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Is Everyone Probably Elsewhere?

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Abstract It has been widely discussed whether the existence of other universes than the known universe is a purely metaphysical consideration. This paper argues to the contrary that the existence of other universes is a meaningful hypothesis, which can be tested based on observable data.

The paper applies the perspective of observer bias as an approach for assigning probabilities to the mutually excluding hypotheses of universe vs. multiverse, i.e., whether the known universe is the only universe, or just one universe in an ensemble of universes, a so-called multiverse. The basic idea of the paper relies on the following consideration. In a multiverse, the vast majority of observers would live in universes that are more life friendly. Thus, conditional on there being a multiverse, we should expect to find ourselves in a universe with values of the fundamental parameters that provide particularly fertile grounds for life. In contrast, if there is only a single universe, it may well be the case that there is still a few observers even if the parameters are far from ideal for life, and in that case, those observers will find values of the parameters that are not ideal for intelligent life. It may well be the case that, among all parameter configurations that allow life at least somewhere, those that are not ideal for life far outnumber those that are ideal for life.

Based on this elementary consideration, the paper proceeds to propose a quantitative framework to determine probabilities for either hypothesis. In particular, it is described how a future *ab initio* determination of some of the factors in the Drake equation may be used to infer which one of the two hypotheses is the more likely. A quantitative approach to this end is proposed.

Expressing these factors of the Drake equation in the context of the two hypotheses, a general likelihood approach is first described. Then, to support intuition, example computations are provided, illustrating how an actual hypothesis test would work in practice.

1 Introduction

'Where is everyone?' asked Enrico Fermi famously [17,6,19,15,25] in 1950, expressing the apparent paradox that our universe, while on one hand apparently containing a large number [18,10,20,24,8,21] of Earth-size, Earth-age, Earth-like orbit planets, on the other hand seems eerily silent, containing to our knowledge at most one example of intelligent life. Since Fermi's original question, the urgency of the paradox, the *Great Silence*, has only increased. The effort to detect non-terrestrial

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artificially generated electromagnetic signals has in the decades since Fermi's question been stepped up considerably [29, 28, 12, 9], only to come up with an even more roaring silence. In this paper, we shall consider the paradox to be simply a statement related to the density of *locations with intelligent life* in the region of space around us.

The probability of life developing in a given region of a universe depends on the size of the region and on the values of the fundamental constants of that universe. We do not currently know how to calculate that probability for our own universe, let alone any other universe. But our understanding of the mechanisms are making great strides forward, and it is not out of the question to imagine at some point in the future a qualified estimate of this probability, an estimate based not on observation, but based on first principles in physics, molecular biology, and possibly on computer simulations.

Such an independently obtained probability value could form the starting point of an interesting probabilistic argument. The fundamental constants observed by the human species in our universe presents a different probabilistic situation if seen *either* on the background of several universes, *or* on the background of a single universe.

In a multiverse, the vast majority of potential observers would live in universes that favor intelligent life. Thus, observers would have a high expectation to find themselves in a universe with values of the fundamental parameters that provide fertile grounds for such life. In contrast, if there is only a single universe, it may well be the case that observers would find values of the parameters that allow intelligent life, but might be far from ideal. In the set of all parameter configurations that allow intelligent life at least somewhere, the subset of configurations that are less ideal for life far might be a larger subset than those that are close to ideal for life. In that case, an observer in a single universe would expect to observe parameters that are less than ideal for intelligent life.

Thus, there is a different expectation for observers in the case of a single universe as compared to an observer in the case of a universe belonging to a collection of universes - a multiverse. In this paper, we shall take this difference in expectation as a point of departure and construct a reciprocal version of the standard 'observer bias' compensation formula.

Indeed, given the probabilities for intelligent life forming in ours and other, possibly hypothetical, universes, we demonstrate how to assign likelihoods to either of the two hypotheses mentioned above being true. If the intelligent life probability is close to its maximum probability, our universe is more likely to be just one among many. If the intelligent life probability is far from the possible maximum, we are more likely to be in a single universe.

We consider different forms of probabilistic reasoning/inference: *Bayesian inference*, *maximum likelihood* approaches and *hypothesis testing*. A prerequisite for the former case is the ability to meaningfully assign prior probabilities and/or cost of error-types. Our inability to make such assignments restrains us from a Bayesian approach. Maximum likelihood approaches are most often used for parameter estimation, where the possible values of a *true* parameter constitutes a set of equitable hypotheses. We give examples below, where the number of universes is approached as a maximum likelihood parameter estimation problem. Finally, in contrast to the symmetry among hypotheses in maximum likelihood approaches, hypothesis testing distinguishes between a *null* and an *alternative* hypothesis. Examples are given below, where the number of universes is approached as an hypothesis testing problem.

Examples illustrate, with reference to the Neyman–Pearson Lemma [23], the possibility of statistical test with low error probability for the selection between hypotheses. The possibility of a test with both low probability of rejecting a true hypothesis (Type I error) as well as a low probability of accepting a false hypothesis (Type II error) and in turn an overall low error probability, supports hypothesis testing as our preferred approach for statistical reasoning.

Different interpretations may be associated to Type I and Type II error probabilities such as objective (frequentist) and subjective/evidential (belief) interpretations. The development of formulae below is indeed inductive, i.e. evidential. Likewise, probabilities are not expected to predict any frequency of occurrence among future observations. That is, not only is statistical reasoning based on a single observation, but also additional observations could never be expected to be available. This also makes any use of the obtained probabilities for operational analysis or decision support impossible, since predicted future loss or gain can never be incurred.

To illustrate the fundamental idea we begin in Section 2 with a very terrestrial example. Taking as point of departure the well known analysis of the damage inflicted on allied World War II planes, we show that an understanding of the basic probabilities involved can inform an observer seeing just a single damaged aircraft whether that plane is more likely to have been on a mission involving just this one plane, or it has been one among several planes all on the same mission.

Next, in Section 3, we present the basic probabilistic set-up for the multiverse/universe analysis. We embark from the Drake equation and introduce densities that describe the occurrence of intelligent life as a function of the set of fundamental constants for a given universe. In Section 4 we discuss selection mechanisms for modeling the stochastic process by which universes are 'drawn' from a potential ensemble of such. We provide formulae that describe probabilities for selecting a universe with a given set of fundamental constants from such a stochastic variable. In Section 5 two numerical examples are given, in order to illustrate conceptually how the approach is applied in practice. The section is concluded with a 'toy' example involving simulations of universes/multiverses, where examples of correct conclusions concerning existence of multiple universes is shown along with false positive and false negative conclusions. Section 6 provides a short discussion on how a future computational scheme can be made feasible by a slight simplification. Finally, Section 7 contains a brief summary and a conclusion.

2 Returning aircrafts

To provide a terrestrial example of the statistics reasoning at play we refer to a famous historical example, where *observer bias* has been applied to provide perhaps surprising conclusions.

During World War II the statistician Abraham Wald applied survival/observer bias to minimization of bomber losses to enemy fire [22] and recommended adding armor to the areas that showed the *least* damage. The reasoning was, that surviving/returning planes were exactly not hit in critical areas. An artist's impression of the damages on the returning planes is shown in Figure 1. Please note that the actual data were much more complex. Readers interested in the details of the fascinating story are referred to e.g. [22].

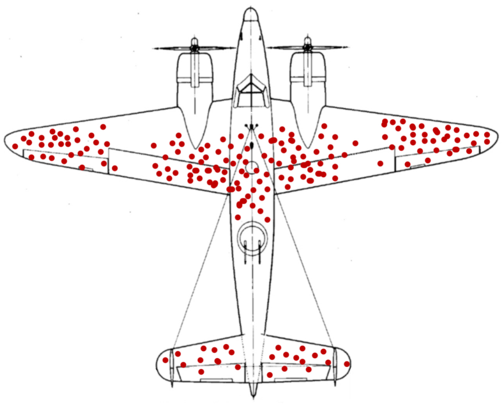

Fig. 1 Artist's impression of hit pattern of returning World War II bomber planes. From https://en.wikipedia.org/wiki/Survivorship_bias#/media/File:Survivorship-bias.png. Can one make a statement about the likelihood of such a plane having been on a single-plane mission, or been part of a group of planes?

We modify the example slightly here and consider a situation where an observer, looking only at a single returned plane, is asked to choose between to complementary hypotheses: $H1$: Only 1 plane was deployed, and $H2$: Exactly 2 planes were deployed (the calculations can be extended to any finite number, for instance a squadron). The observer's conclusion is based on the hit pattern of the returning plane. The probabilistic setup, which is assumed known to the observer, is the following: Planes have a probability of:

– Being hit only in non-critical areas of $P^N = P_N(1 - P_C)$
– Being hit only in critical areas of $P^C = (1 - P_N)P_C$
– Being hit in both critical and non-critical areas of $P^{NC} = P_N P_C$
– Not being hit at all of $1 - P^N - P^C - P^{NC} = 1 - P_N - P_C + P_N P_C$

It is assumed that critical and non-critical hits are independent. If being hit, the various probabilities of returning are

– if hit only in non-critical areas: $P_{R|N}$
– if hit only in critical areas: $P_{R|C}$
– if hit in both critical and non-critical areas: $P_{R|N}P_{R|C}$
– If not hit: $1 - P_{R|N} - P_{R|C} - P_{R|N}P_{R|C}$

If the plane is not hit, the probability of returning is set to 1. For simplicity we set $P_N = P_C = 0.5$, and $P_{R|N} = 0.9, P_{R|C} = 0.1$. We also assume, that in the case two planes return, and only one of them returns with only non-critical hits, this plane is selected for observation. This selection mechanism is also known to the observer.

We consider specifically the case, where the observer finds the plane returning with hits in only non-critical areas.

We choose, as our likelihood function, the conditional probability of observing only non-critical hits given at least one plane has returned.

The probability for a particular plane returning with non-critical hits only is $P_p = P_N(1 - P_C)P_{R|N} = 0.5^2 \cdot 0.9 = 0.23$, whereas the probability of a particular plane returning altogether is $P_{ret} = P_N(1 - P_C)P_{R|N} + (1 - P_N)P_C P_{R|C} + P_N P_C P_{R|N} P_{R|C} + (1 - P_N)(1 - P_C) = 0.25 \cdot 0.9 + 0.25 \cdot 0.1 + 0.25 \cdot 0.09 + 0.25 = 0.52$. The probability of returning with everything other than only non-critical hits is $P_n = P_{ret} - P_p = 0.30$.

– Under $H1$ the conditional probability of a returned plane observed with non-critical hits only - given a plane returned is therefore $P_{ret|w} = P_p/P_{ret} = 0.23/0.52 = 0.43$.
– Under $H2$ the probability that a plane is observed with only non-critical hits is $P_p^2 + 2P_p(1-P_p) = 0.40$. The probability of no planes returning is $(1-P_{ret})^2 = 0.23$. Thus, the probability of at least one plane returning is $1 - 0.23 = 0.77$. The conditional probability of a returned plane observed with only non-critical hits - given at least one plane returned is therefore $0.40/0.77 = 0.52$.

Since our chosen likelihood function is larger under $H2$, likelihood reasoning would favor this hypothesis. The observer can reason that the plane observed is more likely to have been one of a pair, than having been on a single-plane mission. This makes sense intuitively: when more planes are deployed, there will be more possibilities of selecting one with only non-critical hits.

It is also easily verified by similar computations that if the observed plane would have been primarily hit in critical regions, likelihood reasoning would favor $H1$.

Wald's approach, like the approach proposed in this work, is an example of observer-bias at work. A successful application of observer-bias in cosmology can be found e.g. in [5] that uses the causal anthropic principle to predict the cosmological constant for our universe, obtaining a result close to observations.

In the present paper, the Wald example above serves as an analogy. I.e., different damage patterns observed for airplanes provide different 'opportunity for flight' just as different natural constants for different universes provide different 'opportunity for life'. A low 'opportunity for flight' would favor a single plane hypothesis, just as a low 'opportunity for life' would favor a single universe hypothesis.

3 Likelihood Approach

In the following, we shall present an approach that follow the same outline as the aircraft example above, but applied to universe/multiverse hypotheses rather than aircraft hypotheses.

Fermi's paradox is quantified through the Drake equation, first proposed by Frank Drake in 1961 [11,30,7,14]. It estimates the current number N of communicating civilizations in the universe as

$$N = R^* \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot L \tag{1}$$

Here, R^* is the rate of formation of stars suitable for the development of intelligent life, f_p is the fraction of those stars with planetary systems; n_e is the number of planets, per solar system, having an environment not hostile for life; f_l is the fraction of suitable planets on which life actually appears; f_i is the fraction of life bearing planets on which intelligent life emerges; f_c is the fraction of civilizations that develop a technology that releases detectable signs of their existence into space; and L is the average length of time that such civilizations release detectable signals into space.

Along with the intensified search for ex-terrestrially generated radio signals, astronomy has provided significant evidence for the product of the three first factors in the Drake equation: $R^* \cdot f_p \cdot n_e$ accelerating with the first discovery of an exo-planet in 1995 (1988), see [31] supported by (June 2021) almost 5,000 confirmed discoveries [27].

The Drake equation quantifies the probability of our civilization receiving detectable signals from other civilizations by computing the density of civilizations able to release such signals. Density is here defined as the ratio between the expectation value of the number of civilizations within a given ball in space, and the volume of such a ball. In principle, such an expectation value can be computed by modeling each of the factors in the Drake equation by the physical laws determining each factor. Clearly, the numerical value of the expectation value will depend on the numerical value of the fundamental constants that enter the laws involved.

This has the implication that in a multiverse theory, the density of civilizations will depend on the values of the fundamental constants related to the universe in consideration. At the time of writing, there does not exist a reliable formula for computing accurately the density of planets with intelligent life in our own universe, let alone densities of such planets in other potential universes having other fundamental constants. Whether such a model can be obtained from first principles, possibly aided by simulation, is outside the scope of this article.

3.1 Notation and framework

For a given universe, let C denote a vector of possible fundamental constants belonging to some probability space $\mathcal{C}$.

We assume that a universe, at the point of its creation, is assigned fundamental constants through a random pick according to some probability density function u defined on $\mathcal{C}$. We assume that u will be established as a result of future physics. For such a universe, every factor in the Drake equation would in some way or another depend on the set of fundamental constants C for that particular universe.

The only available statistical evidence about u as well as the dependence of fundamental constants on the factors of the Drake equation is the observation of known natural constants in our own universe, evidently hosting intelligent life. We assume that the observed fundamental constants C_O of our own universe are drawn as a sample from a subset of universes which all allow *intelligent life*. Thus probabilities regarding $C_O \in A \subseteq \mathcal{C}$ are inherently conditional on $C_A \in \mathcal{L} \subseteq \mathcal{C}$, where $\mathcal{L}$ denotes the subset of fundamental constants allowing intelligent life to evolve. We shall therefore restrict considerations to the subset $\mathcal{L}$ and redefine u to be a density on $\mathcal{L}$. The above reasoning can be summarized as:

1. Our universe exhibits some spatial density ρ of intelligent life. The density ρ can be given either as habitats (planets) per volume or probability of intelligent life per planet. Here we adopt the former definition.
2. Since fundamental constants determine all characteristics of the universe, the spatial density of intelligent life, ρ is determined by the fundamental constants of the universe. Generally ρ varies with the fundamental constants C of the universe in which C is observed.
3. The observation of one example of intelligent life is now considered on the background of two different hypotheses:
   H1. The observation is obtained from a random iterative selection of planets from a single universe until intelligent life is found
   H2. The observation is obtained from an iterative selection of planets from an infinite multitude of universes until intelligent life is found
4. Under H1 the distribution of C follows exactly $L1 = u$, whereas under H2, C is distributed according to $L2 = c\rho u$ (c is a normalization contant). $L1(C)$ and $L2(C)$ are therefore *likelihood functions* under their respective hypotheses.
5. Since $L1$ and $L2$ are both probability densities, their values differ in order, i.e. for some values of x the one is bigger and for other the opposite order is attained. This is determined directly from $c\rho \neq 1$.
6. If the observer finds $L1(C) > L2(C)$ likelihood reasoning favors $H1$, whereas $H2$ is preferred in the opposite case.

It should be noted at this point that several alternative processes leading to the observation of fundamental constants could be imagined. Instead of an iterative selection of planets as in **H2**, e.g. a process of selecting volumes of a fixed, sufficiently small size could be considered, where the process would stop once intelligent life has been found. Admittedly, the choice proposed above to some extent is taken for mathematical convenience. The impact of the modeling process on the final result would be a relevant but extensive study, which is omitted here in part due to space limitations.

Altogether, knowing ρ, u and C allows an intelligent observer to make qualified probabilistic discrimination between the two hypotheses H1 and H2, i.e. whether there is more likely only one, or there is more likely an infinity of universes.

It should be emphasized that the paper does not pretend to make any assumptions on the nature or origin of the (potential) multitude of universes discussed below. In the literature, attempts has been made, e.g., to relate quantum branching in wave functions to parallel universes. No such attempt is made in this paper, and no relationship between the considered multitude of universes is assumed.

4 Selection mechanism and basic formulae

We now consider stochastic mechanisms associated with a multiverse, modeled with the following settings:

We first assume a finite number N of universes. N is unknown and subject to probabilistic inference. The string of N sets of fundamental constants (one set for each universe) are random variables $C_i \in \mathcal{C}, i \in \{1, .., N\}$. We let A be a ball of volume ε and center a in $\mathcal{C}$. Fundamental constants are drawn independently according to some probability measure P_u on $\mathcal{C}$, where we assume the existence of a density function u, such that $P_u(A) = \int_A u(x)dx$.

We assume the multiverse with its N elements is generated according to some random process, and N and $\{C_i, i \in \{1, .., N\}\}$ remains fixed subsequently. We comprise the sequence of fundamental constants into a random vector $\underline{C} = [C_1, .., C_N]$, such that $P_{\underline{C}}$ is indeed the product measure P_u^N on $\mathcal{C}^N$. First we consider the pair (C, L) drawn randomly among all habitats in the union of universes $U_1, .., U_N$, where the random variable $C \in \mathcal{C}$ denotes the (found) fundamental constants and L can take the values 0 or 1 indicating whether the habitat has intelligent life. We consider the selection process being split into first drawing a particular universe indexed i and with fundamental constants C_i. This is followed by randomly selecting a volume V from U_i and looking for intelligent life within that volume. For given fundamental constants C the probability of finding intelligent life in the selected volume is denoted $P(L = 1 \mid C) = P_L(C)$. $P_L(C)$ depends on the size of the selected volume. In particular, for sufficiently small volumes V, we have $P_L(C) \approx V\rho$.

The process of drawing (C, L) is repeated (for a fixed multiverse) following an *accept/reject* procedure until $L = 1$ and the final observed fundamental constants defines a random variable denoted C_O.

One can show that

$$P(C_O \in A) = \int_{\mathcal{C}^N} \frac{\sum_{j=1}^{N} P(i=j)P(L=1 \mid C_j)I_{C_j \in A}}{\sum_{k=1}^{N} P(i=k)P(L=1 \mid C_k)}\, dP_{\underline{C}} \tag{2}$$

where $P(i = j)$, $P(i = k)$ denotes the probability of selecting a specific universe, and $P(L = 1 \mid C_j)$, $P(L = 1 \mid C_k)$ denotes the probability of observing life, given the observation of fundamental constants C_j, C_k. $I_{C_j \in A}$ is the indicator function for C_j belonging to A.

The interpretation of (2) is that if we observe C_O, we can compute the conditional probability of those fundamental constants being picked from an ensemble of N universes, $P(C_0 \in A \mid N \text{ universes})$. This allows us to compare hypotheses for varying numbers in the ensemble, where $N = 1$ would correspond to a universe hypothesis and $N > 1$ would represent a multiverse hypothesis. We first present a simple numerical example of this.

4.1 A Finite Multiverse Example

We provide here a simple numerical example for a finite multiverse (a finite ensemble of universes) to guide intuition. We compare a multiverse with one element to a multiverse with two elements. For simplicity, we will assume a model with only one fundamental constant.

Let $\mathcal{C} = [0, 1]$ and P_u be uniform, i.e. $P_u(A) = \int_{A\cap[0,1]} dx$. Also Let $P(L = 1 \mid C) = C$, i.e. higher fundamental constant gives higher density of life. Finally, assume that $P(i = j) = \frac{1}{N}$, i.e. universes are drawn uniformly from $\{1, .., N\}$.

Then for $N = 1$

$$P(C_O \in A) = \int_{[0,1]} \frac{C_1 I_{C_1 \in A}}{C_1}\, dx = \int_{[0,1]\cap A} dx = P_u(A) \tag{3}$$

For $A = [a - \varepsilon/2, a + \varepsilon/2]$, $0 < \varepsilon << 1$ and a and C_O in $[\varepsilon/2, 1 - \varepsilon/2]$

$$P(C_O \in A) = P_u(A) = \varepsilon \tag{4}$$

from which we infer a constant likelihood $P_1(a) \equiv 1$, i.e., $P(C_O \in A) = P_1(a)\varepsilon = \varepsilon$.

Similarly, for $N = 2$

$$P(C_O \in A) = \int_{\mathcal{C}} \int_A \frac{2P(L=1 \mid C_1)}{P(L=1 \mid C_1) + P(L=1 \mid C_2)}\, dC_1 dC_2 \tag{5}$$

yielding

$$P(C_O \in A) = 2\varepsilon a \log\left(\frac{1+a}{a}\right) = P_2(a)\varepsilon \tag{6}$$

i.e., the likelihood $P_2(a) = 2a \log\left(\frac{1+a}{a}\right)$.

Inspecting reveals that $P_1(C_O) \geq P_2(C_O)$ for $C_O \leq 0.4$ and $P_2(C_O) \geq P_1(C_O)$ for $C_O \geq 0.4$ (0.4 is the approximate root of $2a \log\left(\frac{1+a}{a}\right) = 1$, see Figure 2).

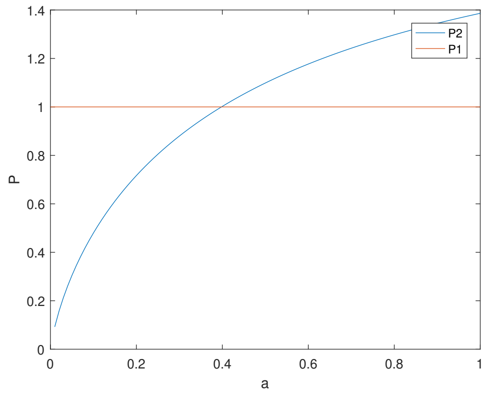

Fig. 2 Comparison of likelihoods in a crude multiverse set-up with $N = 1$ or $N = 2$ universes. The parameter a is the (in this example single) fundamental constant and P_1 and P_2 are the corresponding likelihoods. Thus if the observed fundamental constant is larger than 0.4, two universes are more likely than one, and vice versa.

Thus, if we find ourselves in the situation that we are asked to estimate the number of universes and allowed to pick between numbers $N = 1$ and $N = 2$, we would (based on *Maximum Likelihood* reasoning) pick $N = 1$, if we find $C_O < 0.4$. Accordingly, we would pick $N = 2$, if we find $C_O > 0.4$. For $a = 0.4$ we shall refrain from making a conclusion.

The assumption of uniformity of P_u is this example is convenient, but the approach proposed in this paper applies to any distribution of P_u.

4.2 An infinite number of universes

We now consider the implications of letting $N \to \infty$ in (2). Let $P_L(C)$ be continuous, $P(i = j) = \frac{1}{N}$. Using a standard likelihood probability computation, it can be shown that for these settings

$$P(C_O \in A) = P_L(a)u(a)\varepsilon \int_{\mathcal{C}^{N-1}} \frac{1}{\frac{1}{N}(P_L(a) + \sum_{k=1}^{N-1} P_L(C_k))} \ \Pi_{q=1}^{N-1} u(C_q)\ dC_q \tag{7}$$

with notation as introduced above.

For $N = 1$, (7) reduces to

$$P(C_O \in A) = P_L(a)u(a)\varepsilon \frac{1}{P_L(a)} = u(a)\varepsilon \tag{8}$$

To obtain a limit for $N \to \infty$ in (7) we need two additional technical assumptions, i.e. we assume

$$E\left[\frac{1}{P_L(C)}\right] < \infty \tag{9}$$

and

$$E\left[\frac{1}{P_L^2(C)}\right] < \infty \tag{10}$$

With these assumptions at hand and with the generalized dominated convergence theorem, see [3], one can prove that the following holds (with probability 1):

$$\lim_{N\to\infty} P(C_O \in A) = \frac{P_L(a)u(a)\varepsilon}{E(P_L(C))} \tag{11}$$

We do not claim that the technical assumptions (9) and (10) to be the only way of obtaining convergence nor do we discuss the physical implications thereof. A simple case covering assumptions (9) and (10) is if there is a positive constant γ such that $P_L(C) \geq \gamma$ whenever $u(C)P_L(C) > 0$.

We observe that the ratio between the probability $P(C_O \in A)$ for $N = 1$ as given by (8) and $P(C_O \in A)$ for $N \to \infty$ as given by (11), amounts to:

$$\frac{E(P_L(C))}{P_L(a)} \tag{12}$$

This implies that determining whether a universe model or an infinite multiverse model is more likely, involves only the two numbers $E(P_L(C))$ and $P_L(a)$.

5 Hypothesis testing

In this section we apply standard Neyman-Pearson [23] hypothesis testing for the following two, mutually excluding, hypotheses:

Hypothesis 0 θ_0*: Only one universe exists*

Hypothesis 1 θ_1*: Infinitely many universes exist*

As demonstrated above conditional distributions $P(C_O \in A|\theta_i)$ given hypothesis θ_i can in principle be established through the development of the mapping $P_L(C)$ from fundamental constant to the probability of intelligent life. Thus, we find ourselves in a *simple versus simple* testing scenario, see [4].

The one available sample C_O is drawn among habitats in a single or an infinite multitude of universes all within $\mathcal{L}$. As described above, we can consider that C_O is the result of an iterative accept-reject mechanism, i.e. candidate habitats are picked randomly until we find a habitat where inhabitants can observe the fundamental constants of our known universe. Under θ_0 the selection procedure is restricted to only one universe. The outcome of C_O is therefore firmly tied to the assignment of fundamental constants at the creation of that universe, i.e. from (8)

$$P(C_O \in A|\theta_0) \equiv \int_A u(x)\, dx$$

so that the conditional density $f(C_O|\theta_0)$ of C_O under θ_0 is given by

$$f(C_O|\theta_0) \equiv u(C_O)$$

Under θ_1, a habitat with intelligent life is picked at random among an infinite collection of habitats from an infinite number of universes, having fundamental constants distributed according to u. Intuitively it would be more likely eventually to draw a habitat from a universe with a high likelihood of life. That is, where fundamental constants give larger value for the product of the last four factors $f_l \cdot f_i \cdot f_c \cdot L$ in the Drake equation. This likelihood is indeed parametrized by $P_L(C)$ denoting the probability for intelligent life in a randomly drawn volume in a universe with fundamental constants C.

The conditional Drake density $f(C_O|\theta_1)$ under θ_1 is given by (11) as

$$f(C_O|\theta_1) = \frac{P_L(C_O)u(C_O)}{\int P_L(x)u(x)\, dx}$$

With the Neyman-Pearson lemma [23] we can identify a *Uniformly Most Powerful* reject region R_α of size/significance level α for the *null*-hypothesis by identifying

$$R_\alpha = \{C_0 \mid \frac{f(C_O|\theta_1)}{f(C_O|\theta_0)} \geq k_\alpha\}$$

such that $P(C_O \in R_\alpha|\theta_0) = \alpha$. Thus, R_α yields the highest reject ratio (power) $1-\beta$ for a true alternative hypothesis θ_1 under a fixed size/significance level α, i.e. probability of rejecting a true *null*-hypothesis.

Following common practices in likelihood reasoning [13] and considering $\theta \in \{\theta_0, \theta_1\}$ as a parameter subject to estimation, the case for $f(C_O|\theta_1) > f(C_O|\theta_0)k_\alpha$ would speak in favor of an infinite multitude of universes, whereas the opposite case would argue in favor of a single universe. It could be noted that k_α corresponds, in Bayesian inference, to the ratio of *a priori* probabilities for the two mutually exclusive hypotheses, i.e. $\frac{P(\theta_0)}{P(\theta_1)}$. In this argument we abandon the Bayesian viewpoint based on the lack of meaningful *a priori* probabilities.

5.1 Example of hypothesis testing

In the following, we shall give a numerical example to illustrate how the hypothesis testing outlined above would work in case we had access to a model for density of intelligent life in different universes as a function of their fundamental constants.

Assume that there are N fundamental constants inside an N-dimensional sphere S_N with center $0.5^{(N)}$ and radius 0.5, and that joint conditional densities (restricted to S_N) can be written as $f(C|\theta_{\{0,1\}}) = K_N \prod_{i=1}^{N} f_i(C_i|\theta_{\{0,1\}})$, where as an example it is assumed that $f(C|\theta_0)$ is uniform in S_N as illustrated in Figure 3 for $N=1$ and where $f(C_i|\theta_1)$ is normal with mean 0.5 and standard deviation 0.0.07

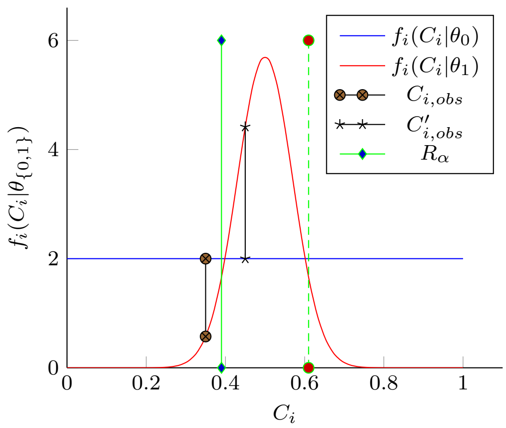

Fig. 3 Conditional densities $f_i(C_i|\theta_0)$ and $f_i(C_i|\theta_1)$. for one (C_i) fundamental constant. Reject region (delimited by green lines) and two imagined observations C_i and C'_i. When the fundamental constant $C'_{i,obs} = 0.45$ is observed the single universe hypothesis is rejected in favour of a multiverse hypothesis, whereas after observing $C_{i,obs} = 0.35$, the single universe hypothesis is accepted.

Figure 3 illustrates the 1-dimensional case for $k_\alpha = 2$, and a reject region of [0.39 0.61] yielding a size $\alpha = 0.61 - 0.39 = 0.22$ and $\beta = 2 \cdot 10^{-6}$. For only one dimension, this size seems too large to be statistically acceptable. However, with a total of, e.g., 13 fundamental constants, the situation turns out quite differently. We let the reject regions R_N for higher dimensions be N-balls with centers $0.5^{(N)}$ and radii 0.11 (consistent with the 1-dimensional case). For the 2-dimensional case we obtain $\alpha = 0.15$ and $\beta = 1.3 \cdot 10^{-5}$. Finally, in the 13-dimensional case we obtain $\alpha = 2.5 \cdot 10^{-9}$ and $\beta = 0.05$, which facilitates a statistically acceptable hypothesis test.

This illustrates how a high entropy (flat) $f_i(C_i|\theta_0)$ and a low entropy $f_i(C_i|\theta_1)$ can pave the way for the design of a powerful hypothesis test with acceptable statistically significance. Indeed, the reduced entropy of $f_i(C_i|\theta_1)$ would come through the model P_L and the information retrieved by the observation of natural constants under the condition of intelligent life. In [16] it is explained how quantum fluctuations in the early universe could create a multiverse with density variations, and it is argued that the existence of a multiverse would explain the low energy density.

This example also shows that with a "flat" likelihood for the (single) universe hypotheses and a low entropy likelihood for the multiverse hypotheses, an observation of fine-tuning would tend to favor multiverse hypotheses and, vice versa, "poor tuning" would tend to favor a single universe hypothesis.

It should be noted that all universes considered are assumed to be infinite. This means that for positive values of the density function, a given universe would harbor life with a probability of 1.

A 2-dimensional case is simulated in the next section.

5.2 Simulation

To make it more explicit how the hypothesis testing would work for randomized data, we present in this subsection a simulation for a 'toy' example.

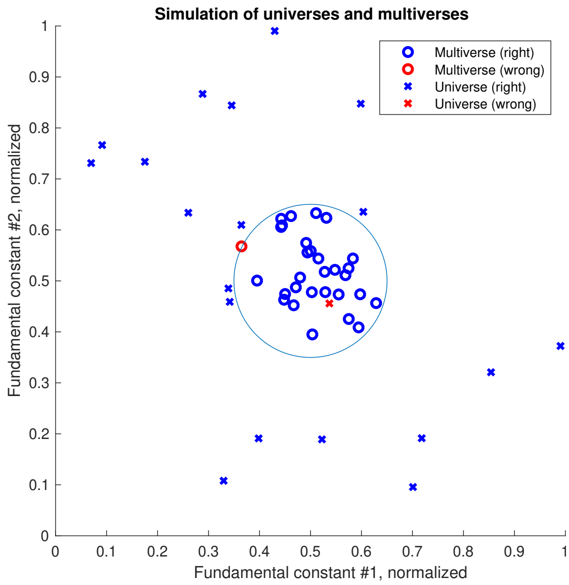

Fig. 4 Result of 50 simulations with universes/multiverses with two fundamental constants. Circle indicates boundary for equal probability for universe/multiverse hypotheses. In the legend, 'Multiverse/Universe' indicates the actual model related to the marker, whereas 'right/wrong' indicates whether the hypothesis with higher probability was correct.

Figure 4 shows the result of 50 simulations for a 'toy' example, where there are only two fundamental constants. Each simulation has been performed using the following algorithm:

1. Decide for either a universe model or a multiverse model, with 50% probability for each
2. If a multiverse model is picked, the number of universes is picked at random (uniform) between 2 and 1,000.

3. For each universe two normalized fundamental constants are picked at random (uniform) from the interval $[0;1]$.
4. Based on values of the fundamental constants, a density of habitats with sapient life is computed by a two-dimensional normal distribution with mean $\mu = \left(\frac{1}{2}, \frac{1}{2}\right)$ and standard deviation $\sigma = (0.07, 0.07)$.
5. For each multiverse case, a universe is picked at random and one small sample taken. A binary stochastic value for sapient life is assigned to the sample with probability given by density times sample size.
6. Repeat Step 5 with a new universe and a new sample, until sapient life is found.

The position of the markers in Figure 4 indicate the fundamental constants for the first time life is found, in a universe or in a multiverse. Blue circles indicates multiverses that are correctly identified as such by the hypothesis testing presented above. Red circles indicate multiverses that are incorrectly identified as universes. Similarly, blue crosses indicate universes that are correctly identified as such. Red crosses indicate universes that are incorrectly identified as universes. By the hypothesis testing, points inside the circle are identified as most likely being multiverses and points outside as most likely being universes. It can be seen in Figure 4 that only one universe is incorrectly identified as a belonging to a multiverse, and that only one multiverse pick is incorrectly identified as a universe.

In this simulation case, error probabilities may be estimated through frequencies, i.e. a frequentist approach is conducted for validation of derivations. This of course is not possible for the presented dichotomy of one versus multiple universes, since experiments can not be repeated as in simulation.

The example in this subsection had just two fundamental constants. If the same numerical framework had been applied with the same parameters to universes/multiverses with a realistic number of fundamental constants, the probability for incorrectly identifying a multiverse as a universe or vice versa, would drop dramatically. It should be emphasized, however, that nothing can be directly inferred about our own universe/multiverse based on this observation, as too little is known at the present time about the dependency on density on the actual fundamental constants.

6 Feasibility discussion

The arguments in this paper rely on the availability of conditional density functions that express how fertile for intelligent life a given universe would be depending on its fundamental constants. Obviously, such density functions are not available in the human knowledge pool at the time of writing.

There are, however, significant epistemological challenges related to determine, whether it is possible at all to extrapolate interpretations of the current laws of physics qualitatively beyond the phenomenological behaviors from which they have been derived.

On one hand, it can not just be contemplated but perhaps even expected that terrestrial science will one day be able to establish a quantitative model that accounts for the full chain of events that led to creation of human life, starting from shortly after the Big Bang. On the other hand, an entirely different matter is to be able to contemplate, let alone to quantitatively model, the origin of fundamentally different intelligent life forms in different universes that might, e.g., not be remotely related to carbon-based life. In the best case, such an endeavor would be extremely optimistic. In the worst case, it could be epistemologically argued that such a program would be fundamentally inconceivable.

There is, however, a potential remedy still to make a program towards one day determining at least reasonable estimates for actual, numerical probabilities for either of the two competing hypotheses (universe vs. multiverse). The pivotal data for the derivations in this paper is the observation of a specific set of fundamental constants in a universe with intelligent life, i.e., the observers. If the data instead is considered to be a specific set of fundamental constants in a universe with *anthropomorphic* life, establishing a computational scheme suddenly becomes much more realistic.

Changing the perspective from *any* intelligent life to *anthropomorphic* life would still comply with the basic idea of this paper: In a multiverse, the vast majority of *anthropomorphic* observers would live in universes that are more friendly for anthropomorphic life. Thus, conditional on the existence of a multiverse, we should expect to find ourselves in a universe with values of the fundamental parameters that provide particularly fertile grounds for anthropomorphic life. In contrast, if there is only a single universe, it may well be the case that there is still a few observers even if the parameters are far from ideal for anthropomorphic life, and in that case, those observers will find values of the parameters that are not ideal for anthropomorphic life.

Modeling the creation of anthropomorphic life for other values of the fundamental parameters is still a monumental challenge. However, relying on arguments proposed in the literature on 'fine tuning', e.g. [1,2,26], it is reasonable to assume that the known laws of physics would apply for universes with parameters in a neighborhood of the values observed in our universe. This means that it might suffice to establish a parameterized model of the terrestrial creation of homo sapiens, as such a model could be assumed to apply for neighboring universes by a continuity argument.

Monte-Carlo simulations for universes based on a model of this type would provide estimates of the density functions on which the derivations in this paper rely.

Clearly, accurate parameterized models for creation of anthropomorphic life will not be available in any near future, if ever. However, a first coarse model could probably be derived within a foreseeable future, given sufficient effort. If such a coarse model would give a sufficiently clear answer to the question, whether a universe or multiverse hypothesis is more likely, this would in turn stimulate hope that humankind could eventually gain (probabilistic) insights that transcends the known universe.

7 Conclusions

The human perception of our cosmos has evolved from a geocentric model via a heliocentric model to the present understanding of an infinite universe with several superstructures. The leap, however, to discuss the existence of several universes is still challenging, and the consideration is often dismissed as purely metaphysical.

This paper does not provide definitive answers to this ultimate challenge for human perception. We do believe, however, that not only have we argued that the question of existence or non-existence of other universes is meaningful, but we have actually provided a framework for recasting this question as a hypothesis, which can be tested by observable data.

Specifically, the paper has introduced a mathematical framework for re-casting our observation of a specific set of fundamental constants for our universe into a statistical hypothesis test for the existence or non-existence of other universes.

Based on density functions for planets with intelligent life in universes with different sets of fundamental constants, we have derived a probabilistic framework, where the known fundamental constants given our own existence is seen as an outcome of a random variable. We have presented a rigorous framework for such a probabilistic reasoning, where the number of universes is assumed unknown and subject to hypothesis testing. The end result is a framework, where observations made in our own universe will lead to a conclusion that either a single universe hypothesis is more preferred in favour of a multiverse hypothesis, or vice versa. Since we cannot foresee any future observations adding to the currently known observations, our probabilistic results should be interpreted as subjective or evidential providing no predictive power onto the statistical behaviour of any sequence of future observations.

Determining conditional probabilities for a universe versus a multiverse hypothesis as proposed in this paper requires two numbers. We need to determine the probability of intelligent life creation in our own universe within a sphere of a given size, and we also need to know the domain of potential creation of intelligent life in the space of fundamental constants. It should be noted that a high-fidelity model is not necessarily needed. As suggested by the numerical examples given above, knowing the two required numbers within an order of magnitude might suffice to providing an inference of whether or not the Great Silence is a likely indicator of an isolated universe, or not.

In other words, if our universe turns out to be relatively poorly tuned, the answer to Fermi's question would be: "Everyone just happens to be really far away, in this (most likely unique) universe". On the other hand, if our universe turns out to be unusually fine-tuned, the answer would be the more mind-boggling: "Everyone is most likely in other universes - the ultimate elsewhere".

Considering the huge progress in advanced simulation platforms based on high performance computing, Monte-Carlo simulations of creation of eukaryotic life, given our set of fundamental constants, might be within reach of the coming generation of computational biologists.

Simulations of the occurrence of *any* type of intelligent life for *any* set of fundamental constants are computationally extremely more demanding, and might not even be within reach in the lifetime of humanity. If intelligent life forms that are completely dissimilar to anthropomorphic life is possible in other universes (or even in ours), such life forms might be less or more sensitive to changes in fundamental constants. Therefore a computation based on an 'anthropomorphic bias' could yield inaccurate results, although it might provide a first, good indication of the final result.

Finally, it has been speculated that the natural constants of our universe could be the result of initial quantum fluctuations. Once mature, such a model could be built into the framework proposed above. This could potentially eliminate the need for assuming distributions of natural constants.

The authors of this paper sincerely hope that the future of scientific endeavor involves an effort to pursue a *probabilistic* answer to this fundamental question: *is our universe the only such?* This paper establishes that an answer of this nature may actually be found.

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