Spring 2023

# Analysis of Material Properties for Wooden Octet Trusses

Julio A. Del Cid , '23

David Mendoza , '23

# Analysis of Material Properties for Wooden Octet Trusses

A thesis presented by

Julio Del Cid and David Mendoza

to

the Faculty of the

Swarthmore School of Engineering

in partial fulfillment of the requirements for

the Bachelor of Science degree of Engineering.

Faculty Adviser: Professor Fiona O'Donnell

Swarthmore College

Swarthmore, PA

May 05, 2023

**Abstract**

The introduction of new construction materials has given rise to different, unique construction projects emphasizing architectural value without sacrificing strength, safety, and durability. This project proposed the design for a wooden octet truss along with an efficient fabrication process; furthermore, it studied the plausibility of implementing columns of octet trusses into everyday structures. Compressive loading tests were conducted on each fabricated truss in order to determine its strength properties and areas that could be improved upon in future iterations. In constructing each truss, priority was placed on maximizing load capacity and overall resistance to deformations while reducing both construction time and cost. Our second octet truss yielded the most satisfactory results, as this iteration achieved the highest maximum load of 14,400 lbs before failure. It had nearly the greatest stiffness of all 3 trusses at 40,800 psi, and presented the highest strength to weight ratio. Members of our trusses were connected using wood glue and punched metal plate fasteners. Additionally, the cross sectional area of members which experienced the most load in the previous iterations were doubled. Although a time efficient fabrication process was created for wooden octet trusses, there still exists opportunities to increase its load capacity to be competitive with other conventional construction materials.

**Table of Contents**


**Introduction** **3**
**Theory** **5**
**Design Constraints, Requirements Developed, Standards, and Codes** **8**
**Methods** **10**
Designing and Verification 10
Implementation: Truss One 13
Implementation: Truss Two 18
Implementation: Truss Three 20
Testing 22
**Results** **23**
**Discussion** **27**
**Conclusion and Future Work** **30**
**Appendices** **31**
Appendix A 31
Appendix B 32
Appendix C 33
Appendix D 34
Appendix E 35
Appendix F 36
Appendix G 37
**Sources** **38**

## Introduction

Octet trusses are three-dimensional structural frameworks composed of triangular elements known for their high strength in compression with conservative amounts of material required for their construction. The centers of octet trusses are hollow and see-through as seen in *Figure 1*, which has allowed for lightweight, yet durable structures to be fabricated; however, due to the complex design of an octet truss, the materials previously used to construct them has been limited. Conventionally, octet trusses have been 3D printed or made using 3D printed molds, methods which both require a significant amount of time and do not utilize widely used construction materials such as: wood, steel, or concrete. This project sought to surpass constructability limitations by developing an efficient wooden octet truss fabrication process which could be implemented in modern construction projects. The primary design goals were to minimize fabrication time and cost, and maximize compressive strength, stiffness, and architectural value. Several iterations of octet trusses were created with the intent of improving each of the aforementioned parameters with each subsequent iteration.

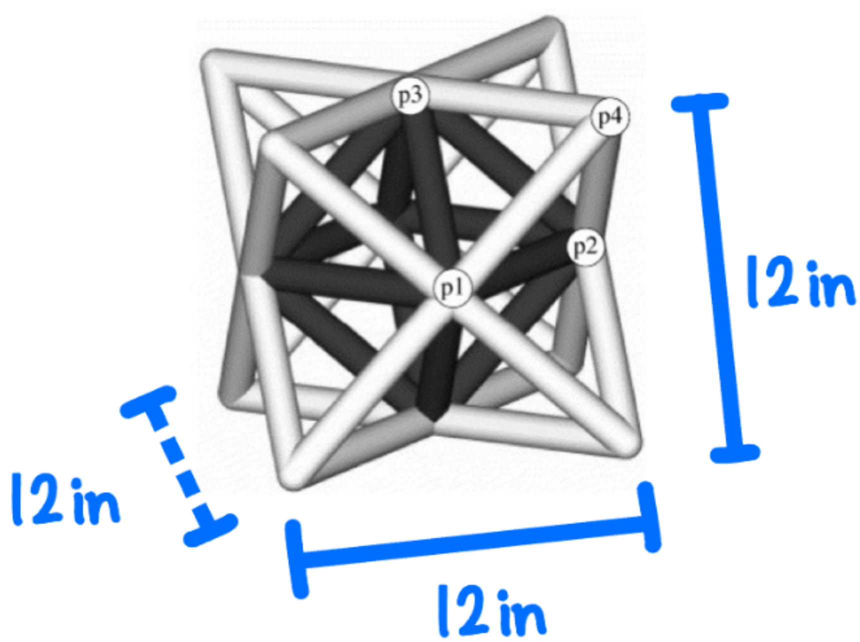



Figure 1: Octet Truss with dimensions of 1 cubic foot (Deshpande, Fleck, Ashby, 2001).[2]

Wood was decided to be the primary building material in this project since it is a natural resource and possesses high strength and stiffness while remaining lightweight. The specific wood utilized was Southern Yellow Pine, a durable, yet cost effective wood, which is easily accessible in the Swarthmore College area, see *Figure 2* – where this project was completed. Since Southern Yellow Pine refers to a set of several wood species which includes Loblolly, LongLeaf, Shortleaf, and Slash, the wood species with the most conversative strength values was assumed in the calculations. The relevant strength properties of Loblolly were obtained from Table 5-3b of "The Wood Handbook" which is referenced in *Table 1*.[6]

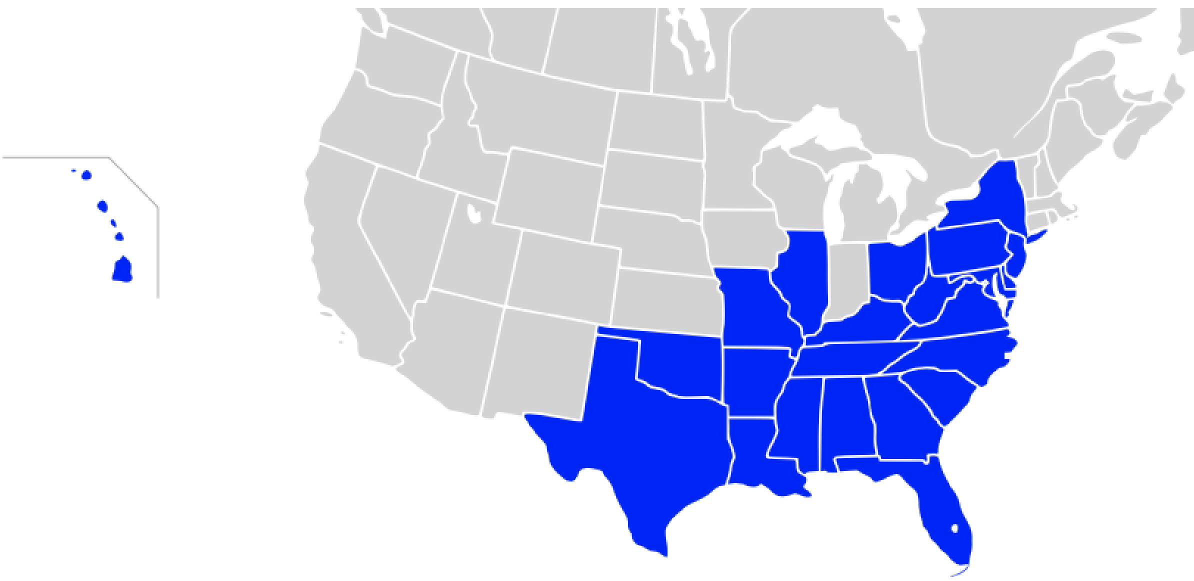


Figure 2: Map of the growth region of Southern Yellow Pine (Trees of North America, n.d.)[8].

Table 1: Table 5-3b from "The Wood Handbook". This table includes the strength properties for Loblolly wood at 12% moisture content.

**CHAPTER 5 | Mechanical Properties of Wood**

**Table 5–3b. Strength properties of some commercially important woods grown in the United States (inch–pound)[a]—con.**

| Common species names | Moisture content | Specific gravity[b] | Static bending: Modulus of rupture (lbf in$^{-2}$) | Static bending: Modulus of elasticity[c] ($\times 10^6$ lbf in$^{-2}$) | Static bending: Work to maximum load (in-lbf in$^{-3}$) | Impact bending (in.) | Compression parallel to grain (lbf in$^{-2}$) | Compression perpendicular to grain (lbf in$^{-2}$) | Shear parallel to grain (lbf in$^{-2}$) | Tension perpendicular to grain (lbf in$^{-2}$) | Side hardness (lbf) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Loblolly | Green | 0.47 | 7,300 | 1.40 | 8.2 | 30 | 3,510 | 390 | 860 | 260 | 450 |
| | 12% | 0.51 | 12,800 | 1.79 | 10.4 | 30 | 7,130 | 790 | 1,390 | 470 | 690 |

## Theory

All truss members are subject to axial compression or tension. Thus, compressive and tensile strength parallel to the wood grain were necessary in determining the cross sectional area of the octet truss members. The only tensile strength data available for Loblolly Pine in "The Wood Handbook" was tensile strength perpendicular to wood grain; therefore, the Modulus of Rupture was used in cross sectional calculations for octet truss members that experienced tensile loading.[6] The Modulus of Rupture is a measure of a simply supported beam's ultimate strength before failure when a transverse load is applied at the beam's midspan. Under these loading conditions, the section of beam which usually experiences failure first is in tension; therefore, the Modulus of Rupture was a suitable substitute for the tensile strength parallel to grain.

To simplify the fabrication process, the size of each octet truss was chosen to be a 1 cubic foot. The octet truss with the dimensions of 1 cubic foot was then modeled as a thin rod truss where the cross sectional area of each member was negligible in order to perform a truss analysis. A compressive distributed loading was applied to the top face of this thin rod truss to simulate the desired loading conditions of future compression tests; however, a truss force analysis assumes loads are only applied at joints. To meet this condition, the magnitude of the distributed loading was then calculated, divided by the number of joints on the surface receiving the compressive loads, and applied to each of the aforementioned joints as shown in *Figure 3*. The simplified thin rod octet truss was modeled with supports at each of its bottom 5 joints to restrict rigid body motion–the pin support prevents horizontal and vertical translation while the roller supports prevent vertical translation. The restriction of rigid body motion signified the truss will not shift sideways or downward when loaded in compression.

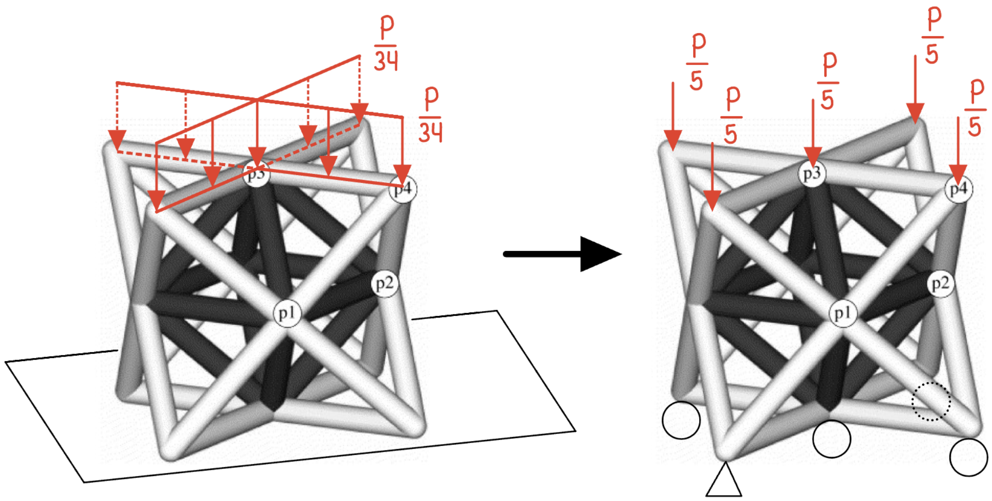



Figure 3: Thin rod octet truss loaded in compression (left) and simplified version of the thin rod octet truss used in 3-dimensional equilibrium force analysis (right). The magnitude of the distributed loads was calculated by multiplying the distributed load by the length it is being applied to. This yielded a magnitude of $P = \frac{P}{34}(17in) + \frac{P}{34}(17in)$ which was then evenly distributed amongst the 5 joints for the simplified thin rod truss. The triangle represented a pin support and the circles represented roller supports.

The method of joints, a 3-dimensional truss force analysis was then performed on the simplified thin rod octet truss to classify members experiencing axial compression versus axial tension.

Once the equilibrium force analysis was completed and members were classified, a thin rod octet truss was designed using a structural analysis program, SAP2000, with the same loading conditions to verify the results.

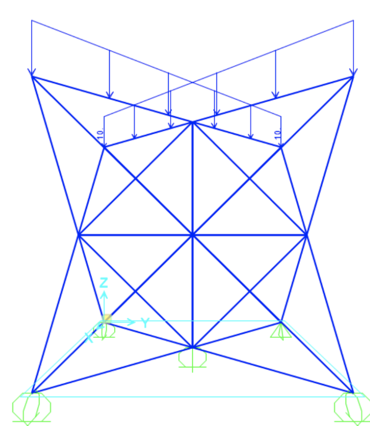


Figure 4: Model of an octet truss in SAP2000 with an arbitrary load applied.

Using the SAP2000 model, the load experienced by each member was calculated, relative to the entire load applied to the truss. By representing the percentage of total load experienced for a member by the variable, *c*, the following equation can be derived:

$$P_{member} = c \cdot P_{applied}\,.$$

The cross sectional area of a member can be calculated if the allowable strength of wood and the load experienced by the specific member is known by rearranging the equation below:

$$\sigma_{allowable} = \frac{P_{member}}{A_{required}}\,.$$

With the foresight of possible error in the fabrication process and given that strength properties of wood vary for unique pieces, a sizable factor of safety was applied to the compressive and tensile strength properties acquired from *Table 1*. In the equation below, $\sigma_u$ represents the applicable compressive or tensile strength, *F.S.* is the factor of safety, and $\sigma_{allowable}$ denotes the allowable strength for a member.

$$\sigma_{allowable} = \frac{\sigma_u}{F.S.},\ where\ F.S. = 3$$

The rearrangement of the previously mentioned equations provides the following relationship, which was utilized to calculate the cross sectional area of individual members.

$$A_{required} = \frac{c \cdot P_{applied} \cdot F.S.}{\sigma_u}$$

The required cross sectional area is directly proportional to the load applied; thus, if higher load capacities for the truss are desired, cross sectional area is increased.

## Design Constraints, Requirements Developed, Standards, and Codes

The building of an octet truss out of wood has several constraints, most important being the fabrication process and wood constraints. The fabrication process included multiple problems that are difficult to solve with a lack of construction and manufacturing experience. The joining of members, the angles not being perfect when the faces are being cut, and the addition of metal plates to combine members together were all challenges that had to be solved. Due to the complexity and scale of the truss, glue was used to combine members together, which led to a longer fabrication time due to the glue needing time to dry. Since the members were so small, the miter saw used in the fabrication process would pull the wood, resulting in "unique" members, when every member was supposed to be identical. Metal plates were introduced to reduce how much needed to be glued, and although it helped the process, a complex system with a hydraulic press had to be created to press them into place. Calculations had to be rounded to the nearest $\frac{1}{2}$" for ease of constructability. Rounding would make the calculations incorrect, which would lead to the truss not carrying load as effectively as possible. Since the properties of the wood are not constant throughout the plank, some of the lumber could perform better or worse and can result in certain parts of the octet truss failing prematurely. The wood planks deformed after very little time, which increased the difficulty of creating the truss.

Requirements for the project would be the strength of a column of octet trusses and the ease of constructing an octet truss out of wood. Traditional buildings use columns made of steel, concrete, and solid wood that can hold hundreds of kips. The objective was to create an octet truss column that can take a load of 5,000 pounds (5 kips) that also provided architectural value. The strength of the column consisting of octet trusses was then compared to columns constructed of steel, concrete, and solid wood to see how the columns differ. Each member of the octet truss should be identical, meaning the production process can be adopted for widespread production.

After the finalized design for an octet truss was found, the design would be tested in the Universal Testing Machine (UTM), which applied a compressive distributed load to the top section of the truss. The UTM would provide the ultimate strength experienced by the truss. From there, a column can be created. The strength and weight of the column will be compared to the strength and weight of other columns (steel, concrete, wood) of similar dimensions, which results in the overall effectiveness being found. The cost of each column was added and then divided by the foot to give a cost per foot ratio for all columns. Lastly, the efficiency of the fabrication process was considered to determine the ease of construction.

The codes that were used were the "Building Code 2018 of Pennsylvania", specifically Chapter 23[1]. Code 2304.10.7 which says "Wood columns and posts shall be framed to provide full end bearing. Column-and-post end connections shall be designed to resist the full compressive loads…". The octet truss was designed to support as much compressive load as possible to be able to compete with the strength of concrete, steel, and solid wood beams. The more load the truss column could support would make the column a more viable option to be used in everyday structures. Another code that was important to the project was Code 2304.11.1.1, which says "columns shall be continuous … throughout all stories and connected in an approved manner." The design of the octet truss is stackable, which means that trusses can be placed on top of each other. In other words, the column could be as tall as possible, depending on the height of the building. Various methods could be used to connect the trusses together, but that was outside the scope of the project. Lastly, American Institute of Steel Construction (AISC) standards were used to determine the max load of a steel column and American Concrete Institute (ACI) standards were used to determine the max load of a concrete column.

## Methods

**Designing and Verification**

To design an octet truss, the first step was to analyze the members as thin rods when a compressive distributed load was applied to the top of the truss. This was done to classify the members in either tension or compression and to find how much force was experienced by each member. To verify our results, SAP2000 was used. An octet truss was created in order to confirm the findings done when analyzing the truss as thin rods. The wood handbook was used to find the Modulus of Rupture (MOR), Modulus of Elasticity (MOE), and the Compressive Strength of the wood species, Loblolly[6]. The desired load the truss was expected to handle was 5,000 lbs. The percent of load of each member of the truss was then calculated using the information from the wood handbook and the findings found in SAP2000. With a factor of safety of 3, the cross sectional area was then calculated. The areas were rounded to the nearest half an inch for ease of construction. The cross section of each member was 1”x0.5”.

After the cross sectional area was found, another software program, Fusion 360, was used to identify the most suitable methods to cut the members of the truss, considering ease of construction and structural performance. Since an octet truss is made of equilateral triangle elements, every member of the truss was to be cut to the same dimensions before coming together to create a completed octet truss. The dimensions of the octet truss was to be a cubic foot. Since each face of the truss was to be 12”x12”, the diagonal of the face was found to be 16.97”, which was rounded to 17” for the ease of construction. Dividing 17” by 2 would become the length of each member, 8.5”. *Figure 5* displays the length of the members of the truss. Since the truss is symmetrical, the lengths apply to all dimensions of the truss. Knowing all three dimensions of the rectangular section, they were put together to form a triangle and then cut so each face would align when put together. Each member was cut at specific angles, which would become the angles each member needed to form a triangle. After the properties of the members were solved, Fusion 360 helped show how a completed truss would look like after the members came together. *Figures 6-10* show what each of the members would look like. *Figure 8* shows how each member comes together to form a triangle. *Figure 9* shows how to combine the triangles to create “pyramids”. *Figure 10* demonstrates how the pyramids are joined together to create a completed truss.

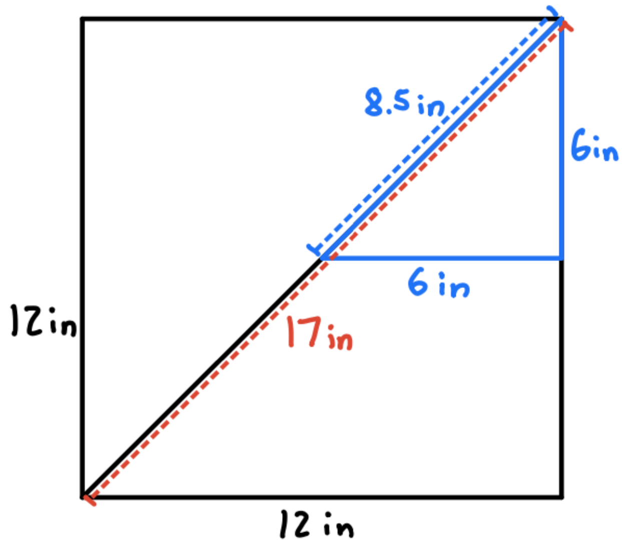



Figure 5: Diagram showing the length of each member.

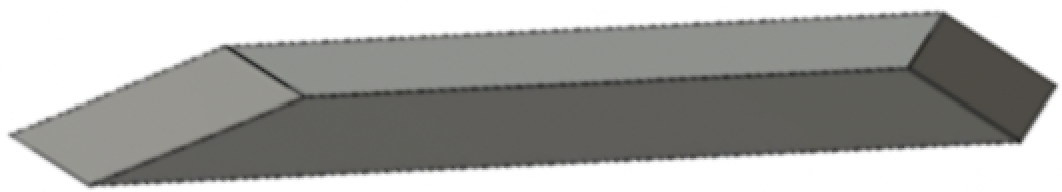


Figure 6: 96 identical rectangular members come together to create an octet truss. The longest part of the member measures 8.5 inches.

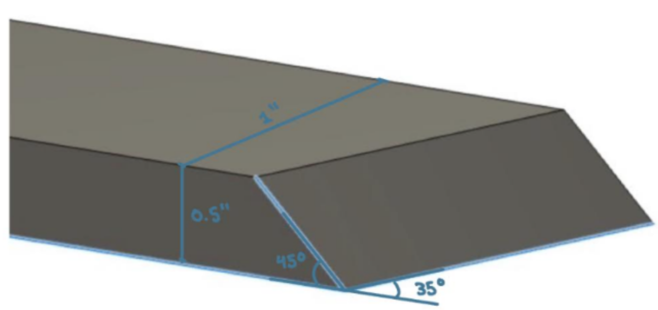



Figure 7: Each rectangular section should be cut at 45° and 35° to get the following face at both ends of the member. Note that the angled faces are mirrored on each member.

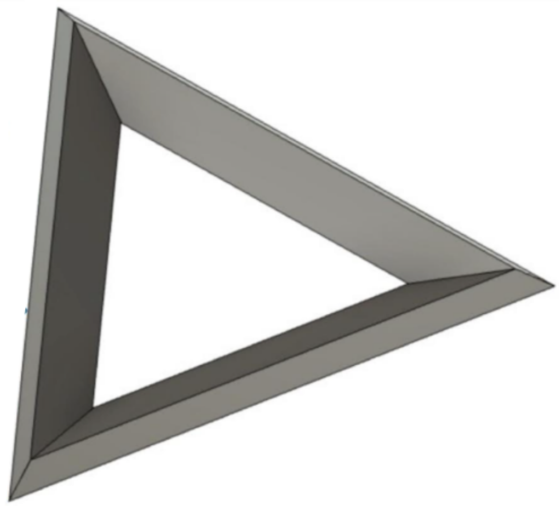


Figure 8: After the members have been cut with angled faces on both sides, a triangle can be formed. Altogether, 32 triangles construct a full octet truss.

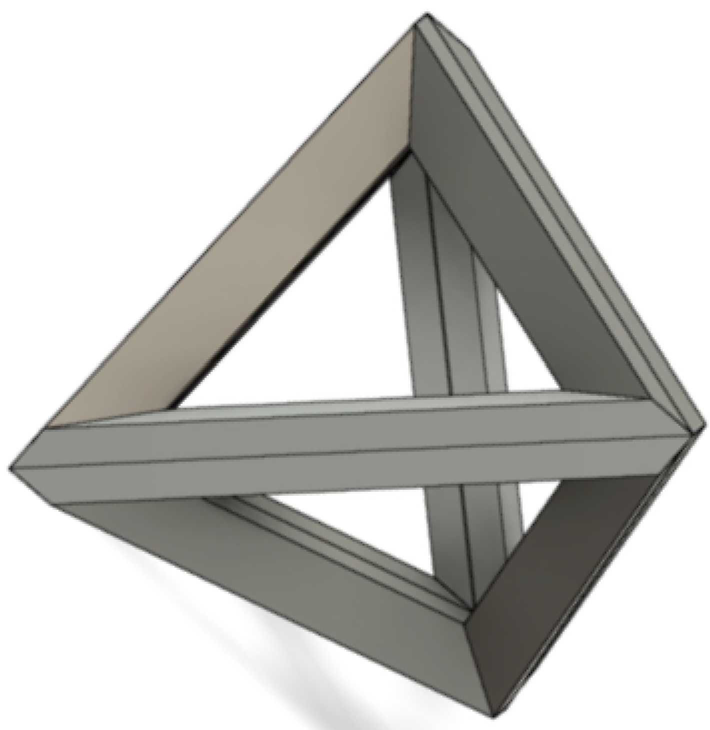


Figure 9: After the construction of the triangles, four of them come together to form a pyramid.

Figure 10: Eight pyramids are joined together to create a completed octet truss.

**Implementation: Truss One**

After sourcing the wood required to fabricate multiple trusses, the first iteration of the octet truss began. Members were created out of the wooden planks on a table saw, with the dimensions of 0.5”x1”x8.5”. While ninety-six members are needed to create an octet truss, more were created in case some members were damaged in the process.

Figure 11: Image of wood members, measuring 0.5”x1”x8.5”, stacked on each other.

Using a miter saw, the ends of each member were cut at an angle so they could come together to create the triangles in each truss. The lower part of the saw is angled at a 55° angle, with the actual saw blade being measured at 30°. This gives the angles needed in each member (see *Figure 7*). A clamping mechanism was created to help secure the wood into place and to reduce the danger of laceration and cutting hazards. First, the left face of the rectangular member is cut to the face needed to join the members into triangles.


Figure 12: Image of the left side of a rectangular member before it is cut.


Figure 13: Image of the left side of the member after it has been cut.

After the left side of the member has been cut, it is taken out of the contraption, flipped 180° vertically (so that the bottom face of the member is now facing upward) and pushed to the left stop so that the right side can be cut.

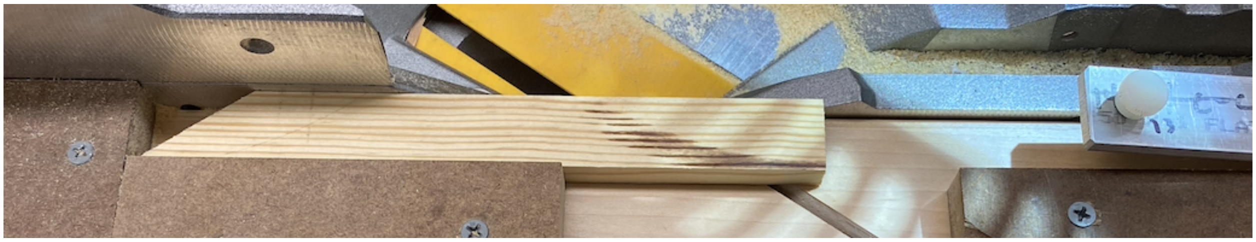


Figure 14: Image of the right side of a rectangular member before it is cut.

Figure 15: Image of the right side of a rectangular member after it has been cut.

Figure 16: Image of wood members, after the faces have been cut, stacked on each other.

After the members have been cut with angled cuts, they are joined together to form triangles. Selecting three members, wood glue was applied to the sloped sides of the member. The members are then combined in a specific configuration to form triangles. After each triangle is formed, rubber bands are used to apply pressure and to prevent the members from separating. Due to imperfections in the wood and how it is cut, there are gaps between the members. To minimize errors, the ends of the members were glued to be flush with each other.

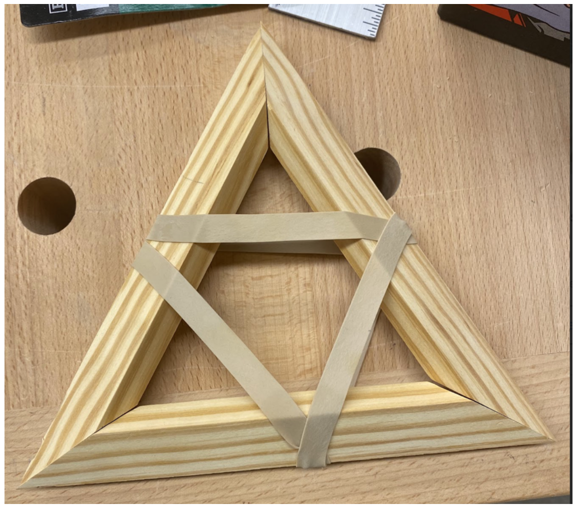


Figure 17: Image of three members coming together to form a triangle.

This process was repeated until no more triangles were able to be formed. After the glue settled for approximately 24 hours, three triangles were selected to form pyramids. Glue was applied to the faces of the triangles that are being joined. C-clamps were then used to apply pressure and keep the triangles together. Two clamps were used to create better connections between the triangles, with one being placed at the top part of the triangle and the other at the bottom. To minimize errors, the ends of the triangle members were glued to be flush with each other.

Figure 18: Image of three triangles coming together to form a pyramid.

This process was repeated until no more pyramids could be formed. After letting the glue dry for approximately 24 hours, the eight pyramids were then glued together. Glue was applied to the faces of the pyramids that would be joined together, using clamps to apply pressure and keep the pyramids together. To help the process go smoother, two pyramids were joined together at a time. This created a quarter of the octet truss. The quarters were then joined to create halves, and finally the halves were joined to form a full octet truss.

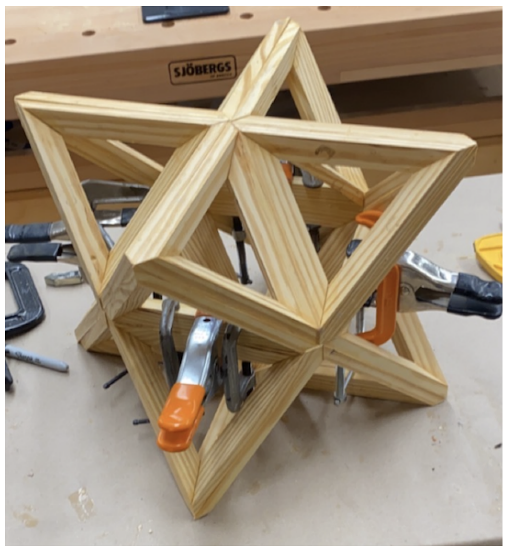



Figure 19: Image of the pyramids being joined together to create an octet truss.

After the octet truss is created, two opposite sides were milled down to be level so that the truss would be flush against the surface of a table. This is done to ensure that the two sides that were milled could lay flush against the top and bottom plates of the UTM and the distributed load was not focused on one specific area when the truss was being tested.

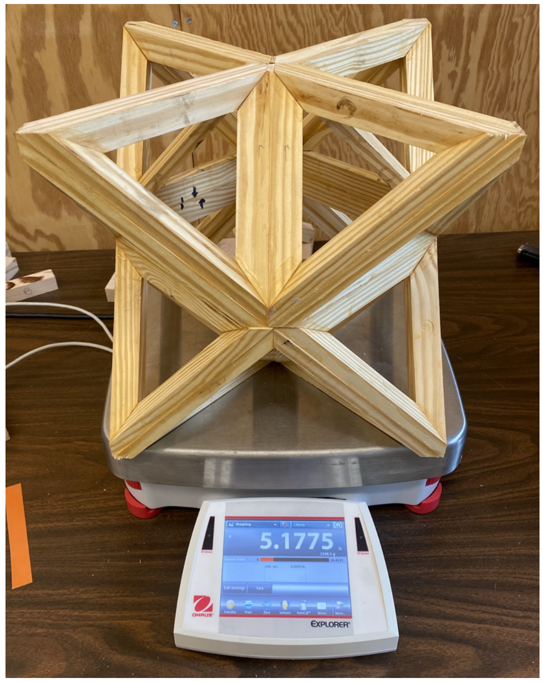



Figure 20: Image of the first completed octet truss, weighing 5.1775 pounds.

Note: There were small holes in the center of the faces that were milled (about 2mm x 2mm). This is due to the fact that when the pyramids were constructed, the edges lined up in the middle of the face of the truss. The edges are very thin, which meant that milling away the imperfections would result in a small square appearing.

**Implementation: Truss Two**

The same initial steps taken when creating the first truss are followed. The difference begins when creating the pyramids. The first iteration used three triangles to create a pyramid. However, joining four triangles would result in a stronger truss, since the diagonal members would have their cross section doubled. Due to the diagonal members having a bigger cross section, the truss in theory would support more weight. To adjust for this improvement, the initial pyramid was created. After the glue dries, a fourth triangle is added to the bottom so that every member of the triangle is matched up with another member of another triangle.

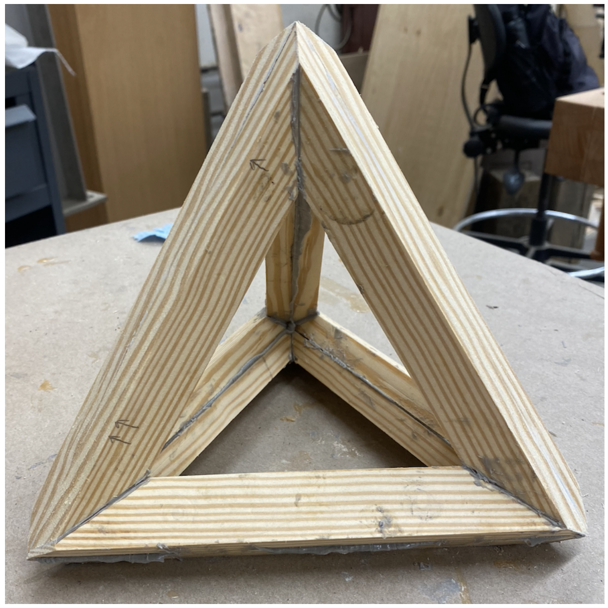


Figure 21: Image of four triangles coming together to form an improved pyramid.

Another improvement was the addition of 2”x4” Galvanized Metal Plates, which helped in the joining of the pyramids. Instead of waiting for the wood glue to dry after the pyramids came together to create a quarter of the truss, half a truss, and finally a completed truss, metal plates were used to join the pyramids together. This change significantly improved the construction time needed to create an octet truss.

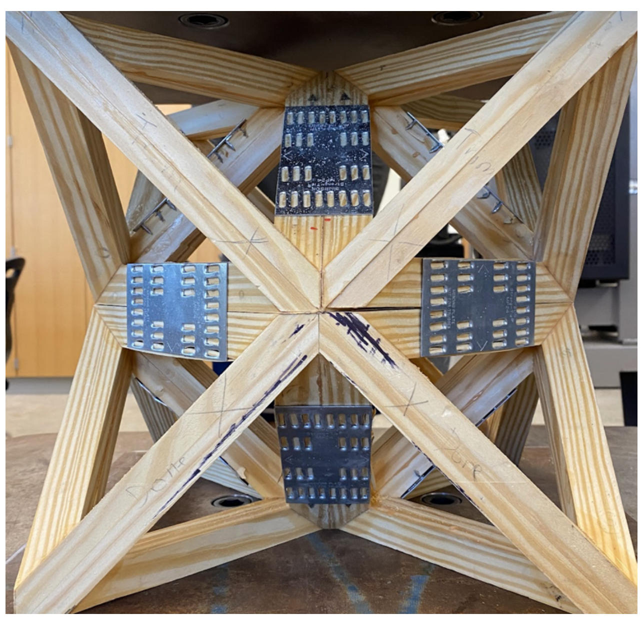


Figure 22: Image that shows how the metal plates were used to combine the pyramids.

After the truss is completed, two opposite sides were milled down to be level, as the first iteration was. However, with the addition of the fourth face in the pyramid, the small holes created in the first iteration did not appear here as the edges increased significantly.

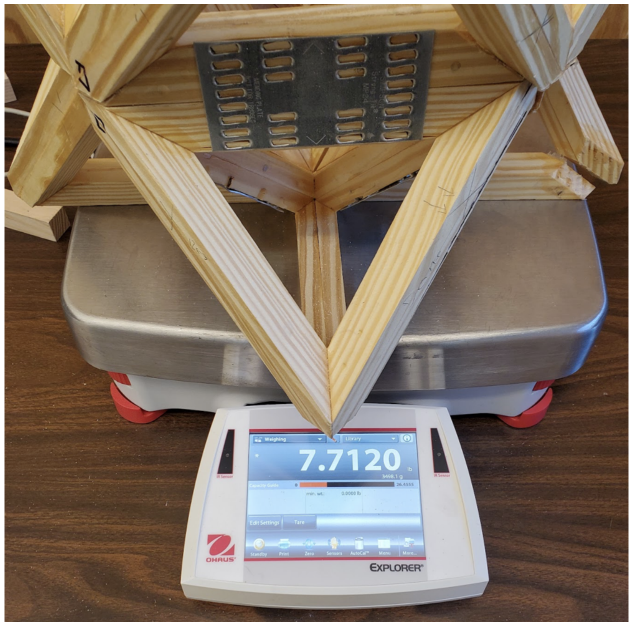



Figure 23: Image of the second completed octet truss, weighing 7.7120 pounds.

**Implementation: Truss Three**

Due to the second truss experiencing ductile failure from a glue joint shearing off, construction adhesive was used instead of wood glue. Whenever triangles were created, gaps were present. Construction adhesive filled the gap between the wood and is also believed to be stronger when bonding two pieces of wood together. The steps followed for the creation of the third iteration were the same as the second iteration.

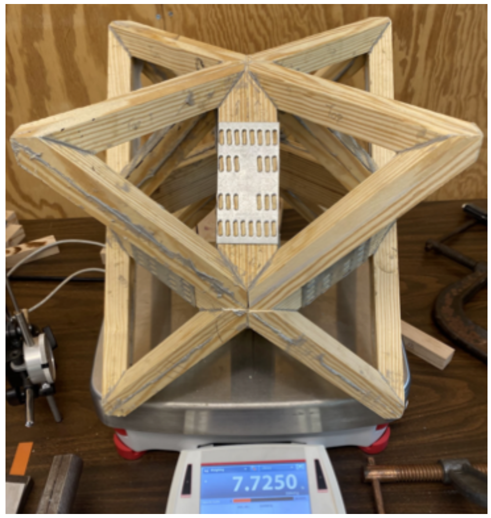



Figure 24: Image of the third completed octet truss, weighing 7.7250 pounds.

**Implementation: Truss Four**

The fourth iteration was created identically to the third iteration. The averages of the data found in the third and fourth iteration was to be found as these are the finalized designs of the octet truss created given the time constraints. The only difference between the third and fourth iteration will be that the metal plates will be placed on the inside of truss four. This was done so the architectural value of the truss would improve due to the metal plates being harder to see.

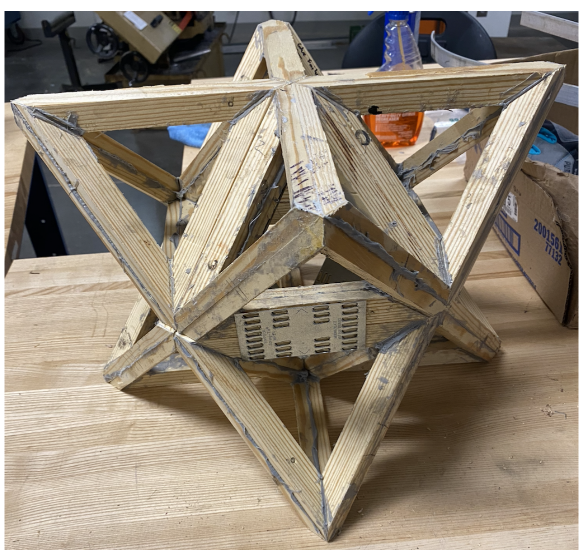


Figure 25: Image of the fourth completed octet truss.

**Testing**

To obtain information on the strength properties of each fabricated wooden octet truss, a compression loading test procedure was designed. In each compression test, the truss was placed into the Universal Testing Machine at Swarthmore College and a compressive distributed load was applied to the top face of the truss. The load was applied to induce deformation at a fixed amount of inches per minute until failure.

Before the test, each truss was weighed in order to calculate its respective strength to weight ratio. The first truss was placed in the UTM with a load rate of 0.01 inch/min. The second and third truss were tested with a load rate of 0.05 inch/min to quicken the testing process due to the first iteration taking 30 minutes to complete. The fourth iteration was not tested due to there being fabrication issues. One of the joint connections in the triangle was not properly glued together and split apart when pressing the metal plates into the pyramids. However, the fourth iteration would have also been tested with a load rate of 0.05 inch/min if this error had not occurred.

For each test performed, the following were recorded: vertical displacement at failure, compressive load capacity, and the mode of failure experienced. With this data, MATLAB was used to create a stress-strain plot and to determine the effective ultimate compressive strength in addition to the effective modulus of elasticity.

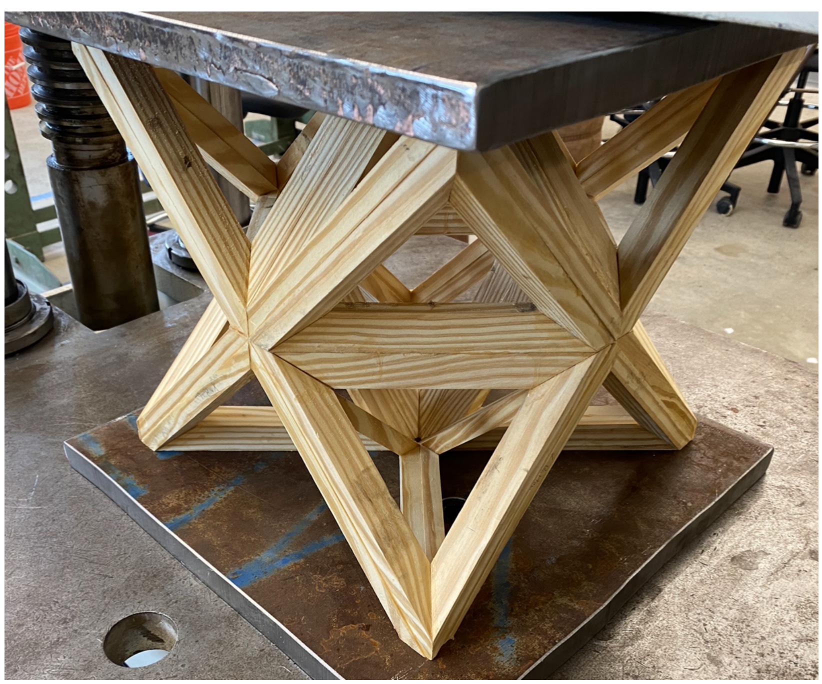


Figure 26: Truss between metal plates before the compression test started.

**Results**

The primary goal for Truss 1 was to achieve a load capacity greater than or equal to 5000 pounds. Using data obtained from its compression test, the strength properties of the octet truss were determined and served as benchmarks which future octet trusses would later surpass through design modifications. Although 4 octet trusses were fabricated, only the first 3 completed trusses were loaded until failure. The fourth and final octet truss had fabrication issues which would have guaranteed premature failure and inaccurate strength data.

The tests provided raw data for the load applied and the corresponding displacement due to compression. The relationship between the change in load and displacement is captured in *Figure 27*.

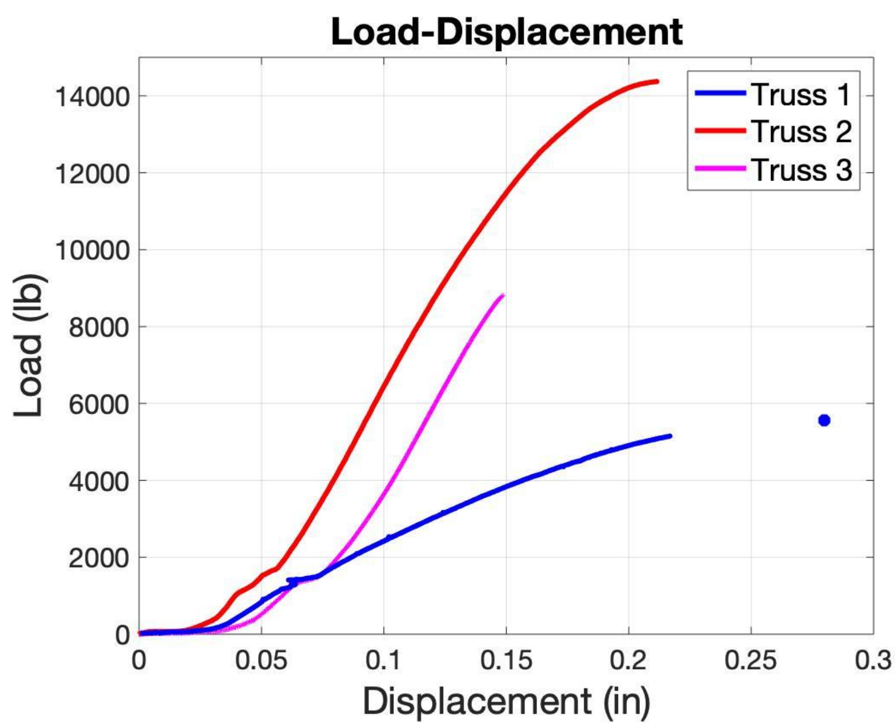



Figure 27: A Load-Displacement plot for Trusses 1, 2, and 3. Load is measured in pounds while strain is measured in inches. The blue star point represents the maximum load and displacement for Truss 1 since the data file is incomplete.

A corresponding stress-strain plot was compiled by applying the equations for normal stress and axial strain to the load and displacement data obtained in the compression tests. Through the analysis of the load-displacement plot and its corresponding stress-strain plot, the following properties were recorded in *Table 2*: vertical displacement at failure, compressive load capacity, yield strength, and effective ultimate compressive strength. The vertical displacement at failure refers to the total decrease in height of each octet truss compressed. The compressive load capacity represents the maximum compressive load each truss experienced before failure was induced. The yield strength is characterized by the stress value at which deformations became permanent when higher loads were applied to the truss. This value is determined by observing when stresses are no longer in the linear elastic region of the stress-strain plot in *Figure 28*. The maximum stress experienced by each truss as a whole is the effective ultimate compressive strength.

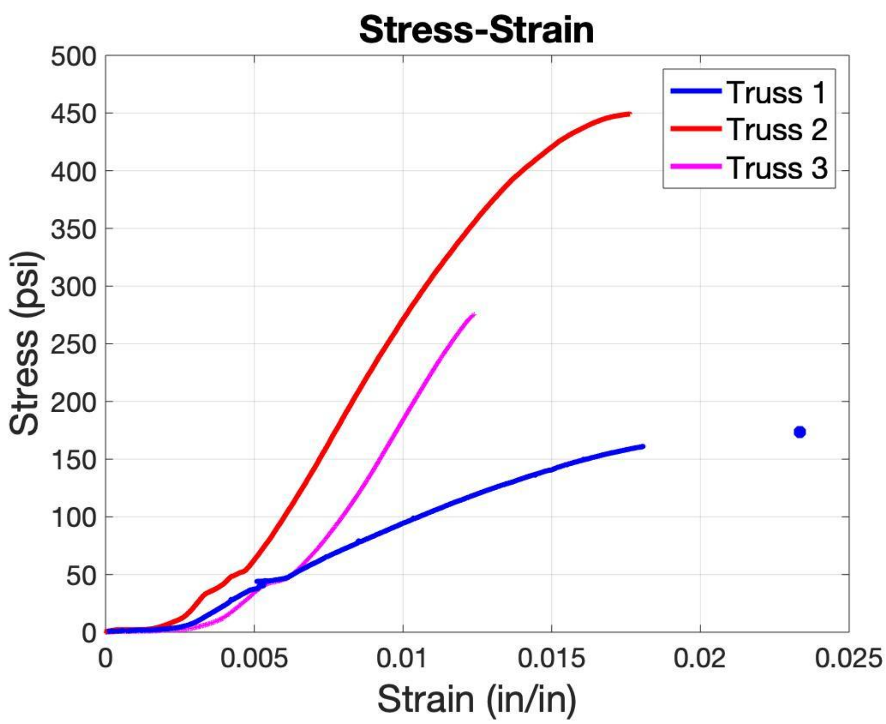



Figure 28: A Stress-Strain plot for Trusses 1, 2, and 3. Stress is measured in pounds per square inch while strain is unitless. The blue star point represents the maximum stress and strain for Truss 1 since the data file is incomplete.

Table 2: The vertical displacement in inches, compressive load capacity measured in pounds, yield strength in pounds per square inch, and effective ultimate compressive strength measured in pounds per square inch for each truss are provided below.

| **Truss** | **Vertical Displacement at Failure (in)** | **Compressive Load Capacity (lb)** | **Yield Strength (psi)** | **Effective Ultimate Compressive Strength (psi)** |
|---|---|---|---|---|
| 1 | 0.217 | 5.56E+03 | 161.0 | 173.6 |
| 2 | 0.583 | 1.44E+04 | 397.5 | 449.1 |
| 3 | 0.149 | 8.81E+03 | 251.2 | 275.2 |

The mode of failure was observed during each compression test conducted. Two cameras were set up to capture any brittle failure that might have occurred within each of the truss' 36 members or 14 joints. This was executed to determine the weak points of the truss for improvements in future iterations. Truss 1 experienced ductile failure; however, no members cracked or joints broke since the data recording software malfunctioned and the test was prematurely stopped. Truss 2 also experienced ductile failure, but was tested until one of the glue joints of a compression member completely sheared off. Further observation revealed cracks in other compression members for truss 2. The failure modes for future trusses will be included following their compression tests.

The stress-strain plots for each truss not only revealed values for the yield strength and ultimate strength, but it also provided information about stiffness: a material or specimen's resistance to bending or deformations. By Young's Modulus, the stiffness or Modulus of Elasticity, $E$, can be calculated using the equation $E = \frac{\sigma}{\varepsilon}$, where $\sigma$ is stress and $\varepsilon$ refers to strain. Alternatively, when given a stress-strain plot, Young's Modulus dictates that the slope of the linear elastic region of the stress-strain curve is also equal to the Modulus of Elasticity. For a complex structure such as an octet truss, the Effective Modulus of Elasticity refers to the entire body's resistance to deformations. Calculations of the linear elastic region for Truss 1's stress-strain line revealed the Effective Modulus of Elasticity to be $E_1$ = 1.10E+04 psi, while for Trusses 2 and 3 the Effective Modulus of Elasticities were $E_2$ = 4.08E+04 psi and $E_3$ = 4.17E+04 psi, respectively.

A unique characteristic of octet trusses is their rigid nature and high compressive strength governed by a lightweight design. To capture this, the normalized strength of each octet truss was calculated by dividing the total load applied with respective self-weight found in *Table 3*.

Table 3: Self-weight recorded in pounds and the maximum strength to weight ratio for each truss..

| **Truss** | **Weight (lb)** | **Maximum Strength to Weight Ratio (lb/lb)** |
|---|---|---|
| 1 | 5.20 | 1068 |
| 2 | 7.73 | 1866 |
| 3 | 7.73 | 1144 |

The relationship between normalized strength and displacement can be seen in *Figure 29.* The Normalized Load-Displacement Plot provided the maximum strength to weight ratio of each octet truss, which is characterized by the greatest normalized load achieved by each truss.

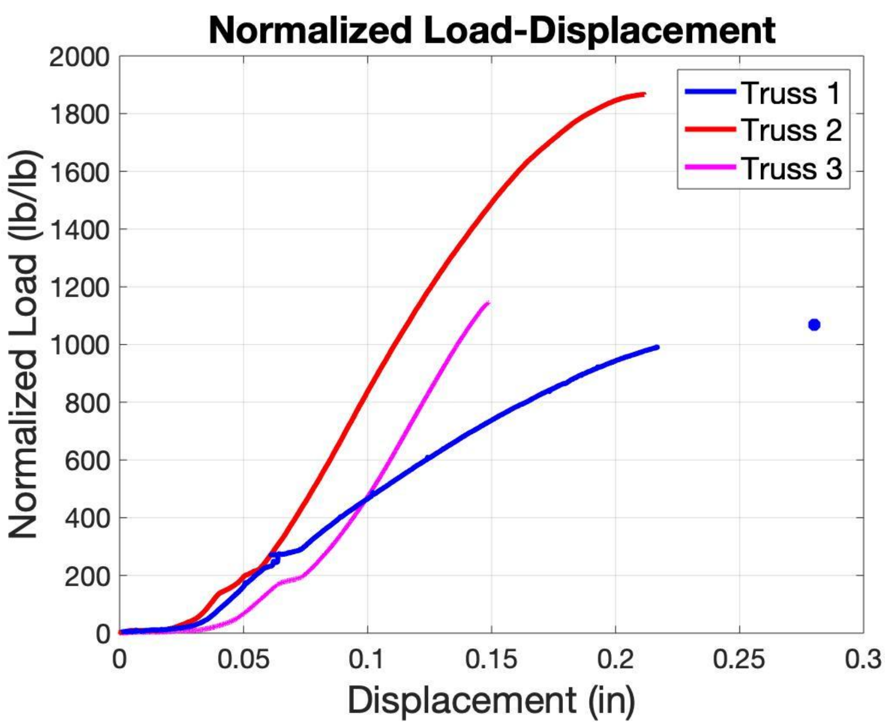



Figure 29: A plot of normalized load versus displacement for trusses 1 and 2. Normalized load is unitless while displacement is measured in inches. The blue star point represents the maximum normalized load and displacement for truss 1 since the data file is incomplete.

**Discussion**

All three trusses exceeded the expected goal, which was to support a load of 5,000 pounds (5 kips). When doing the calculations to find the cross sectional area for each member, the area was increased to the nearest half an inch to ease the construction process. A big factor of safety was also applied in these calculations to factor in wood imperfections and fabrication errors. It makes sense that each truss did better than expected when taking the previous statements into account. Nonetheless, theoretical calculations say that an octet truss truss should be able to support 33 kips, which is more than double the max load of the best truss iteration that was tested. The trusses mostly failed due to joint connections. Joint connections were small surface areas that were glued together, which would start to sheer off as the load continued to increase. The wood itself never failed, but the connections between wood pieces did. These connections could also be stronger if there were no fabrication errors during the construction of an octet truss. *Figure 30* shows a gap between the connection of two pyramids. When constructing triangles, the members did not always align, which would produce errors that only grew as more of the truss was constructed. The members did not always align because of how the angles were cut on the miter saw. The miter saw would pull the wood while it was being cut. This led to the supposedly identical pieces to become unique. Fixing this issue would help the truss support a higher max load.

Figure 30: Gap between connections due to a fabrication error.

The best iteration of the truss created was Truss 2. The second iteration of the truss had major improvements, such as the doubling of the cross sectional area for members experiencing the most load and the addition of Galvanized Metal Plates to connect members rather than wood glue. These modifications led to a 3 times increase in the stiffness of the octet truss while only adding 2.5 pounds to the total truss weight. In addition, the fabrication process was significantly improved since metal plates required less time to insert than the application of coats of wood glue to connect members. Iterations 3 and 4 underwent an additional improvement, which was the replacement of wood glue with construction adhesive. In truss 2, the wood glued joints caused members to shear off when they seemingly could have carried higher loads. It was thought that construction adhesive would be a stronger bonding agent than regular glue. However, the construction adhesive achieved less than the wood glue did. Truss 3, while having the highest MOE, had a max load that was nearly 5.5 kips less than Truss 2, which used wood glue instead. Truss 4 could not be tested due to the construction adhesive not drying properly, after it was left to cure for more than 48 hours. Findings show that construction adhesive was not as strong as wood glue when it came to constructing an octet truss and should not be used if a column was to be constructed.

Given the complex geometry of an octet truss, comparing the global stress to the global strain of the entire truss does not capture the lightweight yet stiff nature of the truss. The load vector data for each truss was normalized by dividing the load experienced by the total respective truss weight. This resulted in the maximum strength to weight ratio of Truss 2 to be 1866 lb/lb, which meant that for every additional pound added to the truss weight, the truss would be able to support 1866 pounds more. Truss 1 and 3 were able to support close to 1000 lb/lb, which demonstrates that the octet truss is a strong design, regardless of its lightweight features.

To compare the strength of an octet truss column to columns made of concrete, steel, and solid wood, the dimensions of the column were first found. It was assumed that the average height of one story in a commercial building is 10 ft, which would make the height of each calculated column also be 10ft. The length and width of an octet truss is 1ft, meaning the other columns would be solved with the same cross section. When comparing the octet truss column to columns of similar proportion (1’x1’x10’), the results in *Table 4* can be found, using calculations used in *Appendix A-G*.

Table 4: Relevant data of commonly used materials for beams.

| Material | Max Load (kips) | Weight (lbs) | Cost | Strength to Weight (k/lb) | Strength to Cost (k/$) |
|---|---|---|---|---|---|
| **Concrete** | 522 | 1620.84 | $166.70 | 0.322 | 3.131 |
| **Steel** | 851 | 650 | $1,164.02 | 1.309 | 0.731 |
| **Loblolly** | 274 | 350 | $595.65 | 0.783 | 0.460 |
| **Octet Truss** | 14.4 | 77.1 | $392.75 | 0.188 | 0.037 |

From *Table 4*, it is noted that a column consisting of octet trusses would support a much smaller max load than the other three columns. This means that the octet truss could not compare with solid columns due to its hollow nature. This is represented by the strength to weight ratio of the truss column being much lower than the strength to weight ratios of the other three columns. Although octet trusses are known for their high strength and for being lightweight, the hollow design meant that the material that should be there to help support the max load is not there. Another downside of building an octet truss column is the cost to create a column that is 10 ft tall. Factoring in the cost of the wood glue, metal plates, and the wood, the octet truss column was outperformed when comparing the strength to cost ratios. This is also sensible since concrete and steel are relatively cheap to create and although the solid wood column would be made of the same material, the octet truss uses more outside material that increases the price tremendously.

Future designs can be improved by knowing the exact species of wood being used. Knowing the species would help determine its properties instead of having to take educated guesses on what the species could possibly be. Increasing the cross sectional area of each member would increase the members stiffness, which would increase the load that the member could carry. This in turn would increase the capacity of the octet truss. However, the downside of doing this would be that the truss would not be as hollow as it was before, taking away some of its aesthetics. If the members were cut at precise angles, the triangles would fit together, which would remove any gaps in the truss when testing. A CNC machine could be used to do this, instead of relying on the miter saw. Lastly, if a better method was found that would not rely on gluing members together, fabrication time would increase dramatically, which would allow for more trusses to be tested. If another method that is not glue was used, then days would not be spent waiting for the glue to cure, which would allow for more trusses to be created, which in turn, could be tested for more results.

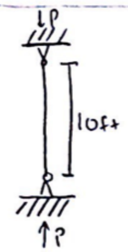
**Conclusion and Future Work**

Additional research is needed in order to determine the full capabilities of a wooden octet truss. Constructing a column using Truss 2's design would allow the column to support a max load of 14.4 kips. However, it is more economical and much simpler to construct a concrete, steel, or solid wood column than to construct a column of octet trusses, which would have a significantly lower load capacity. Instead of having a column every 40-50 ft apart for general columns, a wall of octet trusses would be needed to support the same load. Although results dictated that the design is not safe and/or strong enough to be used in commercial buildings, a simple fabrication method was successfully developed for the construction of wooden octet trusses; in addition, this fabrication process can be scaled for size. Since every member is identical, the process can be streamlined for ease of construction. If the cross sectional area of the members used were to increase, this process could create octet trusses with sufficient strength to be used in everyday structures. Alternatively, if a denser wood was used, such as a hardwood with higher strength values, then the truss would be able to support a heavier load. This increase in strength could help octet trusses appear in the public. Regardless of the length of every member, as long as the ends are cut at specific identical angles, an octet truss can be created with them. In theory, any material could be used to create an octet truss with the method found as long as the proper angles can be cut at the ends of each member. Another improvement which would increase the load capacity of the designed octet truss is the implementation of stronger adhesives or joint connections. The failure mechanism of Truss 2 was shearing at the joints due to the insufficient strength of wood glue. Construction adhesive was utilized in Trusses 3 and 4 to mitigate this issue; however, this bonding agent resulted in weaker connections. Further experimentation with the joint connections would result in greater strength properties for the octet truss. In conclusion, octet trusses are unlikely to be implemented in large scale construction projects unless Truss 2's design limitations are addressed.

## Appendices

### Appendix A

Find the max load for the column if 50 ksi and W 12x65 is used.




Properties of W12x65: Area (A) = 19.1 in² web thickness ($t_w$) = 0.39 in width ($b_f$) = 12.0 in

depth (d) = 12.1 in flange thickness ($t_f$) = 0.605 in $k_{des}$ = 1.20 in r = 3.02 in

First, the slenderness of the flange and web are calculated:

For the flange: $\frac{b}{t} = \frac{b_f/2}{t_f} = \frac{12.1\,in/2}{0.605\,in} = 9.917$

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}} = 13{,}487$$

Since $\frac{b}{t} < \lambda_r$, the flange is considered to be nonslender.

For the web: $\frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{12.1 - 2(1.20)\,in}{0.39\,in} = 24{,}872$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 35{,}884$$

Since $\frac{h}{t_w} < \lambda_r$, the web is considered to be nonslender.

Since both the web and flange are nonslender the critical stress, $F_{cr}$, can be found.

The column is simply supported, meaning the effective length is: $L_c = kL = (1.0)(10\,ft \times 12\text{ in/ft}) = 120\text{ in}$

$$\frac{kL}{r} = \frac{120\text{ in}}{3.02\text{ in}} = 39.735 < 4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Eq 3-2: $F_{cr} = \left[0.658^{F_y/F_e}\right]F_y = 44.544\text{ ksi}$

$$F_e = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = 181.28\text{ ksi}$$

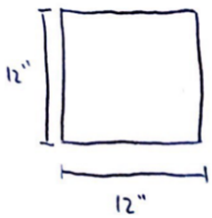
$$F_{cr}A_g = (44.544\text{ ksi})(19.1\text{ in}^2) = 851\text{ k}$$

Capacity of an I steel beam is **851 k.**

Calculation for max load of steel I column with the dimensions of 1'x1'. The max load is 851k.

## Appendix B

Designing an axial loaded square tied column with $f_c' = 4000$ psi, $f_y = 60{,}000$ psi. Find the max load




Gross Area $= (12\text{ in})(12\text{ in}) = 144\text{ in}^2 = A_g$

Design Formula for tied columns $\rightarrow \phi P_n = 0.8\,\phi\left[0.85 f_c'(A_g - A_{st}) + f_y A_{st}\right]$ Eq (1)

where $\phi = 0.65$ and $A_{st}$ = Area of steel

$A_{st} = 0.02 A_g$

Substitute values into Eq (1) and solve for the capacity of the beam, $P_n$:

$$\text{Eq}(1) \rightarrow 0.65\,P_n = 0.8(0.65)\left[0.85(4000\text{psi})(144\text{in}^2 - 0.02(144\text{in}^2)) + (60{,}000\text{psi})(0.02(144\text{in}^2))\right]$$

$$0.65\,P_n = 339.356\text{ k}$$

$$P_n = 522.086\text{ k}$$

$\boxed{P_n = 522\text{ k}}$ ← Capacity of a 12"x12" concrete beam (Max Load)

Plug values into Eq (1) to solve for the area of the steel, $A_{st}$:

$$0.65(522\text{ k}) = 0.8(0.65)\left[0.85(4\text{ ksi})(144\text{ in}^2 - A_{st}) + (60\text{ ksi})(A_{st})\right]$$

$$A_{st} = 2.88\text{ in}^2$$

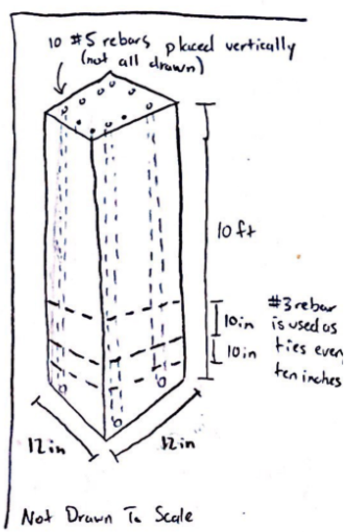
With $A_{st} = 2.88\text{ in}^2$, 10 #5 rebar (area $= 3.07\text{in}^2$) are used to support the concrete beam.

To find the spacing between ties, the minimum of the following calculations are chosen:
Note, #3 rebar was used for the ties.

$S = 16$ (diameter of #5 rebar) $= 16(\frac{5}{8}) = 10$ in ✱

$S = 48$ (diameter of #3 rebar) $= 48(\frac{3}{8}) = 18$ in

$S =$ minimum of lateral dimension $= 12$ in




Calculation for max load of concrete column with the dimensions of 1'x1'. The max load is 522k.

**Appendix C**

The max load of a solid beam of Southern Yellow Pine can be determined by:

$$f_c = \frac{P}{A}$$

where $f_c$ is the actual compressive stress parallel to grain, $P$ is the axial compressive force, and $A$ is the cross sectional area. Using the National Design Specifications for Wood Construction (NDS), $f_c$ = 1,900 psi[9]. The cross sectional area is: $A = (12in) * (12in) = 144in^2$. Multiplying both values together will give $P = (1,900\,psi) * (144in^2) = 273,600lbs$, which can be rounded to 274 kips.

**Table 4B Reference Design Values for Visually Graded Southern Pine Dimension Lumber (2" - 4" thick)[1,2,3,4,5]**

(Tabulated design values are for normal load duration and dry service conditions, unless specified otherwise. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4B ADJUSTMENT FACTORS

| Species and commercial grade | Size classification | Design values in pounds per square inch (psi) | | | | | | | Specific Gravity[4] | Grading Rules Agency |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Bending | Tension parallel to grain | Shear parallel to grain | Compression perpendicular to grain | Compression parallel to grain | Modulus of Elasticity | | | |
| | | $F_b$ | $F_t$ | $F_v$ | $F_{c\perp}$ | $F_c$ | $E$ | $E_{min}$ | G | |
| **SOUTHERN PINE** | | | | | | | | | | |
| Select Structural | | 2,350 | 1,650 | 175 | 565 | 1,900 | 1,800,000 | 660,000 | | |

Table 5: Table 4B from the "2018 National Design Specifications". This table includes the strength properties of Southern Yellow Pine, which Loblolly Pine is included in.

**Appendix D**

## <u>Steel - Cost and Weight</u>

W12X65 weighs 65 lb/ft. The beam is 10 ft long. The **total weight** of the steel beam is 650 lb.

The cost of a W12X65 I beam differed based on the website that was being used.

The average **cost** found between two websites was $1164.02.

| Stock Number | Item Size & Description | Weight / Ft. | Select Size | QTY | Price | Add to Cart |
|---|---|---|---|---|---|---|
| B21265 | **W 12 x 65 lb (12.12" x .390" x 12.00")** A572/A992 Steel H Beam | 65.00 lb | 10 Ft. | 1 | **$1316.20** ea. IN STOCK *Ships 1 - 5 days | Add To Cart |

Figure 31: Cost of W12X65 I beam on Metals Depot website.[4]

**STEEL BEAM**

| Size | Size | Weight | Qty | Estimated unit price | Estimated total price | |
|---|---|---|---|---|---|---|
| W12 x 65 lb (12.12" x .390" x 12.00") ST-BM-1265 | 10 Ft. | 650.00 Lb | 1 | $1,011.84 | $1,011.84 | Add |

Figure 32: Cost of W12X65 I beam on Metals Net website.[5]

**Appendix E**

## Concrete - Cost and Weight

The average weight of concrete is 150 $lb/ft^3$.

The volume of the concrete beam is 1ft*1ft*10ft = $10ft^3$.

The **weight** of the concrete needed for the beam is 150 $lb/ft^3$ * $10ft^3$ = 1,500 lb.

On average, concrete is priced at $\$135/yd^3$. This translates to $\$5/ft^3$.

The **cost** of the concrete needed for the beam is $10ft^3$ * $\$5/ft^3$ = $50.

The **weight** of #5 rebar is 1.043 lb/ft. Multiplying by 100 ft, the weight becomes 104.3 lb.

The **lowest cost** of #5 rebar is $18.75/20 ft. 100 ft of #5 is needed. The cost is $93.75.

The **weight** of #3 rebar is 0.376 lb/ft. Multiplying by 44 ft, the weight becomes 16.54 lb.

The **lowest cost** of #3 rebar is $7.65/20 ft. 11 rows of ties are needed. Each row is 4 ft long. The length of #3 rebar needed is 44ft. That means 3 #3 rebars are needed. The cost is $22.95.

The **total weight** of the concrete beam is 1620.84 lbs.

The **total cost** of the concrete beam is $166.70.

**Appendix F**

**Solid Wood - Cost and Weight**

A 12”x12” solid wooden beam weighs 35 lb/$ft^3$. The beam is 10 ft long.

The **total weight** of the steel beam is found to be 350 lb.

The cost of a 12”x12” solid wooden beam differed based on the website that was being used.

The average **cost** found between two websites was $595.65.

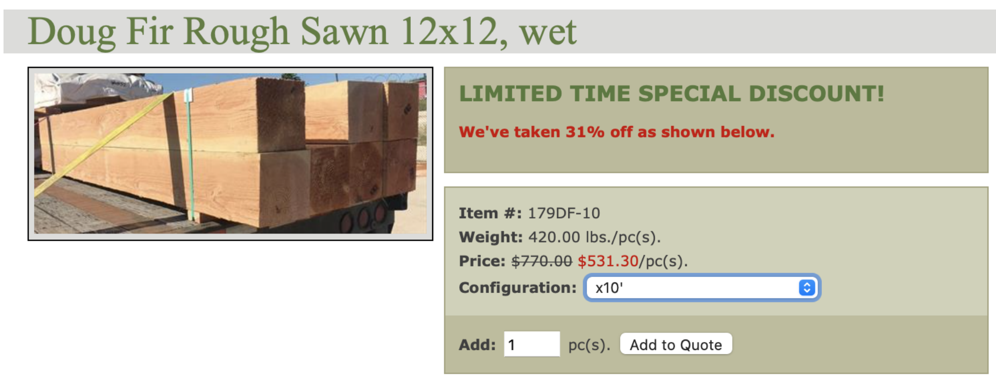



Figure 33: Cost of lumbar on Jimmys-Cypress website.[3]

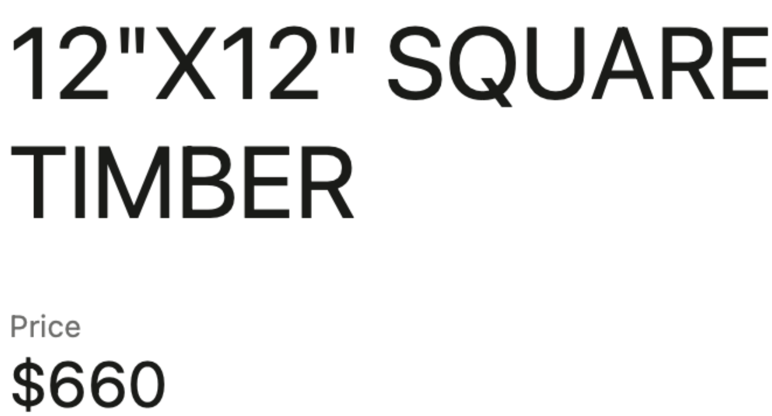



Figure 34: Cost of lumbar on Rustic Lumber Store website.[7]

**Appendix G**

**Octet Truss - Cost and Weight**

To create a 10 ft column of octet trusses, ten trusses are needed. The following is needed:

The **weight** of the best iteration of octet truss we created is 7.71 lb.

The **total weight** of ten trusses would be 77.1 lbs.

The **cost** of wood glue is $13.99 per bottle. ½ of one is used per truss. Five bottles would be needed to create a column. The total becomes $69.95.

The **cost** of a galvanized metal plate $0.98. 12 are used per truss. 120 total would be needed for the column. The total for a column is $117.60.

The **cost** of a 16'x12"x1" plank board is $68.40. 3.7 trusses can be created per plank. 3 planks would be needed to create 10 octet trusses (and have extra pieces). Total cost of wood is $205.20.

The **total cost** of a 10 ft column of octet trusses would be $392.75.

**Sources**


1. "Chapter 23: Wood, Building Code 2018 of Pennsylvania." *UpCodes*, from https://up.codes /viewer/pennsylvania/ibc-2018/chapter/23/wood#2301.
2. Deshpande, V.S., Fleck, N.A., Ashby, M.F. (2001). *Effective properties of the octet-truss lattice material*. Cambridge University Department of Engineering.
3. Jimmy's Cypress. (n.d.). *Douglas Fir Rough Sawn*. Jimmy's Cypress. Retrieved May 5, 2023, from http://www.jimmys-cypress.com/product/179DF.aspx
4. *Metals depot® - buy steel beams online!* Metals Depot® - Buy Steel Beams Online! (n.d.). Retrieved May 5, 2023, from https://www.metalsdepot.com/steel-products/ steel-beams
5. MetalsNet (n.d.). Retrieved May 5, 2023, from https://metalsnet.com/steel-en/steel /steel-beams/
6. Ross, R. (2021). *Wood handbook: Wood as an engineering material.* Forest Products Laboratory.
7. Rustic Lumber Store. (n.d.). *12"X12" Square timber*. Rustic Lumber Store. Retrieved May 5, 2023, from https://rusticlumberstore.com/products/12x12-square-timber ?variant=41098785980599
8. Trees of North America. (n.d.). *Southern Yellow Pine Tree Information*. Retrieved May 5, 2023, from https://www.treesofnorthamerica.net/show/tree/Southern-Yellow-Pine /108
9. Wood Design Standards Committee. (2018). *National Design Specification (NDS) for Wood Construction*. American Wood Council.