What Pre-Service Teachers Want "Math" to Know: Examining Self-Identified Relationships and Critical Experiences With Mathematics

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Claudia M. Bertolone-Smith[1], Alison Puliatte[2], Samantha Sommers[2], Michelle Unigarro[2], Danielle Vantassell[2]

Abstract
This study examines the self-reported critical experiences that undergraduate pre-service teachers (PSTs) choose to share when writing a letter directly to "Math" and creating a self- portrait of a math learning experience. The letters sought to initiate a personification of math and the self-portraits to further explore math learning experiences of the PSTs. The letters and portraits were examined to understand the types of math relationships and critical events PSTs reported and their impact on PST identity and agency as a future teacher. Portraits were analyzed by PST research partners. The relationship between the categories of critical events and mindset was explored.

Keywords
mathematics education, math identity, pre-service teacher education, math anxiety

Our pre-service teachers (PSTs) were in an undergraduate cohort for their education degrees, so we knew our students very well. Before our math methods course started, many began telling us their math stories, such as: "*Is next semester the math methods course? I was never good at math, and I'm really scared about this course."* Or "*I hated math when I was growing up and don't think I will be good at teaching it.*" For some PSTs, enrolling in a math methods or math content course for elementary educators may re-engage memories of critical negative experiences and emotional reactions that

[1] California State University, Chico
[2] SUNY Plattsburgh

Corresponding Author:
Claudia Bertolone-Smith, California State University Chico, School of Education, 400 West First Street, Tehama Hall, Room 432, Chico, CA 95929
Email: cmbertolone-smith@csuchico.edu

developed their mathematics identity and confidence in mathematics. Our students' minds appeared fixed about math and their future success as a math teacher based on past events with mathematics (Lutovac & Kaasila, 2014; Wood, 2013) and their mindset towards the mathematics methods course requirements in our teacher education program (Bekdemir, 2010; Dweck, 2016).

To address this problem of practice, we endeavored to encourage our PSTs to share their math learning experiences. Drawing from bell hooks' work on engaged pedagogy, we recognized our students to be humans with complex lives and experiences, we sought to create a space where students could share their math stories, and both teacher and peers can critically listen and hear each other (hooks, 1994). To elicit an honest and unique reflection of PSTs' math experiences, we created an assignment building on the work from Zazkis (2015) who asked students to personify math, and Cohen (2016), who asked students to write a "letter, or poem, or temper tantrum" (p. 18) to math. We asked the students to write a letter directly to math and create a portrait of themselves learning math to further evoke and illustrate PSTs math experiences (Cappello et al., 2019; Gracin & Kuzle, 2018).

To increase face validity, the extent to which a research project validates the voices and concerns of those being studied (Cai, 2017), we asked our PSTs for volunteers to be part of the research team. Cai (2017) recommends that research - regardless of outcomes- can take a culturally sensitive approach and "in some cases involve the participants actively shaping the research project" (p. 163). At the conclusion of the course during which the data was collected, we invited all students to participate in the research project with us through a google form. Three PSTs volunteered to join us in analyzing data for this research, specifically the self-portraits. The data was de-identified over the summer, and the PST volunteers began analyzing the data when they returned in the fall for their senior year, and our collaboration continued for one more year. Because this study was conducted with PST research partners, we feel it added a depth and dimension possibly unrealized without including their perspectives. This article shares the results of our study, aimed at understanding the self-reported and illustrated critical events which impact PSTs' perception of and confidence in knowing and teaching mathematics.

Literature Review

Research in teacher education has focused on the need for prospective teachers to understand the mathematical content they will teach and knowledge of how to teach mathematics well. Both skills are required to be effective mathematics educators, particularly in the elementary grades (Ball & Bass, 2003; Ball et al., 2005; Carpenter et al., 1988; Hill et al., 2008; Shulman, 1986). Students seeking elementary teacher certification from higher education institutions can, in some cases, be certified to teach kindergarten through eighth grade. This would require a PST to deeply understand the math content for each grade level as well as the nuanced approaches to teaching children at each stage and age in their learning (Hill et al., 2008). Additionally, education programs (i.e., undergraduate, masters, credential) are short on time and long on topics- PSTs need to learn both content and pedagogy in science, social studies, art, and language arts as well. Many students enroll in math methods courses for teacher education with great hesitancy and anxiety, and this can be a result of their past experiences with learning mathematics (Brady & Bowd, 2005; Novak & Tassell, 2017). Below, we discuss further how math identity, anxiety, and mindset influence PSTs' dispositions towards teaching mathematics. We then discuss strategies for eliciting PSTs experiences in efforts to increase self- awareness of influences from episodes of math learning to support the rehabilitation of negative or traumatic

experiences.

Mathematics Identity

The stories we tell about ourselves in relation to learning mathematics often reveal our mathematics identity (Boaler & Greeno, 2006; Sfard & Prusak, 2005). A mathematics identity develops in social contexts and can change over time (Langer-Osuna, 2017). Wood (2013) proposed that micro-identities may develop based on events which inform the student who they might be as a math learner. A thickening or thickened identity is one that the mathematics learner believes is true and is less malleable (Wood, 2013). Bishop (2012) argues that mathematics identity is learned and often stable and predictable, "yet malleable, and is both individual and collective" (p. 39). A mathematics teacher educator can witness and work to rehabilitate a negative and thickened math identity (Drake, 2006) while seeking to provide further math experiences that can transition attitudes (Ellsworth & Buss, 2010).

Mathematics Anxiety

Math identities that include a highly emotional reaction to a critical event can manifest as math anxiety. Math anxiety is often caused by negative experiences involving math, which can greatly hinder working memory (Mutlu, 2020). PSTs entering teacher education for the elementary grades have the highest rates of math anxiety, below average math proficiency, and self-reported poor experiences in K-12 math courses (Bekdemir, 2010; Novak & Tassell, 2017). PSTs' experiences in mathematics can also impact their mindset towards mathematics methods courses in a teacher education program (Bekdemir, 2010; Dweck, 2016). Unresolved and traumatic experiences from past mathematics learning experiences can develop early math anxiety which can remain well into adulthood and impact the quality of mathematics instruction (Schubert, 2019). Jackson and Leffingwell (2020) surveyed 157 PSTs and found that 93% of students reported a negative experience learning math. They found three grade bands where "anxiety producing problem(s)" (p. 583) tended to occur: grades 3 and 4, 9 and 11, and freshman year of college. Much of the anxiety was caused by instructor behaviors such as hostile comments (i.e., anger when students need help), gender bias (i.e., telling girls they do not need mathematics), and embarrassing students in front of their peers. There is a connection, therefore, to a teacher's instructional choices and anxiety that can develop in students. Bekdemir (2010) found that mathematics anxiety is persistent in PSTs and negative mathematics classroom experiences affect their level of anxiety. It is recommended that teacher educators are aware of how PSTs' anxiety can affect their success in the mathematics classroom and provide ways in which they can neutralize traumatic past experiences (Bekdemir, 2010). Math teacher educators can address PSTs' anxiety by communicating high expectations, sustaining a supportive class atmosphere, and responding to PSTs' emotions- these are considered caring mathematical relations and are essential in helping PSTs overcome math anxiety (Hackenberg, 2010). Intervention in this way may stop the mathematics anxiety cycle where anxious PSTs become anxious teachers, who create mathematically anxious students and so on.

Mindset towards Mathematics

Dweck (1975, 2016) posits that how we react to events and our success in life, including school, is influenced by how we perceive the relationship between our behavior and the occurrence of such events; this is our mindset. Mindset exists on a spectrum with a fixed mindset on one end and a growth mindset on the opposite end. Individuals with a fixed mindset tend to avoid challenges, give up easily, see effort as fruitless, ignore useful negative feedback, and feel threatened by the success of others. Fixed mindset orientated people believe that talent alone creates success, are reluctant to take on challenges, prefer to stay in their comfort zone, take less personal responsibility for their successes and failures, and are fearful of making mistakes.

Individuals with a growth mindset tend to embrace challenges, persist in the face of setbacks, see effort as the path to mastery, learn from criticism, and find lessons and inspiration in the success of others. Growth mindset orientated people view mistakes as an opportunity to develop, are resilient, believe that effort creates success, and believe that talents can be developed. When faced with challenging critical experiences, having a fixed mindset may cause an individual to avoid the situation and view the experience as a negative. While having a growth mindset may cause the individual to approach the critical experience with a positive attitude and view it as an opportunity to learn.

Approximately 40% of students have a growth mindset related to math and science, 40% have a fixed mindset, and the remaining 20% fall somewhere between growth and fixed (Dweck, 2013). The type of mindset a student has determines how they approach math and to what they attribute their math performance (Dweck, 1975). A student with a fixed mindset tends to avoid math and attribute performance to their ability. While a student with a growth mindset tends to persist in math and relate their performance to effort. The relationship between mindset and math achievement may be attributed to critical experiences such as their teachers' own mindset and their teachers' feedback practices (Dweck, 2013). Additionally, the math ability groups and level of math classes (advanced, remedial) students were assigned to influence their mindset (Boaler, 2013). Whereas fixed ability grouping reinforces students' beliefs that mistakes and low grades equate to low ability. Promising research has demonstrated students' perception of the relationship between their behavior and occurrence of their failure in math can be altered (Dweck, 1975; Yeager et al., 2019). When students were trained to reattribute their failure to lack of effort and to take responsibility for their failure, they were able to persist after experiencing failure and improve their math performance (Dweck, 1975). Additionally, increasing students' growth mindset resulted in improved GPA, math grades, and increased enrollment in advanced math courses for lower achieving students (Yeager et al., 2019).

Eliciting Math Stories for Rehabilitation Purposes

Research has focused on math stories told by PSTs to investigate how lived math learning experiences may impact a young teacher's relationship with teaching math (i.e., Drake, 2006, Drake et al., 2001; Lutovac & Kaasila, 2014; McCulloch et al., 2013; Mukhopadhyay, 1996; Zazkis, 2015). Several methods have been used to gather and analyze the stories of pre- and in- service teachers' lived experiences while learning math. According to Drake et al. (2001) when compared with literacy learning stories, math learning stories were dominated by disappointment and discouragement while literacy stories tended to remain positive with reported continual progress. Math stories tend to include turning points in learning (i.e., hating math to loving it) and a positive learning experience at any age

fostered a more positive outlook on mathematics (Drake et al., 2001). Relationships with math are often described as roller coaster ride with up and down achievement which coincided with a love/hate relationship with math itself (Cohen, 2016; Zazkis, 2015). Lutovack and Kaasila (2014) analyzed the narrative stories focused on school time memories and found that when PSTs had a negative future projection of themselves as a successful mathematics teacher, it was related to negative past experiences as a student.

Narrative tools and narrative rehabilitation promoted a more "positive predicted future-self and a goal-oriented focus on learning and self-development as a future teacher" (Lutovac & Kaasila, 2014, p. 12). Assigning mathematical autobiographies is also used to get to know students and to better understand their dispositions to mathematics (von Renesse, 2014). Autobiographies have been used to research teacher identity and as a reflection tool to support narrative rehabilitation (Drake et al., 2001; Drake, 2006; Sfard & Prusak, 2005). Ellsworth and Buss (2010) argued that written autobiographies "allow a broader opportunity for description of experiences" and authors are able to record "transitions in attitude based on further experiences" (p. 357).

Another approach to investigating PSTs' math experiences is through self-portraits. Mukhopadhyay (1996) characterized a self-portrait as a "narration and representation of self- study" and a process which "attempts to reveal the inner self to the world outside" (p. 107). Mukhopadhyay asserts that a self-portrait is a powerful means of metacognition and self- assessment and may remove the barrier of language relying on a visual and emotional perception of impactful math learning experiences. Drawings can purposefully be used to provide insights into students' perceptions (of self, learning, social dynamics, etc.) and lived experiences and understandings (Capello et al., 2019; Gracin & Kuzle, 2018). Researchers (e.g., Gracin & Kuzle, 2018; Rolka & Halverscheid, 2011) have used external representations such as drawings to investigate beliefs and emotions about mathematics for over a decade. According to Rolka and Halverscheid (2011), relying only on drawings can be subjective and the inclusion of participants' writing, or interview data is warranted.

Study Framework

Vygotsky (1978) stated that "every function in the child's cultural development appears twice, first on the social level and later on the individual level" (p. 57). We expand this phenomenon to include social learning events that happen at home, in classrooms, among peers, and with teachers. We are drawn to the significance of self-identified events and their impact at both the social and individual level. Boaler and Greeno (2000) found that different math learning environments (social level) where one focused on group problem-solving and the other on lecture, developed different mathematical identities (individual level). School-aged children encounter an array of events while learning mathematics. An event can be considered "leading" or critical, not because of its dominance in the present, but its role in shaping a person's thinking processes and how a person's development is dependent on such activities (Leont'ev, 1981; as cited in Black et al., 2010).

For example, a PST in the current study shared:

> One night my father helped me with my homework and showed me a different way to solve the problem. The next day at school Mrs. __ called me out in front of the class and gave me a zero for my homework because I didn't do her way. Her calling me out made me hate you (math) even more.

This qualifies as a critical event because of the effect and shaping it had on the student's relationship with learning mathematics.

Woods (1993) noted that critical incidents are "highly charged moments" which can have "enormous consequences for personal changes and development" (p. 356) and these incidents can be both positive and negative. Another PST in the current study wrote:

> In high school I was especially discouraged by math tests and there was one time my friend and I were studying, and I actually understood the topic so well that I taught her how to do all of it. My friend received a 100 and I received a 55. That was the day I lost all hope in my ability to do math.

Thus, this critical experience had a high emotional charge and an enormous consequence; the loss of hope in one's ability to do math. The National Research Council (2001) stated that a "students' disposition toward mathematics is a major factor in determining their educational success" and that "they (students) must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out" (p.131). This study is framed in the social construction of an individual's relationship with mathematics, as informed by the critical events which shape PSTs' anxiety level, identity, and mindset.

Methodology

Participants

The study was conducted at a public comprehensive college in the northeast United States. The 54 participants had a mean age of 20.71 years (range 19-23 years) and they were all enrolled in a junior level six credit course focusing on the teaching methods of elementary math and science. All participants were pursuing a degree to be eligible to teach elementary education. The course the PSTs were enrolled in during the study included an early field experience where the PSTs spent two days a week for five weeks in an elementary classroom. The minimum required math content courses for all PSTs in the program includes one general education math course and one course in the fundamental of mathematics for elementary school teachers. This is in addition to the six credit methods of math and science course the students were enrolled in during the study. All PSTs enrolled in the program are required to select a 15-credit specialization in one of six areas: STEM, English Language Arts, Social Studies, the Arts, Modern Languages and Cultures, and Interdisciplinary. Ten of the study participants were enrolled in the STEM specialization.

Participants self-identified their own K-12 school setting as rural (small town), suburban (outskirts of a large city), or urban (city). As K-12 students, 25 participants attended a rural K-12 school, four participants attended an urban K-12 school, and 22 attended a suburban K-12 school, three participants did not share their prior school experience. Ten participants took at least one accelerated math course in middle school and 20 participants took an accelerated math course in high school. The average number of college math courses the participants completed was 2.35 (range 1-6). The participants were recruited for the study by a teacher education faculty member who was not involved in the study. The faculty member visited the classrooms to present an overview of the study and distribute the consent forms. The course instructors, who were also the study researchers, left the room during this time to avoid influencing the PSTs' decision to consent. All participants signed consent to

participate in the study and to have their letters and pictures published. Additionally, all data was de-identified before it was reviewed by the researchers. The first two authors were instructors of the course and co-PIs of the research study. In this role, they coded and analyzed all the de-identified letters. Additionally, three PSTs who participated in the study coded and analyzed all the de-identified self-portraits.

In designing our methods for analyzing the data, we took into account the hierarchy in the teacher student relationship; and we did not want to inhibit the PST co-researchers' participation and confrontation of anxieties arising from their teacher preparation program. We addressed this concern by providing the PST co-researchers with full control over the analysis of the pictures. As the main researchers, we guided the PSTs in their research methodology during weekly meetings and the PSTs developed their own strategies for analyzing the pictures. They wrote and published an article as first authors on their full analysis of the pictures (Sommers et al., 2022).

Data Collections Procedures

The letters and portraits created by the PSTs were based on remembered past experiences, interpretations, and thoughts. As Kelchtermans (2017) states, "human beings are gifted in the capacity to remember and make sense of past experiences...and actions in the present are influenced by their experiences from the past and expectations for the future" (p. 10). To examine PSTs' relationship with math and the critical events that shaped this relationship, at the beginning of the semester, the PSTs were given a homework assignment asking them to: "Write a letter to math, including experiences you have had that have shaped your relationship with math. Discuss your future with math as you become an elementary school teacher. Draw a picture of yourself LEARNING math."

This assignment was given during the first week of the course without any related readings or discussions on the topic prior to submission of the assignment. The goal was to obtain the participants' personal reflection on their K-12 math learning experiences without being influenced by others' views. After the assignment was submitted and reviewed by the instructors, time was devoted to class discussions related to mathematical mindset and the impact of PSTs' past experiences on their pedagogical choices and self-efficacy as future teachers.

Students were put into pairs and given time to share their letters and describe their portraits to their peers. Afterwards, a class discussion was conducted about how the PSTs envisioned their own math identity, anxiety, and mindset. This assignment did not receive an evaluative grade; PSTs received credit for completing the assignment and no credit if they did not complete the assignment. An evaluative grade was not assigned because the goal of the assignment was for the PSTs to openly express their personal experiences and to begin to reflect on their math identity (hooks, 1994). The instructors did not want to hinder the PSTs' reflections by having them focus on a grade nor did they want to contribute to the development of a fixed mindset by placing a positive or negative evaluation on their personal experiences.

During the semester, we used this assignment to indicate teaching strategies and dispositions to foster healthy math identities in students. For example, many letters contained critical events surrounding timed tests for math fact fluency. Alternatives to this strategy were discussed and practiced empowering PSTs to choose a different approach for their future students. This connection to PSTs' experiences proved to be a salient way to encourage the implementation of current and effective math education strategies.

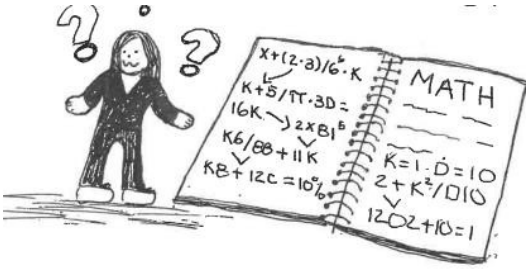
Data Analysis

Data from the letters were open-coded (Corbin & Straus, 2014) initially framing the analyses into three main themes - critical experiences (Leont'ev, 1981), mathematics relationships (Drake, 2001), and becoming a future teacher of mathematics. Each letter contained multiple data points depending on what participants included in their letter. For example, some participants included information in their letter addressing all three themes while others may have addressed only one theme. Additional open coding within each of these themes generated subcategories for each theme. To investigate evidence of fixed or growth mindsets in PSTs writing, participants' statements were again coded and evaluated as showing a fixed or growth mindset (Dweck, 1978, 2016).

PST research partners participated in the analysis of the data by analyzing the self- portraits included with the letters to math. The portraits were open coded to identify themes and commonalities among portraits. See Figure 1 for an example analysis of one portrait. PST research partners generated three themes based on portrait analysis: emotions exhibited, self- placement in the portrait (including others in the portrait), and response to math.

Figure 1.

This self-portrait indicates a negative emotion, the student alone, math depicted as larger than the student, the writing (question marks) indicates confusion, the writing and picture are related, there is not a setting, change is not indicated, and the interpretation of the picture was classified as frustration.



Additionally, guiding questions were developed for each theme to support research partners in analysis (Table 1).

Table 1

Themes and guiding questions developed from self-portrait analysis

Theme	Guiding Questions
Emotions Exhibited	What emotions are shown in the portraits? Who else is in the portrait with PSTs during the emotion?
Self-Placement	Where do PSTs place themselves? Where do PSTs place representations of math?
Response to Math (captions and text analysis)	What feelings are expressed in portrait captions? What feelings are present when portrait and text are examined as one?

Results

The results of the study are presented in two main sections. First, the letter analysis is discussed by presenting the findings of the themes and subcategories for each theme. Additionally, the mathematical mindset of the PSTs is explored through the letter analysis. Next, the self-portrait analysis is discussed by presenting the findings of the themes that were identified in the self-portrait analysis.

Letter Analysis

The letter analysis began by coding written comments from each PST into three themes and subcategories that further defined the theme (see Table 2).

Table 2
Themes and subcategories for letter analysis

Theme	Subcategories for Letter Analysis
Critical Experiences	Struggle and perseverance
	Positioned by a teacher
Math relationships	Character traits
	Feelings/emotions
	Relationship characterization
	Change of Heart
As a future teacher of mathematics	Hopes

Here, we will discuss theme and subcategory results from the letter analysis by providing the frequency in which the themes were mentioned and examples from the PST letters to math to further reflect their voice about their critical events and relationships with math.

The Theme of Critical Experience.

Experiences of Struggle, Perseverance. Critical experiences of struggle or perseverance related to mathematics were identified in 48 (85.7%) PSTs' letters. PSTs identified both positive and negative experiences of struggle or perseverance. The positive experiences highlight how PSTs turned their struggles into success through perseverance. For example, a PST wrote, "*Finding a positive from a dark spot on my education makes all the struggling worth it. So thank you math for being my greatest challenge*." The negative experiences demonstrate students giving up when faced with struggles in math. An example of a negative experience was seen in this PSTs' letter: "*When I was younger, I struggled with you. I had a hard time memorizing your basic skills. Multiplication and division took time to click. I remember dreading timed tests and times table games*." Several PSTs reported both positive and negative critical experiences from home situations when mentioning family members that worked to support them in mathematics. One PST commented that math caused "*my dad and I to get into small arguments because I couldn't understand his explanations*." This PST ends the letter with an emotional

response to math: "*I really, really, hated you and all the misery you caused me*."

Positioned by a Teacher. Interactions with a teacher which impacted PST's relationship with math were identified by 17 (31.5%) PSTs and included both positive and negative situations regarding class placement and teacher comments or actions, similar to Ellsworth & Buss (2000). Class placement refers to a teacher's decision to place a student in a special education, remedial, general education, or advanced math course. The positive experiences demonstrate how positioning by a teacher resulted in a successful math experience both academically and emotionally for the student. One PST wrote, "*I moved up to advanced math classes every year and scored high on every Regents exam, this made me feel confident and proud in school*." Another PST reported being diagnosed with dyscalculia at a young age and reflected that in 7th grade their "*math dreams came true, I was placed in a self-contained classroom. I was passing with flying colors!*" The negative experiences often resulted in instilling low self-esteem related to the student's self-identified math ability. For example, one PST recalled, " *When I was in third grade you [math] made me cry, and when I was in sixth grade you pushed me into an AIS [Academic Intervention Services] class that made me feel inferior to the rest of my classmates*." PST's interpretation of teaching decisions also emerged from the letters, one PST reflected:

> I will never forget the day in Mrs. K's first grade classroom when she gave me three chances to correct you (math). Each chance I failed, and got the massive F circled at the top of my page. From that day forward I knew that it was going to be a rough relationship.

The Theme of Mathematics Relationships. An interesting outcome of asking PSTs to write a letter directly to "math" was how often letters would characterize a deeply personal and long-term relationship with mathematics. PSTs assigned character traits to math, discussed the feelings and emotions generated, defined the type of relationship they experienced, and admitted to changes of heart. The results of the data analysis of mathematics relationships are organized into the following subcategories: character traits, feelings/emotions, relationship characterization, and change of heart. Specific examples and occurrences within the letters are discussed below.

Character Traits. Math was personified by assigning it a character trait by 32 (59.3%) PSTs. Descriptions of the relationships PSTs had with math characterized math in positive, neutral, and negative terms. Positive relationships were characterized by PSTs writing, "*I really enjoy your company. Math, you helped me feel powerful*." Neutral relationships were expressed in examples such as, "*It's like some people get along with you great and others just aren't your type.*" Negative relationships included statements such as, "*You are so complex, and my little brain didn't know what to do. All of your parts have so many steps and only about half would make sense*," and "*You have been a burden in my life. You sabotaged me a few times*."

Feelings/Emotions. Math elicited feelings/emotions for 42 (77.8%) PSTs. The PSTs attributed their feelings and emotions elicited by math to their perceived math ability and their personal comparisons of their ability to their peers. Positive feelings were depicted as, "*Math, you helped me feel powerful. I was always excited when I walked into my math classes*." Neutral feelings included, "*To me you are a tool and not much else*." Negative examples include, "*You stressed me out, and I could never figure you out*," and "*You frustrated me, you made me cry*."

Relationship Characterization. A description of the relationship with math was included for 52 (96.3%) PSTs. The PSTs described the relationship they had with math in terms of a personal relationship they would have with another person. Math's positive characteristics included always having a solution, making sense, being a loyal friend who makes one feel confident and proud. The dependent nature of math (i.e., based on rules to follow) emerged as a noble characteristic for those who also felt successful in math. For example, one letter shared. "*I love how there is always some kind of solution to all of the problems I'm solving with you. You just make sense to me*." Another mentions, "*I love you, I have almost always loved math,*" in conjunction with, "*I have always had a math oriented mind*." Another PST told math, "*You helped me feel powerful,*" and "*When others were not able to understand your problems, I was able to help them*." This also indicates a positive relationship with math can create feelings of pride, being smart, and capable. Neutral friendships were less commonly reported but existed. One PST stated, "*I don't find you to be particularly exciting, but you are not boring. To me you are a tool and not much else*." Some PSTs mentioned math can be "straightforward," and that "*honestly, some people like you and others don't.*" However, the same PST also stated, "*I've watched you have good and bad relationships with the people around me which I totally understand. I just have the whole, what does Peggy have that I don't?*" Perhaps these students attempted to neutralize their reactions to math, similar to when a child ignores being left out in a social group at school, but truly feels upset by it. Another PST told math, "*Sixth grade is when my battle with you seemed to get worse. Why couldn't you be a part of my brain like the other students in my class?*" This portrays math as choosing certain friends and leaving out others, potentially causing confusion and deeply hurt feelings.

PSTs displayed more negative emotions (55.3%) than positive (39.2%) or neutral (11.8%), math was most often described as a poor friend. This was often due to the changing nature of math curriculum (i.e., from algebra to geometry to statistics) or the ways in which math was practiced (i.e., timed tests, group work, word problems). For example, taking a statistics course was mentioned nine times in the coded letters, with three positive comments and six negatives.

The change in curriculum is mentioned in a letter, "*If there is a tiny speck of any common hatred for you- it was trigonometry*." Another PST told math, "*You took a turn for the worse toward the later years of elementary school. You decided to become word problems and you got worse and worse*." Yet another reported, "*sometimes you were easy and sometimes you were downright mean,*" while one told math, "*There are other times where you make it so complicated that it stresses me out and I want to burst into tears.*" The increase in perceived difficulty or performance as one got older significantly shifted the relationship as evidenced in this statement, "*As I got older, you became more complicated*" and "*The thought of you used to make me apprehensive and feel as if I was dumb.*"

Change of Heart. A change of heart regarding their feelings toward math was identified by 30 (55.6%) PSTs. This is akin to the roller coaster of achievement in narrated teacher's math stories found by Drake et al. (2001). PSTs described how their feelings toward math changed both from positive to negative and from negative to positive. The PSTs attributed these changes of heart to critical experiences. A PST who changed from a negative to positive outlook wrote, "*I used to really dislike you. You made me easily frustrated when I didn't understand you. Once I got older, though, I began to work harder and understand more."* A PST who changed from a positive to negative outlook wrote, "*When I was younger, I really liked you, but once you started adding in letters mixed with numbers and symbols, I had a change of heart."*

The Theme of As a Future Teacher. Part of the assignment asked PSTs to discuss their future with math in the letter. A discussion of their hopes regarding their role as a future teacher of mathematics was included in 51 (94.4%) PST's letters. Comments included resolving conflict with past experiences, arrogance and anger at mathematics, a willingness to give math another chance, and giving themselves another chance to learn math without blaming it for its perceived characteristics. A PST who had a positive attitude regarding their future teaching wrote, *"I hope as a teacher, I can dissolve those frustrations for my future students and highlight the importance. Hoping for the best."* A PST who did not express the same level of hope wrote, "*The thought of having to teach you to others causes my insecurities about you to come zooming back."*

Mindset. PSTs' comments coded for each theme were additionally labeled as evidencing a fixed or growth mindset (see Table 3). For example, the comment, "*I had a hard time with math when I was in HS, the teacher blamed us for not doing well, not her teaching"* was coded as a fixed mindset as it indicated the student was taking less personal responsibility for the successes and failures. A growth mindset was coded for comments such as, "*This is me becoming excited about learning math and working to better understand the concepts,*" or "*Once I got older, though, I began to work harder and understand more.*" These growth mindset comments show students facing challenges and seeing effort as the path to mastery. Comments that did not relate to a fixed or growth mindset did not receive a code for the mindset analysis.

PSTs tended to exhibit a more fixed mindset in their letters when discussing critical experiences in mathematics learning; 67.5% of the comments regarding critical experiences were coded as fixed, while 32.5% were coded as showing a growth mindset. Interestingly, when discussing their relationship with math, fixed and growth mindset were somewhat similar (fixed 52% and growth 48%). Growth mindset comments were more common (84.3%) than fixed mindset (15.6%) when discussing becoming a future teacher. When PSTs discussed being a future teacher, comments like, "*I plan to use my negative experience in math as a drive to make math fun and understandable to students. I will have multiple ways to present the information*" shows a transition in thought relying on the mastery of knowing multiple ways to show math to future students.

Table 3
Preservice Teachers' Type of Mindset Relative to Theme

	Fixed Mindset		Growth Mindset	
	Frequency	Proportion	Frequency	Proportion
Critical Experiences	54	0.675	26	0.325
Relationship with Mathematics	44	0.523	40	0.476
Future Teacher Perspectives	10	0.156	54	0.843
Total	108	0.474	120	0.526

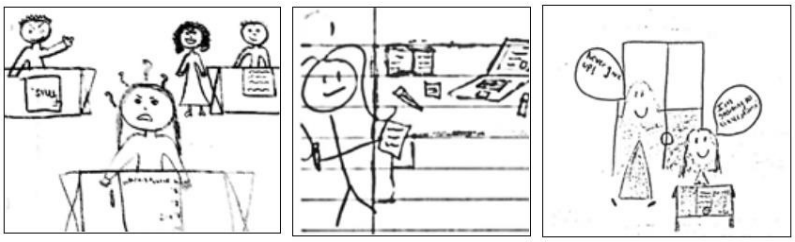
Self Portrait Analysis Results

Three themes were identified in the self-portrait analysis: emotions exhibited, self- placement in the portrait (including others in the portrait), and response to math.

Emotions Exhibited. For most portraits, the PSTs showed emotions while engaging in learning mathematics. The portraits were categorized by negative (55.3%), positive (32.9%), and neutral (11.8%) emotions. Negative emotions included unaddressed confusion which led to prolonged negative views toward math. Positive emotions included happiness and love. Portraits were categorized as neutral when the picture showed a focused student, showing neither distress nor elation. See Figure 3 for PST negative, neutral, and positive portrait examples.

Figure 3

The left portrait (negative) shows a frustrated student with question marks above her head and an angry look on her face. The middle portrait (neutral) shows a student working. The right portrait (positive) shows PST working with the comment "Never give up!" from adult, and PST saying, "I am starting to understand!"

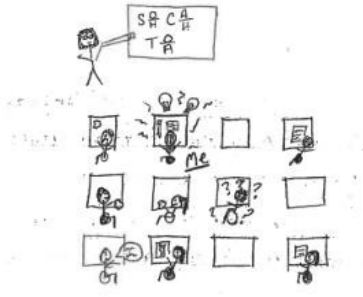


The portrait analysis examined the people who were included in the portraits. Portraits included PSTs with other people (63.0%), by themselves (37.0%), or with no person (0.02%). The most common people in the PSTs' self-portraits were teachers (24.1%), the entire class (7.4%), and classmates (5.6%). All the portraits that included other students in class with no teachers present, displayed negative emotions, which may indicate that a negative mindset and identity are further thickened when poor math learning experiences occur in a social setting such as a classroom and among peers.

Self-Placement. PST portraits depicting positive experiences often showed PSTs smiling, lightbulbs above the head, hearts, the PST sitting in front of the class, and had the teacher present (see Figure 4 as an example).

Figure 4

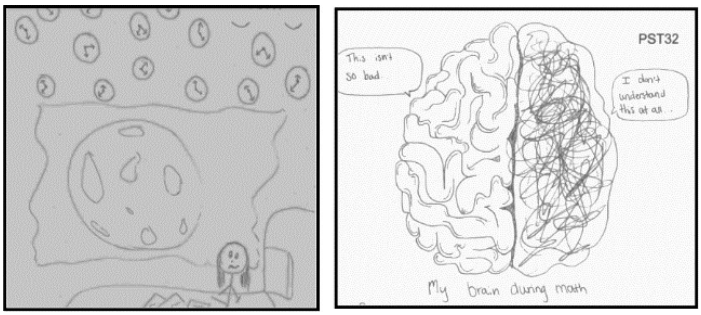
This self-portrait shows the participant having a positive experience in their classroom. The student is sitting in front of the room with light bulbs above their head.



PSTs placed themselves at desks and tables in their dining room, their bedroom, or drew one object that symbolized their experience. One portrait showed the PST sitting on their bed with a large moon outside the window, clocks everywhere, papers spread about, and the participant looking sad and tired. Another PST chose to draw their brain "on math". On the right side of the brain it says, "I don't understand this at all" and on the other side it says, "This isn't so bad". On the bottom it clarifies, "My brain during math." (see Figure 5).

Figure 5

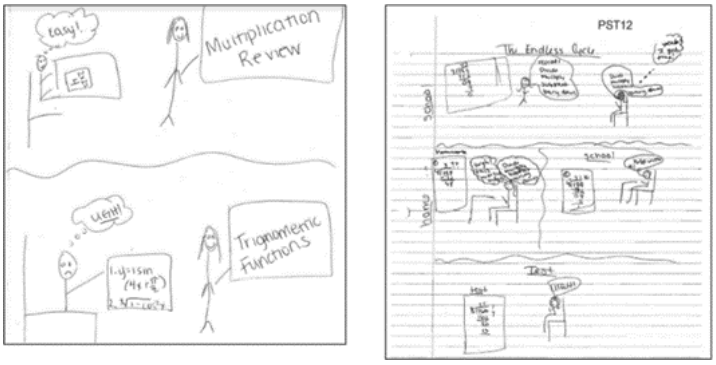
Alternative setting in portraits as described above.



Some PSTs drew a time span of mathematics learning, some indicating a change that occurred over time (Drake et al., 2001). For example, one participant expressed that math came easy to them throughout elementary and middle school but when they were introduced to trigonometry in high school, they became discouraged and less confident about math (Figure 6). The PST's facial expression went from smiling to sad and the text bubble says, "*Ugh!*" Another PST drew their math journey through the lens of learning long division (Figure 6). The participant was shown struggling with division in school, at home, and during a test. This portrait shows repeated episodes of frustration.

Figure 6

The left portrait shows a change due to increasingly difficult topics (multiplication to trigonometry), while the right shows an "endless cycle" of learning at school, practicing at home, and then taking a test. In the final panel the PST says, "UGH!"



Response to Math: Captions and Text. Many of the self-portraits (57.7%) included captions or text that further explained the drawing. For example, a PST showed herself sitting at a desk attempting to solve a problem, with a thought bubble that mentions that although it would take them a while to

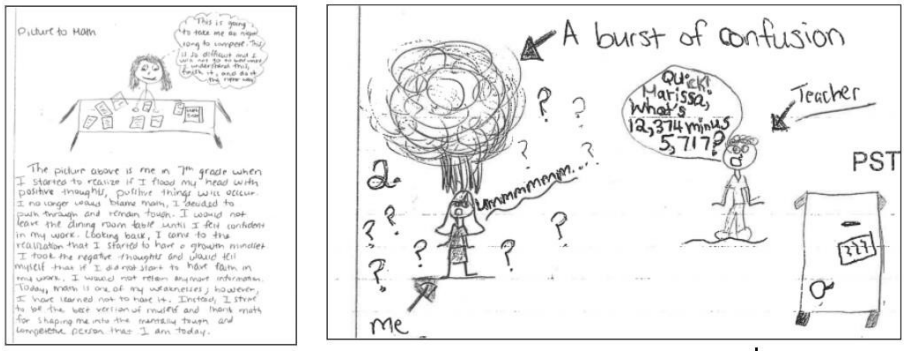
solve the problem, they will sit there until they get it. This illustration showed the PST struggling and showed their determination to succeed. This participant said:

> Looking back, I came to the realization that I started to have a growth mindset. I took the negative thoughts and would tell myself that if I did not start to have faith in my work, I would not retain any more information. Today, math is my weakness, however, I have learned not to hate it (Figure 7).

The overall portrait analysis identified seven types of responses to mathematics: indifference (19.2%), hopeful (9.6%), anxious (9.6%), love (19.2%), frustration (30.8%), positive outlook caused by a positive change (5.8%), and a negative outlook caused by a negative experience (5.8%).

Figure 7

Left portrait, PST appears positive although writing indicates math as a weakness that she is persevering through. Right portrait, PST shows confusion.



Discussion

This study explored the problem of practice of how PSTs' critical experiences in mathematics can impact their mindset towards mathematics methods course requirements in a teacher education program (Bekdemir, 2010; Dweck, 2016; Olson & Stoehr, 2019; Wilkins, 2008). Critical experiences were identified by asking the PSTs to write a letter to math in order to uncover emotions, relationships, and critical events through the personification of math. PSTs also illustrated their math experiences through self-portraits to reveal their inner self by removing the barrier of language (Mukhopadhyay, 1996). The purpose of the study was to develop a space for PSTs to examine and understand their reactions to their math learning experiences to be able to address the emotional reactions in the teacher education classroom. Past research suggests that teacher education programs should provide opportunities for PSTs to explore and develop an understanding of the causes of their math anxiety helping them reduce this anxiety as future teachers (Olson & Stoehr, 2019) and to improve student math achievement (Hadley & Dorward, 2011).

Three themes emerged from the analysis of the data: (a) the theme of critical experience; (b) the theme of mathematics relationships; and (c) the theme of as a future teacher.

The Theme of Critical Experience

Taken together, PSTs' letters and portraits of themselves learning mathematics depicted a wide range of

emotions, from positive to negative, that are associated with math learning. These emotions were the result of critical experiences that influence development (Woods, 1993). Additional analyses showed that adverse critical events tended to generate more fixed and negative mindset stances. This finding aligns to past research demonstrating that learning experiences faced by students influence their relationship with math (Drake, 2006; Drake et al., 2001; Lutovac & Kaasila, 2014; McCulloch et al., 2013; Mukhopadhyay, 1996; Zazkis, 2015). One interesting result is how the experience of being positioned by a teacher (i.e., placed in an advanced or support class) impacted student mindsets differently. For some, the opportunity to learn in a different environment allowed a growth mindset (confidence, "passing with flying colors"). However, others reported that "math" forced them into a different setting causing further dislike and resistance to learning. These findings differ from Boaler's (2013) work that found fixed ability grouping reinforces students' beliefs that mistakes and low grades equate to low ability. This may warrant the need for more teacher communication and possibly student choice about being placed in different learning environments. PSTs entering our classes with negative emotions connected to math learning will need our guidance and nurturance to address their emotional responses before we can begin to address methods of mathematics instruction. Similar to elementary school teachers who need to first build a classroom community and address the safety of their students before jumping into content-based instruction, teacher educators need to address the social emotional needs of PSTs (Hackenberg, 2010). Addressing the emotions associated with math learning will help to ease the PSTs' anxiety as well as help them to understand that their future students will also experience the same range of emotions (Bekdemir, 2010). Additionally, having PSTs use narrative tools to depict their own experiences can help them develop a positive outlook regarding their role as a future teacher (Lutovac & Kaasila, 2014; Zazkis, 2015).

The Theme of Mathematics Relationships

The analysis of the letters and portraits indicated that the emotions PSTs associate with math learning are, in part, due to the social relationships between the student and teacher as well as the student and other students. This analysis indicated a social constructivist view of cognitive development (Vygotsky, 1978). Student and teacher relations were identified as having a positive impact on learning when teachers encouraged and supported students. However, student and teacher relations were identified as having a negative impact on learning when teachers displayed low expectations for students or when their teaching style did not match the students' needs. Some critical events such as being placed in a higher-level math class or performing poorly on timed math tests often led to PSTs comparing their math performance to peers. This comparison resulted in either the PST feeling superior or inferior to their peers. Superior feelings resulted in the PST developing positive views toward math and identifying as a "math person." On the other hand, feelings of inferiority led to students developing a dislike for math and identifying as being a poor math student. Similar results were identified by Olson and Stoehr (2019) where PSTs attributed their math anxiety in part to instances when they compared themselves to their peers.

Together, the letters and portraits revealed the PSTs' math identity and math mindset through dialogue and components of composition, setting, emotions, picture, or captions. Both narrative tools indicated a growth mindset when PSTs showed that talent and ability can be developed over time; viewed mistakes as an opportunity to develop; were resilient; and believed that effort creates success. A fixed mindset emerged when PSTs believed that talent, not effort, creates success; were reluctant to take

on challenges; were fearful of making mistakes; thought it was important to 'look smart' in front of others; and believed that talents and abilities are set in stone (Dweck, 2016). As math teacher educators, we can support our PSTs by engaging them in math learning experiences that leverage collaboration, proposing solutions to problems in a safe community of learners, and providing space for changing our answers along the way. We can also create assignments that promote these practices (i.e., math lesson plans and practicing new approaches with peers or in field placement classrooms).

The Theme of As a Future Teacher

It is critical for teacher education programs to address math anxiety in their students because it contributes to math teacher anxiety (Hadley & Dorward, 2011; Olson & Stoehr, 2019). PSTs with math teacher anxiety lack confidence, prefer not to teach math, and may impact their future students by communicating "negative expectations for learning and performance, while trying to protect students from challenge [which] will certainly reduce academic rigor and opportunities to learn" (Olson & Stoehr, 2019, p. 79). Additionally, teachers' attitudes toward math impacts their instructional practices. Teachers with a positive math attitude tend to use more effective practices such as inquiry-based practices (Wilkins, 2008), whereas a teacher with a negative attitude towards teaching mathematics may shy away from more cognitively demanding approaches to mathematical instruction. This can have a detrimental impact on learning as students have lower mathematics achievement when their teachers experience math teaching anxiety (Hadley & Dorward, 2011).

The messages PSTs shared related to their future teaching provide recommendations for teacher education programs. For example, social influences of math instruction should be included in teacher education coursework. Future teachers would benefit from developing an understanding of how social relationships impact students' development of their math identity (Boaler & Greeno, 2006; Sfard & Prusak, 2005). Additionally, including episodes of instruction that foster a growth mindset for PSTs can increase reflection on learning and behavior and may improve their own performance in math (Yeager et al., 2019). PSTs can learn how to help themselves and their future students change from a fixed to a growth mindset which is important because dispositions toward math is a factor in determining student success (National Research Council, 2001).

Future Research

Future research examining PSTs' critical experiences in math should expand on the current study by exploring activities and assignments for teacher education programs to implement to address the anxiety and negative attitudes toward math that are prevalent in elementary PSTs. For example, Gibbons et al. (2018) identified four practices for teacher educators to implement to help PSTs develop productive math identities. The four practices include emphasizing reasoning, promoting broader engagement, shifting responsibility for learning, and developing a supportive classroom community. Future research should examine if the four practices help to overcome negative critical experiences.

Additionally, future research should address the limitations of the current study, primarily the lack of diversity in the sample. The PSTs in the study were predominantly white and female. Future studies might investigate how the critical identities of race, gender, socio-economic status, sexual orientation, and rural vs. urban upbringing interact with reported critical experiences in learning math by PSTs. One additional aspect for future research, is how PST family-school dynamic can affect their math identity

and anxiety, specifically when parents cannot fully or freely participate in students' learning. The math letters and portraits approach could be expanded to include this demographic information and be used as a further analytic to help inform the mathematics education field about how math identities develop in our PSTs.

Conclusion

The Dear Math letter and self-portrait tasks elicited critical experiences and events that helped to shape mathematics identities in the PSTs. It is important to elicit PSTs' critical experiences that shaped their math identities to help them reduce math anxiety as future teachers (Olson & Stoehr, 2019) and to improve student math achievement (Hadley & Dorward, 2011). Many of the letters that were analyzed offered apologies and regret for their relationship with math, some voiced that it was unusual, but they actually loved math.

The analysis of the letters in the current study add to the dimensionality of the friendship and perceived characteristics of math. One cannot read the letters without also acknowledging the joy, pain, pride, jealousy, and anger emoting from the pages. According to the PSTs' letters and self-portraits, math can be a loyal and useful friend and a tyrant in their lives. The combination of a social/emotional relationship with the multifaceted nature of math builds a thickened identity, entwining the two. Perhaps this explains why simply asking students to forgive and accept the "math tyrant" can be more problematic than imagined.

Mathematics teacher educators must foster and attend to a growth mindset and help PSTs rehabilitate and neutralize critical experiences by sharing their stories and taking a teaching perspective. The Association of Mathematics Educators *Standards for Preparing Teachers of Mathematics* state that PSTs must "understand their role as ethical advocates for elementary children to have access to and advance in mathematics" and as well as "nurturing children's proficiency with, and sense-making of, mathematical ideas, processes, and practices (AMTE, 2017, p. 94).

We engaged our students in this assignment to hear their voices, to give them a platform to openly discuss their experiences while forward thinking towards becoming a math educator. We also sought to help them identify the critical experiences that happened in their math past to help develop an understanding of what has influenced their perceptions of their math abilities, and how this can be transformed. We wanted our PSTs to re-examine their needs as a math learner, how teachers' choices in the classroom had a positive or negative impact, and how this can be applied to their own future as an educator. We hope to foster a greater understanding that under different teaching circumstances; they may have had a different relationship with math, thus breaking the cycle of math anxiety. Our PSTs exhibited courage and hopes for their future as math educators, and we can leverage this positive outlook. As researchers and educators, taking time to address the confluence of story, mindset, identity, and healing broken relationships continues to be an area of discovery and investigation. We close this article with the hopes of one of our PSTs shared with math:

> I know we won't always get along, but I promise I won't give up on you. I know I will have to teach my future students about you, and I will try to make it interesting and fun. I think our future is bright. We will always stick together.

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ORCID iD
Claudia M. Bertolone-Smith 0000-0003-1821-9556
Alison Puliatte 0000-0001-9620-9500

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