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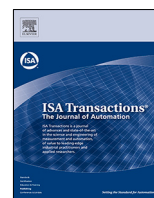
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Research article

Inspection and maintenance optimization for heterogeneity units in redundant structure with Non-dominated Sorting Genetic Algorithm III

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ABSTRACT

Redundant structure has been widely deployed to improve system reliability, as when one unit fails, the system can continue to function by using another one. Most existing studies rely on the similar assumption that the heterogeneous units are subject to periodic inspections and identical in terms of their aging situations and the numbers of resisted shocks. In practice, it is often adequate to trigger a unit individually in the event of a single shock, which intensifies the degradation of that unit, accordingly, requiring a sooner inspection to ensure its safety. In this study, the stochastic dependency among units is addressed firstly by introducing a novel activation sequence. Secondly, an adaptive system-level inspection policy is proposed by prioritizing the unit with a worse state. Finally, we take advantage of Monte Carlo methods to simulate the whole process and estimate two objectives, referring to the average system unavailability and maintenance cost, in a designed service time. It is found that the two objectives are contradictory through numerical examples. The Non-dominated Sorting Genetic Algorithm III (NSGA-III) algorithm, therefore, has been employed to find the optimal solutions in system unavailability and cost, which provide clues for practitioners in decision-making.

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1. Introduction

With the impacts from working conditions, aging, many industrial systems, especially mechanical parts, are suffering from degradation mechanisms and getting vulnerable along with time, which eventually leads to system failures [1–3]. Typical degradation mechanisms of mechanical items include corrosion, wear and fatigue, etc. [4,5]. The failure of these systems brings not only economic loss to the enterprise, but also detrimental impacts on the people, assets, and environment [6,7]. Thus, multiple measures have been widely acknowledged to reach/maintain better system performance in its whole lifecycle. In the design phase, redundant structures are widely applied in engineering with the intention to increase system reliability or improve actual system performance, e.g., final elements subsystem consisting of two or more elements in safety-instrumented systems (SISs) [6]. The main advantage of a redundant structure is that the system is still functional by using the other unit(s) if one fails. 1-out-of-2 (1oo2) configuration is a

typical example. Besides the structural fault-tolerance, inspection and maintenance are typical add-on activities in the operational phase in maintaining system performance at the required level. These interventions are carried out to limit the failure probability, thereby improving system availability and safety [8].

To date, a considerable amount of literature is available in terms of performance/reliability assessment of the redundant structures, covering topics, including common cause failure [9–11], testing and maintenance [12–16], reliability allocation [17,18], cascading failures [19] etc. Exposing to the quite same working conditions, the dependence, especially failure dependence, is attracting more attention [20,21]. Also, many systems are subject to degradation and random shocks simultaneously [22,23]. For example, the required function of the high integrity pressure protection system (HIPPS) is to close the flow in the pipeline in case the pressure is beyond the specialization. As expected, the safety valve is designed to respond to potential occasional high pressures, which might intensify internal stress and, consequently, leave non-neglected damages on itself [12]. It indicates that the continuous degradation makes the system more vulnerable to random shocks and the shocks furtherly accelerate the degradation process [22,24–26]. These random shocks

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contribute to the failure dependence between units. Existing studies address the dependence by assuming the same amount of shocks or same damage [12,27]. This assumption is acceptable in conservatively evaluating system performance but disregarding the design purpose of the redundant structure. Taking one step more in thinking, since the redundant structure is designed with the purpose to improve system performance, and it implies that the single unit should have sufficient ability to complete the required function. Taking the aforementioned 1oo2 safety valves in HIPPS as an example, each valve should be allocated sufficient ability, e.g., maximum allowable pressure, in most cases to stop the flow when the higher pressure in the pipeline occurs. Thus, a question arises regarding how to trigger the 1oo2 configuration system to maximize its performance assuming each shock only exerts on one unit? The potential choices could be (1) Continuing to activate the same unit until it fails; (2) alternating the uses of the two units each time; (3) randomly activating the two units.

Besides the activation sequences of redundant units, the development of sensor technologies boosts the application of condition monitoring technologies in asset management. It provides more opportunities to utilize collected system performance data to predict in real-time system health and to enable maintenance decision-making [1]. Time-based inspection and maintenance, which is still the mainstream currently, falls behind the needs of modern industries. The obvious shortcoming is that it strictly obeys a predefined inspection and maintenance scheme, independent of the system's current actual state. Such an approach can result in unnecessarily expense intervention of the production process. To release the assumption of the periodic inspection, several researchers conduct the condition-based maintenance with adaptive inspection policy, indicating the upcoming inspection interval is upon the current state [1,28,29]. Generally, the inspection interval decreases along with the accumulation of degradation, the closer to the failure threshold, the sooner the next inspection would be arranged. Yet, these studies mainly focus on single-unit systems. Considering the interruption of the process, it might be not economic-friendly to apply the unit-level adaptive inspection policy on redundant structures directly. For the sake of system performance and economics, thus, it is reasonable to prioritize the unit with a worse current state in determining the upcoming inspection interval. That is, the system-level inspection interval is assumed to be consistent with the prioritized unit. When it comes to maintenance, the maintenance action, either preventive maintenance (PM) or corrective maintenance (CM), is well-accepted as the assumption of perfect in existing literature [28,30–32]. It means the system is in an as-good-as-new condition. The assumption might be applicable for CM which normally refers to the replacement of the failed unit, but quite questionable for PM. For a PM, unit degradation can be mitigated but not be eliminated. The maintenance effect in the degradation degree mitigation can be reasonably assumed to have a positive correlation with the cost of single PM action [33,34].

As a response, in this paper, we intend to study the 1oo2 configuration system performance considering the aforementioned factors, including the activation sequence, adaptive inspection policy, and imperfect PM. Specifically, each unit is subject to unavoidable aging and potential damage caused by random shocks that exert on units following a specific activation probability. Meanwhile, on the unit-level, the interval to the forthcoming inspection is determined following the current revealed unit state. Furthermore, the inspection interval of the unit with a worse state would be taken as the interval value for the simultaneous inspection of the redundant system. Unit-level CM and PM interventions are conducted at the inspection time when the accumulative degradation exceeds the preset threshold.

Normally, it is insufficient to find the optimum inspection and maintenance policy relying on a single optimization criterion separately, either the system performance or maintenance cost [35–37]. The frequent and effective inspections and maintenance interventions are beneficial in reducing the probability of the system being in an undetected failed state, but paying the prices with higher relevant costs. Thus, these aforementioned factors are incorporated into an optimization problem for the inspection and maintenance of a 1oo2 configuration by proposing two objective functions. The first objective function minimizes the unavailability of the system, while the second objective minimizes the cumulative cost in its service time. The specific objectives, therefore, include:

- Modeling and quantifying the dependence of units in redundant structure caused by the activation sequence in withstanding random shocks;
- Merging the unit-level inspection and system-level inspection into an adaptive and aperiodic inspection policy;
- Multi-objective optimization in seeking the optimal inspection and maintenance strategies.

The remainder of this paper is organized as follow: Section 2 illustrates the system description and lists required assumptions; Section 3 conducts the analytical formulas of system unavailability and maintenance cost, presents and discusses sensitivity analysis of inspection and maintenance parameters as well; Section 4 generalizes the multi-objective optimization problem and seeks the optimal solutions relying on NSGA-III algorithm; Concluding remarks are presented in Section 5.

2. Problem description

Notations

$Z_i(t)$	The accumulative degradation caused by aging of unit i by time t
$Y_i(t)$	The accumulative degradation caused by random shocks of unit i by time t
$X_i(t)$	The sum of accumulated degradation by aging and shocks of unit i by time t
p	The activation probability of unit 1 for shocks
$[T_{n-1}, T_n]$	The starting and ending time of n th inspection
$T_n^- (T_n^+)$	The immediate time just before (pre-) (after (post-)) an inspection time T_n
$X_{T_n^-}^i (X_{T_n^+}^i)$	The degradation level of unit i at time T_n^- , before (after) the inspection
L	Failure threshold
H	The designed service time

It is well-accepted that redundant structure is helpful in the improvement of system performance. It means that the 1-out-of-2 (1oo2) configuration is capable to be functional even one of the two units fail. The 1oo2 configuration is employed in system performance evaluation. On unit level, the single unit i ($i = 1, 2$) suffers from two degrading processes: one is the continuous degrading process related to the impact of aging $\{Z_i(t), t \geq 0\}$; and the other is random shocks with damage $\{Y_i(t), t \geq 0\}$ due to the implementation of the designed function. Here, the single unit loses its function when the sum of two degrading processes exceeds a preset failure threshold L_i . The system is regarded as in a failed state only if both two units fail. The system is inspected at certain epochs. The n th inspection interval is denoted as $[T_{n-1}, T_n]$ and $n \in \mathbb{N}$, $T_0 = 0$.

2.1. Degradation modeling

2.1.1. Aging degradation

The continuous aging degradation of unit i ($i = 1, 2$) at time t is assumed following a Wiener process $\{Z_i(t), t \geq 0\}$. Then, we can have [38],

$$Z_i(t) = \mu_i t + \sigma_i B(t) \tag{1}$$

where μ_i is the drift parameter, σ_i is the diffusion coefficient, and $B(t)$ is the standard Brownian motion. Here, we assume $\mu > 0$. Then process $Z_i(t)$ has independent and normally distributed increments in (t_1, t_2) , $t_2 > t_1$: $Z_i(t_2 - t_1) \sim N(\mu(t_2 - t_1), \sigma^2(t_2 - t_1))$.

2.1.2. Damage caused by shocks

Besides the aging degradation, the system is exposed to damage resulting from random shocks. Their arrival is assumed as a Homogeneous Poisson Process (HPP) $\{N(t), t \geq 0\}$ with the occurrence rate λ . Then the probability mass function [38] of having n shocks in $(0, t]$ is calculated as:

$$Pr(N(t) = n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}, n = 0, 1, \dots \tag{2}$$

For shocks, two units are activated at random. For the sake of convenience in the following discussion, assuming the activation probability of unit 1 is p ($p \geq 0$), then $1-p$ is on unit 2. Following the splitting property of a Poisson process, before the failure of any unit, the arrival of shocks on unit-level in $(0, t]$ also would be independent HPPs, $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ with rates λp and $\lambda(1-p)$, respectively [38]. But if one unit is failed before the next inspection and maintenance time, the other has to taking the forthcoming shocks with a probability of 1. When the two units fail, the 1oo2 configuration ceases to function.

Furthermore, the independent damage magnitude of j th shock y_j , for $j = 0, 1, 2, \dots, N(t)$, is gamma-distributed with parameters (α, β) . The cumulative damage of unit i caused by random shocks by times t , denoted by $Y_i(t)$, can be given as

$$Y_i(t) = \sum_{j=0}^{N(t)} y_j \cdot \mathbf{1}_i(j), j = 0, 1, \dots \tag{3}$$

where $\mathbf{1}_i(j)$ is the indicator function. It equals to 1 if the shock j is activated on unit i ; otherwise, it equals to 0.

The reliability of unit i at time t equals to the probability that its total degradation $X_i(t)$ is less than the failure threshold L_i .

$$R_i(t) = Pr(X_i(t) = Z_i(t) + Y_i(t) < L_i) \tag{4}$$

2.2. Adaptive inspection and maintenance policy

Inspection and maintenance activities are critical to ensuring that the unit/system performs satisfactorily. Inspection is assumed to be non-destructive with negligible time in this study. Here, we take T_n^- and T_n^+ to indicate the moment just before (pre-) and after (post-) the inspection at T_n , respectively. Then, the unit/ system state is known at such time moment.

Usually, there are two types of maintenance actions on the unit-level, CM and PM. When the accumulative degradation of a unit exceeds a failure threshold L , the unit would be failed with respect to the certain function, thus the CM action follows. Meanwhile, the actual unit performance deteriorates along with the degradation process, which may lead to poor production quality. It is, thus, reasonable to conduct PM when the actual unit performance exceeds a threshold M , and $0 < M < L$. Here, we assume the preventive threshold M is a random variable that needs to be optimized.

Based on the unit state at inspection time T_n , maintenance follow-ups can be expressed in three potential scenarios:

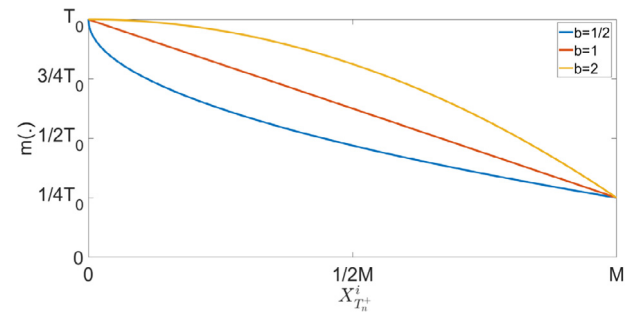


Fig. 1. Illustration of function $m(\cdot)$ with $a = 3/4$.

- (1). If $X_{T_n^-}^i \geq L_i$, meaning that unit i is failed, thus CM is performed incurring a cost (C_{CM}). The unit after CM is as-good-as-new (AGAN), namely, $X_{T_n^+}^i = 0$;
- (2). If $M_i \leq X_{T_n^-}^i < L_i$, imperfect PM is performed with a cost (C_{PM}). The degradation level $X_{T_n^-}^i$ is partially eliminated after PM, represented as $X_{T_n^+}^i = k \cdot X_{T_n^-}^i$. Thanks to the imperfect PM factor k , if the expected degradation level after PM still exceeds the threshold M_i , $X_{T_n^+}^i > M_i$, the CM intervention will be conducted instead of the planned PM, accompanying with cost C_{CM} . In addition, the incurred C_{PM} cost is directly relevant to the imperfect PM factor k . Referring to the proposed degradation improvement factor in imperfect maintenance actions in [39], here we assume the $C_{PM}(k)$ has a function with the imperfect PM factor k and the perfect PM cost $C_{PM,0}$, that is, $C_{PM}(k) = (1 - k) \cdot C_{PM,0}$.
- (3). If $X_{T_n^-}^i < M_i$, the unit i state is left unchanged with $X_{T_n^+}^i = X_{T_n^-}^i$, but will induce an inspection cost C_T .

Following the maintenance intervention, the current unit state at post-inspection $X_{T_n^+}^i$ becomes the starting point of the follow-up inspection epoch and simultaneously serves as the basis for the determination of the next inspection time T_{n+1}^i . The next inspection time T_{n+1}^i is determined by the rule [28], as

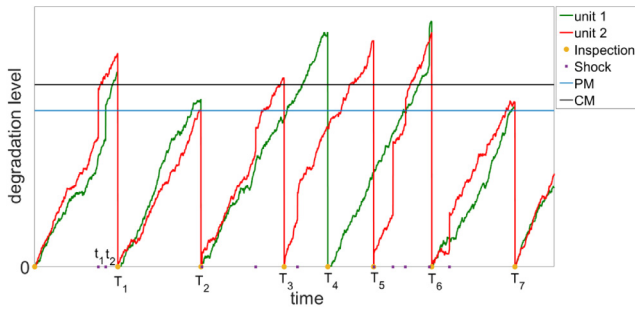
$$T_{n+1}^i = T_n^i + m\left(X_{T_n^+}^i\right) \tag{5}$$

where $m(\cdot)$ is a decreasing function that might be a convex, concave, or linear function to represent the next inspection interval corresponding to the current state. The initial inspection interval $T_0 = m(0)$. The interval $m\left(X_{T_n^+}^i\right)$, which corresponds to the known degradation level $X_{T_n^+}^i$ of unit i at T_n , has a generic relation with T_0 , as

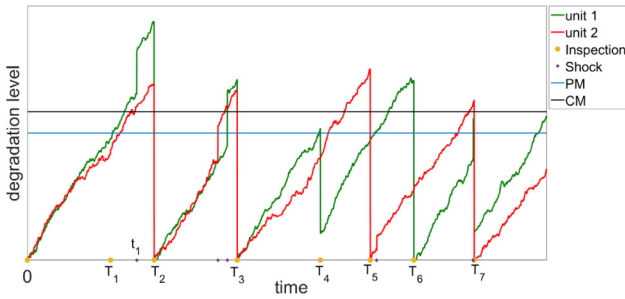
$$m\left(X_{T_n^+}^i\right) = \left[1 - a \left(\frac{X_{T_n^+}^i}{M_i}\right)^b\right] \cdot T_0 \tag{6}$$

where $0 \leq a < 1$ and $b > 0$, and $m\left(X_{T_n^+}^i\right)$ thus has a range $[(1 - a) \cdot T_0, T_0]$. It is noticed that the unit is subject to periodic inspection with interval T_0 when $a = 0$. An illustration of function $m(\cdot)$ is depicted here in Fig. 1.

In terms of the 1oo2 configuration, here, a system-level inspection policy is considered. That is, the unit with a worse state takes precedence and determines the upcoming system-level inspection. It indicates that the two units are inspected at the same time with the interval $\Delta T_n^S = \min\left[m\left(X_{T_n^+}^1\right), m\left(X_{T_n^+}^2\right)\right]$.



(a) Perfect PM



(b) Imperfect PM ($k = 0.2$)

Fig. 2. Illustration of the proposed model with $a = 0.5$ and $b = 1$.

For the sake of a better understanding of the proposed model, an illustrative example is depicted in Fig. 2. The capital T_i ($i = 1, 2, \dots$) on the x-axis stands for the system-level inspection time while random shocks arrive at time t_i . When $p = 0$, it implies that unit 2 will withstand all the shocks. But if unit 2 fails, then unit 1 has to take the shock until the next inspection time when the failed unit 2 is maintained. The shock-activated unit's shift is automatically carried out following the predefined algorithms. For example, in Fig. 2(a), the first shock at t_1 exerts the abrupt increment on the degradation path of unit 2. But for the second shock at t_2 , given the accumulated degradation level of unit 2 exceeds the failure (CM) threshold, the increment automatically appears on unit 1 to indicate the effect of the second shock. A similar process can be found in $[T_5, T_6]$ in Fig. 2(a) and $[T_1, T_2]$ in Fig. 2(b).

Also, the system-level inspection intervals are inconstant, as depicted in Fig. 2. As aforementioned, the system-level inspection interval adapts to the unit which has a worse degradation level at the current time, e.g., the interval length $[T_3, T_4]$ in Fig. 2(a) is determined by the state of the unit 1 at T_3 . When the imperfect PM is conducted, the existing degradation is partially mitigated from a specific value between the PM and CM threshold to below the PM threshold. Here, parameter k stands for the result of the imperfect PM. The corresponding degradation level before and after imperfect PM, in this example, is $X_{T_n}^i$ and $0.2 \cdot X_{T_n}^i$, respectively. The potential degradation paths with imperfect PM are shown in Fig. 2(b), e.g., T_4 and T_7 for unit 1.

3. Performance analysis

This section focuses on the theoretical analysis for the proposed model. The inspection is assumed to be perfect in revealing the current degradation level of each unit. For the convenience of further discussion, we adopt a vector $S(T_n^+) = (X_{T_n^+}^1, X_{T_n^+}^2)$ to stand the system state (degradation level) at the post-moment at

T_n after inspection and maintenance activities and $S(T_n^-)$ for the pre-moment similarly.

3.1. System unavailability analysis

For $t \in [T_n, T_n + \Delta T_n^S]$, or $[T_n, T_{n+1}]$, the reliability of unit i can be calculated as

$$R_i(t|T_n) = \Pr(X_i(t) = X_i(T_n^+) + Z_i(t) + Y_i(t) < L_i) \quad (7)$$

When both units fail, the system ceases to function. By classifying into three cases: (1) both units survive until by time t ; (2) unit 1 fails before time t while unit 2 survives; (3) unit 2 fails before time t while unit 1 survives, the instantaneous system reliability probability can be quantified as

$$R_S(t|T_n) = R_1(t|T_n) \cdot R_2(t|T_n) + (1 - R_1(t|T_n)) \cdot R_2(t|T_n) + (1 - R_2(t|T_n)) \cdot R_1(t|T_n) \quad (8)$$

The average system failure probability, for $t \in [T_n, T_n + \Delta T_n^S]$, is

$$F_{S,T_n}^{avg} = \frac{1}{\Delta T_n^S} \int_{T_n}^{T_n + \Delta T_n^S} F_S(t|T_n) dt = \frac{1}{\Delta T_n^S} \int_{T_n}^{T_n + \Delta T_n^S} (1 - R_S(t|T_n)) dt \quad (9)$$

The average system unavailability in a finite horizon length (H) with total T_H inspections can be calculated as

$$F_S^{avg} = \frac{1}{H} \sum_{i=0}^{T_H} F_{S,T_i}^{avg} \quad (10)$$

3.2. Maintenance costs

The total cost on unit-level, as indicated in Section 2.2, comprises of inspection cost C_T , PM cost C_{PM} and the replacement cost C_{CM} , and $0 < C_T < C_{PM} < C_{CM}$. With the known information $S(T_n^+)$ at T_n and the calculated next inspection interval length ΔT_n^S , the expected cost in $[T_n, T_{n+1}]$, can be discussed in different scenarios as follows:

- (1) No maintenance at T_{n+1}

$$P_1 = \Pr\{S(T_{n+1}^-) < M | S(T_n^+)\}$$

- (2) PM on two units

$$P_2 = \Pr\{M_1 < X_{T_{n+1}}^1 < L_1 \text{ and } M_2 < X_{T_{n+1}}^2 < L_2 | S(T_n^+)\}$$

- (3) CM on two units

$$P_3 = \Pr\{X_{T_{n+1}}^1 > L_1 \text{ and } X_{T_{n+1}}^2 > L_2 | S(T_n^+)\}$$

- (4) Single PM + no maintenance

$$P_4 = \Pr\left\{\left[M_1 < X_{T_{n+1}}^1 < L_1 \text{ and } X_{T_{n+1}}^2 < M_2\right] \text{ or } \left[M_2 < X_{T_{n+1}}^2 < L_2 \text{ and } X_{T_{n+1}}^1 < M_1\right] | S(T_n^+)\right\}$$

- (5) Single CM + no maintenance

$$P_5 = \Pr\left\{\left[X_{T_{n+1}}^1 > L_1 \text{ and } X_{T_{n+1}}^2 < M_2\right] \text{ or } \left[X_{T_{n+1}}^2 > L_2 \text{ and } X_{T_{n+1}}^1 < M_1\right] | S(T_n^+)\right\}$$

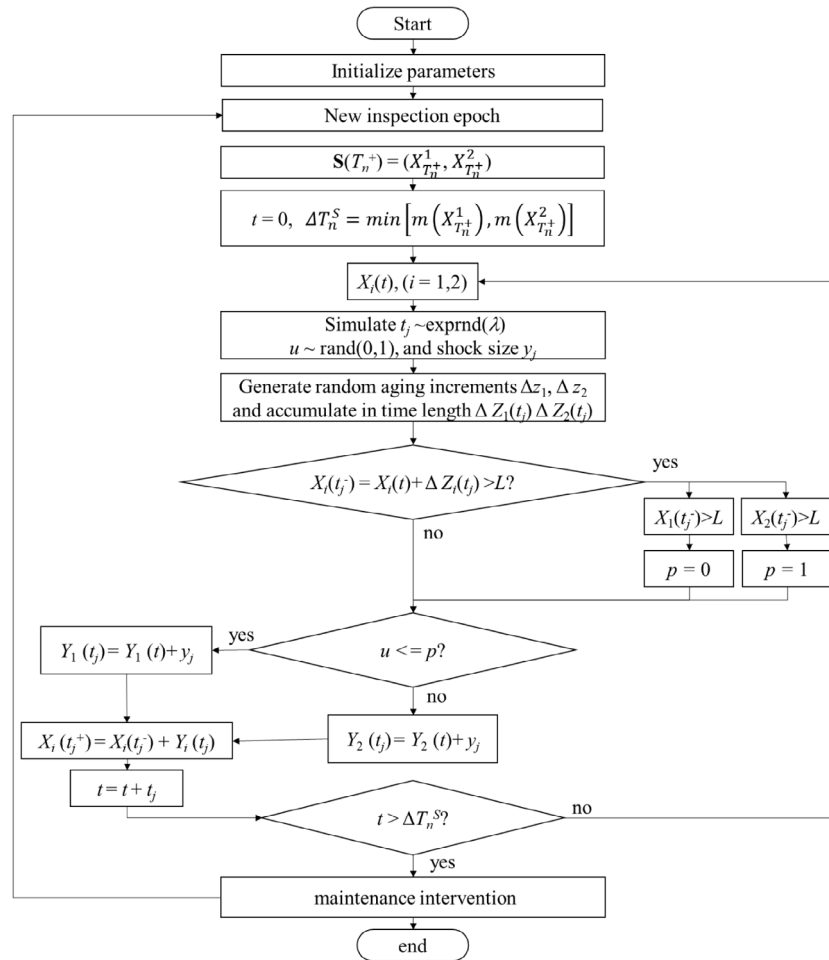


Fig. 3. Simulation procedure in the inspection interval $[T_n, T_n + \Delta T_n^S]$.

(6) Single PM + single CM

$$P_6 = \Pr \left\{ \left[M_1 < X_{T_{n+1}}^1 < L_1 \text{ and } X_{T_{n+1}}^2 > L_2 \right] \text{ or } \left[M_2 < X_{T_{n+1}}^2 < L_2 \text{ and } X_{T_{n+1}}^1 > L_1 \right] \mid \mathbf{S}(T_n^+) \right\}$$

The corresponding cost in $[T_n, T_{n+1}]$ thus can be calculated as

$$TC_{T_n} = P_1 \cdot 2C_T + P_2 \cdot 2C_{PM} + P_3 \cdot 2C_{CM} + P_4 \cdot (C_{PM} + C_T) + P_5 \cdot (C_{CM} + C_T) + P_6 \cdot (C_{PM} + C_{CM}) \quad (11)$$

Hence, the total cost (TC) of the 1oo2 configuration in the finite horizon length (H) with total T_H inspections is given by:

$$TC = \sum_{i=0}^{T_H} TC_{T_i} \quad (12)$$

3.3. Monte Carlo simulation

It is not straightforward to derive the explicit analytical formulas for system unavailability and relevant costs based on the discussions in Sections 3.1 and 3.2. In this view, the Monte Carlo method stands out given its capability of achieving a closer adherence to reality for obtaining estimates of the solution of complex problems utilizing random numbers [40]. It has been widely used in research issues related to reliability, availability, maintainability, and safety (RAMS) problems [41–43].

The main simulation procedure in the inspection interval $[T_n, T_n + \Delta T_n^S]$ is depicted in Fig. 3. The whole simulation in the

given finite time length $[0, H]$ includes 6 main steps described as follows.

Step 1 – Initialization: Start the procedure firstly by initializing the parameters, and the current time is set as 0.

Step 2 – Inspection interval generation: At the beginning of each inspection cycle, determine the system-level inspection interval ΔT_n^S by Eq. (6) with the minimum value in the current state vector $\mathbf{S}(T_n^+)$ as the input. And reset the time elapsed in this interval to $t = 0$.

Step 3 – Simulation of the unit-level degradation processes: Divide the inspection interval ΔT_n^S into small time steps. Randomly generate the increment of aging in each time step with the pdf of the Wiener process. Generate the time interval of two random shocks with exponential distribution with mean λ . Then, make the j th random shock at time t_j equals to the cumulative sum of the previous shock occurrence time intervals, and reject the shocks whose occurrence times is beyond the inspection interval. Besides, generate the relevant shock size y_j with Gamma distribution with parameter (α, β) .

Step 4 – Shock allocation and failure check: The allocation criterion is upon the comparison of a random uniform value u with the original value p of unit 1. The shock is supposed to exert unit 1 if $u \leq p$, and on unit 2 otherwise. Furthermore, the cumulative degradation level of the chosen unit at t_j is compared with the failure threshold L . If the cumulative degradation level is less than L , the j th shock will count on the chosen unit as arranged. But the other unit will automatically take the upcoming shocks if the chosen unit 1 failed at some before t_j . It means

Table 1
Parameters for the numerical example.

Parameter	μ	σ	λ	α	β	p	C_T	k	$C_{PM,0}$	C_{CM}	C_D
Value	1.5	5	0.4	2	0.6	0.5	10	0	80	100	500

the activation probability of unit 1 to withstand shocks after the j th is switched to 1 (unit 2 fails) or 0 (unit 1 fails) upwards. It is mentioned that either the failure of the unit or the system is unknown unless either revealed by the failure in dealing with a shock or revealed at the next inspection.

Step 5 – Maintenance intervention and end of inspection cycle: The accumulated degradation of each unit is known at the scheduled inspection time. If the elapsed time t is beyond the scheduled inspection time ΔT_n^S , relevant maintenance interventions are conducted following the details in Section 2.2. The current time is updated to the cumulative sum of the previous inspections. If the cumulative time is bigger than the designed service time H , the simulation will go to step 6. Otherwise, the simulation continues for the next inspection and restarts from Step 2.

Step 6 – Termination and system performance calculation: The whole simulation process terminates when the maximum number of iteration N_{sim} is reached. System performance and the relevant cost of the j th simulation path are calculated based on Eqs. (10) and (12), respectively. Finally, their mean values are employed as the expected average system unavailability and maintenance cost.

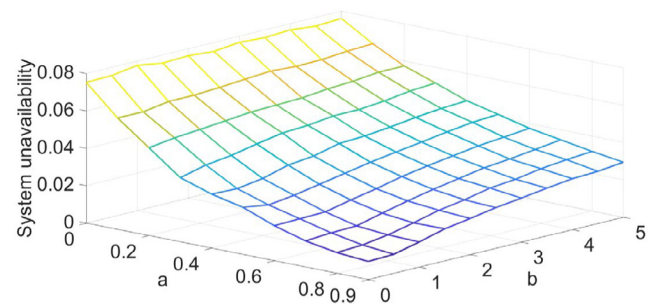
3.4. Adaptive inspection policy

An adaptive inspection policy proposed here is expected with the potential benefits to improve the system performance given the utilization of collected information. To address the proposed policy, a sensitivity analysis on the parameter (a, b) of adaptive inspection length function in Eq. (6) is conducted here. The designed service time H is 25, and the initial inspection interval is 4. Other relevant parameters are listed in Table 1.

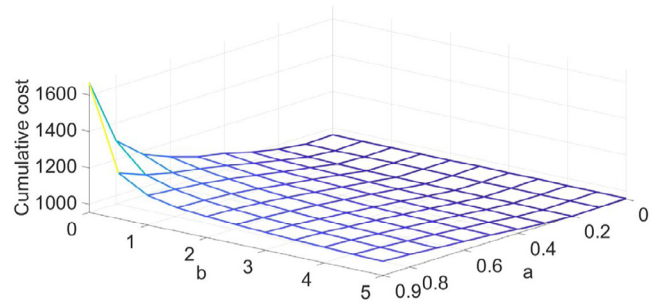
The evaluation criteria are the average system unavailability and maintenance cost in the designed service time calculated by Eqs. (10) and (12), respectively.

From Eq. (6), the upcoming inspection interval apparently tends to 0 as parameter a tends to 1. The proposed inspection policy loses its adaptiveness with these relatively small intervals. Additionally, considering the unavoidable inspection cost, too short an inspection interval might not be the most economic-friendly, which reversely determines the upper boundary of parameter a . Without losing generality, parameters a, b in this section are assumed in ranges $[0, 0.9]$ and $[0, 5]$, respectively. Fig. 4 presents the effects of parameters (a, b) of the proposed inspection policy on the average system unavailability and cumulative cost.

When $a = 0$, the system is subject to periodic inspection with an interval of 4. System unavailability is thus independent with the value of parameter b , as depicted in Fig. 4(a). System performance presents a positive correlation, meaning decreasing unavailability, with the increment of parameter a . When a is a specific value, system unavailability negatively correlates with parameter b , and the differences become more obvious when parameter a approaches the upper boundary. System unavailability reaches a minimum value with $a = 0.9$ and $b = 0$ in this example; on the contrary, system cumulative cost has a maximum value. This peak is mainly caused by excessive inspections. The same as system unavailability, the cumulative cost is independent with parameter b when $a = 0$ given the periodic inspection. To conclude, through this numerical example, it is obvious that the application of adaptive inspection policy is beneficial for the



(a) System unavailability with parameters (a, b)



(b) The cumulative cost with parameters (a, b)

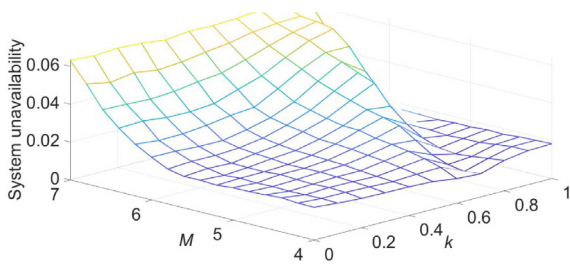
Fig. 4. Effects of adaptive inspection policy on system performance and maintenance cost.

improvement of system performance but induces higher costs to some extent as well. These two contradictory objects, thus for the practitioners, call for a balance, which reduces the system unavailability with a reasonable economic cost.

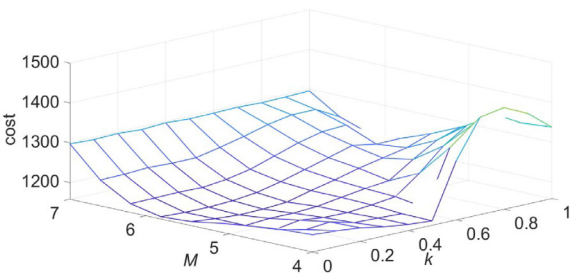
3.5. Maintenance policy

As stated in Section 2.2, PM action might be imperfect and partly mitigate the unit degradation. Here, the main discussion centers on the PM threshold M , factor parameter k , and its effects on relevant cost and system unavailability. To avoid the too early intervention of PM, the starting PM threshold M is set from 4, and the upper boundary is the same as the CM threshold. Thus, the parameter M, k in this section has its specific ranges $[4, 7]$ and $[0, 1]$, respectively. Relevant parameters are adopted from Table 1. The adaptive inspection policy is assumed with $a = 0.5$ and $b = 1$. Results are shown in Fig. 5.

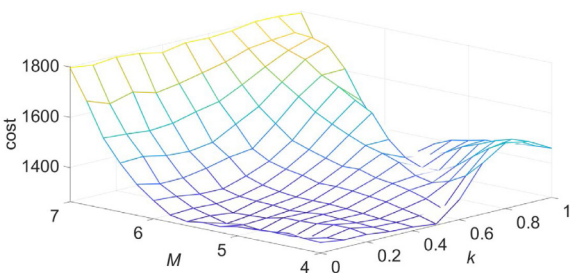
When PM threshold M equals 7, it implies that no PM action will be conducted, system unavailability shows a maximum value, and is theoretically independent with parameter k . Generally, in Fig. 5(a), system unavailability decreases with the increasing value of parameter M . The system possesses a better performance with the assistance of earlier PM intervention, reflected as the decrement of system unavailability along with the smaller value of M in Fig. 5(a). If parameter k takes the same value, yet the cumulative cost presents a two-phase curve. The cumulative cost decreases first along with the lower value of threshold M and then increases. The decrement of the cumulative cost comes from the benefits of PM in reducing the system downtime, but too earlier PM (smaller PM threshold M) will heighten PM cost reversely. The PM policy has limited effects on system unavailability when the threshold M is relatively small, e.g., 3 or 4 in Fig. 5(a). In terms of parameter k , system unavailability curves show dissimilar tendencies correlated with the value of M . When



(a) Parameter (M, k) on system unavailability



(b) Parameter (M, k) on the cumulative cost with $C_{CM} = 200$



(c) Parameter (M, k) on the cumulative cost with $C_{CM} = 500$

Fig. 5. Effects of maintenance policy on system performance and cost.

M is quite large, system unavailability increases along with k to some point and then decreases afterward. When k is quite small, implying a better PM effect, the unit degradation level is partly mitigated thanks to PM actions. But as for the worse PM effect, reflected as the higher value of k , it contributes to the increment of system unavailability. Then after a turning point, system unavailability shows a different tendency. This shift is caused by the compensation measure of imperfect PM described in Section 2.2. When parameter k closes to 1, the effects of PM action, $X_{T_n}^i = k \cdot X_{T_n}^i$, might be insufficient to lower the unit degradation level than PM threshold M , then, in this circumstance, a CM action with higher cost C_{CM} is taken rather than a PM. It complies with the cumulative cost tendency depicted in Fig. 5(b). When M is quite small, system unavailability decreases with k first and bounces thereafter. It says that the early and efficient PM actions are good for system performance improvement to some extent. Imperfect PM actions will weaken the advantage, the proportion of compensation CM is getting bigger along with k . Subsequently, the proposed imperfect PM policy fails to fit the expectations in system performance improvement in the end but induces high costs instead. Based on these considerations, the lower boundary of M should be set more reasonably.

This section analyzes the system unavailability and maintenance cost through constant factors to illustrate the effectiveness of adaptive inspection policy and maintenance interventions.

Considering the incorporation of these factors, a more systematic method is needed to get a whole picture of two objective functions.

4. Multi-objective optimization

The intervention actions of the 1002 configuration in the operational phase, referring to the inspection and maintenance policies, need to consider multiple objectives, including the aforementioned system performance and maintenance cost, from the discussions in Sections 3.4 and 3.5. The change of each input is bound to enhance cost and impact the system performance to varying degrees simultaneously. Therefore, a multi-objective optimization model is considered here to address the proposed problem in Section 3: the optimization of inspection and maintenance of 1002 configuration consisting of heterogeneous units subject to adaptive inspection and imperfect maintenance.

4.1. Description of the problem

The optimization problem is made for two objectives: system unavailability F_S and the total cost TC in a finite time horizon H , calculated based on Eqs. (10) and (12), respectively. Decision variables cover activation probability (of unit 1) p , PM threshold M , the ratio k of imperfect PM, parameters for the adaptive inspection policy a and b . This study aims to achieve better trade-offs between the two dependability attributes of average system unavailability and maintenance cost.

The vector of decision variables is, therefore:

$$\mathbf{x} = \{p, k, a, b, M\}$$

Each variable has a certain range from the real perspective as in Eq. (14).

$$F_{goal} = \min(F_S^{avg}, TC) \tag{13}$$

s.t.

$$0 \leq p \leq 1$$

$$0 < k < 1$$

$$0 < a < 0.9 \tag{14}$$

$$0 \leq b \leq 5$$

$$4 < M \leq L = 7$$

A comparison study following the time-based inspection and maintenance scheme is conducted to verify the advantage of the proposed model. In this case, the variable a and b will be excluded from the decision vector \mathbf{x} . The other variables, the activation probability p , the ratio k of the PM and PM threshold M , are assumed in the range as described in Eq. (14).

4.2. NSGA-III algorithm

This study adopts the NSGA-III algorithm in solving the bi-objective optimization problem. NSGA-III [44] is an extension to the standard NSGA-II [45] with a similar framework but significant improvements by introducing a reference-point-based selection mechanism. These reference points are scattered across the normalized hyperplane [46]. The flowchart of NSGA-III is shown in Fig. 6.

4.2.1. Generation of the initial population

The first step of the NSGA-III algorithm is to generate the initial values of decision variables. Each solution individual consists of 5 decision variables, namely $\mathbf{x} = \{p, k, a, b, M\}$, generated randomly following the given lower and upper boundaries, as displayed in Eq. (14). In sum, a population with N individuals is formed. The population size is normally assumed between 60 and 100 solutions to balance the efficiency and accuracy [47].

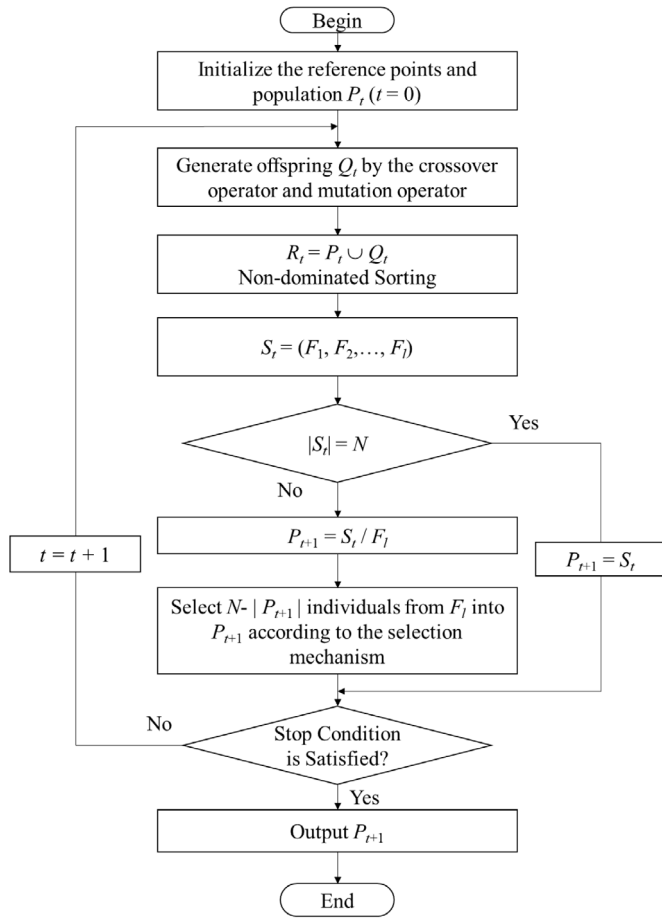


Fig. 6. The main process of NSGA-III.

4.2.2. Crossover and mutation

Crossover and mutation operators intend to create new solutions. The following equation can generate those new values:

$$x_{0,i} = \alpha_i x_{p_1,i} + (1 - \alpha_i) y_{p_2,i} \quad (15)$$

where $x_{p_1,i}$ and $x_{p_2,i}$ are the genes from the one and the other parent (indices p_1 and p_2), respectively, and the α_i is in the range [0,1].

While mutation is to create one child from only one parent. The most common mutation operator treats each gene individually and allows each bit to flip at a low mutation rate. The mutation rate is typically set to a very small value from 0.001 to 0.01 [48].

4.2.3. Elite selection mechanism

The elite selection mechanism of the NSGA-III algorithm is depicted in Fig. 7.

At a generation t , the child population Q_t is generated through crossover and mutation operators of the parent population P_t . The P_t and Q_t have N individuals. Following that, a population $R_t = P_t \cup Q_t$ with the size of $2N$ is obtained by combining the two populations P_t and Q_t . The following step is to select the best N members from R_t to form the parent population for the next generation. Specifically, R_t is firstly categorized into non-domination levels (F_1, F_2 , and so on). Then the fourth step is to construct a new S_t , starting from F_1 , by taking members from different non-domination levels sequentially, terminating at the first hitting the size N . Assuming the last level included is the

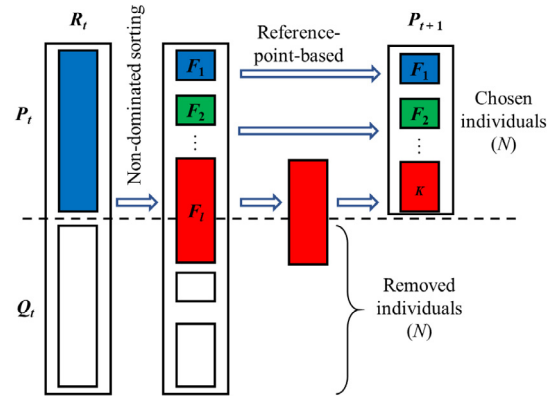


Fig. 7. The main selection procedure of NSGA-III.

Table 2 Set of NSGA-III algorithm related parameters.

Parameter	Value
Population size, pop	100
Evolution times, gen	100
Cross percentage	0.5
Mutation percentage	0.5
Mutation rate, μ	0.02

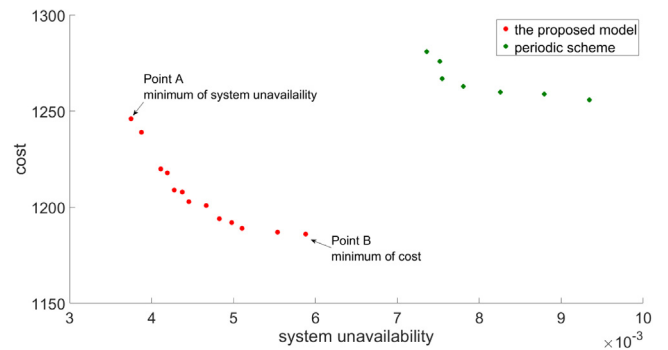


Fig. 8. Pareto frontier of multi-objective optimization of the proposed model.

l th level, the individuals from non-dominated front level $l + 1$ onward are thus rejected. If $|S_t| = N$, then $P_{t+1} = S_t$, if $|S_t| > N$, then all the individuals from level one to $l - 1$ are chosen, and the remaining slots to be occupied by the individual from the last level F_l is $K = N - |P_{t+1}|$. The selection mechanism, relying on a reference-point-based method, is the primary characteristic of NSGA-III. The use of a widely distributed reference point results in a well-distributed set of trade-off points at the end. The algorithm parameters, in this case, are listed in Table 2.

4.3. Discussions of results

The codes of the NSGA-III algorithm and simulation results are completed in the Matlab R2019b program. The calculated Pareto optimal solution sets for the system unavailability and maintenance cost with periodic inspection scheme with $T = 3$ and the proposed model with parameters in Table 1 are depicted in Fig. 8.

The initial inspection interval length for the proposed model is $T_0 = 3$. Compared to the periodic inspection with $T_0 = 3$, the proposed model improves system performance and reduces cost. The details of the Pareto frontiers with the periodic policy and the proposed policy are shown in Tables 3 and 4, respectively.

Table 3
Pareto optimal solutions for the periodic policy.

No.	x			System unavailability	Cost
	p	k	M		
1	0.514	0.110	4.068	7.518E-03	1276
2	0.664	0.130	4.065	7.546E-03	1267
3	0.583	0.149	4.071	7.804E-03	1263
4	0.864	0.168	4.069	8.258E-03	1260
5	0.815	0.201	4.069	9.341E-03	1259
6	0.601	0.089	4.068	7.360E-03	1281
7	0.901	0.207	4.074	9.344E-03	1256

Table 4
Pareto optimal solutions for the proposed policy.

No.	x					System unavailability	Cost
	p	k	a	b	M		
1(A)	0.855	0.373	0.758	0.444	5.006	3.751E-03*	1246
2	0.862	0.580	0.765	0.470	4.818	4.111E-03	1220
3(B)	0.858	0.603	0.702	0.513	4.768	5.880E-03	1186*
4	0.868	0.425	0.771	0.493	5.039	3.879E-03	1239
5	0.859	0.600	0.714	0.509	4.797	5.537E-03	1187
6	0.863	0.576	0.750	0.462	4.811	4.374E-03	1208
7	0.863	0.554	0.751	0.559	4.877	4.670E-03	1201
8	0.853	0.589	0.761	0.565	4.743	4.455E-03	1203
9	0.842	0.600	0.752	0.627	4.724	4.977E-03	1192
10	0.866	0.550	0.755	0.506	4.881	4.277E-03	1209
11	0.863	0.581	0.732	0.625	4.693	5.104E-03	1189
12	0.858	0.569	0.788	0.581	4.885	4.193E-03	1218
13	0.866	0.582	0.743	0.618	4.701	4.827E-03	1194

It is apparent that, with the periodic inspection and maintenance policy, early PM is encouraged since the M value (PM threshold) in solutions closes to the lower boundary with value 4.

In terms of the proposed policy, it is worth mentioning that the optimal solutions for minimizing system unavailability (point A) and minimizing the cost (point B) locate on two extreme points in the Pareto frontier, which are marked with the * symbol in Table 4. Furthermore, the system unavailability and cumulative cost are contradictory. When the system unavailability is reduced, that is, when the system performance is improved, the relevant cumulative cost is increased. The Pareto frontier in Fig. 8 is helpful for practitioners to make decisions for the inspection and maintenance policies considering various factors. Briefly, if the system performance is the priority, point A is the best choice; conversely, point B is the best choice when the inspection and maintenance budget is the priority. Moreover, for example, if the tolerable maximum cost rate is 1200 (economic units), from the results in Table 4 and Fig. 8, there are 5 points, with No. 3, 5, 9, 11 and 13, in Pareto frontier meeting the requirement. Under the budget constraint, system unavailability should be as low as possible. Then the optimal point is No. 13 with variable pair is [0.866, 0.582, 0.743, 0.618, 4.701], the corresponding system unavailability is 4.827E-3, and the cost is 1194. This multi-objective inspection and maintenance optimization is thus expected to provide more feasible solutions to meet the practitioners' preferences or other particular requirements according to the practical situation. However, there are uncertainties in these results from Monte Carlo simulations. The NSGA-III algorithm shows its ability to be leveraged in the decision-making of redundant structure with the balance of maintenance cost and system unavailability.

5. Concluding remarks

Considering the stochastic dependence in redundant structure, this paper has proposed a novel activation probability to determine which unit to withstand the damage from random

shock. On the unit-level, the total degradation process consists of an aging degradation process and abrupt damage caused by random shocks (if it is activated). To release the widely used time-based scheme, an adaptive inspection and maintenance scheme depending on the current state revealed in the inspection has been proposed.

Merging these factors, unit-level for system unavailability and cumulative cost has been proposed. The Monte Carlo method has been employed to study the average system unavailability and cumulative maintenance cost in a designed service time. Numerical examples for the 1002 configuration have been conducted to describe the effects of adaptive inspection policy and imperfect maintenance policy. Briefly, the system unavailability and maintenance cost are two contradictory objectives. Taking advantage of the NSGA-III algorithm, a generalized optimal on these decision variables has been conducted to simultaneously achieve the minimum system unavailability and maintenance cost. It provides clues for practitioners in choosing the optimal inspection and maintenance policy to holistic system performance and economics.

In this paper, the typical 1002 configuration has been employed as the research objective for the sake of simplification and understanding. However, in the future, it would be interesting to extend the proposed model to a more complex structure, e.g., 2003 or a generalized KooN structure. Meanwhile, more kinds of dependence among units in redundant structures could be considered, such as the uncertainties in the failure thresholds and degradation rates. For the proposed activation sequence, a more dynamic and state-dependent probability might be a potential research topic.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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