.3026	0.75
	(0.4833)	(0.4053)	(0.4713)	(0.405)	(0.4595)	(0.5)
Co-Borrower	0.3911	0.4502	0.3558	0.6368	0.3835	0.75
	(0.4881)	(0.4976)	(0.4788)	(0.4813)	(0.4863)	(0.5)
Prob. Not Paying	60.11	44.44	60.1	46.43	67.6	33.07
	(24.72)	(25.13)	(29.18)	(28.18)	(32.12)	(38.51)
Prob. Curtailment	7.91	12.44	6.743	9.654	4.833	8.947
	(7.507)	(12.18)	(9.099)	(12.16)	(9.382)	(8.085)
Prob. Scheduled Payment	31.98	43.13	33.16	43.91	27.57	57.98
	(24.09)	(26.55)	(27.61)	(27.19)	(29.23)	(36.14)
Proportion	0.6118	0.3882	0.8160	0.1840	0.9986	0.0014
N	3196	2028	2771	625	2915	4

Table 2.10: Determinants of Exits by Borrower Types
(standard errors are in parentheses)

	$P < 0.5$		(2) $P \geq 0.5$	
	Estimate	Marginal Effect	Estimate	Marginal Effect
Prepayment				
Prob. Curtailment	0.0609	0.9888	0.0952	1.1394
	(0.0072)	(0.1298)	(0.0122)	(0.1734)
Prob. Not paying	-0.0379	-0.6389	-0.0499	-0.5152
	(0.0020)	(0.0389)	(0.0037)	(0.0675)
Reinstatement				
Prob. Curtailment	0.0946	0.7612	0.1055	0.8973
	(0.0075)	(0.0555)	(0.0124)	(0.1112)
Prob. Not paying	-0.0530	-0.4126	-0.0635	-0.6292
	(0.0026)	(0.0243)	(0.0040)	(0.0551)
Trial/Modification				
Prob. Curtailment	-0.0077	-0.4668	0.0391	-0.0574
	(0.0126)	(0.1274)	(0.0308)	(0.0556)
Prob. Not paying	0.0039	0.2715	-0.0103	0.0528
	(0.0029)	(0.0304)	(0.0083)	(0.0154)
Repayment				
Prob. Curtailment	0.0301	-0.0014	0.0676	-0.0006
	(0.0199)	(0.0304)	(0.0403)	(0.0118)
Prob. Not paying	-0.0112	0.0119	-0.0349	0.0010
	(0.0047)	(0.0072)	(0.0158)	(0.0047)
Payment Deferral (Base)				
Prob. Curtailment		-1.2819		-1.9788
		(0.1640)		(0.2445)
Prob. Not paying		0.7682		1.0906
		(0.0412)		(0.0710)
Other Controls	Y	Y	Y	Y
Original Year	Y	Y	Y	Y
FB Age	Y	Y	Y	Y
Log Likelihood	-7107.72		-1915.18	
N	8882		2657	

2.8 Appendix

2.8.1 Tables

Table 2.11: Summary Statistic

Variable	Mean	SD	Min	Max	N
Current FICO Score Before Covid	702.7	80.15	400.0	825.0	68,313
Original FICO	727.1	46.86	620.0	829.0	68,313
Original Debt-to-Income	38.83	7.887	0.000	50.0	68,313
Original Loan Size	260,578	136,423	13,000	1350,000	68,313
Number of Delinquency Spell	0.212	0.667	0.000	8.000	68,313
Average Duration of Delinquency Spell	0.371	1.526	0.000	18.000	68,313
First-Time Home Buyer Dummy	0.314	0.464	0.000	1.000	68,313
Co-Borrower Dummy	0.401	0.490	0.000	1.000	68,313
Same Servicer and Seller Name Dummy	0.691	0.462	0.000	1.000	68,313

Time	Benef. Refin	Benef. Deferral	EIP Amt	CFICO	MLTV	No.Disaster	Covid Case	Covid Death	UNRATE
2020m3	126.860	17.392	0.000	705.069	70.795	0.005	0.047	0.001	5.084
2020m4	212.595	45.193	1017.267	707.994	69.710	0.013	0.274	0.014	14.54
2020m5	174.431	80.893	393.239	708.277	69.626	0.032	0.465	0.026	13.96
2020m6	191.682	119.254	17.528	707.798	69.640	0.051	0.692	0.032	12.05
2020m7	197.962	118.575	17.537	707.709	68.551	0.100	1.218	0.039	11.43
2020m8	229.860	155.528	31.852	705.232	68.711	0.330	1.599	0.047	9.284
2020m9	220.787	184.963	31.858	704.599	68.828	0.444	1.880	0.052	8.347
2020m10	214.470	236.144	7.486	703.327	67.480	0.576	2.342	0.057	7.148
2020m11	206.890	294.354	7.487	701.729	67.604	0.590	3.472	3.984	6.877
2020m12	273.414	347.649	7.495	699.939	67.721	0.594	5.204	0.084	6.942
2021m1	269.352	450.149	892.235	697.877	66.187	0.620	6.835	0.108	7.227
2021m2	244.321	577.885	25.244	696.554	66.213	0.698	7.418	0.126	6.945
2021m3	224.341	800.200	1826.668	695.962	66.318	0.708	7.880	0.136	6.566
2021m4	224.890	867.483	305.510	695.362	63.274	0.835	8.389	0.142	6.151
2021m5	229.703	944.840	45.556	695.912	63.902	0.865	8.634	0.147	5.831
2021m6	220.991	953.967	37.824	696.076	64.235	0.865	8.721	0.149	6.435
2021m7	233.540	876.684	16.878	694.310	61.270	0.941	9.087	0.151	5.997
2021m8	210.469	902.568	12.643	692.400	61.344	0.959	10.110	0.156	5.594
2021m9	191.310	1028.260	14.141	690.637	61.741	0.979	11.106	0.171	4.826
2021m10	168.336	1098.660	9.181	686.529	60.225	0.966	11.791	0.184	4.454
2021m11	146.104	975.803	10.062	672.841	60.905	0.914	12.672	0.195	4.000
2021m12	122.665	883.772	3.548	663.699	61.993	0.919	14.283	0.207	3.743
2022m1	73.859	1028.264	0.000	658.914	60.874	0.908	19.631	0.223	4.513
2022m2	8.778	1077.040	0.000	654.416	62.368	0.906	20.569	0.238	4.228
2022m3	-57.532	1118.412	0.000	655.071	63.452	0.940	20.788	0.246	3.846
Average	212.964	397.391	250.856	700.845	67.035	0.470	4.407	0.081	8.698

Table 2.12: Results for DMNL and DNL: Determinants of Payment Behavior (standard errors are in parentheses)

	FB Age 6			FB Age 12			FB Age 18		
	MNL	DMNL	DNL	MNL	DMNL	DNL	MNL	DMNL	DNL
Not Making Payment									
Num. Delinquency Spell	0.1715	0.2923	0.0464	0.2791	0.4352	0.0684	-0.0037	0.3577	0.0308
	(0.0496)	(0.1179)	(0.0240)	(0.0412)	(0.0410)	(0.0351)	(0.0354)	(0.0605)	(0.0360)
Avg DUR of Delinq Spell	0.0330	-0.0265	-0.0042	0.0134	-0.0333	-0.0010	-0.0221	-0.0508	-0.0048
	(0.0123)	(0.0304)	(0.0098)	(0.0092)	(0.0186)	(0.0068)	(0.0086)	(0.0221)	(0.0057)
Other Disaster	-0.0038	-0.2017	-0.0272	0.1926	0.2324	0.0409	0.0312	-0.1569	-0.0366
	(0.0505)	(0.0841)	(0.0304)	(0.0476)	(0.0869)	(0.0418)	(0.0524)	(0.0908)	(0.0411)
High Covid	-0.0231	-0.2193	-0.0357	0.0752	0.0662	0.0065	0.0969	0.1083	0.0112
	(0.0214)	(0.0856)	(0.0213)	(0.0311)	(0.0767)	(0.0237)	(0.0312)	(0.0728)	(0.0182)
High UNRATE	-0.0492	-0.0258	-0.0046	-0.0868	-0.0915	-0.0009	-0.0955	0.0015	0.0108
	(0.0238)	(0.0525)	(0.0149)	(0.0225)	(0.0384)	(0.0167)	(0.0214)	(0.0515)	(0.0132)
Fair FICO	-0.1765	-0.1041	0.0012	-0.2910	-0.1183	-0.0065	-0.3937	-0.3831	-0.0125
	(0.0469)	(0.0784)	(0.0152)	(0.0423)	(0.0485)	(0.0228)	(0.0364)	(0.0535)	(0.0189)
Good FICO	-0.2012	-0.0451	0.0183	-0.4066	-0.1548	0.0001	-0.5576	-0.4727	-0.0186
	(0.0467)	(0.0815)	(0.0174)	(0.0422)	(0.0521)	(0.0252)	(0.0364)	(0.0556)	(0.0248)
Very Good FICO	-0.4941	-0.3428	-0.0116	-0.6418	-0.3219	-0.0297	-0.7250	-0.6280	-0.0261
	(0.0482)	(0.0866)	(0.0194)	(0.0435)	(0.0552)	(0.0281)	(0.0379)	(0.0601)	(0.0328)
Exceptional FICO	-0.5657	-0.3823	-0.0216	-0.6528	-0.2640	-0.0513	-0.8670	-0.7342	-0.0399
	(0.0603)	(0.0993)	(0.0230)	(0.0553)	(0.0666)	(0.0339)	(0.0524)	(0.0727)	(0.0467)
MLTV	0.9116	1.7118	0.3459	0.0544	0.6637	0.1199	-0.6743	0.4227	0.0571
	(0.1215)	(0.2649)	(0.0856)	(0.1112)	(0.1780)	(0.0719)	(0.1055)	(0.1773)	(0.0691)
Medium ODTI	0.0375	0.0213	0.0074	0.0332	-0.1763	-0.0316	0.0486	-0.0911	0.0055
	(0.0287)	(0.0577)	(0.0170)	(0.0254)	(0.0533)	(0.0220)	(0.0241)	(0.0444)	(0.0113)
High ODTI	0.0702	0.0070	0.0078	0.0292	-0.0221	0.0149	0.0720	0.0127	0.0006
	(0.0262)	(0.0506)	(0.0158)	(0.0234)	(0.0491)	(0.0185)	(0.0222)	(0.0425)	(0.0098)
Medium Loan Size	-0.1480	-0.1110	0.0034	-0.0882	-0.1589	-0.0431	0.0991	-0.0912	-0.0039
	(0.0274)	(0.0525)	(0.0160)	(0.0240)	(0.0464)	(0.0200)	(0.0227)	(0.0601)	(0.0114)
Large Loan Size	-0.1784	-0.2715	0.0082	-0.0802	-0.0448	0.0244	0.3622	0.0266	-0.0016
	(0.0286)	(0.0514)	(0.0168)	(0.0254)	(0.0456)	(0.0205)	(0.0228)	(0.0608)	(0.0104)
Enter FB Immediately	-0.6044	-0.4371	0.0002	-0.5551	-0.2375	-0.0491	-0.2885	-0.4047	-0.0260
	(0.0445)	(0.0829)	(0.0284)	(0.0403)	(0.0803)	(0.0275)	(0.0700)	(0.1671)	(0.0419)
Enter FB (3-6 months)	-0.5645	-0.5912	-0.0438	-0.3410	-0.2693	-0.0438	-0.3922	-0.5144	-0.0269
	(0.0472)	(0.0843)	(0.0316)	(0.0433)	(0.0836)	(0.0300)	(0.0707)	(0.1681)	(0.0439)
Co-borrower	0.0064	0.0407	0.0128	-0.1195	-0.0117	-0.0033	-0.2386	-0.0799	-0.0122
	(0.0230)	(0.0473)	(0.0143)	(0.0200)	(0.0397)	(0.0160)	(0.0187)	(0.0413)	(0.0146)
First-Time Home Buyer	0.0417	-0.0477	-0.0299	0.0912	0.0083	0.0100	0.0446	-0.0535	-0.0126
	(0.0263)	(0.0486)	(0.0161)	(0.0238)	(0.0472)	(0.0170)	(0.0222)	(0.0476)	(0.0149)
Curtailment Payment									
Num. Delinquency Spell	0.3231	0.3871	0.0594	0.3688	0.4364	0.0745	0.2334	0.4574	0.0365
	(0.0872)	(0.1103)	(0.0237)	(0.0796)	(0.0403)	(0.0325)	(0.0495)	(0.0643)	(0.0387)
Avg DUR of Delinq Spell	-0.0544	-0.1009	-0.0130	-0.0070	-0.0580	-0.0069	0.0356	0.0116	-0.0006
	(0.0280)	(0.0325)	(0.0083)	(0.0187)	(0.0209)	(0.0063)	(0.0134)	(0.0215)	(0.0026)
Other Disaster	-0.0837	-0.2192	-0.0303	0.1124	0.0991	0.0193	0.1981	-0.2302	-0.0318
	(0.0911)	(0.0732)	(0.0265)	(0.1018)	(0.1158)	(0.0333)	(0.0907)	(0.0777)	(0.0351)
High Covid	0.1324	-0.0455	-0.0162	0.1345	0.0274	0.0016	0.0873	0.1146	0.0107
	(0.0428)	(0.0869)	(0.0182)	(0.0323)	(0.0824)	(0.0204)	(0.0646)	(0.0655)	(0.0153)
High UNRATE	0.1107	0.1077	0.0120	-0.0434	-0.0028	0.0089	-0.0692	0.0702	0.0101
	(0.0309)	(0.0549)	(0.0135)	(0.0403)	(0.0323)	(0.0142)	(0.0483)	(0.0709)	(0.0117)
Current FICO	1.3208	1.5068	0.2567	0.9439	1.8055	0.3002	0.6608	0.8993	0.0796
	(0.2779)	(0.3662)	(0.0791)	(0.2805)	(0.3764)	(0.1076)	(0.2899)	(0.2525)	(0.0786)
MLTV	-1.5225	-0.2499	0.0975	-0.7941	0.0235	0.0054	-0.5846	0.2122	0.0236
	(0.1552)	(0.2192)	(0.0526)	(0.1715)	(0.1844)	(0.0540)	(0.1977)	(0.2066)	(0.0375)
Medium ODTI	-0.0798	-0.0757	-0.0050	-0.0991	-0.2854	-0.0443	-0.2758	-0.2142	-0.0073
	(0.0455)	(0.0584)	(0.0152)	(0.0489)	(0.0561)	(0.0197)	(0.0520)	(0.0451)	(0.0119)
High ODTI	-0.0773	-0.1118	-0.0079	-0.0223	-0.0436	0.0066	-0.0580	-0.0980	-0.0058
	(0.0414)	(0.0553)	(0.0144)	(0.0429)	(0.0514)	(0.0150)	(0.0456)	(0.0474)	(0.0095)

Table 2.12: Results for DMNL and DNL: Determinants of Payment Behavior, cont'd (standard errors are in parentheses)

	FB Age 6			FB Age 12			FB Age 18		
	MNL	DMNL	DNL	MNL	DMNL	DNL	MNL	DMNL	DNL
Medium Loan Size	-0.1546	-0.0814	0.0022	-0.1971	-0.2557	-0.0512	-0.2328	-0.2919	-0.0187
	(0.0431)	(0.0520)	(0.0141)	(0.0449)	(0.0468)	(0.0184)	(0.0412)	(0.0707)	(0.0203)
Large Loan Size	-0.2202	-0.2017	0.0068	-0.4000	-0.2424	-0.0161	-0.3243	-0.5649	-0.0395
	(0.0474)	(0.0570)	(0.0151)	(0.0469)	(0.0524)	(0.0171)	(0.0415)	(0.0720)	(0.0387)
Enter FB Immediately	-1.2343	-1.9425	-0.2853	-1.5659	-2.5418	-0.3500	-1.6502	-2.5797	-0.1680
	(0.2380)	(0.3198)	(0.0812)	(0.2441)	(0.3146)	(0.1103)	(0.2606)	(0.1725)	(0.1597)
Enter FB (3-6 months)	-1.2707	-2.1600	-0.3324	-1.5689	-2.6883	-0.3668	-1.7188	-2.5419	-0.1628
	(0.2406)	(0.3229)	(0.0881)	(0.2452)	(0.3150)	(0.1140)	(0.2607)	(0.1713)	(0.1557)
Enter FB (>6 months)	-1.1419	-2.0930	-0.3419	-1.7172	-2.8860	-0.3937	-1.9174	-2.6893	-0.1727
	(0.2435)	(0.3225)	(0.0878)	(0.2536)	(0.3055)	(0.1241)	(0.3012)	(0.1324)	(0.1627)
Co-borrower	-0.0148	0.0049	0.0077	-0.0246	0.0557	0.0071	0.0195	0.0862	0.0024
	(0.0390)	(0.0494)	(0.0128)	(0.0371)	(0.0429)	(0.0136)	(0.0420)	(0.0477)	(0.0071)
First-Time Home Buyer	0.2125	0.0384	-0.0144	0.1653	0.0424	0.0137	0.2239	0.1142	0.0034
	(0.0388)	(0.0533)	(0.0142)	(0.0438)	(0.0464)	(0.0147)	(0.0476)	(0.0530)	(0.0084)
Regular Payment									
Num. Delinquency Spell		0.1170	-0.0771		0.1294	-0.1262		0.3108	0.1030
		(0.1126)	(0.0459)		(0.0396)	(0.0576)		(0.0606)	(0.0899)
Avg DUR of Delinq Spell		-0.0523	-0.0305		-0.0444	-0.0188		-0.0138	0.0052
		(0.0277)	(0.0165)		(0.0185)	(0.0170)		(0.0196)	(0.0194)
Other Disaster		-0.1537	-0.0199		0.0669	-0.0457		-0.1444	-0.0667
		(0.0773)	(0.0735)		(0.0788)	(0.0934)		(0.1002)	(0.1252)
High Covid		-0.1948	-0.0778		-0.0416	-0.0880		0.0136	-0.0393
		(0.0840)	(0.0184)		(0.0775)	(0.0396)		(0.0679)	(0.0737)
High UNRATE		0.0062	0.0062		0.0031	0.0571		0.0519	0.0507
		(0.0519)	(0.0279)		(0.0409)	(0.0384)		(0.0464)	(0.0534)
Fair FICO		0.1262	0.1652		0.2070	0.2490		0.0818	0.2525
		(0.0811)	(0.0526)		(0.0479)	(0.0862)		(0.0402)	(0.0845)
Good FICO		0.2149	0.2083		0.3016	0.3513		0.1384	0.3406
		(0.0873)	(0.0498)		(0.0504)	(0.0886)		(0.0426)	(0.0845)
Very Good FICO		0.2109	0.3630		0.3639	0.4836		0.1478	0.4180
		(0.0911)	(0.0517)		(0.0596)	(0.0890)		(0.0535)	(0.0891)
Exceptional FICO		0.1987	0.3575		0.4166	0.4802		0.1786	0.4970
		(0.1076)	(0.0689)		(0.0804)	(0.1011)		(0.0638)	(0.1273)
MLTV		0.8822	0.1100		0.6266	0.3325		0.9590	0.7801
		(0.2540)	(0.1719)		(0.1135)	(0.2243)		(0.2009)	(0.2307)
Medium ODTI		-0.0143	-0.0139		-0.2004	-0.0790		-0.1364	-0.0706
		(0.0568)	(0.0390)		(0.0508)	(0.0545)		(0.0395)	(0.0603)
High ODTI		-0.0576	-0.0372		-0.0569	-0.0250		-0.0366	-0.0358
		(0.0500)	(0.0346)		(0.0439)	(0.0450)		(0.0428)	(0.0551)
Medium Loan Size		0.0291	0.1049		-0.0638	0.0258		-0.1588	-0.0958
		(0.0514)	(0.0296)		(0.0429)	(0.0373)		(0.0523)	(0.0560)
Large Loan Size		-0.1000	0.1046		0.0389	0.1005		-0.2878	-0.2642
		(0.0514)	(0.0293)		(0.0485)	(0.0440)		(0.0538)	(0.0561)
Enter FB Immediately		0.1334	0.3783		0.3198	0.3877		-0.1216	0.0537
		(0.0751)	(0.0688)		(0.0824)	(0.0849)		(0.1491)	(0.1040)
Enter FB (3-6 months)		-0.0542	0.2941		0.0847	0.1968		-0.1264	0.0967
		(0.0791)	(0.0723)		(0.0833)	(0.0879)		(0.1496)	(0.1085)
Co-borrower		0.0265	0.0073		0.0947	0.0929		0.1431	0.1739
		(0.0473)	(0.0311)		(0.0402)	(0.0373)		(0.0418)	(0.0458)
First-Time Home Buyer		-0.0978	-0.0971		-0.0765	-0.0760		-0.0555	-0.0449
		(0.0484)	(0.0338)		(0.0447)	(0.0437)		(0.0488)	(0.0527)
$\boldsymbol{\eta}$			-2.1150			-1.8002			-2.7021
			(0.2022)			(0.2491)			(1.0224)
Original Year	Y	Y	Y	Y	Y	Y	Y	Y	Y
Log Likelihood	-28183.40	-25094.22	-24940.86	-44770.65	-28159.85	-28085.35	-57723.28	-27935.44	-27905.20

Notes:

Table 2.13: Results of the MNL: Determinants of Exits
(standard errors are in parentheses)

	FB Age 6			**FB Age 12**			**FB Age 18**		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Prepayment									
Monthly Benefit of Refinance	6.5085	3.9223	4.0451	10.3030	7.8023	7.7791	10.5656	7.8980	7.8969
	(0.5379)	(0.4997)	(0.4990)	(0.7996)	(0.8060)	(0.7955)	(0.8576)	(0.8934)	(0.8940)
Value of Deferral	-6.6412	-1.6609	-1.9690	-2.9804	-1.8614	-1.8588	-2.6786	-1.8511	-1.8507
	(0.2317)	(0.2830)	(0.2810)	(0.1089)	(0.1338)	(0.1326)	(0.1065)	(0.1258)	(0.1250)
Eeconomic Impact Payment	0.3240	0.1786	0.2562	-0.0290	-0.0941	-0.0974	0.0000	0.0000	0.0000
	(0.1618)	(0.1564)	(0.1604)	(0.0625)	(0.0706)	(0.0754)	(0.0000)	(0.0000)	(0.0000)
Delinquncy Spell Dummy	-0.4048	-0.6371	-0.6914	-0.9145	-0.9050	-0.9278	-0.5573	-0.6444	-0.6568
	(0.1868)	(0.1965)	(0.1936)	(0.2282)	(0.2453)	(0.2372)	(0.2667)	(0.2887)	(0.2870)
Avg Duration of Delinquency Spell	0.1571	0.1854	0.1979	0.1252	0.1269	0.1284	0.0280	0.0095	0.0079
	(0.0467)	(0.0462)	(0.0463)	(0.0519)	(0.0630)	(0.0566)	(0.0606)	(0.0711)	(0.0711)
Other Disaster	-0.0986	-0.0608	-0.0197	0.1711	0.2017	0.2016	-0.0287	0.0296	0.0275
	(0.0846)	(0.0896)	(0.0888)	(0.1181)	(0.1337)	(0.1280)	(0.1450)	(0.1622)	(0.1623)
High Covid Rate US	-0.0186	0.0180	0.0086	0.0124	-0.0398	-0.0403	0.1999	0.1796	0.1818
	(0.1139)	(0.1234)	(0.1221)	(0.1292)	(0.1470)	(0.1431)	(0.1635)	(0.1798)	(0.1785)
Unemployment Rate	-0.1196	-0.1283	-0.1402	-0.1268	-0.1476	-0.1483	-0.0764	-0.0788	-0.0788
	(0.0206)	(0.0224)	(0.0229)	(0.0316)	(0.0401)	(0.0381)	(0.0481)	(0.0540)	(0.0560)
Current FICO	5.3729	4.3983	3.3841	3.3474	3.1503	3.0280	2.3750	2.2463	2.2735
	(0.6231)	(0.6712)	(0.6725)	(0.8048)	(0.8706)	(0.8540)	(0.9714)	(1.0293)	(1.0164)
Marked-to-Market LTV	-2.2509	-1.4714	-1.2475	0.2609	0.4338	0.4621	-3.3577	-2.3607	-2.3557
	(0.4606)	(0.4929)	(0.4891)	(0.6497)	(0.7072)	(0.6709)	(0.8225)	(0.8881)	(0.8879)
Original Debt-to-Income	-1.3768	-0.8709	-0.7013	-0.0892	-0.0335	-0.0239	1.2679	1.0209	1.0134
	(0.5106)	(0.5483)	(0.5394)	(0.7165)	(0.7762)	(0.7723)	(0.9048)	(0.9473)	(0.8513)
ln(Original Loan Size)	0.8767	0.4098	0.4156	0.4945	0.3766	0.3887	0.7363	0.7057	0.7085
	(0.1135)	(0.1202)	(0.1193)	(0.1530)	(0.1604)	(0.1625)	(0.1494)	(0.1609)	(0.1602)
First-Time Home Buyer	-0.0604	-0.0769	-0.0731	-0.1124	-0.1259	-0.1303	-0.0607	-0.2307	-0.2345
	(0.0988)	(0.1043)	(0.1035)	(0.1317)	(0.1505)	(0.1414)	(0.1594)	(0.1831)	(0.1819)
Co-Borrower	-0.1538	-0.1222	-0.1292	-0.0312	-0.0891	-0.0916	0.0563	-0.0210	-0.0227
	(0.0845)	(0.0886)	(0.0879)	(0.1199)	(0.1359)	(0.1234)	(0.1457)	(0.1559)	(0.1605)
Prob. Curtailment		0.0995	0.1215		0.0247	0.0320		-0.0088	-0.0087
		(0.0068)	(0.0084)		(0.0109)	(0.0114)		(0.0106)	(0.0110)
Prob. Not paying		-0.0460	-0.0480		-0.0338	-0.0338		-0.0310	-0.0311
		(0.0027)	(0.0026)		(0.0035)	(0.0035)		(0.0038)	(0.0037)
Reinstatement									
Monthly Benefit of Refinance	6.7687	3.8499	3.9665	10.1387	6.9504	7.0066	13.6590	10.3401	10.3489
	(0.5759)	(0.5443)	(0.5435)	(0.9189)	(0.9721)	(0.9296)	(1.2270)	(1.2638)	(1.2640)
Value of Deferral	-10.3606	-2.0719	-2.5557	-3.3858	-2.2497	-2.2925	-1.6025	-0.5381	-0.5429
	(0.3587)	(0.4077)	(0.4076)	(0.2284)	(0.2819)	(0.2559)	(0.1683)	(0.1842)	(0.1852)
Eeconomic Impact Payment	0.3245	0.2522	0.3145	0.0080	-0.0469	-0.0498	0.0000	0.0000	0.0000
	(0.1667)	(0.1659)	(0.1710)	(0.0865)	(0.0928)	(0.1004)	(0.0000)	(0.0000)	(0.0000)
Delinquncy Spell Dummy	0.0697	-0.1549	-0.1906	-0.5712	-0.6653	-0.6585	0.1698	-0.0246	-0.0316
	(0.1757)	(0.1967)	(0.1930)	(0.2902)	(0.3080)	(0.3095)	(0.3774)	(0.3935)	(0.3873)
Avg Duration of Delinquency Spell	0.0606	0.0826	0.0905	0.1682	0.1768	0.1776	0.0654	0.0478	0.0457
	(0.0506)	(0.0534)	(0.0537)	(0.0558)	(0.0606)	(0.0612)	(0.0701)	(0.0833)	(0.0892)
Other Disaster	-0.0929	-0.0610	-0.0216	0.1729	0.2610	0.2700	0.2385	0.2835	0.2663
	(0.0884)	(0.0972)	(0.0952)	(0.1682)	(0.1764)	(0.1762)	(0.2667)	(0.2809)	(0.2805)
High Covid Rate US	0.0243	0.0742	0.0758	0.3186	0.2693	0.2665	0.9339	0.8545	0.8547
	(0.1189)	(0.1326)	(0.1317)	(0.1900)	(0.2009)	(0.2048)	(0.3247)	(0.3378)	(0.3333)
Unemployment Rate	-0.0240	-0.0405	-0.0516	-0.0135	-0.0277	-0.0291	-0.1331	-0.1337	-0.1326
	(0.0225)	(0.0237)	(0.0240)	(0.0451)	(0.0508)	(0.0508)	(0.0864)	(0.0929)	(0.0961)
Current FICO	0.9683	-0.4683	-1.5548	-1.5771	-1.9958	-2.1464	-0.6227	-1.3443	-1.3038
	(0.6067)	(0.6742)	(0.6814)	(1.1027)	(1.1459)	(1.1156)	(1.5855)	(1.6557)	(1.6024)
Marked-to-Market LTV	-4.1488	-2.9136	-2.4572	-3.8904	-3.2101	-3.1679	-6.3603	-4.6707	-4.6546
	(0.4826)	(0.5369)	(0.5337)	(0.8880)	(0.9363)	(0.9231)	(1.2903)	(1.4205)	(1.4201)
Original Debt-to-Income	0.3362	1.0180	1.2580	-0.7518	-0.5903	-0.5732	3.1402	3.1180	3.0864
	(0.5358)	(0.5963)	(0.5896)	(1.0192)	(1.0689)	(1.0691)	(1.6254)	(1.6194)	(0.8511)
ln(Original Loan Size)	0.0132	-0.3979	-0.3708	-0.0107	0.1007	0.0999	-0.1289	-0.1782	-0.1764
	(0.1119)	(0.1270)	(0.1247)	(0.2021)	(0.2184)	(0.2144)	(0.2721)	(0.2876)	(0.2818)

Table 2.13: Results of the MNL: Determinants of Exits, cot'd
(standard errors are in parentheses)

	FB Age 6			FB Age 12			FB Age 18		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
First-Time Home Buyer	0.3509	0.3054	0.2776	0.2553	0.1507	0.1494	0.9626	0.7200	0.7126
	(0.0993)	(0.1107)	(0.1093)	(0.1874)	(0.1980)	(0.1953)	(0.2512)	(0.2704)	(0.2708)
Co-Borrower	-0.1813	-0.1526	-0.1499	-0.1630	-0.2327	-0.2339	0.1072	-0.0038	-0.0105
	(0.0890)	(0.0963)	(0.0951)	(0.1678)	(0.1813)	(0.1706)	(0.2425)	(0.2479)	(0.2514)
Prob. Curtailment		0.1152	0.1415		0.0520	0.0582		0.0149	0.0148
		(0.0067)	(0.0084)		(0.0115)	(0.0122)		(0.0120)	(0.0128)
Prob. Not paying		-0.0630	-0.0653		-0.0243	-0.0242		-0.0393	-0.0393
		(0.0028)	(0.0028)		(0.0046)	(0.0047)		(0.0064)	(0.0063)
trial/Modification									
Monthly Benefit of Refinance	3.1190	3.3386	3.3307	4.7824	4.6258	4.6368	8.9132	9.1884	9.1958
	(0.6906)	(0.6731)	(0.6858)	(0.7079)	(0.7288)	(0.7668)	(0.5633)	(0.6230)	(0.6233)
Value of Deferral	0.9872	1.0030	0.9933	0.2635	0.3017	0.2992	0.2454	0.1028	0.1011
	(0.2297)	(0.2481)	(0.2511)	(0.0986)	(0.1096)	(0.1199)	(0.0562)	(0.0667)	(0.0674)
Economic Impact Payment	-0.0684	-0.0623	-0.0561	-0.4074	-0.4082	-0.4076	0.0000	0.0000	0.0000
	(0.1985)	(0.1949)	(0.1962)	(0.0813)	(0.0879)	(0.0860)	(0.0000)	(0.0000)	(0.0000)
Delinquency Spell Dummy	0.3001	0.2887	0.2878	0.4903	0.4841	0.4894	0.3003	0.3592	0.3638
	(0.2338)	(0.2302)	(0.2321)	(0.2208)	(0.2202)	(0.2326)	(0.1821)	(0.1875)	(0.1876)
Avg Duration of Delinquency Spell	0.1397	0.1413	0.1421	0.0952	0.0942	0.0935	0.0110	0.0168	0.0168
	(0.0421)	(0.0369)	(0.0372)	(0.0426)	(0.0425)	(0.0449)	(0.0390)	(0.0451)	(0.0452)
Other Disaster	0.0746	0.0727	0.0777	0.0638	0.0548	0.0564	-0.1686	-0.1722	-0.1781
	(0.1630)	(0.1582)	(0.1586)	(0.1635)	(0.1548)	(0.1652)	(0.1149)	(0.1193)	(0.1184)
High Covid Rate US	-0.2752	-0.2753	-0.2768	0.2034	0.1988	0.1985	0.3066	0.3125	0.3109
	(0.1975)	(0.1892)	(0.1895)	(0.1759)	(0.1742)	(0.1820)	(0.1295)	(0.1343)	(0.1362)
Unemployment Rate	-0.0966	-0.0984	-0.0989	-0.0265	-0.0256	-0.0254	-0.0728	-0.0739	-0.0733
	(0.0388)	(0.0367)	(0.0368)	(0.0468)	(0.0447)	(0.0442)	(0.0398)	(0.0411)	(0.0413)
Current FICO	-1.4972	-1.6121	-1.6914	-3.5581	-3.6200	-3.6268	-3.5107	-3.4364	-3.4196
	(0.9590)	(0.9388)	(0.9526)	(0.9136)	(0.9264)	(0.9243)	(0.6954)	(0.7052)	(0.7053)
Marked-to-Market LTV	-2.4023	-2.4182	-2.4073	-4.4949	-4.4902	-4.5043	-6.1439	-6.1655	-6.1697
	(0.8107)	(0.8204)	(0.8152)	(0.7858)	(0.7968)	(0.8046)	(0.6175)	(0.6528)	(0.6508)
Original Debt-to-Income	-0.4542	-0.4998	-0.4770	1.4740	1.5026	1.5023	0.3328	0.3127	0.3081
	(1.0173)	(0.3765)	(0.3202)	(0.9870)	(0.9828)	(0.9849)	(0.6955)	(0.7025)	(0.6947)
ln(Original Loan Size)	-0.8296	-0.8339	-0.8338	-1.1067	-1.1114	-1.1133	-0.7431	-0.5925	-0.5931
	(0.2025)	(0.1979)	(0.2050)	(0.2139)	(0.2012)	(0.2178)	(0.1541)	(0.1594)	(0.1597)
First-Time Home Buyer	0.1112	0.1073	0.1099	0.1575	0.1583	0.1609	0.2214	0.2282	0.2284
	(0.1792)	(0.1769)	(0.1759)	(0.1678)	(0.1739)	(0.1738)	(0.1218)	(0.1274)	(0.1297)
Co-Borrower	-0.1403	-0.1291	-0.1309	-0.1035	-0.1089	-0.1093	-0.0005	0.0014	0.0010
	(0.1607)	(0.1576)	(0.1574)	(0.1496)	(0.1541)	(0.1507)	(0.1055)	(0.1076)	(0.1142)
Prob. Curtailment		0.0118	0.0120		0.0038	0.0005		-0.0434	-0.0458
		(0.0187)	(0.0209)		(0.0174)	(0.0199)		(0.0176)	(0.0186)
Prob. Not paying		0.0018	0.0011		-0.0036	-0.0039		0.0087	0.0087
		(0.0054)	(0.0050)		(0.0055)	(0.0053)		(0.0047)	(0.0047)
Repayment									
Monthly Benefit of Refinance	0.6536	-2.2402	-2.3083	5.2352	2.3391	2.3267	12.0172	12.6129	12.6054
	(0.9633)	(2.0419)	(2.0402)	(3.2340)	(1.3174)	(3.2946)	(0.8700)	(0.9278)	(0.9262)
Value of Deferral	-5.8259	-2.3586	-2.1651	-1.9318	-0.6349	-0.6434	-0.3135	-0.4978	-0.4974
	(0.4952)	(1.2542)	(1.2312)	(0.6458)	(0.6742)	(0.8079)	(0.0773)	(0.0967)	(0.0941)
Economic Impact Payment	0.6622	0.5714	0.6331	-0.5018	-0.5554	-0.5572	0.0000	0.0000	0.0000
	(0.3928)	(0.3831)	(0.3828)	(0.3597)	(0.3623)	(0.3858)	(0.0000)	(0.0000)	(0.0000)
High Covid Rate US	-0.2090	-0.1906	-0.2068	-0.2788	-0.3369	-0.3390	0.0018	0.0033	0.0017
	(0.4456)	(0.4449)	(0.4448)	(0.6986)	(0.6788)	(0.6955)	(0.2219)	(0.2436)	(0.2362)
Unemployment Rate	-0.1438	-0.1532	-0.1633	-0.1819	-0.2187	-0.2180	0.0852	0.0990	0.0996
	(0.0901)	(0.0869)	(0.0875)	(0.1919)	(0.1866)	(0.1923)	(0.0666)	(0.0684)	(0.0648)
Current FICO	-2.8371	-3.9109	-4.7013	-6.5891	-7.7620	-7.7930	-2.2734	-2.1042	-2.0876
	(1.9006)	(1.8736)	(1.8764)	(2.0064)	(1.8183)	(3.4079)	(1.2460)	(1.2642)	(1.2381)
Marked-to-Market LTV	-1.2241	-2.1458	-1.9999	-0.3211	-0.9199	-0.9387	-0.9601	-0.6407	-0.6428
	(1.1397)	(1.3634)	(1.3688)	(2.2361)	(2.4050)	(2.7782)	(1.0020)	(1.0119)	(0.9953)
Prob. Curtailment		0.0662	0.0904		0.0106	0.0184		-0.0269	-0.0278
		(0.0211)	(0.0258)		(0.0392)	(0.0434)		(0.0300)	(0.0301)
Prob. Not paying		-0.0378	-0.0416		-0.0444	-0.0432		0.0229	0.0227
		(0.0088)	(0.0089)		(0.0159)	(0.0160)		(0.0085)	(0.0083)
Original Year	Y	Y	Y	Y	Y	Y	Y	Y	Y
Log Likelihood	-4734.41	-4328.46	-4368.06	-2376.52	-2279.17	2281.33	-2243.16	-2154.43	-2154.61

Table 2.14: How Do Borrowers Exit Forbearance Differently by Their Types? (standard errors are in parentheses)

	$P < 0.5$	$P \geq 0.5$
Prepayment		
Monthly Benefit of Refinance	5.1153	4.9795
	(0.3979)	(0.2277)
Value of Deferral	-1.5519	-3.9204
	(0.0797)	(0.2895)
Delinquncy Spell Dummy	-0.6861	18.2995
	(0.1349)	(0.2447)
Avg Duration of Delinquency Spell	0.1010	-19.2031
	(0.0331)	(0.2395)
Other Disaster	0.0445	-0.0580
	(0.0750)	(0.1375)
High Covid	-0.0676	0.4404
	(0.0889)	(0.1428)
Unemployment Rate	-0.1301	-0.1197
	(0.0218)	(0.0332)
Current FICO	3.7371	1.1815
	(0.5139)	(0.6121)
Marked-to-Market LTV	-1.1211	0.0110
	(0.4049)	(0.1300)
Original Debt-to-Income	-0.2773	-0.6589
	(0.4796)	(0.1035)
ln OUPB	0.4451	0.5283
	(0.0883)	(0.0576)
First Home Buyer	-0.2135	0.1193
	(0.0845)	(0.1545)
Coborrower	-0.1341	0.0085
	(0.0751)	(0.1308)
Prob. Curtailment	0.0609	0.0952
	(0.0072)	(0.0122)
Prob. Not paying	-0.0379	-0.0499
	(0.0020)	(0.0037)
Reinstatement		
Monthly Benefit of Refinance	4.9714	4.8301
	(0.4872)	(0.2995)
Value of Deferral	-1.0997	-3.4800
	(0.1689)	(0.3483)
Delinquncy Spell Dummy	-0.2875	-3.0509
	(0.1537)	(0.9926)
Avg Duration of Delinquency Spell	0.1091	2.2651
	(0.0365)	(0.8774)
Other Disaster	0.0967	-0.1016
	(0.0915)	(0.1475)
High Covid	0.1356	0.4368
	(0.1139)	(0.1503)
Unemployment Rate	-0.0555	0.0131
	(0.0257)	(0.0343)

	$P < 0.5$	$P \geq 0.5$
Current FICO	-0.1829	-1.5908
	(0.5973)	(0.6318)
Marked-to-Market LTV	-3.3607	-1.8007
	(0.4865)	(0.1372)
Original Debt-to-Income	0.8008	0.6499
	(0.6035)	(0.1008)
ln OUPB	-0.2195	-0.3359
	(0.1127)	(0.0549)
First Home Buyer	0.3264	0.2291
	(0.0999)	(0.1659)
Coborrower	-0.2168	-0.0297
	(0.0922)	(0.1412)
Prob. Curtailment	0.0946	0.1055
	(0.0075)	(0.0124)
Prob. Not paying	-0.0530	-0.0635
	(0.0026)	(0.0040)
Trial/Modification		
Monthly Benefit of Refinance	5.6393	5.2967
	(0.3574)	(0.6158)
Value of Deferral	0.2455	0.7111
	(0.0452)	(0.1836)
Delinquncy Spell Dummy	0.3162	-0.9386
	(0.1203)	(2.2234)
Avg Duration of Delinquency Spell	0.1050	0.5023
	(0.0250)	(1.9584)
Other Disaster	-0.0595	-0.1382
	(0.0811)	(0.2666)
High Covid	0.0583	1.0386
	(0.0943)	(0.4946)
Unemployment Rate	-0.0861	-0.1172
	(0.0232)	(0.0784)
Current FICO	-3.0169	0.0222
	(0.4808)	(0.9372)
Marked-to-Market LTV	-5.0529	-3.3384
	(0.1933)	(0.4764)
Original Debt-to-Income	0.5898	-2.9184
	(0.5164)	(0.7894)
ln OUPB	-0.8198	-0.8960
	(0.1017)	(0.1381)
First Home Buyer	0.1901	0.4790
	(0.0827)	(0.2238)
Coborrower	-0.0447	-0.4953
	(0.0804)	(0.2108)
Prob. Curtailment	-0.0077	0.0391
	(0.0126)	(0.0308)
Prob. Not paying	0.0039	-0.0103
	(0.0029)	(0.0083)

	$P < 0.5$	$P \geq 0.5$
Repayment		
Monthly Benefit of Refinance	6.0970	0.6246
	(0.6441)	(2.7396)
Value of Deferral	-0.1055	-2.5109
	(0.0798)	(2.4859)
High Covid	-0.1945	-0.5359
	(0.2027)	(0.2241)
Unemployment Rate	-0.0395	-0.2626
	(0.0540)	(0.1091)
Current FICO	-2.3628	-6.4217
	(1.0032)	(1.8389)
Marked-to-Market LTV	-1.5556	-4.0332
	(0.7864)	(1.4025)
Prob. Curtailment	0.0301	0.0676
	(0.0199)	(0.0403)
Prob. Not paying	-0.0112	-0.0349
	(0.0047)	(0.0158)
Original Year	Y	Y
FB age	Y	Y
Log Likelihood	-7107.72	-1915.18
N	8882	2657

2.8.2 Variance Correction in Two-Step Models

The two-step estimation is consistent but asymptotically less efficient than the maximum likelihood estimator. The standard errors have to be adjusted for the first stage estimation error, as in Murphy and Topel (2002):

$$\begin{aligned}
R_1(\beta) =& E(\frac{\partial log(l_1)}{\partial \beta})(\frac{\partial log(l_1)}{\partial \beta})' \\
R_2(\omega) =& E(\frac{\partial log(l_2)}{\partial \omega})(\frac{\partial log(l_2)}{\partial \omega})' \\
R_3(\beta,\omega) =& E(\frac{\partial log(l_2)}{\partial \beta})(\frac{\partial log(l_2)}{\partial \omega})' \\
R_4(\beta,\omega) =& E(\frac{\partial log(l_1)}{\partial \beta})(\frac{\partial log(l_2)}{\partial \omega})'
\end{aligned} \tag{45}$$

$$Var(\omega) = R_2^{-1} + R_2^{-1}(R_3^{'}R_1^{-1}R_3 - R_4^{'}R_1^{-1}R_3 - R_3^{'}R_1^{-1}R_4)R_2^{-1} \tag{46}$$

where l_1 is the likelihood function for the first step, l_2 is the likelihood function for the second step. In the first step we estimate the parameters $\beta = [\beta_1\, \beta_2\, \beta_3\, \theta]$ by maximizing the likelihood function l_1. And in the second step we estimate the parameters $\omega = [\psi\, \rho]$ by maximizing the likelihood function l_2.

2.8.3 Marginal Effect of the MNL

The multinomial logit model that estimates the probability of forbearance exit type can be written as

$$P(C_{jmT}) = \begin{cases} \frac{\exp(W_{jT}^{'}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})}{1+\sum_{j=1}^{J-1}\exp(W_{jT}^{'}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & ,j = 1, ...J-1 \\ \frac{1}{1+\sum_{j=1}^{J-1}\exp(W_{jT}^{'}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & ,j = J \end{cases}$$

where C_{jmT} for $j = 1, \ldots, J$ represents the exit type, $\hat{f}_{1T}$ is the estimated predictive probability of not making a payment at terminate time T, and $\hat{f}_{2T}$ is the estimated predictive probability of making curtailment payment at terminate time T.

The marginal effect of estimated payment behaviors on mean monthly probabilities of forbearance exits are calculated as:

$$G(\boldsymbol{\rho}_1) = \frac{\partial \bar{\pi}(C_j)}{\partial \hat{f}_{A_1}} = \bar{\pi}(C_j)(\rho_{1j} - \sum_r \bar{\pi}(C_r)\rho_{1r}) \tag{47}$$

and

$$G(\boldsymbol{\rho}_2) = \frac{\partial \bar{\pi}(C_j)}{\partial \hat{f}_{A_2}} = \bar{\pi}(C_j)(\rho_{2j} - \sum_r \bar{\pi}(C_r)\rho_{2r}), \tag{48}$$

where $\bar{\pi}(C_j)$ is the mean probability for exits event, C_j.

The delta method are used to find the variance of the marginal effect. The variance of the marginal effect on the probability of not making a payment is given by

$$var(G(\boldsymbol{\rho}_1)) = \frac{\partial G(\boldsymbol{\rho}_1)}{\partial \boldsymbol{\rho}_1} cov(\boldsymbol{\rho}_1) \frac{\partial G(\boldsymbol{\rho}_1)'}{\partial \boldsymbol{\rho}_1} \quad , \tag{49}$$

and the variance of the marginal effect on the probability of making curtailment payment is given by

$$var(G(\boldsymbol{\rho}_2)) = \frac{\partial G(\boldsymbol{\rho}_2)}{\partial \boldsymbol{\rho}_2} cov(\boldsymbol{\rho}_2) \frac{\partial G(\boldsymbol{\rho}_2)'}{\partial \boldsymbol{\rho}_2} \quad , \tag{50}$$

where $\frac{\partial G(\boldsymbol{\rho}_1)}{\partial \boldsymbol{\rho}_1}$ is the partial derivative of the predictions with respect to coefficients and evaluated at $\hat{\boldsymbol{\rho}}_1$, $\frac{\partial G(\boldsymbol{\rho}_2)}{\partial \boldsymbol{\rho}_2}$ is the partial derivative of the predictions with respect to coefficients and evaluated at $\hat{\boldsymbol{\rho}}_2$, and $cov(\boldsymbol{\rho}_1)$ and $cov(\boldsymbol{\rho}_2)$ are variance and covariance matrix.

2.8.4 Simulation Study on the Dirichlet Nested Logit Model

The simulation study was motivated by testing our coefficients are unbiased estimators. Moreover, we want to discuss how the θ changes the shape of the Dirichlet distribution given coefficients (βs) are fixed under our estimation. Four sets of βs will be analyzed to cover most types of shapes for the Dirichlet distribution. The process for simulation can be outlined as follows:

1. Set $\boldsymbol{\beta} = \{1.25, 0.7, 1.4\}$, $\theta = 0.5$, and $z \sim N(1, 0.25)$. Group choice 1 and choice 2 as one nest, and choice 3 by itself. Then, calculate marginal probability at period $t = 1$[17], using equation (47).

2. Using the probabilities in step 1 to generate multinomial random vectors for $t = 1$.

3. Recalculate the marginal probabilities given the choices made in previous periods, and use it to generate multinomial random vectors for this period.

4. Repeat step 3 until $t = 6$.

5. Repeat step 1 to step 4 for $M = 50000$ times to generate panel data.

6. Calculate x_i (a cumulative number for choice i has been chosen within 6 period) for each individual, and then transform panel data into cross-section data.

7. Estimate $\boldsymbol{\beta}$ and θ by maximizing the likelihood function in equation (43).

8. Repeat step 1 to step 7 for $\theta = 0.3$, $\theta = 0.8$, and $\theta = 1$ separately.

9. Repeat step 1 to step 8 for different sets of $\boldsymbol{\beta}$: $\boldsymbol{\beta} = \{-0.46, -0.46, -0.11\}$, $\boldsymbol{\beta} = \{0.8, -0.35, 2.3\}$, and $\boldsymbol{\beta} = \{1.15, 1.15, -0.7\}$.

Table 2.15 shows the simulation results. The results show that the estimates ($\boldsymbol{\beta}$ and θ) are unbiased because the magnitude of estimate parameters and real parameters are very close under 16 cases.

[17]For a single period, the Dirichlet nested logit model is the nested logit model.

Table 2.15: Simulation Results

Actual $\boldsymbol{\beta}$	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.8$	$\theta = 1$
Case 1				
β_1 =1.25	1.2496	1.2555	1.2696	1.2565
β_2 =0.7	0.7011	0.7029	0.6954	0.7084
β_3 =1.4	1.4019	1.4045	1.4288	1.4059
θ	0.3023	0.5009	0.8343	0.9936
Case 2				
β_1 =-0.46	-0.4295	-0.4600	-0.4476	-0.4534
β_2 =-0.46	-0.4308	-0.4567	-0.4410	-0.4547
β_3 =-0.11	-0.1008	-0.0921	-0.1059	-0.1062
θ	0.2771	0.5000	0.7806	0.9948
Case 3				
β_1 =0.8	0.7743	0.8020	0.8053	0.7584
β_2 =-0.35	-0.3650	-0.3293	-0.3635	-0.3835
β_3 =2.3	2.2747	2.3001	2.3071	2.2548
θ	0.2963	0.4964	0.8109	0.9985
Case 4				
β_1 =1.15	1.1535	1.1368	1.1618	1.1408
β_2 =1.15	1.1528	1.1387	1.1540	1.1405
β_3 =-0.7	-0.7071	-0.6997	-0.6868	-0.6956
θ	0.3053	0.4957	0.810	0.9991

To show the shape of the beta distribution for choice 1 and choice 2 within nest, we evaluate the parameters of the beta distribution using mean values of the exogenous variables[18]: $\alpha_1 = \exp \frac{\bar{z}_1'\beta}{\theta}$ and $\alpha_2 = \exp \frac{\bar{z}_2'\beta}{\theta}$. And the shape of the beta distribution for group 1 and group 2 can be evaluated by the mean values of the exogenous variables[19]:

[18] α_1 and α_2 are the within nest shape parameters for beta distribution.
[19] α_{G_1} and α_{G_2} are the between nests shape parameters for beta distribution.

$\alpha_{G_1} = \exp(\theta ln(\exp\frac{\bar{z}_1'\beta}{\theta} + \exp\frac{\bar{z}_2'\beta}{\theta}))$ and $\alpha_{G_2} = \alpha_3 = \exp\bar{z}_3'\beta$. Different values of $\boldsymbol{\beta}$ and θ allow discovering how the θ changes the shape of the Dirichlet distribution given different sets of $\boldsymbol{\beta}$. As we discussed in section 2.3.2, the Dirichlet nested logit model can be derived by two beta-logistic models: a probability of choice within the nest and a probability of choice between nests. In Figure 2.10 - 2.13, we show how different values of θ change the probability density function of beta distribution within the nest and between nests keeping the value of $\boldsymbol{\beta}$ unchanged.

Table 2.16 shows how within nest shape parameters, α_1 and α_2, change with θ. α_i increases as θ decreases if $\alpha_i > 1$, and α_i decrease as θ decreases if $\alpha_i < 1$. Table 2.17 shows how between nests shape parameters, α_{G_1} and α_{G_2}, change with θ. α_{G_1} increases (decreases) as θ increases (decreases), and α_{G_2} keeps unchanged.

Table 2.16: Within Nest: α_1 vs α_2

Initial α		α **if** $\theta\uparrow$		α **if** $\theta\downarrow$	
α_1^0	α_2^0	α_1^1	α_2^1	α_1^1	α_2^1
>1	>1	Decrease	Decrease	Increase	Increase
<1	<1	Increase	Increase	Decrease	Decrease
>1	<1	Decrease	Increase	Increase	Decrease

Table 2.17: Between Nests: α_{G_1} vs α_{G_2}

Initial κ		κ **if** $\theta\uparrow$		κ **if** $\theta\downarrow$	
κ_1^0	κ_2^0	κ_1^1	κ_2^1	κ_1^1	κ_2^1
>1	>1	Increase	No Change	Decrease	No Change
<1	<1	Increase	No Change	Decrease	No Change
>1	<1	Increase	No Change	Decrease	No Change

Figure 2.10: The Dirichlet Nested Logit Distribution for Case 1

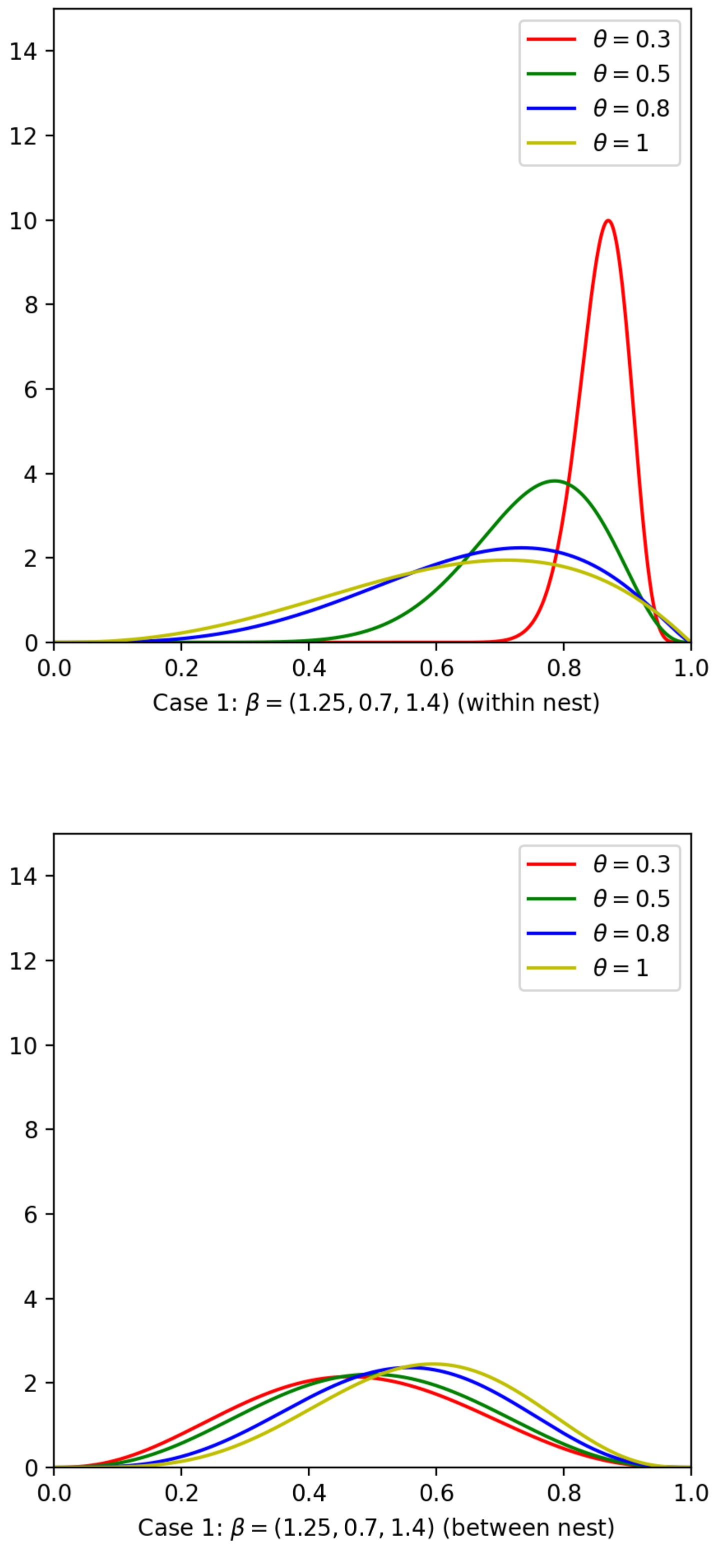
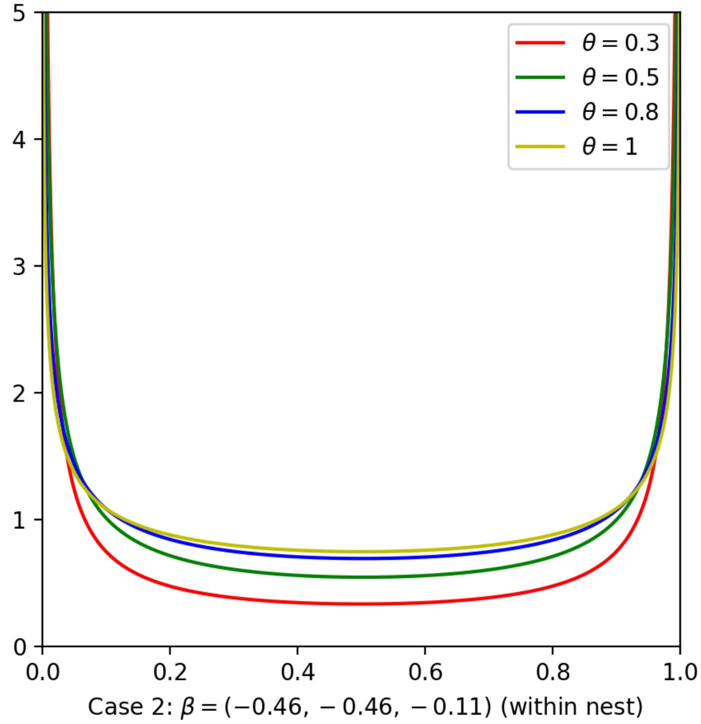


Figure 2.11: The Dirichlet Nested Logit Distribution for Case 2



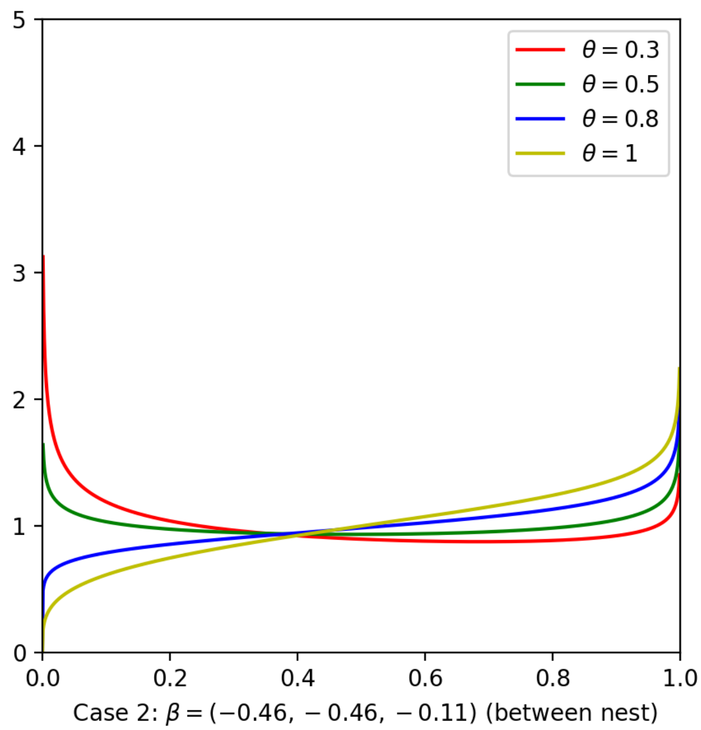


Figure 2.12: The Dirichlet Nested Logit Distribution for Case 3

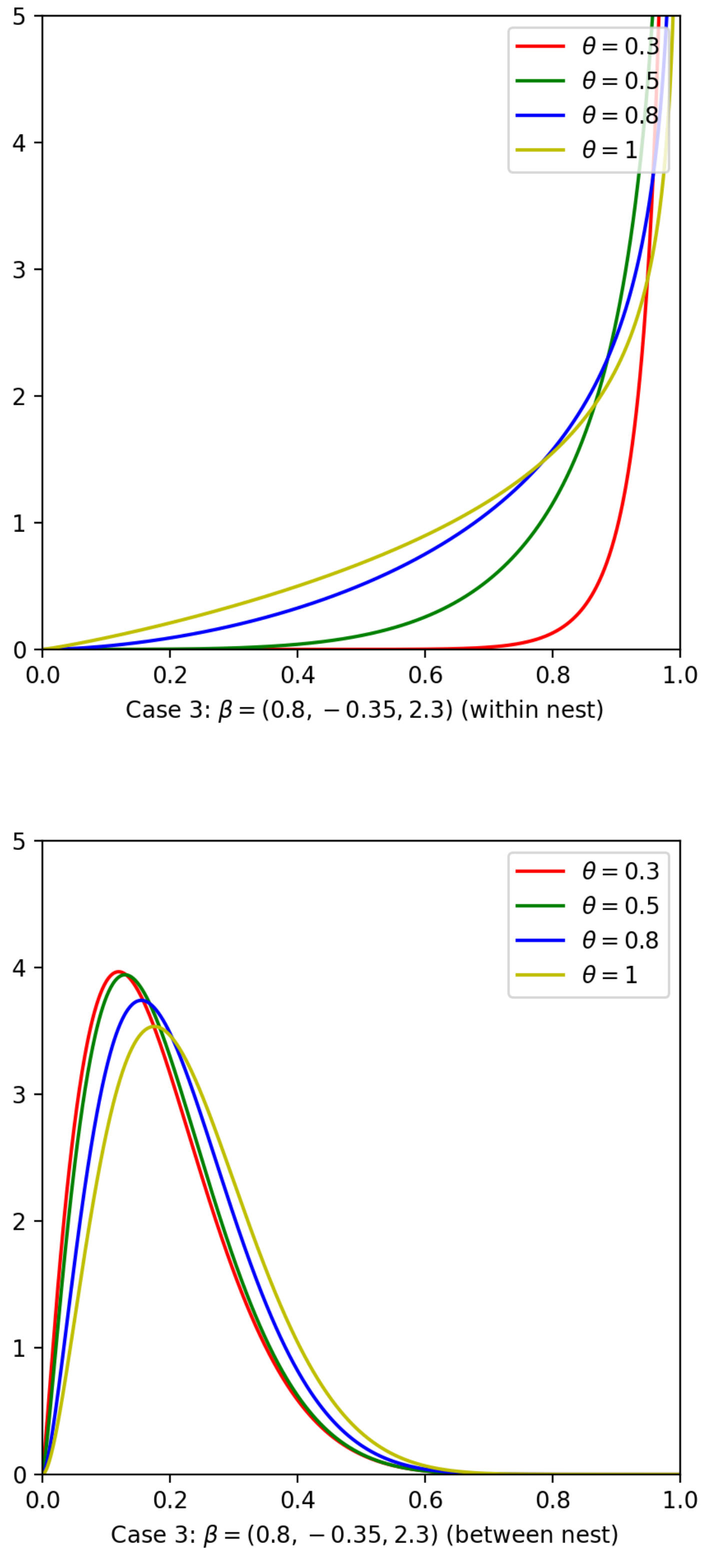
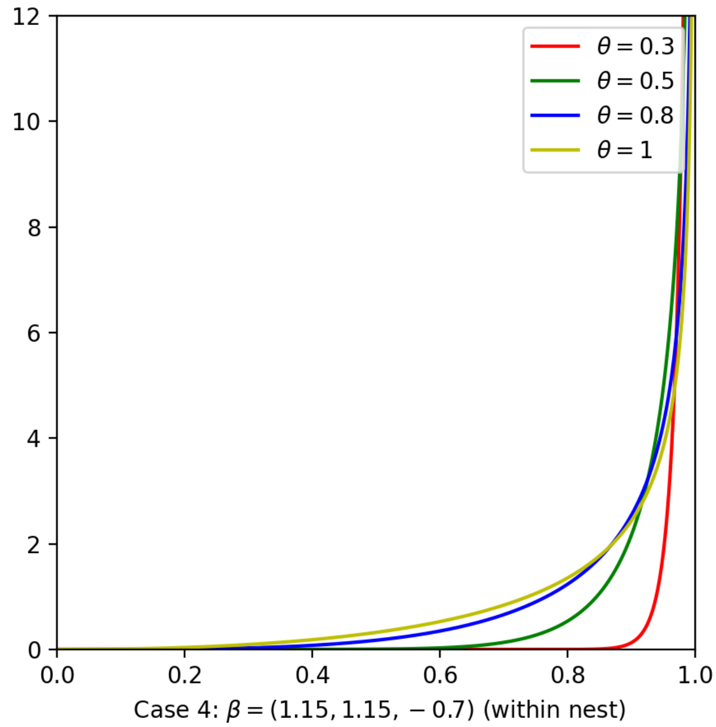
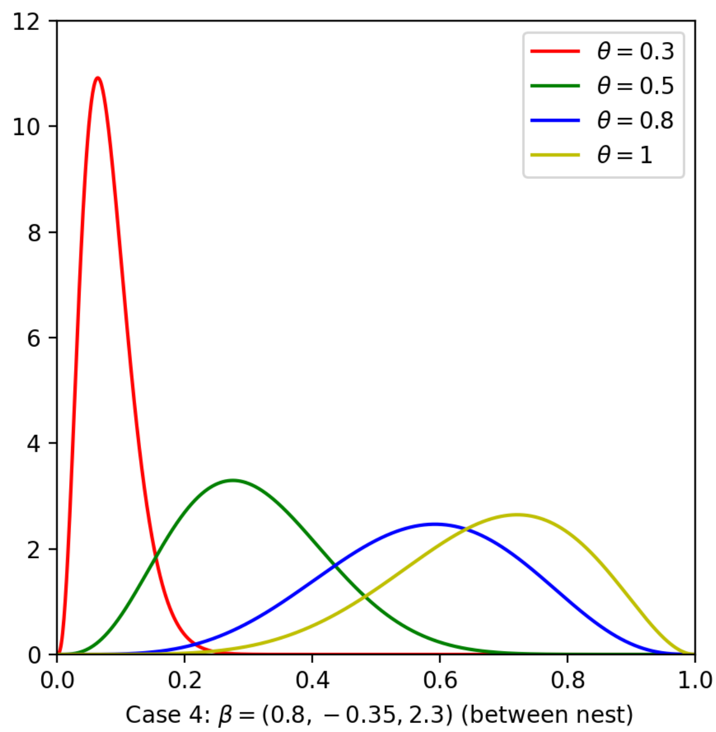


Figure 2.13: The Dirichlet Nested Logit Distribution for Case 4





Chapter 3: Estimating Borrower Behavior in the CARES Act Forbearance Program: Sequential and Full Sample Approaches

3.1 Introduction

The Coronavirus Aid, Relief, and Economic Security (CARES) Act forbearance program was passed by the United States Congress on March 27, 2020. This program offers short-term relief to homeowners with federally-backed mortgages who have been impacted by the COVID-19 pandemic. Homeowners could request the forbearance program by reaching out to their servicers. During the forbearance period, homeowners can temporarily pause their mortgage payments without incurring fees, penalties, or unscheduled interest and without negative effects on their credit history. The borrower needs to attest to a hardship related to the pandemic to qualify for forbearance; no documentation of income loss is required. The initial forbearance term lasted for six months and then was prolonged up to 12 months. Due to the continuous impact of the COVID-19 pandemic, the forbearance period has been extended to a maximum of 18 months[20].

Between April 2020 and December 2021, approximately 16% of individuals who have mortgage loans have entered forbearance[21]. The forbearance program has many potential benefits for lenders, borrowers, and the broader economy relative to alternatives. Farrell et al. (2020) find that forbearance helps families with low levels of liquid assets to maintain their cash buffers. Capponi et al. (2021) argue that this program significantly decreases the refinancing cost of households and relaxes their refinancing eligibility con-

[20] To qualify for an extension of the forbearance program to 18 months for Fannie Mae or Freddie Mac backed mortgages, the initial forbearance must have been received on or before February 28, 2021. For mortgages backed by HUD/FHA, USDA, or VA, the initial forbearance must have been received on or before June 30, 2020, to be eligible for the extension.

[21] It is estimated from monthly data of the New York Fed Consumer Credit Panel/Equifax.

straints. An et al. (2022) show that the forbearance program reduces the delinquency rates of minority and lower-income borrowers, as well as reduced inequality.

Despite being advantageous for borrowers, one possible drawback of the forbearance program is that it may be accessed by individuals who do not actually require them. As a result, this can lead to an increase in the overall cost of the program beyond what would be necessary if only those truly in need participated. But according to the legislation no documentation is required. The reasons for entering the program are unobserved. Loewenstein and Njinju (2022) indicate that the CARES Act forbearance program has largely been used by borrowers who did actually need it. On the other hand, Farrell et al. (2020) find that borrowers experiencing income declines were more likely to enter into forbearance, but many borrowers in the forbearance program continued making all their payments. Anderson et al. (2022) show that borrowers who entered the program may not have experienced a hardship associated with or due to Covid-19. The existence of latent variables among borrowers in the forbearance program presents significant estimation challenges.

Every month, borrowers are faced with the decision of how much of their budget to allocate towards mortgage payments, and must make a discrete choice of whether to make a regular payment, a curtailment payment or not make a payment at all. The reasons for entering the forbearance program are not observable, leading to an issue of unobserved heterogeneity that can affect their payment behavior in the program. To address this, Chapter 2 extended the beta-logistic model in Heckman and Willis (1977) to the Dirichlet nested logit model, which allowed the state dependence. A two-step estimation technique was used to estimate the impact of payment behaviors on exit types. In the first step, the probability of each payment option was estimated using the Dirichlet multinomial logit and nested logit model, and these predictions were used in a multinomial logit model in the second step to estimate the probability of forbearance exit type.

In Chapter 2, the likelihoods contained information on only those mortgages that failed at that exit time for a given exit time. Results were presented for three exit times: 6, 12, and 18 months. The first improvement in this chapter involves incorporating data regarding all mortgages that survive up to a specific time into the likelihood functions. By doing so, it allows to predict borrower payment and exit behaviors for the next period as the program continues through time. This approach also expands the range of choices available to borrowers during these periods. This includes determining whether borrowers intend to exit the program or remain in it, as well as their payment behavior in the following period if they choose to stay, and their exit type if they opt to leave.

In this chapter, I show that the estimation can be accomplished in a single step. The two-step estimation technique used in Chapter 2 has the drawback of being asymptotically less efficient than the one-step maximum likelihood estimator (Amemiya, 1978; Murphy and Topel, 2002). The one-step estimation technique directly evaluates the impact of payment behavior on exit type without the need for predicting marginal probabilities in a separate step. The one-step estimation results in significantly smaller variances for the predicted probabilities of payment behaviors compared to the two-step estimation. Additionally, the accuracy rate for the last period is much higher in the one-step estimation.

The second improvement in this chapter is that it constructs a single model, estimated in a single step, for borrowers with different termination times. It provides a more comprehensive understanding of borrower behavior by using information for all of the first six months. The new model's estimation accuracy is significantly higher than the model that only considers a single survival time of six months. Further research will focus on expanding the estimation to encompass the entire program duration.

This chapter is structured as follows: Section 3.2 presents the data and provides a basic analysis. Section 3.3 outlines the two-step model, the one-step model, and the one-step model with varying terminal times. Section 3.4 discusses the results of the empirical analysis. Finally, Section 3.5 provides a brief conclusion.

3.2 Data

This chapter uses the loan-level data sourced from the publicly available Fannie Mae Credit Insurance Risk Transfer (CIRT)/Connecticut Avenue Securities (CAS) data. Since 2013, government-sponsored enterprises (GSEs) have been selling credit risk to private investors through credit risk transfer (CRT) programs in consultation with the Federal Housing Finance Agency (FHFA). As of Q2:2022, Fannie Mae reports that $681B of the unpaid principal balance of single-family mortgage loans has been partially covered through CIRT transactions. This program aims to reduce credit risk by attracting additional private capital to the single-family housing market. Specifically, CIRT transfers credit risk for a pool of loans to an insurance provider.

This study focuses on a sample of 68,313 loans that participated in the CARES Act Mortgage forbearance program between March 2020 and March 2022, drawn from the public use Fannie Mae Credit Insurance Risk Transfer (CIRT)/Connecticut Avenue Securities (CAS) data. This dataset contains borrower and mortgage characteristics, as well as loan performance information, including current unpaid balance, payment status, forbearance indicator, and servicer and seller names. Additionally, the data includes standard loan information at origination, such as credit score, loan-to-value, loan amount, mortgage rate, debt-to-income, year, occupancy status, and property location (three-digit ZIP code). Notably, the CIRT data also includes updated credit scores, which are a crucial determinant of borrowers' payment behavior and termination events. Including current FICO scores is particularly noteworthy, as very few academic papers consider their application in mortgage-related research.

In each period within the program, borrowers have the option to choose from three payment options: curtailment payment, scheduled monthly payment, and no payment. Curtailment payment refers to borrowers paying more than the expected monthly payment, scheduled monthly payment refers to borrowers paying the exact amount of the required monthly payment, and no payment means borrowers pause their payment for that month. Borrowers have five options for exiting the program: reinstatement, repayment plan, payment deferral, modification, or prepayment. Under reinstatement, the borrower pays the forborne amount before they exit forbearance; a repayment plan enables the borrower to pay off the forborne amount over a period of time; payment deferral is when the forborne balances are placed into a balloon loan payable at the liquidation date of the loan; loan modification reduces the size of the monthly payment by extending the term of the loan or reducing the mortgage rate; finally, the exit is defined as prepayment if the borrower pays off the entire remaining balance, generally by refinancing the mortgage. Among the sample of forborne loans in this study, as of March 2022, 23,775 loans (34.8 percent) had exited forbearance by prepayment, 12,502 loans (18.3 percent) had exited by reinstatement, 690 loans (1.01 percent) had exited by repayment plan, 19,237 loans (28.1 percent) had exited by payment deferral, and 3,782 loans (5.54 percent) had exited by trial/modification.

The CIRT data was merged with external data to capture local and macroeconomic factors. The FHFA's All-Transactions House Price Index (HPI) was utilized to capture fluctuations in single-family housing prices at the most detailed level feasible, i.e., the three-digit ZIP code level. This index is a weighted repeat-sales index that measures the average price shifts in sales or refinancing of the same properties. Information on Economic Impact Payment (EIP) was obtained from the U.S. Department of the Treasury, which publishes the disbursement of the EIP payments at the national level over calendar time. The IRS provided the total dollar amount and the number of receipts in aggregate at the state level for the first, second, and third rounds of payments.

3.2.1 Explanatory Variables and Definition

Borrower behavior during the forbearance program and their decisions regarding program exit are influenced by various factors, including borrower-specific characteristics such as income, FICO score, and age; loan-specific characteristics such as loan amount, loan-to-value ratio, and note rate; and financial market conditions such as the unemployment rate and house prices.

The definitions of the explanatory variables are presented in Table 3.1. In this study, the monthly benefit of refinancing is defined as the monthly reduction in payment resulting from refinancing in the current month. A positive effect of the monthly benefit of refinancing on the probability of prepayment is anticipated. The payment deferral value is defined as the returns on investing the missed payment values in a 10-year Treasury bond until the mortgage maturity date. It is expected that the value of payment deferral will positively impact the probability of payment deferral.

Many mortgages have a minimum down payment requirement, and the loan-to-value (LTV) ratio is defined as the loan amount divided by the property value at origination. Banks typically require an LTV below a certain threshold, which is calculated as Monthly Debt divided by Stable Monthly Income. Three LTV groups have been identified: < 80, $= 80$, and > 80. A higher LTV ratio is associated with a higher probability of not making payments in the program and a lower chance of prepayment and reinstatement, as it indicates a lower initial down payment, holding all other factors constant. The original Debt-to-Income Ratio (DTI) is another critical variable that may influence borrower behavior. We expect that a higher DTI will increase the probability of not making payments during the program.

Borrower behavior can be influenced by their payment history, and this study controls for various factors such as the Fair Isaac Corporation (FICO) Score, the number of delinquency spells, and the average duration of delinquency spells before the Covid-19 pandemic. The FICO score is a number between 300 and 850 that is used to assess the creditworthiness of borrowers. Mortgages are grouped by the current FICO scores in February 2020 (before the pandemic) into poor (300-579), fair (580-669), good (670-739), very good (740-799), and exceptional (800-850) categories. It is expected that the probability of prepayment increases with the current FICO score while the probability of modification and payment deferral decreases with the current FICO score. Furthermore, the higher the number of delinquency spells, the more frequently the borrower made payments during the program and exited with payment deferral and modification.

As part of the CARES Act, Congress enacted the Economic Impact Payment (EIP) program to provide broad relief to Americans from the economic shock of Covid-19. To be eligible for the program, one must either be a U.S. citizen or resident with a valid Social Security number and have an income of up to \$75,000 for individuals and \$150,000 for couples filing jointly. Receipt of EIP might assist borrowers in making their payments and exiting forbearance with prepayment or reinstatement.

3.2.2 Data Descriptive

The summary statistics of various subgroups are presented in Table 3.2. The first column displays the summary statistics for borrowers who exited the forbearance program within six months, and columns 2 to 4 show the summary statistics for borrowers who remained in the program until each extension point (6, 12, and 18 months). During the forbearance period, borrowers must determine how to allocate their mortgage payments or whether to leave the program.

The Table 3.2 shows that those who exited the program within six months had a higher likelihood of making payments than those who stayed in the program for a longer period of time. Borrowers who remain in the forbearance program for 12 months are more likely to have a higher value of refinance and a greater chance of exiting the program with prepayment. Borrowers who stay in the forbearance program for 18 months have a higher value of payment deferral, a higher probability of residing in areas that have experienced other disasters, a higher likelihood of being a first-time homebuyer, a lower probability of having a co-borrower, and a higher probability of entering the program immediately. Furthermore, as the length of time in the program increases, the probability of exiting with payment deferral and modification also increases, while the likelihood of exiting with reinstatement decreases.

Borrowers' payment behavior plays a crucial role in their decision to continue in the forbearance program or exit it for the next period. Figures 3.1-3.3 present the average proportion of borrowers who transition from making curtailment payments, making scheduled payments, and not making payments at time $t-1$ to time t. Borrowers who did not make payments after enrolling in the forbearance program at time $t-1$ are more likely to continue not making payments or exit forbearance with payment deferral or trial/modification. Conversely, borrowers who made payments after enrolling in the forbearance program at time t are more likely to continue making payments or exit forbearance with prepayment or reinstatement.

3.3 Models

3.3.1 Two-Step Sequential Approach

The pattern of payment behavior during the forbearance program predicts how borrowers choose to exit forbearance. For example, borrowers with more forborne payments are more likely to exit the forbearance program with a payment deferral or with a trial/-modification, while borrowers who always make scheduled or curtailment payments are more likely to exit with a prepayment or a reinstatement. The study employs a two-step estimation method from Chapter 2 to investigate how the payment behavior of borrowers during the forbearance program influences their exit choices. However, unlike Chapter 2, which examined only those borrowers who exited the program at precisely 6, 12, or 18 months, this chapter analyzes borrowers who remained in the program at these intervals. This approach expands the range of choices available to borrowers during these periods, as they can now decide to stay in the program or exit. This study's findings should aid in predicting borrower behavior in the next period as the program continues through time.

The described approach involves two steps. Firstly, the Dirichlet nested logit model is used to estimate payment behavior patterns up to time $t = T_m - 1$. The likelihood function for this step is:

$$\begin{aligned} l(\beta_1, \beta_2, \beta_3, \theta) = \prod_{m=1}^{M} & \frac{(x_{1m} + x_{2m})!}{x_{1m}! x_{2m}!} \frac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \frac{\Gamma(\alpha_1 + x_{1m})\Gamma(\alpha_2 + x_{2m})}{\Gamma(\alpha_1 + \alpha_2 + x_{1m} + x_{2m})} \\ & \times \frac{(T_m - 1)!}{(x_{1m} + x_{2m})! x_{3m}!} \frac{\Gamma((\alpha_1 + \alpha_2)^\theta + \alpha_3)}{\Gamma((\alpha_1 + \alpha_2)^\theta)\Gamma(\alpha_3)} \\ & \times \frac{\Gamma((\alpha_1 + \alpha_2)^\theta + x_{1m} + x_{2m})\Gamma(\alpha_3 + x_{3m})}{\Gamma((\alpha_1 + \alpha_2)^\theta + \alpha_3 + T_m - 1)} \quad , \end{aligned} \tag{51}$$

where $\alpha_1 = \exp \frac{z'\beta_1}{\theta}$, $\alpha_2 = \exp \frac{z'\beta_2}{\theta}$, and $\alpha_3 = \exp z'\beta_3$. And x_{im} is the cumulative number of times that choice i has been chosen in the previous $T_m - 1$ periods for m^{th} borrower.

The Dirichlet nested logit model is used to estimate the payment behavior patterns up to time $T_m - 1$, and then to predict the marginal probability at time T_m. These predictions are used in a multinomial logit model to estimate the probability of forbearance exit type and payment type in the second step. Assuming that borrowers remain in the forbearance program until $T_m - 1$ months, the corresponding likelihood function for period T_m can be expressed as:

$$\begin{aligned} l_T = \prod_{m=1}^{M} & [(1 - h_{Tm}) P(B_{1Tm})^{y_{1,Tm}=1} P(B_{2Tm})^{y_{2,Tm}=1} P(B_{3Tm})^{y_{3,Tm}=1}]^{\eta=0} \\ & \times [h_{Tm} \prod_{j=1}^{J} P(C_{jTm})^{q_{j,Tm}=1}]^{\eta=1} \end{aligned} \tag{52}$$

Here, y_{1,T_m}, y_{2,T_m}, and y_{3,T_m} are binary variables, where y_{i,T_m} equals 1 if choice i is selected at time T_m, and zero otherwise. q_{j,T_m} is an indicator of exiting type, where q_{j,T_m} equals one if the borrower exits the forbearance program for type j, and zero otherwise. Additional, B_{1Tm}, B_{2Tm}, and B_{3Tm} correspond to the payment behaviors of making curtailment payments, not paying, and making regular payments, respectively. The probability of these payment behaviors is expressed as follows:

$$
\begin{aligned}
P(B_{1Tm}) = P(y_{1,Tm} = 1) &= \frac{e^{z'\beta_1}}{1 + e^{z'\beta_1} + e^{z'\beta_2}} \\
P(B_{2Tm}) = P(y_{2,Tm} = 1) &= \frac{e^{z'\beta_2}}{1 + e^{z'\beta_1} + e^{z'\beta_2}} \\
P(B_{3Tm}) = P(y_{3,Tm} = 1) &= \frac{1}{1 + e^{z'\beta_1} + e^{z'\beta_2}}
\end{aligned}
\tag{53}
$$

And C_{jmT} for $j = 1, \ldots, J$ represents the exit type. The corresponding probabilities of these exit types are formulated as follows:

$$
P(C_{jTm}) = \begin{cases} \frac{\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})}{1 + \sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & , j = 1, ...J - 1 \\ \frac{1}{1 + \sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}\hat{f}_{1T} + \rho_{2j}\hat{f}_{2T})} & , j = J \end{cases}
\tag{54}
$$

And h_{Tm} denotes the probability of exit, which can be calculated using the following formula:

$$
h_{Tm} = P(\eta_{Tm} = 1) = \frac{\exp(x'\beta + \rho_3\hat{f}_{1T} + \rho_4\hat{f}_{2T})}{1 + \exp(x'\beta + \rho_3\hat{f}_{1T} + \rho_4\hat{f}_{2T})}
\tag{55}
$$

where W_{jT} 's are vectors of exogenous variables that affect how borrowers exit forbearance for the exiting type j. $\hat{f}_{1T}$ is the estimated predictive probability of not making a payment at terminate time T, and $\hat{f}_{2T}$ is the estimated predictive probability of making curtailment payment at terminate time T_m. According to Amemiya (1978), the two-step model for multinomial logit is consistent but asymptotically less efficient than the maximum likelihood estimator. In the next section, this chapter will employ a one-step estimation method to obtain a more efficient estimator.

3.3.2 One-Step Sequential Approach

Although the two-step estimation method is consistent, it is known to be asymptotically less efficient than the one-step maximum likelihood estimator. Moreover, Murphy and Topel (2002) argue that the two-step procedure fails to consider the fact that imputed regressors are measured with sampling error, leading to biased hypothesis tests based on the estimated covariance matrix of the second-step estimator, even in large samples. Comparing with two-step approach, the one-step model directly estimates the impact of payment behavior on exit type without requiring the prediction of marginal probabilities in a separate step.

The likelihood function for the one-step model can be expressed as:

$$\begin{aligned} l(\beta_1, \beta_2, \beta_3, \theta) = \prod_{m=1}^{M} &(E[p(x_1, x_2, x_3, T_m)](1 - h_{Tm}))^{\eta=0} \\ &\times (E[p(x_1, x_2, x_3, T_m - 1)]h_{Tm} \prod_{j=1}^{J} P(C_{jTm})^{q_{j,Tm}=1})^{\eta=1} \end{aligned} \tag{56}$$

$E[p(x_1, x_2, x_3, T_m)]$ represents the expected probability of not making a payment for x_1 months, making curtailment payments for x_2 months, and making regular payments for x_3 months within T_m months in the forbearance program, using the Dirichlet Nested Logit model. The probability of exit behavior is denoted by $P(C_{jTm})$, while h_{Tm} is a logit model that captures the decision to leave or stay in the forbearance program at time $t = T_m$. Furthermore, the indicator $q_{j,tm}$ takes a value of one if the borrower exits the forbearance program for type j and zero otherwise. Finally, η_{Tm} is an indicator of exiting behavior, where $\eta_{Tm} = 1$ if the borrower exits the forbearance program at time $t = T_m$ and zero otherwise.

The likelihood function can also be written as

$$l(\beta_1, \beta_2, \beta_3, \theta) = \prod_{m=1}^{M} \Big(\prod_{t=1}^{Tm-1} [(f_{1t})^{y_{1,tm}=1}(f_{2t})^{y_{2,tm}=1}(f_{3t})^{y_{3,tm}=1}]$$
$$\times [(1-h_{Tm})(f_{1Tm})^{y_{1,Tm}=1}(f_{2Tm})^{y_{2,Tm}=1}(f_{3Tm})^{y_{3,Tm}=1}]^{\eta=0} \quad (57)$$
$$\times [h_{Tm} \prod_{j=1}^{J} P(C_{jTm})^{q_{j,Tm}=1}]^{\eta=1} \Big)$$

where

$$f_{1t} = P(y_{1t} = 1|x_1, x_2, x_3, t-1) = \frac{E[p(x_1+1, x_2, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} = \frac{x_1 + e^{z'\beta_1}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}}$$

$$f_{2t} = P(y_{2t} = 1|x_1, x_2, x_3, t-1) = \frac{E[p(x_1, x_2+1, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} = \frac{x_2 + e^{z'\beta_2}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}}$$

$$f_{3t} = P(y_{3,t} = 1|x_1, x_2, x_3, t-1) = \frac{E[p(x_1, x_2, x_3+1, t)]}{E[p(x_1, x_2, x_3, t-1)]} = \frac{x_3 + e^{z'\beta_3}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \ ,$$

$$P(C_{jTm}) = \begin{cases} \frac{\exp(W'_{jT}\psi_j + \rho_{1j}f_{1T} + \rho_{2j}f_{2T})}{1+\sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}f_{1T} + \rho_{2j}f_{2T})} & , j = 1, ...J-1 \\ \frac{1}{1+\sum_{j=1}^{J-1}\exp(W'_{jT}\psi_j + \rho_{1j}f_{1T} + \rho_{2j}f_{2T})} & , j = J \end{cases}$$

$$h_{Tm} = P(\eta_{Tm} = 1) = \frac{\exp(x'\beta + \rho_{3j}f_{1T} + \rho_{4j}f_{2T})}{1 + \exp(x'\beta + \rho_{3j}f_{1T} + \rho_{4j}f_{2T})}$$

The variables used in the model include f_{1t}, f_{2t}, and f_{3t} representing the marginal probabilities of payment behavior, Additionally, $y_{1,tm}$, $y_{2,tm}$, and $y_{3,tm}$ are binary variables where $y_{i,tm}$ equals 1 if choice i is selected at time t;

3.3.3 One-Step Approach for Full Sample

Both the two-step and one-step estimations discussed above enable the analysis of mortgage holders who are currently enrolled in the forbearance program at a specific month (T_m is fixed) and can predict if they will continue in the program or exit in the following period. However, the one-step model with varying terminal times offers a more comprehensive examination of borrower behavior by not imposing any restrictions on those who are still enrolled in the program at a specific month.

The likelihood function for the one-step model with varying T_m is

$$\begin{aligned} l(\beta_1, \beta_2, \beta_3, \theta) = \prod_{m=1}^{M} \prod_{t=1}^{Tm} [(1 - h(t))(f_{1t})^{y_{1,tm}=1}(f_{2t})^{y_{2,tm}=1}(f_{3t})^{y_{3,tm}=1}]^{\eta_{tm}=0} \\ \times [h(t) \prod_{j=1}^{J} P(C_{jmt})^{q_{j,tm}=1}]^{\eta_{tm}=1} \end{aligned} \tag{58}$$

where

$$\begin{aligned} f_{1t} = P(y_{1t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1 + 1, x_2, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_1 + e^{z'\beta_1}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \\ \\ f_{2t} = P(y_{2t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1, x_2 + 1, x_3, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_2 + e^{z'\beta_2}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \\ \\ f_{3t} = P(y_{3,t} = 1 | x_1, x_2, x_3, t-1) &= \frac{E[p(x_1, x_2, x_3 + 1, t)]}{E[p(x_1, x_2, x_3, t-1)]} \\ &= \frac{x_3 + e^{z'\beta_3}}{t - 1 + e^{z'\beta_1} + e^{z'\beta_2} + e^{z'\beta_3}} \ , \end{aligned} \tag{59}$$

$$P(C_{jmt}) = \begin{cases} \frac{\exp(W'_{jt}\psi_j+\rho_{1j}\hat{f}_{1T}+\rho_{2j}\hat{f}_{2T})}{1+\sum_{j=1}^{J-1}\exp(W'_{jt}\psi_j+\rho_{1j}\hat{f}_{1T}+\rho_{2j}\hat{f}_{2T})} & ,j=1,...J-1 \\ \frac{1}{1+\sum_{j=1}^{J-1}\exp(W'_{jt}\psi_j+\rho_{1j}\hat{f}_{1t}+\rho_{2j}\hat{f}_{2t})} & ,j=J \end{cases} \tag{60}$$

$$h(t) = P(\eta_{mt}=1) = \frac{\exp(x'\beta+\rho_{3j}\hat{f}_{1t}+\rho_{4j}\hat{f}_{2t})}{1+\exp(x'\beta+\rho_{3j}\hat{f}_{1t}+\rho_{4j}\hat{f}_{2t})} \tag{61}$$

The variables used in the model include f_{1t}, f_{2t}, and f_{3t} representing the marginal probabilities of payment behavior, $P(C_{jmt})$ representing the probability of exit behavior, and $h(t)$ representing the logit model for leaving or staying the program. And η_{mt} indicates exiting behavior where $\eta_{mt} = 1$ if the borrower exits the forbearance program, and zero otherwise. Additionally, $y_{1,tm}$, $y_{2,tm}$, and $y_{3,tm}$ are binary variables where $y_{i,tm}$ equals 1 if choice i is selected at time t; and $q_{j,tm}$ is an indicator of exiting type, where $q_{j,tm}$ equals one if the borrower exits the forbearance program for type j, and zero otherwise.

3.4 Results

3.4.1 Two-Step Sequential Approach Results

Table 3.3 displays the regression outcomes of the Dirichlet nested logit model for three borrower groups who remain in forbearance during 6, 12, and 18 months. The covariate values are fixed to those of the first month when the borrowers enrolled in the forbearance program.[22] This assumption is not expected to significantly impact the final results as the forbearance program's duration is relatively brief compared to the loan term.

Table 3.3 presents evidence that the payment behavior of borrowers enrolled in the program is influenced by several factors, including borrower characteristics such as current FICO score; loan characteristics such as loan amount, loan-to-value ratio and debt-

[22] 76.87 percent of borrowers entered the forbearance program in March, April, and May 2020.

to-income ratio, as well as broader financial markets conditions like the unemployment rate and Covid cases.

The results uncover that higher current FICO scores are associated with a decreased probability of not making payments and an increased probability of making scheduled payments and curtailment payments. The borrower's previous delinquency status reduces the chance to make scheduled payments but increases the probability of not paying at the 6^{th} and 12^{th} month. The original debt-to-income ratio has a larger negative impact on making regular payments and curtailment payments. Interestingly, borrowers with a high marked-to-market loan-to-value ratio have a higher probability of making regular payments. In addition, mortgages with co-borrowers are less likely to suspend payments and more likely to make regular payments. Moreover, whether a house is the borrower's first home influences their payment behavior, as individuals tend to make curtailment payments for their primary residence.

Borrowers' payment behavior impacts their decision to continue with the forbearance program or exit it for the next period. Borrowers who have been directly or indirectly affected by Covid-19 are more likely to miss payments after enrolling in the program. Additionally, these borrowers face difficulty in repaying the forborne amount before exiting the program and are more likely to exit with payment deferral or trial/modification. Conversely, borrowers who enroll in the forbearance program but are not experiencing Covid-19 related hardship are likely to make their payments and exit with reinstatement or prepayment. These results can also be observed in Figures 3.1 - 3.3 in Section 3.2, which shows the average proportion of making curtailment payments, making scheduled payments, and not making payments for each exit type.

Table 3.4 presents the results of step 2 of the borrowers' behavior. Borrowers with a higher probability of making curtailment payments are more likely to exit forbearance with prepayment or reinstatement, which diminishes as the forbearance program duration increases. Conversely, borrowers with a higher probability of not making payments are more likely to exit forbearance with payment deferral, as expected.

The results indicate that additional factors may influence borrowers' exit behavior. The likelihood of prepayment and reinstatement rises when the benefit of refinancing is positive; similarly, a higher benefit of payment deferral decreases the probability of prepayment and reinstatement. The estimates suggest that the probability of prepayment increases with the current FICO score, which is also supported by Capponi, Jia, and Rios, 2021, while the probability of trial/modification decreases with the current FICO score. In the twelfth month, borrowers who received stimulus checks are more likely to exit forbearance with prepayment, reinstatement, or repayment plan. A rise in Covid cases is associated with a lower likelihood of borrowers leaving the program in the sixth and twelfth month. Similarly, a higher unemployment rate is linked to a decreased likelihood of borrowers exiting the forbearance program.

3.4.2 One-Step Sequential Approach Results

The two-step estimation is consistent but asymptotically less efficient than the one-step maximum likelihood estimator. In the two-step model, the Dirichlet nested logit model is used to estimate the payment behavior patterns up to time $T_m - 1$, and then to predict the marginal probability at time T_m. Subsequently, in the second step, these predictions are used as explanatory variables to analyze borrower behaviors in the last period.

The one-step model estimates the effect of payment behaviors on exit type directly, without the need for predicting marginal probabilities in a separate step. The estimation of the one-step model is displayed in Table 3.5. The magnitude and sign of variables in the one-step model are consistent with those in the two-step model. However, the variance of the predicted probabilities of payment behaviors, "Prob. Curtailment" and "Prob. Not paying," is significantly smaller in the one-step model compared to the two-step model.

Table 3.6 displays the monthly accuracy rates of the two models, which can be used to assess their goodness of fit. These rates represent the proportion of times the predicted payment behavior with the highest probability aligns with the borrower's actual behavior. The results indicate that the accuracy rates of the one-step and two-step models are similar, except for the final period, where the one-step model has a substantially higher rate than the two-step model.

3.4.3 One-Step Full Sample Approach Results

The estimations, both two-step and one-step, describe in the previous section, enable an examination of the borrower behaviors in a given month (the sixth month, twelfth month, and eighteenth month), and provide forecasts on whether they will continue their enrollment or leave the program in the next period. Here, I use the one-step model with varying terminal times to provide a more extensive investigation of borrower behavior. Table 3.8 exhibits the regression outcomes of the Dirichlet multinomial logit model and nested logit model, both with and without considering the control for the predictive probability of borrower payment behavior for the first six months in the forbearance program.

To manage the computational time, I randomly sampled the data from the entire set of CIRT loans that were part of the forbearance program. Specifically, I selected 10% of the loans from the dataset using a random sampling method. The algorithm we used was as follows: I first randomly assigned a number between 0 and 1 to each loan using a uniform distribution. Next, I selected only those loans whose assigned numbers fell between 0.3 and 0.4. Table 3.7 displays the average values of important variables for the complete data set and its subsets. The table indicates that there is no significant difference in the means of these variables. Over the initial six months of the forbearance program, the average proportions of time that mortgagors choose to make curtailment payments, regular payments, and no payments were 7.7%, 32%, and 61%, respectively.

According to the findings presented in Model 4 of Table 3.8, borrowers who have experienced delinquency before enrolling in the program are more likely not to make payments and curtailment payments, while the probability of making regular payments decreases. On the other hand, a higher current FICO score is associated with a lower probability of non-payment and a higher probability of making curtailment and regular payments. Notably, controlling for borrower payment behavior significantly impacts both models. As anticipated, borrowers who are more likely to make curtailment payments have a higher likelihood of exiting the forbearance program with prepayment or reinstatement. Conversely, borrowers who are more likely not to make payments are more likely to remain in the forbearance program. Additionally, a higher probability of not making payments reduces the chances of exiting the program with prepayment or reinstatement.

The results indicate that there are additional factors that influence the exit behavior of borrowers. Specifically, the benefits of refinancing positively impact the probability of prepayment and reinstatement. However, this effect is reduced after considering the predictive probability of borrower payment behavior. Similarly, a higher benefit of payment deferral decreases the likelihood of prepayment and reinstatement, and this effect is substantially diminished after controlling for the predictive probability of borrower payment behavior. The analysis also suggests that borrowers with higher current FICO scores are more likely to opt for prepayment than payment deferral. Moreover, borrowers who received stimulus checks are more likely to exit the forbearance program through prepayment, reinstatement, or repayment plans, reducing the likelihood of exiting with the repayment plan.

Table 3.9 exhibits the monthly accuracy rates for the Dirichlet Nested Logit model with controls for the predictive probability of payment behavior, enabling the assessment of the model's goodness of fit. Then use the coefficients to predict the probability and calculate the accuracy rate for the complete data set. These rates signify the proportion of instances in which the projected payment behavior with the highest probability matches the borrower's actual behavior. The findings reveal that the accuracy rates of the model start at approximately 54% in the first period and rise to 67% in the sixth period. Moreover, the level of precision in estimating for the new model is notably higher than that of the model with a single survival time of six months, exhibiting an increase of 5%. Further work involves expanding the estimation to encompass the entire program duration.

3.5 Conclusion

The CARES Act forbearance program has temporarily relieved homeowners impacted by the COVID-19 pandemic. By allowing borrowers to temporarily pause their mortgage payments without incurring penalties, fees, or negative credit history, the program has offered several advantages to lenders, borrowers, and the wider economy. However, a possible disadvantage is that it could be utilized by individuals who do not actually need it, and the reasons behind why someone enrolls in the program may not be evident. This presents a significant estimation challenge due to the presence of latent variables among borrowers enrolled in the forbearance program.

To solve the unobserved heterogeneity problem, Chapter 2 used Dirichlet multinomial logit and nested logit model to study borrower payment behavior in the program. Then it uses the predictive payment behavior to investigate how borrowers exit the program. However, Chapter 2 only focused on borrowers who exit the program at exactly 6, 12, and 18 months. This chapter expands on the analysis by including borrowers who survive in the program at those intervals and predicting their payment and exit behavior for the next period. This chapter estimates the probability of each choice directly in a single step, which is found to produce more efficient estimates and achieve a higher accuracy rate for the last period. Furthermore, this chapter constructs a one-step model using all information from the first six months. The new model's estimation accuracy is significantly higher than the model that only considers a single survival time of six months.

The forbearance program has been extended up to a maximum of 18 months due to the continuous impact of the COVID-19 pandemic. However, its effectiveness in providing relief to those in need remains uncertain. The research presented in this chapter sheds light on borrower behavior during the forbearance program, offering valuable insights for policymakers as they consider future relief measures.

3.6 Figures

Figure 3.1: FB Age ≥ 6

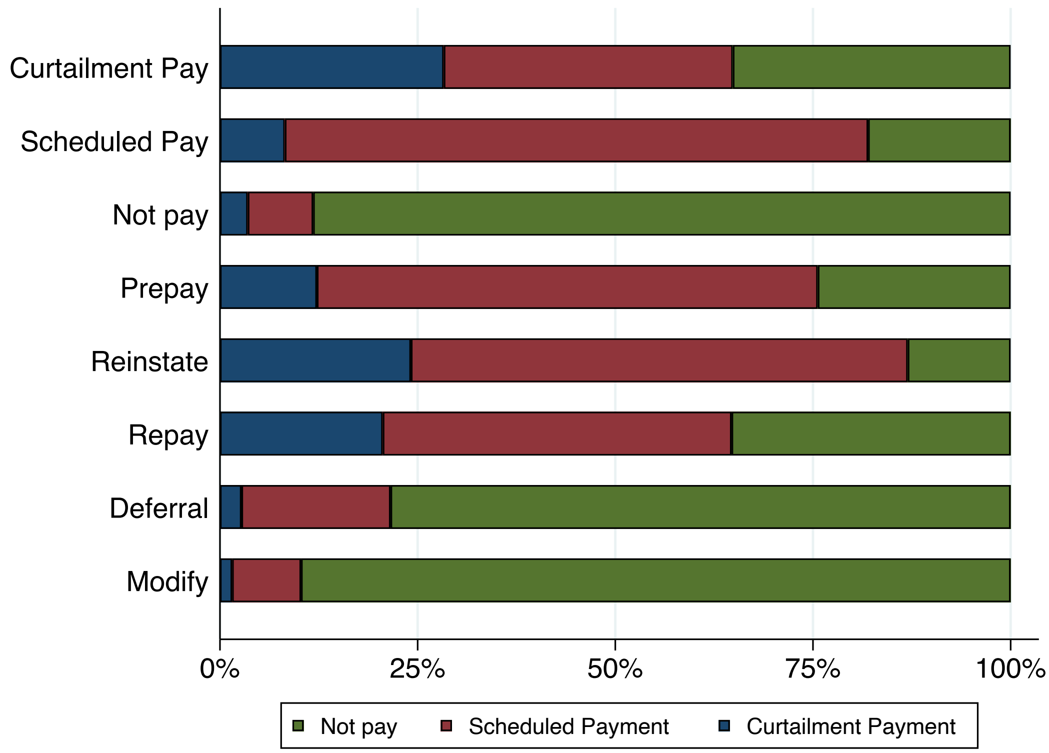


Figure 3.2: FB Age $\geq$ 12

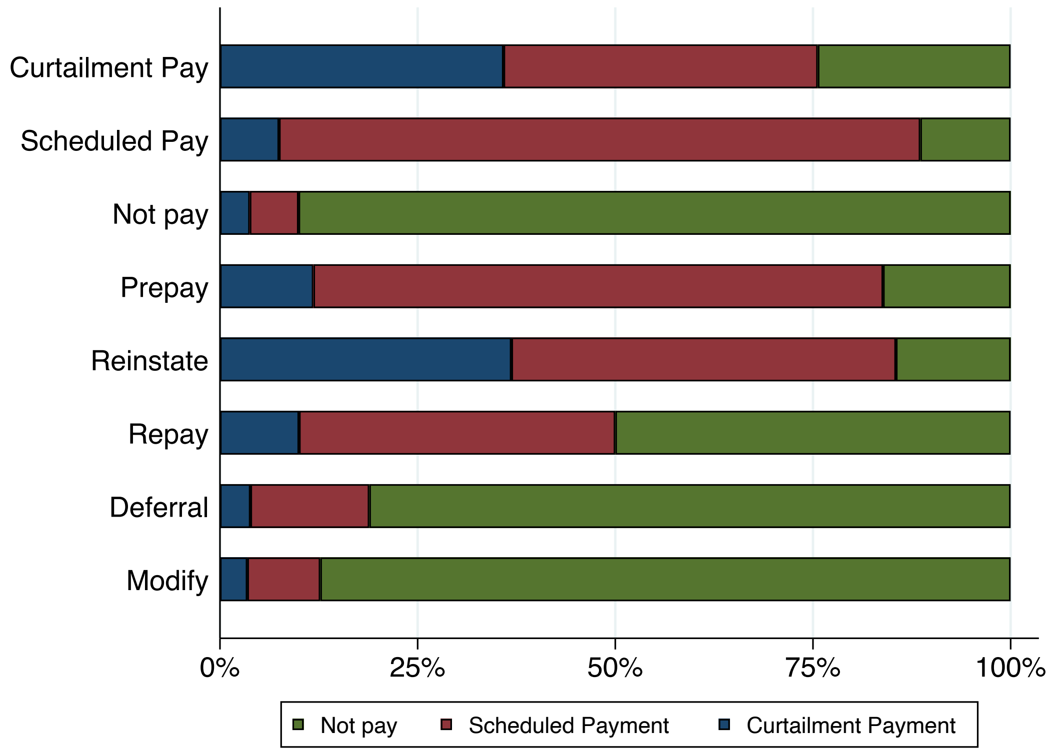
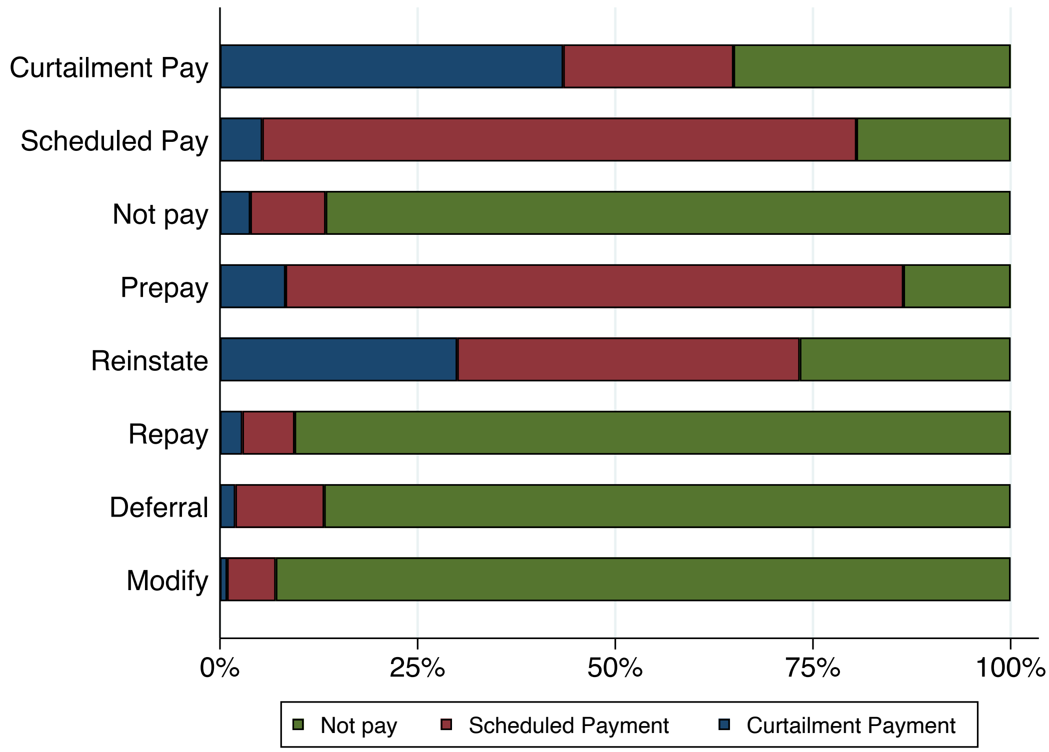


Figure 3.3: FB Age $\geq$ 18



3.7 Tables

Table 3.1: Explanatory Variables and Definition

Variables	Definition
Monthly Benefit of Refinance	The difference between the current and monthly payments after refinancing. The variable is divided by a thousand in the estimation.
Value of Payment Deferral	The gains of investing the values of missed payments into a 10-year treasury bond until the end of the loan[a]. The variable is divided by a thousand in the estimation.
Current FICO Score Before Covid	The score has a minimum value of 300 and a maximum value of 850. Group mortgages by the current FICO in Feb 2020 (before Covid-19) into: poor (300-579), fair (580-669), good (670-739), very good (740-799), and exceptional (800-850).
Marked-to-Market Loan-to-Value (MLTV)	It is calculated by $\frac{(Original\,Loan-to-Value)(Current\,Unpaid\,Balance)}{(Original\,Unpaid\,Balance)(\frac{House\,Price\,Index_t}{House\,Price\,Index_0})}$[b]. The variable is divided by a hundred in the estimation.
First-Time Home Buyer Dummy	If the borrower or co-borrower bought the house as a first-time home buyer, the dummy equals 1, and zero otherwise.
Co-Borrower Dummy	If the borrower has a co-borrower, the dummy equals 1, and zero otherwise.
Original Debt-to-Income	It is calculated by Monthly Debt/Stable Monthly Income.
Original Loan Size	The original loan amount has been grouped by $0-33^{th}$, $33^{th}-66^{th}$, and $66^{th}-100^{th}$ quantiles.
The Number of Delinquency Spell	The number of times the mortgage had been delinquent within 18 months before Covid-19 started.
Average Duration of Delinquency Spell	Define as (number of delinquent months within 18 months before Covid-19 started)/18 if the loan age is greater than 18 when Covid-19 started; (number of delinquent months within 18 months before Covid-19 started)/(loan age) if the loan age was less than 18 when Covid-19 started.
Economic Impact Payment	The average amount of receipts for Economic Impact Payments authorized by CARES Act[c]. The variable is divided by a thousand in the estimation
High Covid Rate US Dummy	An indicator that denotes if $\frac{Covid-19\,Cases\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the national level $\frac{Covid-19\,Cases\,in\,the\,US}{US\,Population}$. And $\frac{Covid-19\,Deaths\,in\,the\,MSA}{MSA\,Population}$ in the MSA is also greater than the national level ($\frac{Covid-19\,Deaths\,in\,the\,US}{US\,Population}$).
High Covid Rate State Dummy	An indicator that denotes if $\frac{Covid-19\,Cases\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the State level $\frac{Covid-19\,Cases\,in\,the\,State}{State\,Population}$, and $\frac{Covid-19\,Deaths\,in\,the\,MSA}{MSA\,Population}$ in the MSA is greater than the State level ($\frac{Covid-19\,Deaths\,in\,the\,State}{State\,Population}$).

Variables	Definition
High Unemployment Rate US Dummy	An indicator that denotes if the monthly unemployment rate in the MSA is greater than the national level.
High Unemployment Rate State Dummy	An indicator that denotes if the monthly unemployment rate in the MSA is greater than the State level.
The Number of Other Disaster[e]	Number of disaster types happens simultaneously when the borrower joints the forbearance program.
Originate Year Dummy	Fannie Mae, and Freddie Mac implemented CRT programs in 2013. CRT data includes loans originating from 2013 to 2021.

[a] This chapter assumes the average life of a mortgage is ten years.
[b] Three-Digit ZIP Codes House Price Index (HPI) from FHFA has been used to calculate MLTV.
[c] The disbursement of the EIP payments over time is based on the US Department of the Treasury.
[d] Approximate monthly income is calculated by (Monthly Mortgage Payment)*12/(Original Debt-to-Income).
[e] Other types of disaster include coastal storm, dam/levee break, earthquake, fire, flood, hurricane, mud/landslide, severe ice storm, severe storm, snow, and tornado.

Table 3.2: Summary Statistics

	FB Age $<$**6**	**FB Age** ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Pct Curtailment	0.103	0.069	0.061	0.068
Pct Regular Pay	0.352	0.309	0.315	0.333
Pct No Paying	0.519	0.622	0.623	0.599
Monthly Benefit of Refinance	0.176	0.217	0.226	0.177
Value of Payment Deferral	0.136	0.311	0.977	1.378
Economic Impact Payment	0.074	0.087	0.423	0.009
Delinquency Spell Dummy	0.098	0.139	0.148	0.139
Avg Duration of Delinquency Spell	0.278	0.418	0.450	0.404
Other Disaster	0.304	0.539	0.631	0.700
High Covid US	0.162	0.146	0.261	0.215
High Covid State	0.474	0.276	0.229	0.212
UNRATE	9.013	7.122	6.089	4.510
Current FICO	0.710	0.701	0.701	0.702
Marked-to-Market LTV	0.672	0.671	0.634	0.599
Original DTI	0.383	0.391	0.394	0.396
Orig 2013 - 2014	0.103	0.073	0.046	0.012
Orig 2015 - 2016	0.234	0.218	0.201	0.170
Orig 2017 - 2018	0.374	0.429	0.457	0.497
Orig 2018 - 2021	0.289	0.280	0.296	0.320
enter (1-2 Months)	0.572	0.651	0.681	0.751
enter (3-6 Months)	0.233	0.242	0.242	0.236
enter (>6 Months)	0.195	0.106	0.077	0.014
First Home	0.324	0.309	0.311	0.314
Co-Borrower	0.408	0.398	0.388	0.373
Exit Type				
Prepayment	0.275	0.462	0.478	0.351
Reinstatement	0.394	0.108	0.043	0.031
Repayment Plan	0.015	0.010	0.015	0.036
Payment Deferral	0.297	0.333	0.337	0.354
Modification	0.0194	0.087	0.128	0.228
Avg Duration	3.415	11.29	14.93	18.00

Table 3.3: Results for Step 1: the DNL Model for Payment Behavior

(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age ≥ 12**	**FB Age ≥ 18**
Not Making Payment			
Num. Delinquency Spell	0.1166	0.0055	0.4574
	(0.0172)	(0.0050)	(0.0596)
Fair FICO	-0.0361	-0.0056	-0.2499
	(0.0133)	(0.0048)	(0.0544)
Good FICO	-0.0463	-0.0073	-0.3997
	(0.0140)	(0.0060)	(0.0524)
Very Good FICO	-0.0938	-0.0124	-0.4566
	(0.0163)	(0.0096)	(0.0569)
Exceptional FICO	-0.1055	-0.0134	-0.4767
	(0.0191)	(0.0105)	(0.0822)
MLTV	0.4595	0.0307	0.3559
	(0.0349)	(0.0231)	(0.1736)
ODTI	-0.0692	0.0045	-0.3215
	(0.0378)	(0.0093)	(0.2187)
Coborrower	0.0035	-0.0031	-0.1730
	(0.0112)	(0.0026)	(0.0379)
First Home	-0.0409	-0.0020	0.0477
	(0.0121)	(0.0022)	(0.0422)
Curtailment Payment			
Num. Delinquency Spell	0.1146	0.0066	0.3472
	(0.0158)	(0.0054)	(0.0660)
Current FICO	0.5294	0.0526	1.1735
	(0.0297)	(0.0377)	(0.2633)
MLTV	0.1245	0.0170	0.2699
	(0.0280)	(0.0134)	(0.1776)
ODTI	-0.1662	-0.0125	-0.6294
	(0.0322)	(0.0122)	(0.1897)
Coborrower	0.0180	0.0008	0.0142
	(0.0099)	(0.0014)	(0.0419)
First Home	0.0079	0.0011	0.0979
	(0.0111)	(0.0017)	(0.0457)
Regular Payment			
Num. Delinquency Spell	-0.0773	-0.1414	0.2024
	(0.0255)	(0.0308)	(0.0557)
Fair FICO	0.1703	0.2172	0.0808
	(0.0291)	(0.0295)	(0.0536)
Good FICO	0.2609	0.3142	0.1500
	(0.0288)	(0.0277)	(0.0516)
Very Good FICO	0.3773	0.4576	0.2517
	(0.0299)	(0.0279)	(0.0557)
Exceptional FICO	0.4247	0.4583	0.2104
	(0.0381)	(0.0436)	(0.0825)
MLTV	0.6555	0.5680	0.8025
	(0.0603)	(0.0849)	(0.1659)
ODTI	-0.1272	-0.1236	-0.5032
	(0.0784)	(0.1116)	(0.2026)
Coborrower	0.1152	0.1472	0.1147
	(0.0156)	(0.0198)	(0.0372)
First Home	-0.0557	-0.0505	-0.0603
	(0.0176)	(0.0219)	(0.0404)
$\boldsymbol{\eta}$	-1.1140	-3.6362	6.4082
	(0.0510)	(0.7342)	(2.6553)
Log Likelihood	-172812.76	-136933.02	-64092.40

Table 3.4: Results of Step 2 — Determinants of Exits

(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Prepayment			
Monthly Benefit of Refinance	1.7117	3.7428	-0.8827
	(0.5082)	(0.6039)	(0.4991)
Value of Deferral	-2.3849	-1.9401	-1.9007
	(0.2652)	(0.1300)	(0.1215)
Economic Impact Payment	0.1837	0.2906	0.0000
	(0.2222)	(0.0915)	(0.0000)
Delinquncy Spell Dummy	-0.2785	-0.8487	-0.8193
	(0.1451)	(0.1767)	(0.2075)
Current FICO	7.3234	8.4747	6.1476
	(0.8576)	(0.8131)	(0.7860)
Marked-to-Market LTV	1.0862	4.4373	3.1696
	(0.4666)	(0.5722)	(0.5849)
Original Debt-to-Income	-0.9830	-0.2896	0.8386
	(0.4981)	(0.6194)	(0.5984)
First Home	-0.1960	-0.2363	-0.3493
	(0.1028)	(0.1312)	(0.1640)
Coborrower	-0.1852	-0.0788	-0.0541
	(0.0904)	(0.1197)	(0.1539)
Prob. Curtailment	0.1048	0.0138	0.0247
	(0.0241)	(0.0149)	(0.0139)
Prob. Not paying	-0.0527	-0.0334	-0.0414
	(0.0046)	(0.0048)	(0.0043)
Reinstatment			
Monthly Benefit of Refinance	1.6210	3.0026	1.6647
	(0.5986)	(0.7213)	(0.5893)
Value of Deferral	-2.9182	-2.3683	-0.5834
	(0.3211)	(0.2636)	(0.1734)
Economic Impact Payment	0.2562	0.3390	0.0000
	(0.2293)	(0.1110)	(0.0000)
Delinquency Spell Dummy	0.0150	-0.4159	-0.0892
	(0.1417)	(0.2226)	(0.3063)
Current FICO	2.4343	3.1516	2.4853
	(0.9159)	(0.8850)	(1.0053)
Marked-to-Market LTV	-0.2654	0.7942	0.6736
	(0.4547)	(0.6449)	(0.5827)
Original Debt-to-Income	0.8522	-0.9352	2.9124
	(0.5244)	(0.8020)	(1.1913)
First Home	0.1417	0.0503	0.6103
	(0.1087)	(0.1768)	(0.2459)
Coborrower	-0.2276	-0.2229	-0.0510
	(0.0991)	(0.1684)	(0.2511)
Prob. Curtailment	0.1243	0.0397	0.0480
	(0.0242)	(0.0153)	(0.0146)
Prob. Not paying	-0.0698	-0.0238	-0.0494
	(0.0046)	(0.0055)	(0.0064)
Repayment Plan			
Monthly Benefit of Refinance	-3.5292	0.7113	4.7862
	(0.5242)	(0.2240)	(0.6545)
Value of Deferral	-2.9212	-0.7311	-0.5347
	(0.4559)	(0.4431)	(0.0957)
Economic Impact Payment	0.5826	-0.0824	0.0000
	(0.3898)	(0.3704)	(0.0000)
Delinquency Spell Dummy	0.2682	-0.6779	-0.1776
	(0.1735)	(0.2288)	(0.2732)
Current FICO	0.8078	-2.1027	2.0170
	(1.5599)	(0.3277)	(0.8537)
Marked-to-Market LTV	0.8615	-1.7941	3.9473
	(0.1732)	(0.3671)	(1.0318)

Table 3.4: Results of Step 2 — Determinants of Exits, cont'd
(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Original Debt-to-Income	-1.3036	-1.4519	-0.6274
	(0.3666)	(0.4649)	(0.4695)
First Home	-0.6588	1.2716	0.0886
	(0.1999)	(0.4786)	(0.2277)
Coborrower	-0.3124	0.1551	-0.2701
	(0.1678)	(0.2829)	(0.2182)
Prob. Curtailment	0.0647	-0.0042	0.0009
	(0.0331)	(0.0422)	(0.0270)
Prob. Not paying	-0.0437	-0.0385	0.0122
	(0.0084)	(0.0133)	(0.0077)
Payment Deferral			
Monthly Benefit of Refinance	-1.5943	-2.4998	-7.7574
	(0.4503)	(0.3292)	(0.4802)
Value of Deferral	-0.7480	-0.1322	0.0000
	(0.1567)	(0.0680)	(0.0000)
Economic Impact Payment	-0.0444	0.4139	-0.0427
	(0.1797)	(0.0773)	(0.0411)
Current FICO	4.0576	5.9259	4.1034
	(0.6994)	(0.5934)	(0.5550)
Marked-to-Market LTV	2.3104	3.3496	4.3518
	(0.4142)	(0.4682)	(0.4277)
Prob. Curtailment	-0.0243	-0.0205	0.0285
	(0.0243)	(0.0162)	(0.0146)
Prob. Not paying	-0.0042	0.0010	-0.0089
	(0.0043)	(0.0044)	(0.0041)
Not Making Payment			
Num. Delinquency Spell	0.4534	0.4617	0.7114
	(0.0399)	(0.0592)	(0.1192)
High Covid	0.0971	0.0493	0.0409
	(0.0238)	(0.0358)	(0.0777)
Fair FICO	-0.0928	-0.1345	-0.2909
	(0.0275)	(0.0416)	(0.0878)
Good FICO	0.4642	0.5609	0.3500
	(0.0461)	(0.0661)	(0.1352)
Very Good FICO	-0.3354	-0.3173	-0.3828
	(0.0288)	(0.0438)	(0.0949)
Exceptional FICO	-7.5728	-0.0902	0.2000
	(0.0016)	(0.0026)	(0.0037)
MLTV	0.1243	-0.0304	1.8418
	(0.1179)	(0.1605)	(0.3967)
ODTI	0.3893	0.5242	0.3077
	(0.1469)	(0.2253)	(0.2377)
Coborrower	-0.2202	-0.2206	-0.4057
	(0.0226)	(0.0330)	(0.0750)
First Home	-0.0059	0.0843	0.0745
	(0.0249)	(0.0385)	(0.0840)

Table 3.4: Results of Step 2 — Determinants of Exits, cont'd
(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Curtailment Payment			
Num. Delinquency Spell	0.2466	0.3576	-0.1809
	(0.0747)	(0.1095)	(0.2306)
Other Disaster	-0.1310	-0.1116	-0.2533
	(0.0419)	(0.0672)	(0.1299)
High Covid	0.1131	-0.0068	0.1036
	(0.0445)	(0.0647)	(0.1314)
Current FICO	2.2127	2.1425	1.8374
	(0.2777)	(0.4308)	(0.5711)
MLTV	-2.1139	-2.0705	-3.2417
	(0.1800)	(0.3003)	(0.5221)
ODTI	-0.4814	-0.5414	1.3572
	(0.2437)	(0.3987)	(0.4074)
Coborrower	-0.0982	-0.1429	-0.3616
	(0.0419)	(0.0663)	(0.1298)
First Home	0.3467	0.3860	0.6525
	(0.0454)	(0.0737)	(0.1353)
Exit			
Covid Case	-2.8906	-0.2859	0.1978
	(0.0016)	(0.0026)	(0.0035)
MLTV	1.6032	-0.2148	
	(0.0647)	(0.1151)	
UNRATE	-4.8088	-2.3613	0.1981
	(0.0016)	(0.0026)	(0.0035)
Current FICO	-0.1168	1.9772	-0.9057
	(0.1505)	(0.2981)	(0.3916)
Other Disaster	-0.0214	0.2923	-0.0679
	(0.0165)	(0.0519)	(0.0787)
Monthly Benefit of Refinance	-0.6985	-0.6123	-2.9207
	(0.1157)	(0.1464)	(0.2152)
Value of deferral	-0.3970	-0.1204	-0.0617
	(0.0570)	(0.0228)	(0.0206)
Prob. Curtailment	0.0564	0.0066	-0.0179
	(0.0017)	(0.0021)	(0.0027)
Prob. Not paying	0.0595	0.0202	0.0072
	(0.0008)	(0.0012)	(0.0011)
Other Controls	Y	Y	Y
Log Likelihood	-49347.53	-23632.20	-8324.25

Table 3.5: Results of One-Step Model

(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Prepayment			
Monthly Benefit of Refinance	1.7074	3.7468	-0.8540
	(0.1143)	(0.2965)	(0.5355)
Value of Deferral	-2.3540	-1.9462	-1.9017
	(0.1569)	(0.1149)	(0.1165)
Economic Impact Payment	0.1354	0.2884	0.0000
	(0.0735)	(0.0925)	(0.0000)
Delinquency Spell Dummy	-0.3049	-0.8388	-0.8423
	(0.1272)	(0.1682)	(0.1383)
Current FICO	7.1985	8.5643	6.0440
	(0.2322)	(0.3145)	(0.3645)
Marked-to-Market LTV	0.9260	4.4121	3.1582
	(0.1301)	(0.2722)	(0.1443)
Original Debt-to-Income	-0.8661	-0.2369	0.8311
	(0.1227)	(0.0425)	(0.0555)
First Home	-0.1910	-0.2379	-0.3518
	(0.0505)	(0.1036)	(0.1422)
Coborrower	-0.1722	-0.0776	-0.0636
	(0.0590)	(0.1163)	(0.1160)
Prob. Curtailment	0.0723	0.01122	0.0274
	(0.0035)	(0.0043)	(0.0118)
Prob. Not paying	-0.0548	-0.0338	-0.0427
	(0.0010)	(0.0030)	(0.0040)
Reinstatement			
Monthly Benefit of Refinance	1.6214	2.9988	1.5499
	(0.0733)	(0.2493)	(0.5298)
Value of Deferral	-2.8401	-2.3572	-0.5821
	(0.3152)	(0.2227)	(0.1740)
Economic Impact Payment	0.2309	0.3361	0.0000
	(0.1041)	(0.1107)	(0.0000)
Delinquency Spell Dummy	-0.0084	-0.4032	-0.0671
	(0.1212)	(0.2028)	(0.1040)
Current FICO	2.4648	3.2721	2.4415
	(0.2309)	(0.2879)	(0.1671)
Marked-to-Market LTV	-0.4456	0.7197	0.6915
	(0.1474)	(0.3072)	(0.3225)
Original Debt-to-Income	1.0813	-0.8332	3.0545
	(0.1550)	(0.0438)	(0.0885)
First Home	0.1467	0.0527	0.6010
	(0.0504)	(0.1067)	(0.2127)
Coborrower	-0.2156	-0.2211	-0.0495
	(0.0596)	(0.1587)	(0.1101)
Prob. Curtailment	0.0906	0.0373	0.0522
	(0.0036)	(0.0047)	(0.0128)
Prob. Not paying	-0.0721	-0.0243	-5.1092
	(0.0010)	(0.0035)	(0.0061)
Repayment Plan			
Monthly Benefit of Refinance	-3.2763	1.0381	4.8324
	(0.1834)	(0.1344)	(0.5600)
Value of Deferral	-3.1736	-0.7593	-0.5418
	(0.0634)	(0.4772)	(0.0745)
Economic Impact Payment	0.4904	-0.0955	0.0000
	(0.1465)	(0.0552)	(0.0000)
Delinqunсy Spell Dummy	0.3104	-0.6458	-0.1591
	(0.0498)	(0.0444)	(0.1533)
Current FICO	0.6162	-1.9496	2.1099
	(0.0227)	(0.1168)	(0.1245)
Marked-to-Market LTV	0.6917	-1.7186	3.8788
	(0.0571)	(0.1568)	(0.1209)

Table 3.5: Results of One-Step Model, cont'd
(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
Original Debt-to-Income	-1.1368	-1.4310	-0.6671
	(0.1421)	(0.0287)	(0.0252)
First Home	-0.6112	1.2727	0.0896
	(0.0841)	(0.1144)	(0.0619)
Coborrower	-0.3035	0.1568	-0.2614
	(0.0140)	(0.1439)	(0.1057)
Prob. Curtailment	0.0273	-0.0024	-0.0121
	(0.0040)	(0.0018)	(0.0037)
Prob. Not paying	-0.0441	-0.0389	0.0101
	(0.0008)	(0.0143)	(0.0040)
Payment Deferral			
Monthly Benefit of Refinance	-1.5790	-2.5198	-7.8458
	(0.1587)	(0.2371)	(0.1736)
Value of Deferral	-0.7526	-0.1294	0.0000
	(0.0746)	(0.0563)	(0.0000)
Economic Impact Payment	-0.0922	0.4148	-0.0420
	(0.0902)	(0.0782)	(0.0386)
Current FICO	3.8722	5.9585	4.1477
	(0.1851)	(0.2937)	(0.3246)
Marked-to-Market LTV	2.3007	3.3442	4.3848
	(0.1269)	(0.2613)	(0.1966)
Prob. Curtailment	-0.0432	-0.0215	0.0259
	(0.0057)	(0.0050)	(0.0099)
Prob. Not paying	-0.0079	0.0005	-0.0104
	(0.0012)	(0.0025)	(0.0035)
Not Making Payment			
Num. Delinquency Spell	0.1128	0.0014	0.4796
	(0.0170)	(0.0008)	(0.0618)
Other Disaster	-0.1086	-0.0009	0.4213
	(0.0153)	(0.0007)	(0.0917)
High Covid	-0.0026	0.0000	0.0459
	(0.0078)	(0.0003)	(0.0375)
High UNRATE	0.0201	0.0003	0.0066
	(0.0090)	(0.0003)	(0.0349)
Fair FICO	0.0077	0.0002	-0.0944
	(0.0063)	(0.0003)	(0.0439)
Good FICO	0.0327	0.0011	0.2736
	(0.0123)	(0.0007)	(0.0688)
Very Good FICO	-0.0247	-0.0007	-0.1668
	(0.0073)	(0.0004)	(0.0457)
Exceptional FICO	7.9360	-0.0134	-0.4767
	(0.0021)	(0.0025)	(0.0034)
MLTV	0.4122	0.0071	0.2067
	(0.0375)	(0.0035)	(0.0964)
ODTI	-0.0138	0.0021	-0.2592
	(0.0435)	(0.0019)	(0.1308)
Coborrower	-0.0017	-0.0005	-0.1778
	(0.0082)	(0.0004)	(0.0367)
First Home	-0.0368	-0.0003	0.0567
	(0.0094)	(0.0003)	(0.0402)
Curtailment Payment			
Num. Delinquency Spell	0.0998	0.0015	0.3124
	(0.0150)	(0.0008)	(0.0577)
Other Disaster	-0.0129	0.0009	0.4192
	(0.0109)	(0.0006)	(0.1015)

Table 3.5: Results of One-Step Model, cont'd
(standard errors are in parentheses)

	FB Age ≥ 6	**FB Age** ≥ 12	**FB Age** ≥ 18
High Covid	0.0011	0.0001	0.1143
	(0.0070)	(0.0002)	(0.0411)
High UNRATE	0.0014	0.0000	-0.0373
	(0.0076)	(0.0003)	(0.0406)
Current FICO	0.5921	0.0131	1.1148
	(0.0492)	(0.0062)	(0.0492)
MLTV	0.0806	0.0032	-0.0051
	(0.0290)	(0.0018)	(0.0639)
ODTI	-0.1228	-0.0019	-0.5794
	(0.0413)	(0.0017)	(0.0934)
Coborrower	0.0108	0.0001	-0.0158
	(0.0073)	(0.0003)	(0.0389)
First Home	0.0106	0.0005	0.1280
	(0.0082)	(0.0003)	(0.0424)
Regular Payment			
Num. Delinquency Spell	-0.1338	-0.1726	0.1669
	(0.0255)	(0.0299)	(0.0574)
Other Disaster	0.2230	0.1751	0.7213
	(0.0262)	(0.0397)	(0.0815)
High Covid	0.0687	0.0981	0.1293
	(0.0144)	(0.0179)	(0.0359)
High UNRATE	-0.0253	-0.0240	-0.0355
	(0.0146)	(0.0170)	(0.0355)
Fair FICO	0.0338	0.0560	0.0605
	(0.0171)	(0.0225)	(0.0428)
Good FICO	-0.2361	-0.2614	-0.1037
	(0.0286)	(0.0358)	(0.0652)
Very Good FICO	0.1600	0.1897	0.1257
	(0.0179)	(0.0234)	(0.0448)
Exceptional FICO	0.1663	0.4583	0.2104
	(0.0021)	(0.0025)	(0.0034)
MLTV	0.5283	0.4270	0.3074
	(0.0692)	(0.0924)	(0.1035)
ODTI	-0.1683	-0.1889	-0.4548
	(0.0823)	(0.1110)	(0.1145)
Coborrower	0.1181	0.1409	0.1075
	(0.0143)	(0.0178)	(0.0354)
First Home	-0.0474	-0.0398	-0.0293
	(0.0161)	(0.0201)	(0.0384)
Exit			
Covid Case	0.9791	-1.8866	-0.3264
	(0.1824)	(0.2856)	(0.2391)
MLTV	0.0239	-0.1751	
	(0.3873)	(0.1468)	
UNRATE	-1.9282	0.0267	2.9547
	(0.6148)	(0.1791)	(1.1607)
Current FICO	0.0886	0.9062	-0.5376
	(0.1518)	(0.2092)	(0.2386)
Other Disaster	-0.0150	1.0189	0.0236
	(0.0156)	(0.3212)	(0.0773)
Monthly Benefit of Refinance	-0.6413	-0.9053	-2.8465
	(0.1131)	(0.1464)	(0.1772)
Value of Deferral	-0.3993	-0.0868	-0.0602
	(0.0542)	(0.0222)	(0.0212)
Prob. Curtailment	0.0025	0.0193	-0.0133
	(0.0057)	(0.0036)	(0.0024)
Prob. Not paying	0.0148	-0.0039	-0.0212
	(0.0098)	(0.0015)	(0.0119)
η	-1.2729	-5.2555	7.5643
	(0.0667)	(0.4658)	(1.4484)
Other Controls	Y	Y	Y
Log Likelihood	-215494.57	-156668.83	-72660.98

Table 3.6: Accuracy Rate

	FB Age ≥ 6		**FB Age** ≥ 12		**FB Age** ≥ 18	
	Two-Step	One-Step	Two-Step	One-Step	Two-Step	One-Step
FB Age 1	0.6305	0.6286	0.6331	0.6330	0.5904	0.5901
FB Age 2	0.7690	0.7686	0.7992	0.7988	0.7592	0.7606
FB Age 3	0.7051	0.7056	0.7441	0.7441	0.7386	0.7381
FB Age 4	0.7237	0.7259	0.7656	0.7661	0.7828	0.7834
FB Age 5	0.7178	0.7180	0.7610	0.7612	0.7781	0.7775
FB Age 6	0.4693	0.6251	0.7542	0.7540	0.7928	0.7931
FB Age 7			0.7681	0.7684	0.8078	0.8072
FB Age 8			0.7677	0.7678	0.8167	0.8270
FB Age 9			0.7722	0.7722	0.8266	0.8268
FB Age 10			0.7778	0.7778	0.8402	0.8396
FB Age 11			0.7628	0.7629	0.8236	0.8238
FB Age 12			0.4326	0.6175	0.8293	0.8297
FB Age 13					0.8359	0.8354
FB Age 14					0.8370	0.8367
FB Age 15					0.8357	0.8357
FB Age 16					0.8392	0.8391
FB Age 17					0.8190	0.8188
FB Age 18					0.3970	0.5164

Table 3.7: Summary Statistics

	Whole Sample	Subsample
Pct Curtailment	0.077	0.077
Pct Regular Pay	0.315	0.318
Pct No Paying	0.608	0.605
Monthly Benefit of Refinance	0.207	0.203
Value of Payment Deferral	0.218	0.214
Economic Impact Payment	0.074	0.069
Delinquency Spell Dummy	0.125	0.121
Avg Duration of Delinquency Spell	0.371	0.354
Other Disaster	0.152	0.144
High Covid US	0.202	0.202
High Covid State	0.277	0.274
UNRATE	11.64	11.09
Current FICO	0.697	0.702
Marked-to-Market LTV	0.691	0.690
Original DTI	0.388	0.387
Orig 2013 - 2014	0.083	0.089
Orig 2015 - 2016	0.221	0.225
Orig 2017 - 2018	0.410	0.406
Orig 2018 - 2021	0.283	0.279
enter (1-2 Months)	0.647	0.635
enter (3-6 Months)	0.240	0.226
enter (>6 Months)	0.136	0.139
First Home	0.314	0.316
Co-Borrower	0.401	0.399
Exit Type		
Prepayment	0.275	0.269
Reinstatement	0.394	0.390
Repayment Plan	0.015	0.014
Payment Deferral	0.297	0.304
Modification	0.019	0.023

Table 3.8: Results of One-Step Model without Fixing T_m
(standard errors are in parentheses)

	Model 1	**Model 2**	**Model 3**	**Model 4**
Prepayment				
Monthly Benefit of Refinance	3.1795	3.1728	2.8945	2.5864
	(0.9606)	(0.0067)	(0.9745)	(0.4924)
Value of Deferral	-11.9234	-11.9225	-4.9951	-5.2689
	(0.7543)	(0.0053)	(0.8335)	(0.5627)
Economic Impact Payment	2.4505	2.4475	1.9509	2.0264
	(0.2063)	(0.0054)	(0.7361)	(0.2491)
Delinquency Spell Dummy	0.1686	0.1693	-0.0051	-0.0599
	(0.2498)	(0.0052)	(0.2499)	(0.1924)
Current FICO	12.1484	12.1470	11.1650	10.1276
	(1.9403)	(0.0052)	(1.1317)	(1.1872)
Marked-to-Market LTV	0.1985	0.1959	0.0661	-0.0224
	(0.5814)	(0.0064)	(0.7190)	(0.1872)
Original Debt-to-Income	-1.8915	-1.8782	-2.4818	-2.4389
	(0.6165)	(0.0084)	(0.4728)	(0.1571)
First Home	-0.5226	-0.5235	-0.4791	-0.4696
	(0.1553)	(0.0052)	(0.1764)	(0.1564)
Coborrower	-0.2822	-0.2822	-0.4193	-0.4504
	(0.1366)	(0.0052)	(0.1520)	(0.1482)
Prob. Curtailment			0.0871	0.0937
			(0.0227)	(0.0091)
Prob. Not paying			-0.0316	-0.0330
			(0.0079)	(0.0031)
Reinstatment				
Monthly Benefit of Refinance	0.8068	0.7970	0.4930	0.1255
	(1.0000)	(0.0067)	(0.9744)	(0.4803)
Value of Deferral	-21.7630	-21.7608	-5.8572	-6.5485
	(1.0647)	(0.0053)	(1.1840)	(0.8197)
Economic Impact Payment	0.6448	0.6412	0.2690	0.3179
	(0.2453)	(0.0054)	(0.7819)	(0.2618)
Delinquency Spell Dummy	0.0806	0.0810	-0.0946	-0.1274
	(0.2474)	(0.0052)	(0.2695)	(0.1963)
Current FICO	9.0621	9.0569	7.0569	5.7112
	(1.8630)	(0.0055)	(1.1497)	(1.1352)
Marked-to-Market LTV	-2.0408	-2.0501	-1.5001	-1.2904
	(0.5946)	(0.0067)	(0.7864)	(0.2844)
Original Debt-to-Income	-0.2834	-0.2634	-1.0595	-0.8995
	(0.5534)	(0.0085)	(0.4393)	(0.1445)
First Home	0.0619	0.0614	0.0572	0.0273
	(0.1465)	(0.0052)	(0.1801)	(0.1594)
Coborrower	-0.1786	-0.1784	-0.4029	-0.4224
	(0.1321)	(0.0052)	(0.1585)	(0.1538)
Prob. Curtailment			0.1234	0.1409
			(0.0227)	(0.0086)
Prob. Not paying			-0.0565	-0.0591
			(0.0078)	(0.0033)
Repayment Plan				
Monthly Benefit of Refinance	4.0678	4.0750	3.8010	3.4535
	(1.8978)	(0.0059)	(1.7260)	(0.5794)
Value of Deferral	-5.2208	-5.2263	-2.0468	-2.0189
	(1.4584)	(0.0061)	(1.0670)	(0.2746)
Economic Impact Payment	-10.1394	-10.1593	-14.7846	-15.2170
	(1.4928)	(0.0121)	(2.2648)	(0.7320)
Delinquency Spell Dummy	0.3625	0.3750	0.1671	0.0744
	(0.4646)	(0.0055)	(0.5821)	(0.4433)
Current FICO	10.4346	10.4383	9.8474	8.4357
	(2.9753)	(0.0058)	(1.7972)	(2.2513)
Marked-to-Market LTV	-0.7719	-0.7732	-0.4824	-0.5083
	(1.0456)	(0.0052)	(1.1777)	(0.3266)

Table 3.8: Results of One-Step Model without Fixing T_m, cont'd
(standard errors are in parentheses)

	Model 1	**Model 2**	**Model 3**	**Model 4**
Original Debt-to-Income	-3.7286	-3.7247	-4.0463	-4.1458
	(0.8775)	(0.0057)	(0.6242)	(0.0640)
First Home	0.1392	0.1387	0.0957	0.1047
	(0.4015)	(0.0052)	(0.3986)	(0.1811)
Coborrower	-0.2735	-0.2743	-0.4142	-0.4531
	(0.4146)	(0.0052)	(0.4000)	(0.2414)
Prob. Curtailment			0.0943	0.1035
			(0.0270)	(0.0134)
Prob. Not paying			-0.0207	-0.0251
			(0.0106)	(0.0058)
Payment Deferral				
Monthly Benefit of Refinance	-0.6209	-0.6298	-0.9245	-1.0710
	(0.9330)	(0.0062)	(0.8983)	(0.3521)
Value of Deferral	-2.3266	-2.3242	-2.7087	-2.7273
	(0.4104)	(0.0052)	(0.5184)	(0.3211)
Economic Impact Payment	0.7051	0.7022	0.6739	0.6691
	(0.2588)	(0.0053)	(0.7075)	(0.2687)
Current FICO	9.2550	9.2556	10.4656	10.3485
	(1.7230)	(0.0053)	(1.0683)	(1.0873)
Marked-to-Market LTV	0.3770	0.3645	0.4035	0.2329
	(0.5641)	(0.0066)	(0.7608)	(0.1805)
Prob. Curtailment			-0.0212	-0.0385
			(0.0240)	(0.0132)
Prob. Not paying			0.0204	0.0194
			(0.0069)	(0.0036)
Not Making Payment				
Num. Delinquency Spell	0.3329	0.0622	0.4062	0.1305
	(0.0979)	(0.0251)	(0.0942)	(0.0458)
Other Disaster	-0.1111	0.0242	-0.2316	-0.1039
	(0.1267)	(0.0266)	(0.1093)	(0.0463)
High Covid	-0.1440	-0.0567	-0.1028	-0.0026
	(0.0867)	(0.0131)	(0.0835)	(0.0333)
High UNRATE	-0.0662	-0.0301	-0.1306	-0.0684
	(0.0571)	(0.0151)	(0.0545)	(0.0258)
Fair FICO	-0.1551	-0.0014	-0.1373	-0.0091
	(0.0980)	(0.0223)	(0.0917)	(0.0281)
Good FICO	-0.1952	-0.0273	-0.2081	-0.0372
	(0.0984)	(0.0226)	(0.0908)	(0.0315)
Very Good FICO	-0.4102	-0.0739	-0.3995	-0.0718
	(0.1039)	(0.0269)	(0.0952)	(0.0370)
Exceptional FICO	-0.3971	-0.1024	-0.3699	-0.0934
	(0.1284)	(0.0330)	(0.1174)	(0.0433)
MLTV	0.9314	0.4865	0.9652	0.4532
	(0.2547)	(0.0344)	(0.2352)	(0.1029)
ODTI	-0.8040	-0.1161	-0.7535	-0.0854
	(0.3374)	(0.0220)	(0.3039)	(0.1201)
Coborrower	-0.0598	0.0019	-0.0578	0.0037
	(0.0553)	(0.0100)	(0.0528)	(0.0217)
First Home	0.0327	-0.0255	0.0075	-0.0291
	(0.0610)	(0.0113)	(0.0549)	(0.0251)

Table 3.8: Results of One-Step Model without Fixing T_m, cont'd
(standard errors are in parentheses)

	Model 1	Model 2	Model 3	Model 4
Curtailment Payment				
Num. Delinquency Spell	0.2870	0.0569	0.3307	0.1126
	(0.0997)	(0.0247)	(0.0971)	(0.0420)
Other Disaster	-0.2112	-0.0162	0.0753	0.0286
	(0.1383)	(0.0250)	(0.1447)	(0.0501)
High Covid	-0.0194	-0.0271	-0.0137	0.0077
	(0.0897)	(0.0127)	(0.0812)	(0.0298)
High UNRATE	-0.0023	-0.0158	0.0081	-0.0246
	(0.0561)	(0.0136)	(0.0557)	(0.0225)
Current FICO	1.8146	0.3706	1.8706	0.4812
	(0.3527)	(0.0686)	(0.3188)	(0.1320)
MLTV	-0.1616	0.1959	-0.2143	0.1154
	(0.2257)	(0.0321)	(0.2180)	(0.0786)
ODTI	-0.5867	-0.0932	-0.6520	-0.1035
	(0.3480)	(0.0251)	(0.3147)	(0.1094)
Coborrower	0.0727	0.0269	0.0770	0.0320
	(0.0585)	(0.0098)	(0.0563)	(0.0207)
First Home	0.1067	-0.0029	0.1067	0.0014
	(0.0651)	(0.0128)	(0.0621)	(0.0230)
Regular Payment				
Num. Delinquency Spell	0.1417	-0.0682	0.1997	-0.0204
	(0.0945)	(0.0358)	(0.0920)	(0.0648)
Other Disaster	-0.1074	-0.0113	-0.0714	-0.0192
	(0.1175)	(0.0548)	(0.1080)	(0.0730)
High Covid	-0.1085	-0.0573	-0.0824	-0.0194
	(0.0849)	(0.0120)	(0.0819)	(0.0572)
High UNRATE	0.0139	0.0391	-0.0146	0.0253
	(0.0529)	(0.0158)	(0.0513)	(0.0398)
Fair FICO	0.1054	0.1900	0.1379	0.2058
	(0.0969)	(0.0311)	(0.0939)	(0.0707)
Good FICO	0.1761	0.2596	0.1819	0.2687
	(0.0961)	(0.0295)	(0.0923)	(0.0699)
Very Good FICO	0.1805	0.3609	0.2025	0.3793
	(0.1010)	(0.0255)	(0.0956)	(0.0722)
Exceptional FICO	0.2627	0.4122	0.2973	0.4370
	(0.1242)	(0.0273)	(0.1181)	(0.0909)
MLTV	1.3125	1.0487	1.3023	0.9843
	(0.2483)	(0.0303)	(0.2328)	(0.1749)
ODTI	-0.4909	0.0507	-0.4845	0.0606
	(0.3261)	(0.0603)	(0.2977)	(0.1919)
Coborrower	0.0969	0.1281	0.1016	0.1324
	(0.0534)	(0.0093)	(0.0510)	(0.0361)
First Home	-0.0620	-0.1134	-0.0742	-0.1105
	(0.0588)	(0.0148)	(0.0547)	(0.0414)
Exit				
Covid Case	0.0289	0.0289	0.0462	0.0478
	(0.0068)	(0.0052)	(0.0067)	(0.0062)
MLTV	-0.4203	-0.4201	-0.1077	0.0851
	(0.1665)	(0.0052)	(0.1689)	(0.1601)
UNRATE	-0.0050	-0.0050	-0.0085	-0.0098
	(0.0068)	(0.0037)	(0.0068)	(0.0069)
Current FICO	2.4464	2.4466	1.2810	1.0114
	(0.2911)	(0.0052)	(0.2932)	(0.1846)
Other Disaster	-0.0792	-0.0792	-0.0822	-0.0980
	(0.0536)	(0.0052)	(0.0521)	(0.0540)
Prob. Curtailment			0.0187	0.0295
			(0.0021)	(0.0028)
Prob. Not paying			-0.0127	-0.0138
			(0.0012)	(0.0011)
θ		0.2089		0.2335
		(0.0263)		(0.0306)
Other Controls	Y	Y	Y	Y
Log Likelihood	-35901.58	-35805.71	-35572.49	-35490.44

Table 3.9: Accuracy Rate

	In-Sample	Whole Sample
FB Age 1	0.5445	0.5408
FB Age 2	0.6115	0.6169
FB Age 3	0.7115	0.7135
FB Age 4	0.6108	0.6129
FB Age 5	0.6696	0.6652
FB Age 6	0.6739	0.6662

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PUBLICATION

"Applying Seasonal Adjustments to Housing Markets" (with William M. Doerner, *Cityscape*, 2022)

House price seasonality has been increasing over the last decade, but adjustments have remained largely unchanged in commonly used public data. This report shows how seasonal adjustments work—both theoretically and applied to observed transactions—when constructing house price indices (HPIs). In this report, the authors find the seasonality in the housing market is not uniform across geographies. Evidence is provided about where adjustments are more necessary, how often they should be recalculated, and how the weather-related variables, social, and industry characteristics impact differences between adjusted and non-adjusted HPIs. Using the Federal Housing Finance Agency's (FHFA's) entire suite of public indices, the authors update adjustments provided by FHFA and offer new adjustments for more than 400 metropolitan areas and other geographies, which haven't been provided before. They find the difference between previous and updated adjusted indices are relatively small, with slight improvement in recent years.

RESEARCH PAPERS

"A Dirichlet Nested Logit Model of Household Mortgage Payment in the CARES Act Forbearance Program" (Job Market Paper)
This paper uses borrower mortgage payment behavior in the CARES Act Mortgage Forbearance program to predict the mode of exit from the program. The CARES Act permits borrowers to postpone mortgage payments without penalty. In the empirical work, this paper extends the beta-logistic model in Heckman and Willis (1977) to the Dirichlet nested logit model, which allows the state dependence of choices to vary across different nests. The results show that the beta distribution of probabilities of choices within the nest and between nests are both *J* shaped, which indicates that the payment behavior probability of relatively few borrowers is near the average. Moreover, borrowers who make curtailment payments are more likely to exit forbearance with prepayment or reinstatement. In comparison, borrowers who frequently forbear payments are more likely to leave with payment deferral or trial/modification.

"New Tests of the Multinomial and Nested Logit Models with an Application to Residential Mortgage Termination." (with Ran An and Jan Ondrich)
In this paper we apply the Ding-Tian-Yu-Guo transformation to multinomial logit duration models with and without non-parametric random effect controls for over-dispersion in the data due to unobserved heterogeneity. Ding, Tian, Yu, and Guo (2012) analyze transformations of the binomial logit duration model for which the results are an exact binomial logit duration model for one value of a shape parameter and an interval-censored proportional hazard model for the limiting value of the shape parameter. We analyze the simultaneous mortgage-termination risks of 90-day delinquency and prepayment for single-family 30 year fixed-rate mortgages securitized by Fannie Mae using the Fannie Mae Public Use Data. We show that the transformation can control for over-dispersion in the data and performs better in both cases than the corresponding models without the transformation.

"The Price of Short-Term Housing: A Study of Airbnb on 26 Regions in the United States." (with Fan Yang)
This study employs sentiment analysis on online review comments to investigate the attributes that affect Airbnb users' experiences. We identified the main attributes of location, amenities, host, and transportation. The analysis suggests a positivity bias in Airbnb users' comments. Besides, the finding indicates that the positive comments are usually concentrated -- the top 5 popular positive words count around 30\% of the positive word. In contrast, the negative words are much more sparsely distributed. This research also combines sentiment analysis with a hedonic model and quantile regression to detect the impact of the price determinants of Airbnb listings, especially the effect of review content on Airbnb listing prices. We find that negative words in review content have a more massive impact on the lower-price listings, while positive comments are more negligible at the tail of the price distribution. Lastly, the result suggests the hysteresis of the impact of reviews on prices -- the host may not adjust the price immediately after the consumers leave comments online.

“Estimating Borrower Behavior in the CARES Act Forbearance Program: Sequential and Full Sample Approaches”

The models in my job market paper estimate the effect of payment behavior in the CARES Act forbearance program as the program continues through time. For a given exit time, the likelihoods contained information on only those mortgages that failed at that exit time. Results were presented for three exit times: 6, 12, and 18 months. A two-step estimation technique was used and standard errors were corrected in the second step. The first improvement in this paper incorporates information on all mortgages that survive until a given time into the likelihood functions. I show that the estimation can be accomplished in a single step. The accuracy of the two-step estimation and single-step estimation results are compared. The second improvement in this chapter is to construct a single model, estimated in a single step, that uses information for all of the first six months. The accuracy rate of the estimation for this new model is substantially higher than the accuracy rate of the estimation for the model with a single survival time of six months. Future work is to extend the estimation to cover the entire length of the program.

RESEARCH PROGRESS

“The Causal Effect of the CARES Act Forbearance Program on the Risk of Prepayment”

In my job market paper, I studied borrowers' payment and exit behavior after enrolling in the forbearance program. This paper uses the public-use Fannie Mae Credit Insurance Risk Transfer (CIRT) dataset to study the effect of foreclosure prevention policies on the prepayment risk from the CARES Act. In my next paper, borrowers are separated into four groups, as follows: (1) forbearance with no missed payments, (2) forbearance with missed payments, (3) no forbearance with no missed payments, and (4) no forbearance with missed payments. Group (1) and group (2) are the treatment groups, and group (3) and group (4) are the control groups. Servicers play an essential role in the CARES Act since borrowers need to contact their servicers to enter the forbearance program. Kim, Lee, Scharlemann, and Vickery (2022) find that small servicers and nonbanks have a lower propensity to provide forbearance. For every forbearance loan in groups (1) and (2), I intend to use the propensity score to match forbearance loans in groups (1) and (2) with nonforbearance loans with the same servicer in groups (3) and (4). I will then estimate the causal effect of forbearance on the prepayment risk.

SKILLS

Languages: English and Chinese.
Programming: R, Stata, GAUSS, LaTex.
Technique: Causal Inference, Difference in Difference, Discrete Choice, Logistic Regression, Instrumental Variable, Machine Learning, Data Mining, Text Mining, Sentiment analysis, Data visualization.